Mathematics and Thinking*

Steven P. Meiring
Ohio Department of Education (retired)

Abstract: *Traditional textbook-centered, teacher-led instruction is based heavily on research about how students best learn mathematics, at what stages ideas can be introduced, and of what kinds of thinking students are capable. Application of student knowledge to tasks not explicitly taught enters a new domain relying on the emotional circuitry of the brain, as well as the cortex where reasoning and problem solving predominate. Cognitive science and neural science provide fascinating insights into non-content related considerations that govern how much of their intellectual resources students will commit to a task and how long they will persist before their knowledge is accessed. Information about how the brain works and how students address mathematics is complementary to more familiar mathematics education research.*

Keywords: *problem solving, inquiry*

> *The joy of suddenly learning a former secret and the joy of suddenly discovering a hitherto unknown truth are the same to me—both have the flash of enlightenment, the almost incredibly enhanced vision, and the ecstasy and euphoria of released tension.* —Paul Halmos, *I Want to be a Mathematician*

Introduction

For every problem there is an action that produces a breakthrough. Suddenly, a keen insight, an original thought, a lucky happenstance comes from the problem solver or someone else. Obstacles to uncovering a solution fall away. It is readily evident the approach will lead to a solution, even though the result may not be in sight and considerable work is yet to be done. The solver experiences keen emotions similar to those described.

Prior to the discovery, the learner is beset by other emotions that affect what happens, including the tension that builds as the solution remains elusive. Not so well known in mathematics teaching are the emotional factors that come into play before any active investigation begins. They depend upon the type of learning the task entails, students' prior experiences, and students' expectations for accomplishing the task. These considerations often determine how students deploy their thinking.

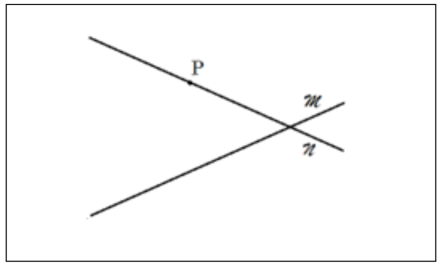
An Example from Alan Shoenfeld (1985)

The following item was given first to geometry students, then entry level college students, and finally a mathematician, for solution within a 15-min observation period.

> Given two intersecting lines m and n and a point P on one of the lines. Construct with compass and straight edge a circle that is tangent to both lines with P as one point of tangency (Figure 1).

*Editor's Note: This article is the second of three linked articles by this author, published by this journal over three consecutive issues. The first article, *Investigation and Discovery* (Fall 2022 OJSM issue) suggested using this framework consistently across courses to emphasize the mindset one employs in doing mathematics, whether learning new content or applying what one already knows to situations yet to be encountered. *A Laboratory for Secondary Math* (next OJSM issue) completes the series of articles about three recommendations for systemic change across the secondary grades

Figure 1: *Original figure from Schoenfeld task.*



The 10th-grade geometry students were given the item shortly after the requisite converse theorem had been discussed, whereby a circle was given tangent to two intersecting lines and properties of the resultant figure deduced. In the allotted time, the geometry students were unable to 'reverse engineer' their prior knowledge for this construction task. At a different time, 1st-year college students, given the same problem, were also unable to do it in the research time frame.

Asked to do the problem, a mathematician made short work of it by sketching in the required circle, drawing radii from the center perpendicular to the two lines, and drawing in a segment from the center to the point of intersection of lines m and n. Recognizing the two triangles were congruent, he constructed the bisector of the angle between m and n and the perpendicular through P to find the center of the required circle.

This problem was not particularly difficult. However, the striking finding was students' **belief** they needed to recall a procedure from their learning which would apply. Failing in that recall, they gave up.

The Schoenfeld problem could be replaced with almost any non-routine problem solving item from current state, national, or international assessments. Most students are not successful with these items (though a few are)—even though they possess the required content knowledge. In our previous article, *Investigation and Discovery* (Meiring, 2022), we discussed how lack of experience and insight into how mathematics is applied contributes to this problem. In this article, we explore how the mind is influenced by factors other than content knowledge—including beliefs, emotions, and habits of mind that we acquire by doing mathematics.

How Emotions Contribute to Doing Mathematics

Most instructional time is devoted to helping students understand how mathematics works. This aspect of thinking occurs mostly in the frontal lobes of the brain—the neocortex. The neural circuits dedicated to emotional processing reside in the limbic system (the amygdala, hippocampus, septal area, preoptic area, and hypothalamus). This is where our mind makes decisions **before** applying knowledge from our neocortex (Coleman, 1995; Panksepp, 1998).

Emotions as a Biological Function

Besides reflecting our state of mind, the evolutionary purpose of emotional circuitry is to prepare us to act. Emotions signal states of the world that may require a response from us or to reinforce behaviors that are in our best interest. Emotions may be precipitated by external events or our own thinking, informed by memories of relevant information (Fridja, 2007). We experience emotions when strong waves of affect overwhelm our sense of self. When the intensity of an emotion overwhelms us, it describes a state of action readiness in which an accompanying physical response may be governed by impulse rather than rational thought. Lesser emotions are described as moods, concerns, or states of mind that lead to the formation of beliefs, preferences, satisfaction, or distaste (Panksepp, 1998).

The Role of Emotions in Thinking

Emotions provide texture to events we experience, and aid in memory encoding and recall when positive. They also offer a value-filter in assessing particular events for immediate resolution or future patterned behavior. If we enjoy something, it is given a positive value, and we file away the memory as something we might like to repeat (attraction). If we dislike something, we stamp that thing with a negative value and remember it for future transactions as something to be avoided. Thus, habits are born (Goleman, 1995).

Human beings are unique among mammals in possessing a **Sense of Self**. This is a higher-level state of awareness, recognizing that we are sentient beings with a notion of our representation in the world and what our self-interests are. We perceive other humans as having their own SOS, which recognizes us and which makes judgments about us. Our SOS is influenced by: (1) events that happen in the world to and around us; (2) our own judgments of our actions and abilities; and (3) perceptions of others about our actions and abilities (Panksepp, 1998).

As we interact with the world or ponder future actions, a panoply of emotionally-related gradations of feelings assail us – representing threats or opportunities. These feelings concern not only our physical selves, but also our SOS. *Concerns* (GPA, health) are events whose outcome has significance for us. *Sentiments* are attitudes over time that we care about (self-esteem, social standing). *Motivations* are psychological forces that propel us to action (penalty for late homework).

Feelings cause us to behave at a particular time in a particular way. Internal conditions (need for achievement, status) that push us in the direction of a goal are called *drives*. External motivations (good grades, avoid parental censure) are called *incentives*. *Moods* (depression, happiness) and *dispositions* (general attitude toward a person, event, or thing) are prevailing frames of mind. In one sense, emotions provide criteria for determining the intellectual resources that one brings to bear toward any anticipated action by ascertaining its importance and consequences against our SOS (Fridja and Mesquita, 2000).

The Importance of Beliefs

From the vantage of learning, **beliefs** are products of the rational and emotional minds that have measured influence on how we approach a task and how long we persist in that endeavor (Schoenfeld, 1985). Whereas an emotion can be of relatively short duration, beliefs are more enduring, like *schemata*. They persist as a kind of framework for determining responses to some form of event encountered more than once. Beliefs can come from personal experience, or they can be transmitted as a part of one's culture. They consist of propositions whose truth is not objectively fixed. Yet, the holder believes the propositions to be true (Fridja and Mesquita, 2000).

- Nearly everyone is capable of understanding mathematics if they work at it.
- If I am stuck on a mathematics problem for more than five minutes, there is no chance I will figure it out on my own.
- Mathematics should be learned as sets of algorithms or rules that cover all possibilities.
- Students should not have to read the text to learn; that is the teacher's job.

Risk Assessment Tied to Emotions

According to Appraisal Theory (Cherry, 2010), emotions result from how the individual believes the world to be, how events are believed to have come about, and what implications events are believed to have. Of singular importance is the notion that an event need not actually to have taken place; but only to be thought about (rational mind) to invoke anticipations, beliefs, and interpretations of hypothetical results. Such imaginings of the future arise as anticipations, foresights, and imaginings of actual emotions that might emerge under certain envisaged circumstances (math anxiety).

In practice, emotional anticipations control most human actions. They involve the mechanisms of emotion regulation. We prevent embarrassment, regret, shame, and guilt by avoiding behavior that might give rise to them. We seek out behaviors that produce positive emotions such as pride, social respect, satisfaction, and reinforcement of our sense of self (Fridja, 2000).

Problem Solving as a Creative Act of Thinking

Schoenfeld's construction findings and the capsule summary of the role that emotions play in thinking augur the complexities of nurturing students to be able to think on their own – Skemp's third goal (Meiring, 2022). Since emotions prompt or inhibit actions before content knowledge is applied, they have to be accounted for in the design of learning opportunities – particularly when a schema for the task under consideration has not been explicitly taught.

Knowledge-centered, textbook-directed, teacher-led study of mathematics is such a pervasive format that it becomes the default setting for most students. Getting students to recognize that the rules of engagement are different for a problem solving or application task is the first hurdle to their thinking. When students believe they are supposed to be able to recall from memory a solution method and can't, they (like the geometry students) understandably quit – appraising that additional effort is futile and thus avoiding further damage to their SOS.

Whereas this is perhaps expected at grade ten, it is sobering to encounter at the college level – calling into question any tenuous response to defend this dilemma as, *'Yes, yes . . . this is important. Just not now; perhaps in the next course. Students are not ready for this level of thinking, yet.'*

Another eventuality also emerges: what of the relatively few students who *do apply* themselves to wrestle with the construction task, perhaps a couple succeeding? What is it they have that their fellow students do not? Again, cognitive psychology provides us insight. These students are usually comfortable with any mathematics task thrown their way, and it is important to their SOS to succeed. They have persisted with this outlook over many grades, developing both a different habit of thinking, and most likely coming up with their own rules for how to attack and persist with a problem challenge until they succeed.

These students have persisted through the tension that arises when investigating a task for which the way forward is not clear. With success, they experience the elation and joy that Paul Halmos describes in the opening quote, as discovery of something acquired by one's own efforts – thereby reinforcing their SOS. The accompanying release of dopamine as part of their elation further encodes the event into memory as both something the solver wants to repeat and which makes the associated memory easier for future recall.

The Role of Depth of Understanding

Now suppose we turn to the actions of the mathematician. Trotter (1986) describes how the expert's thinking is qualitatively different than that of students. Students are likely at the *novice* stage of mastery (respecting constructions). At the beginning stage of learning a new skill, the actions of the novice are directed by recognizing relevant facts and features of the situation. They then act on the given situation according to recently-learned rules prescribing how the situation should be handled. The context of the actual task is ignored. The student is so driven by following rules that he has no sense of the overall task.

By comparison, the mathematician seemingly accomplishes his work in what appears to be an effortless flow. Rule-making, deliberation of thought, even conscious selection of a plan or approach are not part of the thought process. Steps are guided by holistic, intuitive thinking that is seldom wrong. Experts are not necessarily analytical. But they do possess a robust repertoire of representations from which they select in how they view a situation. They draw upon recognition of patterns and similarity to situations in their experience. And that experience is organized as hierarchical structures within their thinking, such that they are seldom conscious of their own performance. It is a part of them that just 'occurs.' (Ibid)

As a consequence, the solution method devised by the mathematician, will likely not satisfy the novice student. While he may follow the logic of the solutions steps, he is still left with the quandary: 'How on earth am I supposed to be able to do that?' The solution does not start at an early enough point in its evolution to connect with where the student's level of thinking is. This is what Hunt (Meiring, 2022) meant when he said of a Pólya solution: 'What Pólya did not do was explain how he found an appropriate way to look at a problem before he knew the solution.'

Teaching Methodologies Suitable for Thinking

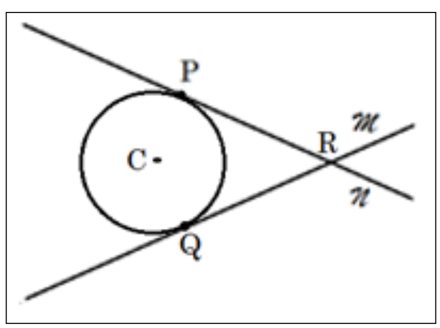
When the Schoenfeld task first came to this author's attention, the immediate question coming to mind was *'Why 15-minutes to complete the task? That sends the wrong message about problem solving.'* This question was subsequently answered with two thoughts: this was probably the length of time the investigator found that students would consider the question; and the time limitations of experimental observation. However, suppose instead that we consider the construction task as a teaching vehicle to deepen student understanding of constructions. It is posed to students overnight as a challenge problem. The next day, the following dialogue might ensue.

How many are ready to turn in the challenge problem solution? (Predictably, few hands will be raised; but we collect those papers, directing their submitters to plow ahead on the next day's homework.)

To the rest of the class, we say, *What seems to be the problem?* After a few random responses, we summarize: *So the difficulty appears to be how to get started. Let me ask this: In a general sense, what does the problem ask you to do*? <response: to construct a circle.> *And what must you know to construct a circle?* <response: the center and its radius.>

Turning now to the intersecting lines and given point P, we say, *Where do you expect the center of the circle to be?* Followed by: *How many of you took a compass and tried to make a rough sketch of the required circle?* Finally, we say, *I'll tell you what. Take a second shot at the problem and turn in your solution tomorrow.*

Figure 2: *Constructed circle*



The following day, the collection process is repeated, and if necessary, the teacher works with an even smaller group by giving yet another hint. This time with the diagram suggested by the previous day's suggestions. The hint: *what theorems have we studied that might apply to this diagram?* Students then have a third opportunity to turn in the assignment.

As a capstone, the entire class is directed to select any construction of their choosing and to identify why the construction accomplishes what it is purported to do.

Changing the Teaching Rules of Engagement

The teacher starts with the perception that most students will not know how to get started. Working with those students, he poses the strategic question they need to ask themselves – *Generally, what are you asked to find?* This is the critical first step to getting started; an observation he will later return to in the eventual solution postmortem. Whereas heuristics are *actions* the solver eventually takes to generate more information than the problem givens (hoping for association to the breakthrough ideas), self-talk questions (modeled by the teacher's hints) are the stepping-stones to discovery *before* the breakthrough is achieved.

Providing more time as needed for further reflection, acknowledges what neuroscience informs us is how the brain sifts through information in the **Preparation-Incubation-Illumination-Verification** model (Wallas, 1926). Moreover, it provides the avenue for eventual discussion about the roles of focus and sleep in the thinking process.

Just as it is important for students to realize their approach to the learning task has different rules of engagement, it is likewise important for the teacher. While this might sound pedantic, how many teachers take the formal steps of: (1) writing out a goal for what they are trying to achieve for student thinking; (2) consider how their roles and methodologies might change; (3) reflect on how their instructional organization, including a time and place, and evaluations should be adjusted; or (4) collect artifacts to assess the quality and type of thinking of which individual students are capable?

We will consider point (3) in our next article. But here is a model definition for how mathematical thought might be defined and how it relates customary instruction to non-routine applications.

> Mathematical thinking is what one does in order to understand, make sense of, and integrate mathematical ideas into a more comprehensive structure of one's thinking. It is what a mathematician does when confronted with a mathematics-related conundrum. It is not driven by somebody else's need or direction, but rather by a personal drive for understanding and reflection. It does not end with a mere answer. It culminates only when any related questions and ideas are put to rest as making sense within the broader framework of what is already known and understood.

Conclusion

Mathematics, perhaps more than most disciplines, is an exercise of the mind. In nurturing that mind, it is not just how the mathematics works that the teacher must plan for. But, also, how the instrument to do the thinking functions – its limitations, its biological predispositions, and the constraints under which it operates best. Equally important are the characteristics of thinking engendered by the developing mastery of students over the content material and ideas they acquire in pursuit of mathematical thinking. This specialty borders on the realm of neuro- and cognitive science, two rich areas of advancing research. For this author, they have provided many answers to why students do not always perform to their potentials.

Their study has proved to be complementary for this author to mathematics education research (e.g., the van Hiele Model of Geometric Thought, (Crowley, 1987)). Lacking the context of specific discipline application, discussions make the reader more consciously aware of the long-term goals for education, how thinking contributes to individual learning, and, for many students, the acquisition of habits of thinking through needed opportunities to learn.

Which brings us to our second recommendation at the secondary mathematics program level. Once more, we choose that level because the kind of learning described in this article takes time to develop with numerous opportunities and time to learn, spread across several teachers and multiple subjects.

Recommendation 2 Enhance teaching knowledge through the career-long study of creative thinking, including how the brain functions, both general and specific to mathematics, to better understand student thinking and how it progresses.

Following this article is **Exhibit 1** for the type of tool the author envisions being applied across subjects to assess changes in students' dispositions and beliefs as more attention is given to mathematical thinking.

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Steven P. Meiring, meiringsteven2@gmail.com, began his education career in Indiana in 1965, teaching junior high school (all subjects) between earning his bachelor's and master's degrees. After eight years, including four teaching college mathematics, he came to Ohio to serve 26 years as a mathematics consultant for the Ohio Department of Education. During that tenure, he served on the prestigious Mathematical Sciences Education Board of the NRC and the American Junior High School Examination committee of the Mathematical Association of America. He is also a textbook author and served eight years after retirement from ODE, working for the Ohio Board of Regents and the Ohio Resource Center in various capacities.

Exhibit 1: Mathematics Disposition and Survey Questions

A subset (about 30) of the following questions can be used to construct a pre-test and post-test to assess changes in student dispositions and beliefs over a given course and mathematics emphasis.

Directions: This questionnaire is anonymous and has nothing to do with your grade. The summary results are intended to provide a class profile of changes that may occur during the course of this school year. Please indicate what you *really* think by putting a in the box that corresponds to: **Strongly Agree**, **Agree**, **Undecided**, **Disagree**, or **Strongly Disagree**. Thank you for your assistance.

	Discipline of Mathematics	**SA**	**A**	**U**	**D**	**SD**
Executing	1. There are several ways to find the correct solution of a mathematics problem.					
	2. Mathematics problems can be done correctly in only one way.					
	3. In mathematics something is either right or it is wrong.					
	4. Mathematics problems have one and only one correct solution.					
	5. Mathematics is a solitary activity, done by individuals alone.					
Requisites	6. Students who understand the subject matter should be able to solve mathematics problems in five minutes or less.					
	7. Being good at mathematics requires talent.					
	8. Solving a mathematics problem is difficult and requires smart.					
	9. Only geniuses are capable of discovering, creating, or really understanding math.					
Discipline Nature	10. Mathematics enables us to better understand the world we live in.					
	11. Mathematics will not be important to me in my career after school.					
	12. Mathematics has very limited relevance to my life.					
	13. One reason I learn mathematics is to help me think more clearly in general.					
	14. Mathematics is a way of thinking using symbols and equations.					
	15. Mathematics is a science.					
	16. Mathematics is a language.					
	17. Knowledge in mathematics consists mostly of disconnected topics.					
	18. Everything important about mathematics is already known by mathematicians.					
	19. There is little					
	20. In mathematics, you can be creative and discover things yourself.					
	21. The form of a mathematical answer is often just as important as the content of the answer.					
Coherence	22. It is possible to explain mathematical ideas without using equations.					
	23. Doing geometry proofs gives me a better understanding of mathematical thinking.					
	taught.					
	25. Geometric and other proofs have little or nothing to do with discovery or invention.					
	26. Mathematical formulas express meaningful relationships among measurable things or amounts.					

	Mathematics Learning	SA	A	U	D	SD
	27. Anyone can learn mathematics.					
	28					
	29. Only the brightest students can learn higher levels of mathematics; students of lesser ability should learn mathematics that is more practical.					
	30. In mathematics you are rewarded for your effort.					
	31 different ideas.					
	32. Nearly everyone is capable of understanding mathematics if they work at it.					
	33. Making mistakes is part of learning mathematics.					
	34. Learning mathematics must be an active process.					
	35. Mathematics learning is mainly memorizing.					
	36. Mathematics is mainly doing lots and lots of problems.					
	37. Mathematics should be learned as sets of algorithms or rules that cover all possibilities.					
	38. To solve mathematics problems you have to be taught the right procedure, or you					
	39. It is important for me to make sense out of formulas and procedures before I can use them correctly.					
	40. In mathematics I try to link new ideas to what I already know rather than just memorizing it the way it is presented					
	41. When the teacher asks a question in mathematics class, there are lots of possible right answers you might give.					
	42. When the teacher asks a question in mathematics class, the students who understand only need a few seconds to answer correctly.					
	43. I learn mathematics by understanding the underlying logical principles, not by memorizing rules.					
	44. The mathematics that I learn in class is thought provoking.					
	45. In mathematics, you can be creative and discover things by yourself.					
	46. Mathematics consists of facts and procedures that others have discovered and that is					
	47. It is important to know why a solution to a mathematics problem works.					
	48					
	49. A demonstration of good reasoning should be regarded even more highly than a					
	50. In doing a mathematics problem, if my calculation gives a result very different from what I expect, I trust the calculation rather than reworking the problem.					
	51. In mathematics, increased emphasis should be given to use of key words to determine which operation to use in problem solving.					
	52. It is a waste of my study time to understand where mathematics formulas come from.					
	53. Mathematics formulas are not helpful for understanding of ideas; they are mainly for doing applications.					
	54 nothing much I can do to come up with it.					

	Mathematics Teaching	SA	A	U	D	SD
Skills & Concepts	55. Mathematics should be taught as a collection of skills and algorithms.					
	56. In mathematics, skill in computations should precede work problems.					
	57. Students should first be taught a set of skills in mathematics before being presented a complex problem.					
	58. A mathematics course should cover a wide range of topics to broadly expose one to different applications and approaches.					
	59. A mathematics course should focus on a core set of principles to really understand those ideas deeply.					
Teaching Reliance	60. Good mathematics teachers show you the exact way to answer the question you will be tested on.					
	61. I cannot learn mathematics if the teacher does not explain things well in class.					
	62. I think it is unfair to expect me to solve a mathematics problem that is not similar to any example given in class or the textbook, even if the topic has been covered.					
	63. Good teachers show students lots of different ways to look at the same problem.					
	64. More than one representation (picture, concrete material, approach, symbol set) should be used in teaching a mathematics concept.					
	65.					
Thinking & Understanding	66. Students learn mathematical concepts best by being prompted with questions to reflect on their own experience, activity, and thinking					
	67. Mathematical ideas should be introduced to students in the context of everyday life and then the mathematics separated or abstracted from it.					
	68. It is a waste of time when the teacher makes us think on our own about how to solve a new mathematical problem.					
	69. Students learn mathematical concepts best by the teacher telling them the meaning of the concept.					
	70. Students understand more mathematics by listening and imitating the teacher.					
	71. Students understand more mathematics by discussing with and teaching each other.					
Methodology	72. Students only retain mathematics concepts and procedural skills when they are constantly repeated and reviewed by the teacher.					
	73. Students best retain mathematics concepts and procedural skills through learning experiences with other students where mathematics is placed in a meaningful context.					
	74. The better teachers solve lots of examples rather than carefully analyzing only a few problems in a given lesson.					
	75. Students should be encouraged, and sometimes required, to justify their solution, thinking and conjectures.					
	76. Developing mastery of skills and procedures in mathematics is more important than teaching students to reason and think mathematically; the latter can come later.					
	77. A major goal of mathematics instruction is to help students acquire the belief that they have the power to control their own success in mathematics.					

	Self Relative to Mathematics	SA	A	U	D	SD
	78.					
	79. Learning mathematics is enjoyable.					
Appeal	80					
	81. Mathematics is my favorite subject.					
	82. Mathematics is a mechanical and boring subject.					
	83. I cannot do difficult mathematics tasks.					
	84. I am not the type who is good in mathematics.					
	85. Mathematics is my weakest school subject.					
	86. I have a mathematical mind.					
	87. I cannot do difficult mathematics tasks.					
	88. I can get good grades in mathematics.					
	89. Though I do my best, math is more difficult for me than for many of my classmates.					
	90. I am sure that I can learn mathematics.					
	91. I have always hated mathematics.					
	92. I feel confident in my abilities to solve mathematics problems.					
	93. I never expect to do well in a mathematics course.					
	94. I have less trouble learning mathematics than other subjects.					
	95. I am a hard worker by nature.					
	96. always do all of the homework that is assigned.					
Effort	97. If I try hard enough, then I will understand the course material in my math class.					
	98.					
	99. I concentrate hard in mathematics.					
	100. Sometimes I solve a math problem more than one way to help my understanding.					
	101. I get a sense of satisfaction when I solve mathematics problems.					
	102. grade in mathematics.					
	103. I want to do well in mathematics to show the teacher and my fellow students how good I am in it.					
	104. When I have the opportunity, I choose math assignments that I can learn from even sure of getting a good grade.					
	105. I expect to get good grades on assignments and tests of mathematics.					
	106. I avoid solving mathematics problems when possible.					
	107. It is useful for me to do lots and lots of problems when learning mathematics.					
	108. If I am stuck on a mathematics problem for more than five minutes, there is no chance that I will figure it out on my own.					
Persistence	109. After I study a topic in mathematics and feel that I understand it, I have difficulty solving problems on the same topic.					
	110. I believe that if I work long enough on a mathematics problem, I will be able to solve it.					
	111. understand how everything works.					
	112.					
	113. I am interested to learn new things in mathematics.					
Interest	114. I have never liked mathematics; it is my most dreaded subject.					
	115. I would like a job that involves using mathematics.					

	Self Relative to Mathematics	SA	A	U	D	SD
Adaptability	116. If I am presented with a new mathematical situation, I can cope with it because I have a good background in mathematics.					
Adaptability	117. When I cannot remember the exact way my teacher taught me to solve a mathematics problem, I know some other methods that I can try.					
Adaptability	118. I do not feel that I can use the knowledge gained in the mathematics courses I have taken so far.					
Adaptability	119. I can draw upon a wide variety of math techniques to solve a particular problem.					
Adaptability	120. Sometimes, when I do not understand a new topic in mathematics initially, I know that I will never really understand it.					
	121. I usually try to just pass a mathematics course.					
	122. in mathematics class					
	123. Mathematics is enjoyable and stimulating to me.					
	124. I have never liked mathematics, and it is my most dreaded subject.					
	125. Mathematics never seems to stick, and after I learn it or even get a good grade on know it.					
	126.					
	127. .					
	128. study hard enough.					
	129. m just not good at math.					
	130.					
	131. O everything if you get good marks on the test.					
	132. To me mathematics is an important subject.					
	133. I think mathematics is necessary for you to be successful in life.					
	134.					
	135. I think I will be able to use what I learn in mathematics in other courses.					
	136. I need to do well in mathematics to get the job I want.					
	137. My family has encouraged me to study mathematics.					
	138. The example of my parent(s) has had a positive influence on my motivation.					
	139. My parent(s) get a good grade in mathematics.					
	140. My parent(s) enjoy helping me with mathematics problems.					
	141. I am anxious before mathematics tests.					
	142. I get flustered if I am presented with a problem different from the problems worked in class.					
	143. When I see a mathematics problem, I become nervous.					
	144.					
	145. Mathematics makes me feel uneasy and confused.					
	146. Group work helps me learn mathematics.					
	147. I prefer working alone rather than in groups when doing mathematics.					
	148. I am not eager to participate in discussions that involve mathematics.					
	149. I enjoy hearing the thoughts and ideas of my peers in mathematics class.					

<u>Open-ended Pre-Assessment Question</u>: If you could change 1-3 things about mathematics teaching or learning this year to make it more interesting and valuable, what would those changes be?

<u>Open-ended Post-Assessment Question</u>: In your study of mathematics, what are the 1-3 beliefs or dispositions you previously held that have changed the most during this year?