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Creative Expression in Mathematics

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CREATIVE EXPRESSION IN MATHEMATICS

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PREFACE

The writer wishes to thank the members of her Committee, and particularly Professor John D. Leith, for the criticisms and suggestions which made this thesis possible.

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February 3, 1933

This thesis, presented by Jessie M. Striegl in partial fulfillment of the requirements for the degree of Master of Arts, is hereby approved by the committee in charge of her work.

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Introduction.

General Purpose: "Knowledge is not power until it is applied; before the application is made it is only potentiality. Facts, principles, and theories are useless unless applied to situations to which they are relevant." The thought expressed in this quotation from A. R. Mead is the key-note of the purpose for which the material presented in this thesis has been gathered and organized. Mathematics, as a means of creative expression, covers a broad field, for it is centered about almost every activity of life. The illustrations herein, have been chosen from a vast wealth of material, with the idea of broadening the reader's vision by touching upon specific examples from many sources.

Applications: The acquisition of knowledge is not a goal in itself. Concrete application or expression of that knowledge is the response which must be given to complete the learning. A small boy asked his brother, who was studying mathematics in college, to help him make a square corner. The brother, could not, although if he had been asked the principle of the Pythagorean theorem, he would have known it perfectly.

Creative response must not be considered as something distinctive and apart from the mere acquiring of knowledge. These are not separate, but one; and it is the responsibility of the teacher to guide and direct the student in such ways as to minimize efforts and waste and still leave the learning process with him. The student who has been trained in creative expression will see the relationships involved in his problems for they will mean more to him than mechanical operations.

The material presented in the following sections is primarily for the teacher of mathematics in the secondary schools, to be used as suggestive of projects in creative expression. It may be made available to students as their ability or progress calls for activities ordinarily not provided for by the texts. For students of superior ability, there is material included to furnish appealing activities for the development of creative and cultural qualities not usually attained from the study of mathematics. For those students who manifest inability to learn or dislike for mathematics, the purpose is an appeal through creative expression; and although the principles involved may not be fully understood, the production of a concrete piece of work will lead towards a definite goal of accomplishment, which in itself is a measure of improved learning.

School Equipment and Facilities for Learning: In order that creative activities may be a part of the regular school work, the curriculum, equipment, and methods of teaching will be called upon to lend themselves to making a favorable environment. The courses offered in mathematics must be looked upon as general modes of thinking, as comprehensive and flexible patterns for the re-arrangement of experiences, and not merely as "tool" subjects. The equipment, which is ordinarily limited, may be added to by purchase, but preferably by hand made constructions, whose planning and execution offer excellent opportunities for creative activities. The methods used, whether they be group or individual instruction, lecture, question-answer, or laboratory, must offer a chance for self expression, creativeness, and practical associations.

Proper facilities for carrying out a program of creative-expression are ideals not generally realized. The rows of desks, the orderly shelves of books, and the inelastic curricula are not conducive to self expression on the part of the student. He needs books, but reading should form only a part of his activities. Instead of rows of desks he needs a group of related laboratories where, under guidance, he can try out and discover the characteristic pursuits of adult life. Old fashioned classes in mathematics must needs disappear, and in an attempt to introduce the student to projects through which he may attain self education, activities approaching life situations must appear.

<u>Sources of Suggested Projects</u>: Creative activities in mathematics offer excellent opportunities to observe concrete forms and situations, as those found in plant life, snowflakes, the planets, light and sound, and so on. These expressions may take the form of designs, constructions, industrial applications, or devices of various kinds. The results obtained may be varied by the use of color and decoration, and be displayed on bulletin boards or in other ways about the class room. Activity may manifest itself in projects such as running a school bank, building a radio set, running a book store or shop, putting on a style show to exhibit geometric figures and designs on clothing, or it may be in forms of entertainment and dramatization which can be used on programs by the Mathematics Club.

Field trips may often facititate observation, for some of nature's most enticing geometric applications are not given to a removal from their out-of-door settings. The use of a scrap book for recording observations and preserving collected examples, specimens, pictures, diagrams, and applications, is a means of bringing together the various activities in connection with some project into a unified whole. The results of such creative expression, may be placed on display, or kept for future use.

The Adaptability of a Creative-expression Program: The activity program is an accepted reality in a large number of progressive school systems. Measured results have been made which show that it is possible to teach through activities and still accomplish normal or better than normal results in the fundamental skills.

A course in Cultural Mathematics was introduced at Antioch College in 1928 for freshmen.[1] No group instruction is given on the principle that creative powers are thus stunted. A laboratory plan of study is followed in which emphasis is placed on the quality of work rather than quantity, and progress is measured by the satisfactory mastery of successive problems. To illustrate: the application of the law of sines to the solution of a triangle is not shown by example, but the student is asked in a given concrete case to explain how the law may be used to find the necessary parts of the triangle. Thus he gains an inward feeling that he is creating ideas of his own. J. D. Dawson voices his opinion, which is tempered by the opinions of associated instructors and students, in regard to the results of the

1-Dawson, J. D.: Cultural Mathematics at Antioch, School and Society (January 24, 1931) Vol XXXIII p. 141-144.

course. He believes that although the average student may cover less ground, a better understanding of the material has been gained, and more adequate applications may be made. An apparent increase of student interest has come with the change in method, teaching interest has also been increased and the work has become a cooperative enterprise in the College.

The question is being asked, "Is Creative Education Compatible with Mastery of the Fundamental Processes?"[1] Willard W. Beatty answers the question affirmatively. He states that a real education must consist of at least two parts--that which transmits to the coming generation the traditions and achievements of the past, and that which provides opportunity for creative expression out of which alone can develop the power to contribute further to the permanent heritage of the race. The schools of Bronxville, New York, under the superintendency of Mr. Beatty, have recognized these two obligations, and while securing creative expression in a notable degree, are also maintaining high standards of accomplishment in the fields of objective learning. The fact is therein recognized that the potentiality for creative expression and the ability to fashion from materials of his environment some expression which shall be stamped with his own personality, lies within each individual. In the formal subjects, goal standards are set which the student must reach in order to be ready for advancement into the next

1-Beatty, Willard W.: Is Creative Education Compatible with Mastery of the Fundamental Processes? National Education Association. Department of Superintendence. Official Report 1930. p. 147.

unit of work.

A method of combining the concrete and abstract in mathematics by projects, use of graphs, recurring to every day realities of students, and other means of approach has been outlined by Fletcher Durell.[1] As indicated by Mr. Durell, the advantages realized by this method lie in the additional values which are given not only to the practical and vocational, but also to the ideal and spiritual side of mathematics. The results of a semester's work on an activity program in a fifth grade English class have been reported by C. C. Crawford and Lillian Gray.[2] The class, which was not a highly selected one, made about 60 per cent more gain in English than was normal for the semester.

In considering vital values in creative expression, Margaret E. Mathias[3] concludes that wholesome physical activity, acquisition of knowledges and skills, development of desirable social habits, and individual satisfaction are provided for the student by the use of materials suitable for creative work. Her experiments were made with children in painting. However, recognition of these qualities there stregthens the fundamental principle of the advantages of creative expression elsewhere.

1-Durell, F.: Making the Concrete and Abstract Help Each Other in Mathematics. School Science and Mathematics. (October, 1929) Vol. XXIX p. 702-713.

2-Crawford, C. C., and Gray, Lillian: Measured Results of Activity Teaching. National Education Association Journal (October, 1931) Vol. XX p. 270.

3-Mathias, M. E.: Vital Values in Creative Expression. National Education Association Proceedings (1930) p. 376-378.

Creative expression through music, art, literature, and drama, is quite generally recognized as an ideal way of awakening in an individual the highest form of response of which he is capable. Working upon the same principle, creative expression through mathematics, will awaken new interests, responses, and abilities for direct application.

Chapter II

The Language of Mathematics as Inducive to Creative Acitivity.

Mathematical Symbolism in Science: The symbolism of mathematics is simply a language by means of which thought is conveyed. It is required for the best expression of scientific method when the relations to be expressed become too involved for ordinary language, which is less precise and complete. There have been periods during which no progress was made in mathematics because the point had been reached at which new symbols were necessary before further expression was possible. In each case the need led to invention and the necessary symbols were forthcoming, which enabled further progress. In order to realize the importance of symbols a student need only undertake to carry out a simple operation, e.g., in algebra, with words alone.

Much of current literature makes free use of mathematical symbols. Workers in science find them convenient and indispensable for conveying their findings to others. The student should be encouraged to collect articles from his readings which contain such symbols, or to list as many as he may be familiar with, adding to the list as his observations broaden his field. The list may be arranged according to the year in which each symbol was first invented, according to the country which contributed each, or according to the branch of mathematics to which it is especially adapted, as, for example, the signs of algebra and the integral and differential signs used in calculus.

Progress of Number Forms: The development of number forms is a long and interesting story. A source of activity for the student lies in tracing these changes from the Chinese, Babylonian, Egyptian, and other early languages, into the Old European and the modern languages. Material of this kind may be gathered from a good dictionary, encyclopaedia, or histories of mathematics. Reference is also made here to Smith's Number Stories[1] in which the progress of number forms is discussed quite in detail.

Modern Arabic and Roman Notation: The ten digits of the Arabic notation as used today may be found in various styles and types of print; e.g. ㄣ and 4. Calendars, bulletins, and other sources furnish illustrations of such types. The Roman notation includes the use of I, V, X, C, L, M, D, and ¯ (the bar) as applied to any of the symbols. This notation is still used quite commonly on corner stones, clocks, some legal documents, and elsewhere. The student should be able to add to the list.

The combination of figures in both types of notation to form large numbers may be made the basis of creative activity. For an example in Arabic notation, the number of molecules in a gram is expressed as 6.06×10^{23}, a form with considerable advantage over the expanded notation. Other forms may be shown.

English and Metric Systems of Weights and Measurements: The systems of weights and measurements as expressed in English and in Metric terms, each uses an organized group of notations from mathematics. Just as the use of the common fractions is gradually giving way to the use of the decimal fraction, so is the English system of weights and measurements, which involves common fractions, giving way to the Metric system, which involves decimal fractions.

1-Smith, David E.: Number Stories of Long Ago Boston--New York: Ginn and Company, 1919.

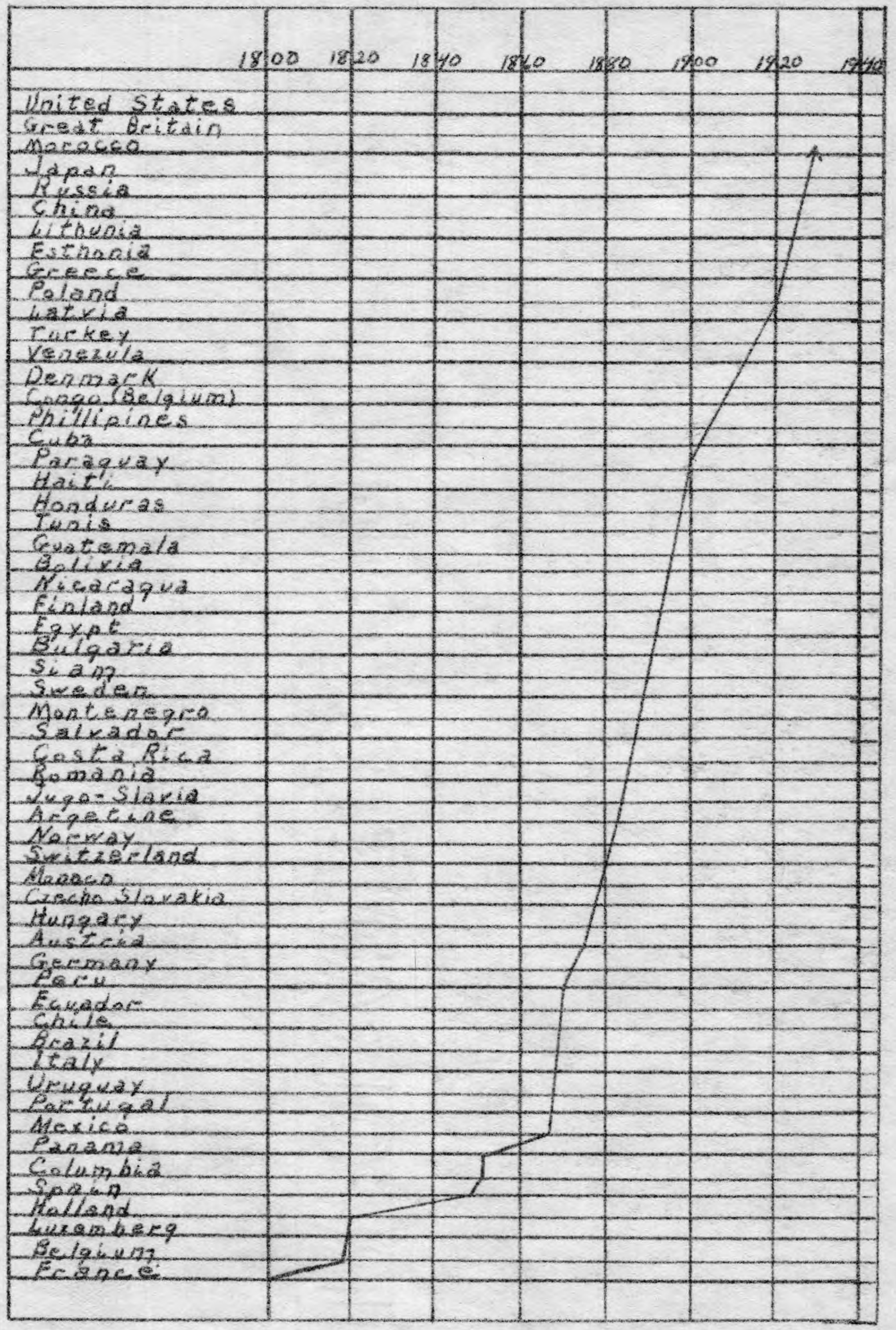
The graph of the Metric Advance, as shown on Plate I, is indicative of the increased usage of the Metric system among the nations of the world. It is presented here that creative activity may be centered about it and interest aroused for its adoption by the United States and Great Britain, which would then mean a one hundred per cent metrical world. The material presented can be re-arranged into other graphical types, as, for example, a time line, a bar graph, or according to geographical location of the countries.

A few other suggestions are given in regard to the metric system about which projects for student activity may be centered: Constructed models will illustrate the commonly used units in length and measures of volume and capacity. These models can be compared with similar units of measure in the English system. A project may call for obtaining bulletins from the United States Bureau of Standards as added sources of information. Another project may include making a collection of labels containing double markings: as, a can of peas--1 pound 3 ounces or 539 grams, a bottle of listerine --7 fluid ounces or 207 cubic centimeters, a package of sterilized gauze -- 1 yard or .9 meter.

Although the English system of weights and measures is in common use, the units involved and their relations to each other could be made the basis of projects to increase one's familiarity with them. Many of the historical facts, which are ordinarily overlooked, would appeal to the student if arrived at through some form of activity. A project may be centered about the terms used, their origin, and derivation. The following are current facts, to be used as suggestion, from mathematical history: The

Plate I

Metric Advance



yard was used by King Henry I of England as a unit, being the distance from the point of his nose to the end of his thumb. The standards for the rod and foot in the 16th Century are given in the following account.[1]

To find the length of a rod in the right and lawful way, and according to scientific usage, you should do as follows: Stand at the door of a church on Sunday, and bid 16 men to stop, tall ones and small ones, as they happen to pass out when service is finished; then make them put their feet one behind the other, and the length thus obtained shall be a right and lawful rod to measure and survey land with, and the 16th part of it shall be a right and lawful foot.

1-Sanford, Vera: A Short History of Mathematics. Cambridge: Riverside Press, 1930 p. 354.

Chapter III

Numbers as Classified by Significant Relations.

General Classifications: Numbers may be grouped with the principle of factoring as a basis, with construction as a basis, in series or progressions, and with other basic classifications. Each group thus enumerated contains various divisions each appropriately named, so that numbers may be referred to by many definite classifications. Collections of as many of these groups as possible, with their sub-divisions, may be made. A few of the classifications are here discussed briefly, as representative of the activity which may be involved.

Factors as a Basis of Grouping: With factors as a basis of grouping, the classification will include such numbers as the perfect, the amicable, the prime, and the composite. The perfect and amicable numbers are few compared to other groupings. Activity for the student in regard to such numbers may be centered around the historical interest in the mathematicians who discovered some of them, as well as in including illustrations for the general grouping of numbers.

A method of discovering prime numbers is by the use of Eratosthenes' Sieve. Eratosthenes inscribed the odd numbers on parchment, and, having cut out all the composite numbers, the remaining parchment had somewhat the appearance of a sieve. The method was as follows: Write the natural odd numbers:

1, 3, 5, 7, ~~9~~, 11, 13, ~~15~~, 17, 19, ~~21~~, 23, ~~25~~, ~~27~~, 29, 31, ~~33~~, ~~35~~,

Counting from 3 strike out every third number; counting from 5 strike out every fifth number; counting from 7 strike out every seventh number, and so on. The numbers that remain are prime.

Numbers Derived from Construction: Numbers derived from construction may include such classifications as commensurable, incommensurable, triangular, square, and oblong numbers.

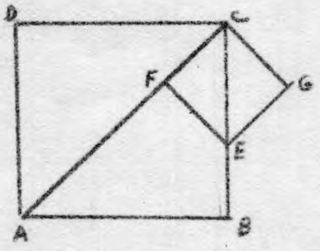
Numbers that are incommensurable may be illustrated by the famous case of the side and diagonal of a square. Let ABCD represent the square. On the diagonal AC, AF is laid off equal to AB. The remaining distance, FC, then becomes the side of a square whose diagonal CE lies on the side BC, EF being perpendicular to AC. See Plate II, Fig. 1. This may be constructed by the student and by taking a definite length for the side of the square, approximations of the lengths of the diagonals derived.

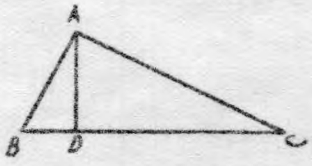
Triangular numbers may be illustrated by cannonballs piled in triangular formation. The formula for finding the number of balls in a pile with triangular base is given by: $1/6\ n(n+1)(n+2)$ where n equals the number on each side. The triangular numbers are: $1+3+6+10+15+21+$ -- -- -- -- etc. The student will find it of interest to use any available spheres, as marbles, and by forming different sized piles verify the triangular numbers. Square and oblong numbers may be found in a similar way by changing the base of the pile to a square or oblong.

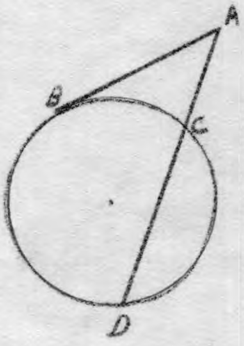
Numbers in Series and Progressions: Numbers in series and progressions will include such classifications as circulating decimals, and arithmetic and geometric series or progressions. Examples of these may be found in various texts or reference books. Activities may center about direct applications or problems to illustrate. The solution of the famous problem of the loaves which is given in Chapter XI is illustrative of an

Plate II

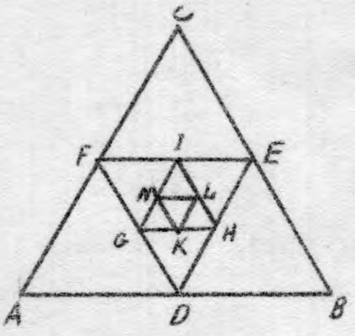
Numbers Derived by Construction

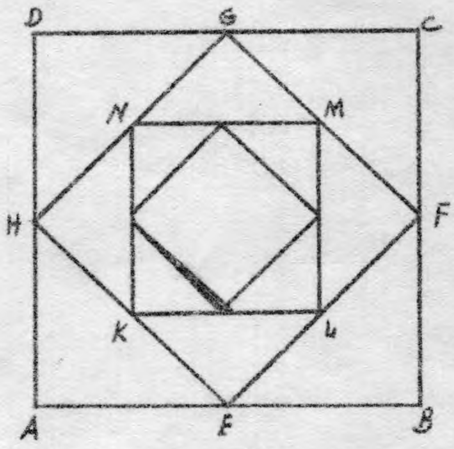




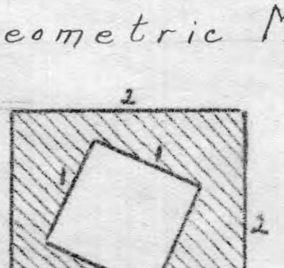


Geometric Series





Geometric Mean



arithmetic progression.

The Geometric and Arithmetic Means: Problems and constructions illustrating specific examples of the geometric mean and progressions follow: Fig. 2, Plate II, shows any right triangle in which AD, the perpendicular to the hypotenuse BC, is always a geometric mean between the lengths BD and DC. In Fig. 3, AB, the tangent, is always a geometric mean between the lengths AC and AD of the secant AD. In Fig. 4, the triangles are equilateral and formed by joining the mid-points of the sides. If AB=1, the perimeters of the triangles in order of their size, 3, 3/2, 3/4, 3/8, 3/16, etc. form a geometric series. A similar construction is shown in Fig. 5, where the comparative lengths AB, EF, LM, etc. may be determined by the student. Fig. 6 is illustrative of the geometric mean, or the single term which is inserted in geometric progression between two numbers. In this illustration the geometric mean is the square root of 4 X 1 or 2.

The arithmetic mean of two numbers is equal to half their sum. To illustrate: A piece of rope when coiled in a round mat is found to have 12 complete turns, or layers. The innermost turn is 4 inches long and the outermost turn is 37 inches long. The mean is 1/2(4+37), or 20 1/2 inches. The length of the rope then is 12 X 20 1/2, or 246 inches. Other illustrations may be made by winding a rope around a cone, or in noting the number of strokes the clock makes in striking the 12 hours of a day or night; e.g. 1/2(12+1)12, or 78.

The arithmetic mean is also used as one of the measures of central tendency in statistics. The column diagram illustrated in Chapter VII is the graphical representation of this measure.

As a project for student activity, the computation of various class averages is suggested. The marks of an individual student may also be used, and the averages computed from time to time be graphed in order to illustrate the progress being made. Actual data should be given to the students to be put in visual form. Chapter VII, on graphical representation, will suggest types of creative expression which it may be found desirable to use in this connection.

Chapter IV

The Acquisition of Short Cuts in Mathematics: An Activity which Lends itself to Cultural as well as Utilitarian Values.

General Purposes: Short cuts in mathematics are a means by which cultural values may be developed. If one must always be busy with the common place things in life there is little time left to develop superior appreciation and aptitude. To be able to reach a goal quickly is an incentive to go farther, to explore, and create, and reach out to include more than the mere attainment of the immediate end in view. By the application of shortened methods, e.g., to the processes of addition and multiplication, in acquiring the utilitarian purpose of mathematics, a feeling of appreciation for the beauty and power therein is created.

Short Cuts Basic in the Four Fundamental Processes: The following are suggestions for "short cuts" which are basic in computations. Other shortened processes are available from various text books which may be added to those given here. The student may apply such rules to specific examples to be used for illustrations, or if of superior ability he may be interested in preparing to some extent exposès of the machinery behind each of the "short cuts."

To square a number ending in 5, multiply the number of tens by one more than itself and annex 25.

To find the product of two numbers whose tens' digits are the same and whose units' digits add to make 10, multiply the tens' digit by one more than itself and annex the product of the units' digits.

The "elevens' rule" may be illustrated by 11 X 4532 = 49852. Write 2 for the right hand figure. Add 2 & 3 for the next figure; 5 &4 for the 4th figure; and write 4 for the 5th figure of the product.

The process of "casting out 9's" may be used as a check for addition, multiplication, and division. For addition, add the digits in each of the addends and divide by 9. The remainder is called the "excess". Find the total sum of the excesses and divide by 9. The excess of the sum of excesses equals the excess of the sum of the addends. For multiplication the excess of 9's in the product equals the excess in the product of excesses of the multiplier and multiplicand. For division, when there is no remainder, the excess in the divident equals the excess in the product of excesses of divisor and quotient. When there is a remainder, the excess of the product of the excesses in the divisor and quotient plus the excess of the remainder is equal to the excess in the dividend.

A number may be squared by the expansion of the binomial theorem as in algebra. Thus, $(96)^2 = (90+6)^2 = 8100+1080+36 = 9216$, or, $(96)^2 = (100-4)^2 = 1000-800+16 = 9216$.

In addition it is sometimes an advantage to be able to add two or more columns at a time. To do this for two column's begin with the number at the bottom and add the units of the number next above, and then add the tens, naming the totals only. Continue in this way until all the numbers are added. To add three or more columns, the method for two columns is extended to include all the columns desired.

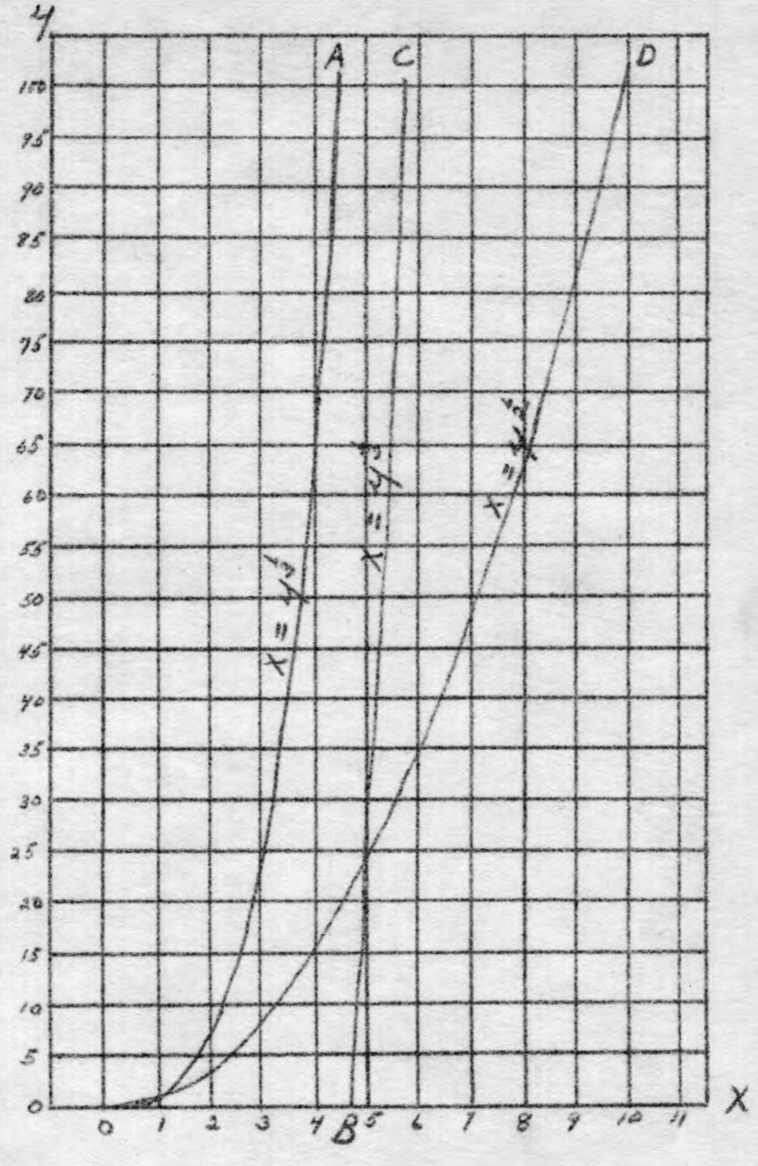
Extraction of Roots: Other "short methods" of computation are used in extracting the square and cube roots of numbers. For more rapid computations, tables have been made of the squares and cubes of numbers. Student competition and verification of solutions, involving the direct application of such tables, would insure skill and efficiency of their use.

The algebraic method insures a rapid calculation of square and cube roots when tables are not available. To show the relationship between this method and the expansion of the binomial theorem, the following illustration for square root is given. The extraction of the cube root involves more complicated computation and is not often used, but to show the similarity is a worth while expression for the student. To extract the square root of 547.56 by algebraic method, apply the binomial expansion: $(f+n)^2 = f^2+2fn+n^2$, where f is the found part of the root at any stage, and n is the next digit to be found. $f_1=20$, and $(f_1)^2$, or 400, subtracted from 547.56 leaves a remainder of 147.56. This contains $2f_1+n_1$, $(n_1=3)$ with a remainder of 18.56. This now contains $2f_2-n_2$, $(n_2=0.4)$ with no remainder, giving 23.4 as the root sought.

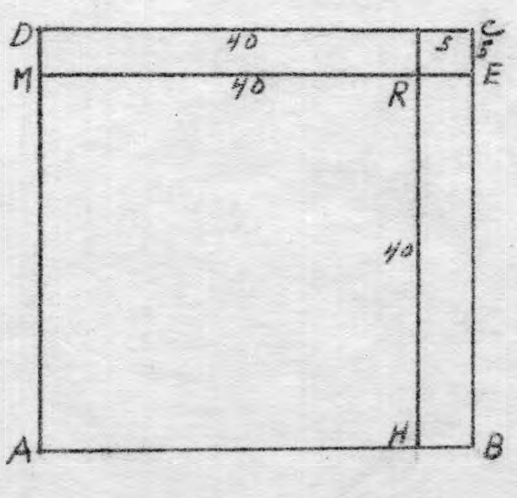
The extraction of the roots by the graphical method, although limited in its usefulness, illustrates the functional relationships involved. To obtain the chart, (See Plate III, Fig. 1), plot the graphs of the equations: $X=\sqrt{y}$ and $X=\sqrt[3]{y}$. To illustrate the validity of the curves, the student may find the roots of various well known squares or cubes on them. From the curve, OD, can be read the square root of any number from 1 to 100 correct to one decimal place. Curve BC is a continuation of OA, and from it can be read the cube root of any number between 1 and

Plate III

Graphical Method of Extracting Roots



Geometric Method of Extracting Square Root



200, correct to one decimal place. The graphical method can also be used to extract fourth and higher roots.

That the student may appreciate the added dexterity given by "short cut" methods of extracting roots, he should determine them in other ways, as by the geometric method, the factoring method, or the method of averages. Fig. (2) PlateIII, diagrams the geometric method, using the square root of 2025 as an example. A square whose area is 1600 is 40 units on a side. A square whose area is 2500 is 50 units on a side. If the square of 40 units on a side is taken out, two rectangles and a square remain. The length of the rectangles is 40 units, so the problem is to find the width. If the width is 6, the combined area is too great, therefore try 5, and the combined areas becomes 2025.

The factoring method may be illustrated by finding the square root of 85,766,121. Its factors are 3, 3, 3, 3, 3, 3, 7, 7, 7, 7, 7, 7, therefore its square root is 3X3X3X7X7X7, or 9261.

To find the square root of 40 by the method of averages, factor it into 5 X 8. Taking the average of 5 and 8, the factors approximate 6.5 X 6.15. Averaging again a better approximation, 6.325 X 6.3241, is derived. The process may be repeated until as many significant places as desired are obtained.

The method of obtaining a result, not being the creative end for which mathematics exists, the pupil should be encouraged to find direct application in specific examples; e.g., What is the shortest distance from the south-west corner to the north-east corner of a township?

Thus the extraction of the roots of numbers means more than a mere following of formula or rule. To be able to do so by one method is sufficient for utilitarian purposes. To be able to reach the end in view by other methods as well, broadens ones appreciation of the application and purpose by connecting through creativeness and self expression what might otherwise be isolated facts.

Logarithmic Computations: There are methods of computations of a more advanced nature which might be thought of, also, as "short cuts". The long and laborious arithmetic computations of astronomers were brought to an end in the early part of the 17th century by the invention of logarithms. These had been forshadowed in the use of exponents by Stifel, but it was for Napier who was in close touch with Briggs to invent them. Student activity may be centered around such suggestions as; historical facts concerning the invention, a comparison of the two types of logarithms, as, their bases, the kind of work in which each is used, and the method of changing from one to the other. Specific instances may be found where either type is used, as in applications in business, economics, and science.

The student may not gain speed by the application of logarithms to specific problems involving simple processes in multiplication, division, involution, or evolution; e.g., those centered about the right angled triangle, but by making such application he will discover some of the beauty of mathematics, and will gain a cultural knowledge and an appreciation of the principles involved. The use of the slide rule in computing values by logarithms involves creative activity. If the student does not have access to a slide rule, he may make one in Bristol board. (See Chapter VII.)

Algebraic Laws Considered as Short Cuts: Algebraic principles, as the Laws of Exponents, Laws of Algebra, and Laws of Signs, are in themselves "short cuts". It is suggested that student activity be centered about grouping these laws, and giving specific examples to illustrate.

The advantage of the Laws of Exponents may be shown by expressing a computation involving them in some other form, as, for example: $7^6 \div 7^3 = 7\cdot7\cdot7\cdot7\cdot7\cdot7 \div 7\cdot7\cdot7$. In similar ways each of the laws may be expressed. The student may learn to manipulate exponents and signs with a great deal of skill, but unless he has gone further than formula and theory would require, and set up for himself a goal which includes the purpose and advantage of such discoveries, he has lost the real value which self discoveries and creativeness offer.

Chapter V

Basic Mathematical Processes Considered for Creative and Cultural Values.

Use of Fractions: Fractions now so easily expressed as common fractions, decimal fractions, or per cents, are a comparatively recent invention. A project based on the development and use of these forms may be made more interesting by including constructions to illustrate them. Pestalozzi's Fraction Sheet, which shows the relation of the parts to the whole, is an early method of studying fractions. Plate IV illustrates its construction and other constructions which may be used as suggestions for student activity.

Measurement of Angles: An angle is a changeable figure usually thought of as being measured in degrees or in radians, i.e., "natural" measure. Euclid in his "Elements" defines an angle of a segment as that angle "contained by a straight line and a circumference of a circle". The Babylonians had, before the time of Euclid, divided the circle into 360 degrees getting their idea, no doubt, from the 360 days of a year. A project for student activity may center around the early development of angular notation. Plate V contains suggestions for ways in which angles may be illustrated by cardboard constructions.

Illustrations for the various kinds of angles, as acute, right, obtuse, straight, supplementary, and complementary, may be noted about the class room. Articles showing them may be collected from various sources; e.g., a carpenter's rule, clock hands, a pocket knife, or a folding fan. Pictures of objects in

Plate IV

Fractional Values Illustrated

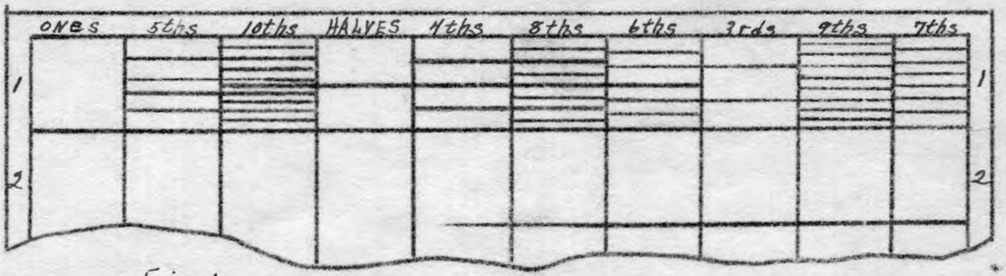


Fig. 1

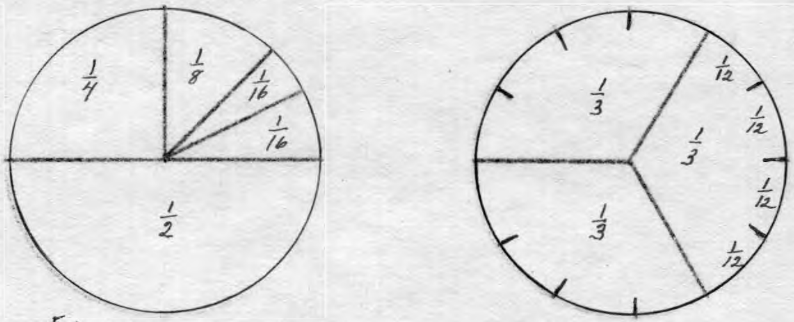


Fig. 2

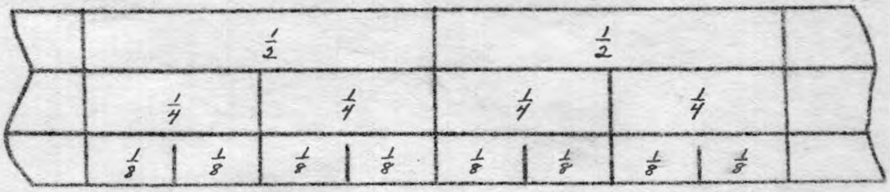


Fig. 3

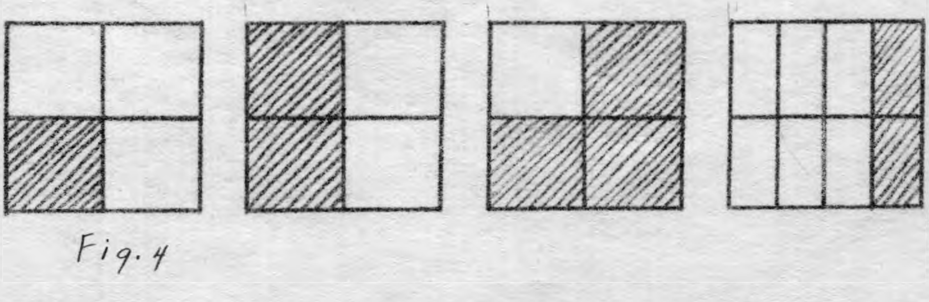
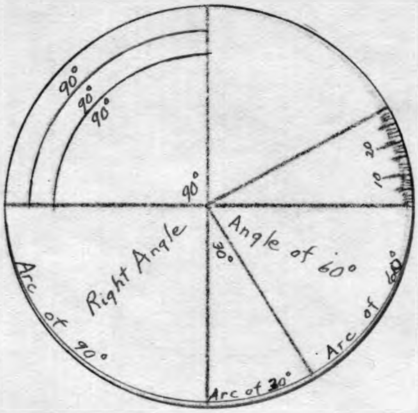


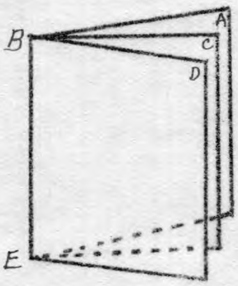
Fig. 4

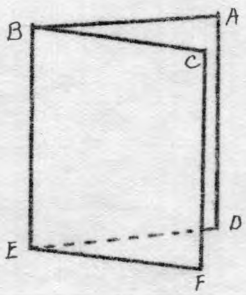
Plate V

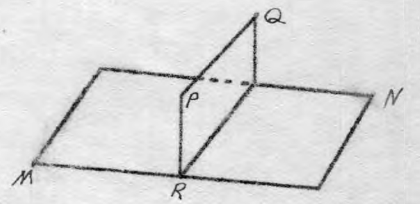
Angles Illustrated



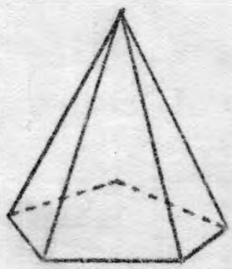
Diedral Angles

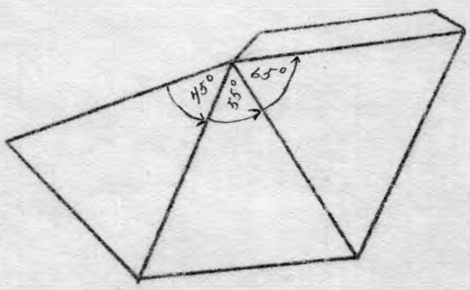






Polyedral Angles





which angles appear may be collected, classified, and mounted as a means of creative expression in this connection.

Development of Trigonometric Functions: If the graphs of the sine, cosine, tangent, and cotangent functions are constructed to the scale of a unit circle, their modes of variation and comparative values may be observed. Plate VI illustrates a project which may be centered upon this principle. If the scale for the unit circle be taken large enough to avoid crowding effects, the graphs may be combined into one diagram, and produced in artistic form.

As a project the student may construct from cardboard a unit circle mounted so as to rotate upon a card. Fastened at any point on the circumference is a swinging pendulum which is scaled in length similar to a scale across the diameter of the circle. As the circle is made to rotate so that the various angles have been turned, as indicated by the scale of degrees about the circle, the sines and cosines of the angles may be read from the vertical and horizontal scales.

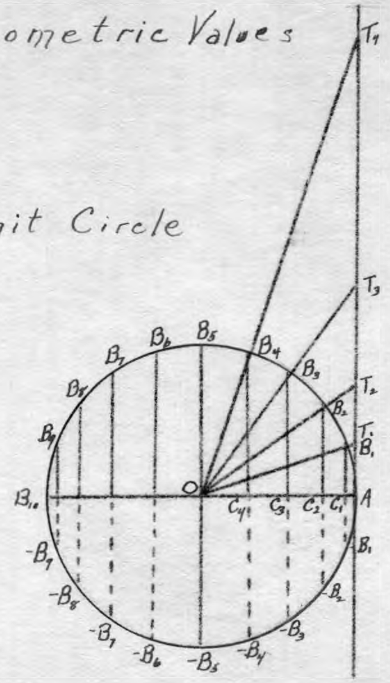
There are notable applications of the sine and cosine curves, as in the study of sound waves and alternating electric currents; e.g., in radio. These suggest further creative and cultural activities for the student.

Differentiation and Integration: Calculus is sometimes called "the mathematics of nature". It deals with change, and "in nature all things change". Calculus is used to simplify many computations which would otherwise be long and complicated. It may be applied to work in many fields where summations, rates of motion, slopes, maxima and minima, and other concepts involving

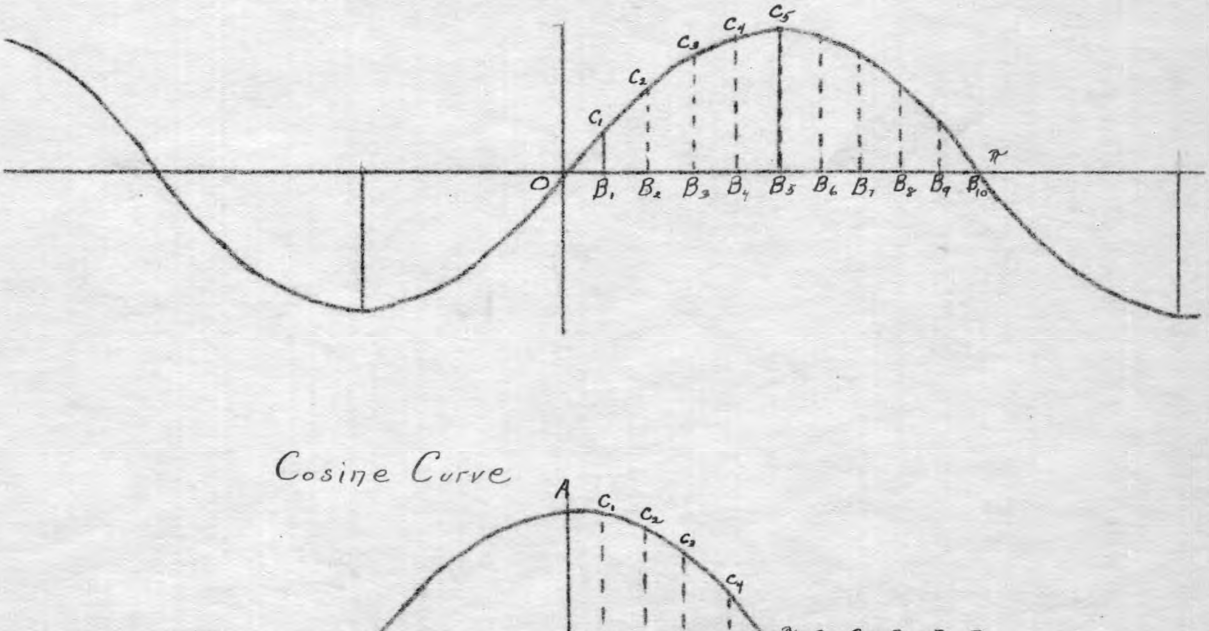
Plate VI (a)

Trigonometric Values

Unit Circle



Sine Curve



Cosine Curve

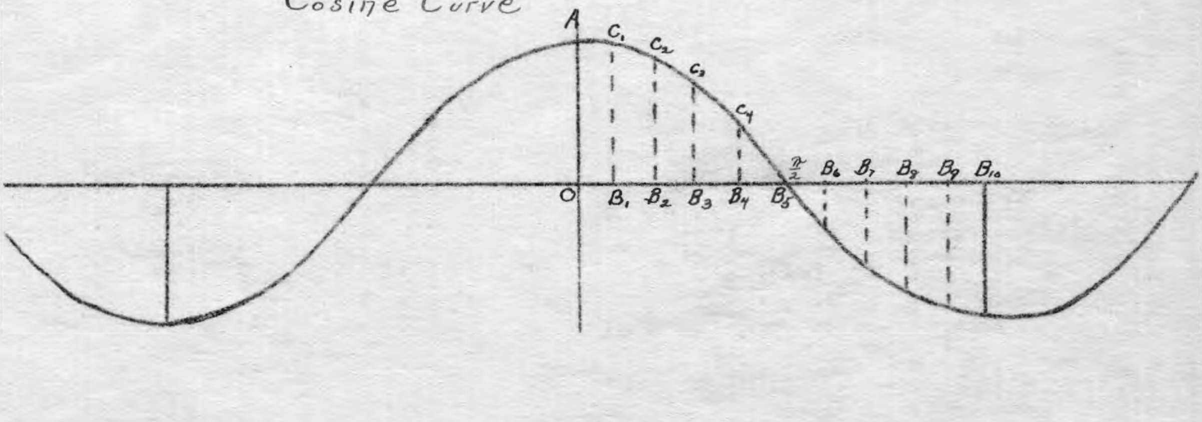
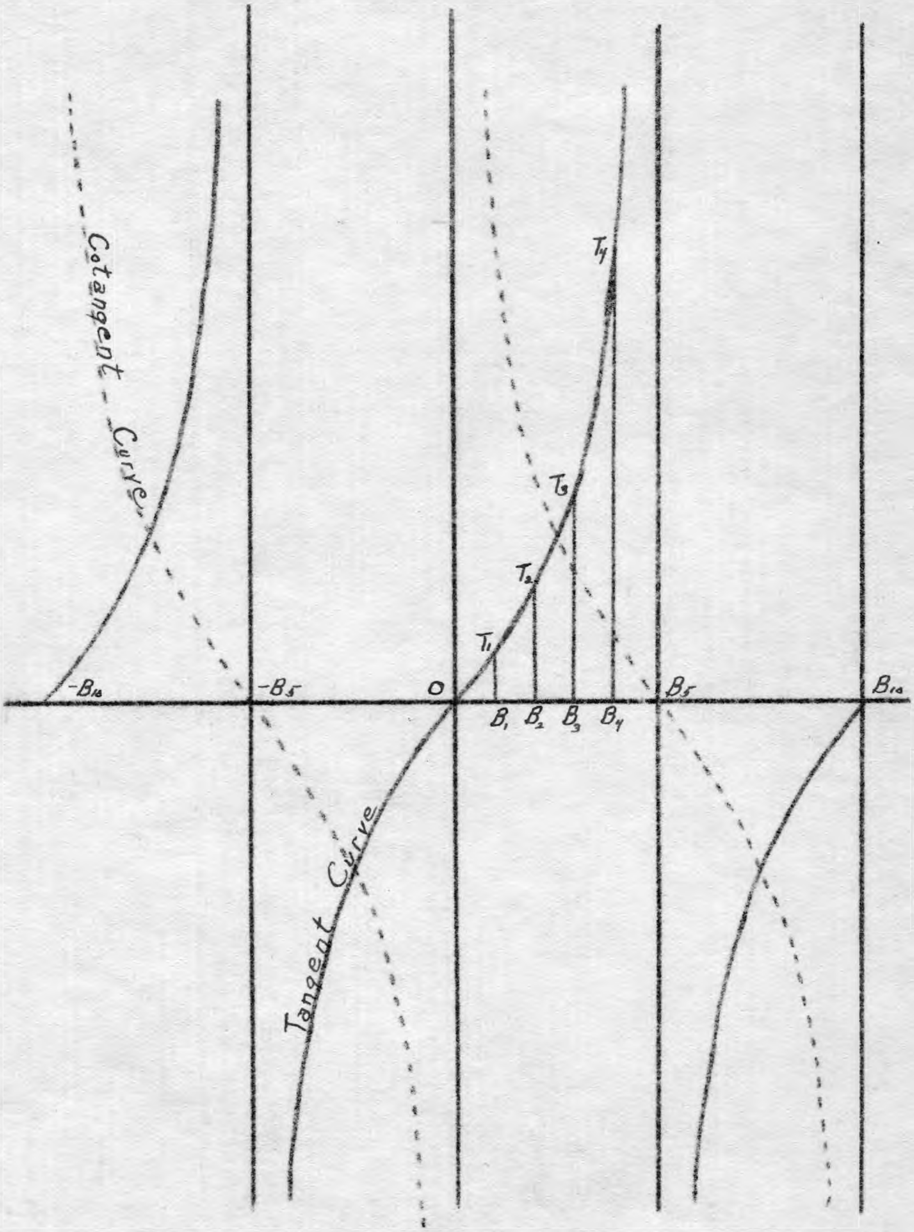


Plate VI (b)

Tangent and Cotangent Curves



change appear. The student may find and list specific applications of the use of differentiation and integration. Articles from current literature showing the use of calculus in various fields may be found. For example, calculus may be used in making computations in aviation, in radio activity, in medicine, and wherever the direction, speed and balancing of forces are involved.

Diagrams showing the principles involved in differentiation and integration may be constructed in forms suitable for display. Such diagrams may be found in books containing the elementary principles of calculus. Other activities may center around the historical facts in regard to the discovery and perfection of calculus.

Chapter VI

Creative Activities Involved in the Construction of Mechanical and Graphical Devices Used in Ancient and in Modern Times.

Early Devices Used in Computation: Some of the devices used in computing in early times, although outmoded now, were indispensable in their day. Their study will afford the student means of creative expression through constructions, diagraming, and making direct applications. Such devices also have an appeal from their historical standpoint, many of them being closely related to the development and progress of the science of mathematics. Glaisher said, "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history".

In the following pages in this chapter, a few computing devices will be discussed in detail that the construction of them may be made directly. Other devices will be mentioned as suggestions for further activity where reference material to them is accessible.

Many of the modern devices may be used by the student in direct applications. It is here again suggested that where the school does not have these devices available, student activity be first centered around constructions for the class room, and then further activities follow in the form of the direct applications.

Many forms of the abacus appeared as early devices used in computations. The suan-pan in China, the soroban in Japan, the s'choty in Russia, the Roman abacus, and the modern abacus are all well known forms. The modern abacus consists of beads strung on a wire, or of buttons sliding in a groove. In the Roman abacus, each groove is divided into two parts, with 4 buttons in the longer part, and 1 in the smaller part of each groove. Each of the 4 buttons stands for I, and the single button for V, in the units groove. In the tens groove, the 4 are X each and the single one L.

The construction of an abacus forms a simple project.

Illustrations of the work done on an abacus may be reproduced by means of a piece of paper marked with 4 or 5 horizontal lines, with coins to represent counters. (See Plate VII) The following describes the addition of 282 and 369: Since 5 counters on the units line are equivalent to 1 on 5's space, 5 of the unit counters are removed and 1 is carried to the next space, leaving 1 counter on the units line. Then 2 counters are removed and 1 is carried to the next line, etc.

In subtraction the subtrahend is literally taken away from the minuend and whenever a line or space in the minuend has an insufficient number of counters, others are borrowed from the space or line next above.

In multiplication, the multiplicand and multiplier are laid out on the left. (See Fig. 2, Plate VII) (In actual work the multiplier counters are removed when each has been used.) The top line of the multiplier is taken as the unit and the multiplicand is laid out on the board as many times as there are counters on this line of the multiplier. These multiplier counters are then removed and the next line is taken as a unit. Parts I, III, IV illustrate this. In multiplying by 5 as in II, the next higher line is taken as the unit and the multiplicand is

Plate VII

Devices Used in Computations

Abacus - Addition

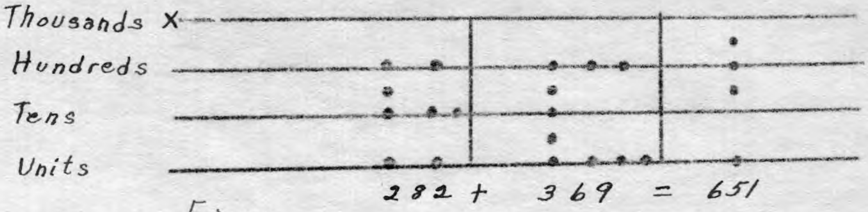


Fig. 1

Abacus - Multiplication

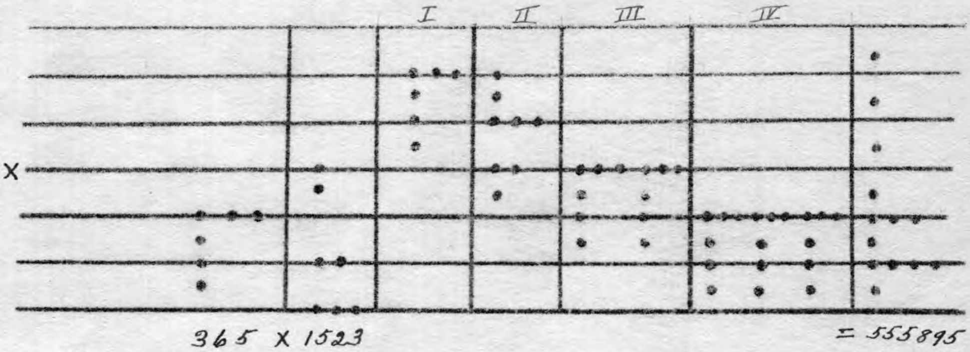


Fig. 2

Pascal's Triangles

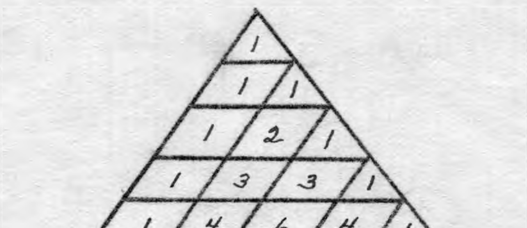


Fig. 3

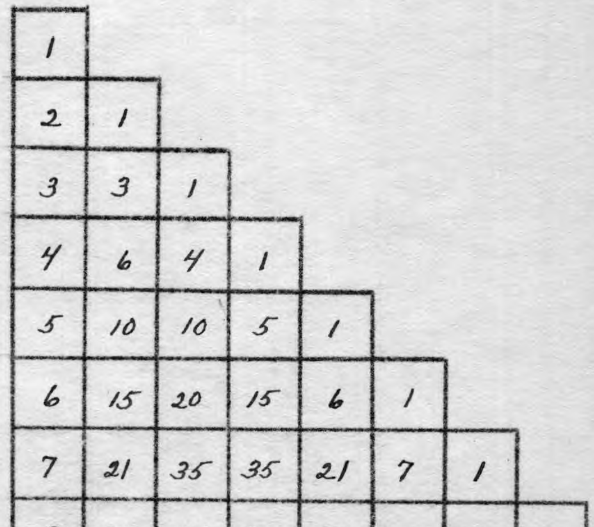


Fig. 4

taken as the unit and the multiplicand is divided by 2. (182.5). At the conclusion, the counters that represent partial products are added and the sum appears on the right.

Division on the abacus is accomplished by repeated subtraction, and the computer uses his counters to record the number of times the divisor has been subtracted. The units line is shifted as the work progresses.

The sand board is an old Greek type of counting board on which the figures are made in sand on a board and obliterated after use.

On the counting board the numbers are represented by loose counters laid on lines. This arrangement has an elementary place value idea. In regard to the permanence of the value at any one position, a Greek writer said: "like favorites of a tyrant, their value is sometimes more and sometimes less".

The psammites is a sand reckoner which was used by Archimedes as a means of reckoning large numbers. By its use he determined the amount of sand required to fill a universe, of radius equal to the distance to the sun, as being less than 10^{63} grains of sand, assuming that 10,000 grains of sand are in a sphere of not less than a finger breadth.

Napier's rods, Korean rods, and the ancient Chinese stick numerals are other devices used to facilitate computations.

The coefficients of the various terms in a binomial expansion are displayed in elegant form by Pascal's triangle. (See Fig. 3, Plate VII). This triangular array first appeared in the form shown in Fig. 4, and is constructed as follows: Write down the numbers 1, 2, 3, etc. as far as desired in a vertical row.

On the right of 2 place 1, add them together and place 3 under the 1; then 3 and 3 are 6, which is placed under the 3; 4 and 6 are 10, which is placed under the 6, and so on. The third vertical row is formed from the second, and in a similar way, the other rows are formed. This triangle gives, without the trouble of calculation, how many combinations can be made, taking any number at a time, out of a larger number. For example; how many selections can be made of 3 at a time out of 8? On the horizontal row commencing with 8, look for the third number; this is 56, which is the answer.

Modern Mechanical Devices: In mathematics, as in other fields of endeavor, work and energy are facilitated by machinery. Thus innumerable inventions have been made which aid the mathematician in his computations. In the small school many of these will be unavailable unless constructed by the student. Neither the list of devices which follows nor the list of applications to which they may be put is complete. It is left for the student to explore, discover, and create.

The straight edge, draftsman's triangles and the T-square are devices used for drawing straight and parallel straight lines. (See Figs. 1 & 2, Plate VIII). The parallel ruler (Fig. 3) is based on a familiar theorem on parallelograms. On this principle, an extension shelf (Fig. 4) which always remains in a horizontal position may be constructed. The carpenter's square (Figs. 5 & 6) may be used to determine a true semicircle or to show where to cut a ring into halves.

A simple compass (Fig. 7) may be constructed from a straight edge. Pivoted at one end, a marker may be inserted in holes at

Plate VIII (a)

Instruments for Computation

Draftsman's Triangles

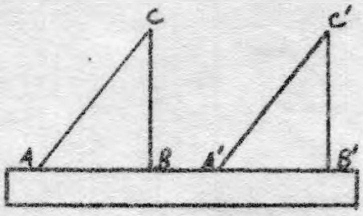


Fig. 1

T-squares

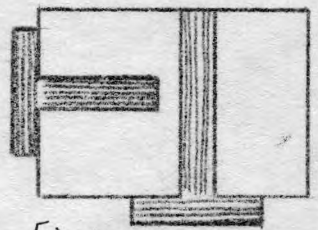


Fig. 2

Parallel Ruler

Fig. 3

Horizontal Shelf

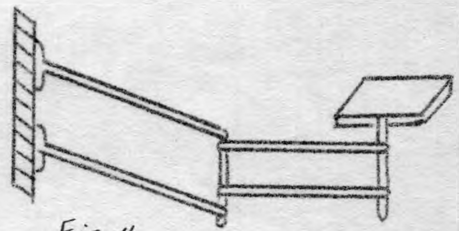


Fig. 4

Carpenter's Square

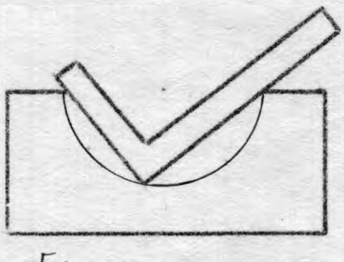


Fig. 5

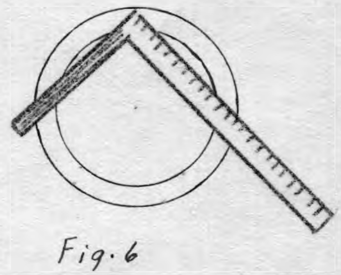


Fig. 6

Compass

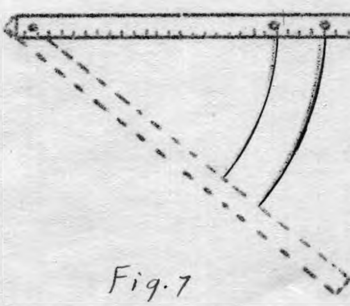


Fig. 7

at various scale distances and the edge swung around in circular motion, thus producing a circle of any desired radius.

Levels may be constructed in a number of ways. Fig. 8 shows an ancient form, and Fig. 9, a means of using a plumb line in a semi-circle to determine a level line.

The 3-4-5 rule is a simple means of constructing a right angled triangle. (See Fig. 10)

The protractor and combination ruler and protractor, if not available may be constructed from card board, or from plywood for blackboard use. (See Figs. 11 & 12)

The diagonal scale (Fig. 13) which is based on the principle of proportional line segments, measures lengths to the hundredths of an inch. This is also easily constructed.

Proportional compasses (Fig. 14) are used to make scale drawings of given figures. By making OB' equal to one half of OB and OA' equal to one half of OA, and then opening the compasses so that AB equals a given line segment, A'B' is equal to one half of AB. This is also the principle of proportional line segments.

On a similar principle the pantograph (Fig. 15) is used to magnify or reduce a given diagram or map. A construction of this kind for blackboard use would be a valuable aid for class room work.

The slide-rule, which has already been mentioned in Chapter IV may be constructed from bristol board. A diagram and directions for construction are given in the Mac Millan Logarithmic and Trigonometric Tables.

The Monroe calculator is a valuable asset to a mathematical department. A simple adding machine may be constructed which is

Plate VIII (b)

Instruments for Computation

Levels

3-4-5 Rule

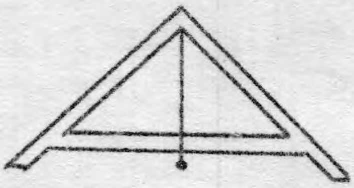


Fig. 8

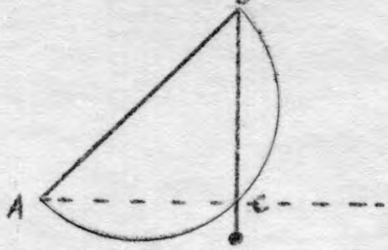


Fig. 9

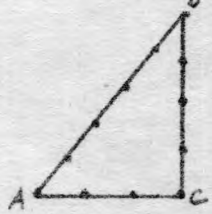


Fig. 10

Protractor

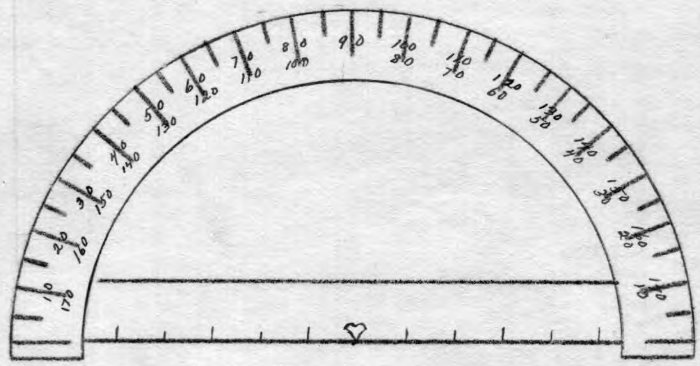


Fig. 11

Combination and Protractor Ruler

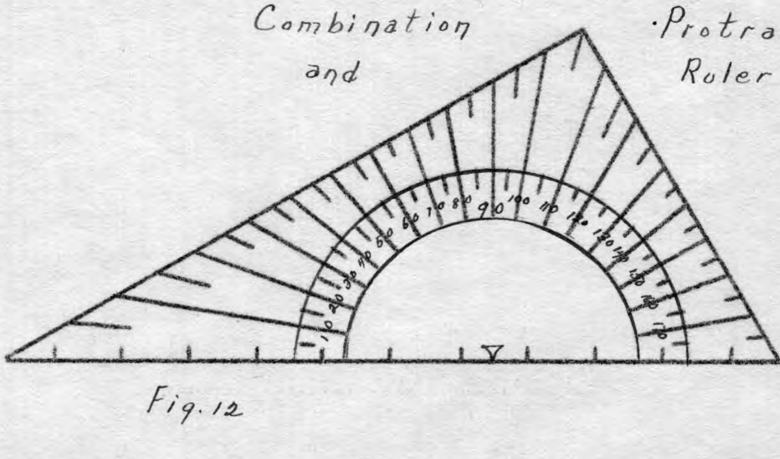


Fig. 12

Plate VIII (c)

Instruments for Computation

Diagonal Scale

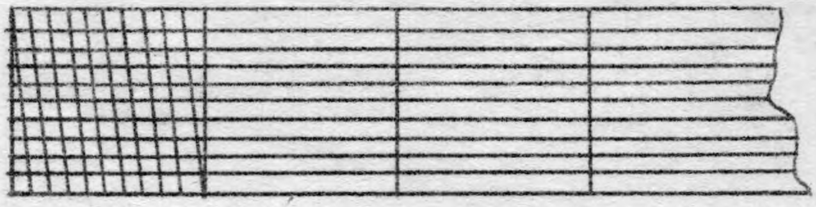


Fig. 13

Proportional Compass

Fig. 14

Pantograph

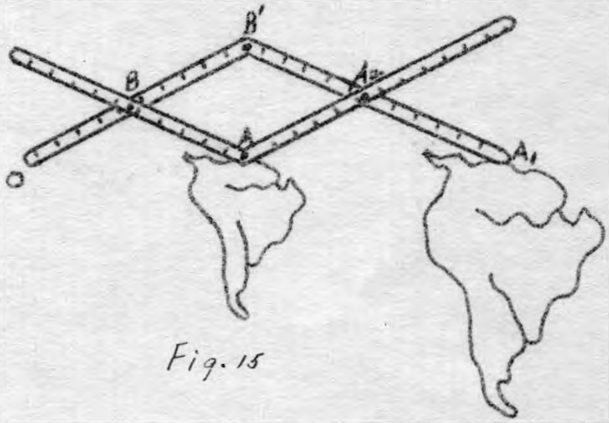


Fig. 15

Principle of Adding Machine

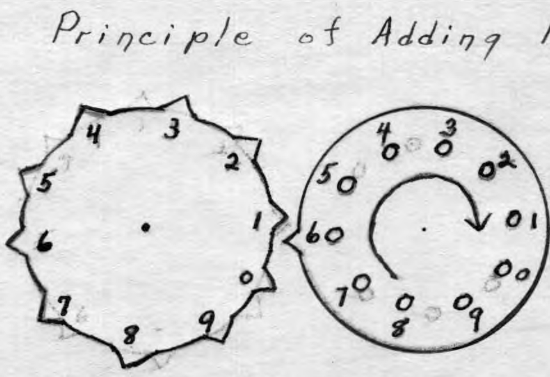


Fig. 16

based on the principle of the now familiar device of a series of wheels each having ten gear teeth, one for each unit from 0 to 9. (See Fig. 16) At each complete turn of any wheel the next higher one is turned through one tenth of a revolution. For the student who may be interested in the mathematics of the calculating machine, reference is here given to two articles in the Mathematics Teacher.[1]

Home-made instruments for measuring horizontal and vertical angles are diagramed in Figs. 1 and 2, Plate IX. Spirit levels are used in setting up the apparatus. A pin placed at O (Fig. 2) is used as a pivot for the ruler, and a pin at A is used to sight over in taking measurements. For further discussion of these instruments, reference is here made to the Teaching of Mathematics in Secondary Schools.[2]

The plane-table (Fig. 3) is used for mapping. A simple outfit may consist of a stool, a drawing board and a chalk box for an alidade. Directions for using the plane-table may be found in "The Third Year Book."[3]

The hypsometer (Fig. 4) is a modern form of a geometric square used to measure heights. It is easily constructed by

1-Locke, Leland: Some Mathematics of the Calculating Machine. Mathematics Teacher (November 1922) Vol. XV p. 423-428. Mathematics of the Calculating Machine. Mathematics Teacher (February 1924) Vol. XVII p. 78-86.

2-Breslich, Ernst R: The Teaching of Mathematics in Secondary Schools. Chicago, University of Chicago Press 1930 Volume I-Technique p. 126-128.

3-Shuster, C.N.: The Use of Measuring Instruments in Teaching Mathematics. Third Year Book 1928 New York Teachers College, Columbia University p. 215

Plate IX (a)

Instruments for Observation

For Measuring Horizontal Angles

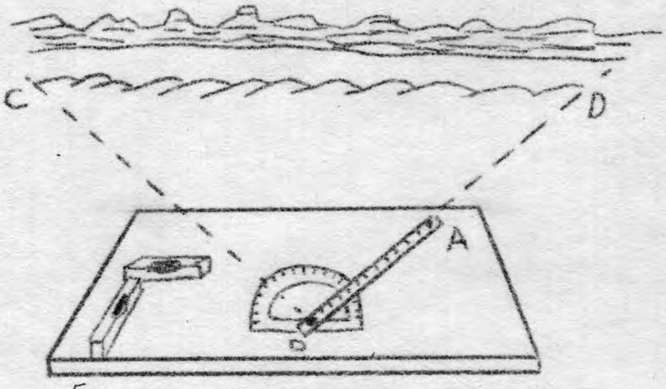


Fig. 1

For Measuring Vertical Angles

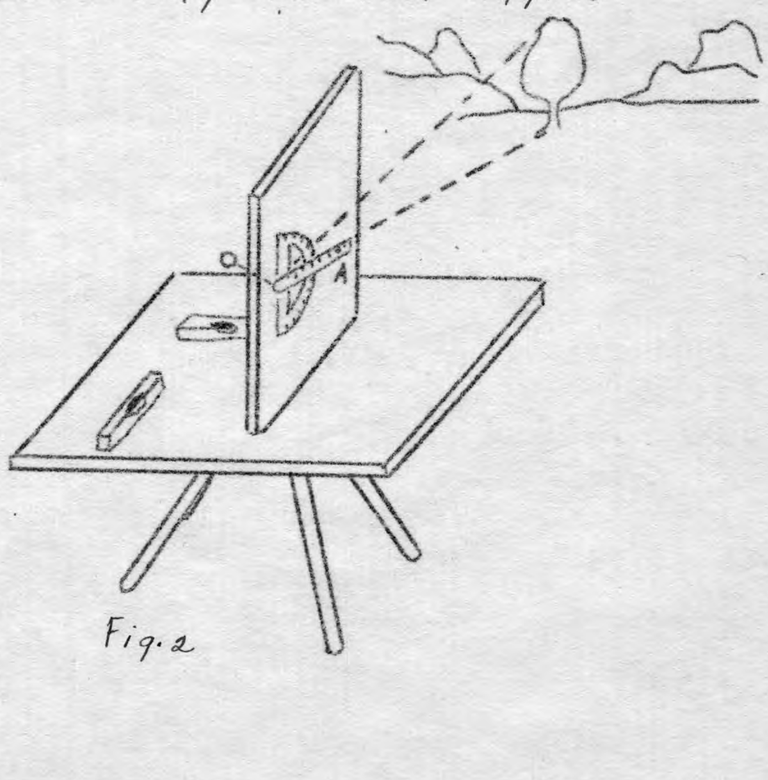


Fig. 2

Plate IX (b)

Plane Table

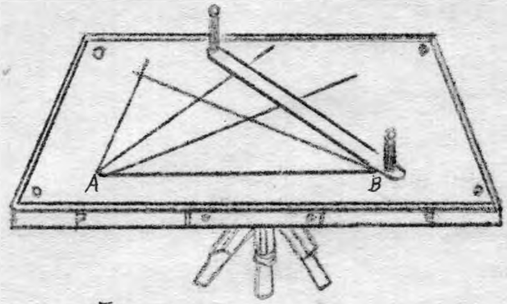


Fig. 3 (a)

Map by Radiation

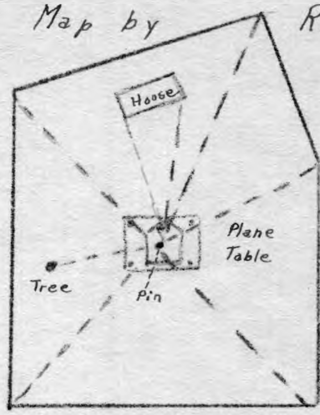


Fig. 3 (b)

For Mapping Areas

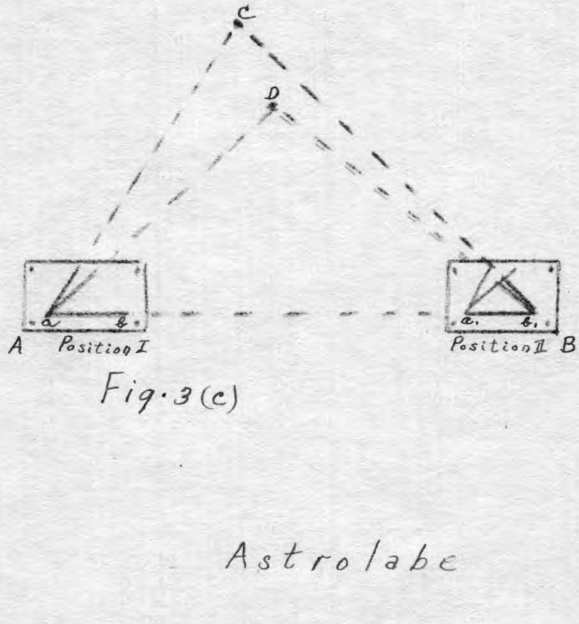


Fig. 3 (c)

Hypsometer

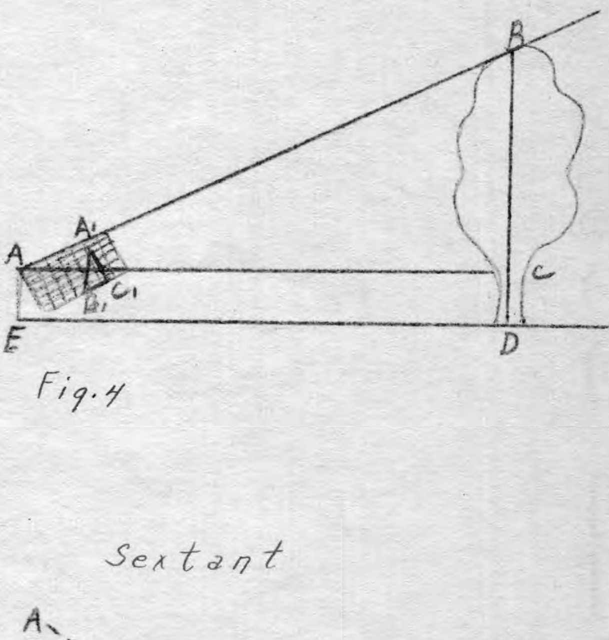


Fig. 4

Astrolabe

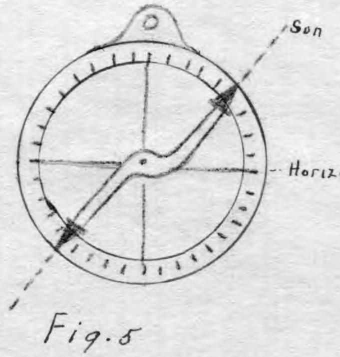


Fig. 5

Sextant

Fig. 6

pasting a sheet of graph paper on a board and fitting plumb bob and sights. By the principle of similar triangles, the desired computations are made.

The astrolabe (Fig. 5) is used to measure angles. It can be constructed of a large circular bristol board protractor, 12 to 14 inches in diameter, which is divided into quarter degrees. This is tacked on to a board and given several coats of varnish and wax. A ring is fitted in the top, and a sighting arm, or alidade, suspended. If a staff is fitted to the back so it can be used in a horizontal position, it is possible to measure both vertical and horizontal angles.

The sextant (Fig. 6) provides a means of measuring angles of elevation, and thereby determining latitudes. It may also be used for measuring horizontal, vertical and inclined angles. Further discussion of the sextant may also be found in the Third Year Book.

The transit, although ordinarily a complicated instrument, may be made or purchased in simple form. For directions for constructing a transit as a project in geometry, reference is here given to the Mathematics Teacher for 1931.[1] Other discussions of the construction and use of the transit may be found in the Third Year Book and in the Teaching of Mathematics in Secondary Schools by Breslich.

Other devices which lend themselves to similar student activity are the angle mirror, cross-staff, clinometer, planimeter,

1-Engle, T. L.: Constructing a Transit as a Project in Geometry. Mathematics Teacher (November 1931) Vol. XXIV No. 7 p. 444-447 and vernier.

Mechanical devices for constructions of the hyperbola (Fig. 1, Plate X), and of the ellipse (Fig. 2) may be used in creative expression by the student. To construct the hyperbola, fasten one end of a ruler at one focus, F', so that it may swing about that point, and to the other end fasten a string. Make the length of the string less than that of the ruler by 2a and fasten the free end of it to the focus, F. Press the string against the ruler with a pencil at P, and turn the ruler about F'.

The method of construction of the ellipse is also called the Gardener's Rule as it is used by gardeners for laying out elliptical flower beds. Fasten pins in the paper at the foci F and F'. Tie a string to them, making the length of the string equal 2a, the length of the major axis. Then press a pencil against the string and move it, keeping the string taut. The sum of the distances of the pencil point from the two pins will at each point be equal to the length of the string.

<u>The Principle of the Golden Section</u>: The principle of the Golden Section provides a method of producing a mean proportion between two given lengths. Its construction was first discovered by Pythagoras and presented as a geometric theorem. (See Fig. 3, Plate X). The straight line, a, is divided into two parts, X and (a-X) so that $a(a-X)=X^2$. By construction AE=ED and EF=EB. $FGKD=(EF+ED)(EF-ED)=EF^2-ED^2=EF^2-AE^2$. Therefore, $FGKD+AE^2=EF^2$. Since EAB is a right triangle, $AB^2+AE^2=EB^2=EF^2$. Then, $AB^2+AE^2=FGKD+AE^2$. Therefore, $AB^2=FGKD$ and $AH^2=HBCK$. The Golden Section is used to construct a pentagon. (See Chapter VIII)

Because of its mystic qualities the pentagon was adapted

Plate X

Devices for Construction

Hyperbola

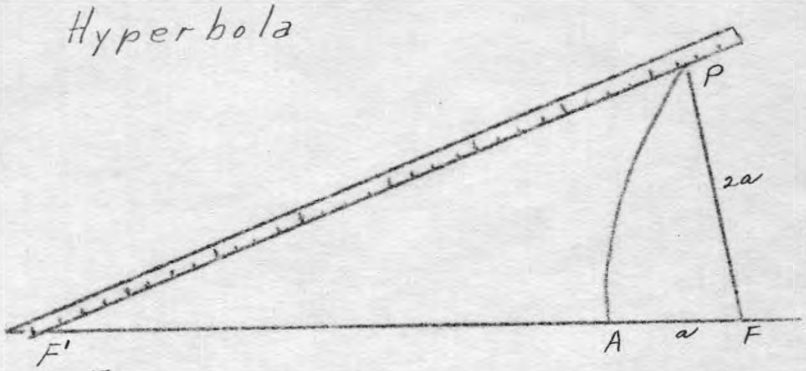


Fig. 1

Ellipse

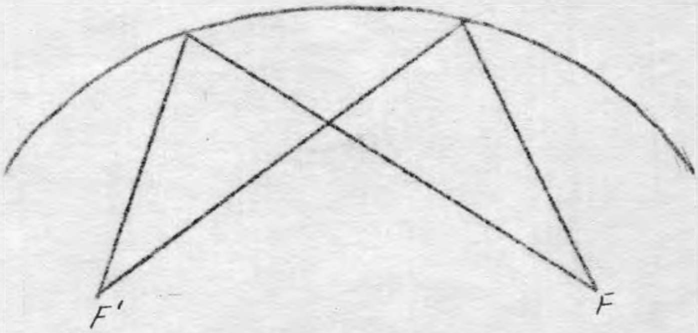


Fig. 2

"Golden Section"

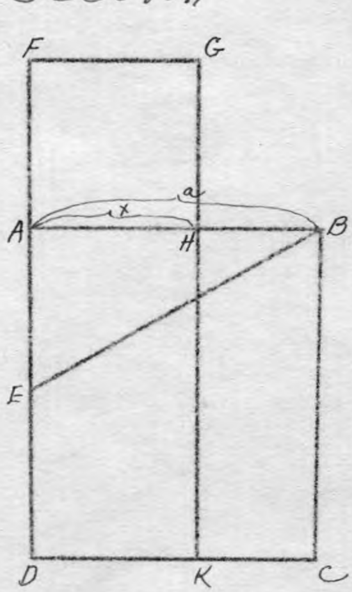


Fig. 3

by the Pythagoreans as the badge of their society. The five sides of the pentagon may be changed to produce various patterns of the five-pointed star.

Devices used in the Solution of the Three Ancient Problems: Because of their inability to square the circle, duplicate the cube, and trisect an angle by means of straight edge and compass, mathematicians ventured farther and discovered other means to solve their problems.

The cissoid of Diocles is a curve used to duplicate the cube, for by it a length is found such that its cube is twice that of the cube of some given length. The conchord of Nicomedes is a curve which also gives the desired length to duplicate a cube, as well as the desired point through which to construct a line to trisect an angle.

The quadratrix is a curve which gives the desired lengths to square a circle, and to also trisect an angle.

The construction of these curves is not included here because of the necessity for rather long explanations. However, the superior student in geometry may find these a source of interesting activity. They may be found in various texts as well as books of mathematical history. Besides the activity involved in constructing the curves, specific applications may be made in duplicating cubes, trisecting angles, and squaring circles.

Devices for Measuring Time and Temperature: Devices used for the measurement of time and of temperature may afford the student a means of creative expression. The gnomen or sun dial, and the water clock, were devices used to tell time before the invention of the hour glass or more modern clock. A project may be centered upon historical interests, and principles involved, or a sun dial may be constructed according to proper mathematical principles and observations made.

Measures of temperature are based on the centigrade and Fahrenheit Thermometers. A chart or graph may be made to illustrate the comparative values, including the boiling point and freezing point. If the two types of thermometers are available direct readings may be made to verify mathematical computations.

Chapter VII

Relative Values Illustrated by Graphs.

<u>Uses of the Graph</u>: As has been quoted by Klein, "The function is the soul of mathematics." The reading world is becoming functionally minded, and consequently current literature uses freely the graph and its companions, the formula and the equation.

The graph is ordinarily thought of as a means of plotting related points which have been calculated as values of unknown quantities. In reverse order, from the graph of a function the value of points may be determined. The zero values of polynomials, the intersection points of geometric curves, interpolated values, and maximum and minimum points are determined by means of the graph.

Graphs are also used to illustrate relationships involved in concrete situations. Rates, areas, volumes, water pressures, and other functions from every-day life are definitely described by means of graphs. The business and economic world, as well as educators, find the graph a satisfactory way of recording information, and more use is being made of it each year.

The graph may be used to illustrate relationships between speed and distance, distance and time, or any two values which vary dependently. For example, interest on money depends on time, the weight of a uniform rod depends on length, the cost of a quantity of sugar depends on weight, or the circumference of a circle depends on its radius. The use of the graph in building problems, railroad time tables, points of maximum net returns, etc., means an unfolding of relationships in a manner easily understood.

These are given as suggestions of dependent relations of which many specific examples may be found and represented graphically. Functional relationships often exist where they are easily overlooked. The following specific example is given as a suggestion for activity in applying the graph.

A total of 151 M. bricks are needed for a building, and they will be used as follows: 4M. per day for the first four days, 10M. per day for the next six days, and 15M. per day for the last five days. As a matter of economy it is desired to have the deliveries made at a uniform rate per day. Find the maximum storage capacity needed to allow uniform deliveries over the fifteen day period. (See Fig. 1, Plate XI) By measuring the ordinate differences, AD and BE, it is found that the maximum storage capacity needed is approximately 24,000 bricks. The varying ordinate differences between the line "Total Used" and the line "Total Deliveries" show the growth and disappearance day by day of the pile of stored bricks.

Types of Graphs Used for Concrete Illustrations: The application of the graph serves a two-fold purpose: 1st, it puts the individual in contact with a wide range of experiences, and 2nd, it enables the individual to comprehend total relationships.

There are various classes and types of graphs used to illustrate concrete situations. They include the bar graph, pictograph, circle graph, line graph or comparative line graph, histogram or column diagram, and the step graph, all of which are popularly used. Many and varied are the applications which can be made involving their use. A project which suggests itself for

Plate XI(a) - Types of Graphs

Comparative Line Graph

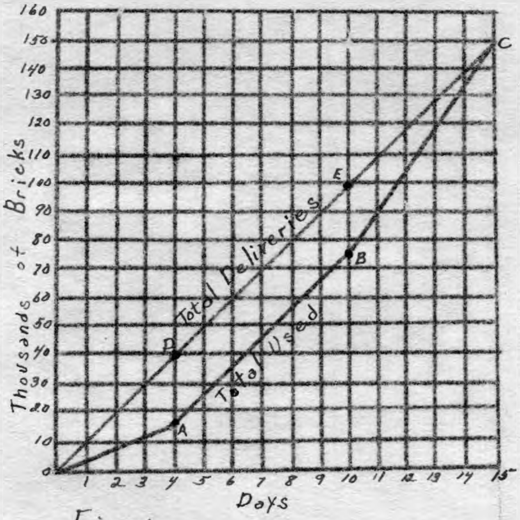


Fig. 1

Learning Curve

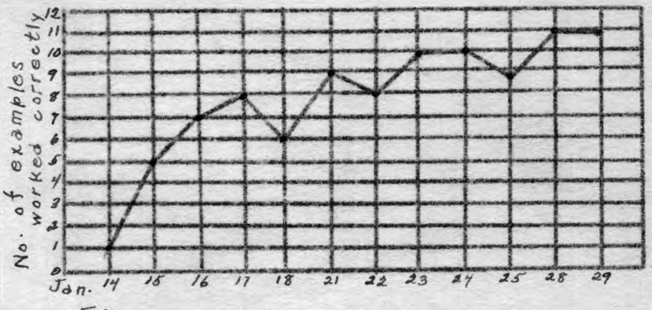


Fig. 3

Circle Graph

Fig. 2 Foreign Born Pop. in U.S.

Bar Graph

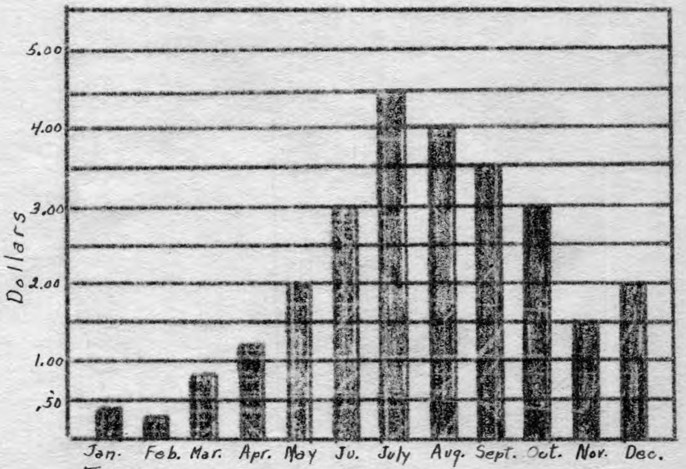


Fig. 5 Ice Bill

Pictogram

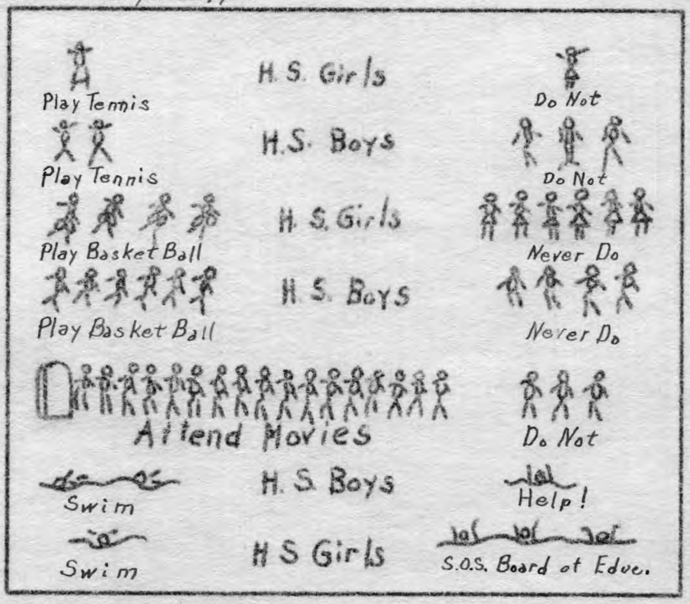


Fig. 4

the student consists in collecting graphs from his own experience, from current literature, or from any available source, endeavoring to find illustrations of the various types and their applications. The following illustrations are given as suggestions to start the student on this tour of investigation and exploration.

The circle graph may be a single dissected circle as shown in Fig. 2, Plate XI, or several concentric circles.

The learning curve may be used by the pupil to show graphically the relationship of his score in tests from day to day over a period of time. Similar applications may be made to progress or achievement made by a class. (See Fig. 3)

The pictogram is perhaps the most easily understood and is often used to create an attitude for the usefulness of graphs, or to introduce a discussion of graphs among a group which is unfamiliar with them. Fig. 4 is an example of a pictogram used in teaching mathematics. It serves to illustrate the elementary principle involved, and suggests many possibilities which would put information of various kinds in an attractive and easily interpreted form.

The bar graph lends itself to a variety of forms. (See Fig. 5) The bars may be horizontal or vertical; a single bar, several bars, each plain or each dissected; or a set of multiple bars. This is a useful as well as an interesting type of graph and the student will find it a means by which he may be creative and original in expressing information.

If a column diagram (Fig. 6) is plotted on cardboard and along the base and outline, the diagram will balance on a knife edge placed at the mean or average. The column diagram is a graph

Plate XI (b)

Types of Graphs

Column Diagram or Histogram

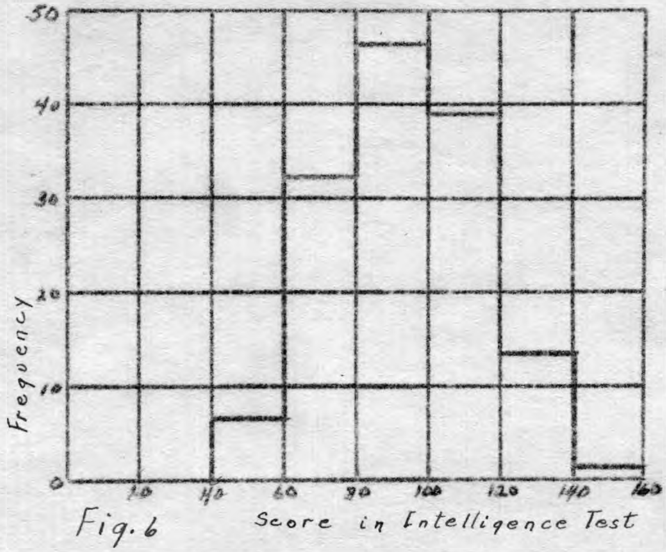


Fig. 6

Frequency Polygon or Line Graph

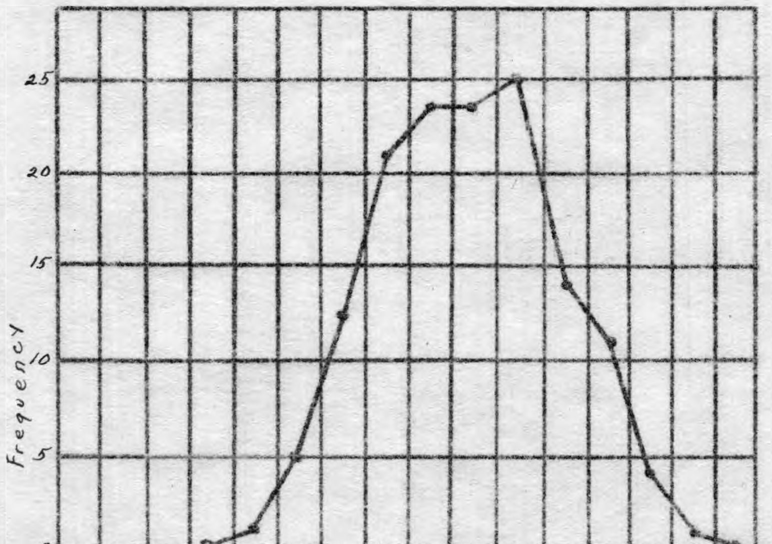


Fig. 7

Step Graph

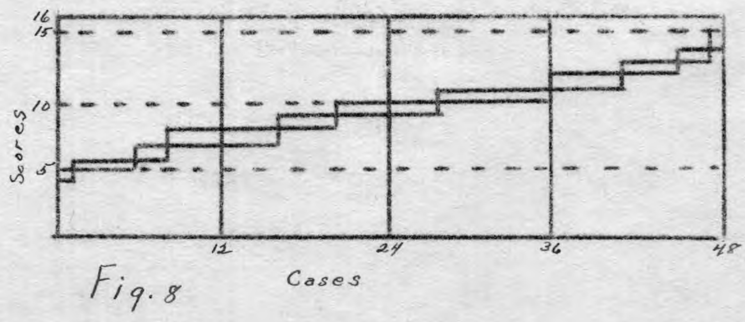


Fig. 8

which may be used in illustrating the grades of a pupil or of a class. If the grades are thus represented, and the average or "mean" grades (See Chapter III.) obtained, the diagram will balance at that point on the scale. Other similar applications may be made.

The frequency polygon (Fig. 7) or line graph is similar to the learning curve already discussed. There are several kinds of line graphs; as, a single line which may be broken or curved, and several comparative lines each of which may be broken or curved. An illustration of a comparative line graph is given by Fig. 1, Plate XI. This type of graph may be used in comparing class grades, or individual achievements, in comparing the price of foods from time to time, in comparing the amount of rainfall from month to month or year to year, and in many other ways where comparisons may be desired between definite functional relationships.

The step graph represents relationships by means of rectangular areas. Fig. 8, Plate XI, represents the distribution of scores in a test. The rectangles may be replaced by their diagonals, thus converting the step graph into a line graph. The student may illustrate this by making the desired construction.

Graphs of Functions Described by Equations: Graphs are used also to illustrate the linear and non-linear functions expressed by equations. Posters, descriptive of various types of curves, may be constructed by the student as a means of creative expression. Plate XII suggests a few types of curves which may be included.

Fig. 1 shows the graph of an equation which has no real root.

Plate XII

Graphs of Functions Described by Equations

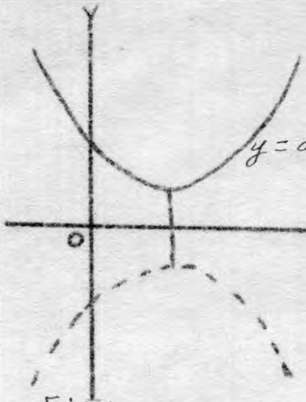


Fig. 1

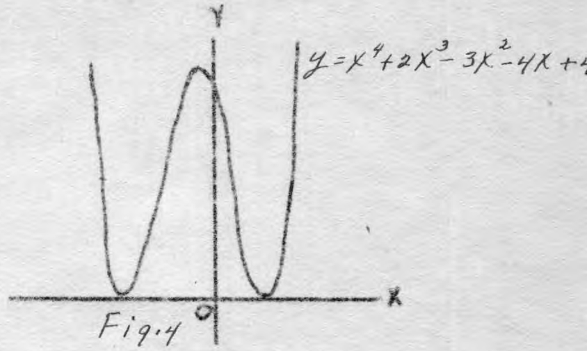


Fig. 4

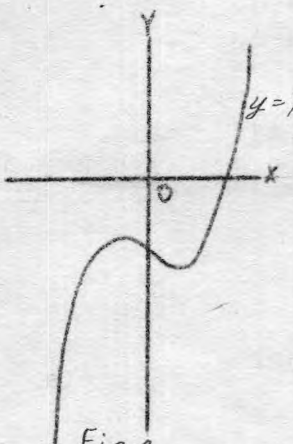


Fig. 2

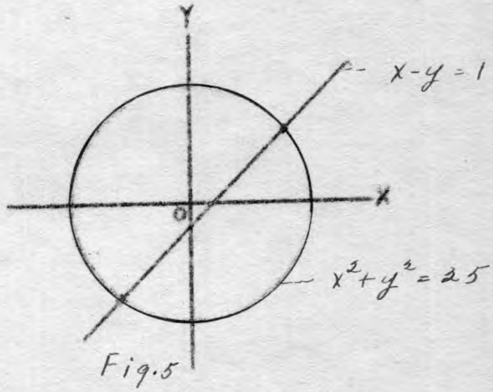


Fig. 5

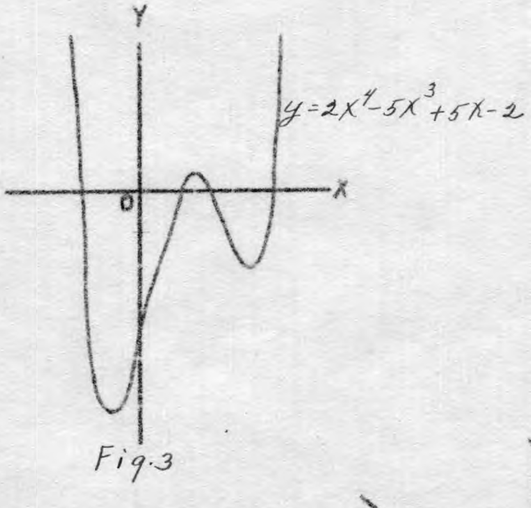


Fig. 3

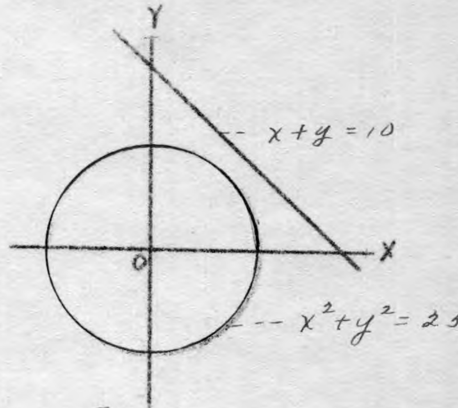


Fig. 6

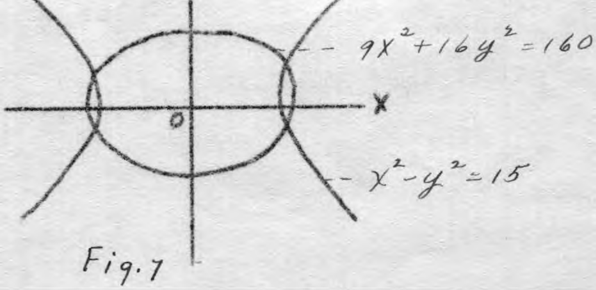


Fig. 7

A cubic equation may have one real root and two imaginary roots, imaginary roots always occuring in pairs. This is illustrated in Fig. 2.

The curve, $y=2x^4-5x^3+5x-2$, as shown in Fig. 3, cuts the X-axis at -1, $\frac{1}{2}$, 1, and 2. Hence there are four real roots.

In Fig. 4, $y=x^4+2x^3-3x^2-4x+4$, illustrates a curve in which x has but two values, -2, and +1. It is thus shown that this function has two double roots.

Fig. 5 shows the graphs of two simultaneous equations, one a circle and the other a straight line. The graph shows that there are two solutions for the equations, points (4,3) and (-3,-4).

Fig. 6 shows that the two equations $x+y=10$, and $x^2+y^2=25$, have no real solutions.

Fig. 7 shows that in the two quadratic equations given, there are four possible pairs of values for x and y; points (4,1), (-4,1), (4,-1), and (-4,-1).

The graph is a convenient way of representing the variation of a function, as the independent variables are given successive different values. For example, in the equation, $y=-ax+b$, as the value of b is changed the straight line crosses the y axis at those given values. If the value of, a, remains unchanged the line retains the same slope for the various assigned values of b. The student may use the same idea for a project to show the effect of assigning different values to the constants in various types of equations; as; the linear equation, quadratic equation, cubic equation, simultaneous equations, and so on.

<u>Conic</u> <u>Sections</u> -<u>Curves</u>: Some of the possibilities have been suggested in the graphical representation of the curves of the

conic sections; i.e., the circle, ellipse, parabola, and hyperbola. These curves may be produced, as their name suggests, from cross sections of a cone. Each section taken at a different angle produces a different curve, or type of curve. For example, the hyperbola is produced by a cross section of a cone shaped like an hour glass. This discovery was made by Apollonius, and a project in this connection might include historical facts in regard to his life and works.

Many curves may be observed by the student in the things about him. Other forms of creative expression which may be centered about such curves, may be in making collections of pictures and diagrams to illustrate them, as well as in graphical productions. A discussion of a few of such curves follows.

The spiral is the name given to an important family of curves. It includes the curve of the tendrils on vines; it is the path in which a comet travels which comes within the sun's atmosphere and eventually falls upon its surface; it includes the Spiral Nebulae of the heavens, the spiral of Archimedes, the hyperbolic spiral, and the logarithmic spiral.

The catenary is the curve in which a chain of uniform weight will hang when supported at both ends.

The cycloid is the curve traced by a point on a circle which rolls along a straight line. Variations of this curve may be produced by the tracings of a point which is less than or more than a radius distance from the center of the circle. The wheel may also be rolled on the inside or outside of a fixed circle instead of a straight line, thus producing other variations in the curve.

The Witch of Agnesi, the lemniscate, and the cardioid are suggestions for other curves which may be included in the students creative activity in this field.

Chapter VIII

The Construction of Figures of Area and Volume as a Source of Creative Activity in Mathematics.

<u>Regular Polygons</u>. Among the regular polygons which are inscribable in a circle are the equilateral triangle, the square, the pentagon, the hexagon, and the octagon. With the exception of the pentagon, the construction of these is not difficult, and from them the construction of polygons having twice the number of sides; as, 10, 12, 16, etc. is easily obtained. The construction of the pentagon depends upon the principle of the Golden Section (See Chapter VI and Fig. 1 Plate XIII). Divide OB by the Golden Section. BC=OD by construction. $OD^2=OB \cdot BD$. By similar triangles BC:OB=BD:BC. CS=BC and BS is a side of the desired pentagon.

Other regular polygons are inscribable, as those of 15, 17, and 257 sides, but their construction is also difficult. The following rule has been established to determine what figures can be inscribed: $(2^n) \cdot P_1 P_2 P_3 - - - - P_n$ equals the number of sides, where 2 is raised to any power and P represents any prime number. Also, (2^n-1) represents the number of sides, if the result is a prime number, as in the case of 3, 17, or 257.

It is of interest to note that in the time of Ptolemy, a figure was discovered which gives the sides of 4 of the regular polygons, See Fig. 2 Plate XIII, in which M is the mid-point of AO and CD is the arc of a circle of radius MC. The length of side for each is given as, OC= side of hexagon, BC= side of square, CD= side of pentagon, and OD= side of decagon.

Plate XIII

Regular Polygons

Pentagon

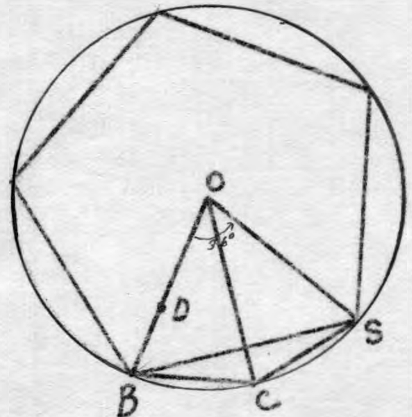


Fig. 1

Ptolemy's Figure

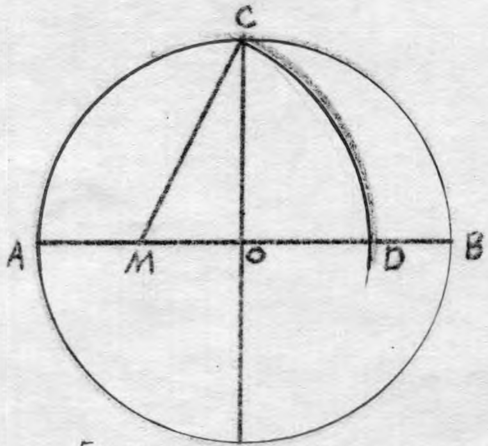


Fig. 2

Figures based on the regular polygons are found in profusion in the snow flakes. These may also be cut from paper and mounted with beautiful effects. To photograph the snowflakes against a black background, is an interesting activity for the student who is handy with the use of a camera.

Regular inscribed polygons involve certain relationships which may be graphed or diagramed as a means of illustration by the student. For example, the side is a function of the radius of the circle, and therefore the area depends on the radius. This suggests also, that there are relations between the perimeter, the area, and the sides. These relations may be charted as a group.

Five Regular Polyedrons: There are but five regular polyedrons; the tetraedron, hexaedron, octaedron, dodecaedron, and icosaedron, having respectfully 4, 6, 8, 12, and 20 regular sides or faces.

The five regular polyedrons may be easily constructed from paper. For patterns, see Plate XIV. The models thus constructed will furnish a valuable background for a clearer concept of these solids. With the constructions built upon a rather large scale, measurements may be taken and a project of determining areas, volumes, and comparisons of the figures centered upon them.

A relationship between the octaedron and the cube is shown in Fig. 1, Plate XV. By joining AC, CD', D'A, D'B', B'A, and B'C (Fig. 2) a regular tetredron is formed within a cube. These figures may be used to verify such relations in the areas as, Cube $= 6S^2$, Tetraedron $= \frac{1}{2} S^2\sqrt{3}$, and Octaedron $= S^2\sqrt{3}$, where 3 represents a side of a cube. Such a project in mensuration may include grouping or

Plate XIV

The Five Regular Polyedrons

Dodecaedron

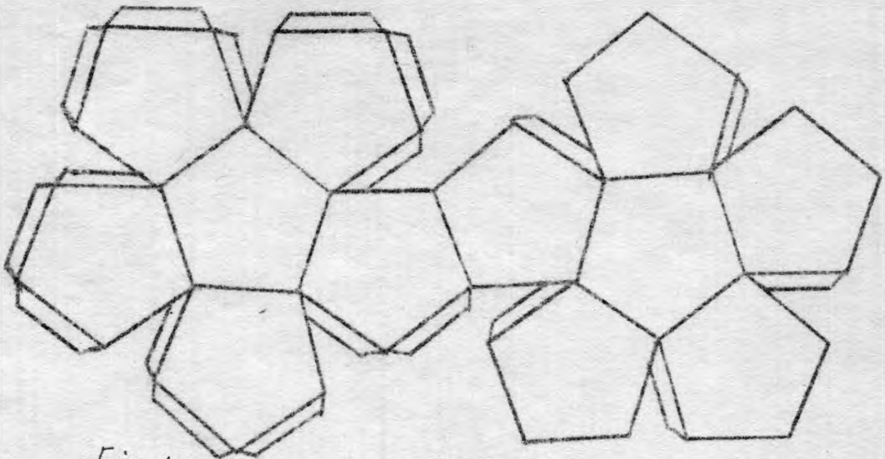


Fig. 1

Icosaedron

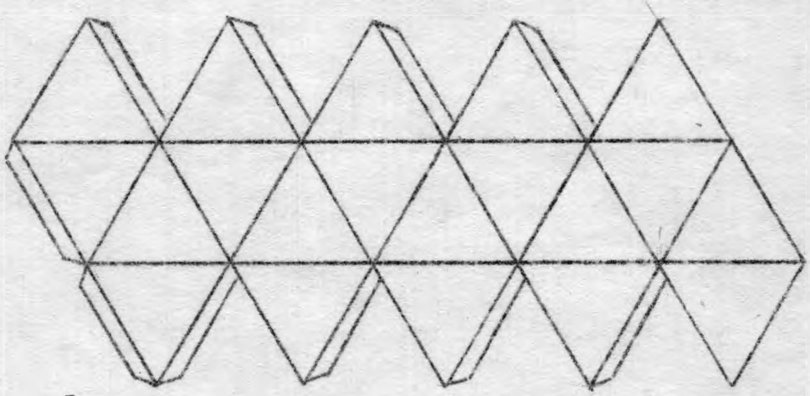


Fig. 2

Octaedron

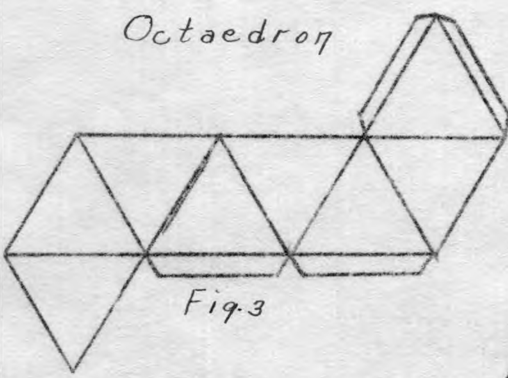


Fig. 3

Hexaedron

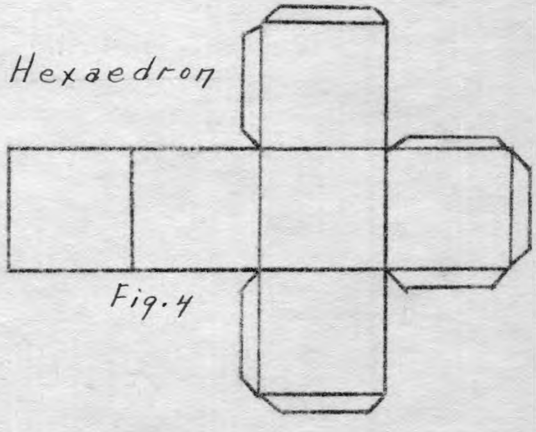


Fig. 4

Tetraedron

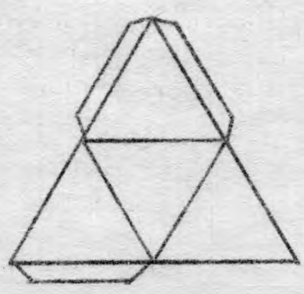


Fig. 5

Plate XV

Regular Polyedrons in Crystal Forms

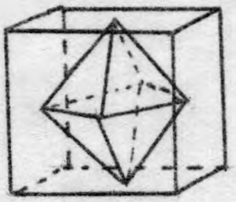


Fig. 1

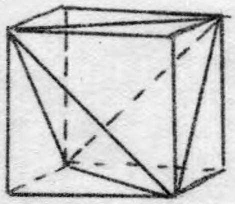


Fig. 2

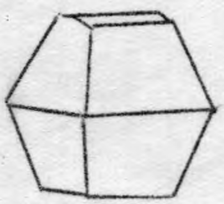


Fig. 3

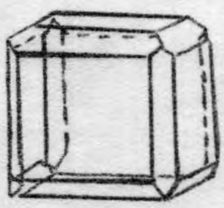


Fig. 4

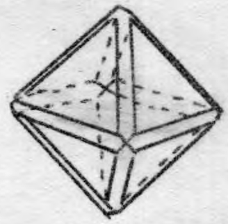


Fig. 5

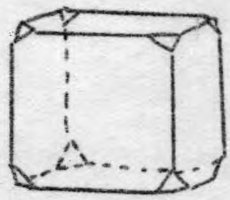


Fig. 6

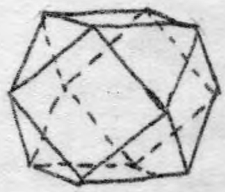


Fig. 7

Fig. 8

charting all of the possible values of area, surface, and volume, with comparisons, of the five regular polyedrons.

Polyedrons are found in nature in the form of crystals. Artificial crystals may be made by modifying the edges and corners of the regular polyedrons. These crystal forms are more difficult of construction but are attractive if carefully made. The student who has designed the patterns and models for the five regular polyedrons may proceed to modify them to obtain some of the results as suggested by Figs. 3, 4, 5, 6, 7, and 8, Plate XV. Beads may also be found to illustrate a number of the various forms of crystals. Other volumes may be made from paper, or modeled in clay or wood. (See Fig. 1, 2, 3, 4, and 5 Plate XVI).

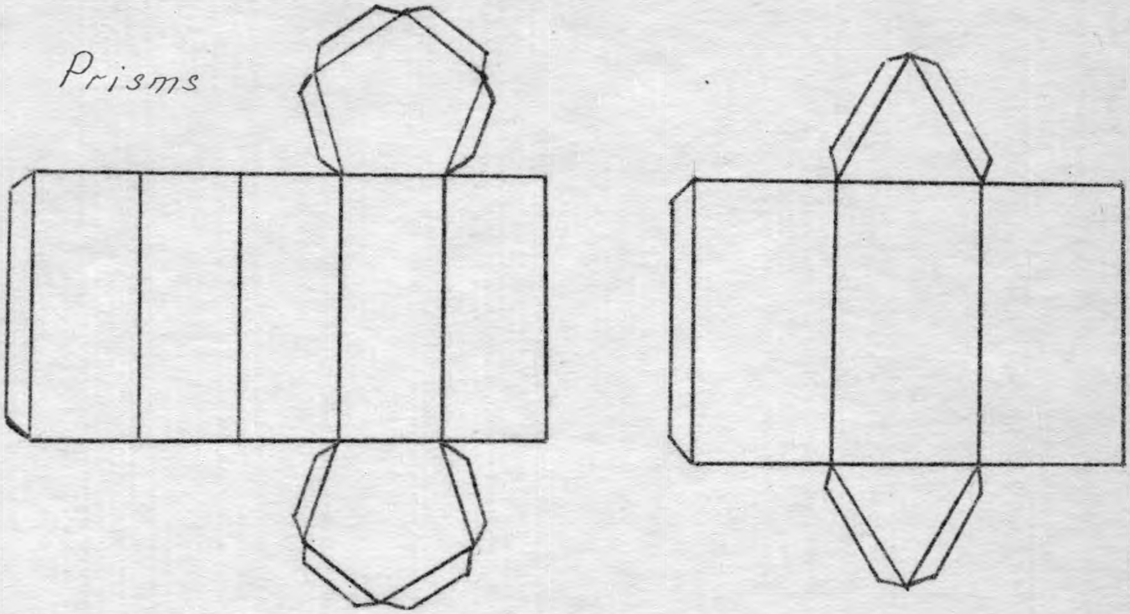
General Volumes: A collection of wooden models to illustrate the basic volume figures would be a valuable possession for any mathematical department. They may be purchased, but a double purpose is served if constructed by students. See Plate XVII for suggestions. Other figures may be added to those illustrated.

Observation reveals areas and volumes in towers, chimneys, pyramids, gas tanks, flower beds, tin containers, road pavements, glass in decorative windows, interior decorations, and many other things. The study of dwellings, as the Eskimo igloo, the Indian tepee, cliff dwellings and the South Sea Island hut affords interesting illustrations of geometric solids. Creative expression may be centered in such projects as laying out a baseball diamond, finding the cost of blackboards, finding the area of floor spaces, computing the air space in the class room, and preparing bills of materials needed for boxes, bins and so forth. A study of excavations for basements about the community, as well as brick

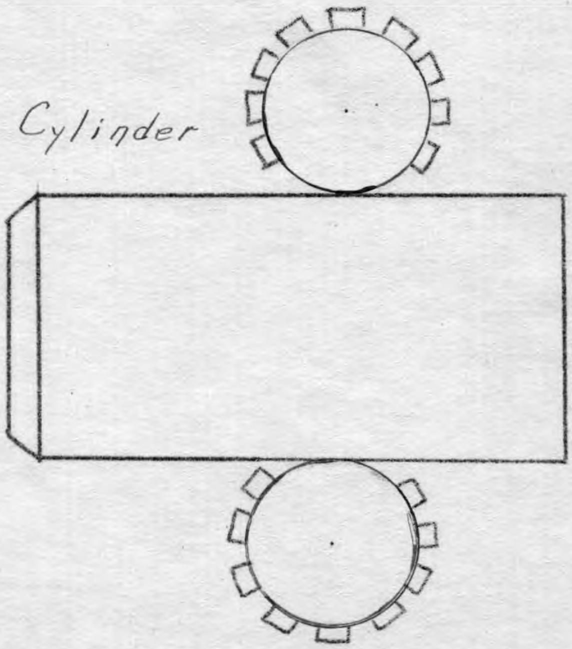
Plate XVI

Construction Patterns for Figures of Volume

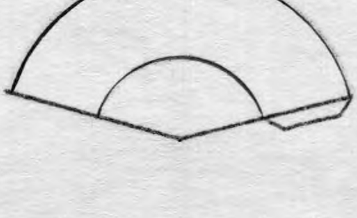
Prisms



Cylinder



Frustrum



Cone

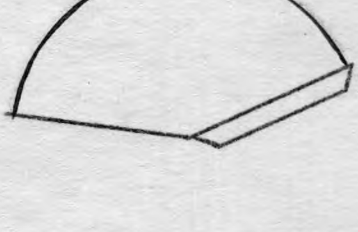


Plate XVII (a)

Figures of Volume

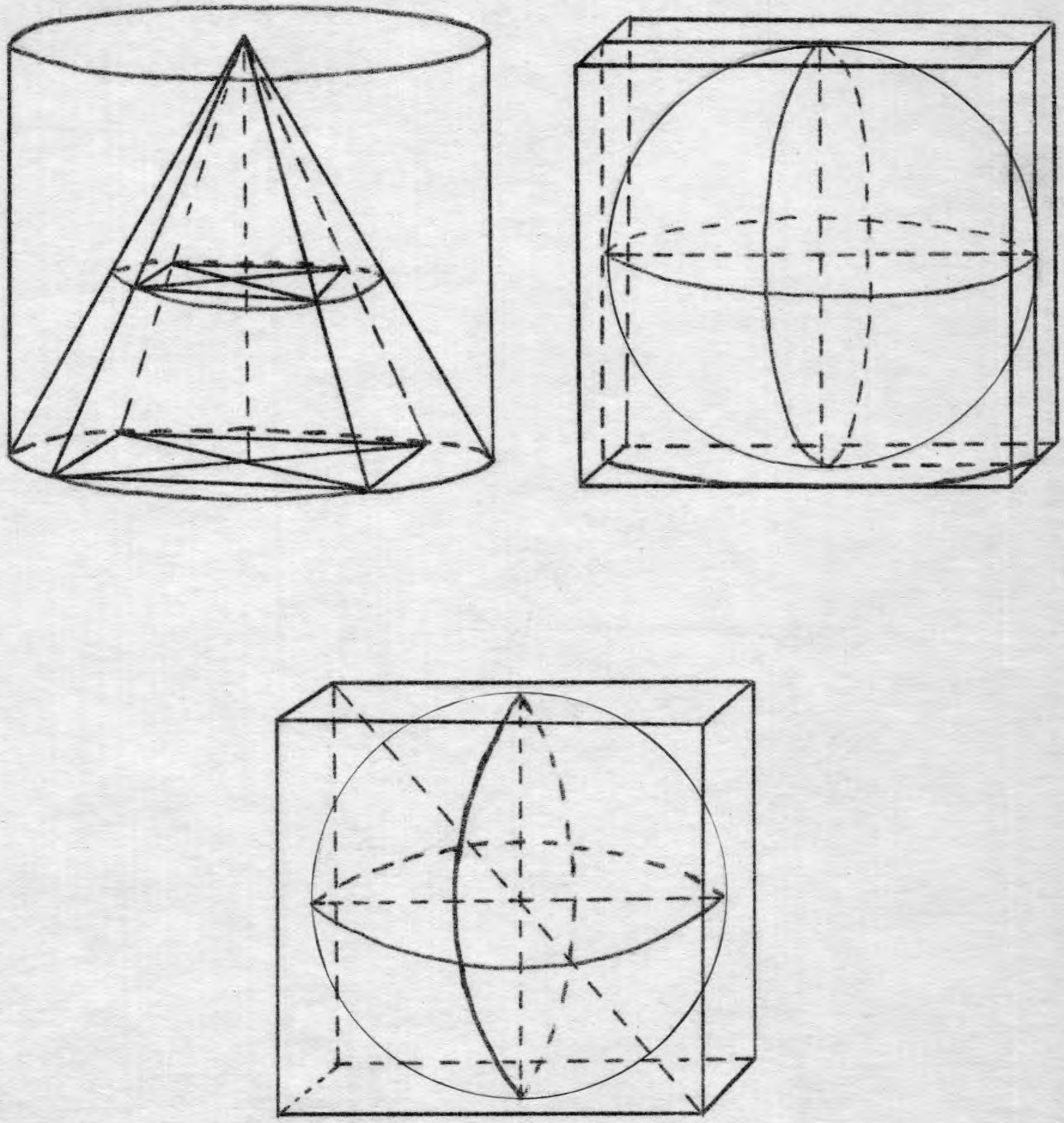
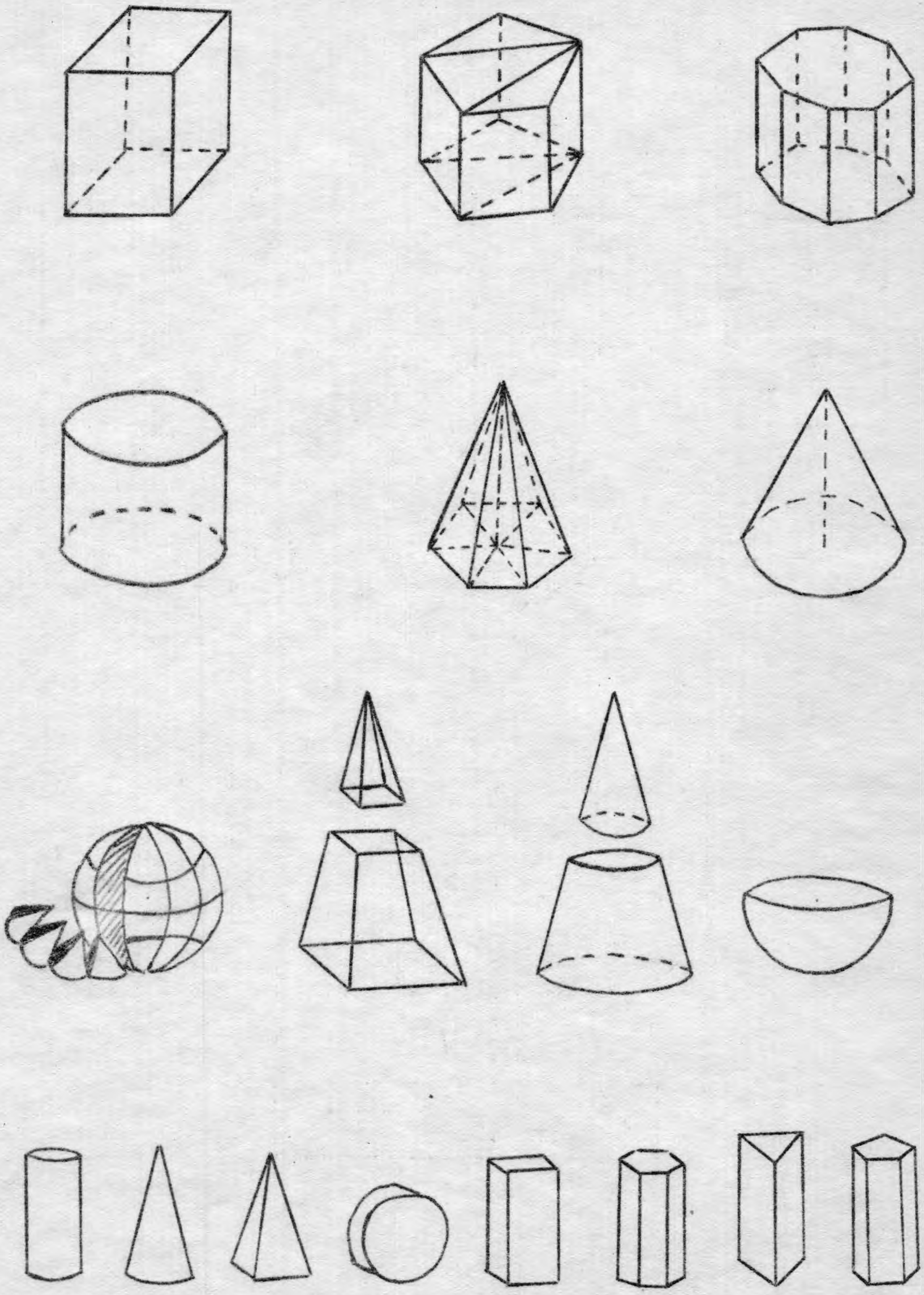


Plate XVII (b)

Figures of Volume



and stone work may also suggest a project to be based upon the principles involved in areas and volumes. As more appealing to girls, a study of the cost of oranges for example in comparison with their sizes may be made and thus decide on the most economical buy. Another project in connection with volumes may be based on the forms found in a honeycomb or hornet's nest. The student may speculate on why the bee and hornet use hexagonal rather than square, circular, or any other shaped bases in the cells of the comb.

General Areas: Figures to illustrate plane areas, may also be constructed of cardboard or wood, or drawings made of them by the student. See Plate XVIII for suggestions of figures which can be thus considered, and others may be added by the student. A chart giving the formulas and principles used in computations of the areas of plane figures may be compiled, and direct applications of them may be made.

Any one of the plane figures may be taken as a basis for more intensive activity and creative expression; as, for example, the triangle. There are many kinds of triangles, and these may be constructed and classified. If in two similar triangles the sides of one are twice the sides of the other, the area of the bigger is equal to 4 times the area of the smaller. This may be shown by constructed blocks. See Fig. 1, Plate XIX. Fig. 2 shows that a triangle is equal in area to one half a parallelogram of the same base and altitude. Fig. 3, in which CD=DO, illustrates another method of forming a parallelogram from a triangle. Areas of triangles may be determined by the application of various formulas which may be listed and illustrated

Plate XVIII

Figures of Area

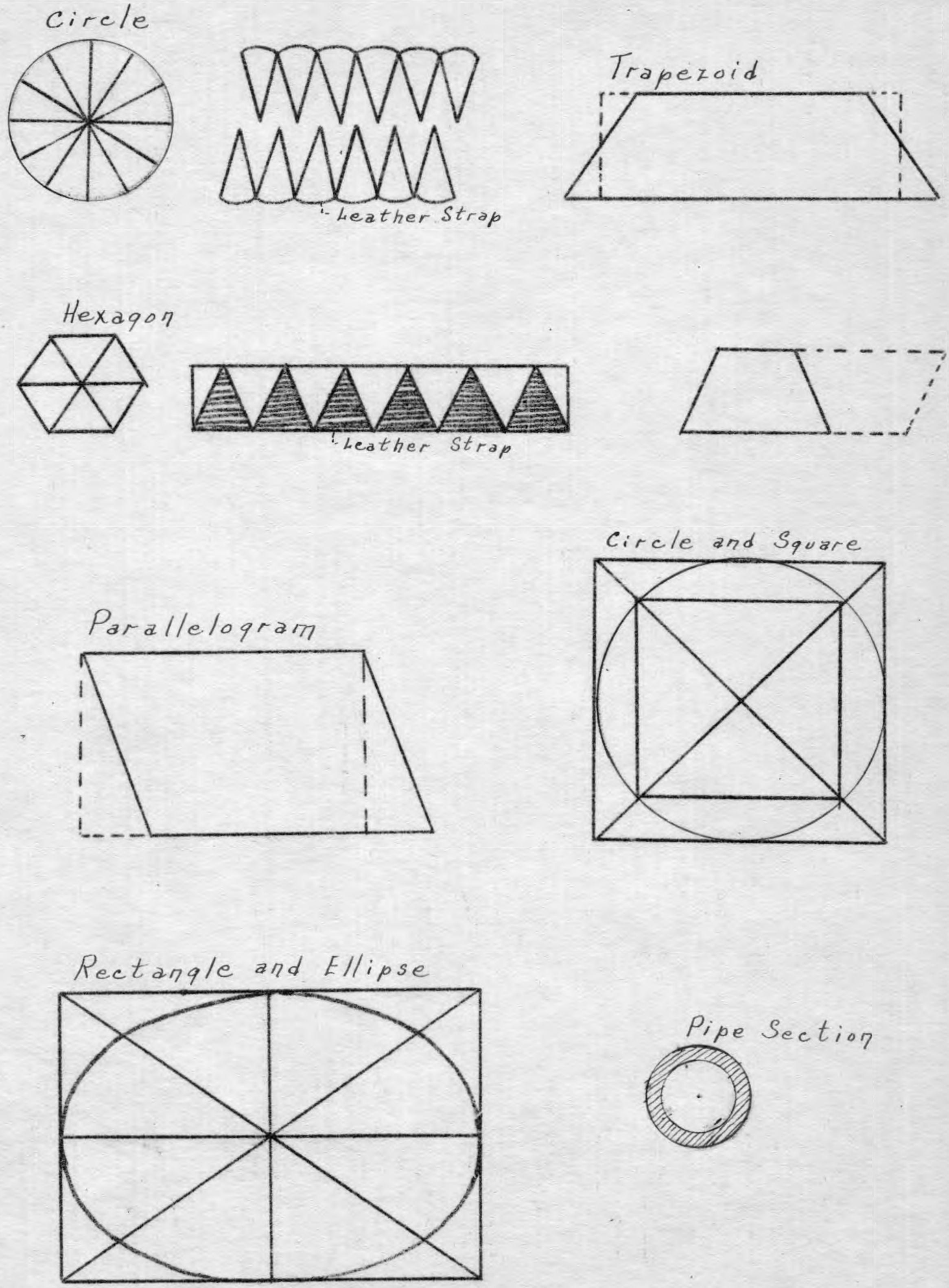


Plate XIX

The Triangle

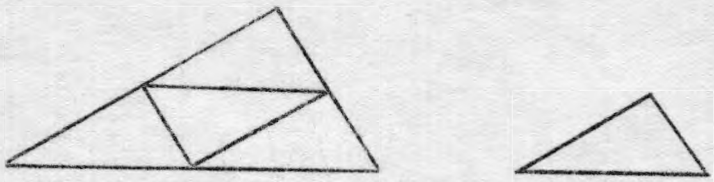


Fig. 1

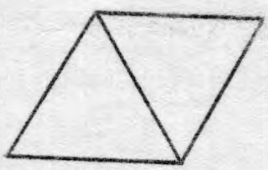


Fig. 2

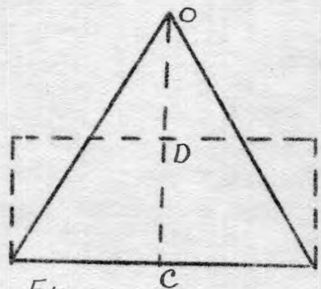


Fig. 3

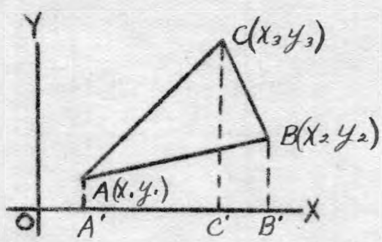


Fig. 4

By Determinants:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Fig. 5

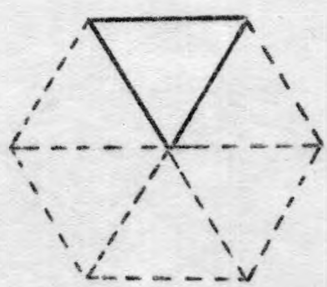


Fig. 6

Triangle in Design
(See Plate XXI)

by the student. Thus, the area of any triangle may be found by the formula, $A=\sqrt{s(s-a)(s-b)(s-c)}$, in which a, b and c represent the sides and $s=\frac{1}{2}(a+b+c)$. The area of a triangle in rectangular coordinates (See Fig. 4, Plate XIX) may be found as follows: Draw $A'A$, BB', and CC' perpendicular to the X-axis. Then, area ABC = Area $A'ACC'$ + area $C'CBB'$ − area $A'ABB'$. This expression may be positive or negative depending upon the order in which the vertices are arranged around the triangle (clockwise or counter-clockwise), but its numerical value is always the area of the triangle. To compute, this may be set up by determinants, as shown on Plate XIX. Areas of triangles depend upon certain relationships; as, the base and altitude, two sides and the included angle, three sides, or as a function of the radius of the inscribed or circumscribed circle. These relationships may be illustrated by the student in diagrams or graphs as an added expression of creative activity in the study of triangles.

The principle involved in the Pythagorean theorem offers many possibilities for creative expression in its application to areas. Fig. 1, 2, 3, and 4, Plate XX show applications which may be illustrated by diagram or construction.[1] To add to generalizations, changes may be made in the figure to show relationships; i.e., keep two sides fixed and change the included angle from acute, to right, to obtuse. The square on the side opposite the changing angle is at first less, then equal to, and then greater than the sum of the squares on the other two sides. (See Fig. 5)

1-Rupert, C. E.: Famous Geometrical Theorems and Problems, Chicago, D. C. Heath and Company 1900

Plate XX

Figures to Illustrate the Pythagorean Theorem

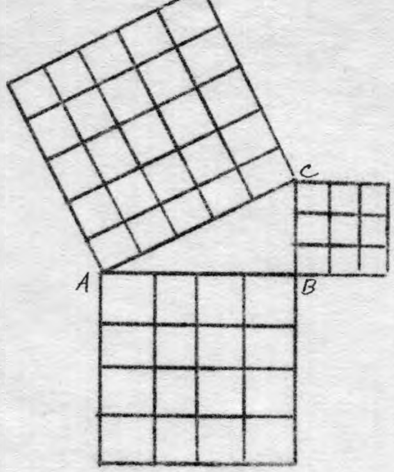


Fig. 1

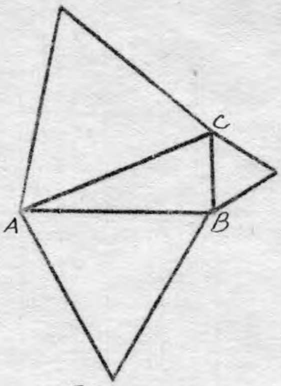


Fig. 2

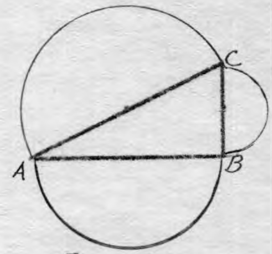


Fig. 3

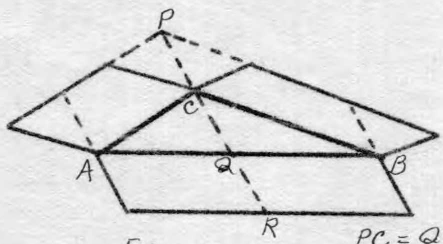


Fig. 5

PC = QR
Applied to any triangle

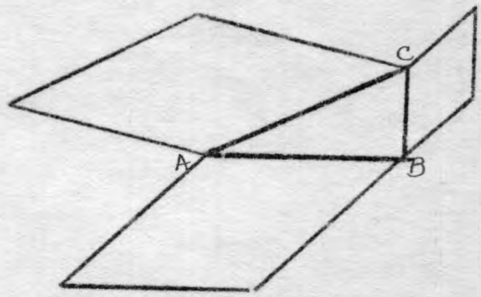


Fig. 4

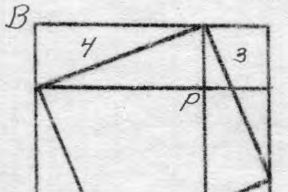


Fig. 6

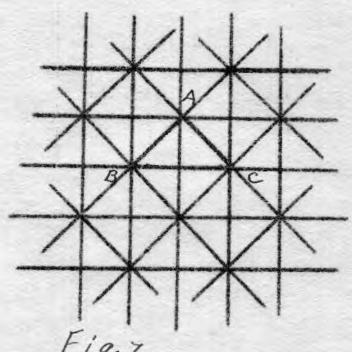


Fig. 7


Fig. 8

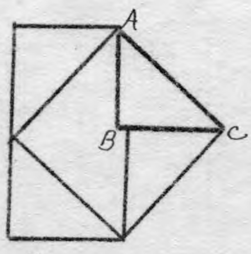


Fig. 9

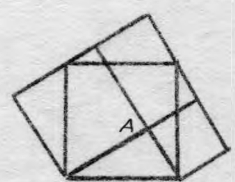


Fig. 10

If in Fig. 6, the triangles 1, 2, 3, & 4 are taken away, the square on the hypotenuse of a right-angled triangle remains; and if the two rectangles AP and PB are taken away from the whole figure, the sum of the squares on the two sides of the triangle remains, but the four triangles all together equal the two rectangles. This may be constructed from blocks to illustrate the Pythagorean theorem.

The Pythagorean theorem as observed in tiles illustrated by Fig. 7. This may be suggestive of various types of creative expression.

Figs. 8, 9, and 10, suggest further block constructions which may be used for illustration. These do not exhaust the possibilities, and the student may wish to add other proofs and applications to these suggested.

By the Pythagorean Theorem distances between two points in rectangular coordinates may be found. For example, the distance between points $x_1 y_1$ and $x_2 y_2$ is equal to the square root of the square of (x_2-x_1) plus the square of (y_2-l_1). The student may set up many concrete examples of this application.

Chapter IX

Mathematical Principles Involved in Design are Conducive to Creative Activities.

<u>The Regular Polygon in Design</u>: Although the properties of the regular polygons have been considered to some extent for construction activities, they are again considered here in relation to design. Forms of design suggested by the possibilities of the regular polygon have been in use since ancient times. Illustrations of such applications may be found in ancient mosaic patterns, Egyptian tapestries and many more modern productions.

Mosaic pavements are constructed upon the principle that all surface about a point in a plane is filled completely by 6 equilateral triangles, 4 squares, or 3 regular hexagons. (See Figs. 1, 2, and 3, Plate XXI). Figs. 4 and 5 are illustrations of the application of that principle. They are suggestive of the historical interest in mosaic pavements, about which projects for student activity may be centered in connection with the uses made of mosaic pavements in both ancient and modern times.

Figs. 6-11 inclusive, Plate XXI, are suggestive of designs which use the regular polygons. Others original designs may be created by the student according to mathematical principle. These may be used for various decorative purposes.

More intensive activity may be centered around any one of the regular polygons. Figs. 1, 2, and 3 Plate XXII, show some of the possibilities of the properties of the square. Fig. 13 illustrates how a regular pentagon may be constructed which is equal in area to a given square. Thus, construct any regular

Plate XXI

Regular Polygons in Design

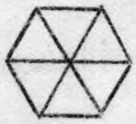


Fig. 1

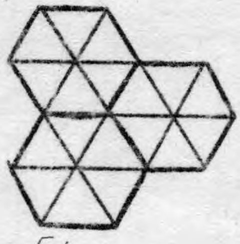


Fig. 3

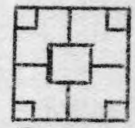


Fig. 2

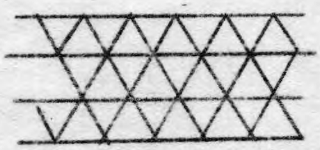


Fig. 4 Roman Mosaic

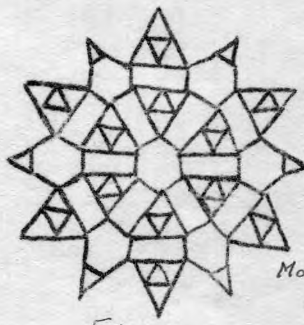


Fig. 5 Mosaic from Byzantine

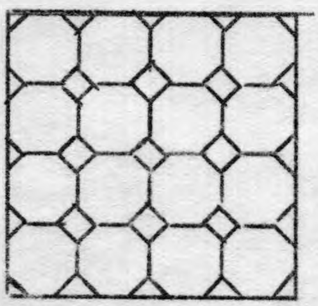


Fig. 6

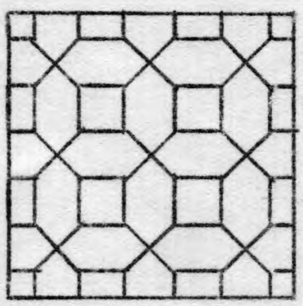


Fig. 7

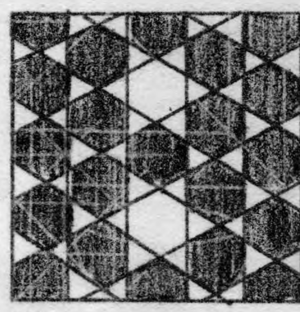


Fig. 8

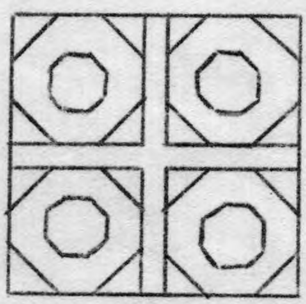


Fig. 9

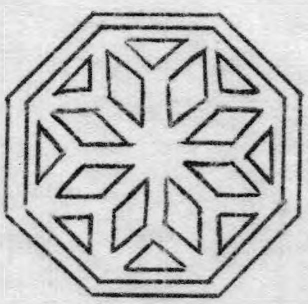


Fig. 10

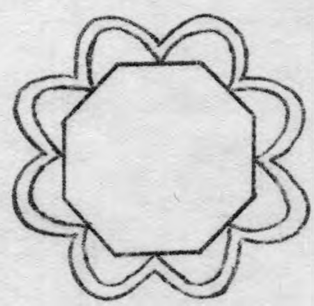


Fig. 11

Plate XXII

Properties of the Square

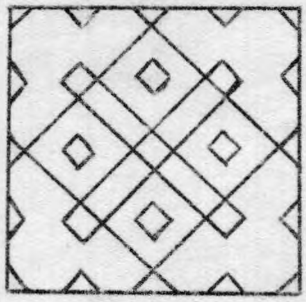
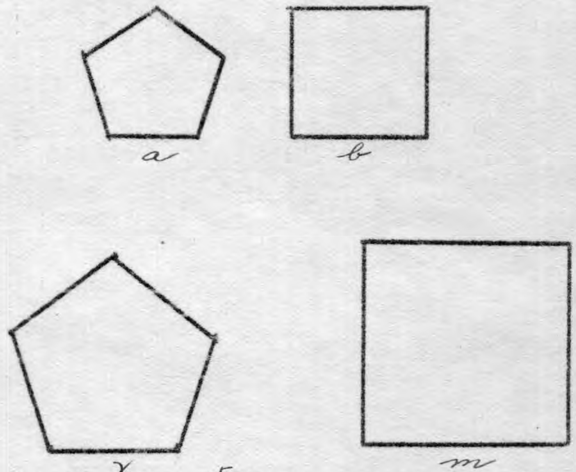


Fig. 1

See Plate XXI



a b

x Fig. 2 m

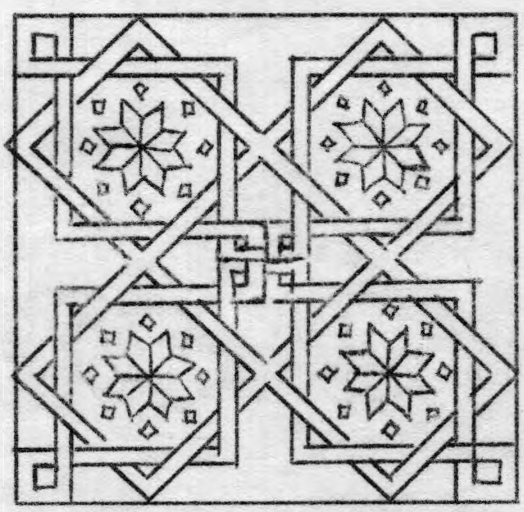


Fig. 3

pentagon of side (a). The square of side (b) is now constructed equal to the pentagon in area. If (m) is the side of the given square, $b^2 : m^2 = a^2 : x^2$, or $b:m = a:x$. Since x is the fourth proportional to b, m, and a, it may now be found and the desired regular pentagon constructed.

<u>The Circle in Design</u>: Design may be based also almost solely upon the properties of a circle. A number of suggestions follow: The results which may be obtained from the circle in design are many and beautiful, and the architect ingeniously weaves them into the construction decorations of his edifices. The beautiful stained glass windows, cornices, engraven wood work finishings, the exquisite arrangement of the visible pipes of the organ, as well as the latice work so often used to cleverly conceal less desired effects, are examples of such works of art. In reality such results must first be constructed in miniature design by the skillful application of mathematical principles. Projects may be worked out in bristol board or construction paper cut-outs, with colored tissue paper backgrounds. Practical applications may be made in constructing lamp shades, waste paper baskets and other decorative pieces. Suggestions for design are found on Plate XXIII.

Figs. 10 and 11 not only illustrate suggestions for design, but they also show the mensuration of the circle. Thus in Fig. 10 the area is divided into 3 equal portions and in Fig. 11, into 4 equal portions by the use of other circles.

<u>Mathematical Principles from Design in Nature</u>: Nature is an unending source of suggestion for design. The student may collect seeds, seed pods, leaves, and flowers, and examine them to find

Plate XXIII

The Circle in Design

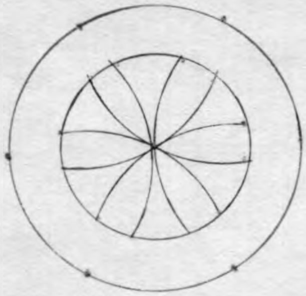


Fig. 2

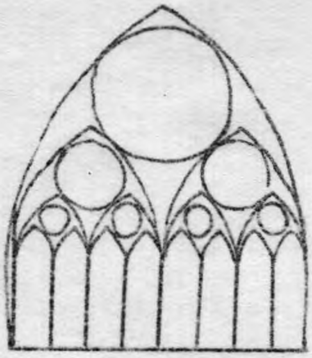


Fig. 1

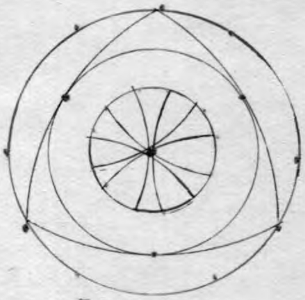


Fig. 3

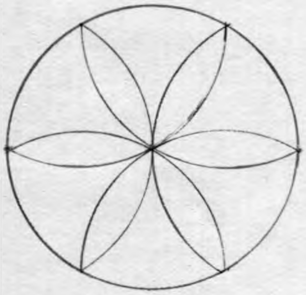


Fig. 4

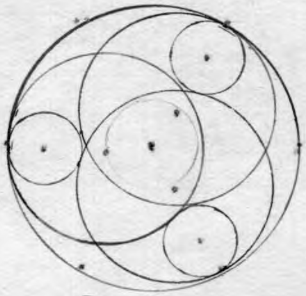


Fig. 5

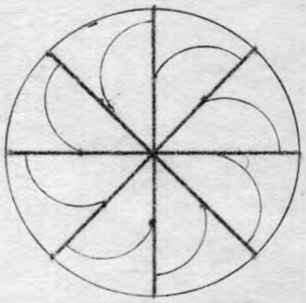


Fig. 6

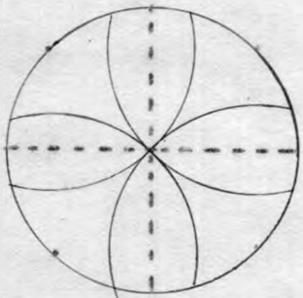


Fig. 7

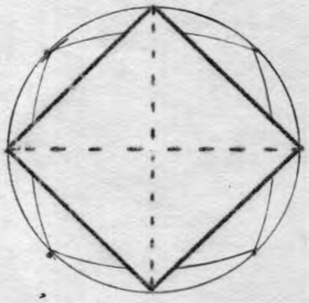


Fig. 8

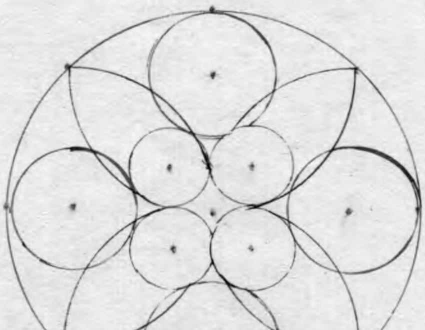


Fig. 9

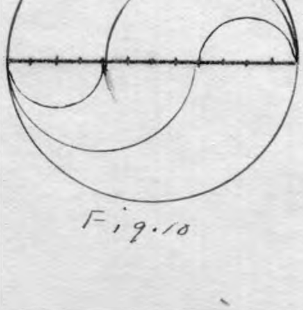


Fig. 10

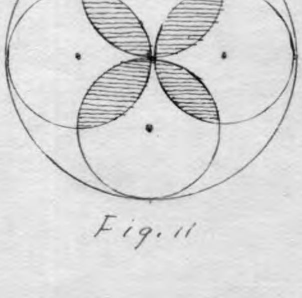


Fig. 11

artistic designs. The beauty of geometric figure found there is almost unbelievable except to him who is explorative and observing.

Field trips will make possible the observation of things incapable of being brought into the room. Not only with plant life are such relations found, but geometry may be correlated with animal biology as well. The star fish is but one example of the possibilities there.

To reap the fullest benefits from such observations, the keeping of notebooks or scrap books of results should be encouraged. It is a means of bringing together as a unit the various forms of activity which may be centered about a single project. Not only drawings, but examples, specimens, graphs, and other creative expressions may be thus preserved.

Illustrations from Nature and Art: "God geometrizes continually" was Plato's reply when questioned as to the occupation of the Deity. In no way is this conviction more firmly established than when a little thought is given to art in nature. Geometric lines and forms of almost every kind are to be found in nature for it was from nature that man conceived his ideas for such. The Fibonacci series, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc., may be found in the arrangement of the leaves about the stalks of a plant. This series is the ratio of the "whirling square" which is constructed as follows: Draw a square ABCD; from O, the mid point of AB draw OC; with O as the center and OC as radius, draw the arc CE meeting AB produced at E; On AD and AE complete the rectangle AEFD, which is a whirling square rectangle whose width is 1 and whose length is 1.618. The butterfly also fits into a whirling square design as well as into a root-five rectangle which is discussed in the following paragraph.

Root-rectangles are a basic factor in art, because of their pleasing effect on the eye. Again man follows after nature, for nature uses the root-rectangle plan. For example the dragon fly and the iris are built on the root-three rectangle pattern, and as has already been stated the butterfly is built on a root-five rectangle. Root-rectangles are constructed as follows: The side of a square and its diagonal are the dimensions of a root-two rectangle; the side of a square and the diagonal of the root-two rectangle are the dimensions of a root-three rectangle; the process is thus continued for other root rectangles, the root-four rectangle being a double square.

The seed pod of the iris does not show dynamic symmetry as does the flower, but rather static symmetry, for it is a regular equilateral triangle with a smaller one within.

These are given as bare suggestions of the fine illustrations which nature offers for mathematical designs. Two courses of activity are open for the student: he may merely observe and collect designs from nature, or he may use such observed designs in productions of his own.

Coloring Designs: Not more than four colors are necessary in coloring a map or design in such a way that no two contiguous sections shall be of the same color. In Fig. 1, Plate XXIV, there is no possible way of drawing another section Y which would be contiguous with A, B, C or X. Any one of the sections may diminish and disappear with out spoiling the color effect. This may

Plate XXIV

The Four Color Theorem

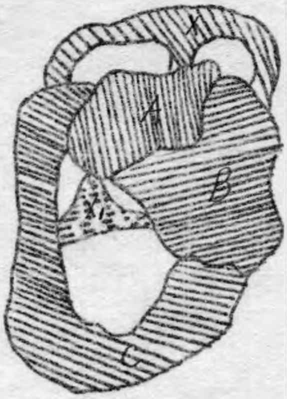
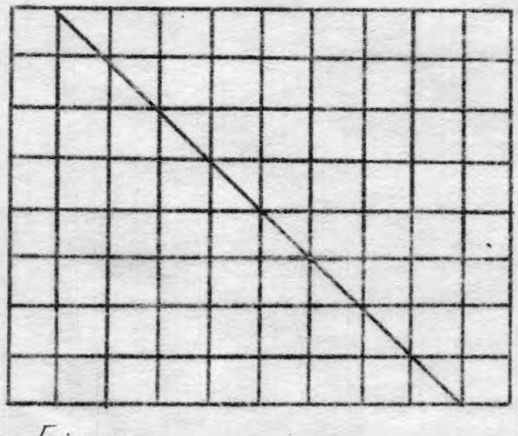


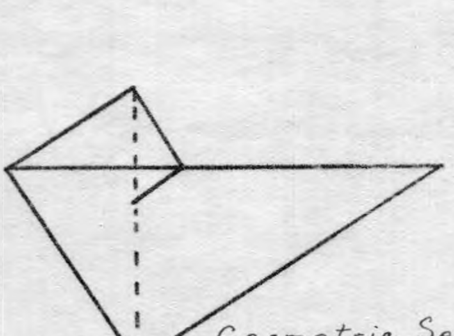
Fig. 1

Paper Folding



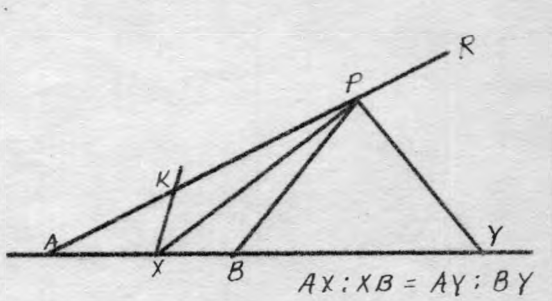
Arithmetic Series

Fig. 2



Geometric Series

Fig. 3



AX : XB = AY : BY

Harmonic Series

Fig. 4

be verified and illustrated by the student.

Paper-folding: Paper-folding offers many opportunities for creative expression.[1] Plane figures which may be easily folded are the square, equilateral triangle, rectangle, pentagon, hexagon, octagon, and nonagon. The decagon and dodecagon are obtainable from the pentagon and hexagon respectively by first obtaining the angles at the center, and the pentedecagon is obtainable from the pentagon. The figures of the conic sections; circle, parabola, ellipse, and hyperbola, may also be illustrated by paper-folding processes.

Fig. 2, Plate XXIV shows how the arithmetic series may be illustrated by paper-folding. The horizontal lines to the left of the diagonal including the upper and lower edges, form an arithmetic series. The initial line being, a, and the common difference, d, the series is: a, a+d, a+2d, etc.

Fig. 3 represents a geometric series, for in a right-angled triangle, the perpendicular from the vertex on the hypotenuse is a geometric mean between the segments of the hypotenuse.

A harmonic series is represented in Fig. 4. Fold any lines AR and PB, P being on AR, and B on the edge of the paper. Fold again so that AP and PR may both coincide with PB. PX and PY are the creases thus obtained. The points A, X, B, and Y form a harmonic range, and any points obtained by the intersection of any other line cutting PA, PX, PB, and PY will also form a harmonic range.

1-Row, Sundara: Geometric Exercises in Paper Folding. Chicago--London: Open Court Publishing Company. 1901

Because of the principle of congruence which is involved in paper-folding, many of the mathematical processes may be thus illustrated. Finite lines, bisected and trisected, rectilineal angles, bisected and divided into other equal parts, and perpendiculars drawn to given lines, are illustrative of such processes.

Chapter X

Mathematical Activities in Occupations.

Introduction: There is practically no occupation of man which does not involve some form of mathematics. It is necessary for progress of any kind in scientific prediction, discovery, and invention. Descriptive sciences need intricate mathematics to compute electrical energy, or speed and balancing forces of any kind. Social sciences need mathematics in coping with community problems and statistics of all kinds. Mathematics is involved in all business affairs. Engineering work, as the design and construction of structures and machines of every kind, is governed by mathematics. Medical science, and basic physics and chemistry involve mathematics. The student may find many specific examples of the use of mathematics in many fields of activity. As suggestive of such applications several fields of human activity are considered in this section.

Architecture and Landscaping: Architecture is based on the practical application of mathematical principles. The blue print is the center of the architect's activities. The student may obtain copies of blue prints and learn to read and interpret them. He may also draw plans for himself using architectural care and accuracy. Plans for a yard, a garden, a house, and many other things may be easily obtained or observed. Current literature contains a wealth of material of which collections may be made. As a further suggestion for motivated activity, the student may make collections of pictures and plans of famous buildings, and of many types of bridges to illustrate different uses of geometric design, and thus formulate the idea of mathematical principles involved in planning constructions.

Surveying: Surveying offers many opportunities for student activities. Instruments used in surveying have been discussed in Chapter VI.

There are a number of methods of determining distances by congruent and similar triangles. A few suggestions follow which may be applied by the student to specific cases. Fig. 1, Plate XXV, illustrates a method of determining the height of a tree, building, or flagpole. Fig. 2 illustrates a method of determining the distance between two inaccessible points. Fig. 3 shows how to determine the width of a river without crossing it, by using an isosceles triangle and a sighting point A, while Fig. 4 determines width by marking off BM perpendicular to AB and erecting MR perpendicular to BM. Fig. 5 illustrates how the location of a distant object may be determined, by using a surveyor's transit in constructing angle C'AB equal to angle CAB and angle C'BA equal to angle CBA.

Mechanics: Mechanics offers many opportunities for creative and constructive activities. The inclined plane, the lever, the pendulum, and the pulley furnish concrete illustrations and require little apparatus to show the mathematical principles involved. Simple examples may be found from text books in mathematics or science which may be used as suggestions in constructive work.

The center of gravity for a circle, a triangle, a square or any other plane surface may be illustrated by cardboard figures; while for a cone and other figures of volume, experiments may be

Plate XXV

Methods of Determining Unknown Distances

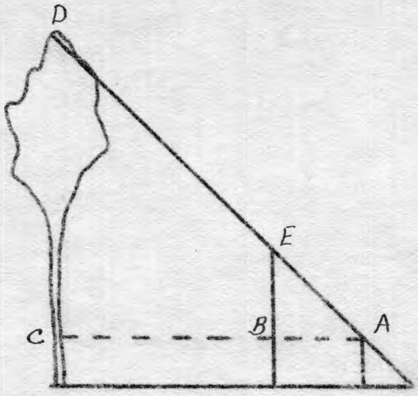


Fig. 1 To Determine Height

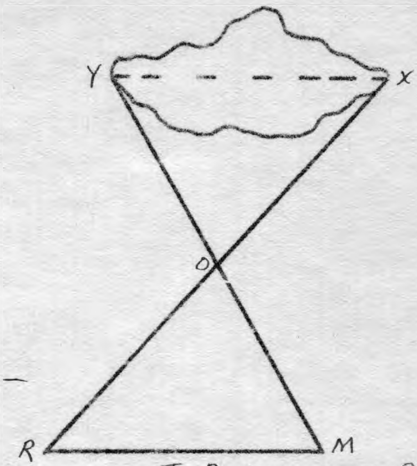


Fig. 2 To Determine Distance

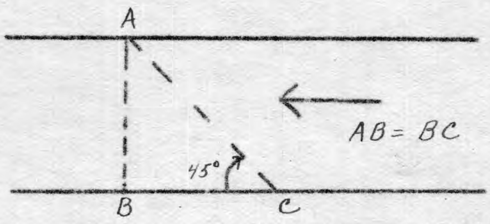


Fig. 3 To Determine Width

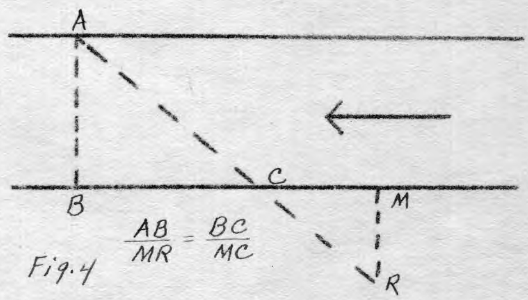
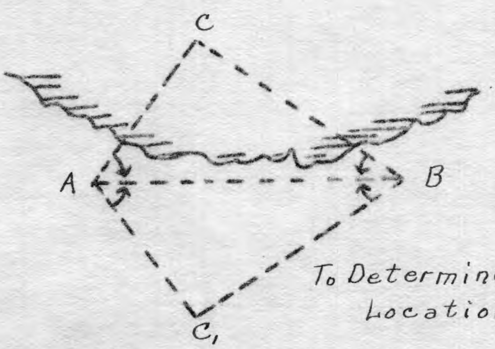
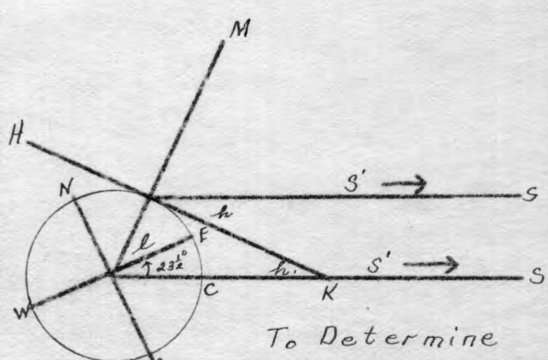


Fig. 4



To Determine Location

Fig. 5



To Determine Latitude

Fig. 6

made to find the incline at which the body may be placed before equilibrium is destroyed. The Leaning Tower of Pisa is an example taking a position in which equilibrium will be destroyed if the incline is increased. Projects in this connection will illustrate the principles involved in finding the center of gravity.

Transportation and Communication: Transportation and communication offer excellent examples of the applications of mathematics. The highway engineer, the electrical engineer, the radio engineer, and the aviator must be familiar with practical applications of mathematics. If magazines of current literature on these subjects are available, instances of mathematical applications may be found. Curves in road building, meters, and other measuring devices are suggestions of some of the applications which may be found.

Music: In music, pitch, time, and the scale, are examples of concepts involving the applications of mathematical principles. Early mathematicians spent a great deal of time with the study of harmonics. A project for student activity should include the historical background, as, for example, Bach's "well-tempered scale," in which the octave is divided into 12 tones with a constant ratio of vibration from one tone to the next, leading to a geometric progression of the ratio 2. This may be compared to the divisions of the various scales used now. Other activities may involve experimenting with a vibrating string to determine the mathematical relationships existing between the harmonics or overtones of a given tone. If tuning forks are available they offer a simple means of verifying the pulse of vibrations. Evaluating notes and rests in written music is a worth while activity if the student is not already familiar with the mathematical divisions of musical notation.

Literature: The importance of mathematics from the viewpoint of language may be suggested by the following quotation: "Mathematics is thinking God's thought after Him. When anything is understood, it is found to be susceptible of mathematical statement. The vocabulary of mathematics is the ultimate vocabulary of the material universe."---White.

There is no better training for accurate verbal self-expression than that provided by theorems in geometry. Clear, definite, and precise in wording and phrasing, they become almost musical in effect. A mere repetition of theorems is not necessary for there are numberless isolated unwritten theorems which may be expressed by the student. These may not be of supreme importance, but they will be of great beauty if carefully worded.

Literature is full of mathematical quotations and references, often notable for their precision and exactness of statement. A collection of some of these suggests an activity for the student which will lend interest to his formal mathematical study.

Astronomy: In astronomy a project may be centered upon the divisions of time. The following suggestions are given as to some of the things which may be considered.

Time is designated by such significant terms as, solar time, sidereal time, equator time, or a sidereal day, and astronomical day, and others. The student may find it of interest to determine the sources of such terms and compare them as to their actual length.

A perpetual calendar may be constructed by the student. This may be a simple arrangement covering a short period of time, or it may be more involved including centuries.

Time signals givenover the radio are sent out in methodical order. Information in regard to when they are given, why they are given, and so forth, may be found by the student to lend interest in this study.

Latitude is determined by aid of the sextant, the transit, or the astrolabe, as have been discussed in Chapter VI. The student may use the principle involved in the following example in determining the latitude in which he lives. (See Fig. 6, Plate XXV). SS' represents the position of the earth on December 21, with relation to the sun's rays. NS represents the earths axis, and EE the equator. An observer at O sees the sun along OS'S, and its angle of elevation above his horizon is h or SOK. An observer at C, 23½ south of the equator, sees the sun at noon along CS'S, or in his zenith, at the same instant that the observer at O sees it h degrees above his horizon. It is necessary to show that angle SOK is equal to angle CKE, or h equals h'. If l is the latitude of the observer at O, 23½ +l+h = 90 degrees; l then equals 66½ minus h. Therefore an observation taken at noon which will determine the angle of elevation h of the sun above the horizon, will enable one to find his latitude.

A project may be centered on the relative distances and sizes of the planets. After finding the necessary values for these, they may be represented by articles ranging in size from a small pea to an eleven inch globe. The distances, ranging on a small scale from inches to rods, may be illustrated, as an out of doors project, by given locations about the school yard.

The following problem will make more impressive the vast distances involved. It may be used as a suggestion for other problems which the student may create:

> Altair, which is our nearest star is 15.5 light years away. A light year is 365.25 days in length. What would be the railroad fare from the earth to Altair at 25 cents for a 24 hour ride? Knowing how fast light travels the distance to Altair may be computed. What would the fare be at 2 cents a mile? If the train goes at 50 miles an hour it would take 50,000,000 years to get to Altair. From this the distance may be computed, also.

Medicine: Mathematics is found in the science of medicine. For the boy who fails to take an interest in mathematics because he intends to be a doctor, creative activity might take the form of going to the drug store to see about having one or more prescriptions filled. He will find that mathematics forms a very necessary foundation for his profession.

Organic growth and decay are now computed by the formulas $y = ae^{kt}$ and $y = ae^{-kt}$, respectively. $(a, k > o)$. By means of the planimeter (See Chapter VIII) the area of a wound is determined and according to the above formula the normal rate of healing is computed. In this connection an article on war wounds may be of interest to the student.[1]

The study of optics, or converging and diverging lens, may be considered here, also, for student activity, as the correction of vision depends upon it. Apart from the field of medicine, the

1-Tuffier, Theodore, and Desmarres, R.: A Note on the Progress of Cicatrization of War Wounds. Journal of Experimental Medicine Vol. XXVII (February 1, 1918) p. 165-178.

student may wish to extend this study into the application to automobile headlights and other specific uses of similar nature.

Social Sciences, Biology, and Economics: Facts are determined by the application of mathematical principles which are of importance in the social sciences. The student may find specific examples from facts drawn from present social conditions: the social status of the country, international relations, immigration, and election returns, are suggestions of topics about which statistical reports may be obtained or compiled by the student and used in various mathematical activities.

Biology also offers many sources of material which lends itself to mathematical treatment; for example, the relation between the age and height, or weight of children. A project based on a chart of that kind will furnish interesting activity for the student.

Economics is filled with concepts based on differential calculus. Suggestions of applications which may be found in current literature might include, the mortality curve, correlations, probability, theory of interest, rate of price change, marginal utility, and statistical values. As a more specific application of what may be included: it has been determined that the sufficiency of a given street car fare, the price of a sandwich, or a pair of shoes involves the formula for the expansion of the binomial theorem. It is applications of such types that may be found by the student.

In Business: Mathematics, as it affects the business world, includes many phases of activity about which projects may be centered.

Banking, insurance, taxation, and investment involve a series of business forms which simplify such proceedings. A collection of blank forms would be a valuable aid to becoming familiar with their uses. These may be obtained at a bank or from county or state officials.

Newspaper clippings of stock market reports, bond issues, tax receipts and expenditures, and international finance proceed- serve to broaden the student's familiarity and ability to appreciate such business relations. Such clippings also provide material for innumerable projects in graphical representation of facts.

School banking gives opportunity for many activities in which the student may participate. Considerable ingenuity may be shown by the students in carrying on such a project. Officers may be chosen to serve a given length of time, bank furniture, such as the cashier's window, may be constructed, and rules and regulations may be established which will lend reality to the activity in general. Artificial situations may be created by the students to make it possible to become familiar with numerous forms of business papers, as: promissory notes, checks, bank drafts, bills, receipts, money orders, cash slips, inventory blanks, pay-rolls, and many others.

A project on insurance may follow suggestions similiar to those given for school banking. Policy blanks of the various kinds of insurance issued may be collected by the students to serve as a basis for further activities.

Investments and finance include stock market reports, liberty bonds, building bonds, amortization and borrowing, the kinds of investments and the various ways of paying obligations. These again suggest various opportunities for student exploration and activity.

Computation of taxes, clippings on tax spenditure, comparison of taxes per capita in different countries, as well as the kinds of taxes, are further suggestions for student activities.

Suggestions for projects in connection with interest and discount include the study of advantages and disadvantages of installment plans and comparison by graphs of the simple and compound interest values on a given sum of money at a given rate. The various methods of computing interest include the formula, interest tables, and graphs. Compound interest may be computed by the binomial theorem as an easy solution as follows: $A = P(1+i)^n$, A being the compound amount and $(1+i)^n$ the accumulation factor. The compound amount of $500 for 5 periods at 2%, a typical savings bank rate, is:

$$A = 500(1.02)^5$$

$$(1+i)^5 = 1+5i+10i^2+10i^3+5i^4+i^5$$

$$(1.02)^5 = 1.1040808032$$

Therefore, $A = \$500 \times 1.1040808032 = \$552.04$

A project may be based on a comparison of methods used for computing interest.

Chapter XI

Mathematics for Leisure and Recreation.

Introduction: Mathematics lends itself to leisure and recreation as well as to scientific thought and calculations. Many mathematical activities have indirect value in that they tend to open the student's horizons by stimulating the imagination through providing interest and amusement, and by emphasizing modes of thinking which can be applied to the affairs of life. Some of these forms of entertainment have had their origin in ancient times. Daily newspapers, almanacs, and advertising pamphlets often contain similar mathematical "brain teasers." Through these and other sources, the student may add to the list of suggestions given herein, and produce a scrap book of interesting material for leisure and recreation. Such material may be used by a Mathematics Club for entertainment, or it may suggest various types of activity to be enlarged upon for different purposes.

Fallicies and Illusions: Many fallicies and illusions have been drawn from mathematical principles. By their absurdity the true conception is sometimes made more impressive. For example, it may be "proved" that any number is equal to any other number: Let, $c = d$. Then, $ac = ad$ and $bc = bd$, a and b being any two given numbers. By subtracting: $ac-bc = ad-bd$. By transposing: $ac-ad = bc-bd$ or $a(c-d) = b(c-d)$. Therefore $a = b$. Since this cannot be, the impropriety of dividing by zero is at once apparent.

To "prove" that every triangle is isosceles, take any triangle ABC. See Fig. 1, Plate XXVI. DE is a perpendicular bisector of AB, and CE is a bisector of the angle C meeting DE at E. Draw EA and EB. Draw EG perpendicular to AC and EF perpendicular to BC. Then triangle ADE is similar and equal to triangle BDE, and AE equals BE. In the same way it is shown that CG equals CF and CG equals CF. This being true, CA equals CB and triangle ABC is isosceles.

It may be "shown" that 64 is equal to 65. Figs. 2, 3, and 4, Plate XXVI, illustrate this fallacy.

Plate XXVII illustrates other figures that appear to be what they are not. The observation and reproduction of such figures should impress the student with the necessity of geometric proofs. In Fig. 1, AB and XY are equal in length, but appear unequal. In Fig. 2, AB equals CD. In Fig. 3, AB and CD are parallel. ABCD and WXYZ, in Figs. 4 and 5, lie on a straight line. In Fig. 6, it is difficult to determine the path of AB beneath the obstruction. AB and CD, in Fig. 7, are straight lines, but appear curved. In Fig. 8, the circle appears flattened at Q, P, R, and S, and in Fig. 9, BD appears a greater distance than AC.

Rowed Constructions: The included figures constructed from rows are suggestive of creative activity. Concrete applications may be made, or constructions made merely in design for entertainment. Fig. 1, Plate XXVIII, shows how 16 trees may be planted in 12 straight rows with 4 trees in every row. Fig. 2 shows how 16 trees may be planted in 15 straight rows with 4 trees in every row. In Fig. 3, 19 trees are planted in 9 rows with 5 trees to a row. Fig. 4 illustrates the arrangement of 12 coins. They are to be moved so as to have 5 on a side instead of 4, and

Plate XXVI

Deceptive Proofs

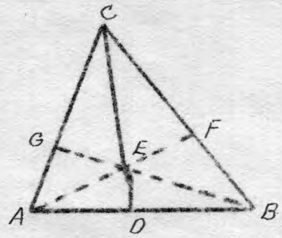
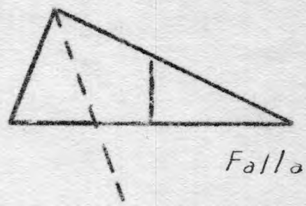
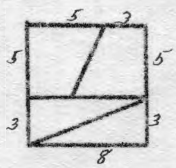


Fig. 1



Fallacy



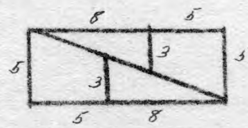


Fig. 2 (a)

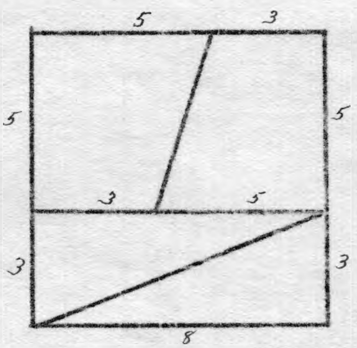
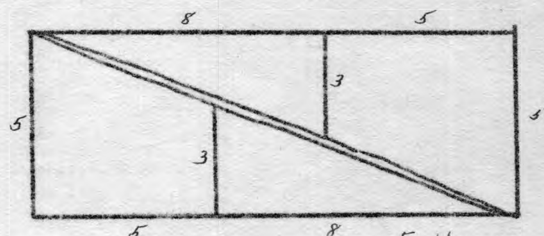


Fig. 2 (b)



Fallacy

Plate XXVII

Deceptive Figures

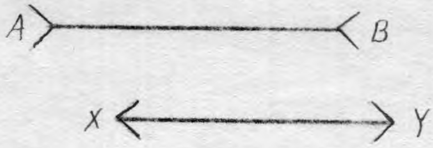


Fig. 1

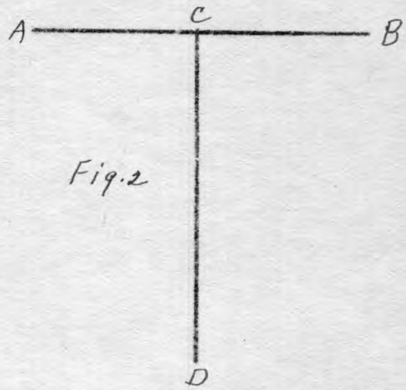


Fig. 2

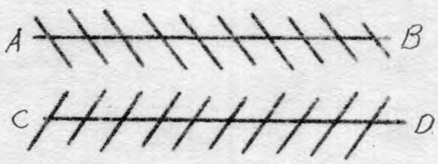


Fig. 3

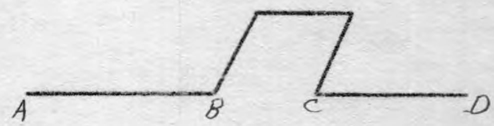


Fig. 4

Fig. 5

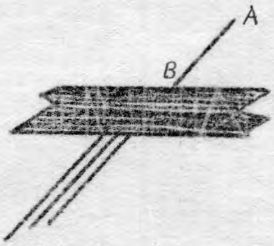


Fig. 6

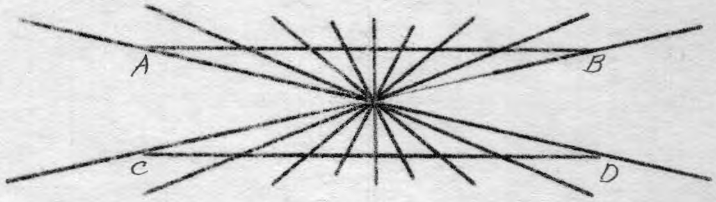


Fig. 7

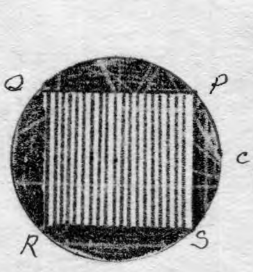


Fig. 8

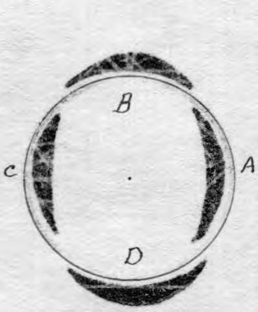


Fig. 9

Plate XXVIII

Miscellaneous Figures

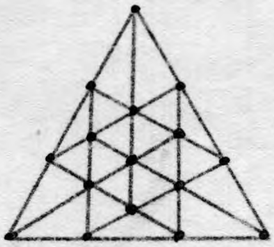


Fig.1

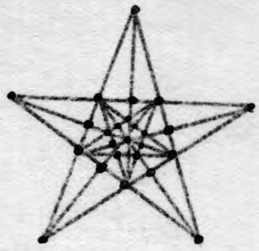


Fig.2

Fig.3

Fig.4

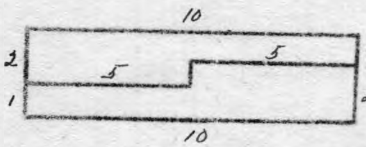


Fig.5

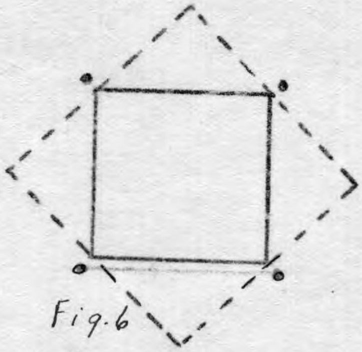


Fig.6

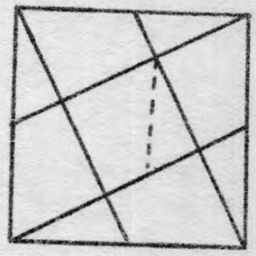


Fig.7

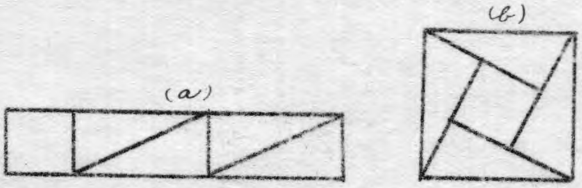


Fig.8

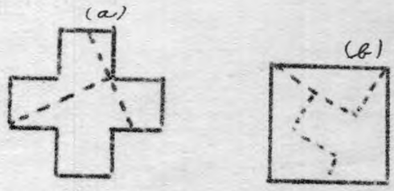


Fig.9

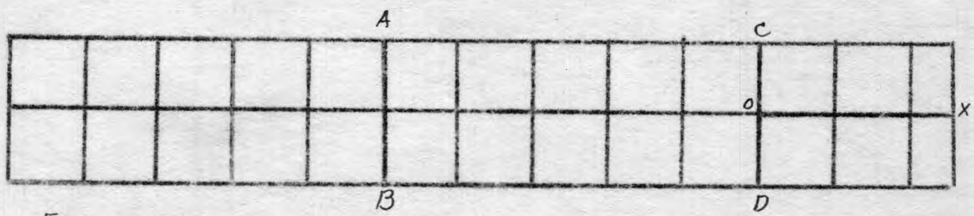


Fig.10

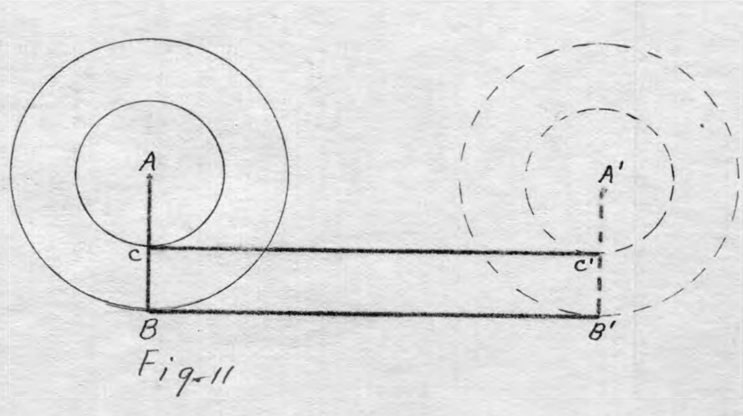


Fig.11

preserve the given square. This may be done by placing 5 on 4, 2 on 1, 11 on 10, and 8 on 7.

Re-arrangement of Areas: The re-arrangement of areas as suggested in the following examples, may be illustrated by cardboard constructions. Other similar constructions may be found or created by the student. Fig. 5, Plate XXVIII illustrates how a board 3 ft. by 10 ft. can be cut once to be made to cover a hole 2 ft. by 15 ft. Fig. 6 illustrates the following problem: Mr. Jackson owns a square farm, the area of which is 20 acres; near each corner stands a large tree which is upon a neighbor's land. How may he add to his farm so as to have a square farm containing 40 acres and still not own the land upon which the trees stand? Fig. 7 shows the arrangement of 5 squares which have been cut so as to form 1 large square. In Fig. 8 is shown how a board 15 inches by 3 inches is cut so that the pieces when arranged shall form a perfect square. Fig. 9 shows how a piece of cardboard in the form of a Greek cross has been cut by two straight cuts so as to divide it in such a way that when re-united they will form a square. Fig. 10 shows how a piece of cardboard 12½ inches long by 2 inches wide may be cut into 4 pieces in such a manner as to form a perfect square without waste.

Fig. 11 is drawn to "prove" that the circumference of all circles are equal. Thus, fasten together the centers of two circles of unequal radii, as per diagram, and let them roll from A to A'. Since the distance BB' and CC' are equal, and the number of revolutions is the same for both, the circumferences must be equal. If not, why not?

<u>Chinese Magic Squares</u>: Mathematical minds have developed the so-called Chinese "magic square" until its "mysteries" now fill a fair sized volume,[1] and still, no doubt, all of its properties have not yet been discovered. The included magic squares in Plate XXIX are simple illustrations of the discoveries which have been made. The magic square is referred to not only in books of mathematics, but also in other books of literature. In early times a great deal of superstition centered about the magic square, and there still exists a superstitious regard for it. It appears as the decorations of fortune-telling bowls, it is used in games of chance or lottery, and is even worn by some as an amulet or charm. The student may be able to find specific applications in his readings.

<u>Properties of Numbers</u>: Curious results are derived from the number 142,857. If this is multiplied by 2, 3, 4, 5, or 6 the same sequence appears in each of the products as in the given number. If, however, it is multiplied by 7 the curious result 999,999 is obtained.

Computations with the numbers 37 and 73 show that upon multiplication with the various numbers from the arithmetical progression, 3, 6, 9, 12, 15, 18, 21, 24, and 27, an odd relation of products is derived. They are but suggestions for a mathematical type of entertainment. Other similar computations may be found in such reference books as Smith's Number Stories of Long Ago, and Jones' Mathematical Wrinkles. (See Bibliography)

1-Andrews, W. S.: <u>Magic Squares and Cubes</u>, Chicago: Open Court Publishing Company 1908

Plate XXIX

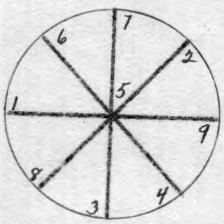
Chinese Magic Squares

6	10	3	15
11	7	14	2
16	4	9	5
1	13	8	12

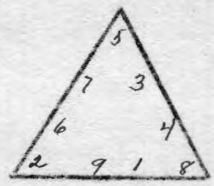
Sum = 34 in every straight line.

8	1	6
3	5	7
4	9	2

Sum = 15 in every straight line.



Sum = 15 in every straight line.



Sum = 20 in every straight line.

25	6	7	24	3
4	10	17	12	22
5	15	13	11	21
8	14	9	16	18
23	20	19	2	1

A bordered magic square.

Sum = 17 in every straight line.

The figure 9 reveals many properties in computations which are of interest. Although many of these have no utilitarian value, they have a place in mathematics in verifying the computations and using them as a means of enjoyment and pastime.

If the number 987,654,321 is multiplied by 18, 27, 36, 45, 54, 63, 72, 81, and 99, respectfully, it gives products in which the first and last figures are the same as those in the multiplier. This is also true of higher multiples of 9, as 108 and 117 and others. If multiplied by 9 and 90 the last figures are the same as those of the multiplier.

If a number consisting of the nine digits--except the 8--in their regular order be multiplied by 9, or any multiple of 9, the product in each case will be a number formed by repetitions of the same digit.

The difference between any certain number and any other number composed of the same figures in a different order is divisible by 9.

If the sum of the digits of any number be subtracted from that number, the remainder is divisible by 9.

The process of "casting out 9's" has already been discussed in Chapter IV.

These computations and facts do not exhaust the possibilities which have been discovered. Other relations may be found from various sources by the student and added to these as given.

Such problems as have been discussed under the properties of numbers, although intrinsically unimportant, have often served to awaken life long interests in the fundamental truths underlying and in the background.

Famous Problems of Antiquity: Problems have been puzzled over by mathematicians until they have become famous in history. A few of these are noted here as a nucleus for an enlarged collection which might be made by the student. Although the solution may not always be readily understood, there is historical interest in the problem itself. A project for student activity may include the preparation of hand made pamphlets on one or more of the problems, the historical setting which made each problem, at the time at which it was originated, a "real life problem," and the life and works of men whose names are connected with the problems.

The following is the famous chess problem of Sessa:

> Sessa, the inventor of chess, presented his game to Scheran, and Indian prince. The latter asked Sessa to name his reward. Sessa asked that the prince give him 1 grain of wheat for the first square on the chess board, 2 for the second, 4 for the third, 8 for the fourth, and so on to the sixty-fourth. The number of grains of wheat thus called for is given by a formula for the sum of a geometric progression as:
>
> $$\frac{1-1.2^{64}}{1-2} = \frac{2^{64}-1}{1} = 2^{64}-1 \quad 18,446,744,073,709,551,615$$
>
> This amount is greater than the world's annual supply at present, since 7000 grqins equal 1 pound Avoirdupois and there are 60 pounds in a bushel. There would thus be approximately 420,000 grains of wheat in a bushel, which amount would give in round numbers 43 trillion bushels in the total amount of wheat required. History does not relate how the claim was settled.

One of the three famous problems of Leonardo of Pisano (Fibonacci) is as follows:

> Three men agree to share money in the ratios of $\frac{1}{2}$, 1/3, and 1/6. Surprised by an enemy each snatched what he could. Later, the first gave up $\frac{1}{2}$ of what he took; the second, 1/3; and the third, 1/6. This was then divided among them equally and each then had the share to which he was entitled. What was the total sum?

The cistern problem of Heron of Alexandria follows:

> I am a brazen bin; my spouts are 2 eyes, my mouth and the flat of my right foot. My right eye fills a jar in 2 days, my left eye in 3, my foot in 4. My mouth is capable of filling it in 6 hours; tell me how long all four together will take to fill it.

A problem which involves the idea of arithmetical progression is recorded in a papyrus of the Egyptian priest Ahmes, who lived nearly two thousand years before Christ:

> Divide 40 loaves among 5 persons so that the numbers of loaves that they receive form an arithmetical progression, and so that the two who receive the last bread together have 1/7 as much as the others.

Unicursal Problems: The unicursal problems given on Plate XXX, Figs. 1, 2, and 3, are of historical interest. Fig. 1, a re-entrant polygon, is a Pythagorean symbol, and Fig. 2, the sign manual of Mohammed, which he traced in the sand by the point of his scimitar without taking it off the ground. Fig. 3 contains only 2 odd nodes, A and B, and therefore can be described unicursally if started from one of them and finished at the other.

The Labyrinth and Maze: The labyrinths and mazes shown in Figs. 4, 5, 6, 7, and 8 on Plate XXX are also of historical interest as well as forming a means of furnishing recreation and entertainment if constructed on a larger scale. These shown are given as suggestive of other patterns which may be found or created by the student. A labyrinth may be made more difficult by increasing the number of nodes or by making them of a higher order, using bridges and tunnels so as to produce the construction in three dimensions.

In Fig. 4 the letters stand for various towns and the lines indicate the only possible paths by which a person may travel. If a person is to start from any town and go to every other town

Plate XXX

Unicursal Problems


Fig. 1


Fig. 2

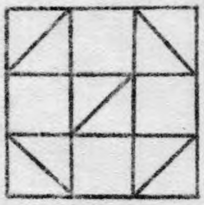


Fig. 3

Labyrinth and Maze

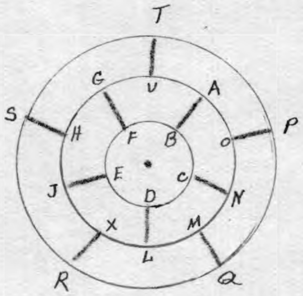


Fig. 4

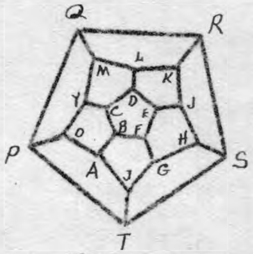


Fig. 5


Fig. 6

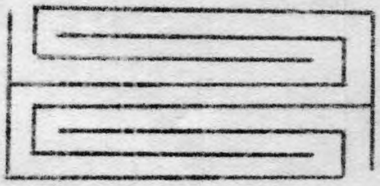


Fig. 7

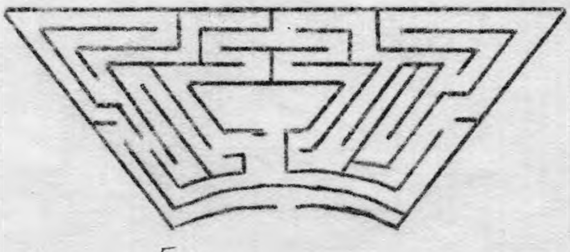


Fig. 8

once and only once, and return, he must take one of only 2 routes. The letter r representing a right turn and l a left turn, the rule for each route is as follows:

1. r r r l l l r l r l r r r l l l r l r l

2. l l l r r r l r l r l l l r r r l r l r

Fig. 5 illustrates what is known as the Hamiltonian Game and follows the same principle as illustrated in Fig. 4.

Fig. 6 illustrates the design on the coins of Cnossus, which is supposed to be a clue to the correct path in the traditional labyrinth constructed for the Minotaur. This design is shown in rectangular measure in Fig. 7. In either arrangement this maze may be run by three different routes: 1st, always follow the wall on the right hand side; 2nd, always follow the wall on the left hand side; or 3rd, when a node is reached, i.e., a point where there is a choice of paths, the path to be taken is that which is next but one to that by which the node was approached.

To run the Hampton Court maze, Fig. 8, the rule is to always follow the wall to the right or to the left.

Dramatizations: Dramatization is another creative expression very appealing to students of secondary school age. Problems of life involving mathematical interpretations may be dramatized. Significant aims and purposes in mathematics may also be interpreted by dramatization. Mathematical plays are issued in such magazines as the Mathematics Teacher, School Science and Mathematics, and the "Texas Outlook." These may be obtained and produced by mathematics classes or clubs or used as suggestions for original plays which may be made more applicable to the circumstances under which they are to be given.

Mathematical Posters: The construction of classroom posters using appropriate mottos and designs is an appealing form of creative expression for the student. To suggest the worth of mathematics, posters may be planned and made to illustrate such mottos as: "What man can do with mathematics, not what mathematics can do with man," or "Chemistry depends on mathematics"; for reminders of class room activities: "Stop! have you checked!" (✓), or "Members of equations are like twins. Treat them alike"; or as incentives: "Can you make your letter in Arithmetic?"

Geometric forms in nature and art are suggested by snow crystals, fruit cut to show symmetrical arrangement of seed pods, or the design of a church window with the motto: "Geometry in the quiet of the Church."

Other mottos which in themselves suggest creative expression, are: "The plan made this house. Mathematics made the plan"; "Mathematics--the master key"; "The men who fired the shot could not see the target, but mathematics helped them hit it"; "Aviation--nine-tenths mathematics"; "Geometry in the home" (showing a girl making a lampshade with hexagonal base). The student may add to these by his own observations.

Chapter XII

Creative Expression Through Mathematics: Summary.

Purpose of Given Suggestions: No effort has been made to present the material in this thesis in text book style. It has rather been the aim to provide suggestions for and means of creative expression which should supplement the work provided by the usual texts.

It is quite generally acknowledged that creative expression allows the student to unfold mental and physical abilities which would otherwise be latent and unproductive to the social world. It is also believed that inhibition means a gradual destruction of that inherent power known as general intelligence. Unless the student is given opportunity and allowed expression when his learning has created a desire, he not only loses the benefit of the advanced learning, but his power ever after to respond is also weakened.

The student may not always be a wise judge in the type of response by which he wishes to express himself. In that he needs guidance and direction. He needs to be brought in contact with problems and projects which will be appealing; i.e., suggestive of creative activities which are to him of a desirable character. In his choice of these activities the student should not be allowed to feel that he is merely completing a task which has been provided for him. The need for the expression must be felt by him, and the form of expression must be his, created from his own ideas, or the real values in the activity are destroyed. He himself must, in some measure, live the life of an artist, an inventor, an explorer, and a discoverer. The fact that the work has already been covered by others, each in his individual way, does not lessen the value to the student of his own creative expression of the same.

Application of Material Suggested: The material in the preceding sections has been drawn from the history of mathematics, observation, direct application, and experience. The general principles involved may be applied to mathematical interests, or interests apart from mathematics. The suggestions herein given are means by which the student may extract further cultural values from his mathematical study; he may be provided with entertainment and recreation; he may acquire greater facilities for expressing his thought and ideas; and he may broaden his interests, for many fields of activity have been touched upon.

Many of the suggestions and examples given are general rather than specific and direct. It is hoped that they are presented in such a manner as to support the underlying aim, that of arousing a larger self-activity in the form of creative expression on the part of the students of secondary school mathematics. The possibilities have not all been suggested, or even touched upon. It may be, too, that those given are not the ones that will be most suitable for all circumstances under which creative expression is desired. However, if the material here presented can be considered representative, and because of it other activities be aroused and participated in, the purpose of this thesis will be fulfilled.

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