

INTEGRATED HUB LOCATION AND CAPACITATED VEHICLE ROUTING PROBLEM OVER INCOMPLETE HUB NETWORKS

Zühal Kartal^{1,*}

¹Department of Industrial Engineering
Eskisehir Technical University
Eskisehir, Türkiye

*Corresponding author's e-mail: zkartal@eskisehir.edu.tr

Hub location problem is one of the most important topics encountered in transportation and logistics management. Along with the question of where to position hub facilities, how routes are determined is a further challenging problem. Although these two problems are often considered separately in the literature, here, in this study, the two are analyzed together. Firstly, we relax the restriction that a vehicle serves between each demand center and hub pair and propose a mixed-integer mathematical model for the single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up and delivery. Moreover, while many studies in hub location problem literature assume that there is a complete hub network structure, we also relax this assumption and present the aforementioned model over incomplete hub networks. Computational analyses of the proposed models were conducted on various instances on the Turkish network. Results indicate that the different capacity levels of vehicles have an important impact on optimal hub locations, hub arc networks, and routing design.

Keywords: Hub Location Problem; Vehicle Routing Problem with Simultaneous Pick-up and Delivery; Hub Location and Routing Problem; Incomplete Hub Network Design, Mixed Integer Programming Formulation.

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1. INTRODUCTION

The problem of determining the locations of hubs arises in various application areas when designing telecommunication and transportation networks, such as airline passenger and postal/cargo delivery networks. This problem appears when traffic between origin and destination nodes in a network has to be routed through a set of nodes called hubs. Hub facilities are used to consolidate and disseminate flows. In these networks, a node is defined as both a supply and a demand point at the same time. Since the flows of multiple nodes are accumulated in the hubs, transportation between the hubs can be performed with larger vehicles, which reduces the unit transportation cost due to economies of scale.

In the literature, hub location problems are divided into three classes according to their objective functions. The first classification is the p -hub median and fixed costs hub location problems, where the aim is to minimize the total transportation cost. The second classification is the p -hub center problem which aims to minimize the maximum distance/time traveled between any pair of nodes. The last classification is the hub covering problem. This aims to locate hubs and assign non-hub nodes to them, subject to the travel time (cost or distance) between origin-destination pairs that do not exceed a predetermined threshold. Some detailed analyses have been conducted on various types of hub problems in the following studies: (Campbell, 1994a; Campbell, 1994b; Alumur and Kara, 2008; Campbell and Kelly, 2012; Farahani *et al.*, 2013; Alumur *et al.*, 2021).

In hub location problems, there are two main assumptions: the first is that there is one dedicated vehicle between each demand center and hub. The second assumption is the hub network design is complete. However, in various applications, these two basic constraints must be modified and relaxed. Often, in reality, nodes usually do not have adequate demand to connect them directly to hubs. Moreover, cargo firms generally offer a 'next day delivery' or 'delivery within 24 hours' guarantee. In such systems, to ensure service quality, when traffic volume is low and distances are relatively short between non-hub nodes, a vehicle has to be assigned to more than one non-hub and hub node. To provide a strong competitive advantage in the market, cargo/logistics firms can design low-cost transportation networks as well as a high level of service by using a strategy that combines hub location with vehicle routing. By reducing the need for direct transportation of the flow between each demand center and hub in this type of network, it is possible to reduce the number of required vehicles. As a result, the total investment cost of vehicles may decrease considerably.

Traditionally, in hub location network designs, vehicles collect the flows from hubs in the morning and deliver them to hubs at the end of the day. Generating collection and distribution routes separately based on such a design structure is one

option. However, this results in increased transportation costs since it requires separate vehicles between each demand center and hub. The relaxation of the first basic assumption in hub location problems by allowing vehicles to visit multiple nodes is known as the hub location and vehicle routing problem in the literature.

The motivation behind this study is the daily operational characteristics of cargo companies in Türkiye. The hub location and routing problem can be seen in same-day cargo service in big cities. In this service, the parcels left at branch offices in the morning are delivered to addresses on the same day. This problem was first studied in the literature by Kartal *et al.* (2017). The authors assumed that the vehicles are uncapacitated because usually only documents and small parcels are transported with this type of service. However, especially since the COVID pandemic, this service has become more popular for transferring a much wider range of cargo, and this has resulted in increased parcel flow in cities. Thus, the necessity of considering the capacities of the vehicles used in urban cargo logistics has arisen.

In this study, vehicle capacities were considered in the single allocation p -hub median and routing problem with simultaneous pick-up and delivery. The cargo companies interviewed advised that they are also searching for an incomplete hub network design for this service to reduce the total traveled distance among hubs in big cities like Istanbul, where the traffic density is very high. Therefore, incomplete hub network design decisions were also included in the same problem. Although we are motivated by a real-life application, our aim with this study is to show the importance of the different capacity levels of vehicles on optimal hub locations, hub arc networks, and routing design.

Kartal *et al.* (2017) proposed four different mathematical models based on three-index and four-index multi-commodity flow formulation for the p -hub median location and routing problem with simultaneous pick-up and delivery. They included symmetry-breaking constraints in their formulations. We start by including the vehicle capacity constraint in the three-index multi-commodity flow formulation of this problem as proposed by Kartal *et al.* (2017) and investigate the effects of vehicle capacities on the network design. After this, we include incomplete hub network design decisions into the mathematical model. To adapt the problem to include the design of incomplete hub networks, the modeling treatment in Alumur *et al.* (2009) was used. In the literature, a similar modeling approach was also used in Campbell *et al.* (2005a; 2005b), where the authors took the p -hub median problem in a multiple allocation scheme into account and introduced the hub arc location problem to the literature. Unlike Alumur *et al.* (2009) and Campbell *et al.* (2005a; 2005b) studies, we allow capacitated vehicles to form routes.

Karimi (2018) proposed the capacitated hub covering location-routing problem for simultaneous pick-up and delivery in the literature. Their work was based on the classic capacitated fixed costs hub location and vehicle routing problem with simultaneous pick-up and delivery. Their objective function included distance-dependent vehicle costs and the fixed costs of vehicles and hubs as in the classic vehicle routing problem literature, which also included inter-hubs transportation costs. The authors considered a complete hub network, and they used three-index multi-commodity flow variables while calculating transportation costs between hubs. Wu *et al.* (2022) worked on a multi-allocation hub location routing problem and proposed the single allocation version of the problem in their study. Motivated by the design of an intra-city express service system, the pick-up and delivery processes are handled simultaneously at the branch offices and hubs. The authors considered both hub capacities and vehicle capacities and assumed that only one vehicle could be assigned to each hub. Four-index multi-commodity flow variables to calculate inter-hub transportations were used in the study. Similar to Karimi (2018), Wu *et al.* (2022) calculated the local tour costs by totaling the travel distance of the arcs visited as in the classic vehicle routing problem, and inter-hub transportation costs were determined as in the classic hub location problem. Unlike Karimi (2018) and Wu *et al.* (2022), the costs in our study are calculated as a function of the distance traversed and the flow volumes by taking into account vehicle capacities and assuming that the flows are picked-up and delivered simultaneously at each node. This approach distinguishes our problem formulation from the classic vehicle routing problem literature. Furthermore, we also allowed non-complete hub networks in our study, which also differentiates our model from these two studies.

To the best of our knowledge, we present the single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up and delivery over *complete* and *incomplete* hub network design for the first time in the literature by incorporating the distance traversed and flow volumes together.

The network design of this problem can also be adapted for other cargo/postal service applications. While visiting non-hub nodes, the distribution of the previous day's flows and collection of the same day's flows can be carried out on the same route. Thus, firms can provide a 'delivery within 24 hours' guarantee to their customers with lower-cost transportation networks. In such a case, vehicle capacities must also be taken into consideration.

The integration of the hub location and vehicle routing problems was discussed for the first time by Kuby and Gray (1993). The authors developed a mathematical model that determined to send the flows from the demand centers to predetermined hubs directly. Aykin (1995) studied a hub location problem by considering whether the flows of a demand center should be collected and distributed via hubs (one or two at most) or without hubs. A heuristic method was suggested for the solution to the problem. Bruns *et al.* (2000) addressed the package distribution problem of the Swedish postal service, where the packages were sent by vehicles from post offices to the package processing centers (hubs) and then to the customers by visiting multiple demand centers. The authors decided the number and location of the hubs in the network, and it was

determined which demand centers should be assigned to the opened hubs. This problem was reduced to the discrete facility location problem, and a branch-bound algorithm was developed. Grünert and Sebastian (2000) worked on the network design details of the German postal service to review tactical decisions regarding the locations of hubs and warehouses. To solve the problem, the authors proposed various optimization techniques. Wasner and Zapfel (2004) discussed the restructuring of the Austrian postal system, and the authors determined the location of the warehouses, hub centers, and vehicle routes. However, in this system, the flows between hubs were transferred via a central hub, as each hub could not send flows directly to another. A hierarchical solution was developed to solve the problem based on an add-drop-shift heuristic. Regarding the need for restructuring the Turkish postal system, Çetiner *et al.* (2010) addressed the multiple allocation hub location and routing problem and developed an iterative algorithm as a solution.

Camargo *et al.* (2013) worked on the single allocation hub location and routing problem where a maximum time limit for each route existed to guarantee the quality of service. The authors developed a mathematical model based on the formulations of the four-index multi-commodity p -hub median problem and the traveling salesman problem. To solve this, a Benders decomposition algorithm was proposed. Unlike Camargo *et al.* (2013), we added a capacity limit on vehicles in this study. Another variant of the single allocation hub location and routing problem was studied by Rodriguez-Martin *et al.* (2014). In their problem, one vehicle was assigned to each hub, and the total number of nodes that each vehicle could visit was limited. The routing cost was considered a distance-dependent function. For the solution of medium-sized problems, a branch and cut algorithm was proposed. Different from Rodriguez-Martin *et al.* (2014), our proposed formulation does not limit the number of vehicles that are assigned to each hub, and it allows heterogeneous vehicles in the routes. Kartal *et al.* (2019) worked on the single allocation p -hub center and routing problem and developed a new discrete particle swarm optimization algorithm (DPSO) that combines the ant colony algorithm and simulated annealing. They also discussed the performance of the DPSO algorithm on the solution by comparing it with the ant colony system algorithm and an iterative local search algorithm. Unlike Kartal *et al.* (2019), our study considers the flow volumes and vehicle capacities. Different from the three studies mentioned in this paragraph, an incomplete hub network design is proposed in this study.

There are other real-life application-based studies in the hub location and routing problem literature. Sariçiçek and Akkuş (2015) worked on the unmanned aerial vehicle hub location and routing problem for monitoring geographic borders. Lopes *et al.* (2016) investigated a hub location and routing problem that emerges in transportation networks, especially in the movement of people between cities using public transportation. A biased random-key genetic algorithm and multi-start variable neighborhood descent algorithm were developed to tackle the problem. Ratli *et al.* (2022) addressed the same problem as Lopes *et al.*'s (2016) study and proposed a general variable neighborhood search algorithm for the solution. A multiple assignment hub locations and routing problem was studied by Wu *et al.* (2022). The authors developed an adaptive large neighborhood decomposition search technique to address the problem based on express delivery within a city. Mokhtar *et al.* (2017) considered the intermodal container distribution problem around the islands of Indonesia. Since containers are transported to/from the hubs and from/to the islands, multiple visits are possible between non-hub nodes within an island. The authors developed a novel mathematical model for the solution of the problem.

Over the years, hub location problems have been changing due to the different applications of the companies, all of which have various needs and specifications (Eghbali-Zarch *et al.*, 2019; Sadeghi *et al.*, 2018). An example of these features is to determine the incomplete hub network design decisions instead of using a complete hub network, which is a factor encountered in most real-life applications (Çalık *et al.*, 2009; Alumur *et al.*, 2009; Campbell *et al.*, 2005a; 2005b; Serper and Alumur, 2016). Çalık *et al.* (2009) proposed the hub covering problem together with an incomplete hub network design and solved the problem with a tabu search algorithm. Serper and Alumur (2016) investigated a problem where vehicle types with different capacities serving between hubs were taken into consideration. A mixed integer mathematical formulation was proposed, and a reduced variable neighborhood search heuristic was developed to solve the problem. Although the authors considered different vehicle capacities between hubs, they did not allow the vehicles to perform local tours. Unlike the studies by Serper and Alumur (2016) and Çalık (2009), in our work here, we allow vehicle capacities to be taken into consideration in local tours by combining flow volumes and the distances traversed by vehicles together.

O'Kelly and Miller (1994) worked on a problem where the connections between hubs were not complete. An urban public transport problem, by determining the locations of the hubs and hub links over the incomplete hub network, was considered in Nickel *et al.* (2001). Alumur *et al.* (2009) presented mathematical models for all hub location problem types in the literature over incomplete hub network designs. The complexity of the mathematical models and decision variables were $O(n^3)$, including the single allocation p -hub median and fixed costs hub location problems, p -hub center problem, and hub covering problem. Alumur and Kara (2009) presented a mathematical model and linearizations for the hub covering problem for a special case in the hub network design. Campbell *et al.* (2005a; 2005b) introduced the hub arc location problem into the literature, and the authors proposed different connection types within the hub network. The multimodal hub location problem over the incomplete hub network design was considered by Alumur *et al.* (2012). The authors developed a mathematical model and a two-stage heuristic for the solution. Alibeyg *et al.* (2016) considered the issue of profit in the hub location problem. The authors developed a mathematical model that determined which hub links should be activated. Alibeyg *et al.*

(2018) developed a mixed-integer mathematical model formulation that was designed to maximize the revenue of the hub network design and proposed a method based on the Lagrangian algorithm. Dai *et al.* (2019) developed a variable neighborhood search heuristic based on the tendency of which hub links to be included in the solution for the multiple allocation hub network design problem. Zhang *et al.* (2022) tackled the stochastic incomplete multimodal hub location problem and proposed two Benders decomposition algorithms to solve it. Oliveria *et al.* (2022) dealt with the profit-maximizing hub location problem by considering incomplete hub network design decisions. The authors developed a Benders decomposition method to solve the problem with up to 150 nodes. An intermodal green p -hub median problem under incomplete hub networks was solved by Ibnoulouafi *et al.* (2022).

The outline of this paper is as follows. In the second section, we present the single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up/delivery and the computational results of this problem on the Turkish network. The third section considers the same problem over incomplete hub networks. We tested our proposed mathematical models using a reduced data set with the optimization software GAMS 34.0 with CPLEX solver. In the last section, we give our concluding remarks and future research directions.

2. p -HUB MEDIAN AND CAPACITATED VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICK-UP AND DELIVERY

In this section, we present a mixed-integer mathematical model formulation for the single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up/delivery. The problem aims to determine the locations of the hubs, the assignment of demand centers (non-hub nodes) to hubs, and the routing of vehicles to visit all the demand centers. In this problem, the vehicles can visit multiple demand centers on a route without exceeding vehicle capacities by picking up (collecting) the **supply** and delivering (distributing) the **demand** simultaneously in such a way that the total transportation cost is minimized.

Formally, we added the capacity of the vehicles used on the routes into the three-index multi-commodity flow formulation proposed by Kartal *et al.* (2017). We note here that Kartal *et al.* (2017) developed a multi-commodity flow formulation based on Ernst and Krishnamoorthy's (1996) study in which the authors proposed a three-index multi-commodity flow formulation for the single allocation p -hub median problem. The proposed mathematical model here could be reduced to the p -hub median problem if there were no capacity limit on the vehicles and if there were only one vehicle dedicated between each demand center and hub.

We start by presenting the problem definition and parameters in the first subsection. We propose the mixed-integer mathematical model formulation in Section 2.1. The computational results of the sensitivity analysis performed on the Turkish network with a reduced data set are presented in Section 2.2.

2.1 Mathematical Model-1

The single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up/delivery can be described as follows: $G = (N, A)$ is a complete network, N consists of n demand nodes and a potential hub set H where $N=H$ for the rest of the paper. A link i - j connects each pair of nodes with a routing cost (distance, time, cost). In the mathematical model, it is assumed that there are homogeneous *capacitated* vehicles that wait empty in the hubs. The tour of a vehicle starts with the loading of the cargo/parcels into the vehicle to be transferred to the demand centers on the route that the vehicle will visit. Each route begins from a hub node and continues throughout the non-hub nodes. The route is completed with the return of the vehicle to the same hub. Each vehicle visits at least one non-hub node. However, the vehicles are allowed to visit multiple nodes if their capacity limits are not exceeded. We also assume that there are dedicated faster vehicles between each pair of hubs. There is not any capacity limitation for these vehicles. Moreover, they are not permitted to make stopovers along their routes at this stage.

In the proposed mathematical model, since the flow variables eliminate sub-tours, there is no need to use sub-tour elimination constraints. The parameters used in the mathematical model are listed below:

Parameters

α	= hub-to-hub transportation cost discount factor
W_{ij}	= flow from node $i \in N$ to node $j \in N$
d_{ij}	= distance between node $i \in N$ and node $j \in N$
n_i	= number of vehicles that are assigned to hub node $i \in N$
cap_v	= capacity of the vehicle v
p	= the number of hubs to be opened
M	= a big number

Also, let,

$$\begin{aligned}
 O_i &= \sum_j W_{ij} \text{ the total amount of flow originating from node } i. \text{ We then define the decision variables of the} \\
 &\text{mathematical model.} \\
 f_{kj}^i &= \text{the amount of flow that originating from node } i \in N \text{ and travels, passing through hub } j \in H \text{ to hub } k \in H. \\
 y_{kj}^i &= \text{the flow that originating at node } i \text{ and travels from node } j \text{ to node } k, \text{ at least one of } j \text{ and } k \text{ must be a demand} \\
 &\text{node, and the other may be a hub.} \\
 x_{ij}^v &= \begin{cases} 1, \text{ if vehicle } v \text{ visits arc } (i, j) \text{ in that order;} \\ 0, \text{ otherwise.} \end{cases} \\
 h_i &= \begin{cases} 1, \text{ if node } i \in H \text{ is opened as a hub node;} \\ 0, \text{ otherwise} \end{cases} \\
 z_{iv} &= \begin{cases} 1, \text{ if hub node } i \in H \text{ is allocated to vehicle } v; \\ 0, \text{ otherwise} \end{cases}
 \end{aligned}$$

We refer to the single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up and delivery problem is Model-1, and it is presented below:

Model-1:

$$\begin{aligned}
 \text{Min} \sum_i \sum_j \sum_k (d_{jk} * y_{jk}^i) + \sum_i \sum_j \sum_k (\alpha * d_{jk} * f_{jk}^i) & \quad (1) \\
 \sum_{k \neq i} (y_{ik}^i + f_{ik}^i) = O_i & \quad \forall i, \quad (2) \\
 y_{jk}^i \leq O_i * \sum_v x_{jk}^v & \quad \forall i, j, k \neq j, \quad (3) \\
 \sum_{j \neq k} (f_{jk}^i + f_{kj}^i) = O_i * h_k & \quad \forall i, k, \quad (4) \\
 \sum_k (y_{kj}^i + f_{kj}^i) = \sum_k (y_{jk}^i + f_{jk}^i) + W_{ij} & \quad \forall i, j \neq i, \quad (5) \\
 \sum_i h_i = p & \quad (6) \\
 \sum_{j \neq i} \sum_v x_{ij}^v - \sum_v z_{iv} = 1 - h_i & \quad \forall i, \quad (7) \\
 z_{iv} \leq \sum_{j \neq i} x_{ij}^v & \quad \forall i, v, \quad (8) \\
 \sum_i z_{iv} = 1 & \quad \forall v, \quad (9) \\
 \sum_v z_{iv} = h_i * n_i & \quad \forall i, \quad (10) \\
 \sum_{j \neq i} x_{ij}^v - \sum_{j \neq i} x_{ji}^v = 0 & \quad \forall i, v, \quad (11) \\
 \sum_{i \neq k} y_{jk}^i \leq \text{cap}_v + M * (1 - x_{jk}^v) & \quad \forall j, k \neq j, v, \quad (12) \\
 z_{iv}, x_{ij}^v, h_i \in \{0, 1\} & \quad \forall i, j, v, \quad (13) \\
 y_{jk}^i, f_{jk}^i \geq 0 & \quad \forall i, j, k, v, \quad (14)
 \end{aligned}$$

The objective function minimizes the total transportation costs on the network (1). In the first term of the objective function, the costs of the capacitated vehicle routes are calculated. In the second term, the costs between the hubs are calculated. Constraint (2) ensures that the total flow sent from any node i is the same as the amount of supply by node i . Constraint (3) guarantees that the i -flow passing through k - j nodes can only occur if there is an assigned vehicle between nodes k and j . Constraint (4) inhibits the hub flows from passing through nodes that are not hub nodes. Constraint (5) assures that the i -flow is maintained at any other node j with demand W_{ij} . Constraint (6) ensures that exactly p number of hubs are opened. Constraint (7) ensures that if node i is a hub node, v vehicles are allocated to it, and if node i is not a hub node, only one vehicle visits this node. Constraint (8) ensures that if vehicle v is assigned to hub node i , the same vehicle serves the arc between node i and node j . Constraint (9) ensures that each vehicle is assigned to only one hub. Constraint (10) ensures that if a node is a hub node, then the total number of vehicles dedicated to that hub is n_i . Constraint (11) is a degree constraint that ensures that the number of incoming and outgoing vehicles from a node is equal.

Constraint (12) relates to the capacity limit of the vehicles and ensures that the amount flow assigned to vehicle v does not exceed the capacity threshold. This constraint has been added to Kartal *et al.*'s (2017) three-index multi-commodity flow formulation mathematical model, as discussed earlier. We note here that with Constraint (12), we also allow each vehicle to have different capacities. If the capacities of the vehicles are different, the problem becomes a p -hub median and *heterogenous capacitated* vehicle routing problem with simultaneous pick-up and delivery. However, for the sake of simplicity, we assume here that the vehicles and their capacity limits are identical. Constraints (13) and (14) show binary and non-negativity requirements.

In the worst case, where there are flows between each pair of the n -nodes (origin/destination), there are $n^3 + n^2v + 2n^2 + 2nv + 3n + v$ constraints and $2n^3 + n^2v + n^2 + n$ variables. In the following subsection, we present some computational analysis of Model-1.

2.2 Computational Results with Model-1

We tested the performance of Model-1 on the Turkish network data set, which was introduced into the literature in Tan and Kara (2007)¹. There are 81 demand centers in this data set. Flow amongst these demand centers and the distances in this network are also given in the aforementioned study. Turkish network data set is also available from the OR library (Beasley, 1990). In our study, we used a reduced dataset based on 10 cities where industry and population are relatively dense. These cities are taken from Yaman *et al.*'s (2007) study. The reasons for choosing these cities are that they are well-separated geographically in Türkiye, and since the complexity of the problem is $O(n^3)$, in our preliminary trials, we saw that larger dataset sizes could not be solved in a reasonable amount of time. During this study, all the experiments were performed on GAMS 34.0 software with CPLEX solver on a computer with a 64-bit Intel i7 (2.80 GHz) processor and 4 GB RAM.

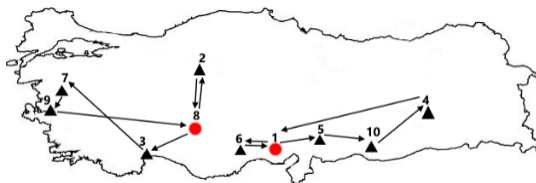
The proposed mathematical model was tested on a data set containing ten nodes of the Aegean, Mediterranean, Central Anatolian, and Southeast Anatolian regions of Türkiye. These cities are 1- Adana (1), 2- Ankara (2), 3- Antalya (3), 5- Diyarbakır (4), 7- Gaziantep (5), 8- Icel (6), 10- Manisa (7), 11- Konya (8), 12- İzmir (9), and 16- Urfa (10), respectively. For this data set, the number of hubs to be opened (the p number) was set as 2 or 3, and the number of vehicles assigned to each hub was 1 or 2.

We report the optimal hub location and route structure results of Model-1 for the 10-node instances with different hub and vehicle sets in Table 1, Table 2, and Table 3. For each problem instance, these tables report the number of nodes (n), the number of hubs to be located (p), and the number of vehicles assigned to each hub (v), shown by 'n.p.v'. In the next columns, the locations of the hub nodes, the routes, the objective function, the required CPU time in seconds, the number of branch-bound nodes, and the vehicle capacities used to solve the models are presented, respectively.

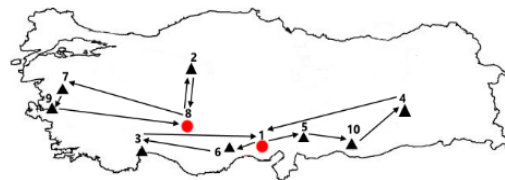
Table 1. p -Hub Median and *Capacitated Vehicle Routing Problem* with Simultaneous Pickup and Delivery on TR10.2.2

n.p.v	Hubs	Routes	Objective Function	CPU (sec)	Nodes	Vehicle Capacity
TR.10.2.2	1,8	1-5-10-4-1; 1-6-1; 8-2-8; 8-3-7-9-8;	4783310818	238	2732	Uncapacitated
TR.10.2.2	1,8	1-5-10-4-1; 1-6-3-1; 8-2-8; 8-7-9-8	5052369041	450.64	7078	1,250,000
TR.10.2.2	1,8	1-5-10-4-1; 1-6-3-1; 8-2-8; 8-7-9-8	5052369041	552	8472	1,200,000
TR.10.2.2	2,8	2-5-10-4-2; 2-7-2; 8-6-1-8; 8-3-9-8	5609194426	2466	41285	1,150,000
TR.10.2.2	2,8	2-5-10-4-2; 2-7-2; 8-6-1-8; 8-3-9-8	5609194426	2153	28842	1,100,000
TR.10.2.2	2,7	2-10-4-2; 2-8-6-2; 7-1-5-7; 7-9-3-7	6961676615	16308	315414	1,050,000
TR.10.2.2	2,7	2-1-6-2; 2-10-4-2; 7-9-3-7; 7-8-5-7	7019240316	9083	169164	1,000,000

a. Uncapacitated



b. 1,200,000-1,250,000 Units



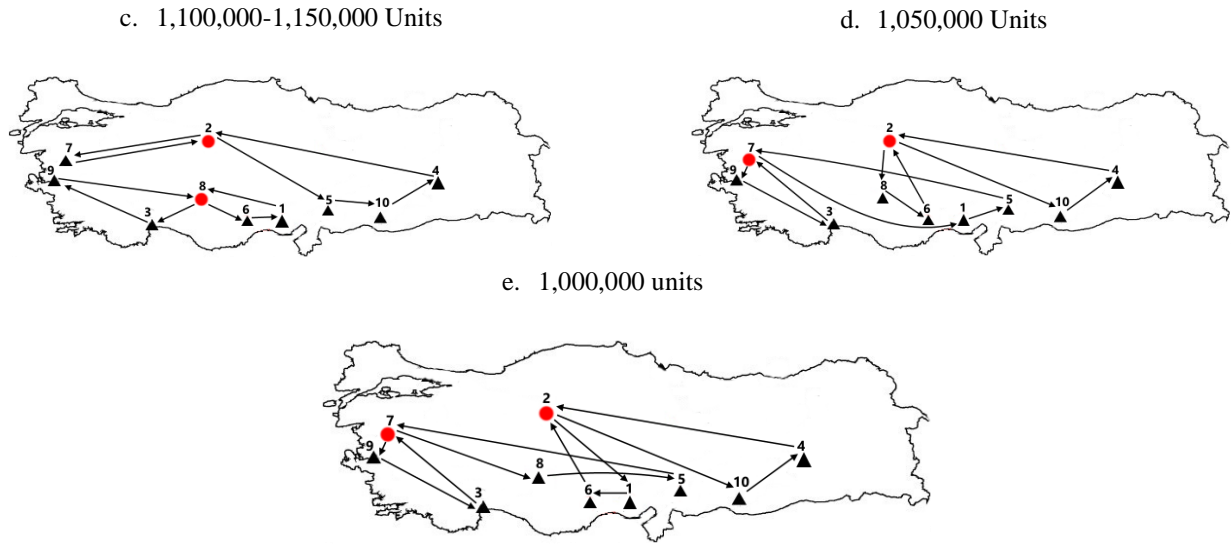


Figure 1. *p*-Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery Solutions on Turkey Map for TR10.2.2

To explain the findings in detail, we first present the solutions of the TR10.2.2 instances using different capacities. Figure 1 illustrates the optimal hub location-routing network structure of the solutions. Looking at Table 1 and Figure 1, the optimal hub locations were observed to change three times in total depending on vehicle capacities. On the other hand, vehicle capacities started from a value that is, on average, one-quarter of the total flow, corresponding to 1,000,000 units. First, this value was reduced to 900,000 units with a 10% decrease, but the solution was unfeasible due to the lack of capacity. The same result was observed in 950,000 units, which indicates a 5% decrease. Therefore, when the vehicle capacities were increased, it was decided that the value of 5% was sufficiently sensitive, and subsequent increases were made in 5% increments.

When we ranked the amount of flow values that were generated from the nodes in the data set, Ankara (2) had the largest flow, Manisa (7), and Konya (8) were second and third, respectively. Accordingly, as can be seen in Figure 1.e., when capacity is at its tightest (1,000,000 units), hub locations were established in Ankara (2) and Manisa (7). The reason for installing hubs on these nodes at the tightest vehicle capacity is that the capacity of the vehicles will only be sufficient to pick up and deliver the cargoes of other nodes where the hubs are established in the centers, and the flow is the most intense. When the vehicle capacities were increased to 1,100,000 units and 1,150,000, it was observed that Ankara (2) remained constant, but another hub was established in Konya (8), which ranks third in terms of flow density (Figure 1.c.). This follows our expectations to observe hubs on nodes that generate less flow as the vehicle capacity constraints become looser.

The effects of vehicle capacities on the route structure were also investigated. Even if the hub locations did not change, there were differences in the route structure with the capacity increase/decrease. As shown in Figure 1. d. and Figure 1.e., although there was no change in the locations of the hubs, Konya (8) was assigned to the Manisa (7) hub in the tightest vehicle capacity, whereas when the capacity was increased, it was assigned to the Ankara hub nearby (Figure 1.d.). Unsurprisingly, as the vehicle capacity constraint is relaxed, the vehicles can carry more flows on them, which enabled Konya to be assigned to a closer hub in the network, causing a further decrease in the objective function.

The results show a large range of CPU times (238 to 16308 seconds). In general, by solving the model under different scenarios, with the increase in capacity constraint, CPLEX required a longer time to find the optimal solution. Another important point is that the number of branch-and-bound nodes produced for the optimal solution of the model increased proportionally to the CPU time required for the solution of the models. Accordingly, in the model where there was no vehicle capacity, the optimum solution was found in 238 seconds, and the number of branch-bound nodes produced by the solver was 2732. However, in the case where the vehicle capacities constraint was the loosest, the required CPU time increased to 450.61 seconds and the number of branch-bound nodes to 7078. When the vehicle capacity was decreased by 5%, it was observed that while the hub locations and routes did not change, the CPU time and the number of branches-bound nodes increased dramatically. This may have resulted from finding routes faster due to the sufficiently tight capacity of the vehicles. The most pronounced effect of vehicle capacity was the instance where the capacity was the second tightest. This problem, with a capacity of 1,050,000 units, was the hardest to solve as it took CPLEX around 16000 seconds, indicating how difficult the capacity constraint makes this model.

The next question we investigated was the effects of vehicle numbers on the solutions. For this purpose, we report the

optimal results of TR10.3.1 in Table 2 and TR10.3.2 in Table 3. In general, the objective function values are higher when one vehicle is assigned to hubs than two. In real-life problems, if decision-makers consider the fixed costs of vehicles, there could be an increase in the objective function. Also, our problem could be reduced to a single allocation p -hub median problem if a separate (uncapacitated) vehicle is dedicated between each demand center and hub. The single allocation p -hub median problem's cost is smaller than our problem's cost (objective function). This is parallel to the decrease in the objective function with the increase in vehicles. The reason for the rise is that the pick-ups and deliveries are cumulative as each node is visited along the route.

Table 2. p -Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery for TR10.3.1

n.p.v	Hubs	Routes	Objective Function	CPU (sec)	Nodes	Vehicle Capacity
TR.10.3.1	3,6,8	3-7-9-3; 6-1-5-10-4-6; 8-2-8	4936427305	471.03	9256	Uncapacitated
TR.10.3.1	3,6,8	3-7-9-3; 6-1-5-10-4-6; 8-2-8	4936427305	477.28	9807	1,500,000
TR.10.3.1	1,2,8	1-5-10-4-1; 2-7-9-2; 8-3-6-8	5053845959	669.07	14506	1,400,000
TR.10.3.1	1,2,8	1-5-10-4-1; 2-7-9-2; 8-3-6-8	5053845959	780.43	13875	1,300,000
TR.10.3.1	1,2,8	1-5-10-4-1; 2-7-9-2; 8-3-6-8	5053845959	798.03	15934	1,200,000
TR.10.3.1	2,7,8	2-5-10-4-2; 7-9-3-7; 8-6-1-8	5397538110	2678.8	51098	1,100,000

As expected, the capacities when two vehicles are allocated to hubs are significantly less than when one vehicle is allocated. While routes can be obtained with a minimum capacity of 750,000 units with two vehicles, this minimum was 1,100,000 units for one vehicle. Therefore, as in real life, cargo company managers and decision-makers can strategically evaluate and decide on the investment and operating costs of more vehicles with lower capacity or fewer vehicles with higher capacity.

Table 3. p -Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery for TR10.3.2

n.p.v	Hubs	Routes	Objective Function	CPU (sec)	Nodes	Vehicle Capacity
TR.10.3.2	5,6,8	5-10-5;5-4-5;6-1-6; 6-3-6;8-2-8; 8-7-9-8	4261097233	86.71	1803	Uncapacitated
TR.10.3.2	5,6,8	5-10-5;5-4-5;6-1-6;6-3-6; 8-2-8;8-7-9-8	4261097233	92.53	1747	1,200,000
TR.10.3.2	1,2,8	1-5-10-1;1-6-1; 2-4-2; 2-9-2; 8-7-8;8-3-8	4454376889	102.29	2733	950,000&1,000,000 1,100,000&1,150,000
TR.10.3.2	1,2,8	1-5-10-1;1-6-1; 2-4-2; 2-9-2; 8-7-8;8-3-8	4454376889	141.73	2744	900,000
TR.10.3.2	1,2,7	1-5-10-1;1-6-1; 2-4-2; 2-8-2;7-3-7;7-9-7	4697132739	53.48	1431	850,000
TR.10.3.2	1,2,7	1-5-10-1;1-6-1; 2-4-2; 2-8-2;7-3-7;7-9-7	4697132739	50.07	1349	800,000
TR.10.3.2	1,2,7	1-5-4-1;1-10-1; 2-6-2; 2-8-2;7-3-7;7-9-7	4876764666	86.45	2140	750,000

We calculated the average CPU times reported in Tables 2 and 3 for TR10.3.1 and TR10.3.2. These averages were 978 and 82 seconds, respectively. Comparing the results for the two problem instances shows that TR10.3.1 was computationally much more demanding than TR10.3.2. This is not surprising because as the vehicle numbers increase, the number of nodes visited on the routes decreases. This makes the problem closer to the single allocation p -hub median problem, which was proven to be solved in much less CPU time (Ernst and Krishnamoorthy, 1996). Therefore, the instances in Table 2 with more node numbers in the routes turned out to be harder. The hardest of these instances was when the capacity was tightest, requiring about 44.6 min of CPU time.

Table 4. p -Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery for TR10.3.2

α	Hub Locations	Routes	Objective Function	CPU (sec)	Nodes
1	1,2,8	1-5-10-1; 1-6-1; 2-4-2; 2-9-2;8-3-8;8-7-8	4454376889	102.29	2733
0.8	1,2,9	1-5-10-1; 1-6-1;2-4-2; 2-8-2;9-3-9;9-7-9	4148898491.6	78.20	2520
0.6	1,2,9	1-5-10-1; 1-6-1;2-4-2; 2-8-2;9-3-9;9-7-9	3629639672.2	33.56	626
0.4	1,2,9	1-5-10-1; 1-6-1;2-4-2; 2-8-2;9-3-9;9-7-9	3110380852.8	19.92	964
0.2	2,5,7	2-6-2; 2-8-2; 5-1-5; 5-10-4-5;7-3-7;7-9-7	2536932465.6	9.09	328

In order to observe the effects of the inter-hub discount factor α on the solution, **TR10.3.2** was solved with 950,000 units capacity (Table 4), and we report here the optimal solutions. The reason for selecting this example is that it represents the situation in which the capacity change (from 950,000 to 1,150,000 units) in a very large range resulted in the same optimal objective value. It can be observed in Table 4 that the values of α cause the hub locations to change three times. Accordingly, when the value of the discount factor α is 0.2, the hubs are established in the provinces of Ankara (2), Gaziantep (5), and Manisa (7). According to the data set under consideration, Ankara and Manisa are the two nodes with the highest flow. When the discount factors were 0.4, 0.6, and 0.8, although the routes and hub locations remained the same, the objective function values increased as the discount factor increased. When $\alpha=1$, the hubs were located in Ankara (2), Adana (1), and Konya (8). These nodes were established in the first, third, and fourth ranks in terms of the demand-generating nodes, respectively. We also observe that the routes vary with each hub change. We concluded that as the inter-hub discount factor increases, the tendency of hubs to be installed in close proximity to each other in more central places, together with the hard capacity constraint, makes the forming of the routes difficult and, thus, the problem becomes computationally more challenging. The minimum CPU time requirement was 9.09 seconds for $\alpha=0.2$, while the maximum was recorded as 102.29 seconds for $\alpha=1.0$.

In the next section, we present the single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pickup/delivery over an incomplete hub network. We formally present the mixed integer mathematical model of the problem in the next subsection.

3. p -HUB MEDIAN AND CAPACITATED VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICK-UP AND DELIVERY OVER INCOMPLETE HUB NETWORKS

The single allocation p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up and delivery over incomplete hub networks aims to determine the locations of the hubs, the allocations of non-hub nodes to the hubs, to form the vehicle routes by considering capacities, and to decide which hub links should be established. In **Section 3.1**, the introduction of the problem and the mixed-integer mathematical model are presented. In **Section 3.2**, we present the computational results of the sensitivity analysis performed on the same reduced Turkish network.

3.1 Mathematical Model-2

For this model, we need to emphasize that the modeling logic for the incomplete part was based on the work in the studies by Alumur *et al.* (2009) and Campbell *et al.* (2005a; 2005b) where, in the former study, the authors determined which hub links were to be established in the single allocation p -hub median problem. In the latter study, the authors worked on establishing hub arcs and bridge arcs in the multiple allocation p -hub median problem. Since we consider the single allocation scheme, we integrated Alumur *et al.*'s (2009) modeling approach into our formulation.

We refer to this formulation as Model-2. Decision variables and parameters used in addition to those of Model-1 are listed below.

Parameter:

q = number of links to be established.

Decision variable:

$$l_{ij} = \begin{cases} 1, & \text{if a hub link is established between hub nodes } i \in H \text{ and } j \in H; \\ 0, & \text{otherwise.} \end{cases}$$

Model-2:

Min (1)

s.t. (2)-(5), (6)-(15)

$$\sum_i \sum_{j>i} l_{ij} = q \quad (15)$$

$$l_{ij} \leq h_i \quad \forall i, j: j>i, \quad (16)$$

$$l_{ij} \leq h_j \quad \forall i, j: j>i, \quad (17)$$

$$f_{kij} + f_{kji} \leq O_k * l_{ij} \quad \forall i, j, k: j>i, \quad (18)$$

$$l_{ij} \in \{0,1\} \quad \forall i, j: j>i, \quad (19)$$

The objective functions of Model-1 and Model-2 are equivalent. The objective function minimizes the total transportation costs on the network. Constraint (15) assures that exactly q number of hub arcs will be located. Constraints (16) and (17) together ensure that the endpoints of established hub links will be the hub nodes. We note here that the hub link variables l_{ij} , $j>i$, as used to establish undirected hub arcs, i.e., they allow flow routing in both directions. Constraint (18) establishes a hub arc for every hub arc flow. Constraint (18) guarantees that if there are established hub links, then f variables must be positive in those connections.

In the worst case, where there are flows between each pair of the n -nodes (origin/destination), there are $2n^3 + n^2v + 4n^2 + 2nv + 3n + v$ constraints and $2n^3 + n^2v + 2n^2 + n$ variables. In the following subsection, we present some computational analysis of Model-2.

3.2 Computational Results

We tested the performance of our model on the same reduced Turkish network 10-node data set as stated in the previous section since instances with $n=10$ could be solved optimally in a reasonable amount of time. We also used the same example structure (e.g., TR10.2.2) to see the effects of an incomplete hub network design compared to a complete hub network. For this purpose, in the first row of Table 5, the experiments are formed as 'n.p.v.q'. As explained in Section 2, n is the total number of nodes, p is the number of hubs to be opened, v is the number of vehicles per hub, and q is the number of links to be established. In the following tables, unlike the previous section, the links that were established are given in the second column. We also present the percentage increase in the objective function in Column 6. The objective function of the incomplete solution is $Obj_{incomplete}$, and for the complete hub network it is $Obj_{complete}$. The percentage increase in the objective function was calculated as follows: $\frac{Obj_{incomplete} - Obj_{complete}}{Obj_{complete}} * 100$.

Table 5. p -Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery over Incomplete Hub Networks for TR10.2.2

n.p.v.q	Hub Locations (Links between hubs)	Routes	Objective Function	CPU (sec)	%Increase in total costs	Nodes	Vehicle Capacity
TR.10.2.2.1	1,8;(1-8)	1-5-10-4-1;1-6-1;8-2-8; 8-3-7-9-8	4783310818	174.15	0	2256	Uncapacitated
TR.10.2.2.1	1,8;(1-8)	1-5-10-4-1;1-6-3-1;8-2-8;8-7-9-8	5052369041	551.92	0	5557	1,250,000
TR.10.2.2.1	1,8;(1-8)	1-5-10-4-1;1-6-3-1;8-2-8;8-7-9-8	5052369041	471	0	3812	1,200,000
TR.10.2.2.1	2,8;(2-8)	2-5-10-4-2;2-7-2;8-3-9-8; 8-6-1-8	5609194426	3665	0	42954	1,150,000
TR.10.2.2.1	2,8;(2-8)	2-5-10-4-2;2-7-2;8-3-9-8; 8-6-1-8	5609194426	1725.97	0	28842	1,100,000
TR.10.2.2.1	2,7;(2-7)	2-8-6-2;2-10-4-2;7-1-5-7; 7-9-3-7	6961676615	25495	0	334878	1,050,000
TR.10.2.2.1	2,7;(2-7)	2-1-6-2; 2-10-4-2;7-8-5-7;7-9-3-7	7019240315	9802	0	178090	1,000,000

In Table 5, the optimal solutions of the TR10.2.2.1 instances using the capacities given in Table 1 are presented. In the table, the value of q is 1, and as there are 2 hubs to be located, the value of q can be at most 1 because we use an undirected hub network. We observe that the objective function values are the same as the complete hub network design. The reason for this is that Model-1 and Model-2 establish the same hub link for this particular instance where there are two hubs and one hub link in the network. This also caused the locations of the hubs and routes not to differ in the optimal solution.

Table 6. *p*-Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery over Incomplete Hub Networks for TR10.3.1

n.p.v.q	Hub Locations (Links between hubs)	Routes	Objective Function	CPU (sec)	%Increase in total costs	Nodes	Vehicle Capacity
TR.10.3.1.2	1,3,8; (1-8;3-8)	1-6-5-10-4-1; 3-7-9-3; 8-2-8	5182094128	1869.81	4.9	12827	Uncapacitated
TR.10.3.1.3	3,6,8; (3-6;3-8; 6-8)	3-7-9-3; 6-1-5-10-4-6; 8-2-8	4936427305	1007.68	0.0	8542	Uncapacitated
TR.10.3.1.2	1,2,8; (1-8;2-8)	1-5-10-4-1; 2-7-9-2; 8-3-6-8	5248623051	2089.42	3.8	13432	1,400,000
TR.10.3.1.3	1,2,8; (1-2;1-8; 2-8)	1-5-10-4-1; 2-7-9-2; 8-3-6-8	5053845959	1417.79	0.0	15340	1,400,000
TR.10.3.1.2	2,7,8;(2-7;2-8)	2-5-10-4-2; 7-9-3-7; 8-6-1-8	5712885428	7350.07	5.8	53994	1,100,000
TR.10.3.1.3	2,7,8; (2-7;2-8; 7-8)	2-5-10-4-2; 7-9-3-7; 8-6-1-8	5397538110	3546	0.0	45797	1,100,000
				Average	2,41		

In Table 6, *q* values for the TR10.3.1 problem were considered as 2 and 3; in addition, a diverse set of capacity values was used. While selecting the capacities, the instances where the hub locations vary in the complete hub network design were considered (Table 2). We first analyzed the increase in transportation costs in the incomplete hub network design decisions with respect to complete hub networks. We observed that the highest increase was found to be 5.8%, and the lowest was 3.8%. The average increase in the objective function was 2.41%. All the increases occurred where the number of links to be established between hubs was equal to the number of hubs. Since the flow between hubs is not always routed via direct hub pairs in incomplete hub networks, this can cause an increase in transportation costs, which is the main disadvantage of establishing such a network design. However, we believe that the increase in transportation costs in the incomplete hub networks corresponding to establishing complete hub location and routing networks can be tolerated in most real-life applications.

On average, the problem instances in Table 6 were solved optimally in 47.9 min CPU time. We observed that instances where we allowed the hub networks to be sparse ($q=2$) were significantly harder to solve than instances with more connections in the hub networks ($q=3$). The most computationally demanding example was TR10.3.1.2, with the tightest capacity (1,100,000 units), requiring just over 2h of CPU time. In contrast, we observed the minimum CPU requirement was 16.6 min for the TR10.3.1.3 instance, which had the loosest capacity (uncapacitated) and three connections in the hub networks. We conclude that when the number of links to be established is lower, it causes the model to be harder to solve, so the required CPU time for the model to reach the optimum solution is expected to increase.

Table 7. *p*-Hub Median and Capacitated Vehicle Routing Problem with Simultaneous Pickup/Delivery over Incomplete Hub Networks for TR10.3.2

n.p.v.q	Hub Locations (Links between hubs)	Routes	Objective Function	CPU (sec)	%Increase in total costs	Nodes	Vehicle Capacity
TR.10.3.2.2	1,5,8 (1-5;5-8)	1-3-1;1-6-1; 5-4-5; 5-10-5;8-2-8; 8-7-9-8	5002884049	278	0	1559	1,200,000& Uncapacitated
TR.10.3.2.3	5,6,8 (5-6; 5-8; 6-8)	5-10-5;5-4-5; 6-1-6; 6-3-6;8-2-8; 8-7-9-8	4261097233	187.96	0,6	2240	1,200,000& Uncapacitated
TR.10.3.2.2	1,2,8 (1-8;2-8)	1-5-10-1;1-6-1; -4-2;2-9-2; 8-7-8; 8-3-8	4610729109	3665	0	42954	950,000
TR.10.3.2.3	1,2,8 (1-2;1-8; 2-8)	1-5-10-1;1-6-1; 2-4-2;2-9-2; 8-7-8; 8-3-8	4454376889	413.875	3,5	2819	950,000
TR.10.3.2.2	1,2,7 (1-2;2-7)	1-5-4-1;1-10-1; 2-6-2;2-8-2;7-3-7; 7-9-7	5069476458	218.5	0	2090	750,000
TR.10.3.2.3	1,2,7 (1-2;1-7; 2-7)	1-5-4-1;1-10-1; 2-6-2;2-8-2;7-3-7;7-9-7	4876764666	145.891	3,9	2891	750,000
				Average	1,33		

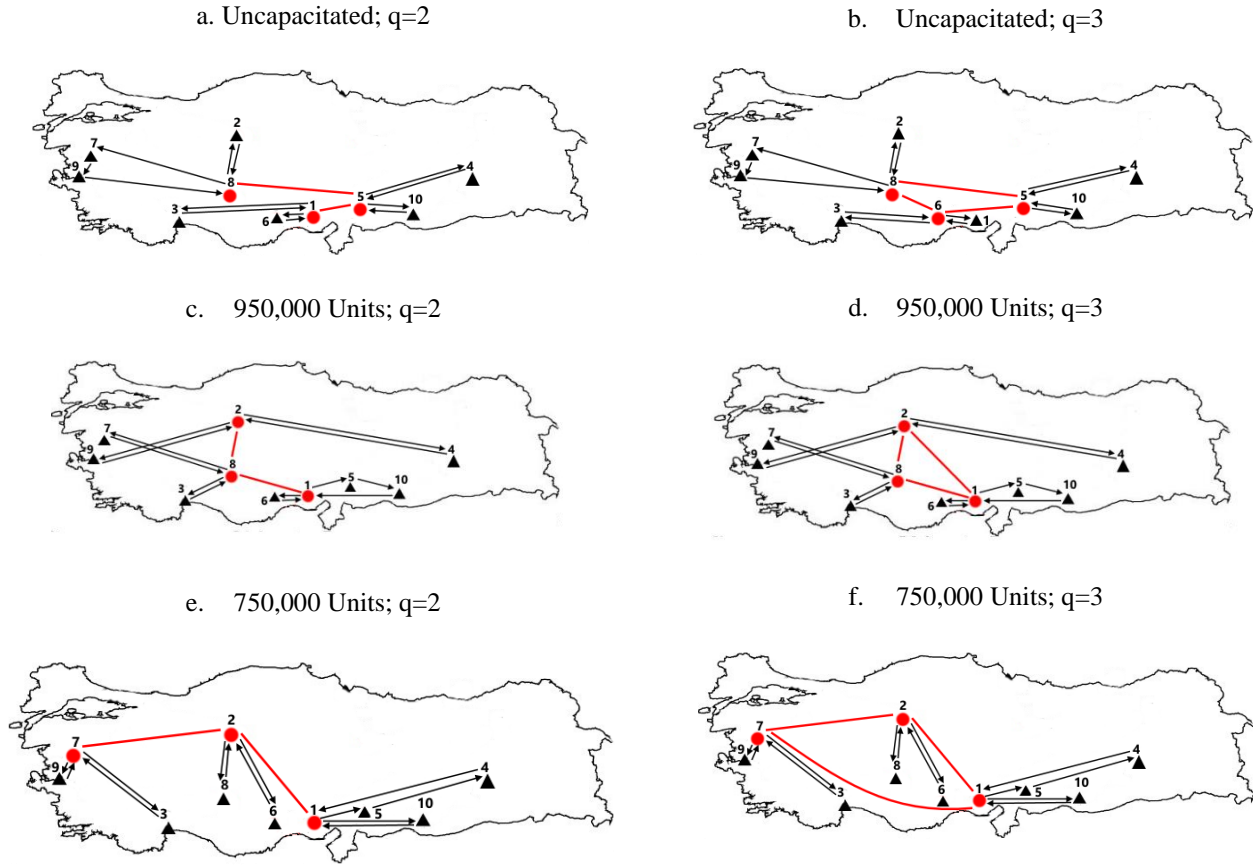


Figure 2. p -Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery Solutions over Incomplete Hub Network on Turkey Map for TR10.3.2

We also tested different q values (2 and 3), with capacities of 750,000 units, 950,000 units, and uncapacitated cases for TR10.3.2. The corresponding results are provided in Table 7. We illustrated the resulting optimal solutions on the hub location-routing network in Figure 2. Here, the links established between the hubs are shown in red, and the routes are shown in black.

We first investigated the effect of the number of hub links when the vehicle capacities are unchanged in the incomplete hub location and routing network design. For this purpose, we first analyzed TR10.3.2.2. with the uncapacitated case. In this case, when $q=2$, the hubs were opened in Adana (1), Gaziantep (5), and Konya (8) (Figure 2.a.). These nodes are fourth, ninth, and third rank in terms of flow-generating nodes in the network. The largest flow in the network was between hubs Gaziantep (5) and Konya (8), and the second largest flow was found between Adana (1) and Gaziantep (5). This was in line with our expectations to obtain high flows on hub links rather than route links. The third, fourth, and fifth route links are İzmir (9)-Konya (8), Konya (8)-Manisa (7), and Manisa (7)-İzmir (9), respectively. Since Manisa is the second-largest flow node in the network, it is not surprising that two links are passing through Manisa in the top five of the links that carry the most flow. In addition, three out of the first five links were on the route where Manisa was present. When $q=3$, a feature of the complete hub network characteristic has been preserved, and the hub in Adana (1) was replaced by Icel (6), which is a lower demand generating node. The total flow value passing through the hub links was also higher when $q=2$ than when $q=3$. The hub link with the highest flow in the network was transferred via Icel (6)-Konya (8); Icel was the sixth and Konya the third node in decreasing order of flows; and the hub link with the least flow arises between Gaziantep (5)-Icel (6). In the examples where the capacity was 750,000 units and 950,000 units, the increase in the number of links to be established did not cause any change in the locations of the hub nodes and routes. However, unlike the uncapacitated case, the amount of flow routed via hub links did not show a similar trend to the flow order that hubs generated. The reason for this can be explained as the difference in flow due to simultaneous pick-up and delivery on the routes by considering capacity constraints.

As can be seen in Table 7, the highest percentage increase in the objective function was 3.9%, where the capacity was 750,000 units, and the number of links to be established was 3. The lowest objective function increase (0.6%) was in the

uncapacitated case in the TR10.3.2.3. The greatest increases in the objective function occurred in cases with the lowest capacity. This can be explained because hubs have to transfer a very large amount of flow due to the low capacity of the vehicle on the routes. Therefore, in an incomplete hub network, this causes a more significant increase in the percentage of the objective function.

The next question we investigated was the effect of vehicle numbers on the incomplete hub design and hub location-routing network design. It can be observed from Table 6 and Table 7 that increasing the number of vehicles caused a decrease in the objective functions. The objective function was higher in the cases where the number of vehicles assigned to each hub was one than when the number of vehicles was two. It was also seen that all the examples with two vehicles required shorter CPU times than the examples with one vehicle. Similarly, when the number of vehicles stays the same and the number of hubs increases, both the objective function value and the required CPU time decrease (Table 5; Table 7).

Lastly, to see the effect of the discount factor α on the hub location-routing network design, TR10.3.2.3 with 1,200,000 units capacity was considered. We note here that this example also fits the uncapacitated case. We varied α from 0.2 to 1.0 as customarily done in the literature. The hubs, the hub links, and the routes are summarized in Table 8. Table 8 shows that the objective function values increased in parallel to α values. Hub locations are prone to change with respect to various inter-hub discount factors and relative to the total demand. For instance, Konya (8) had the third largest flow node in the data set, and it was chosen as a hub in every instance in Table 8. Another reason for this is that Konya has a central location in Türkiye. Again, it was ranked third when we summed each node's distances to the other nodes in the network.

Table 8. p -Hub Median and *Capacitated* Vehicle Routing Problem with Simultaneous Pickup/Delivery over Incomplete Hub Network for TR10.3.2

e	Hub Locations (Links between hubs)	Routes	Objective Function	CPU(sec)	Nodes
1	5,6,8 (5-6;5-8;6-8)	5-10-5;5-4-5;6-1-6;6-3-6; 8-2-8;8-7-9-8	4261097233	187.96	1747
0.8	5,6,8 (5-6;5-8;6-8)	5-10-5;5-4-5;6-1-6;6-3-6; 8-2-8;8-7-9-8	3953204425	208.46	2603
0.6	5,8,9 (5-8;5-9;8-9)	5-4-5;5-10-5;8-2-8; 8-6-1-8;9-3-9;9-7-9	3598929059	110.76	2668
0.4	8,9,10 (8-9;8-10; 9-10)	8-2-8; 8-6-1-8; 9-3-9; 9-7-9; 10-4-10;10-5-10	3081052249.4	81.68	1646
0.2	7,8,10 (7-8;7-10; 8-10)	7-3-7;7-9-7;8-2-8;8-6-1-8; 10-4-10;10-5-10	2507432755.4	52.641	1480

Figure 3. illustrates the network structure of optimal solutions from TR10.3.2.3 instances with different discount factor values. Five inter-hub discount factor values give rise to four different solutions both in the hub networks and also in the routing structure. Unsurprisingly, the discount factor causes a change in the route structures when the optimal hub set varies in the solution. However, when the same hubs are included in the optimal solution, the routes for these hubs generally remain the same. The routes for Konya (8) are stable in Figure 3 (Figure 3.b.; 3.c.; 3.d). Interestingly, good routes and hub links are preserved, although there are some changes in other parts of the network.

When the discount factor $\alpha=0.2$, the hubs were located far from each other: Manisa (7), Konya (8), and Urfa (10), with the sum of the distances of the three hub links being 2494 km (Figure 3.d.). In contrast, when α was 1.0 and 0.8 (Figure 3.a.), the hubs were established centrally; in Gaziantep (5), Icel (6), and Konya (8), with a total distance of 1185 km. Parallel to the changes in hub locations, and there were also differences in the routes. As the discount factor increases, hubs can serve more distant cities since they are centrally located, and when this value decreases, as can be observed in Figure 3, hubs serve nearby cities. In conclusion, as the discount factor decreases, the benefit obtained from established hub links is greater than the gains obtained through simultaneous pick-up and delivery. Therefore, while hubs are spread farther apart in the hub network with greater inter-hub discount factors, optimal vehicle routes are shortened.

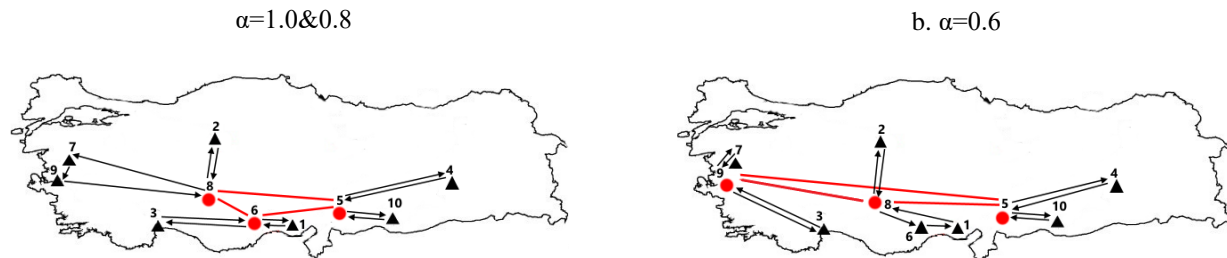




Figure 3. TR10.3.2.3 problem solutions with respect to different discount factors with 1,200,000 units capacity.

We conclude that the location of nodes and hubs, the capacities of the vehicles, and the number of hub links in forming the hub location-routing network has a significant effect on such a challenging problem. The problems discussed in this study are especially important for cargo and logistics companies for urban deliveries. In general, the total transportation cost tends to decrease with the increasing number of hubs and vehicles. In contrast, the costs of the solutions are likely to increase with the increasing number of hub links and higher vehicle capacities. Higher capacity vehicles with simultaneous pick-up and delivery can visit more nodes, and this causes an increase in costs.

Since the fixed costs of vehicles both operating on the routes and between hubs are not explicitly considered in our models, this is a trade-off against the fixed costs. Managerial insights can be gained from the decrease in costs that can be obtained by investing in more hubs, hub links, and/or dedicated vehicles in the network design. While a more flexible operation network can be achieved with more vehicles, it is inevitable that if the fixed costs are included in the models, the total transportation costs will increase. Moreover, decision-makers would also consider using more vehicles in big cities causes traffic jams, pollution and more CO₂ emission.

We believe that decision-makers for cargo delivery companies could also use our second model to analyze the best trade-off between building an incomplete hub network and an increase in transportation costs. We observed from the solutions that building complete hub networks in practice to provide a cost-effective service is not necessary. In general, when there are low-capacity vehicles in the routing network, hubs transfer a reasonably higher amount of flow, and this results in a higher increase in the percentage of the objective function. However, the increase in total transportation costs to build incomplete hub networks is usually tolerable. Therefore, the advantage in service quality provided by complete hub network designs may not be sufficient to justify the resulting higher transportation costs.

4. CONCLUSIONS

In this study, we first proposed a mathematical model for the p -hub median and *capacitated* vehicle routing problem with simultaneous pick-up and delivery by motivating from real-life observations of daily operational characteristics of cargo services. This mathematical model was designed to consider the capacities of different vehicles, as in real life. The solutions of the mathematical model were tested on a reduced data set based on the Turkish network. The capacities of the vehicles used in routing were observed to play a significant role in the design of hub locations and routes.

We then extended our mathematical model to include incomplete hub network design decisions. In the incomplete hub network, the hub links were generally established sensitively to the amount of flow passing through the hubs since the route structure is heavily dependent on vehicle capacity. We also showed that the service that is provided with a complete hub network might also be delivered by an incomplete hub network while considering capacities on the routing network. Establishing incomplete hub networks will usually result in an increase in overall transportation costs, but this is generally tolerable. Especially in urban transportation, an incomplete hub network design can be preferred by cargo/logistics companies as it will reduce both the total distance traveled among hubs and CO₂ emissions.

Acquiring managerial insights is possible from the savings in costs that can be achieved by deploying further hubs, hub connections, or different vehicles. Our results revealed that our mathematical models are helpful tools for cargo/logistics company decision-makers in the design of hub location, hub arc and routing networks, particularly in metropolitan areas.

The complexity of the mathematical models proposed in this paper is $O(n^3)$, and the mathematical models could solve only relatively small-sized problems optimality in a reasonable amount of time. However, to solve realistic-sized instances, advanced exact or heuristic algorithms are required.

Both the single allocation p -hub median location and *capacitated* vehicle routing problem with simultaneous pick-up and delivery and the incomplete hub network version of this problem could be extended to consider time constraints resulting

from cargo firms' commitments to make deliveries within the same day. Another possible extension might also be focused on developing multiple allocation versions to make the models more flexible.

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