# PROMOTION OPTIMIZATION IN COMPETITIVE ENVIRONMENTS BY CONSIDERING THE CANNIBALIZATION EFFECT 

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#### Abstract

This study proposes a new model to optimize sales promotion in competitive markets and examines the impact of competition on sales promotion planning and business performance in retail chains. The model can be used to determine the best promotional discount for different products with a cannibalization effect when competitors are present in the retail market and offer the same products with different discounts. An integer nonlinear programming problem is proposed to model the above issue. To solve the model, it is reformulated as a mixed-integer linear programming problem. Consequently, a MIP solver can be used to solve the model in a reasonable CPU time. Several examples are solved and a sensitivity analysis of the model parameters is performed. The results of our numerical study show interesting findings that considering different competitors is very important in promotion planning and optimization. Failure to take them into account can lead to loss of profits.


Keywords: Competitive environment; Promotion; Cannibalization effect; Mixed integer linear programming.
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## 1. INTRODUCTION

In today's competitive world, sales and profit growth depend heavily on understanding consumer behavior. Retailers can use various methods to increase sales and profits. Sales promotion is one of the best methods. Some studies have examined the impact of sales promotions on retail purchase decisions, customer loyalty and profit growth (e.g., Andreti et al., 2013; Sagala et al., 2014). Promotional techniques include many retailers temporarily lowering product prices as part of their promotional efforts. Promotional discounts are used to encourage customers to buy. Witell (2011) has shown that loyal customers appreciate price promotions and are more optimistic about the brand. The benefits of promotions include increased sales, customer loyalty and improved cash flow. In addition, promotions are critical in a sector with a very low-profit margin as they can directly impact profitability. There is ample evidence that promotions drive the most sales in certain sectors, such as fast-moving consumer goods (FMCGs).

Planning promotions is a challenge in retail because the more a product is advertised, the more likely it is to attract customers, and the profit margin also decreases. Several factors should be considered when determining the optimal promotional discounts. For example, discounts of $20 \%$ and $30 \%$ may have a similar effect on customer growth, but a $30 \%$ discount will result in a $10 \%$ decrease in profit margin without increasing sales. Despite the complexity of the process, most managers still plan promotions manually (Cohen et al., 2021). Our goal is, therefore to develop and analyze a promotional optimization model that helps them plan promotions more efficiently and, at the same time, more profitably.

As a unit of analysis, this paper examines sales promotions at the store level. Therefore, we only consider consumer promotions. Trade promotions, on the other hand, are marketing activities conducted between manufacturers and retailers or those promoted by brands and wholesalers to their business customers. Therefore, trade promotions were not considered in this study.

Next to the quality and quantity of products and services, decisions about sales promotions are perhaps the most important issue to consider in competition (Shao, 2015). Since decisions about promotions and discounts are strongly influenced by competition, it is impossible to ignore the issue of competition (Tsao et al., 2022). As far as we know, there is no research on optimizing sales promotions in competitive retail environments. It is important to note that previous studies on retail sales promotion, whether a descriptive approach or mathematical models, only highlighted one retail chain and neglected competition between multiple competitors. In a competitive market, the customer can buy the products in another shop at better conditions, and this competition should not be ignored. By introducing a new model, we present an effective
way to address the problem of promotion optimization. With our proposed model, we are able to consider the real-world situation and find optimal discounts for each retail store in a competitive environment with multiple competitors.

Since our problem formulation has a nonlinear objective function and is NP-hard, we apply a linearization technique to make the model linear, which speeds up the optimization process. Our model helps managers test different scenarios to explore different situations and make the best decisions.

Promotions usually have two additional effects:

- Cannibalization: as demand for a promoted item increase, demand for the alternative products decreases.
- Market-boosting: when stores reduce their product prices, customers may experience better quality or utility, and demand volumes will increase accordingly.

In this paper, we optimize promotional discounts for various products with cannibalization and market-boosting effects for the first time in a competitive retail environment.

The rest of the paper is organized as follows: Section 2 discusses related studies. Section 3 describes the problem. Section 4 presents the solution method. Section 5 presents the computational experiments, Section 6 presents the management implications, and Section 7 presents the conclusions and recommendations for future research.

## 2. LITERATURE REVIEW

Sales promotion research has been conducted in the field of marketing for many years. However, most of the research in marketing is devoted to analyzing and estimating dynamic sales models so that the company's management can gain deeper insights into the business. Several studies empirically examine retail sales promotion (e.g., Felgate \& Fearne 2015). These works are primarily descriptive. They apply techniques such as case studies, observations, and questionnaires to identify the factors involved in promotions and the casual relationship between them. Recently, operations research has also begun to examine the optimization of promotional effects. For example, Cohen et al. (2017) investigated the best method for promoting a single item in an operations research setting.

In this section, the literature review is based on four research areas: 1) retail sales promotion, 2) sales promotion planning in a competitive retail environment, 3) cannibalization effect in retail sales promotion, and 4) solution methods in sales promotion planning. Finally, the research gaps in this area are explained.

### 2.1. Retail sales promotion

Several studies have examined retail sales promotions and shown that they significantly influence purchase decisions and customer loyalty (e.g., Amini et al., 2012; Hanaysha, 2018). Mendez et al. (2015) suggest that sales promotions increase customer loyalty and brand reputation over time. Greenstein-Messica \& Rokach (2020) proposed an approach to predict the price elasticity of products in e-commerce retail stores. Breiter \& Huchzermeier (2015) investigated how retailers and supplier collaboration can increase sales promotion efficiency. They offered strategies for predicting demand during multiple promotional periods and hedging risk through a portfolio of supply contracts. Agu (2021) studied and surveyed the effect of perceived transparency of sales promotions on customers' intention to participate in a sales promotion campaign. Joshi \& Bhatt (2021) focus on measuring the impact of advertising and sales promotions on grocery purchase intentions via online grocery delivery services, as well as examining various mediating factors and their indirect and direct effects on the decision to purchase groceries via online grocery delivery services.

Many retail promotion studies have focused on a descriptive approach, particularly in marketing, to identify variables that may influence consumer attitudes and behaviors toward promotional activities. In contrast, several researchers in recent years have focused on the optimization of promotional activities.

### 2.2 Sales promotion planning in a competitive retail environment

Several marketing research studies have addressed the question of which pricing format is most appropriate and viable under different conditions and regardless of competition. For example, Glaeser et al. (2019) study show that prices vary by product category, store location, and customer. Kienzler \& Kowalkowski (2017) conducted an analysis of 515 papers on the development of pricing strategies. These studies did not consider competitors with a different pricing format in the market.

Previous research has questioned the influence of competitive factors on retailers' pricing decisions. Some studies found competitive factors to be insignificant (e.g., Rao et al., 1995). They assumed that retailers should set their pricing policies independently of their competitors' prices because information about their competitors' price changes was considered a hidden factor. In contrast to this approach, Dhar and Hoch (1997) suggested that retailers can easily observe their competitors' actions and estimate their effects. Similarly, Nijs et al. (2007) found that retail prices fluctuate more than competitors' prices due to price trends, wholesale prices, and brand demand.

In contrast, some studies argue that competition strongly influences retail pricing policies and firms are generally vulnerable to the activities of their competitors. For example, Ailawadi et al. (2001) found that competition also affects firms' pricing decisions as long as these activities affect firms' market share. In an empirical study by Chintagunta (2002), it is shown that competitors' factors significantly affect retailers' pricing decisions. Due to the lack of data, their study used store traffic as a surrogate variable to operationalize retail competition. The main reason for these discrepancies in previous studies on the effects of competitor pricing strategies was the unit of analysis used. Many studies were based on brand- or productlevel analyzes that were heterogeneous and variable. This type of brand- or product-level analysis may have resulted in ambiguous information about price changes. Some studies examined the effects of competitors' promotions by using the store level as the unit of analysis, which is more understandable from a price perspective (e.g., Park et al., 2020). Consumers know about promotions before sales, and other competitors can also see the price and promotion information. Therefore, the logic that competitors' prices are a hidden factor and pricing decisions should not consider competitors' conditions can no longer be applied.

Customers consider several factors before choosing a store in a competitive market, not just the price of the products offered there. For example, distance to the store, quality of service, etc. To our knowledge, this issue has not yet been studied in the promotion literature. While pricing policy has been studied in great depth in the competitive location literature, this study is the first to analyze it from a sales promotion perspective, taking into account all competitive factors, not just product prices.

There is intensive research activity on pricing and competition, but only a few papers have studied pricing decisions in a competitive retail environment. Lueer-Villagra \& Marianov (2013), for example, developed a location and pricing model for the hub in a competitive environment. Their study models competition between a new entrant and a market leader whose prices are based on mill prices. At the same time, the former defines its locations and charges based on the multinomial logit model (MNL) analyzes of customers' decisions. In a competitive setting with an incumbent retailer, Zhang (2015) proposed a location and pricing model for a retailer selling a homogeneous product based on the MNL model to estimate customer flows to stores. Zambrano-Rey et al. (2019) discussed the retail store location choice problem using strategic pricing for a retailer selling a homogeneous product. Their work extended the model described by Zhang (2015). Although the literature on competitive retail location is scarce, there is no research on promotion as pricing.

It should be noted that there is a pricing policy called mill pricing, where prices vary across different stores or facilities. Many industries apply this policy in practice. For example, the mill price is used in the fast-food industry. Another option is uniform pricing, where the same price is charged in all stores or facilities. Such a policy is also common in the real world. Stamps, for example, are usually sold at the same price in all post offices. In the case of mill pricing, the type of goods offered at different locations may vary due to various factors, such as quality. Therefore, the products are not necessarily the same, which may result in different prices. On the other hand, a product that is identical in different stores and should have the same "consumer price" can be promoted using special offers to increase sales. This work is the first to find promotional pricing with cannibalization and market elevation effects in a competitive environment.

Table 1 lists the main previous papers on the study of retail sales promotions by categorizing them according to the type of research and the consideration of competition.

### 2.3. Cannibalization effect in retail sales promotion

Retail promotions have been associated with cannibalization since at least 1972 when the notion of incremental shares was calculated using a constant share (Little, 1972). Blattberg \& Wisniewski (1989) point out that customers who buy premium products switch to lower-priced brands only when the lower quality is justified by a substantial price reduction, while customers who typically buy cheap brands try premium brands when they can afford them. Based on 9659 observations of market research data, Mason and Milne (1994) found pairwise cannibalization in cigarettes. Lomax (1996) measured cannibalization using deviation from expected sales. Using a cannibalization approach, Srinivasan et al. improved on Lomax's approach by allowing for cannibalization across product families (Srinivasan et al., 2005). Yuan et al. (2009) conducted a pairwise cannibalization analysis for the orange juice category in new product introductions. Based on the ratio of unit prices between cannibalized and cannibalizing products, Abere et al. (2002) converted volume cannibalization into sales cannibalization. Previous research on cannibalization has been mostly conceptual, but the use of sales data can help estimate pairwise cannibalization rates. In addition, the studies on cannibalization emphasize the importance of understanding the phenomenon from a managerial perspective, which underscores the importance of the research. In this paper, the cannibalization effect is modeled mathematically in the demand function.

Table 1. Overview of the most related past studies on retail sales promotion


### 2.4. Solution methods in sales promotion planning

Different solution methods have been used in promotion studies depending on the requirements of the model. For example, the dynamic programming model proposed by Rao and Thomas (1973) determines how much discount and how often an individual brand should be promoted within a fixed planning horizon. The work of Nobibon et al. (2011) provides a model to solve the optimization problem related to promotion campaigns based on integer programming. Their approach is to present a set-covering formulation and implement a branch-and-price algorithm to solve the model. The above models do not take into account many of the assumptions of our proposed problem, such as the competitive environment and the cannibalization effect, and we cannot use their methods.

In this paper, we use methods from the literature on nonlinear and integer optimization. A mixed-integer nonlinear program (MINLP) is used to solve the promotion optimization problem in this paper. Since the demand and utility functions we consider are highly nonlinear, such MINLPs are computationally very complex. There are polynomial-time algorithms for solving MINLPs under certain structural conditions (for example, Hemmecke et al., 2010). Grossmann (2002) states that many MINLPs do not satisfy these conditions and are solved using strategies such as branch and bound, outer approximation, generalized benders, and extended cutting planes.

In this paper, we have developed a method to convert the MINLP model into a mixed integer linear problem and solve the proposed model. This method has already been used by some researchers in other areas of optimization (Zhang et al., 2012; Elhedhli, 2005; Mehmanchi et al., 2019).

### 2.5. Contribution of the paper

The following is a summary of our main contributions:

1. There are many studies in the literature on sales promotion optimization, but most of them have looked at the competitive environment of manufacturers. They considered competition at the product/brand level, while the competitive environment for retailers is completely different. Retailers need to consider inter-brand competition as well as inter-store competition because customers can easily buy goods and services from a competitor's store. To the best of our knowledge, this is the first work that examines store-level and product-level competition simultaneously in promotion optimization research.
2. Using a special technique, the proposed nonlinear model was transformed into a linear model. Thus, a new method is presented for a class of optimization problems in sales promotion, which can be widely used in a competitive environment.

## 3. MODEL DESCRIPTION

Suppose that in a competitive market, there are m stores competing with different and alternative products. Our analysis examines how a store competing with other $\mathrm{m}-1$ stores can increase its market share and profit. In this competitive market, there are n customers with different base demands for each product. Based on the attractiveness of the different stores, customers allocate their demand between them so that the more attractive a particular store is to a particular customer, the more it will be considered.

The following notations will be used throughout:

## Indices:

$i$ : Index of customers; $i=1, \ldots, n$
$j$ : Index of stores; $j=1, \ldots, m ; j=1$ is related to the store under study
$k$ : Index of products; $k=1, \ldots, \mathrm{~N}$
$s$ : Index of alternative products for $k^{\text {th }}$ product; $s=1, \ldots, \mathrm{~N}-1$
$l$ : Index of promotion discount options for products; $l=1, \ldots, r$
$l^{\prime}$ : Index of promotion discount options for alternative product $k ; l^{\prime}=1, \ldots, r$

## Data:

$d_{i j}$ : Distance between $i^{\text {th }}$ customer and $j^{t h}$ store
$b_{i k}$ : Demand of $i^{\text {th }}$ customer for $k^{\text {th }}$ product when there is no promotion
$D_{i k}$ : Effective demand of $i^{\text {th }}$ customer for $k^{t h}$ product under the cannibalization and market-boosting effects of promotion
$f_{i}$ : Fixed utility of stores for $i^{\text {th }}$ customer even there is no promotion
$q_{i j}$ : Quality of $j^{\text {th }}$ store perceived by $i^{t h}$ customer
$C_{k}$ : Consumer Price of the $k^{t h}$ product
$M_{k}$ : Profit margin of the $k^{t h}$ product
$P_{j k}$ : Promotion discount of the $j^{\text {th }}$ store for $k^{t h}$ product $j=2, \ldots, m$
$\alpha_{k l}: l^{\text {th }}$ Promotion discount option for $k^{\text {th }}$ product in store under study $(j=1)$
$\lambda_{i k l}$ : Market-boosting coefficient for the $i^{t h}$ customer due to the promotion of the $k^{t h}$ product with the $l^{t h}$ promotion discount option
$\beta_{i k s l}$ : Cannibalized percentage of $i^{\text {th }}$ customer in the share of $\mathrm{s}^{\text {th }}$ product due to the promotion of $k^{\text {th }}$ product with the
$l^{\text {th }}$ promotion discount option
$A_{i j}$ : Attractiveness of $j^{\text {th }}$ store to $i^{\text {th }}$ customer
$M S_{i j}$ : Market share of $j^{\text {th }}$ store for $i^{\text {th }}$ customer

## Variable:

$x_{k l}$ : a binary variable that is equal to 1 if the promotion discount $\alpha_{k l}$ is selected for the store under study
The total profit of the business under study is composed of three parts:

- The first part: the profit generated by each product.
- The second part: the effective demand for each product.
- The third part: the market share of the business for each customer.


### 3.1. The first part: the profit generated by each product

If there is no promotion, $C_{k} M_{k}$ is the profit per unit of product for the store. Now, if a promotional discount is considered for product k , its profit is reduced by the promotional value. Therefore, the profit per unit of product is equal to $C_{k}\left(M_{k}-P_{j k}\right)$. Since the amount of product promotion in the store under study is variable, the above relationship is shown as follows:

$$
\begin{equation*}
C_{k}\left(M_{k}-\sum_{l=1}^{r} \alpha_{k l} x_{k l}\right) \tag{1}
\end{equation*}
$$

### 3.2. The Second part: the effective demand for each product

Two promotion effects can be observed in the demand for promoted products and their alternatives. In a cannibalization effect, the promoted item feeds some of the demand for alternatives. In addition, promotions may induce consumers to buy larger quantities than usual. The following relationship can be used to construct the practical demand function:

$$
\begin{equation*}
D_{i k}=b_{i k}+\lambda_{i k} P_{j k} b_{i k}+\sum_{s} \beta_{i k s} P_{j k} b_{i s}-\sum_{s} \beta_{i s k} P_{j s} b_{i k} \tag{2}
\end{equation*}
$$

The first term of the relation (2) is basic demand. Market-boosting is the second term, where a percentage is added to product demand in proportion to the level of discount. In the third term, product $k$ cannibalizes a portion of the share of alternative goods. Fourth, the cannibalization of alternative goods reduces the demand for product $k$. Since the amount of product promotion in the store under study is not known, the above relationship is shown as follows:

$$
\begin{equation*}
D_{i k}=\left(1+\sum_{l=1}^{r} \lambda_{i k l} \alpha_{k l} x_{k l}-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}\right) b_{i k}+\sum_{s=1}^{N-1} \sum_{l=1}^{r} \beta_{i k s l} \alpha_{k l} x_{k l} b_{i s} \tag{3}
\end{equation*}
$$

### 3.3. The third part: the market share of the business for each customer

Based on Reilly's Law of Gravity in retailing, customers will seek out any store that provides adequate service. The probability increases proportionally to the attractiveness of the store compared to other facilities. We hypothesize that customer attractiveness is influenced by the perceived quality, distance to the store, and promotional discounts, supporting the findings of Huff (1964), Nakanishi \& Cooper (1974), and Jain (1979).

According to Huff's rule, higher quality (or better design) of the store and closer proximity to the customer is more attractive to the customer (Tóth et al., 2009; Drezner \& Drezner, 2004; Fernández et al., 2007; Fernández et al., 2021). The quality of a store includes everything related to the facility. There are several factors to consider in the quality of a store, such as accessibility, parking, queues, friendliness of the staff, cleanliness of the facility, etc. Since price is one of the most important decision factors for customers in today's competitive market, facilities can reduce their product prices compared to their competitors and attract more customers by offering a promotional discount. For this reason, in this study, we included the percentage of promotions in the Huff model for customer patronizing behavior.

Promotional discounts have a direct impact on the attractiveness of a facility to customers, as does the quality of the facilities. So, the attractiveness of $\mathrm{j}^{\text {th }}$ store to $\mathrm{i}^{\text {th }}$ customer can be:

$$
\begin{equation*}
A_{i j}=\frac{q_{i j}\left(f_{i}+\sum_{k=1}^{N} P_{j k}\right)}{d_{i j}^{2}+1} \tag{4}
\end{equation*}
$$

There is a one in the denominator because if the distance between the customer and the facility were zero, the fraction would not be infinite. Also, $f_{i}$ in the numerator means that the attractiveness of the facility to customers is not zero if no discount is offered. The more attractive the facilities are, the more customers they will attract.

It is important to note that $A_{i j}$ and $D_{i k}$ are independent functions. " $A_{i j}$ " measures the attractiveness of the facility, which changes as a function of the amount of promotion. The amount of promotion discount affects the probability of choosing a store. That is, the more promotions a store has, the more attractive it is to customers. On the other hand, " $D_{i k}$ " is the demand function whose value changes with product promotion. The amount of product promotion influences the choice of the product compared to other products.

Therefore, the market share of a store for a given customer is equal to its attractiveness divided by the total attractiveness of all facilities, as follows:

$$
\begin{equation*}
M S_{i j}=\frac{A_{i j}}{\sum_{j} A_{i j}} \tag{5}
\end{equation*}
$$

Finally, the total profit is as follows:

$$
\begin{equation*}
\sum_{i=1}^{n} \sum_{k=1}^{N} C_{k}\left(M_{k}-\sum_{l=1}^{r} \alpha_{k l} x_{k l}\right) \cdot D_{i k} M S_{i 1} \tag{6}
\end{equation*}
$$

### 3.4. The mathematical model

We must solve the Promotion Optimization Problem in Competitive Environment (POPCE) as follows:

## Problem P1:

$$
\begin{align*}
& \operatorname{Max} z(x)=\sum_{i=1}^{n} \sum_{k=1}^{N} C_{k}\left(M_{k}-\sum_{l=1}^{r} \alpha_{k l} x_{k l}\right)\left(\left(1+\sum_{l=1}^{r} \lambda_{i k l} \alpha_{k l} x_{k l}-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}\right) b_{i k}+\right. \\
& \left.\sum_{s=1}^{N-1} \sum_{l=1}^{r} \beta_{i k s l} \alpha_{k l} x_{k l} b_{i s}\right)\left(\frac{\frac{q_{i 1}\left(f_{i}+\sum_{k=1}^{N} \sum_{l=1}^{r} \alpha_{k l} x_{k l}\right)}{d_{i 1}^{2}+1}}{\frac{q_{i 1}\left(f_{i}+\sum_{k=1}^{N} \sum_{l=1}^{r} \alpha_{k l} x_{k l}\right)}{d_{i 1}^{2}+1}+\sum_{j=2}^{m} \frac{q_{i j}\left(f_{i}+\sum_{k=1}^{N} P_{j k}\right)}{d_{i j}^{2}+1}}\right) \tag{7}
\end{align*}
$$

s.t.

$$
\begin{array}{ll}
\sum_{l=1}^{r} x_{k l}=1 & ; k=1, \ldots, N \\
x_{k l} \in\{0,1\} \tag{9}
\end{array} \quad k=1, \ldots, N \text { and } l=1, \ldots, r
$$

Here, equation (7) represents the profit of the chain that should be maximized. Equation (8) ensures that one of the various promotion options should be selected for each product.

The problem is an integer nonlinear programming problem. In the following section, we will describe how to solve problem P1 using some properties of the objective function.

## 4. SOLUTION METHODS

In this section, two possible solutions to problem P1 are discussed. In the first, the problem is reformulated as a mixed-integer linear programming problem, followed by a technique for finding the upper bound of the objective function to compare and validate the solution method.

### 4.1. Integer linear formulation from fractional programming

To solve problem P1, artificial variables can be used to replace quadratic terms. To achieve this, we need to linearize the problem in three steps. They are shown in Fig. 1 and are explained below.


Figure 1. The steps of the proposed method

### 4.1.1 Step 1: Linearization of market share

The market share function has a special structure: Numerator and denominator of the ratio differ only by the constants. Similar problems in various fields have been linearized using this approach (Zhang et al., 2012; Elhedhli, 2005; Mehmanchi, 2019).

Assume $B_{i}=\frac{q_{i 1}}{d_{i 1}^{2}+1}, B_{i}^{\prime}=\frac{q_{i 1} f_{i}}{d_{i 1}^{2}+1}, B_{i}^{\prime \prime}=\sum_{j=2}^{m} A_{i j}$ for $i=1, \ldots, n$.
As a result, we have
$\mathrm{MS}_{i 1}=\frac{B_{i}^{\prime}+B_{i} \sum_{k=1}^{N} \sum_{l=1}^{r} \alpha_{k l} x_{k l}}{B_{i}^{\prime}+B_{i}^{\prime \prime}+B_{i} \sum_{k=1}^{N} \sum_{l=1}^{r} \alpha_{k l} x_{k l}}$
Due to the positive denominator, this is equivalent to:
$\operatorname{MS}_{i 1}\left(B_{i}^{\prime}+B_{i}^{\prime \prime}+B_{i} \sum_{k=1}^{N} \sum_{l=1}^{r} \alpha_{k l} x_{k l}\right)=B_{i}^{\prime}+B_{i} \sum_{k=1}^{N} \sum_{l=1}^{r} \alpha_{k l} x_{k l}$
A variable is now introduced:

$$
w_{i k l}=x_{k l} \mathrm{MS}_{i 1}
$$

where the following inequalities exist:

$$
\begin{aligned}
& w_{i k l} \leq \mathrm{MS}_{i 1} \\
& w_{i k l} \leq x_{k l} \\
& \text { And } \\
& w_{i k l} \geq \mathrm{MS}_{i 1}-\left(1-x_{k l}\right)
\end{aligned}
$$

As a result,
$\mathrm{MS}_{i 1}\left(B_{i}^{\prime}+B_{i}^{\prime \prime}\right)-B_{i}^{\prime}+\sum_{k=1}^{N} \sum_{l=1}^{r} B_{i} \alpha_{k l} w_{i k l}-\sum_{k=1}^{N} \sum_{l=1}^{r} B_{i} \alpha_{k l} x_{k l}=0$
So that

$$
\begin{equation*}
\mathrm{MS}_{i 1}=\frac{1}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime}+\sum_{k=1}^{N} \sum_{l=1}^{r} B_{i} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)\right) \tag{13}
\end{equation*}
$$

### 4.1.2. Step 2: Linearization of effective demand multiplied by profit

This section discusses a standard linearization method for the multiplication of binary variables (Asghari et al., 2022).
Assume that

$$
\begin{align*}
& M D_{i k}=\left(M_{k}-\sum_{l=1}^{r} \alpha_{k l} x_{k l}\right)\left(\left(1+\sum_{l=1}^{r} \lambda_{i k l} \alpha_{k l} x_{k l}-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}\right) b_{i k}+\right. \\
& \left.\sum_{s=1}^{N-1} \sum_{l=1}^{r} \beta_{i k s l} \alpha_{k l} x_{k l} b_{i s}\right) \tag{14}
\end{align*}
$$

After simplification, we have

$$
\begin{align*}
& M D_{i k}=M_{k} b_{i k}+\sum_{l=1}^{r}\left(M_{k} b_{i k} \lambda_{i k l} \alpha_{k l} x_{k l}-b_{i k} \alpha_{k l} x_{k l}-\alpha_{k l}{ }_{k l} \lambda_{i k l} x_{k l} b_{i k}\right)- \\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}+\sum_{s s=1}^{N-1} \sum_{l=1}^{r}\left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha_{k l}^{2} x_{k l}\right)+  \tag{15}\\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l} \alpha_{s l^{\prime}} x_{k l} x_{s l^{\prime}}
\end{align*}
$$

Now introduce the following variables

$$
u_{k l s l^{\prime}}=x_{k l} x_{s l^{\prime}}
$$

where the following inequalities exist:

$$
\begin{aligned}
& u_{k l s l^{\prime}} \leq x_{k l} \\
& u_{k l s l^{\prime}} \leq x_{s l^{\prime}} \\
& \quad \text { And } \\
& u_{k l s l^{\prime}} \geq x_{k l}+x_{s l^{\prime}}-1
\end{aligned}
$$

Therefore,
Relation (15) is reformulated as follows:

$$
\begin{align*}
& M D_{i k}=M_{k} b_{i k}+\sum_{l=1}^{r}\left(M_{k} b_{i k} \lambda_{i k l} \alpha_{k l} x_{k l}-b_{i k} \alpha_{k l} x_{k l}-\alpha_{k l}{ }_{k l} \lambda_{i k l} x_{k l} b_{i k}\right)- \\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}+\sum_{s=1}^{N-1} \sum_{l=1}^{r}\left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}\right)+  \tag{16}\\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l} \alpha_{s l^{\prime}} u_{k l s l^{\prime}}
\end{align*}
$$

### 4.1.3. Step 3: Linearization of expression obtained in Step 1 multiplied by the expression obtained in Step 2

$z=\sum_{i=1}^{n} \sum_{k=1}^{N} C_{k} M D_{i k} \mathrm{MS}_{i 1}$

Considering relations (13) and (16) for $\mathrm{MS}_{i 1}$ and $M D_{i k}$, respectively, we have

$$
\begin{aligned}
& Z=\sum_{i=1}^{n} \sum_{k=1}^{N} \frac{c_{k}}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime}+\sum_{k=1}^{N} \sum_{l=1}^{r} B_{i} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)\right)\left(M_{k} b_{i k}+\sum_{l=1}^{r}\left(M_{k} b_{i k} \lambda_{i k l} \alpha_{k l} x_{k l}-b_{i k} \alpha_{k l} x_{k l}-\right.\right. \\
& \left.\alpha^{2}{ }_{k l} \lambda_{i k l} x_{k l} b_{i k}\right)-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}+\sum_{s=1}^{N-1} \sum_{l=1}^{r}\left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha_{k l}^{2} x_{k l}\right)+ \\
& \left.\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l} \alpha_{s l^{\prime}} u_{k l s l^{\prime}}\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& Z=\sum_{i=1}^{n} \sum_{k=1}^{N} \frac{C_{k}}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime} M_{k} b_{i k}+\sum_{l=1}^{r} B_{i}^{\prime}\left(M_{k} b_{i k} \lambda_{i k l} \alpha_{k l} x_{k l}-b_{i k} \alpha_{k l} x_{k l}-\alpha^{2}{ }_{k l} \lambda_{i k l} x_{k l} b_{i k}\right)-\right. \\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} B_{i}^{\prime} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}+\sum_{s=1}^{N-1} \sum_{l=1}^{r} B_{i}^{\prime}\left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha^{2}{ }_{k l} x_{k l}\right)+ \\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r} B_{i}^{\prime} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l} \alpha_{s l^{\prime}} u_{k l s l^{\prime}{ }^{\prime}+\sum_{l=1}^{r} B_{i} M_{k} b_{i k} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)+\sum_{l=1}^{r} B_{i}\left(M_{k} b_{i k} \lambda_{i k l} \alpha^{2}{ }_{k l} x_{k l}-\right.}^{b_{i k} \alpha^{2}{ }_{k l} x_{k l}-b_{i k} \alpha^{3}{ }_{k l} \lambda_{i k l} x_{k l}-M_{k} b_{i k} \lambda_{i k l} \alpha^{2}{ }_{k l} w_{k l}+b_{i k} \alpha^{2}{ }_{k l} w_{k l}+} \\
& \left.b_{i k} \alpha^{3}{ }_{k l} \lambda_{i k l} w_{k l}\right)-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r} B_{i} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} \alpha_{k l}\left(u_{k l s l^{\prime}}-w_{i k l} x_{s l^{\prime}}\right)+ \\
& \sum_{s=1}^{N-1} \sum_{l=1}^{r} B_{i}\left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha^{3}{ }_{k l} x_{k l}-M_{k} b_{i s} \beta_{i k s l}{ }^{2}{ }_{k l} w_{k l}+b_{i s} \beta_{i k s l} \alpha^{3}{ }_{k l} w_{k l}\right)+ \\
& \left.\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r} B_{i} b_{i k} \beta_{i s k l^{\prime}} \alpha^{2}{ }_{k l} \alpha_{s l^{\prime}}\left(u_{k l s l^{\prime}}-u_{k l s l^{\prime} w_{i k l}}\right)\right)
\end{aligned}
$$

A variable is now considered:
$\varphi_{i k l s l^{\prime}}=x_{s l^{\prime}} W_{i k l}$,
where the following inequalities exist:
$\varphi_{i k l s l^{\prime}} \leq w_{i k l}$
$\varphi_{i k l s l^{\prime}} \leq x_{s l^{\prime}}$
$\varphi_{i k l s l^{\prime}} \geq w_{i k l}-\left(1-x_{s l^{\prime}}\right)$
Therefore,
$Z=\sum_{i=1}^{n} \sum_{k=1}^{N} \frac{C_{k}}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime} M_{k} b_{i k}+\sum_{l=1}^{r}\left(\left(B_{i}^{\prime}\left(M_{k} b_{i k} \lambda_{i k l} \alpha_{k l} x_{k l}-b_{i k} \alpha_{k l} x_{k l}-\alpha_{k l}^{2} \lambda_{i k l} x_{k l} b_{i k}\right)\right)+\right.\right.$
$\left(B_{i} M_{k} b_{i k} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)\right)+\left(B_{i}\left(M_{k} b_{i k} \lambda_{i k l} \alpha^{2}{ }_{k l} x_{k l}-b_{i k} \alpha^{2}{ }_{k l} x_{k l}-b_{i k} \alpha^{3}{ }_{k l} \lambda_{i k l} x_{k l}-M_{k} b_{i k} \lambda_{i k l} \alpha^{2}{ }_{k l} w_{k l}+\right.\right.$ $\left.\left.\left.b_{i k} \alpha^{2}{ }_{k l} w_{k l}+b_{i k} \lambda_{i k l} \alpha^{3}{ }_{k l} w_{k l}\right)\right)\right)-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} B_{i}^{\prime} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}+\sum_{s=1}^{N-1} \sum_{l=1}^{r}\left(\left(B_{i}^{\prime}\left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-\right.\right.\right.$
$\left.\left.\left.b_{i s} \beta_{i k s l} \alpha^{2}{ }_{k l} x_{k l}\right)\right)+\left(B_{i}\left(M_{k} b_{i s} \beta_{i k s l} \alpha^{2}{ }_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha^{3}{ }_{k l} x_{k l}-M_{k} b_{i s} \beta_{i k s l} \alpha^{2}{ }_{k l} w_{k l}+b_{i s} \beta_{i k s l} \alpha^{3}{ }_{k l} w_{k l}\right)\right)\right)+$ $\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r}\left(\left(B_{i}^{\prime} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l} \alpha_{s l^{\prime}} u_{k l s l^{\prime}}\right)-\left(B_{i} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} \alpha_{k l}\left(u_{k l s l^{\prime}}-\varphi_{i k l s l^{\prime}}\right)\right)+\right.$ $\left.\left.\left(B_{i} b_{i k} \beta_{i s k l^{\prime}} \alpha^{2}{ }_{k l} \alpha_{s l^{\prime}}\left(u_{k l s l^{\prime}}-\varphi_{i k l s l^{\prime}}\right)\right)\right)\right)$

Finally, Problem P1 is reformulated as P2 as follows:

## Problem P2:

$$
\begin{align*}
& \operatorname{Max} Z=\sum_{i=1}^{n} \sum_{k=1}^{N} \frac{C_{k}}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime} M_{k} b_{i k}+\sum_{l=1}^{r}\left(\left(B_{i}^{\prime}\left(M_{k} b_{i k} \lambda_{i k l} \alpha_{k l} x_{k l}-b_{i k} \alpha_{k l} x_{k l}-\alpha_{k l}^{2} \lambda_{i k l} x_{k l} b_{i k}\right)\right)+\right.\right. \\
& \left(B_{i} M_{k} b_{i k} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)\right)+\left(B _ { i } \left(M_{k} b_{i k} \lambda_{i k l} \alpha^{2}{ }_{k l} x_{k l}-b_{i k} \alpha^{2}{ }_{k l} x_{k l}-b_{i k} \alpha^{3}{ }_{k l} \lambda_{i k l} x_{k l}-M_{k} b_{i k} \lambda_{i k l} \alpha^{2}{ }_{k l} w_{k l}+\right.\right. \\
& \left.\left.\left.b_{i k} \alpha^{2}{ }_{k l} w_{k l}+b_{i k} \lambda_{i k l} \alpha^{3}{ }_{k l} w_{k l}\right)\right)\right)-\sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} B_{i}^{\prime} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} x_{s l^{\prime}}+\sum_{s=1}^{N-1} \sum_{l=1}^{r}\left(\left(B _ { i } ^ { \prime } \left(M_{k} b_{i s} \beta_{i k s l} \alpha_{k l} x_{k l}-\right.\right.\right.  \tag{21}\\
& \left.\left.\left.b_{i s} \beta_{i k s l} \alpha^{2}{ }_{k l} x_{k l}\right)\right)+\left(B_{i}\left(M_{k} b_{i s} \beta_{i k s l} \alpha^{2}{ }_{k l} x_{k l}-b_{i s} \beta_{i k s l} \alpha^{3}{ }_{k l} x_{k l}-M_{k} b_{i s} \beta_{i k s l} \alpha_{k l}^{2} w_{k l}+b_{i s} \beta_{i k s l} \alpha^{3}{ }_{k l} w_{k l}\right)\right)\right)+ \\
& \sum_{s=1}^{N-1} \sum_{l^{\prime}=1}^{r} \sum_{l=1}^{r}\left(\left(B_{i}^{\prime} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l} \alpha_{s l^{\prime}} u_{k l s l^{\prime}}\right)-\left(B_{i} M_{k} b_{i k} \beta_{i s k l^{\prime}} \alpha_{s l^{\prime}} \alpha_{k l}\left(u_{k l s l^{\prime}}-\varphi_{i k l s l^{\prime}}\right)\right)+\right. \\
& \left.\left.\left(B_{i} b_{i k} \beta_{i s k l^{\prime}} \alpha_{k l}^{2} \alpha_{s l^{\prime}}\left(u_{k l s l^{\prime}}-\varphi_{i k l s l^{\prime}}\right)\right)\right)\right)
\end{align*}
$$

s.t.

$$
\begin{align*}
& \sum_{l=1}^{r} x_{k l}=1  \tag{22}\\
& w_{i k l} \leq \frac{1}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime}+\sum_{k=1}^{N} \sum_{l=1}^{r} B_{i} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)\right)  \tag{23}\\
& w_{i k l} \leq x_{k l}  \tag{24}\\
& w_{i k l} \geq \frac{1}{B_{i}^{\prime}+B_{i}^{\prime \prime}}\left(B_{i}^{\prime}+\sum_{k=1}^{N} \sum_{l=1}^{r} B_{i} \alpha_{k l}\left(x_{k l}-w_{i k l}\right)\right)-\left(1-x_{k l}\right)  \tag{25}\\
& u_{k l s l^{\prime}} \leq x_{k l}  \tag{26}\\
& u_{k l s l^{\prime}} \leq x_{s l^{\prime}}  \tag{27}\\
& u_{k l s l^{\prime}} \geq x_{k l}+x_{s l^{\prime}}-1  \tag{28}\\
& \varphi_{i k l s l^{\prime}} \leq w_{i k l}  \tag{29}\\
& \varphi_{i k l s l^{\prime}} \leq x_{s l^{\prime}}  \tag{30}\\
& \varphi_{i k l s l^{\prime}} \geq w_{i k l}-\left(1-x_{s l^{\prime}}\right)  \tag{31}\\
& x_{k l}, x_{s l^{\prime}}, u_{k l s l^{\prime}} \in\{0,1\} ; \quad w_{i k l}, \varphi_{i k l s l^{\prime}} \geq 0 \tag{32}
\end{align*}
$$

Problem P2 is a mixed-integer linear programming problem, and MIP solvers can be applied directly to this type of model.

### 4.2. The continuous and boosted demand relaxation of the objective function of $P 1$ is concave.

Suppose the amount of product promotion is equal to the maximum promotion for that product in competing stores. Then this value is multiplied by $\lambda_{i k}$. We call it $D^{\prime}{ }_{i}$. In this case, this constant number can be used in the objective function.

It is possible to prove that the continuous relaxation of the following function is concave.

$$
\begin{equation*}
z^{\prime}=\sum_{i=1}^{n} \sum_{k=1}^{N} C_{k}\left(M_{k}-x_{k}\right) D_{i}^{\prime} M S_{i 1} \tag{33}
\end{equation*}
$$

Since there are no singularity points in the continuous relaxation of the above function, the second cross-derivative can be calculated by standard methods. It suffices to show that each term in the objective function is concave because it is a sum of ratios. To illustrate this, consider each ratio as the term $i k$ :

$$
\begin{equation*}
f\left(x_{1}, x_{2}, \ldots, x_{n}\right)=C_{k}\left(M_{k}-x_{k}\right) D_{i}^{\prime}\left(\frac{B_{i}^{\prime}+B_{i} \sum_{k} x_{k}}{B_{i}^{\prime}+B_{i}^{\prime \prime}+B_{i} \sum_{k} x_{k}}\right) . \tag{34}
\end{equation*}
$$

In the Hessian matrix

$$
H=\left[h_{m w}\right]
$$

where

$$
h_{m w}=\frac{\partial f\left(x_{1}, x_{2}, \ldots, x_{n}\right)}{\partial x_{m} \partial x_{w}}=-2 \frac{B_{i} B_{i}^{\prime \prime}}{\left(B_{i}^{\prime}+B_{i}^{\prime \prime}+B_{i} \sum_{k} x_{k}\right)^{2}}-2 C_{k}\left(M_{k}-x_{k}\right) D_{i}^{\prime} \frac{B_{i}{ }^{2} B_{i}^{\prime \prime}}{\left(B_{i}^{\prime}+B_{i}^{\prime \prime}+B_{i} \sum_{k} x_{k}\right)^{3}}
$$

Therefore, $h_{m w}$ is always negative. The determinant of every submatrix of order two is 0 , while the diagonal elements are negative. Thus, $f\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ is concave. We can compute the upper bound using gradient methods since the continuous relaxation of the domain is a convex set. This upper bound is called UB.

## 5. COMPUTATIONAL RESULTS

In this section, we perform computational experiments to evaluate the performance of the proposed method and model. First, the proposed method is examined using synthetic data. Then, a typical problem is solved, and the results are analyzed. Then, the effectiveness of the proposed model is illustrated by solving a series of problems of different sizes.

### 5.1 MIP solver solutions quality

In this section, five synthetic data problems are solved using both methods, and their results are compared. The information is presented in Table 2.

Table 2. Results for the different problems

| $\boldsymbol{n}$ | $\mathbf{n}$ | $\mathbf{m}$ | $\mathbf{N}$ | $\boldsymbol{b}_{\boldsymbol{i} \boldsymbol{k}}$ | $\boldsymbol{d}_{\boldsymbol{i} \boldsymbol{j}}$ | $\boldsymbol{q}_{\boldsymbol{i} \boldsymbol{j}}$ | $\boldsymbol{f}_{\boldsymbol{i}}$ | $\boldsymbol{P}_{\boldsymbol{j} \boldsymbol{k}}$ | $\boldsymbol{M}_{\boldsymbol{k}}$ | $\boldsymbol{C}_{\boldsymbol{k}}$ | $\boldsymbol{\lambda}_{\boldsymbol{i} \boldsymbol{k}}$ | $\boldsymbol{B} \boldsymbol{\boldsymbol { B } _ { \boldsymbol { i } \boldsymbol { k } }}$ | Optimal <br> Solution |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 20 | 4 | 2 | $\mathrm{U}(1,100)$ | $\mathrm{U}(1,80)$ | $\mathrm{U}(1,8)$ | 0.01 | $\mathrm{U}(0,0.3)$ | $\mathrm{U}(0.3,0.6)$ | $\mathrm{U}(1,8)$ | 0.2 | 0.6 | 636 | 636 |
| 2 | 15 | 10 | 3 | $\mathrm{U}(1,50)$ | $\mathrm{U}(1,20)$ | $\mathrm{U}(1,4)$ | 0.02 | $\mathrm{U}(0,0.2)$ | $\mathrm{U}(0.2,0.6)$ | $\mathrm{U}(1,4)$ | 0.5 | 0.7 | 167 | 167 |
| 3 | 12 | 2 | 4 | $\mathrm{U}(1,30)$ | $\mathrm{U}(1,50)$ | $\mathrm{U}(1,5)$ | 0.03 | $\mathrm{U}(0,0.2)$ | $\mathrm{U}(0.4,0.6)$ | $\mathrm{U}(1,5)$ | 0.9 | 0.6 | 266 | 266 |
| 4 | 16 | 8 | 5 | $\mathrm{U}(1,80)$ | $\mathrm{U}(1,90)$ | $\mathrm{U}(1,2)$ | 0.01 | $\mathrm{U}(0,0.4)$ | $\mathrm{U}(0.4,0.8)$ | $\mathrm{U}(1,2)$ | 0.3 | 0.4 | 395 | 395 |
| 5 | 11 | 5 | 6 | $\mathrm{U}(1,70)$ | $\mathrm{U}(1,60)$ | $\mathrm{U}(1,6)$ | 0.05 | $\mathrm{U}(0,0.2)$ | $\mathrm{U}(0.3,0.6)$ | $\mathrm{U}(1,6)$ | 0.5 | 0.2 | 587 | 587 |

We show that the optimal solution of the MIP method is the same as the MINLP.

### 5.2. An illustrative example

Suppose a market with five stores, one of which is under investigation and the others are competitors. Assume that the 25 customers in this market have two products with different basic demands that are offered in the five stores at a consumer price of $\$ 30$ and $\$ 35$, respectively. Table 3 shows the coordinates of the stores, the quality perceived by the customers, and the percentage of special offers offered at that time.

Table 3. The stores' information

| \# Store | Coordinates | Ownership | Quality perceived by <br> customers | Promotional discount |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Product 1 | Product 2 |  |
| 1 | $(2,0)$ | Under study | 8 | Model determines | Model determines |
| 2 | $(0,4)$ | Competitor | 7 | $10 \%$ | $10 \%$ |
| 3 | $(1,3)$ | Competitor | 9 | $20 \%$ | $15 \%$ |
| 4 | $(3,2)$ | Competitor | 8 | $15 \%$ | $10 \%$ |
| 5 | $(4,4)$ | Competitor | 9 | $5 \%$ | $10 \%$ |

Table 4 shows the spatial coordinates of customers and their demands.
Table 4. The customers' information

| \# Customer | Coordinates | $b_{i 1}$ | $b_{i 2}$ | $F_{i}$ | $\lambda_{i 11}$ | $\lambda_{i 2 l}$ | $\boldsymbol{\beta}_{\text {i12l }}$ | $\boldsymbol{\beta}_{\mathbf{i 2 1 1} \mathbf{l}^{\prime}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $(0,0)$ | 66 | 93 | 0.04 | 0.94 | 0.82 | 0.5 | 0.8 |
| 2 | $(0,1)$ | 98 | 89 | 0.03 | 0.87 | 0.72 | 0.1 | 0.1 |
| 3 | $(0,2)$ | 64 | 77 | 0.07 | 0.72 | 0.72 | 0.4 | 0.9 |
| 4 | $(0,3)$ | 56 | 86 | 0.08 | 0.95 | 0.90 | 0.5 | 1.0 |
| 5 | $(0,4)$ | 73 | 55 | 0.08 | 0.88 | 0.97 | 0.2 | 0.4 |
| 6 | $(1,0)$ | 78 | 92 | 0.09 | 0.75 | 0.92 | 0.4 | 0.3 |
| 7 | $(1,1)$ | 76 | 51 | 0.04 | 0.83 | 0.86 | 0.1 | 0.2 |
| 8 | $(1,2)$ | 66 | 71 | 0.05 | 0.72 | 0.99 | 0.1 | 0.4 |
| 9 | $(1,3)$ | 85 | 98 | 0.02 | 0.87 | 0.85 | 0.0 | 0.2 |
| 10 | $(1,4)$ | 58 | 84 | 0.02 | 0.95 | 0.94 | 0.3 | 1.0 |
| 11 | $(2,0)$ | 86 | 98 | 0.08 | 0.89 | 0.88 | 0.2 | 0.3 |
| 12 | $(2,1)$ | 68 | 69 | 0.07 | 0.93 | 0.75 | 0.7 | 0.1 |
| 13 | $(2,2)$ | 98 | 73 | 0.08 | 0.94 | 0.96 | 0.8 | 0.3 |
| 14 | $(2,3)$ | 94 | 60 | 0.04 | 0.93 | 0.84 | 0.7 | 0.2 |
| 15 | $(2,4)$ | 84 | 85 | 0.05 | 0.93 | 0.82 | 0.1 | 0.1 |
| 16 | $(3,0)$ | 57 | 68 | 0.06 | 0.71 | 0.90 | 0.3 | 0.1 |
| 17 | $(3,1)$ | 91 | 100 | 0.07 | 0.74 | 0.83 | 0.4 | 0.1 |
| 18 | $(3,2)$ | 54 | 68 | 0.00 | 0.85 | 0.89 | 0.9 | 0.1 |
| 19 | $(3,3)$ | 92 | 60 | 0.07 | 0.78 | 0.72 | 0.0 | 0.3 |
| 20 | $(3,4)$ | 93 | 87 | 0.04 | 0.99 | 0.70 | 0.7 | 0.5 |
| 176 |  |  |  |  |  |  |  |  |


| \# Customer | Coordinates | $\boldsymbol{b}_{\boldsymbol{i} \mathbf{1}}$ | $\boldsymbol{b}_{\boldsymbol{i} \mathbf{2}}$ | $\boldsymbol{F}_{\boldsymbol{i}}$ | $\boldsymbol{\lambda}_{\boldsymbol{i 1} \boldsymbol{l}}$ | $\boldsymbol{\lambda}_{\boldsymbol{i} \mathbf{l} \boldsymbol{l}}$ | $\boldsymbol{\beta}_{\boldsymbol{i \mathbf { 1 2 \boldsymbol { l } }}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | $(4,0)$ | 79 | 86 | 0.08 | $\boldsymbol{\beta}_{\boldsymbol{i 2 1} \boldsymbol{l}^{\prime}}$ |  |  |
| 22 | $(4,1)$ | 68 | 58 | 0.07 | 0.85 | 0.96 | 0.7 |
| 23 | $(4,2)$ | 86 | 69 | 0.04 | 0.75 | 0.6 |  |
| 24 | $(4,3)$ | 83 | 65 | 0.04 | 0.89 | 0.1 | 0.1 |
| 25 | $(4,4)$ | 71 | 54 | 0.06 | 0.75 | 0.77 | 0.6 |
| 0.9 | 0.9 | 0.4 |  |  |  |  |  |

Fig. 2 depicts the locations of customers and facilities.


Figure 2. The locations of customers and facilities
The promotion options for both products are $0 \%, 5 \%, 10 \%, 15 \%, 20 \%, 25 \%$ and $30 \%$. With a profit margin of $50 \%$ and $40 \%$ for products 1 and 2, respectively, the best promotional discount (compared to competitors' promotions) is currently $25 \%$ for product 1 and $10 \%$ for product 2 if the model is applied. In this case, the store would make $\$ 11,001$ in profit.

Table 5 shows how the level of the promotional discount and the store's profit will change under different margin conditions.

Table 5. Optimal solution at different profit margins

| Profit Margin |  |  | Optimal promotional discount |  | Optimal Store Profit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Product 1 | Product 2 |  | Product 1 | Product 2 |  |
| $30 \%$ | $30 \%$ |  | $15 \%$ | $5 \%$ | 5,919 |
| $30 \%$ | $40 \%$ |  | $15 \%$ | $10 \%$ | 7,734 |
| $40 \%$ | $30 \%$ |  | $20 \%$ | $5 \%$ | 7,388 |
| $40 \%$ | $40 \%$ |  | $20 \%$ | $10 \%$ | 9,287 |
| $40 \%$ | $50 \%$ |  | $20 \%$ | $15 \%$ | 11,428 |
| $\mathbf{5 0 \%}$ | $\mathbf{4 0 \%}$ |  | $\mathbf{2 5 \%}$ | $\mathbf{1 0 \%}$ | $\mathbf{1 1 , 0 0 1}$ |
| $50 \%$ | $50 \%$ |  | $25 \%$ | $15 \%$ | 13,210 |

Table 5 shows that the higher the store's profit margin, the more promotion it can offer, thus increasing its profit by attracting more customers. Many managers currently perform this activity manually, which may be far from ideal. However, the most important thing is that the model can be used to easily determine the optimal discount based on market conditions. Another example: If all conditions remain the same, but store 4 changes the discount of product 1 from $15 \%$ to $0 \%$, the optimal discount for store 1 goes from $25 \%$ to $20 \%$.

### 5.3. Non-Optimal solutions comparison

This section examines the losses that occur when the proposed model is not considered. A comparison of some feasible solutions with an optimal solution is given in Table 6.

Table 6. Comparison between different solutions

| Solution | Promotional discount |  | Store Profit | \% Loss |
| :---: | :---: | :---: | :---: | :---: |
|  | Product 1 | Product 2 |  |  |
| $\mathbf{1}$ | $25 \%$ | $10 \%$ | 11,001 | - |
| $\mathbf{2}$ | $0 \%$ | $0 \%$ | 4,265 | $61.2 \%$ |
| $\mathbf{3}$ (Store 2 promotions) | $5 \%$ | $5 \%$ | 8,039 | $26.9 \%$ |
| $\mathbf{4}$ | $10 \%$ | $10 \%$ | 10,019 | $8.9 \%$ |
| $\mathbf{5}$ | $15 \%$ | $15 \%$ | 10,799 | $1.8 \%$ |
| $\mathbf{6}$ | $20 \%$ | $20 \%$ | 9,750 | $3.2 \%$ |
| $\mathbf{7}$ | $25 \%$ | $25 \%$ | 8,095 | $11.6 \%$ |
| $\mathbf{8}$ (Store 3 promotions) | $30 \%$ | $30 \%$ | 10,955 | $26.4 \%$ |
| $\mathbf{9}$ (Store 4 promotions) | $20 \%$ | $15 \%$ | $0.4 \%$ |  |
| $\mathbf{1 0}$ (Store 5 promotions) | $15 \%$ | 10,634 | $3.3 \%$ |  |

According to Table 6 , line 1 is the optimal solution, and the remaining seven solutions are compared. Table 6 shows that the percentage loss in profit can be substantial if the model is not used. For example, profit would decrease by $8.9 \%$ if we behave like store 2 . If we behave like store 5 , the reduction is $17.4 \%$. In summary, the era of experience-based promotion planning is over, and mathematical models should replace it. While some of the non-optimal solutions, such as behaving like store 2 , differ in profit by only $0.4 \%$, for a retailer with $\$ 100$ million in sales, the result is a loss of $\$ 0.4$ million. So, relying on personal experience rather than mathematical models can lead to many losses from a management perspective.

Table 7 compares the optimal solution of the original problem with the optimal solution for the cases where some competitors are not present.

Table 7. Comparison between different scenarios of competitors presence

| Scenario | Description | Optimal promotional discount |  | Store Profit |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Product 1 | Product 2 |  |
| $\mathbf{1}$ | All competitors exist | $25 \%$ | $10 \%$ | 11,001 |
| $\mathbf{2}$ | Store 2 is not present. | $25 \%$ | $10 \%$ | 11,747 |
| $\mathbf{3}$ | Store 3 is not present | $20 \%$ | $10 \%$ | 14,823 |
| $\mathbf{4}$ | Store 4 is not present | $20 \%$ | $10 \%$ | 14,576 |
| $\mathbf{5}$ | Store 5 is not present | $25 \%$ | $10 \%$ | 11,907 |
| $\mathbf{6}$ | Store 3 and 4 are not present. | $20 \%$ | $5 \%$ | 22,235 |
| $\mathbf{7}$ | Store 2, 3 and 4 are not present | $15 \%$ | $5 \%$ | 59,699 |
| $\mathbf{8}$ | All competitors are absent | $0 \%$ | $0 \%$ | 59,136 |

Table 7 shows that a competitive environment significantly affects promotions. Interestingly, the optimal solution may even change when a competitor is not present. Moreover, a smaller number of competitors means that less promotion is needed.

Consequently, the strong influence of competition on promotion decision-making is one of the issues that can be mentioned from the management's point of view.

### 5.4. Sensitivity Analysis

In this section, we change the model parameters of the example and then analyze the percentage change in the optimal solution.

### 5.4.1. Demand changes

In Table 8, we show how the optimal solution changes with some changes in the base demands of the customer.
Table 8. Demand changes effect

| \# Scenario | Change | Optimal promotional discount |  | Optimal Profit | Obtained Profit | $\%$Change |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Product 1 | Product 2 |  |  |  |
| 1 | 20\% Inc. product 1 | 20\% | 15\% | 11,996 | 11,909 | 0.7\% |
| 2 | 20\% Dec. product 1 | 25\% | 10\% | 10,245 | 10,245 | - |
| 3 | 20\% Inc. product 2 | 25\% | 10\% | 12,294 | 12,294 | - |
| 4 | $20 \%$ Dec. product 2 | 20\% | 15\% | 9,996 | 9,924 | 0.7\% |
| 5 | $100 \%$ Inc. product 1 | 15\% | 20\% | 16,387 | 15,541 | 5.2\% |
| 6 | $100 \%$ Dec. product 1 | 30\% | 5\% | 8,895 | 8,732 | 1.8\% |
| 7 | $100 \%$ Inc. product 2 | 30\% | 5\% | 17,789 | 17,464 | 1.8\% |
| 8 | $100 \%$ Dec. product 2 | 15\% | 20\% | 8,194 | 7,770 | 5.2\% |

In Table 8 , the "\% change" column is the percentage difference between the optimal solution and the original solution. Using Table 8, we have shown that when demand changes by $20 \%$, which can be caused by an incorrect forecast, the optimal solution does not change, or if it does, the change is small. Accordingly, it is necessary to constantly monitor changes in demand, but to some extent, an error in the demand forecast does not primarily affect the optimal solution.

The ability to accurately forecast demand with the least margin of error is, therefore, critical for managers to achieve optimal promotion.

### 5.4.2. Store's quality changes

Table 9 shows how the optimal solution changes when the quality of the store varies.
Table 9. Store's quality changes effect

| $\#$ | Change | Optimal promotional discount |  | Optimal Profit | \% Change |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Product 2 |  |  |  |
| $\mathbf{1}$ | $q_{i 1}=8$ | $25 \%$ | $10 \%$ | 11,001 | Original Problem |
| $\mathbf{2}$ | $q_{i 1}=9$ | $25 \%$ | $10 \%$ | 11,727 | $7 \%$ |
| $\mathbf{3}$ | $q_{i 1}=10$ | $20 \%$ | $10 \%$ | 12,397 | $13 \%$ |
| $\mathbf{4}$ | $q_{i 1}=11$ | $20 \%$ | $10 \%$ | 13,040 | $19 \%$ |
| $\mathbf{5}$ | $q_{i 1}=7$ | $25 \%$ | $10 \%$ | 10,208 | $-7 \%$ |
| $\mathbf{6}$ | $q_{i 1}=6$ | $25 \%$ | $10 \%$ | 9,333 | $-15 \%$ |
| $\mathbf{7}$ | $q_{i 1}=5$ | $25 \%$ | $10 \%$ | 8,355 | $-24 \%$ |

As shown in Table 8, store quality did not affect the optimal promotional discounts, but it changed the objective function by about $6-7 \%$ when 1 unit was increased or decreased. It is also clear that the higher the facility's quality, the less promotion can be required to attract customers and vice versa. Promotions and store quality are weighed against each other here. If a store invests in higher quality, it can attract more customers and must spend less on promotion. If it cuts back on quality, it must spend more on promotion.

Considering that both factors have to be weighed against each other, the question arises of how managers decide what is better to spend their time and money on: quality or promotion - this could be a case where management concepts can be applied.

### 5.4.3. Distance decay changes

Our goal is to determine how the decay function of the distance $d_{i j}^{\omega}$ affects the solution about the exponent $\omega$. Four cases are considered: $\omega=1,2,4,8$.

Table 10. Distance decay changes effect

| $\#$ | Change | Optimal promotional discount |  |  | Optimal Profit | Obtained Profit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

As the $\omega$ increases, customers are less willing to use facilities that are farther away, so they choose only facilities that are closer, which means that the effect of promotion on customer attraction is rather negligible, as shown in Table 10. The distance function directly affects the optimal promotion and the objective function. Therefore, estimating how important the distance is to the customer is extremely important. Otherwise, the business would suffer a significant loss. For example, suppose $\omega$ was considered to be 8 but was set at 2 . As a result, instead of $15 \%$ and $0 \%$ promotion, $25 \%$ and $10 \%$ promotion were chosen, and instead of a profit of 13,617 , the store will make a profit of 12,169 , which is $10.6 \%$ less profit than the optimum. For this reason, it is important to consider the importance of distance for the customer when choosing a facility.

Field studies can help management gain a deeper understanding of customer behavior. Misjudging customer behavior can lead to an incorrect decision when promoting a product or service.

### 5.4.4. Cannibalization percentage changes

Here we study the effects of cannibalization changes on optimal solutions. To this end, we present two additional scenarios in Table 11 where one product is extremely vulnerable, and the other has a strong structure.

Table 11. Cannibalization percentage changes effect

| $\#$ | Change | Optimal promotion |  | Optimal Profit | Obtained Profit | \% Change |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Product 1 | Product 2 |  |  |  |  |
| $\mathbf{1}$ | Original $\beta_{i 12}$ and $\beta_{i 21}$ | $25 \%$ | $10 \%$ | 11,001 | 11,001 | Original Problem |
| $\mathbf{2}$ | $\beta_{i 12}=0.99, \beta_{i 21}=0.01$ | $20 \%$ | $15 \%$ | 10,871 | 10,587 | $2.6 \%$ |
| $\mathbf{3}$ | $\beta_{i 12}=0.0 .1, \beta_{i 21}=0.99$ | $30 \%$ | $5 \%$ | 11,374 | 11,370 | $0.04 \%$ |

Table 11 shows that the cannibalization coefficient of commodities strongly influences the optimal solution. For example, if product 1 becomes more vulnerable than the current state and customers buy the product only if it is heavily discounted, and they prefer product 2, the promotional discount becomes $30 \%$ instead of $25 \%$. Product 2 can be interpreted in the same way.

To make a more accurate promotional decision, field studies should be conducted to determine customer preferences. Otherwise, the optimal solution will not be realized.

The results of the study show that, in addition to the discussion of the importance and dependence of the two areas of promotion and competition in the previous section, the model parameters, especially those that may have a greater impact on the optimal solution, must be carefully estimated in order to make the right decisions. There is no question of how the level of competitors' promotion or the profit margins of stores affects the optimal solution. It is possible to test other model parameters, but some have obvious effects.

The final issue in managing sales promotion is effective supplier and consumer notification. Notifying the supplier is critical because an increase in demand can lead to shortages if the supplier is not properly notified. For example, product 1 has a demand of 1922 at the base state and reaches a demand of 2443 at the optimal promotion, so the demand for product 1 has increased by $27 \%$, and if the supplier cannot supply the product, the store will lose profit instead of earning more. When consumers are unaware of the supply, the profit margin decreases without increasing demand.

### 5.5. The test problems

Table 12 contains the values of the objective function and the time taken to solve 30 problems of different sizes to test the efficiency of the model. Each instance consists of a different number of customers ( $\mathrm{n}=50,100,200$, 1000), a different number of facilities $(\mathrm{m}=10,20,100)$, and a different number of products $(\mathrm{N}=2,5,10)$. The parameters of the problems were randomly selected from the following intervals for each setting:

$$
\begin{gathered}
b_{i k} \sim \mathrm{U}(1,100), d_{i j} \sim \mathrm{U}(1,150) q_{i j} \sim \mathrm{U}(1,10), f_{i} \sim \mathrm{U}(0.01,0.1), P_{j k} \sim \mathrm{U}(0.05,0.3), \mathrm{M}_{k} \sim(0.3,0.6), \mathrm{C}_{k} \sim(10,50), \lambda_{i k l} \sim \mathrm{U}(0.1, \\
1), \beta_{i k s l} \sim \mathrm{U}(0.1,1) .
\end{gathered}
$$

Table 12. Results for the different problem size

| \# | n | m | N | CPU time (sec) |  | Objective function |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | MINLP (P1) | MIP (P2) | MIP (P2) | UB | MIP/UB |
| 1 | 50 | 10 | 2 | 232 | 9 | 7,395 | 8,354 | 0.89 |
| 2 | 50 | 20 | 2 | 498 | 19 | 3,953 | 4,218 | 0.94 |
| 3 | 100 | 10 | 2 | 238 | 10 | 15,352 | 17,141 | 0.90 |
| 4 | 100 | 20 | 2 | 511 | 20 | 4,072 | 4,512 | 0.90 |
| 5 | 200 | 10 | 2 | 250 | 10 | 16,994 | 19,060 | 0.89 |
| 6 | 200 | 20 | 2 | 537 | 21 | 11,563 | 12,311 | 0.94 |
| 7 | 200 | 100 | 2 | 3,153 | 106 | 2,229 | 2,623 | 0.85 |
| 8 | 1000 | 10 | 2 | 374 | 17 | 121,698 | 148,447 | 0.82 |
| 9 | 1000 | 20 | 2 | 801 | 34 | 45,152 | 48,878 | 0.92 |
| 10 | 1000 | 100 | 2 | 4,703 | 172 | 17,184 | 18,303 | 0.94 |
| 11 | 50 | 10 | 5 | 6,273 | 32 | 13,717 | 15,534 | 0.88 |
| 12 | 50 | 20 | 5 | 13,447 | 65 | 7,943 | 9,223 | 0.86 |
| 13 | 100 | 10 | 5 | 6,432 | 33 | 35,850 | 38,995 | 0.92 |
| 14 | 100 | 20 | 5 | 13,787 | 66 | 9,847 | 11,318 | 0.87 |
| 15 | 200 | 10 | 5 | 6,762 | 35 | 34,079 | 38,271 | 0.89 |
| 16 | 200 | 20 | 5 | 14,494 | 71 | 26,189 | 31,387 | 0.83 |
| 17 | 200 | 100 | 5 |  | 359 | 3,309 | 4,074 | 0.81 |
| 18 | 1000 | 10 | 5 | 10,086 | 57 | 473,316 | 537,024 | 0.88 |
| 19 | 1000 | 20 | 5 | 21,620 | 114 | 139,865 | 165,366 | 0.85 |
| 20 | 1000 | 100 | 5 | - | 580 | 25,389 | 32,511 | 0.78 |
| 21 | 50 | 10 | 10 | - | 243 | 16,781 | 18,269 | 0.92 |
| 22 | 50 | 20 | 10 | - | 490 | 14,509 | 16,297 | 0.89 |
| 23 | 100 | 10 | 10 | - | 251 | 15,636 | 18,085 | 0.86 |
| 24 | 100 | 20 | 10 | - | 505 | 30,895 | 36,647 | 0.84 |
| 25 | 200 | 10 | 10 | - | 266 | 105,719 | 155,627 | 0.68 |
| 26 | 200 | 20 | 10 | - | 536 | 27,843 | 35,828 | 0.78 |
| 27 | 200 | 100 | 10 | - | 2,723 | 16,624 | 22,877 | 0.73 |
| 28 | 1000 | 10 | 10 | - | 430 | 668,616 | 773,741 | 0.86 |
| 29 | 1000 | 20 | 10 | - | 866 | 549,671 | 661,310 | 0.83 |
| 30 | 1000 | 100 | 10 | - | 4,401 | 52,933 | 70,879 | 0.75 |

Table 12 shows that as the size of the problem increases, the solution time of the MINLP solver increases significantly, while the impact on the MIP solver is small. In the column MINLP CPU time, "-" indicates that the method is not able to solve the problem. Thus, we conclude that the proposed method is efficient and provides the optimal solution in a short time, even for large problems. On the other hand, as shown in Table 12, the method provides solutions of reasonable quality even for large data sets.

### 5.6. Case Study

In this section, we describe a concrete application of the model to find the best promotional discount for a store in the city of Tehran, Iran.

Farmaniyeh, a neighborhood in Tehran, was selected for this purpose. In an area with a population of 10,000 , there are about 2,500 customers if each household has four members. In order to get the optimal promotion, we selected the product hamburger. In this area, there are seven different brands of this product. The promotion issue should be considered from the point of view of one of the stores. There are a total of 39 stores in this area. With the help of CRM software, the hamburger purchases of 1854 customers in the last two years in the studied store were analyzed. Since the data of the other customers (if any) are not accessible, their demand is considered as zero. Information about promotions for all brands was displayed in
each store. We used questionnaires randomly distributed to customers to estimate the parameters $f_{i}, q_{i j}, \lambda_{i k l}, \beta_{i k s l}$, which can be generalized to the rest of the customers.

Table 13-Information about the case study

| Data | $\mathrm{n}=1854$ |  |
| :---: | :---: | :---: |
|  | $\mathrm{~m}=39$ |  |
|  | $\mathrm{~N}=1$ |  |
|  | $\mathrm{r}=7$ |  |
| Optimal Solution | Alternative Products | Optimal Promotional Discount |
|  | 202 | $0 \%$ |
|  | CPU time $(\mathrm{Sec})=1907$ | B-A foods |

The result of applying the model proposed in this paper is that the promotion in the studied store yields more profit than the different promotion scenarios. Table 14 compares the optimal solution with a number of feasible solutions.

Table 14. Comparison between different solutions

| Solution | Promotional discount |  |  |  |  |  |  |  | Store Profit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{2 0 2}$ | B-A foods | Pakdam | Pemina | Sadak | Kimbal | Mam |  |  |
| $\mathbf{-}$ | $0 \%$ | $5 \%$ | $15 \%$ | $0 \%$ | $20 \%$ | $10 \%$ | $5 \%$ | 1,115 |  |
| $\mathbf{1}$ | $20 \%$ | $25 \%$ | $5 \%$ | $20 \%$ | $25 \%$ | $5 \%$ | $25 \%$ | 1,002 | $10.1 \%$ |
| $\mathbf{2}$ | $15 \%$ | $25 \%$ | $10 \%$ | $5 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | 984 | $11.7 \%$ |
| $\mathbf{3}$ | $20 \%$ | $30 \%$ | $30 \%$ | $20 \%$ | $20 \%$ | $5 \%$ | $25 \%$ | 1,026 | $8.0 \%$ |
| $\mathbf{4}$ | $25 \%$ | $0 \%$ | $15 \%$ | $10 \%$ | $25 \%$ | $25 \%$ | $25 \%$ | 1,009 | $9.5 \%$ |
| $\mathbf{5}$ | $15 \%$ | $15 \%$ | $25 \%$ | $15 \%$ | $25 \%$ | $15 \%$ | $20 \%$ | 993 | $10.9 \%$ |

Table 14 shows that the business may suffer losses when the calculations of promotions are done manually.

## 6. MANAGERIAL IMPLICATIONS

Our promotion optimization model provided several insights, which we briefly discussed. Most retailers are interested in finding out how various promotional items perform. It is easy to use our solution to test different promotional strategies to better understand the impact of retail promotions. As we mentioned earlier, our problem is affected by several factors: cannibalization effects, market-enhancing effects during the promotion period, the customer choice rule, and competitiveness. The retailer should learn how these different effects influence his promotional decisions and how they are consistent with each other. The following is a list of management effects followed by a description of them:

1. The strong impact of competition on promotional decisions
2. The need to accurately predict demand to calculate the optimal scale of promotion.
3. The impact of a trade-off between the quality of the business and the amount of the promotion as two factors in attracting customers.
4. The need to pay attention to how customers choose stores and products
5. The effect of cannibalization of goods among themselves and the need to correctly estimate their scale using field studies
6. Paying attention to all influencing factors in the promotion and the need to use mathematical models to find an optimal solution
7. Informing product suppliers before conducting the promotion to ensure the supply of the product facing increased demand
8. Properly informing consumers to take advantage of the promotion and increase demand.

Each of the above cases is explained below:

1. Competition was one of the factors examined in this study, and the authors found that competitors have a significant impact on the level of promotion discounts. Consequently, retailers need to know the quality of competitors' services and their prices, as these factors influence customers' choices. Moreover, the closer customers are to their competitors or the better their services are, the more difficult it is for retailers to offer more discounts, and vice versa.
2. Careful prediction of demand is necessary to determine the optimal amount of promotion. Incorrect forecasting will result in less-than-optimal levels of promotion. By using demand forecasting techniques, managers should be able to predict demand with the least amount of error.
3. The quality of a store plays an important role in the decision to promote. The higher the quality of the store, the easier it will be to attract customers, and the lower the need for a promotional discount is likely to be in competition with other stores. On the contrary, high discounts should be offered to attract customers if the store is not highquality.
4. The promotion of a product depends heavily on what factors are important to consumers and the weight they give to each factor. Therefore, retailers should constantly communicate with consumers to get their opinions on the selection of the store and the products on the shelf because this selection rule changes depending on the product and the economic situation of consumers.
5. Cannibalization can have a significant impact on promotion. Therefore, retailers should never ignore this effect when planning their promotions. Moreover, the extent of cannibalization of products varies. For example, premium products lose their share only when the alternative and weaker product experiences a significant price reduction. When the premium product enters the promotion, a weaker product cannot compete. For this reason, the cannibalization coefficient must be determined precisely.
6. The results show that even promotion values close to the optimum can lead to significant losses on a large scale. In summary, if retailers accurately estimate the parameters in our proposed model (such as customer demand, cannibalization of goods, determining the importance of various factors in attracting customers to the store, etc.) and apply the proposed methodology, they can determine the exact level of promotion.
7. Increasing the demand for promoted products is one of the most important points in sales promotion. Retailers must inform their suppliers about the promoted products. There is a possibility that this increase in demand may not be compatible with the supplier's production capacity and may lead to a shortage of stock for the promoted product, which may have a negative impact on both the supplier's and the retailer's credit.
8. Before starting the promotion, retailers should provide sufficient information to customers. Otherwise, consumers will go to the store as usual, and the profit that should have been made by the store by selling more products will be virtually absent, and only the profit margin will be lost.

## 7. CONCLUSION

This paper presents a new concept for optimizing promotional activities in the literature. A model for determining the best promotional pricing has been proposed in a competitive environment is proposed. In this model, we assume a static competitive environment, i.e., competitors are already present in the market and compete with each other for the same products. We assume customers behave according to Huff's rule and distribute their demand among all stores. According to this rule, the more attractive a store is, the more likely customers are to visit it. In this paper, we studied the situation of promotional discounts for different products with cannibalization and market-boosting effect.

Our model is based on integer nonlinear programming. Then, the model is reformulated as a mixed-integer linear program so that standard optimization programs can be used to find optimal solutions. Several examples were solved to evaluate the efficiency of the model and the proposed method, and the results show that the developed method is efficient. In addition, the results show the importance of considering competition and promotion simultaneously.

A possible extension of our work is considering the proposed model for the case of leaders and followers. The model can also be applied to future studies when the customer chooses the closest facility. Other variables, such as location, facility size, assortment, and shelf space, can be considered for further research.

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