

## *Original Paper*

# Keynes's Theory, Based on an Imprecise, Interval Valued Approach to Probability, Rejected Ramsey's Emphasis on the Importance of the Use of Mathematical Expectations in Decision Theory

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### ***Abstract***

*Keynes spent chapter 12 of the General Theory emphasizing two major points that were extremely important in long run decision making, confidence and expectations. Keynes saw that the technical analysis of the role of confidence in decision making had been overlooked in economics. Keynes corrected this lacuna in the General Theory. Confidence was defined as a function of Keynes's evidential weight of the argument,  $V$ , where  $V = V(a/h) = w, 0 \leq w \leq 1$ , just as the term "very uncertain", used three times on p.148 of the General Theory, was defined as a function of a slight amount of information, a definition that is identical to Keynes's definition of "very uncertain" on p.310 of his A Treatise on Probability.  $w$  equaled the degree of the completeness of the relevant information upon which the probabilities were based. Keynes's definition of  $V$  can be found on p.315 of his A Treatise on Probability in chapter 26, titled "The application of probability to conduct". His discussion of the completeness of the evidence can be found on pp.313-315.*

*The second major point was Keynes's completely overlooked discussion of the reasonable calculation of probabilities, based on approximate and inexact measures like interval valued probability and his decision weight,  $c$ , which he called a conventional coefficient of weight and risk,  $c$ , versus the*

*unreasonable calculation of probabilities based on strict or exact mathematical expectations calculations as advocated by Frank Ramsey. The heart of Ramsey's theory is a reliance on betting quotients and mathematical expectations based on precise probability.*

### **Keywords**

*relational, propositional logic, approximation, inexact measurement, non-numerical (interval valued) probability, reasonable calculation, unreasonable calculation, mathematical expectation*

## **1. Introduction**

The paper will be organized in the following manner. Sections Two through Four will cover Keynes's views on mathematical expectations before the General Theory (GT, 1936), in the GT, and after the GT. Keynes's view never changed. It stayed the same his entire life. Keynes ignored Ramsey's claims because Ramsey's 1922 and 1926 reviews incorporate grave and severe errors (See Arthmar & Brady, 2016, 2017, 2018; Brady, 2004a, 2004b, 2014; Brady & Arthmar, 2012). B. Russell demonstrated the nature of the errors made by Ramsey in his refutation in his review of Keynes's 1921 book in the July, 1922 issue of the *Mathematical Gazette* in a footnote on p.120. Russell showed that Ramsey's examples violated the relevance-irrelevance logic contained in chapter Four of the *A Treatise on Probability* (TP, 1921). Section Five will examine the many severe errors made by B Bateman as he continues to assert that Keynes capitulated to Ramsey's 1922 and 1926 reviews of the TP, repudiated his logical theory of probability in his 1931 comments on Ramsey's theory of subjective probability, and then adopted some version of a subjectivist-intersubjectivist approach to probability upon which he based the GT.

The Townshend-Keynes exchanges of 1937-38 showed that Bateman's claim is an impossibility, as Keynes reaffirms to Townshend that the Liquidity Preference Theory of the rate of interest in the GT is based solely on (a) nonnumerical (interval) probability and (b) evidential weight. Ramsey's name appears nowhere in the Keynes-Townshend exchanges, as the core of Ramsey's approach, precise probability, betting quotients and mathematical expectations, played no role at all in the GT. Section Six concludes the paper.

A minor point made by Keynes in his GT chapter 12 attack on mathematical expectations was to point out that "animal spirits", first discussed by G. Boole in his discussion of expectations on p.272 of his *The Laws of Thought* (1854) were needed to serve as a supplement/support to the reasonable calculations of expectations based on inexact probability assessments and the estimate of the degree of confidence based on the assessments of evidential weight. On page 161 of the GT, Keynes does state that "animal spirits" would offer a better understanding of decision making under uncertainty (a situation of partial knowledge-partial ignorance) *if and only if* the only alternative is strict, exact, mathematical expectations, which can offer NO worthwhile explanation as the information needed to calculate such exact and precise outcomes in the future does not exist:

“Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits—of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. Enterprise only pretends to itself to be mainly actuated by the statements in its own prospectus, however candid and sincere. Only a little more than an expedition to the South Pole, is it based on an exact calculation of benefits to come. Thus if the animal spirits are dimmed and the spontaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die;—though fears of loss may have a basis no more reasonable than hopes of profit had before.

It is safe to say that enterprise which depends on hopes stretching into the future benefits the community as a whole. But individual initiative will only be adequate when reasonable calculation is supplemented and supported by animal spirits’ (Keynes, 1936, pp. 161-162).

An excellent example of the current, wide spread confusion prevalent in the economics profession, as regards Keynes’s analysis on pp.161-163 of the GT on animal spirits, is R. Shiller (2021; see also 2017, 2019; Akerlof & Shiller,2009):

In a famous passage from John Maynard Keynes’s *The General Theory of Employment, Interest and Money* (1936), the founding volume of modern Keynesian economics, Keynes gives the concept of animal spirits a central place in his theory. After commenting on the obstacles people face in making economic decisions because of the difficulty of predicting very far into the future, and of even trying to ascertain all the possible long-term implications of any economic decision, he wrote:

Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits—spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities (Keynes, 1936, p. 161) (Shiller, 2021, p. 1).

Of course, Keynes is NOT talking about his non numerical (interval) probabilities, inexact calculation approach, approximate outcomes or evidential weight, all of which are reasonable calculation, in this quote. Keynes is talking about strict, exact mathematical expectations calculation which are unreasonable calculations as there is no access to future data upon which to base the calculations. Keynes NEVER EVER gave the “...concept of animal spirits a central place in his theory”. Pace Shiller, he only gave it a supplemental and supporting role in his discussions dealing with reasonable calculation on pp.161-163 of the GT. However, this can be figured out only if Keynes’s *A Treatise on Probability* has been read. Shiller never read the *A Treatise on Probability*.

Many economists have, since the late 1970’s and early 1980’s, made confused responses to Keynes’s discussions in chapter 12 of the *General Theory*, due to their ignorance of Keynes’s analysis of the evidential weight of the argument in chapter 6 (a logical analysis) and chapter 26 (a mathematical analysis) of his *A Treatise on Probability*.

Akerlof and Shiller (2009), Shiller (2017, 2019, 2021), and dozens of Post Keynesians both before and after the publication of the work of Shiller and Akerlof and Shiller was published, have completely mixed up Keynes's unreasonable calculation (strict mathematical expectations) with Keynes's reasonable calculation (approximation, nonnumerical (interval) probabilities and inexact measurement). The claim that Keynes gave "...the concept of animal spirits a central place in his theory" (Shiller, 2021, p. 1) has no textual support and is much like J. Robinson's "fundamental uncertainty" claims that Keynes also rejected in his exchanges with her in the September-November, 1936 time period when he was reviewing her 1937 book (See Vol. 14, pp. 134-148, CWJMK, 1973).

The latest erroneous presentation of this confused and confusing position is by B. Bateman in 2021. Bateman bases his discussions of Ramsey's supposed contributions to Keynes's General Theory as resting on Ramsey's approach to mathematical expectations based on Ramsey's exact and precise subjective, probability approach. However, he still does not realize that Ramsey's position on mathematical expectations is in direct and complete conflict with Keynes's position on mathematical expectations in the *A Treatise on Probability*, as well as in the *General Theory* and after the *General Theory*. What this means is that Bateman's entire argument about the *General Theory* being based on Ramsey's 1922 and 1926 reviews makes no sense, given that Keynes rejected the applicability of the concept of mathematical expectations before 1936, in 1936, and after 1936.

## 2. Method-Keynes on Mathematical Expectations in the TP before the GT

Keynes is absolutely clear that strict or exact mathematical expectations is based on the two assumptions of the purely mathematical laws of the probability calculus, additivity and linearity (linear probability preferences), that severely limit its applicability in a real world of non additivity and non-linearity.

Additivity requires that the evidential weight of the argument supporting the estimated probabilities,  $V$ , where

$$V = V(a/h) = w=1, \text{ where } 0 \leq w \leq 1, .$$

Linearity requires all probabilities to be raised to the first degree only, so that

$$p_a^1 + p_b^1 + \dots + p_n^1 = 1, \text{ where } n=0, \dots, n+1.$$

Keynes's theory generalizes the purely mathematical theory of probability used by mainstream neoclassical economists, so that it is a limiting case only. Therefore,  $V=w$ , where

$$w \leq 1$$

and

$$p_a^1 + p_b^1 + \dots + p_n^1 \leq 1.$$

Mathematical expectations thus becomes an extreme, limiting case for Keynes, but is, as we shall see in Section Five below, the general case for Ramsey, where subjective probability is defined as the degree of confidence a decision maker has in a subjective probability assessment. There can be no evidential

weight of the argument,  $V$ , for Ramsey. However, if Ramsey were to have ever entertained such a concept, than it would have to always be equal to one on the unit interval  $[0,1]$ , so that the degree of the completeness of the information on which the probability is based,  $w=1$ , where  $0 \leq w \leq 1$ , occurs intertemporally over time, as it is for the case for rational expectations, which are actually just disguised mathematical expectations.

Keynes shows how to correct these severe deficiencies with his conventional coefficient of weight and risk,  $c$ , which Keynes viewed as a complement or alternative to his interval valued approach to probability. It was developed in Part II of the TP and was based on Boole's interval valued theory.

Keynes maximizes  $cA$ , whereas Ramsey maximizes  $pA$ ,

where

$$cA = p \left[ \frac{1}{1+q} \right] \left[ \frac{2w}{1+w} \right] A,$$

and  $A$  is the outcome or the utility of the outcome.

It should be obvious that Ramsey's theory is simply the extreme, limiting point of Keynes's  $c$  coefficient, which means Ramsey's theory is a very special case of Keynes's general theory of decision making under risk, uncertainty and ignorance.

The same results can be shown with Keynes's preferred theory of probability, his theory of interval valued probability. If the lower probability bound equals the upper probability bound, then you have an exact, precise, definite probability that can be maximized a la Ramsey; on the other hand, if the lower probability bound does not equal the upper probability bound, then you are dealing with non additivity and non-linearity a la Keynes and must use interval valued probability or the  $c$  coefficient.

The reader is advised to very carefully read pp.310-315 of chapter 26 of the TP for Keynes's discussion, which can then be compared to Keynes's discussion on pp.148-153 and 161-163 of the GT. Both books deal with the term "very(or greatly)uncertain", as opposed to uncertainty per se.

We can sum up Keynes's rejection of the applicability of mathematical expectations in this quote from his *A Treatise on Probability*:

"In Chapter III. of Part I. I have argued that only in a strictly limited class of cases are degrees of probability numerically measurable. It follows from this that the "mathematical expectations" of goods or advantages are not always numerically measurable; and hence, that even if a meaning can be given to the sum of a series of non-numerical "mathematical expectations," not every pair of such sums are numerically comparable in respect of more and less... The second difficulty, to which attention is called above, is the neglect of the "weights" of arguments in the conception of "mathematical expectation."(Keynes,1921,pp.312-313).

It is important to note that by non-numerical probability, Keynes is referring to interval valued probabilities (See Keynes, 1921, pp. 160-163).

### 3. Method-Keynes on Mathematical Expectations in the GT

I have already gone over the material from pp.161-163 of the GT in my discussions of Keynes's comparison-contrast between his reasonable calculations approach and the unreasonable calculations approach of classical and neoclassical economists and philosophers above. However, I will do so again leaving out of the picture Keynes's very important distinction between reasonable and non-reasonable calculations:

"Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits—of a spontaneous urge to action rather than inaction, *and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities*. Enterprise only pretends to itself to be mainly actuated by the statements in its own prospectus, however candid and sincere. *Only a little more than an expedition to the South Pole, is it based on an exact calculation of benefits to come*" (Keynes, 1936, pp. 161-162; italics added).

and

"Thus if the animal spirits are dimmed and the spontaneous optimism falters, *leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die*;—though fears of loss may have a basis no more reasonable than hopes of profit had before" (Keynes, 1936, p. 162; italics added)

and

"We should not conclude from this that everything depends on waves of irrational psychology. On the contrary, the state of long-term expectation is often steady, and, even when it is not, the other factors exert their compensating effects. We are merely reminding ourselves *that human decisions affecting the future, whether personal or political or economic, cannot depend on strict mathematical expectation, since the basis for making such calculations does not exist*; and that it is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best we are able, *calculating where we can*, but often falling back for our motive on whim or sentiment or chance" (Keynes, 1936, p. 163; italics added).

Keynes's position on mathematical expectations in his GT is identical to his position in his TP.

The problem is that economists, social scientists and philosophers, who have written on Keynes's approach to expectations, have completely overlooked Keynes's chapter Four on measurement, in which Keynes discusses reasonable calculation on pp. 39-40 and pp. 43-44 of the GT. Of course, reasonable calculation is none other than Keynes's inexact measurement and approximation as contained in chapters VI, XV, XVII and XXVI of the TP. All academicians have been misled by their mistake in believing Ramsey's canard that Keynes's theory of logical probability in the TP was an ordinal theory.

#### 4. Method-Keynes on Mathematical Expectations after the GT

Consider the following statement made by Keynes in his 1937 *Eugenics Review* article, which was published at the same time as Keynes's 1937 reply article in the QJE to some of his GT critics who never understood the difference between reasonable and unreasonable calculation. The same situation exists in 2023:

"...we should hide from ourselves how little we foresee. Yet we must be guided by some hypothesis. We tend, therefore, to substitute for the knowledge which is unattainable certain conventions, the chief of which is to assume that the future will resemble the past. This is how we act in practice. Though it was, I think, an ingredient in the complacency of the nineteenth century that, in their philosophical reflections on human behaviour, they accepted an extraordinary contraption of the Benthamite School, by which all possible consequences of alternative courses of action were supposed to have attached to them, first a number expressing their comparative advantage, and secondly another number expressing the probability of their following from the course of action in question; so that multiplying together the numbers attached to the possible consequences of a given action and adding the results, we could discover what to do. In this way a mythical system of probable knowledge was employed to reduce the future to the same calculable status as the present. No one has ever acted on this theory. But even to-day I believe that our thought is sometimes influenced by some such pseudo-rationalistic notions" (Keynes, 1937, p. 13; taken from Keynes's speech of Feb., 16<sup>th</sup>, 1937).

Keynes's attack on the concept of mathematical expectations in his *Eugenics Review* article of 1937 reached far beyond his attacks as contained in the TP and GT. Keynes expresses outright scorn and contempt as regards any possible application of the concept of mathematical expectations, in general. It is a pseudo rationalistic notion that can play no part in decision making by rational human beings.

Yet it is this pseudo rationalistic notion that is at the heart of Ramsey's theory of decision making.

B. Bateman, none the less, claims that Keynes's GT is based on a concept of mathematical expectations as deployed by Ramsey, that Keynes rejected his entire life.

#### 5. Discussion-Ramsey on Mathematical Expectations in 1926

Consider the fact that Ramsey's entire decision making approach in "Truth and Probability" (1926) depends on the application of strict or exact mathematical expectations without exceptions:

The question then arises how we are to modify this simple system to take account of varying degrees of certainty in his beliefs. *I suggest that we introduce as a law of psychology that his behaviour is governed by what is called the mathematical expectation, that is to say that, if p is a proposition about which he is doubtful, any goods or bads for whose realization p is in his view a necessary and sufficient condition enter into his calculations multiplied by the same fraction, which is called the "degree of his belief in p". We thus define degree of belief in a way which presupposes the use of the mathematical expectation.*

We can put this in a different way. Suppose his degree of belief in  $p$  is  $m/n$ ; then his action is such as he would choose it to be if he had to repeat it exactly  $n$  times, in  $m$  of which  $p$  was true, and in the others false [Here it may be necessary to suppose that in each of the  $n$  times he had no memory of the previous ones.]” (Ramsey, 1926. In Kyburg and Smokler (eds.), 1980, 2<sup>nd</sup> ed., p. 36; italics added).

and

“Some concluding remarks on this section may not be out of place. First, it is based fundamentally on betting, but this will not seem unreasonable when it is seen that all our lives we are in a sense betting. Whenever we go to the station we are betting that a train will really run, and if we had not a sufficient degree of belief in this we should decline the bet and stay at home. The options God gives us are always conditional on our guessing whether a certain proposition is true. *Secondly, it is based throughout on the idea of mathematical expectation*; the dissatisfaction often felt with this idea is due mainly to the inaccurate measurement of goods. *Clearly mathematical expectations in terms of money are not proper guides to conduct. It should be remembered, in judging my system, that in its value is actually defined by means of mathematical expectation in the case of beliefs of degree  $1/2$ , and so may be expected to be scaled suitably for the valid application in the case of other degrees of belief also*” (Ramsey, 1926. In Kyburg and Smokler (eds.), 1980, 2<sup>nd</sup> ed., p. 42; italics added).

Ramsey’s approach to mathematical expectations requires linearity and additivity, which must always be the case. It is, therefore, basic and fundamental to his system of analysis. It has been very simple for me to show that Ramsey’s approach is in direct conflict with Keynes’s approach before the GT, in the GT and after the GT.

## 6. Discussion-Bateman on Mathematical Expectations in the GT

Consider the confusions about the concept of mathematical expectations impacting Bateman in the following unclear and confused discussion of Keynes’s alleged use of mathematical expectations in his mathematical modeling in chapters 11 and 12 of the GT, which depends on his reasonable calculation approach, which EXCLUDES mathematical expectation in general. Bateman is simply ignorant of Ramsey’s commitment to strict mathematical expectations:

“Ramsey, however, offered Keynes a way forward for his modeling. Nowhere is this more clear than in the way that he models investment in The General Theory. On the one hand, Keynes can work out in great detail in Chapter 11 (“The Marginal Efficiency of Capital”) the model for rationally optimizing profits, including how investors weigh expected future profits, which are formed by weighting each possible future return by its probability of occurrence; on the other hand, he can step back in Chapter 12 (“The State of Long Term Expectations”) and declare that investors act on animal spirits, not mathematical expectation. That is, investors do not actually line up each outcome and its probability to make their decisions. Just as Ramsey had demonstrated in “Truth and Probability,” you can calculate the probabilities for different outcomes from the bets that a person is willing to make, even if they



make the bets for reasons of enthusiasm, fear, or excitement. From a pragmatic perspective, probabilities in human action reflect beliefs and commitments; they are not objective facts (or logical relations). But it is not just that Keynes employs Ramsey's subjective probabilities in his model in *The General Theory*<sup>19</sup>. The entire style of model building reflects Ramsey's pragmatism" (Bateman, 2021, p. 630).

Let us consider Keynes's actual discussion of expectations in chapter 11 and not Bateman's confusions. We will see that Bateman has substituted "mathematical expectations" for Keynes's use of the term "expectations" in chapter 11. Nowhere does Keynes apply strict mathematical expectation in chapter 11 of the GT:

"The reader should note that the marginal efficiency of capital is here defined in terms of the expectation of yield and of the current supply price of the capital-asset. It depends on the rate of return expected to be obtainable on money if it were invested in a newly produced asset; not on the historical result of what an investment has yielded on its original cost if we look back on its record after its life is over" (Keynes, 1936, p. 136).

Keynes sets up what the problem is here. You can't use past data to calculate what the expected future returns will be when dealing with *mec* calculations.

This then leads to Keynes's explicit discussion of the problem :

"Finally, there is the distinction, the neglect of which has been the main cause of confusion and misunderstanding, between the increment of value obtainable by using an additional quantity of capital in the existing situation, *and the series of increments which it is expected to obtain over the whole life of the additional capital asset;—i.e., the distinction between  $Q_1$  and the complete series*

*$Q_1, Q_2, \dots, Q_T, \dots$ . This involves the whole question of the place of expectation in economic theory.*

Most discussions of the marginal efficiency of capital seem to pay no attention to any member of the series except  $Q_1$ . Yet this cannot be legitimate except in a Static theory, for which all the  $Q$ 's are equal. *The ordinary theory of distribution, where it is assumed that capital is getting now its marginal productivity (in some sense or other), is only valid in a stationary state. The aggregate current return to capital has no direct relationship to its marginal efficiency; whilst its current return at the margin of production (i.e., the return to capital which enters into the supply price of output) is its marginal user cost, which also has no close connection with its marginal efficiency*" (Keynes, 1936, pp. 138-139; italics added).

Keynes answers this question in chapter 12. You can't use mathematical expectations to calculate

"...the series of increments which it is expected to obtain over the whole life of the additional capital asset;—i.e., the distinction between  $Q_1$  and the complete series  $Q_1, Q_2, \dots, Q_T, \dots$ . This involves the whole question of the place of expectation in economic theory..."

At best, you can calculate strict mathematical expectations for the short run period  $Q_1$  or if all of the future values are identical. The entire series of  $Q$ 's involves long run expectations. Here Keynes brings

in his reasonable calculation techniques that were presented in Parts II and IV of the TP and discussed in chapter 4 of the GT on pp. 39-40 and pp. 43-44. What Keynes is saying is that you CAN'T make a technical optimization calculation as claimed by Bateman.

Finally, Keynes emphasizes his point again:

“The most important confusion concerning the meaning and significance of the marginal efficiency of capital has ensued on the failure to see that it depends on the prospective yield of capital, and not merely on its current yield. This can be best illustrated by pointing out the effect on the marginal efficiency of capital of an expectation of changes in the prospective cost of production, whether these changes are expected to come from changes in labour cost, i.e., in the wage-unit, or from inventions and new technique” (Keynes, 1936, p. 141).

What accounts for the errors made by Bateman, not just in this paper, but in all of his work going back to 1987, regarding the use and application of either precise probability and/or mathematical expectations by Keynes? (For instance, see Bateman, 1987, 1989, 1990, 1992, 2016, 2021a, 2021b).

Hishiyama (1969) was the first to point out that Keynes's 1921 contribution was not read. The answer that follows from Hishiyama is that Bateman never read Keynes's TP. Therefore, Bateman does not know what Keynes is talking about when he mentions inexact measurement, approximation, and non-numerical probabilities. Bateman does not know what Keynes means by reasonable calculation. Bateman does not know that Ramsey's mathematical expectations lie at the heart of his subjective approach, whereas Keynes rejects, in general, the concept of mathematical expectations.

Nowhere in Chapter 11 does Keynes rely on or use the concept of mathematical expectations calculations. Keynes explicitly rejects mathematical expectations except in the special case where all of the Q's are equal.

Bateman's claim, that Keynes is making use of Ramsey's theory of subjective probability, which is based on mathematical expectations, in chapters 11 and 12 of the GT, has no support, given that Keynes REJECTED mathematical expectation calculations in chapters 11 and 12.

However, contrary to the Heterodox, Institutionalist and Post Keynesian schools of thought, Keynes never adopted the anti-formalist, anti-mathematical nihilist stance of Joan Robinson, G L S Shackle and T. Lawson. Instead, he substituted his non-numerical probabilities, inexact measurement and approximation approaches for strict mathematical expectation, all of which he derived from Boole's 1854 *The Laws of Thought*.

## 7. Conclusions

Bateman's central claim, that “As we celebrate the centenary of John Maynard Keynes's *A Treatise on Probability* (1921), we are still faced with unresolved, fundamental questions about his foray into the philosophy of probability. One of these unresolved questions, perhaps the most fundamental one, concerns whether Keynes (1931) later changed his mind in response to intense criticism from Frank

Ramsey (1922, 1931) and abandoned the logical theory of probability he had advocated in Probability<sup>1</sup>. The purpose of this essay is to argue that Ramsey had an even wider influence on Keynes's work than has been recognized, and that this influence was not just on his philosophy of probability but also on his economics" (Bateman, 2021, p. 619) is false. None of Ramsey's definitions of Keynes's formal, relational, propositional, Boolean logic are correct. Ramsey's claim about some axiom I(Ramsey,1922,p.3) in the TP, which does not exist, is nonsense. All of Ramsey's examples (Ramsey, 1926), used to illustrate his claimed refutation of Keynes's Boolean mathematical logic, are based on irrelevant propositions that contain no evidence to support a conclusion, as pointed out by Russell in 1922, over 100 years ago.

How is it that Bateman made these mistakes about Keynes and his approach to measurement and mathematics? I. Hishiyama gave the answer back in 1969. Economists and philosophers, indeed all academicians working on Keynes's TP, never actually read Keynes's TP. What they read were articles and book reviews written about Keynes's TP that were written by confused confusers like Ramsey and Richard B. Braithwaite, which are filled with many dozens of obvious errors.

Ramsey "s work is not cited, mentioned or used by Keynes after 1931. Keynes's exchanges with Townshend present an overwhelming case against B. Bateman, which he can't answer, since all of Townshend's 1937 and 1938 exchanges center around how Keynes used his TP to form the foundation for his liquidity preference theory of the rate of interest in the GT. There is no mention of subjective probability, intersubjective probability, Ramsey, or the use of mathematical expectations anywhere in these exchanges or in the GT.

There is nothing in chapters 11 and 12 of the GT that support Bateman's belief that Keynes is using Ramsey's subjective probabilities or intersubjective probabilities or mathematical expectations or anything related to or written by Ramsey about Keynes, especially Ramsey's error filled reviews of 1922 and 1926, which are intellectually worthless as regards any assessment of the content of Keynes's logical theory of probability regarding decision theory.

## References

- Ramsey, F. P. (1926). Truth and probability. In D. H. Mellor (Ed.), *Foundations: Essays in Philosophy, Logic, Mathematics, and Economics*. London: Routledge & Kegan Paul [1978].
- Brady, M. E. (2004a). J. M. Keynes' Theory of Decision Making, Induction, and Analogy. *The Role of Interval Valued Probability in His Approach*. Philadelphia. Pennsylvania: Xlibris Corporation.
- Brady, M. E. (2004b). *Essays on John Maynard Keynes and...* Philadelphia, Pennsylvania: Xlibris Corporation.
- Keynes, J. M. F. P. Ramsey. *In Essays in Biography, CWJMK* (pp. 335-346, vol. X). London: Macmillan for the Royal Economic Society (reprinted from *The New Statesman and Nation*, 3 October 1931).

- Akerlof, G. A., & Shiller, R. J. (2009). *Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism*. Princeton, NJ: Princeton University Press.
- Arthmar, R., & Brady, M. E. (2017). Reply to Feduzi, Runde, and Zappia. *History of Economic Ideas*, XXV(1), 55-74.
- Arthmar, R., & Brady, M. E.. (2016). The Keynes-Knight and the de Finetti-Savage's Approaches to Probability: An Economic Interpretation. *History of Economic Ideas*, XXIV(1), 105-124.
- Bateman, B. (1987). Keynes's changing conception of probability. *Economics and Philosophy*, 3(March), 97-119.
- Bateman, B. (1989). "Human Logic" and Keynes's Economics: A Comment. *Eastern Economic Journal*, 15(1) (Jan.-Mar.), 63-67.
- Bateman, B. (1990). Keynes, Induction, and econometrics. *History of Political Economy*, 22(2), 359-380.
- Bateman, B. (1992). Response from Bradley Bateman. *Journal of Economic Perspectives*, 6(4) (Fall ), 206-209.
- Bateman, B. (2016). Review of Frank Ramsey (1903-1930): A Sister's Memoir. *History of Political Economy*, 181-183.
- Bateman, B. (2021a). Pragmatism and Probability: Re-examining Keynes's thinking on probability. *Journal of the History of Economic Thought*, 43(4), December, 619-632.
- Bateman, B. (2021b). Review of Zachary Carter's "The Price of Peace". *History of Political Economy*, 53(4) (August), 795-797.
- Boole, G. (1854). *An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probability*. New York: Dover Publications, [1958].
- Brady, M. E. (2014). *A Study of Ramsey's Extremely Poor Reading of Chapter III of J. M. Keynes's A Treatise on Probability and the Refutations Made by J. M.*
- Brady, M. E., & Arthmar, R. (2012). Keynes, Boole, and the Interval Approach to Probability. *History of Economic Ideas*, 20(3), 65-84.
- Hishiyama, I. (1969). The Logic of Uncertainty according to J. M. Keynes. *Kyoto University Economic Review*, 39(1), 22-44.
- Keynes & Bertrand, R. (February 17, 2014). Retrieved from <http://ssrn.com/abstract=2397404>
- Keynes, J. M. (1921). *A Treatise on Probability*. London: Macmillan.
- Keynes, J. M. (1921). *A Treatise on Probability*. *The Collected Writings of John Maynard Keynes (CWJMK)* (vol. VIII). London: Macmillan, 1973.
- Keynes, J. M. (1937). Some Economic consequences of declining population. *Eugenics Review*, XXIX(1), 13-17.
- Keynes, J. M. (1937). The General Theory of Employment. *Quarterly Journal of Economics*, 51, 209-223.

- Keynes, J. M. (1964). *The General Theory of Employment, Interest and Money*. New York: Harcourt, Brace and World.
- Keynes, J. M. (1972). Essays in Biography. In D. Moggridge (Ed.), *The Collected Writings of John Maynard Keynes* (CWJMK) (Vol. X). London: Macmillan.
- Misak, C. (2020). *Frank Ramsey: A Sheer Excess of Powers*. Oxford: Oxford University Press.
- Ramsey, F. P. (1922). Mr. Keynes on Probability, Cambridge Magazine, XI, 1, (Jan) 3-5. Reprinted in *the British Journal of the Philosophy of Science*, 40(1989), 219-222.
- Ramsey, F. P. (1926). "Truth and Probability", in Ramsey, 1931. In R. Braithwaite (Ed.), *The Foundations of Mathematics and other Logical Essays*, Ch. VII (pp.156-198).
- Russell, B. (1922). Review of a Treatise on Probability, by J. M. Keynes. *Mathematical Gazette*, 11(159) (July, 1922), pp. 119-125.
- Russell, B. (1931). Review of Ramsey. *Mind*, 40, 476-482. Reprinted in Russell, 1996.
- Russell, B. (1932). Review of Ramsey. *Philosophy*, 7, 84-86. Reprinted in Russell, 1996.
- Shiller, R. J. (2021). Animal spirits and viral popular narratives. *Review of Keynesian Economics*, 9(1) (Spring), 1-10.
- Shiller, Ro. J. (2019). *Narrative Economics: How Stories Go Viral and Drive Major Economic Events*. Princeton, NJ: Princeton University Press.
- Townshend, H. (1979 [1937]). Letter to Keynes. In D. E. Moggridge (Ed.), *CWK* (XXIX, pp. 236-237).
- Townshend, H. (1979 [1938]). Letter to Keynes. In D. E. Moggridge (Ed.), *CWK* (XXIX, pp. 240-245).
- Townshend, H. (1979 [1938]). Letter to Keynes. In D. E. Moggridge (Ed.), *CWK* (XXIX, pp. 257-258, pp. 288-294).