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The Application of Dynamic Models in Operations Management

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The Application of Dynamic Models in Operations Management
by
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Dedicated to my parents and my grandmother.

ABSTRACT OF THE DISSERTATION

The Application of Dynamic Models in Operations Management

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Professor Panos Kouvelis, Chair

The dynamic program is a principal method for analyzing stochastic optimization problems. This dissertation studies three operations management problems that arise in the dynamic environment. The principal motivation behind these comes from the applicability in three areas: the agricultural supply chain, the container shipping industry, and supply chain financing. In the first chapter, we consider the hog production industry, where the hog raising farm should decide the selling strategy among several selling options. The farm also faces the uncertain yield of different weights of hogs and spot price volatility from other interactive markets. In the second chapter, we formulate a blockchain-based cargo reservation system, where a token is designed to be used as a booking deposit to compensate the contractual party if the other side fails to honor the booking, i.e., the overbooking from the service provider and customer no-show. In the third chapter, we study advance payment as a financing instrument in a multitier supply chain to mitigate the supply disruption risk and compare the traditional system (with limited visibility) with the blockchain-enabled system (with perfect visibility). The main goal of this chapter is to shed light on how blockchain adoption impacts agents' operational and financial decisions and profit levels in a multitier supply chain.

We apply the genre of dynamic models to formulate all three problems, but we address them by different methodologies because of the difference in the contexts. The first two problems possess structural properties adequate to find the optimal structural policy for a dynamic program, whereas the last problem can be applied to game theory. In the hog production chapter, we find that the optimal selling strategy for the hog farm is non-monotone. The counter-intuitive situation, namely, the farm does not fulfill the long-term contract but sells to the open market to speculate the high spot price, happens when the open market is good enough. We also propose a newsvendor-like heuristic policy that improves the profit of the hog farm by over 25%. We find the service provider has different acceptance strategies for the maritime container shipping problem with and without overbooking. He always prefers reliable customers without overbooking but prefers unreliable customers with overbooking in some circumstances. In the deep-tier supplier chain finance, take a game-theoretic approach to compare how blockchain-enabled deep-tier financing schemes affect a financially constrained supply chain's optimal risk-mitigation and financial strategies. We find that although improved visibility via blockchain adoption can help the manufacturer make informed supply chain financing decisions, whether it can benefit all supply chain members depends on the financing schemes in use. Blockchain-enabled delegate financing increases risk-mitigation investments and benefits all three tiers of the supply chain only when tier-2 is severely capital-constrained with the working capital below a threshold. Because delegate financing endows the intermediary tier-1 supplier leverage over the manufacturer, the inefficiency inhibits an all-win outcome when the tier-2 is not severely capital-constrained. Blockchain-enabled cross-tier *direct financing* exhibits a compelling performance as it always leads to win-win-win outcomes (and thus ubiquitously implementable) regardless of the supplier's working capital profile.

Chapter 1

Managing Operations of a Hog Farm Facing Volatile Markets: Inventory and Selling Strategies

We study a dynamic finishing stage planning problem of a pork producer who gets to see how many market-ready hogs she has available for sale at the beginning of each week and the current market prices. Then, she must decide how many hogs to sell to a meatpacker and on the open market and how many to hold until the following week. The producer has a contract to deliver a fixed quantity to the meatpacker each week priced according to a predetermined formula that depends on market commodity indices. She pays a penalty if she fails to deliver to the meatpacker. The numbers of hogs that become market-ready every week, all costs, and all prices fluctuate over time. We use a dynamic programming approach to derive an optimal policy and a one-period look-ahead heuristic informing the farmer what hogs to sell at the beginning of each week. We show there are sell/hold thresholds that depend on the available hogs' weights, quantities, and prices. Unfortunately, identifying the thresholds requires messy computations. So, we propose an approximate dynamic programming approach that preserves the optimal policy structure and produces a sharp heuristic that is easy to implement. Numerical experiments calibrated to a pork producer's data (The Maschhoffs) reveal the optimal policy is a substantial improvement over the existing practice (around 25% on average), and the one-period look-ahead policy is as close as 1.76% to the optimal. The majority of the improvement value over the current practice is recognizing and exploiting the "real option" value of under-weight hogs in hedging supply uncertainty and stochastic prices.

1.1 Introduction

The U.S. is the world's third-largest producer and consumer of pork and pork products globally. In 2019, the value of U.S. pork and pork product exports to the world reached a record $7.0 billion, accounting for over 15% of the entire world's trading volume [103]. Figure 1.1 illustrates a typical U.S. pork supply chain. Upstream, we see grain producers (e.g., raw crops agribusiness firms in corn and soybeans, etc., such as ADM, Bunge, and Cargill) whose output is crucial in animal feed. Grain producers supply animal feed to pork producers (also called hog farmers), who raise live pigs to sell to meatpackers and food producers. Meatpackers and food producers then deal with food distributors and retailers that sell to consumers.

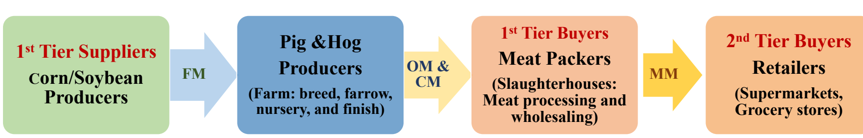


Figure 1.1: Hog Supply Chain

This paper focuses on a planning decision of the second tier in the above chain – the pork producer (hog farmer). The authors of this paper were introduced to the problem when working with The Maschhoffs, the largest family-owned pork producer in the U.S. and a top-ten pork producer globally. The Maschhoffs' pig operation employs about 1500 employees and sells about 5 million live hogs annually. Their buyers are mostly packing plants and food producers such as Cargill, Hormel Foods, and Farmland.

In modern hog farming, hogs at different stages of growth are raised at specialized farm sites. It all starts at sow (or breeding) facilities. After three weeks, the piglets are transferred to weaning farms specially equipped to care for young pigs. At about ten weeks, the young pigs are moved to feeding and finishing facilities that accommodate larger-sized animals. The planning decision we will analyze involves the finishing stage that lasts for 13 to 16 weeks. The finishing farms are often located close to a major buyer. As one can imagine, pig farming is a complex logistics operation with a continuous flow of pigs from sow facilities to weaning, feeding, and finally finishing operations. The Maschhoffs contract with over 70 sow farms, and they transport 700 to 800 trailer truckloads of live pigs each week.

When the pigs are between 23 and 26 weeks old, the pork producer divides the pigs into two pools: an under-weight pool and a regular-weight pool. Each week, the producer must decide how many regular-weight and under-weight hogs to sell through a contract with a meatpacker and how many regular-weight and under-weight hogs to sell on the open market where farmers can auction off livestock. Each week, the producer (the finishing farm) is under a contractual obligation to sell a fixed quantity of regular-weight hogs to a meatpacker for a contractually pre-determined price. The price is set via a formula that depends on relevant commodity market indices (e.g., fodder and pork prices), and as a result, the price fluctuates over time. (We describe contracts in §1.4.1 and provide more details in §1.3.2.)

At the beginning of each week, the producer observes how many regular-weight and under-weight hogs are market-ready, the open market and contract prices of regular-weight and under-weight hogs, and the default penalty she must pay the meatpacker for each undelivered hog. (The terms of the contract with the meatpacker state that the producer must pay the penalty for each undelivered regular-weight hog.) If the producer delivers an under-weight hog as a substitute for a regular-weight hog, the penalty is waived, but the contract price for the substitute hog is lower than the regular-weight price. In contrast, if the producer keeps a regular-weight hog for one additional week, it incurs a feeding cost. However, hogs that stay on the farm can satisfy future contractual obligations with the meatpacker and help avoid the penalty. The numbers of hogs that become market-ready every week, all costs, and all prices are stochastic. Flows between weaning and feeding farms may be affected by pig mortality rates (especially vulnerable to viruses and illnesses earlier in their lives), the weight growth rates (vary depending on diets at different contracted farms and supply availability of different feed mixes due to agribusiness markets), weather (slower growth rates in the summer, with animals eating less in the summer heat), and other environmental, market, and logistics factors.

Our research question – a question that Maschhoffs have to answer as part of their sales and operations planning cycle – is how many market-ready hogs should the farm sell to the meatpacker and on the open market at the beginning of each week. We are looking to devise a decision framework and a rule that the farmer can employ to make her weekly planning decision. From a managerial perspective, the farmer needs to understand better how to exercise her "real options." That is, the farmer needs to know the value of holding an under-weight (or even regular-weight) hog for another week and the value of selling hogs in the open market.

Currently, the producer uses an always fulfill (AF) policy. Under this policy, the farm fulfills the meatpacking contract, no matter what, and sells all remaining excess hogs on the open market. If the number of regular-weight hogs is insufficient to fulfill the contract, then the producer substitutes under-weight for regular-weight ones. It is an open question of how good the AF policy is compared to an optimal policy, which has to be established as is unknown in the current literature.

We view the pork producer's problem as a dynamic, multi-item (e.g., hogs of different weights) inventory model with supply uncertainty (uncertain number of available under-weight and regular-weight hogs) and stochastic prices in the contractual and spot markets. We answer the research question using a conventional dynamic programming approach. Although inventory policies in environments that we just described are generally not simple or easily implementable, we can derive an optimal policy structure. The optimal policy's key feature is that there are two different types of thresholds reflecting decision switches (holding to selling to the contract or holding selling to the open market). For each threshold type, there are separate thresholds for the under-weight and the regular-weight hogs. Whenever the number of hogs in a particular weight pool – under-weight or regular-weight – is below any applicable threshold, the producer should do nothing, i.e., she should let the hogs feed for at least one more period. When the number of market-ready hogs exceeds the relevant threshold, the producer should sell the excess either on the open market or to the meatpacker, depending on the prevailing market prices. The thresholds (of two different decision switch types and specialized by weight pool, four in total) are sensitive to the current and the future number of available hogs and prices. As such, they are not straightforward to derive in practice.

To overcome the computational difficulties around identifying the thresholds, we derive a sharp one-period look-ahead heuristic. Calibrated numerical experiments reveal that the optimal policy substantially improves the existing practice (between 22.55 and 25.89% on average). At the same time, the optimal policy outperforms the one-period look-ahead policy by as little as 1.76%. This heuristic's performance is noteworthy because the one-period look-ahead algorithm is easy to implement and significantly improves the farm's existing practice.

In summary, the current paper makes the following contributions to the literature:

(1) We formalize the pork producer's finishing sales planning problem and devise a rule that tells her how many under- and regular-weight hogs to hold and sell every week.

(2) We prove the optimality of a threshold policy in an environment with many sources of uncertainty. An interesting sidelight of the optimal policy that we present is that it does not necessarily have the simple form that intuition might lead one to predict. Specifically, the optimal policy can have two disjoint hold regions. As such, the farmer may find it optimal to hold some quantity hogs and sell the excess. However, if the excess is too large, she might want to hold some of the excess hogs as well.

(3) We demonstrate that a one-period look-ahead policy can be implemented as a practical, near-optimal heuristic. Using data from The Maschhoffs pork producer and publicly available commodity prices, we demonstrate the benefits of the optimal policy over the currently used "Always Fulfill" policy (around 25%), and the near optimality of the easy to implement one-period look-ahead policy (within 2%).

(4) From a managerial insights perspective, we provide an economic interpretation of the thresholds underlying the decision switches between holding for another week and fulfilling the contract or selling to the open market. We fully characterize the marginal value of holding, contract fulfillment, and spot market selling and the relevant factors affecting them. The optimal policy structure and the numerical results on the near-optimal performance of the look-ahead policy informed us on the high "real option"

value of holding under-weight hogs as a valuable hedge for both supply and price uncertainty. Our pork producer heavily underestimated the value of such an option in her planning formula. The majority of the improvement value over the current policy comes from better managing the under-weight hog pool, occasionally exercising the option to hold regular-weight hogs.

The rest of the paper is organized as follows. §1.2 reviews the most relevant literature. §1.3 describes the data pf this project. §1.4 presents the model and describe the current policy. §1.5 derives optimal policy structure. §1.6 describes the one-period look-ahead heuristic. §1.7 extends our baseline model in two directions. §1.8 numerically investigates the optimal and heuristic policies to infer the practical implications of our results. §1.9 concludes. All proofs, data calibration details are provided in Appendices.

1.2 Related Literature

The pork producer's finishing stage planning problem that we study has three notable features:

(a) The farmer sells market-ready hogs via a long-term contract (at contractually pre-determined prices pegged to market indices) and in the spot market. Both the contract and spot prices are stochastic.

(b) The farmer's problem is dynamic because the long-term contract binds her to deliver a pre-determined quantity and quality of market-ready hogs every week. Any shortfall triggers a penalty that is pegged to stochastic market indices.

(c) Finally, the sizes of the under-weight and over-weight pools are stochastic due to the uncertainty in early sourcing flows from sowing and weening farms. Uncertainty in birth rates and illnesses drive fluctuations in ween death rates. Weather conditions yield hard-to-control variations in the weight-gain process.

A stream of literature related to Part (a) includes research on procurement/selling strategies involving simultaneous trading in spot and forward markets. Firms in these papers participate in both markets while looking either for the best price or increased operational flexibility while facing uncertain demand. Examples of static models include [21, 63], and [72]. [63] study a game-theoretic model where two competing OEMs procure inputs from a single supplier and provide a strategic-based rationale for why firms source in both forward and spot markets. In [21] an OEM orders components under demand uncertainty, processes to order, and sells any unused components in a secondary market once demand uncertainty is resolved. [72] show how forward selling can mitigate risk in product quality while producing wine.

Notable examples of papers that explore spot and forward trading in a dynamic environment include [27, 39, 61, 64]. [61] show how firms can utilize a portfolio of procurement contracts to increase expected profits and reduce procurement risk over time. [64] study how online business-to-business (B2B) exchanges affect buyer-supplier relationships where an exchange takes the role of a secondary market in which buyers of the initial product can trade excess inventory to address supply and demand imbalances. [27, 39] consider integrated optimization problems of procurement, processing, and trading of commodities. [39] focus on applications in petroleum refining, [27] concentrate on soybean production and processing. All papers mentioned above essentially characterize optimal policy structures in their respective environments.

Although the models in the papers that we just mentioned are dynamic, they do not necessarily cover all the complexities our farmer faces. In [64] selling prices are fixed rather than stochastic, in [27, 39] there is no long-term contract that the firm must satisfy, and in [61] there is no uncertainty about the output's quality. For additional details, see Table 1.1.

Table 1.1: Summary of the Literature

11 Research Article	Mashhoffs' Problem Complications: Contract and Spot Markets	Multiple Product Qualities	Default Penalties	Dynamic Model	Stochastic Costs and Prices
[42]		✓	✓		
[48]			✓		✓
[61]	✓			✓	✓#
[106]	✓(option +spot market)		✓⋆		✓#
[63]	✓			✓	
[64]	✓			✓	✓#
[73]			✓	✓	
[98]		✓			
[21]	✓	✓(only multiple products)	✓		
[44]			✓	✓	
[5]			✓	✓	✓
[8]	✓	✓			✓
[27]	✓	✓		✓	✓
[39]	✓		✓§	✓	✓
[4]		✓	✓†		
[14]		✓			
[56]	✓			✓	✓#
[31]		✓			
[84]	✓		✓‡		✓
[7]	✓	✓			
[72]	✓	✓			✓#
[10]		✓		✓	✓
[40]	✓		✓	✓	✓
[15]		✓			
[9]		✓		✓	✓
This paper	✓	✓	✓	✓	✓

⋆ [106] only assume that sellers face stringent penalties for nonperformance under contract to set as a capacity constraint.

† [4] only did sensitivity analysis w.r.t. penalty cost.

‡ [84] treat spot market as emergency sourcing, penalty cost is the same as a spot market price.

§ [39] consider the penalty cost for the downstream retailers.

[61], [106], [64], [56] and [72] only consider the random price in the spot market not in the long-term contract.

Table 1.2: The following items may be included in your dissertation or thesis, in the order in which they are listed. Any optional components, if used, must be included in the table of contents, unless noted below.

Major Part	**Thesis Component**	**Required**	**Optional**	**Page Numbering**
Front Matter	Title page	✓		counted, not numbered
	Copyright page		✓	neither counted, nor numbered
	Table of Contents	✓		begins on page number ii
	List of Figures		✓	[lowercase Roman numerals continue]
	List of Illustrations		✓	[lowercase Roman numerals continue]
	List of Tables		✓	[lowercase Roman numerals continue]
	List of Abbreviations		✓	[lowercase Roman numerals continue]
	Acknowledgments	✓		[lowercase Roman numerals continue]
	Dedication*		✓	[lowercase Roman numerals continue]
	Abstract page		✓	[lowercase Roman numerals continue]
	Preface		✓	[lowercase Roman numerals continue]
Body	Epigraph*		✓	begins on a page numbered 1
	Chapters	✓		[Arabic numerals begin or continue]
Back Matter	References**	✓		[Arabic numerals continue]
	Appendices		✓	[Arabic numerals continue]
	Curriculum Vitae***		✓	[Arabic numerals continue]

* Do not include in the table of contents.
** There are two options for the placement of references; they can be listed at the end of each chapter, or at the end of the document.
*** Do not put your Social Security Number, birthdate, or birthplace on your CV.

1.3 Data Description

When making the selling decision for the Maschhoffs farm, our model includes both the production uncertainty caused by the variance of hogs' growth velocity, the mortality rate and the variability of market prices. Therefore, we categorize data required to calibrate our model into the following two parts: 1) Maschhoffs' hog production data and 2) price data.

The first data set describes the inputs and outputs of the farm's hog production. The input production data includes

- the wean pigs' production schedule (total head of wean pigs and starting date, projected total head, and date to be market-ready);
- weekly feed consumption;
- feed cost; and
- weekly feed conversion ratio (pounds of meals consumed to grow one pound of weight), and weekly yardage cost.

The output production data includes the daily quantity and the average weight of marketable hogs from different batches. Most batches are about 150 to 200 heads of marketable hogs. The farm's production data starts from July 3, 2017, and ends on June 7, 2019. We aggregate the daily data into weekly data for both the under-weight and regular-weight hogs from all sites to match the farm's decision epoch.

The second data set includes the vector $\boldsymbol{P}_t$, which are prices at which producers (farmers) and processors (meatpackers) transact *without* a central exchange. We refer to such a market as the "over-the-counter" (OTC) and the prices, $\boldsymbol{P}_t$, as OTC prices. In (1.4a), we need to take expectations over these prices without necessarily knowing their joint distribution. Having data will allow us to estimate that distribution.

The OTC price data, which includes meatpacking contract prices, open market transaction prices, holding costs, penalties (see §1.4.1) also, come from the Maschhoffs farm. The contract price data comes from 5 to 10 year-long contracts the farm signed with its major downstream meatpackers. On average, 92.04% of the hogs are sold through a meatpacking contract. Each contract specifies producer and packer's obligations and default conditions, including default penalties. Examples include laws and regulations relating to facility operations, delivery schedule (due dates and hog counts), health condition, and weight requirement for the delivered hogs. The contract also defines the pricing formula, which will be provided in §1.3.2. We use a consistent time frame for the OTC price data to match the production data.

1.3.1 Production Data

The farm's production data includes the input quantity of to-market-weans, production quantity and hogs' weight. Recall that in each week, the farm needs to put a certain quantity of piglets into the production pipeline, which grow into marketable hogs around 20 to 23 weeks later. The input decision is not closely related to the market conditions since the production lead time is relatively long, which is evidenced in §1.3.3. As per the meatpacker's contractual terms, hogs at 205 pounds or higher are regular-weight; lighter hogs are classified as under-weight. We find that the number of new incoming marketable hogs is a random variable that follows a normal distributions[1] for both regular-weight and under-weight hogs each week. The weekly mean quantity of regular-weight hogs is 62,906, with a standard

[1]The number of new incoming under-weight and regular-weight hogs passes the normality test. We use the Lilliefors test for normality, with the null hypothesis saying that the data comes from a normal distribution. The p-Value for the test is 0.2411 for under-weight hogs and 0.4210 for regular-weight hogs, respectively. Since the p-value for the test is higher than 0.05, we cannot reject the null hypothesis that the data is normal distribution at 95% or higher confidence level.

deviation of 9,551. The weekly mean of under-weight hogs is 28,959, with a standard deviation of 7,839. Hence, we use those normal distribution parameters to simulate the random yield of incoming marketable hogs in our numerical experiment in §1.8.2.

1.3.2 OTC Price Data

As per Equation (1.3), the contractually pre-determined base price that the farm receives from the focal meatpackers consists of livestock prices, pork prices, and prices of agricultural foodstuff used to produce animal feed. We refer to them as *factor prices*, denote them as Π_t^O, Π_t^M and Π_t^F. These prices are publicly available from both the Chicago Mercantile Exchange and the USDA (United States Department of Agriculture) [104].[2]

$$P_t^C = \frac{103.8\%}{3}\underbrace{\text{LM\_HG201}}_{\text{OM Price}} + \frac{91.8\%}{3}(\underbrace{\text{LM\_PK602}}_{\text{MM Price}} + 0.57) + \frac{1}{3}\underbrace{\max\{\text{Floor}, 103.8\%\text{LM\_HG201}\}}_{\text{Mix of OM and FM Price}}.$$

The first one-third of the base price is the average livestock price negotiated between slaughterhouses and packers in the open market before the delivery day, which is reported in the National Daily Slaughtered Swine Report (LM_HG201 from [102])[3]. The second one-third comes from the pork market, which considers the pork cutout value. This pork price is obtained from the National Daily Negotiated Pork Report (LM_PK602). The last one-third is determined by the greater of the open market price, or the corn and soybean meal average prices (reported in SJ_GR850 and GX_GR117 reports from USDA), which essentially accounts for the feeding cost to compensate the producer. The meatpacking contract price works as a pass-through price, which is a standard way of pricing commodity-based contracts in many industries [57].

[2]LM_HG201 stands for National Daily Direct Hog Prior Day Report-Slaughtered Swine. LM_PK602 stands for National Daily Pork FOB Plant-Negotiated Sales. The "Floor" contains Corn/Soybean Meal Component: $\$36.44346 + (4.40235 \cdot \text{Corn}) + (0.044845 \cdot \text{SBM})$ where Corn is from report SJ_GR850, Soybean Meal is from report GX_GR117. For detailed explanations of these price indexes, see Appendix B.

[3]The report excludes Saturdays and Sundays and six holidays (New Year's Day, Memorial Day, Independence Day, Labor Day, Thanksgiving Day, and Christmas Day).

The fixed contract price discount for under-weight hogs is a constant contractually pre-determined by each meatpacker. The penalty cost, C_t^P, is calculated as a set percentage of the open market price. Again, the rates vary from one meatpacker to another.

We obtain the open (or "spot") market price for under-weight hogs per head, P_t^1, by aggregating the weekly spot market trading prices of slaughtered hogs that were under 205 pounds. We apply similar logic to calculate the spot market price for the regular-weight hog per head, P_t^2. We consider both the yardage cost and feeding cost per head to calculate the holding cost, C_t^H. The feeding cost can be obtained from the unit price of corn, soybean, and soybean meal components multiplied by the corresponding consumption volume of hogs each week, using the weekly feed conversion rate recorded by the Maschhoffs.

1.3.3 Relationship Between Production Data and OTC Price Data

Before we proceed to fit the OTC prices, we examine the relationship between the input decision and market prices. We want to investigate if the input decision is correlated with market prices. Figure 1.2 demonstrates the weekly piglets input quantity of the farm, which is different from the pattern of the OTC market prices in Figure 1.13. Figure 1.3 shows that there is no correlation between the number of to-market weans and open market prices and contract price. Indeed, the correlation coefficient between the number of input weans and the open market price of the regular-weight hogs is 0.255 (0.268 for the under-weight hog), and the correlation coefficient with the contract price is 0.313. This is due of the long lead time, which makes the hog producer hard to make any adjustment far in advance according to the market condition.

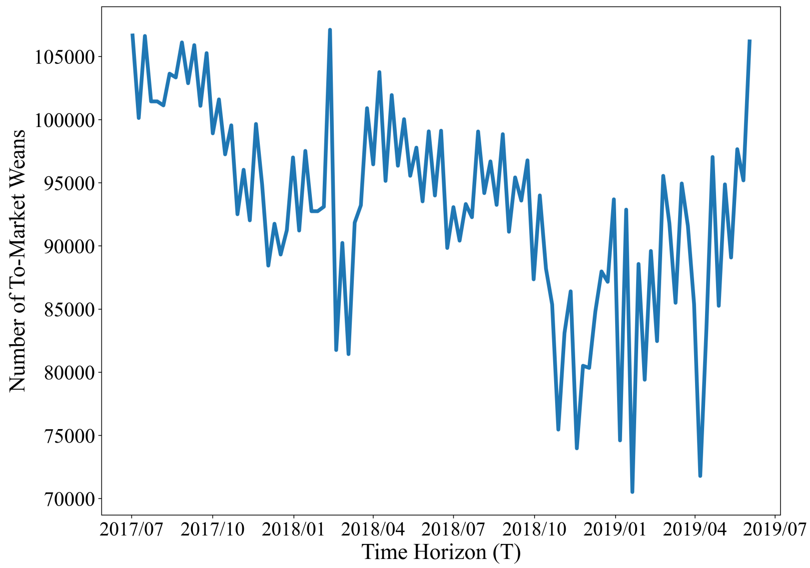


Figure 1.2: Weekly Wean-to-market Quantity

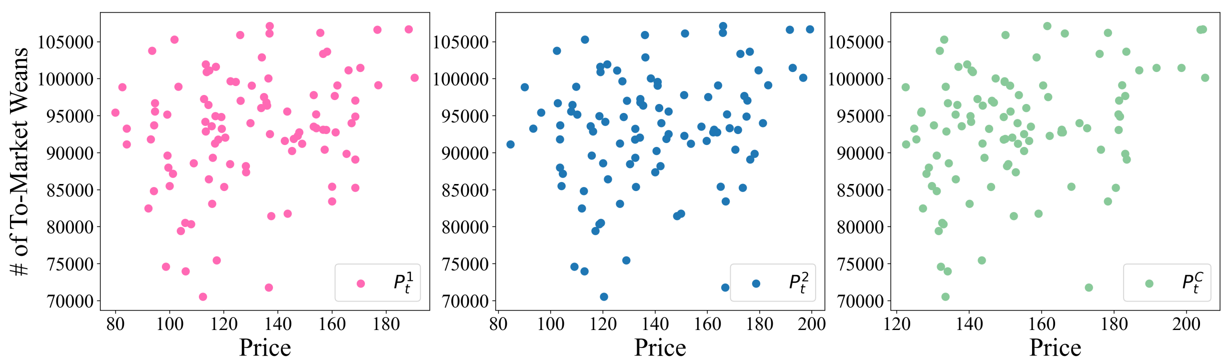


Figure 1.3: Weekly Wean-to-market Quantity

1.4 Analytical Model of Wean-to-Finish Hog Farm

We study a discrete-time, multi-period model of a wean-to-finish hog farm, which sells market-ready hogs on the open market and via a long-term contract with meatpackers.

At the beginning of each week, t, roughly ten-week-old hogs ("weans") arrive at the farm to feed and grow. The hogs become market-ready approximately thirteen to sixteen weeks later, at which point the farmer divides them into two pools according to weight. Market-ready hogs that weigh less than 205 pounds are labeled as under-weight and assigned to Pool 1; the remaining hogs are labeled as regular-weight and assigned to Pool 2. Due to natural fluctuations in birth rates and mortality, the number of hogs that arrive at the farm to become market-ready is random. In addition, the rate at which hogs gain weight depends on weather and other environmental conditions. So, if $\boldsymbol{W}_t = [W_t^1, W_t^2]$ denote the numbers of under-weight and regular-weight hogs that the farmer transitions from the growing stage to the finishing stage in each period t, then $\{\boldsymbol{W}_t, t \in \mathbb{N}\}$ is a stochastic process defined over some filtered common probability space $[\Omega_w, \mathcal{A}_w, \mathbb{P}_w]$. Throughout the paper, we use uppercase letters to denote random variables and lowercase letters to denote realizations of random variables. Bold letters denote vectors.

The total number of hogs that are available for sale in each period t, however, consists not only of hogs that the farmer transitions to the market-ready stage at the beginning of period t but also of market-ready hogs that she did not sell in period $(t-1)$. We use the notation z_{t-1}^i to denote market-ready hogs that stayed on the farm at the end of period $(t-1)$, where the superscripts indicate the pools into which the farmer assigned them at the beginning of period $(t-1)$. Any market-ready hog that stays on the farm incurs a feeding cost of c_t^H per week. Feeding, however, affects the hogs in Pools 1 and 2 differently: whereas the under-weight hogs gain lean weight and grow to become regular-weight between periods $(t-1)$ and t, the regular-weight hogs gain mainly fat without gaining market value. (For this reason, we will say that regular-weight hogs do not gain weight when they feed. However, we mean that they do not gain economic value.)

Taken together, the flow of market-ready hogs is subject to the following conservation constraints:

$$S_t^1 = W_t^1 \qquad \text{and} \qquad S_t^2 = W_t^2 + z_{t-1}^1 + z_{t-1}^2, \tag{1.1}$$

where S_t^i, $i = 1, 2$ is the number of market-ready hogs available in Pool $i = 1, 2$ in period t. The left side constraint in (1.1) says that the number of under-weight hogs at time t equals the number of hogs assigned to Pool 1 at the beginning of t. The right side constraint says that the number of regular-weight hogs in period t includes everything left over from period $(t-1)$ *plus* the number of hogs assigned to Pool 2 at the start of period t. Henceforth, whenever we write $\boldsymbol{S}_t$, we refer to the vector $[S_t^1, S_t^2]$, which directly inherits its stochastic nature from the vector $\boldsymbol{W}_t$.

The farmer sells the available hogs in two principal ways. First, she has a long-term contract with a maturity date of T to deliver q regular-weight hogs to a meatpacker for a price of p_t^C each week $t \leq T$. (The maturity date of this obligation is T, where T is large, typically 200 or more weeks.) The farmer can wriggle out of the contract by paying a penalty of c_t^P for each undelivered hog *or* delivering an under-weight hog instead of a regular-weight one. However, the meatpacker pays a (lower) price of $(1-\alpha)p_t^C$ for the under-weight hogs, where $0 < \alpha < 1$ is a contractually pre-determined constant. We let $\boldsymbol{y}_t = (y_t^1, y_t^2)$ represent the quantities of hogs that the farmer sells to the meatpacker from each pool $i = 1, 2$. It follows that

$$y_t^1 + y_t^2 \leq q,$$

which is a contract capacity constraint.

Second, the farmer has the option to sell any market-ready hogs on the open market for spot prices of p_t^1 and p_t^2, where $p_t^1 \leq p_t^2$. (Incidentally, because the farmer can also buy hogs on the open market, the open market price, p_t^2, represents an upper on the default penalty, c_t^P.) If $\boldsymbol{x}_t = (x_t^1, x_t^2)$ denote the quantities of hogs that the farmer sells on the open market, then preservation of mass requires

$$x_t^i = s_t^i - y_t^i - z_t^i, \quad i = 1, 2. \tag{1.2}$$

In other words, the number of hogs to be sold on the open market is fully determined after the farmer decides how many hogs to hold until the next period and how many to sell to the meatpacker. So, our model's *decision vector* is $(\boldsymbol{y}_t, \boldsymbol{z}_t)$, if we take $\boldsymbol{s}_t$ to be the state vector.

1.4.1 Stochastic Prices, Costs, and Penalties

Much like prices of other commodities, contract prices, open market prices, fodder prices, and the default penalty, $\boldsymbol{P}_t = [P_t^C,\ P_t^1,\ P_t^2,\ C_t^H,\ C_t^P]$, are all stochastic, where $\{\boldsymbol{P}_t, t \in \mathbb{N}\}$ is defined over some common filtered probability space $[\Omega_p, \mathcal{A}_p, \mathbb{P}_p]$. The farmer gets to see the realization of the vector $\boldsymbol{p}_t = [p_t^C,\ p_t^1,\ p_t^2,\ c_t^H,\ c_t^P]$ at the beginning of each period, t.

An interesting aspect of the farmer's selling decision is the nature of the long-term contracts with the food producers (meatpackers). The contract specifies the delivery of a fixed quantity but at a stochastic future price expressed as a pricing formula that depends on market price indices. These market price indices reflect the feeding cost (fodder commodity prices, such as soybeans and corn, etc.), the live pig open market prices, and the pork meat market prices. To give an example, the contract price in a recent Maschhoffs' contract with Farmland Foods, Inc., specifies the contract price as

$$\begin{aligned} P_t^C = & \frac{1}{3} \cdot 103.8\% \cdot \text{LM\_HG201} + \frac{1}{3} \cdot (\text{LM\_PK602} + 0.57) \cdot 91.75\% \\ & + \frac{1}{3} \cdot \max\left\{\text{Floor Price}, 103.8\% \cdot \text{LM\_HG201}\right\}. \end{aligned} \tag{1.3}$$

Equation (1.3) reveals that P_t^C is a price index with a contractually pre-specified floor. Without getting into institutional market details, the index LM_HG201 reflects live hog open market prices, and LM_PK602 reflects pork market prices. In the third term, as part of the floor price, fodder market costs are captured via corn and soybean market prices. See section 1.3.2 for more details on the nature of the contracts and the commodity indices used.

Although the stochastic process, $\{\boldsymbol{P}_t, t \in \mathbb{N}\}$, can be quite general (e.g., auto-correlated and non-stationary), we do impose standard no-arbitrage conditions.

Assumption 1.1 (No Arbitrage). $\mathbb{E}_t P_{t+1}^C - p_t^C \leq c_t^H$ and $\mathbb{E}_t P_{t+1}^2 - p_t^2 \leq c_t^H$.

The no arbitrage assumption above asserts that expected increases in the regular-weight hog's prices on the open and the contract markets never surpass the feeding costs. (Thus the farmer cannot make a risk-less profit by simply holding regular-weight hogs.) The other assumption is that $\{\boldsymbol{P}_t, t \in \mathbb{N}\}$ and $\{\boldsymbol{W}_t, t \in \mathbb{N}\}$ are independent, which reflects that the farm is too small to affect U.S. pork and fodder prices.

1.4.2 Optimization Objective and Dynamic Program

If β denotes a single-period discount factor, then the optimization of the expected proceeds from sales of market-ready hogs corresponds to the following dynamic program:

$$V_t(\boldsymbol{s}_t, \boldsymbol{p}_t) = \max_{\boldsymbol{y}_t, \boldsymbol{z}_t} \left\{ v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) : y_t^1 + y_t^2 \leq q, y_t^1 + z_t^1 \leq s_t^1, y_t^2 + z_t^2 \leq s_t^2 \right\}, \tag{1.4a}$$

$$v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) = r_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) + \beta\, \mathbb{E}_t V_{t+1}(\boldsymbol{S}_{t+1}, \boldsymbol{P}_{t+1}), \ \ t = 1, 2, \ldots, T \tag{1.4b}$$

$$r_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) = p_t^C \left(y_t^2 + (1-\alpha)\, y_t^1 \right) + \sum_{i=1}^{2} p_t^i \left(s_t^i - y_t^i - z_t^i \right) - \sum_{i=1}^{2} c_t^H z_t^i - c_t^P \left(q - y_t^1 - y_t^2 \right), \tag{1.4c}$$

where $\boldsymbol{S}_{t+1}$ is given by (1.1). The subscript on the expectation operation denotes filtration.

The best period t payoff is related to the best period $(t+1)$ payoff through the transition probabilities that may *evolve over time* and that are encapsulated in the expectation operator $\mathbb{E}_t[\ldots]$, $t = 1, 2, \ldots, T$. Thus, the MDP may be non-stationary, reflecting changing market conditions and prices. The maturity date of the meatpacking contract, T, provides a natural stopping time for the problem because the best period t payoff reflects the contractual obligation that the farmer has with the meatpacker *until* time T at which time she has to enter into a new – and a different – contract.

Lemma 1.1 (Farmer's Incentive Compatibility of the Contract)**.** *There exists a $q > 0$ such that the farmer can obtain more expected profit than $q = 0$ if $\mathbb{E}\sum_t^T p_t^C \geq \mathbb{E}\sum_t^T p_t^2$.*

Proof. It can be proved as follows. When $q = 0$, the farmer's only option is to sell all hogs to the open market. Holding is never an option because of the implicit no-arbitrage assumption for the open market (otherwise, there exist an arbitrage that the farmer purchase from the open market and hold to the next period). Then we use the contradictory to prove the incentive compatibility.

Suppose that for all $q > 0$, we have signing the long-term contract always leads to a less expected profit. However, for a very small $q > 0$ such that $s_t^2 > 0$ holds for all t, we can find the following policy that the farmer always fully fulfills the contract and sells the remaining to the open market. Then the expected profit of this policy is less than the maximal one of $q = 0$ only if the expected average contract price is less than the open market, which is a contradictory. We complete the proof.

□

Lemma 1.1 shows the farmer's incentive compatibility to sign the long-term contract with the meatpacker when the average contract price is higher, which is a realistic assumption according to the data from the Maschhoffs. The farmer can choose the higher option between fulfilling the contract and selling to the open market in each period, which guarantees the positive profit surplus.

Although extending the problem into an infinite horizon is not one of our goals in this paper, [68, 69] gives a procedure for doing that. Specifically, in [68, 69], there is a T-period problem for every possible terminal salvage function which might occur. By taking an expectation over these salvage values, one can infer the infinite-horizon policy. Later in the paper (§1.8.2), we propose a surprisingly sharp one-period look-ahead heuristic, which reveals that a solution that considers only two periods can be near-optimal for a planning horizon $T >> 2$.

1.4.3 The Current Policy

Before we characterize the optimal policy, we report what the farm behind this research – the Maschhoffs – uses currently. The current policy works as follows:

> If $q \leq s_t^2$, the farm fulfills the contract with regular-weight hogs only; all other hogs go to the open market.
>
> If $s_t^2 < q < s_t^1 + s_t^2$, all regular-weight hogs are used to fulfill the contract, and the contractual shortage is satisfied with under-weight hogs. The farm sells all remaining under-weight hogs on the open market.
>
> If $q > s_t^1 + s_t^2$, all market-ready hogs are used to fulfill the contract.

The current policy should be expected to work quite well whenever all four markets are deterministic, and the meatpacking contract pays more than the open market. Data that we present in §1.3, however, reveals that market prices are far from deterministic and that the open market sometimes pays more than meatpacking contact. In §1.8.2), we use the same data to calibrate our model and evaluate the gap between the current and the optimal policies.

1.5 An Optimal Policy

Using T as the maturity date, in this section, we utilize the standard backward induction algorithm to find a policy that is optimal in (1.4). If one wanted to know the optimal policy for an infinite horizon, [68, 69] give a computational procedure. However, as we show later in the paper (§1.8.2), a surprisingly sharp one-period look-ahead heuristic can be near-optimal for planning horizons that far exceed two periods. (The standard values of T we see in practice are around 200 weeks.) For this reason, we decided not to pursue the infinite horizon analysis in this paper.

The thrust of our analysis in this section is as follows. In Lemma 1.2, we show that the value function is jointly concave in the decision variables for all states. In Lemma 1.4 (1.3), we exploit the concavity property of Lemma 1.2 to describe the trade-off between keeping hogs on the farm until the next decision period and selling them either to the meatpacker or on the spot market in the current period. We conclude with Proposition 1.1, where we combine Lemmas 1.3 and 1.4 to arrive at an optimal policy. Where convenient, the mnemonics **F**, **S**, and **H** will indicate that the farmer is fulfilling the meatpacking contract, selling on the spot market, and holding hogs in inventory, respectively.

Lemma 1.2 (Concavity)**.** *The value function $v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \cdot)$ is jointly concave in $(\boldsymbol{y}_t, \boldsymbol{z}_t)$ for all $t = 1, 2, \ldots, T$.*

The lemma asserts that the value function is concave in the decision vector, $(\boldsymbol{y}_t, \boldsymbol{z}_t)$, which expresses how many hogs are sold to the meatpacker and held on the farm for one more period. (Recall that the number of hogs to be sold on the open market is entirely determined after $(\boldsymbol{y}_t, \boldsymbol{z}_t)$ are set).

The concavity property reflects that marginal contributions from selling to the meatpacker in the current period and holding market-ready hogs on the farm until the next decision period – the **F**- and **H**-margins – are decreasing in the number of hogs. (The marginal contribution from selling the hogs on the open market in the current decision period – the **S**-margin – is constant.) All three margins can be seen graphically in Figures 1.5 and 1.6 and are formally quantified in Table 1.3.

The **F**-margin in each period t has two components. First, the farmer "receives back" the penalty of c_t^P, which the meatpacker would otherwise charge her for each undelivered hog. Second, the meatpacker pays the farmer a unit price of either p_t^C or $(1-\alpha)p_t^C$, depending on the hog's weight. The **F**-margin is constant up to the meatpacking contract's capacity, q, and zero after that.

The decreasing **H**-margin reflects that the number of selling alternatives open to the farmer in the subsequent decision period diminishes in the number of hogs that the farmer decides to hold in the current period. Viewed from period t, consider what happens if the farmer's next period's total inventory, S_{t+1}^2 (see Equation 1.1), is smaller than the meatpacking contract's size, q. Then, she can sell all her hogs to the meatpacker or on the spot market or continue to hold them, depending on which option yields the highest expected payoff. However, any quantity that exceeds the meatpacking contract's capacity of q must be sold on the spot market or stay on the farm for one more period.

Mathematically, the **H**-margin is a messy expectation bounded on an interval that we characterize in the last column of Table 1.3. The lower and upper interval bounds depend on the lowest and highest prices the farmer can hope to get in the subsequent decision period *minus* the feeding cost, which she must expend to hold the hogs for one more period.

Table 1.3: Marginal Values of Selling Hogs and Holding Them in Inventory

	Marginal Values of Selling		Marginal Values of Holding
Hog Pool	**F**-margin ($= \nabla_{y^i} V_t$)	**S**-margin ($= \nabla_{x^i} V_t$)	**H**-margin ($= \nabla_{z^i} V_t$)†
Under-weight	$\left((1-\alpha)\, p_t^C + c_t^P \mid y_t^1 + y_t^2 \leq q\right)$	p_t^1	$\left[\beta \underline{p}_{t+1}^H - c_t^H, \beta \bar{p}_{t+1}^H - c_t^H\right]$
Regular-weight	$\left(p_t^C + c_t^P \mid y_t^1 + y_t^2 \leq q\right)$	p_t^2	$\left[\beta \underline{p}_{t+1}^H - c_t^H, \beta \bar{p}_{t+1}^H - c_t^H\right]$

† The expressions for $\bar{p}_{t+1}^H$ and $\underline{p}_{t+1}^H$ are as follows:

$$\bar{p}_{t+1}^H = \lim_{\boldsymbol{w} \to 0} \mathbb{E}\left[V_{t+1}\left(\left[w^1, w^2 + s_t^1 + s_t^2 + 1\right], \boldsymbol{P}_{t+1}\right) - V_{t+1}\left(\left[w^1, w^2 + s_t^1 + s_t^2\right], \boldsymbol{P}_{t+1}\right)\right], \quad \text{(1.5a)}$$

$$\underline{p}_{t+1}^H = \lim_{\boldsymbol{w} \to \infty} \mathbb{E}\left[V_{t+1}\left(\left[w^1, w^2 + s_t^1 + s_t^2 + 1\right], \boldsymbol{P}_{t+1}\right) - V_{t+1}\left(\left[w^1, w^2 + s_t^1 + s_t^2\right], \boldsymbol{P}_{t+1}\right)\right]. \quad \text{(1.5b)}$$

Because the **F**-margin and **S**-margin are constants, they cannot cross. (Although, in a special case, they can equal.) The H-margin, however, is *decreasing* in quantity, implying that it will cross the **F**- and **S**-margins *at most once from above.* In Lemmas 1.3 and 1.4, we define $\underline{z}_t^i$, and $\bar{z}_t^i$, $i = 1, 2$ as the smallest quantities of hogs from pools 1 and 2 for which the **H**-margin crosses the **F**- and **S**-margins and give conditions under which the crossing points exists. We show $\underline{z}_t^i$, and $\bar{z}_t^i$, $i = 1, 2$ graphically in Figures 1.5 and 1.6

For instance, if $\underline{z}_t^i \in (0, \infty)$ (see the conditions in Lemma 1.3), then it follows that the farmer is better off holding the first $\underline{z}_t^i$ hogs until the next period and selling the excess to the meatpacker (up to q hogs). This strategy reflects that contract prices are low in the current period, in which case the farmer has an incentive to let the hogs gain weight while she is waiting to see the next period's prices. However, holding too many hogs in the current period becomes economically futile for reasons that we already explained earlier (see our explanation of why the **H**-margin decreases in quantity).

Lemma 1.3. *Let $t = 1, 2, \dots, T$. There exist two, unique break-even stocking levels, $\underline{z}_t^i$, $i = 1, 2$ such that if $S_t^i = \underline{z}_t^i$, then the farmer is indifferent between selling the marginal, $\underline{z}_t^i$-th, hog from pool $i = 1, 2$ through the contract in period t and holding it in inventory until period, $t+1$. Moreover:*

1. *$\underline{z}_t^2 = 0$ for any prices, i.e., the **H**- and **F**-margins do not cross.*

2. *$\underline{z}_t^1 \in (0, \infty)$ iff $(1-\alpha)p_t^C + c_t^P \in \left[\beta \underline{p}_{t+1}^H - c_t^H, \beta \bar{p}_{t+1}^H - c_t^H\right]$, where $\underline{p}_{t+1}^H$ and $\bar{p}_{t+1}^H$ is defined in Equations (1.5). (If $(1-\alpha)p_t^C + c_t^P < \beta \underline{p}_{t+1}^H - c_t^H$, then $\underline{z}_t^1 = \infty$. If $(1-\alpha)p_t^C + c_t^P > \beta \bar{p}_{t+1}^H - c_t^H$, then $\underline{z}_t^1 = 0$. In neither case, the **H**- and **F**-margins do not cross.)*

Lemma 1.4. *Let $t = 1, 2, \dots, T$. There exist two, unique break-even stocking levels, $\bar{z}_t^i$, $i = 1, 2$ such that if $S_t^i = \bar{z}_t^i$, then the farmer is indifferent between selling the marginal, $\bar{z}_t^i$-th, hog from pool $i = 1, 2$ on the open market in period t and holding it in inventory until period, $t+1$. Moreover:*

1. *$\bar{z}_t^2 \in (0, \infty)$ iff $p_t^2 < \beta \bar{p}_{t+1}^H - c_t^H$, where $\bar{p}_{t+1}^H$ is defined in Equation (1.5a). (If $p_t^2 \geq \beta \bar{p}_{t+1}^H - c_t^H$, then $\bar{z}_t^2 = 0$, i.e., the **S**- and **H**-margins do not cross.)*

2. *$\bar{z}_t^1 \in (0, \infty)$ iff $p_t^1 \in \left[\beta \underline{p}_{t+1}^H - c_t^H, \beta \bar{p}_{t+1}^H - c_t^H\right]$, where $\underline{p}_{t+1}^H$ and $\bar{p}_{t+1}^H$ is defined in (1.5). (If $p_t^1 < \beta \underline{p}_{t+1}^H - c_t^H$, then $\bar{z}_t^1 = \infty$. If $p_t^1 > \beta \bar{p}_{t+1}^H - c_t^H$, then $\bar{z}_t^1 = 0$. In neither case, the **S**- and **H**-margins cross.)*

Lemmas 1.3 and 1.4 are optimal policies for Pools 1 and 2, provided that the farmer can trade only in a single market – either on the open market or with the meatpacker.

In the next Proposition 1.1, we exploit Lemmas 1.2 through 1.4 to develop a period-t decision rule, given state vectors, $\boldsymbol{s}_t$ and $\boldsymbol{p}_t$, which reveal the current period's hog availability in each weight pools, prices, feeding costs, and penalties.

Proposition 1.1 (An Optimal Policy).

1. *Let* $p_t^2 > p_t^C + c_t^P$, $t = 1, 2, \cdots, T$

   (a) *If* $p_t^1 > (1-\alpha)p_t^C + c_t^P$, *the optimal solution is* $y_t^1 = y_t^2 = 0$, $z_t^1 = s_t^1 \wedge \bar{z}_t^1$, *and* $z_t^2 = 0$.

   (b) *Otherwise, the optimal solution is* $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge q$, $z_t^1 = (s_t^1 - q)^+ \wedge \bar{z}_t^1$, $y_t^2 = 0$, *and* $z_t^2 = 0$.

2. *Let* $p_t^2 \le p_t^C + c_t^P$, $t = 1, 2, \cdots, T$.

   (a) *If* $p_t^1 > (1-\alpha)p_t^C + c_t^P$, *the optimal solution is* $y_t^1 = 0$, $z_t^1 = s_t^1 \wedge \bar{z}_t^1$, $y_t^2 = s_t^2 \wedge q$, *and* $z_t^2 = (s_t^2 - q)^+ \wedge \bar{z}_t^2$.

   (b) *else if* $0 \le (1-\alpha)p_t^C + c_t^P - p_t^1 \le p_t^C + c_t^P - p_t^2$, *the optimal solution is* $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge (q - s_t^2)^+$, $z_t^1 = (s_t^1 - y_t^1)^+ \wedge \bar{z}_t^1$, $y_t^2 = s_t^2 \wedge q$, *and* $z_t^2 = (s_t^2 - q)^+ \wedge \bar{z}_t^2$.

   (c) *else, the optimal solution is* $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge q$, $z_t^1 = (s_t^1 - q)^+ \wedge \bar{z}_t^1$, $y_t^2 = s_t^2 \wedge (q - (s_t^1 - \underline{z}_t^1)^+)^+$, *and* $z_t^2 = (s_t^2 - y_t^2)^+ \wedge \bar{z}_t^2$.

Case 1a

In this case, the open market pays the farmer more than the meatpacker does for both regular- and under-weight hogs. Thus, the **S**- dominates the **F**-margin. The rational farmer responds to these open market prices by "defaulting" on the meatpacking contract and selling her entire period-t inventory of regular-weight hogs on the open market. Trading all regular-weight hogs is optimal because the farmer cannot expect them to gain weight or price in period $(t+1)$.

In contrast, with the under-weight hogs, the farmer faces a risky trade-off. On the one hand, open market prices are high, making it attractive to sell these hogs. On the other hand, under-weight hogs trade at a discount compared to the regular-weight hogs, and the farmer knows that they will become regular-weight if she lets them continue to feed. Thus, by waiting, the farmer will be able to sell these for a full price in the next period, $(t+1)$. Waiting, however, is not without risk because the loss due to a drop in prices between periods t and $(t+1)$ may more than offset the gain from the associated increase in weight.

Part 1a of Proposition 1.1 asserts that the optimal way of resolving this risky trade-off is by hedging, which involves selling some under-weight hogs now and selling the rest later. Specifically, the farmer optimally sells $(s_t^1 - \bar{z}_t^1)$ hogs[4] on the open market (to take advantage of the high prices) and lets the remaining $\bar{z}_t^1$ hogs continue to feed (to bet that the weight gain offsets a potential drop in price). As such, if there are fewer than $\bar{z}_t^1$ under-weight hogs available in period t, then all under-weight hogs optimally stay on the farm until the period $(t+1)$.

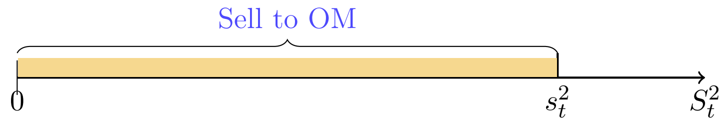
Figure 1.4 summarizes the farmer's optimal decisions of Case 1a graphically.

Case 1b

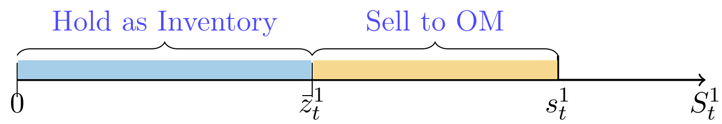
In Case 1b, the open market (contract with the meatpacker) pays more for the regular-weight (under-weight) hogs than the contract with the meatpacker (open market) does. That is, the **S**-margin is higher than the **F**-margin, which is something that we show graphically in Figure 1.5(b).

In this situation, the farmer sells her entire inventory of regular-weight hogs on the open market, which is what she does in the previous Case 1a.

[4] Recall that $\bar{z}_t^1$ is the break-even stocking level of Lemma 1.4.



(a) Selling Strategy for Regular-Weight Hogs



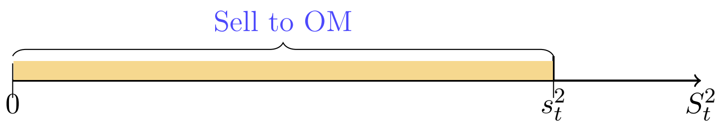
(b) Selling Strategy for Under-Weight Hogs

Figure 1.4: Selling Strategy for Case 1a

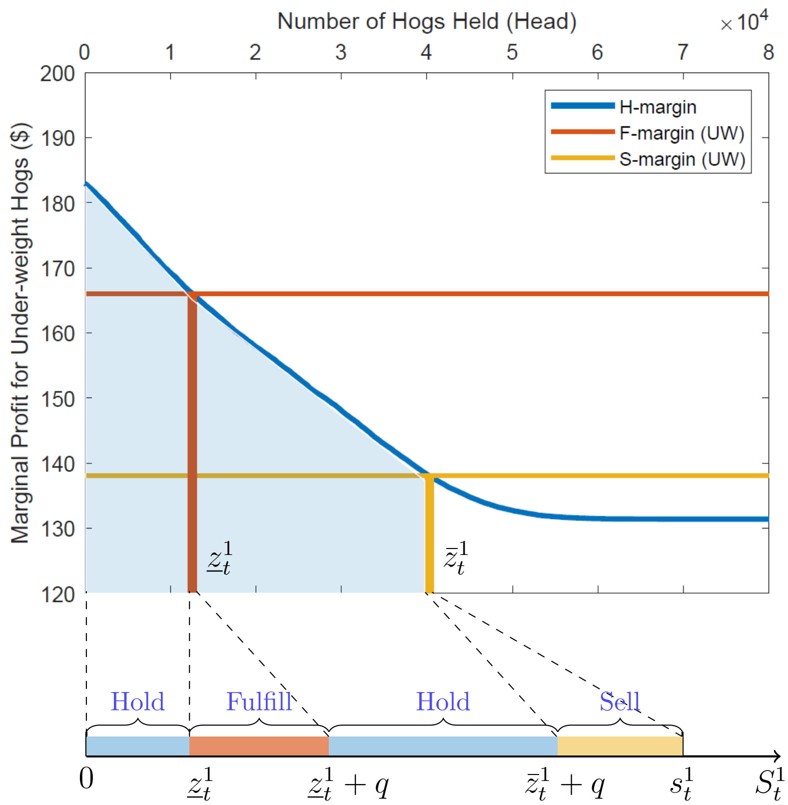
For under-weight hogs, trading with the meatpacker takes priority over trading on the open market. To decide how many hogs to trade with the meatpacker, the farmer computes the break-even level, $\underline{z}_t^1$ of Lemma 1.3. Then she keeps the first $\underline{z}_t^1$ hogs until the next decision period and sells $\min\{s_t^1 - \underline{z}_t^1, q\}$ to the meatpacker. (Following Lemma 1.3, the **H**-margin is higher than the **F**-margin on the first $\underline{z}_t^1$.)

In a situation, in which the number of the under-weight hogs is high, i.e., when $\min\{s_t^1 - \underline{z}_t^1, q\} = q$, the farmer needs to decide what to do with the remaining $(s_t^1 - q - \underline{z}_t^1)$ hogs. She computes the second break-even level of Lemma 1.4, $\bar{z}_t^1$. Following Lemma 1.4, whatever quantity she has on hand above the break-even quantity, $\bar{z}_t^1$, she sells on the open market. Any under-weight hogs that remain on the farm in period t continue to feed to become regular-weight in period $(t+1)$.

Interestingly, if $q < (s_t^1 - \underline{z}_t^1)$ and $\bar{z}_t^1 < (s_t^1 - q)$, then the farmer's policy becomes non-monotone – the policy transitions from **H** to **F** to **H** to **S** – which is something that we show graphically in Figure 1.5(b).



(a) Selling Strategy for Regular-Weight Hogs



(b) Selling Strategy for Under-Weight Hogs

Figure 1.5: Selling Strategy for Case 1b

Case 2a

In Case 2a, the meatpacker (open market) pays more for the regular-weight (under-weight) than the open market (meatpacker) does. That is the **F**-margin is higher than the **H**-margin on the regular-weight hogs. For the under-weight hogs, the **H**-margin dominates the **S**-margin.

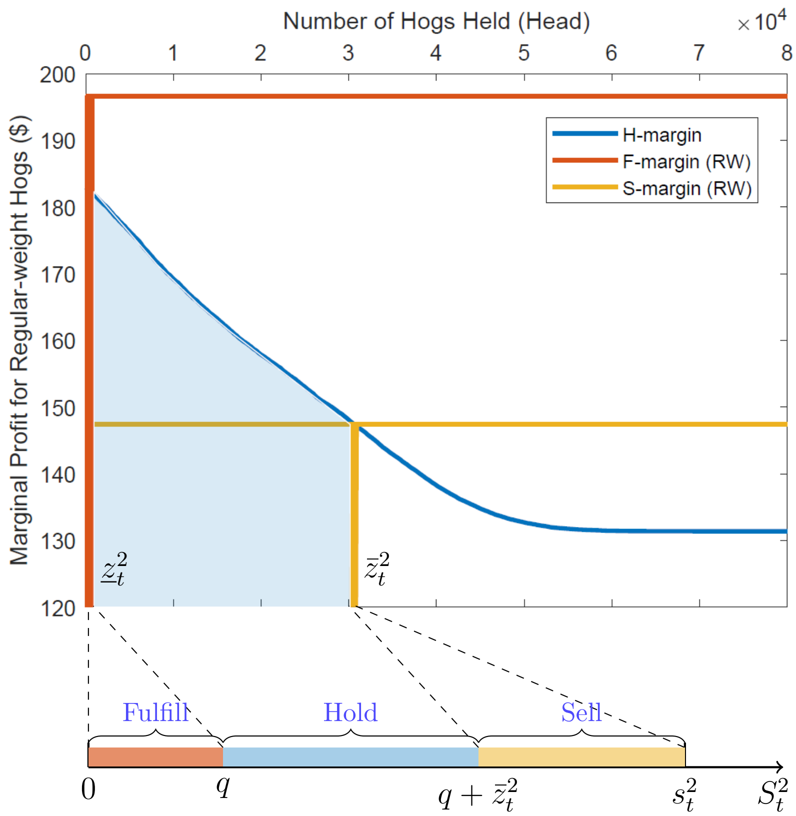
The farmer optimally responds to these prices by selling $\min\{q, s_t^2\}$ of regular-weight hogs to the meatpacker. If $s_t^2 > q$, i.e., if there are regular-weight hogs left over, she computes the break-even level $\bar{z}_t^2$ of Lemma 1.4. Then, she keeps up to $\bar{z}_t^2$ regular-weight hogs in inventory until the next period $(t+1)$ and sells the excess (if any) on the open market. We show these thresholds graphically in Figure 1.6.

The rationale for holding regular-weight hogs is that the open market prices are low in the current period. The farmer cannot sell more hogs to the meatpacker because of the meatpacking contract's capacity. However, the open market might improve in the next period. Alternatively, the farmer might sell some regular-weight hogs to the meatpacker if an insufficient number of hogs are available at the beginning of the next period. Thus, by holding some regular-weight hogs, the farmer also hedges against the risk of paying the default penalty meatpacking contract.

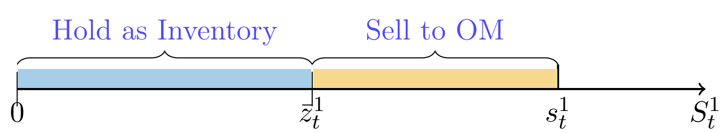
The treatment of under-weight hogs is straightforward. The farmer computes the break-even level of Lemma 1.4, keeps the first $\bar{z}_t^1$ hogs on the farm until the next decision period, and sells the excess on the market. The rationale behind this policy towards the under-weight hogs is already explained in our discussion of Lemma 1.4. Figure 1.6 summarizes the Case 2a graphically.

Case 2b and 2c

In the last two cases of Proposition 1.1, the meatpacker pays more than the open market for both the under-weight and the regular-weight hogs. The policy towards the regular-weight hogs is the same as in the Case 2a. The approach towards the under-weight hogs is the same as in the Case 1b. We skip the discussion to avoid unnecessary repetition.



(a) Selling Strategy for Regular-Weight Hogs

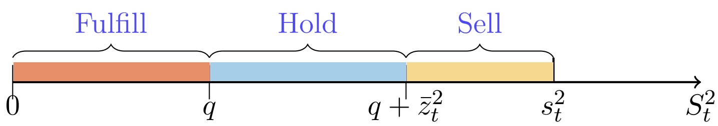


(b) Selling Strategy for Under-Weight Hogs

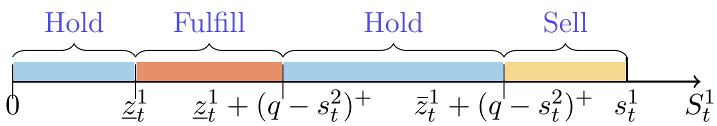
Figure 1.6: Selling Strategy for Case 2a

The critical difference between the Cases 2b and 2c is that in the former, the difference between the contract prices and open market prices is more significant for the regular-weight hogs and vice versa. Thus, in the former Case 2b, the farmer prioritizes regular-weight hog sales to the meatpacker over under-weight hog sales. She sells the under-weight hogs to the meatpacker only if an insufficient quantity of the regular-weight hogs is available. In contrast, she prioritizes under-weight hog sales over regular-weight hog sales in the latter Case 2c.

Figures 1.7 and 1.8 summarize the cases graphically. Also, for convenience, Table 1.4 condenses the policy of Proposition 1.1 on the entire state space.

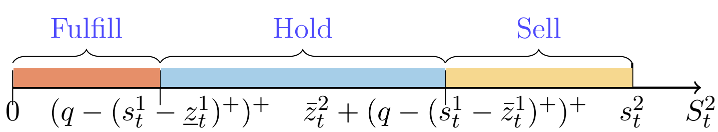


(a) Selling Strategy for Regular-Weight Hogs

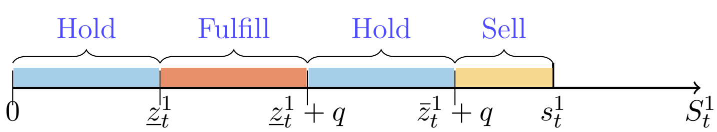


(b) Selling Strategy for Under-Weight Hogs

Figure 1.7: Selling Strategy for Case 2b



(a) Selling Strategy for Regular-Weight Hogs



(b) Selling Strategy for Under-Weight Hogs

Figure 1.8: Selling Strategy for Case 2c

An interesting sidelight of the optimal policy that we just presented is that it does not necessarily have an easy to predict form. Classical single-item dynamic inventory policies with fixed prices are often monotone [78]. Here, we have two items and two markets with stochastic pricing. Moreover, one of the items – the under-weight hogs – becomes more

Table 1.4: Overview of the Optimal Policy Structure

Case (State)	Regular-Weight (Action)	Under-Weight (Action)
1a	Only **S**	**H** → **S**
1b	Only **S**	**H**→ **F** →**H** →**S**
2a	**F** →**H** →**S**	**H** →**S**
2b	**F** →**H** →**S**	**H**→ **F** →**H** →**S** ⋆
2c	**F** →**H** →**S**	**H**→ **F** →**H** →**S** †

⋆ Use regular-weight hogs to fulfill contract first.
† Use under-weight hogs to fulfill contract first.

valuable when held in inventory. This setup can lead to *two disjoint hold regions*, which is something that we show graphically in Figure 1.5.

1.6 One-Period Look-Ahead Policy

In Lemmas 1.3 and 1.4, we introduce the thresholds, $\underline{z}_t^i$ and $\bar{z}_t^i$, $i = 1, 2$, which happened to be an important building block of the optimal policy of Proposition 1.1. With so many random variables in our state space, however, identifying these thresholds – even numerically – is complicated because of the usual explosion of problem size when optimizing over a long horizon, T. Because the farmer must re-derive these thresholds at the beginning of each decision period to reflect to new market conditions, executing the optimal policy in practice would be cumbersome.

That is why in this section, we look for a sharp approximation that considers a shorter horizon, one for which the thresholds are easier to compute. In picking the new horizon length, we exploit the fact that all under-weight hogs grow to become regular-weight between periods t and $(t+1)$ and gain no additional lean weight beyond that. With this insight, we solve the problem optimally from the beginning of period t to the beginning of period $(t+2)$. We then repeat the process by optimizing over the interval $(t+1)$ to $(t+3)$, etc. In this sense, we are "rolling" the horizon one time period forward. In the literature, such a policy is known as one-period look-ahead policy [76]. (Other simplification approaches are discussed in [70].) Formally, the one-period look-ahead policy labeled with the superscript "OL" is optimal in

$$V_t^{OL}(\boldsymbol{s}_t, \boldsymbol{p}_t) = \max_{\boldsymbol{y}_t^{OL}, \boldsymbol{z}_t^{OL}} \left\{ v_t^{OL}(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) : y_t^1 + y_t^2 \le q, y_t^1 + z_t^1 \le s_t^1, y_t^2 + z_t^2 \le s_t^2 \right\},$$

$$v_t^{OL}(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) = r_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) + \beta\, \mathbb{E}_t \max_{\boldsymbol{y}_{t+1}, \boldsymbol{z}_{t+1}} \left\{ r_{t+1}(\boldsymbol{y}_{t+1}, \boldsymbol{z}_{t+1}; (S_{t+1}^1, S_{t+1}^2 + \sum_{i=1}^{2} z_t^i, \hat{\boldsymbol{P}}_{t+1}) \right\}, \tag{1.6}$$

where r_t follows Equation (1.4c), and $\hat{\boldsymbol{P}}_{t+1}$ is a price forecast (see §1.8.1).

Proposition 1.2 (An Optimal One-Period Look-Ahead Policy). *In Proposition 1.1, for* $i = 1, 2$ *let*

$$\underline{z}_t^1 = G_{t+1}^{-1}\left(\frac{\beta(\hat{P}_{t+1}^C + \hat{C}_{t+1}^P) - c_t^H - (1-\alpha)p_t^C - c_t^P}{\beta(\hat{P}_{t+1}^C + \hat{C}_{t+1}^P) - \beta\hat{P}_{t+1}^2}\right), \quad \bar{z}_t^i = G_{t+1}^{-1}\left(\frac{\beta(\hat{P}_{t+1}^C + \hat{C}_{t+1}^P) - c_t^H - p_t^i}{\beta(\hat{P}_{t+1}^C + \hat{C}_{t+1}^P) - \beta\hat{P}_{t+1}^2}\right),$$

where $G_{t+1}(\xi) \in [G_{t+1}^{II}(\xi), G_{t+1}^{I}(\xi)]$, $\xi \in [0, q]$,

$$G_{t+1}^{I}(\xi) \doteq \mathbb{P}\left\{W_{t+1}^2 \leq q - \xi\right\}, \qquad \textit{and} \qquad G_{t+1}^{II}(\xi) \doteq \mathbb{P}\left\{W_{t+1}^1 + W_{t+1}^2 \leq q - \xi\right\}.$$

Above, $G(\cdot)$ is the shortfall distribution defined as the probability that the farmer will be unable to deliver ξ hogs when fulfilling the meatpacking contract of size q in the next decision period. By writing $G_{t+1}(\xi) \in [G_{t+1}^{\mathrm{II}}(\xi), G_{t+1}^{\mathrm{I}}(\xi)]$, we mean that the shortfall distribution function is a mixture of $G_{t+1}^{\mathrm{I}}(\xi)$ and $G_{t+1}^{\mathrm{II}}(\xi)$. The mixing is required because the exact specification of the shortfall distribution depends on how the farmer chooses to fulfill the meatpacking contract. There are two principal methods: (1) she can use only regular-weight hogs, or (2) she can utilize all hogs. Of course, her ultimate decision depends on which method makes her better off based on the prevailing market conditions in the given decision period. Therefore, when viewed from time t, the probability the farmer will short some hogs when fulfilling the contract in period $(t+1)$ is a mixture distribution that reflects the optimal use of both methods.

Proposition 1.2 reveals that our one-period look-ahead policy is the same as the optimal policy of Proposition 1.1 with closed-form expressions for the break-even thresholds $\underline{z}_t^i$ and $\bar{z}_t^i$, $i = 1, 2$. Proposition 1.1 does not actually specify the thresholds. Instead, it appeals to Lemmas 1.3 and 1.4, which merely assert that these thresholds exist without giving a specific recipe for how to compute them. Having the closed-form expressions for the thresholds makes the implementation of the optimal policy of Proposition 1.1 relatively straightforward.

The second point worth mentioning is that the closed-form thresholds in Proposition 1.2 have an appealing interpretation. Viewed from period t, if we interpret the number of market-ready hogs at the beginning in period $(t+1)$ as “demand;" the contract price plus the penalty cost for regular-weight hogs as the “retail price;” the spot price for regular-weight hogs as the “salvage value,” and the period-t holding cost as the “cost,” then the thresholds $\underline{z}_t^1$ and $\bar{z}_t^i$ resemble standard newsvendor critical fractiles. Thus, the one-period look-ahead policy effectively reduces the complicated multi-period, non-stationary problem of Equation (1.4) to an analytically appealing “newsvendor-like” solution.

Next, we would like to produce a meaningful comparison of the farm’s current practice (§1.4.3) and the optimal policy. For this purpose, we use the farm’s data and publicly prices to calibrate our model and perform numerical experiments.

1.7 Model Extensions

Our model can be implemented to other industries where multiple items are interchangeable held as inventory. We extend our model in the following directions to adapt the environment of different industries.

1.7.1 Endogenize the Input Decision

Even though the hog producer makes independent weans procurement decision from the market volatility, production planners in other industries might determine the input quantity because of shorter production lead time [13]. Let $\tau < T$ denote the production lead time representing the number of periods from the input decision to the selling decision. The farm should decide how many weans to order denoted by u_t at the beginning of period t at price c_t, and we add c_t to the price vector $\boldsymbol{P}_t$. The procured weans will be market-ready at period $t+\tau$, so the supply is prepared for the sales at that time. We assume uncertain proportional yield ratios for different weights of hogs. Let γ_t^1 (resp. γ_t^2) denote the proportion of weans that grow to be under(regular)-weight hogs at time t. They follow a joint distribution $\mathcal{F}(\gamma_t^1, \gamma_t^2)$ in the space $(0,1)^2$ and we have $\gamma_t^1 + \gamma_t^2 \in (0,1)$. Both yield rations depend on the season and weather conditions. A common knowledge is that the yield ratios are higher is winter, but lower is summer. Therefore, we can endogenize the physical randomness by using $\gamma_t^1 u_{t-\tau}$ (resp. $\gamma_t^2 u_{t-\tau}$) to substitute $W_{t+\tau}^1$ ($W_{t+\tau}^2$) in the original model. We also need to track the supply decision log during the lead time in the state, which can be characterized by a $(\tau-1)$-vector, $\boldsymbol{u}_t = (u_{t-\tau+1}, \cdots, u_{t-1})$ like [13]. The input decision is also related to the market volatility. We use subscription $t|\tau$ to represent the time t's forecast for the market price at $t+\tau$, e.g., $\hat{\boldsymbol{P}}_{t|\tau}$ is the time t's price forecast τ periods forward. The dynamic program can be re-written as follows:

$$V_t(\boldsymbol{s}_t, \boldsymbol{u}_t, \boldsymbol{p}_t) = \max_{u_t, \boldsymbol{y}_t, \boldsymbol{z}_t} \left\{ v_t(u_t, \boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{u}_t, \boldsymbol{p}_t) : y_t^1 + y_t^2 \le q, y_t^1 + z_t^1 \le s_t^1, y_t^2 + z_t^2 \le s_t^2 \right\}, \tag{1.7a}$$

$$v_t(u_t, \boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{u}_t, \boldsymbol{p}_t) = r_t(u_t, \boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) + \beta\, \mathbb{E}_t V_{t+1}(\boldsymbol{S}_{t+1}, \boldsymbol{u}_{t+1}, \boldsymbol{P}_{t+1}), \;\; t = 1, 2, \ldots, T \tag{1.7b}$$

$$r_t(u_t, \boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) = p_t^C \left(y_t^2 + (1-\alpha)\, y_t^1 \right) + \sum_{i=1}^{2} p_t^i \left(s_t^i - y_t^i - z_t^i \right) - \sum_{i=1}^{2} c_t^H z_t^i - c_t^P \left(q - y_t^1 - y_t^2 \right) - c_t u_t, \tag{1.7c}$$

$$\text{s.t.} \quad S_t^1 = \gamma_t^1 u_{t-\tau}, S_t^2 = \gamma_t^2 u_{t-\tau} + z_{t-1}^1 + z_{t-1}^2. \tag{1.7d}$$

Note that the one-period reward function is independent of the supply history vector.

Lemma 1.5 (Concavity)**.** *The value function* $v_t(u_t, \boldsymbol{y}_t, \boldsymbol{z}_t; \cdot)$ *is jointly concave in* $(u_t, \boldsymbol{y}_t, \boldsymbol{z}_t)$ *for all* $t = 1, 2, \ldots, T$*.*

Proof. We have proved that v_t is concave in $(\boldsymbol{y}_t, \boldsymbol{z}_t)$ in Lemma 1.2. So, we only need to show that v_t is also concave in u_t by backward induction. For $t = T$, the concavity holds since $v_T = r_T$ for any $(u_T, \boldsymbol{y}_T, \boldsymbol{z}_T)$, which is linear in u_T with the first-order derivative $-c_t$. For any other t, we have the value function's second-order partial derivative with regard to u_t as follows,

$$\frac{\partial^2 v_t}{\partial u_t^2} = \beta \frac{\partial^2 \mathbb{E}_t V_{t+1}}{\partial u_t^2} = \beta^\tau \frac{\partial^2 \mathbb{E}_t V_{t+\tau}}{\partial u_t^2} \leq 0,$$

since $\frac{\partial^2 v_{t+\tau}}{\partial u_t^2} \leq 0$ and the concavity preserves in expectation and maximization. □

Proposition 1.3 (The Optimal Policy with Supply Decision)**.** *The supply decision* u_t^* *is the solution of the following FOC,*

$$\beta^\tau \frac{\partial \mathbb{E}_t V_{t+\tau}(\boldsymbol{S}_{t+\tau}, \boldsymbol{u}_{t+\tau}, \boldsymbol{P}_{t+\tau})}{\partial u_t} - c_t = 0, \tag{1.8}$$

where the first term is the discounted margin of a wean τ *periods later.*

1. *Let* $p_t^2 > p_t^C + c_t^P$, $t = 1, 2, \cdots, T$
   (a) *If* $p_t^1 > (1-\alpha)p_t^C + c_t^P$*, the optimal solution is* $y_t^1 = y_t^2 = 0$, $z_t^1 = s_t^1 \wedge \bar{z}_t^1$, $z_t^2 = 0$, *and* $u_t = u_t^*$*.*
   (b) *Otherwise, the optimal solution is* $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge q$, $z_t^1 = (s_t^1 - q)^+ \wedge \bar{z}_t^1$, $y_t^2 = 0$, $z_t^2 = 0$, *and* $u_t = u_t^*$*.*
2. *Let* $p_t^2 \leq p_t^C + c_t^P$, $t = 1, 2, \cdots, T$*.*

(a) *If* $p_t^1 > (1-\alpha)p_t^C + c_t^P$, *the optimal solution is* $y_t^1 = 0$, $z_t^1 = s_t^1 \wedge \bar{z}_t^1$, $y_t^2 = s_t^2 \wedge q$, $z_t^2 = (s_t^2 - q)^+ \wedge \bar{z}_t^2$, *and* $u_t = u_t^*$.

(b) *else if* $0 \leq (1-\alpha)p_t^C + c_t^P - p_t^1 \leq p_t^C + c_t^P - p_t^2$, *the optimal solution is* $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge (q - s_t^2)^+$, $z_t^1 = (s_t^1 - y_t^1)^+ \wedge \bar{z}_t^1$, $y_t^2 = s_t^2 \wedge q$, $z_t^2 = (s_t^2 - q)^+ \wedge \bar{z}_t^2$, *and* $u_t = u_t^*$.

(c) *else, the optimal solution is* $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge q$, $z_t^1 = (s_t^1 - q)^+ \wedge \bar{z}_t^1$, $y_t^2 = s_t^2 \wedge (q - (s_t^1 - \underline{z}_t^1)^+)^+$, $z_t^2 = (s_t^2 - y_t^2)^+ \wedge \bar{z}_t^2$, *and* $u_t = u_t^*$.

The future margin of a wean consists of two parts: 1) the lump-sum reward at period $t+\tau$ and 2) the effect on the remaining horizon after that. Even though it is intractable because of 2), we can interpret some insights from 1) since it is the main component of the future value. The future margin of a wean can be estimated by the future market prices. But the coefficients are not deterministic since the optimal supply decision interacts with the finishing decisions $\boldsymbol{y}_{t+\tau}$ and $\boldsymbol{z}_{t+\tau}$. However, according to Proposition 1.1, the finishing decision is divided into five different cases, and each of them have different effects on the supply decision. So, we summarize the coefficients of future market prices on the marginal value of a wean in Table 1.5 and 1.6. We cannot obtain the closed form of the margin because they contain the holding stock thresholds generated by Proposition 1.1. Therefore, we propose a τ-period look-ahead policy to find the supply decision effectively based on the forecast prices $\hat{\boldsymbol{P}}_{t|\tau}$.

Let $\mathcal{F}_t^1(\cdot)$ and $\mathcal{F}_t^2(\cdot)$ denote the marginal cdf of γ_t^1 and γ_t^2, respectively, then we summarize the τ-period look-ahead policy in the following Proposition 1.4.

Proposition 1.4 (An Optimal τ-Period Look-Ahead Policy). *For any* $t = 1, \cdots, T-\tau$, *the optimal procurement quantity for weans* u_t^* *is the solution to the following equation,*

$$u_t^* = \frac{q}{{\Gamma_{t+\tau}^2}^{-1}\left(\frac{c_t + \sum_{l=0}^{\tau-1} \hat{c}_{t+l}^H - \beta^\tau \hat{p}_{t+\tau}^2}{\beta^\tau \hat{p}_{t+\tau}^C - \beta^\tau \hat{p}_{t+\tau}^2}\right)}, \tag{1.9}$$

Table 1.5: Coefficients of Future Prices on Marginal Value of A Wean

Case (State)	$p^C_{t+\tau}$	$p^1_{t+\tau}$	$p^2_{t+\tau}$
1a	0	$0, \gamma^1_{t+\tau}$	$\gamma^2_{t+\tau}$
1b	$0, (1-\alpha)\gamma^1_{t+\tau}$	$-\gamma^1_{t+\tau}, 0, \gamma^1_{t+\tau}$	$\gamma^2_{t+\tau}$
2a	$0, \gamma^2_{t+\tau}$	$0, \gamma^1_{t+\tau}$	$0, \gamma^2_{t+\tau}$
2b	$0 \to (1-\alpha)\gamma^1_{t+\tau} + \gamma^2_{t+\tau}$	$0, \gamma^1_{t+\tau}, \gamma^1_{t+\tau} + \gamma^2_{t+\tau}$	$0, \gamma^2_{t+\tau}$
2c	$0, \alpha\gamma^1_{t+\tau}, (1-\alpha)\gamma^1_{t+\tau} + \gamma^2_{t+\tau}, \gamma^2_{t+\tau}$	$-\gamma^1_{t+\tau}, 0, \gamma^1_{t+\tau}$	$0, \gamma^2_{t+\tau}, \gamma^1_{t+\tau} + \gamma^2_{t+\tau}$

Table 1.6: Coefficients of Future Costs on Marginal Value of A Wean

Case (State)	$c^H_{t+\tau}$	$c^P_{t+\tau}$
1a	$-\gamma^1_{t+\tau}, 0$	0
1b	$-\gamma^1_{t+\tau}, 0$	$0, \gamma^1_{t+\tau}$
2a	$-\gamma^1_{t+\tau}, -\gamma^2_{t+\tau}, -\gamma^1_{t+\tau} - \gamma^2_{t+\tau}, 0$	$0, \gamma^2_{t+\tau}$
2b	$-\gamma^1_{t+\tau} - \gamma^2_{t+\tau}, -\gamma^1_{t+\tau}, -\gamma^2_{t+\tau}, 0$	$0 \to \gamma^1_{t+\tau} + \gamma^2_{t+\tau}$
2c	$-\gamma^1_{t+\tau} - \gamma^2_{t+\tau}, -2\gamma^1_{t+\tau} - \gamma^2_{t+\tau}, -\gamma^1_{t+\tau}, -\gamma^2_{t+\tau}, 0$	$0, \gamma^1_{t+\tau} + \gamma^2_{t+\tau}, \gamma^2_{t+\tau}$

where $\Gamma^2_{t+\tau}(\xi) \coloneqq \mathbb{E}[\gamma|\gamma \leq \xi]$, *represents the truncated expectation of the yield proportion for regular weight hogs.*

1.7.2 An Extension to Infinite Horizon

In this subsection, we extend our model to an infinite horizon. We follow the fashion of [69], which investigated a nonstationary period review inventory problem with a infinite horizon. The following Proposition 1.5 shows the sufficient condition such that the optimal structural policy preserves for the infinite horizon extension.

Proposition 1.5. *The probability that hog producer holds REGULAR weight hogs approaches to zero, i.e.,* $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{P}_w\{\bar{z}^2_t > 0\} \to 0$*, is sufficient to guarantee that the policies uniquely satisfy the infinite horizon analogues of Proposition 1.1.*

A sufficient condition of for the preservation of optimal policy to the infinite horizon is that hog producer never hold the regular weight hogs. The reason is that only the regular weight pool contains hogs from previous period, whereas the under-weight pool only has hogs newly become market ready this period. If the the hog producer does not hold regular weight hogs, the hogs stay at the finishing stage for at most two weeks, and thus, the optimal policy, which has the "one-period lookahead" fashion will not be influenced by long-stay hogs.

It is not an unrealistic assumption because holding regular weight hogs does not add any value to the product. Compared with holding under weight hogs, which can be raised to regular weight and gaining extra market price, holding a regular weight hog creates the same option value but loses more opportunity profit in the current period. It is not the best option for the hog producer as long as she has excessive under-weight hogs that can be stored for future use. Therefore, we can conclude that our model can be extended to infinite horizon in practice.

1.8 Empirical Study for The Maschhoffs

1.8.1 OTC Price Fit

In this section, we use a regression model to estimate the implied distribution of the price vector $\boldsymbol{P}_t$ of §1.4.1, which will allow us to take expectations in (1.4) and (1.6).

Figure 1.9 provides us an overview of the steps when fitting OTC prices. The model we use is a seasonal autoregressive (AR) moving average (MA) model with exogenous regressors, denoted as VARMAX$(p, q) \times (P, Q)_m$ model, where p is the number of AR terms, q is the number of MA terms, P is the number of seasonal autoregressive (SAR) terms, Q is the number of seasonal moving average (SMA) terms, m is the number of periods in a season (for

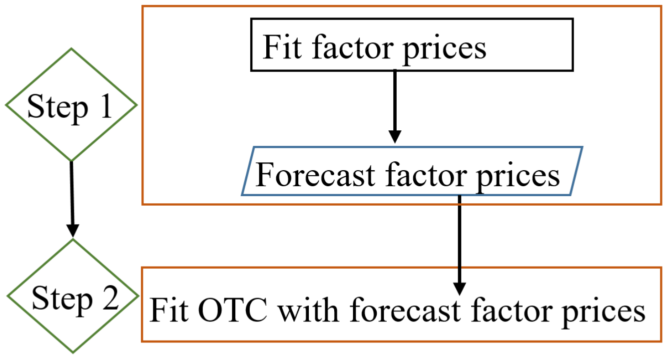


Figure 1.9: Logic Flow of Fitting OTC Price

ARIMA model details, see [86] and [110]. For more details of model calibration, see [50] and [31], which also use ARIMA model framework to simulate prices). Our model specification is VARMAX$(1,1) \times (0,1)_{52}$ formulated as follows,[5]

$$(\boldsymbol{I} - \boldsymbol{\phi}_1 B)\boldsymbol{P}_t = \boldsymbol{\epsilon}_0 + (\boldsymbol{I} + \boldsymbol{\theta}_1 B))(\boldsymbol{I} + \boldsymbol{\Theta}_1 B^{52})\boldsymbol{\epsilon}_t + \boldsymbol{\psi}_1 \hat{\boldsymbol{\Pi}}_t^O + \boldsymbol{\psi}_2 \hat{\boldsymbol{\Pi}}_t^M + \boldsymbol{\psi}_3 \hat{\boldsymbol{\Pi}}_t^F, \tag{1.10}$$

where $\boldsymbol{\epsilon}_t$ is an i.i.d. normal vector with zero mean and a covariance matrix of $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$.

Note the "ˆ" symbol above the factor price vectors $\hat{\boldsymbol{\Pi}}_t^O$, $\hat{\boldsymbol{\Pi}}_t^M$, and $\hat{\boldsymbol{\Pi}}_t^F$.

Forecasting current period's OTC price requires knowing the current period's factor price. We address this issue by forecasting the factor prices $\boldsymbol{\Pi}_t^O$, $\boldsymbol{\Pi}_t^M$, and $\boldsymbol{\Pi}_t^F$ using a standard VARIMA(1,1,1) model and feeding the forecasts, $\hat{\boldsymbol{\Pi}}_t^O$, $\hat{\boldsymbol{\Pi}}_t^M$, and $\hat{\boldsymbol{\Pi}}_t^F$, into the model (1.10). (For additional details, see Appendix A.2.)

In Equation (1.10), the coefficients we need to estimate from data are the AR coefficients ($\boldsymbol{\phi}_1$), the vector of constants ($\boldsymbol{\epsilon}_0$), the MA coefficients ($\boldsymbol{\theta}_1$), the seasonal coefficients ($\boldsymbol{\Theta}_1$), and the vectors ($\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$, and $\boldsymbol{\psi}_3$), which link OTC prices to factor prices (see A.2.1 for additional details).

[5] See Appendix A.2 for additional details.

Table 1.7 gives an overview of the estimated parameters. Table 1.7 indicates that the factor prices are significant at the 5% level.

Table 1.7: Parameter Estimates for OTC Prices Model VARMAX(1,1)× $(0,1)_{52}$

Dep. Var. Parameter	P_t^C	P_t^1	P_t^2	C_t^H	C_t^P
$\boldsymbol{\epsilon}_0$	14.617***	−34.846**	−8.940	1.037*	−0.089
	(4.957)	(14.511)	(9.613)	(0.547)	(0.096)
$\boldsymbol{\phi}_1$	0.165***	0.225*	−0.183**	0.806***	−0.183***
	(0.096)	(0.122)	(0.106)	(0.176)	(0.106)
$\boldsymbol{\theta}_1$	0.358***	0.342*)	0.454***	−0.253	0.454***
	(0.096)	(0.122)	(0.106)	(0.176)	(0.106)
$\boldsymbol{\Theta}_1$	0.110	0.006	−0.027	0.026	−0.027
	(0.166)	(0.183)	(0.184)	(0.223)	(0.184)
$\boldsymbol{\psi}_1$	0.876***	2.207***	2.284***	0.005**	0.023***
	(0.085)	(0.332)	(0.159)	(0.003)	(0.002)
$\boldsymbol{\psi}_2$	0.673***	0.471*	0.328**	−0.007*	0.003**
	(0.074)	(0.244)	(0.144)	(0.004)	(0.001)
$\boldsymbol{\psi}_3$	0.237	1.101***	0.191	0.001	0.002
	(0.159)	(0.292)	(0.257)	(0.009)	(0.003)

Note: Standard errors are given in parentheses under coefficients. * indicates p-value at the 10% level; ** at the 5% level; and *** at the 1% level.

We evaluate the fitness of our time-series model using the following two criteria:

(1) The model's residuals should be normally distributed with a mean of zero. We use Royston's multivariate normality test and the Chi-square quantile-quantile (Q-Q) plot to check this.

(2) There should be no autocorrelation in the residuals. We check for auto-correlation function (ACF) as per [49]. The ACF= 1 indicates a perfect (positive) correlation, whereas ACF= 0 indicates no autocorrelation.

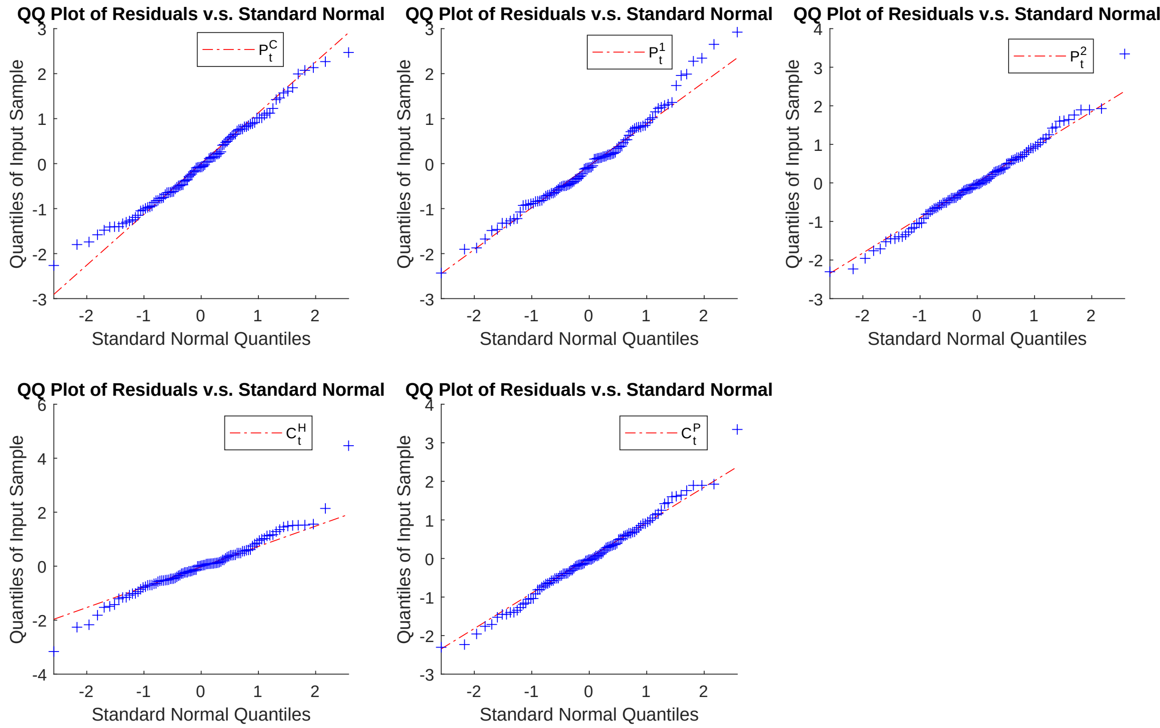


Figure 1.10: QQ Plot of Residuals of VARIMAX(1,0,1)$\times$ $(0, 0, 1)$ with Forecast Factor Price Using ARIMA(1,1,1) Model

We include the corresponding Q-Q plots and ACF plots in Figures 1.10 and 1.11. All indicated that the model passes based on the fitness criteria outlined above.

Two points deserve a discussion regarding our choice of the VARIMAX model. First, the exogenous regressors play an essential role in the estimation of the OTC prices. Without them, the model would reduce to a standard VARIMA model for which, as per Figure 1.12, the ACFs of some residuals are not within the required confidence bounds. Thus, the VARIMA model alone is inadequate to explain our data.

Second, if we tried to fit OTC prices by only regressing on factor prices, the model would fail to capture the seasonal and autoregressive nature of our data. We illustrate this point graphically in Figures 1.13 and 1.14.

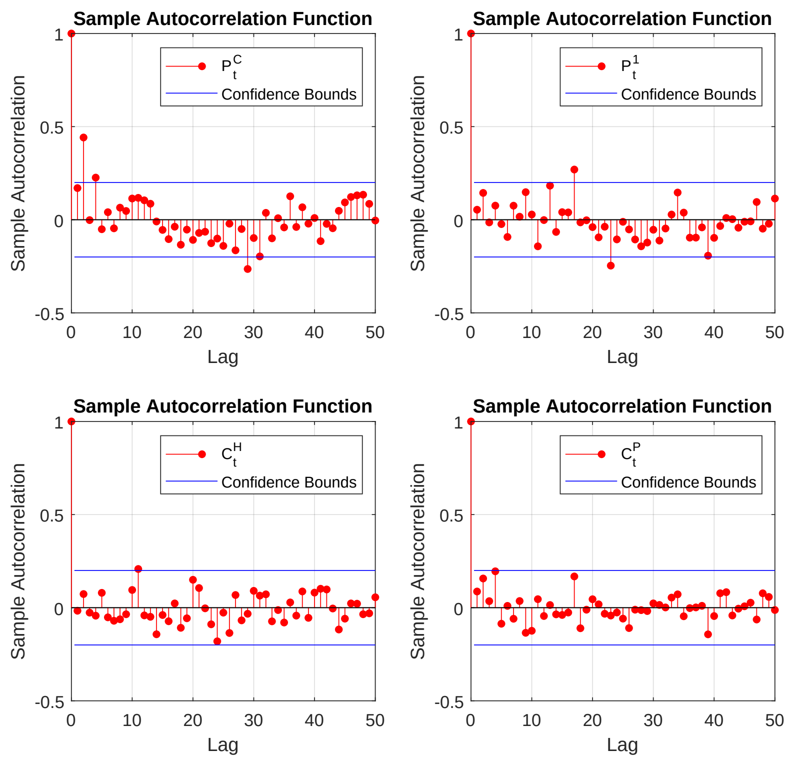


Figure 1.11: Sample ACF of Residuals of OTC Prices Using ARIMAX(1,0,1)× $(0, 0, 1)_{52}$ Model with Forecast Factor Prices Using VARIMA(1,1,1) Model.

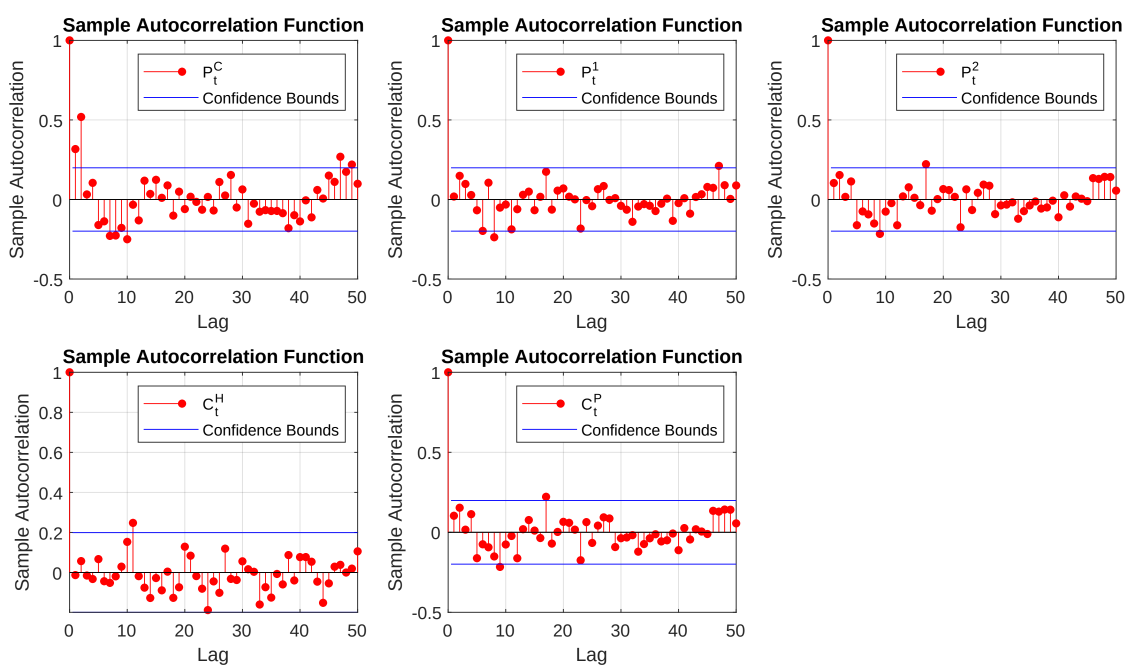


Figure 1.12: ACF for the Residuals of OTC Prices ($(\text{VARIMA}(1, 0, 1) \times (0, 0, 1)_{52})$

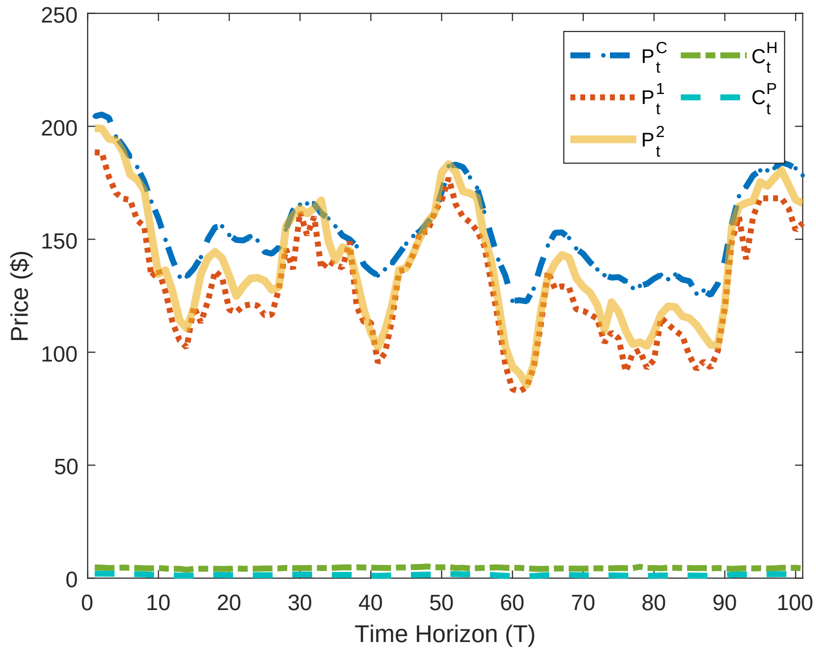


Figure 1.13: OTC Prices in the Time Horizon.

1.8.2 Policy Performance

Based on the data of the previous section, we now use the following inputs to perform a numerical comparison of the policies presented earlier in the paper. Our data inputs are:

- The contract size (q) is 98,107 hogs/week.
- W_t^1 and W_t^2 follows normal distributions with weekly means of 62,906 and 28,906 and standard deviations 9,551.4 and 7,839.2 (§§1.3.1).
- OTC prices are governed by Equation (1.10) of §§1.8.1.
- The discount factors α and β are 15.4% and 2% respectively.

With these inputs, we run 10,000 simulations over a 100-week horizon to compare the following policies:

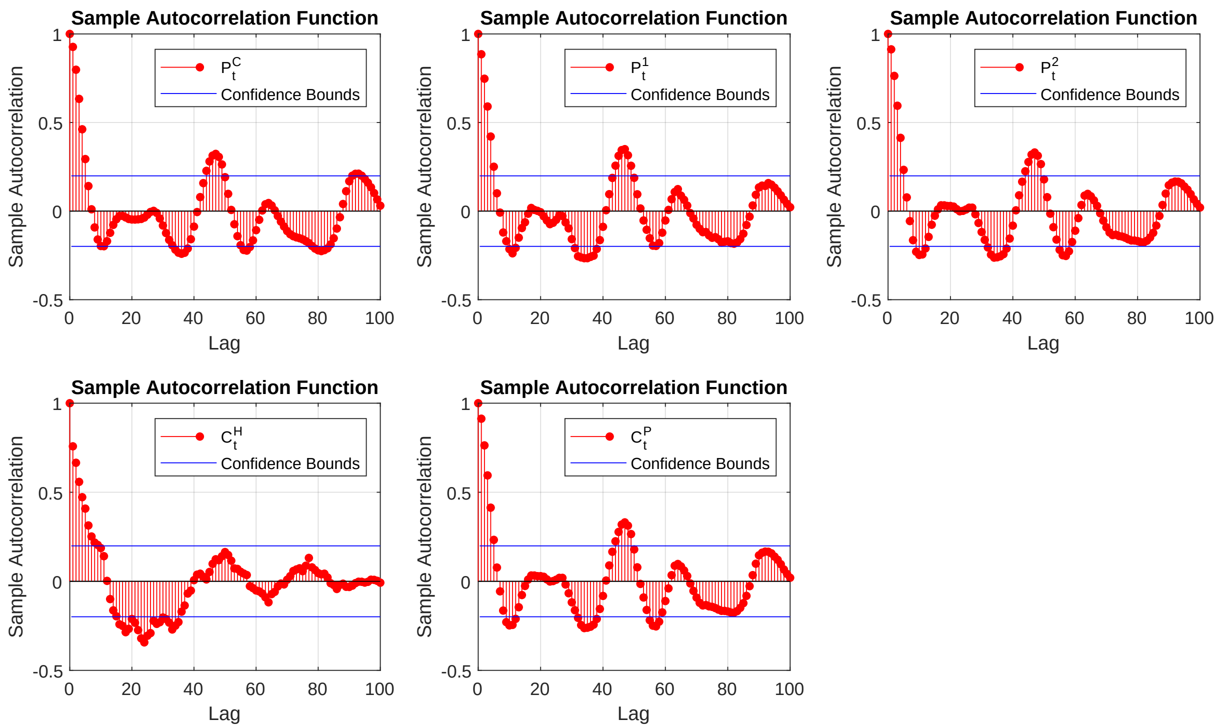


Figure 1.14: ACF of OTC Prices in the Time Horizon.

- The existing always fulfill policy (AF) of §1.4.3
- The optimal policy (OP) of §§1.5
- One-period look-ahead policy (OLNV) of §1.6
- τ-period look-ahead policy (τ-LNV) of §§1.7

Figure 1.15 presents the profit for each policy graphically. Table 1.8 benchmarks each heuristic policy against the optimal policy, OP. (Although in §1.6, we argue that the OP is not trivial compute, here we exploit the farm's assumed short 100-week operating horizon to gain numerical tractability.)

Our calibrated numerical study's central observation is that the optimal policy represents a substantial improvement over the existing practice (25.89% on average). At the same time,

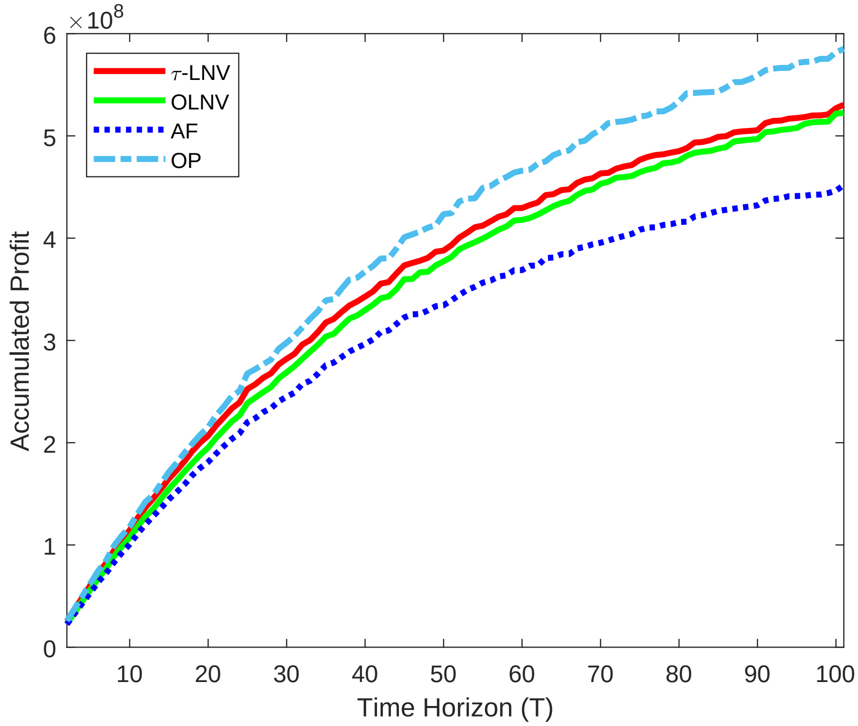


Figure 1.15: Performance of Heuristics with Endogenous Supply

the optimal policy outperforms the one-period look-ahead policy by as little as 1.76%. This performance is noteworthy because the one-period look-ahead is relatively easy to implement and delivers a significant improvement over the farm's existing practice.

From the managerial point of view, the most significant factor behind the optimal policy's improvement over the existing practice is the treatment of the under-weight hogs. Currently, the farm uses the under-weight hogs to either fulfill meatpacking contract or even sell them on the open market (see §1.4.3). As we explain in our discussion of Lemmas 1.3 and 1.4, the optimal strategy is to keep some of these under-weight hogs on the farm until the next decision period to capitalize on their inevitable weight and price gains.

Table 1.8: Performance of Heuristic Policies

Change†	τ-LNV	OLNV	AF
Average⋆	**9.33%**	**10.69%**	**22.77%**

† $\Delta^\eta \doteq \dfrac{V_0^*(\boldsymbol{s}_0, \boldsymbol{p}_0) - V_0^\eta(\boldsymbol{s}_0, \boldsymbol{p}_0)}{V_0^*(\boldsymbol{s}_0, \boldsymbol{p}_0)}$, where $V_0^*(\boldsymbol{s}_0, \boldsymbol{p}_0)$ and $V_0^\eta(\boldsymbol{s}_0, \boldsymbol{p}_0)$ denote the total expected profit over the planning horizon with benchmark policy (OP) and a policy η.

⋆ For each heuristic policy, "Average" denotes the average percentage profit loss in all numerical instances.

1.9 Conclusion

Our paper investigates a challenging weekly operations-sales planning problem at the wean-to-finish hog farm (farms raising larger-sized pigs close to market-ready weight, typically 23-26 weeks old). 23- to 26-week old hogs are combined into pools of under-weight and regular-weight hogs that can be used to either fulfill fixed quantity long-term contracts with food producers or be sold on the open market. The farmer can use either the regular-weight or under-weight hogs to fulfill the food producer contract, although the under-weight hogs fetch a lower price. Pork producers can essentially pick the best time to sell because they have the flexibilities to hold hogs and let them continue to feed instead of selling them, although the decision to hold means they incur additional feeding costs. While the hogs feed, they gain weight. Under-weight (regular-weight) hogs gain lean weight (fat). A rise in lean weight leads to a rise in the economic value of the hog. The research questions are what hogs to sell and when. To the best of our knowledge, this is the first paper that addresses dynamic multi-item inventory and selling policy in contractual and spot markets under-supply uncertainties, stochastic costs, and both markets governed by stochastic prices.

Currently, the producer (the most prominent family-owned US hog farmer, The Maschhoffs) uses an always fulfill (AF) policy. The spirit of the current practice is that the long-term contract has the utmost priority, and the regular-weight hogs are used to fulfill the contract. The under-weight hogs are mostly there to account for shortages in the contract obligations, and if in excess, they are sold in the open market. We improve the AF policy by viewing the farm's problem as a dynamic, multi-item inventory model with stochastic supply and prices and answering the research question using a conventional dynamic programming formulation to derive an optimal policy structure. The optimal policy is non-monotone with four thresholds. The thresholds characterize action switching for the hogs (under- or regular-weight hogs as reflected by the threshold) from holding to selling to the open market or from holding to fulfilling the contract depending on market prices. All thresholds are conceptually explained through the marginal value comparison of holding, fulfilling the contract, or selling to the spot market. The major drawback of directly applying the optimal policy is that it is difficult to calculate the theoretically optimal thresholds. To overcome the computational difficulties in identifying the thresholds, we also derive a one-period look-ahead heuristic based on the optimal policy's structure and thresholds with a newsvendor-like interpretation.

Calibrated numerical experiments reveal that the optimal policy substantially improves the existing practice (25.89% on average). At the same time, the optimal policy outperforms the one-period look-ahead policy by as little as 1.76%. This performance is noteworthy because the one-period look-ahead algorithm preserves the optimal policy structure, is relatively easy to calculate the needed thresholds, and delivers a significant improvement over the farm's existing practice. The success of the proposed solution (policy structure and heuristic thresholds) is attributed to recognizing the value of holding under-weights hogs and effectively hedging supply uncertainty and future prices – an insight missed in the planning actions of the current practice.

Chapter 2

A New Class of Revenue Management Problems with Overbooking and No-Shows: Shoring up Trust between Shippers and Carriers in Maritime Container Shipping

In the airline industry, the practice of overbooking has been a celebrated operational tool that has lead to revenue gains exceeding hundreds of millions of dollars when implemented correctly. By contrast, in the container shipping industry, the story of overbooking is filled with tales of chronic mistrust between shippers and carriers. Specifically, loose and unenforceable contracting practices have led to a failed market where shippers constantly renege on their agreement to produce containers as promised, and as a result, carriers overbook too frequently in an effort to hedge against this no-show behavior. The cost of such behaviors has been estimated to be in the range of $30-40 billion annually, which highlights the glaring need for a remedy to this issue.

In this paper, we propose and study a deposit-based booking system that draws inspiration from current practices that have been shown to be successful in mitigating no-show behavior and overbooking in the container shipping industry. Specifically, we consider a reservation system where inquiring shippers book cargo space using a customized deposit. The carrier, upon accepting the shipper's booking request, matches the shipper's deposit with a deposit of their own of equal size. If either party reneges on the agreement, the defaulting party loses their deposit to the more trustworthy party. However, if both parties uphold their side of the deal, the deposits are returned in full to both sides. Under this booking mechanism, we study the carrier's sequential online booking problem, which gives rise to a new class of revenue management problems with overbooking and no-show behavior that share only superficial commonalities with existing frameworks. Our main algorithmic finding is the development of a simple and easy to implement booking policy, which we show to be $\frac{1}{6}$-competitive against a clairvoyant benchmark that knows the full sequence of deposits. Additionally, in certain settings that are likely to arise in practice, we maintain that this policy rewards reliable shippers by guaranteeing them a service slot if their booking request is accepted.

2.1 Introduction

The field of revenue management is no stranger to the practice of overbooking, where airlines [47], hotels [71], hospital clinics [33] and the like adopt booking policies that hedge against the prospect of cancellations and no-shows by accepting more reservations than they have physical capacity to serve. Indeed, there is a rich and diverse history of research on this topic that dates back over 60 years, beginning with the seminal works of [97], [80] and [81], and extending to the more modern-day takes of [92], [37], and [35], to name a few. While each of these previous works has its own distinguishing features, at their core, they are all fueled by the following fundamental trade-off that is inherent to the practice of overbooking: book too few reservations and there will be unused capacity, but book too many reservations and customers will either have to be turned away or rescheduled, and then compensated for this inconvenience. The ability of airlines, in particular, to find a booking limit "sweet spot" has led to considerable improvements in their bottom line. [87] reports that American Airlines estimates that 15 percent of seats on sold-out flights would be unused if it were not for overbooking, which equates to what would be around $225 million[6] in lost revenue.

Maritime container shipping and its pain points. A lesser-studied and intriguing application of overbooking has been steadily evolving in maritime container shipping, which is the service of transporting goods by means of truck-sized intermodal containers via cargo ships. The Twenty-foot Equivalent Unit (TEU), whose name was derived from the dimensions of a 20 foot standardized shipping container, is the standard unit of measurement used to determine cargo capacity for container ships. The big players in this industry are container shipping companies like Maersk Line, China Ocean Shipping Company (COSCO), and Orient Overseas Container Line (OOCL), who offer regular service on fixed routes and schedules to many multi-national companies (e.g. Apple and Walmart). The container shipping industry as a whole accounts for approximately 60 percent of all seaborn trade, which equates to a $14 trillion valuations. [7]

To initiate the discussion surrounding how overbooking and no-show behavior have shaped the maritime container shipper industry, we begin with a high-level description of its sequential

[6] All dollar figures are in USD.
[7] https://www.statista.com/topics/1367/container-shipping/#dossierKeyfigures

booking problem. Abstracting away from the finer details for the moment, we first note that each carrier's liner (ship) naturally has a fixed number of TEUs that it can transport. We henceforth refer to the available capacity on a liner as service slots, i.e., a service slot is the physical space needed to transport a single container or TEU. Shippers in need of transport for their cargo send booking requests to a carrier, who can either accept or reject this request. For each accepted request, the carrier sets aside one of its available service slots for use by the shipper, who agrees to deliver its cargo, ready for transport, on a specific date. For reasons that will be made clear shortly, the carriers experience relatively high no-show and cancellation rates among shippers, which forces them to accept excessive bookings in relation to their available shipping capacity. Unlike the airline industry, the result of such overbooking practices has not been financial success, but rather, has resulted in a downward competitive cycle that has caused profit losses for carriers, and supply chain inefficiencies for shippers. There are two central, and thus far unmentioned, features of the carrier-shipper booking process that are the culprit for the pain points caused by overbooking and no-show behavior. First, unlike airline passengers who pay for their ticket upon booking, shippers are only required to make a payment to the carrier once their cargo is loaded on the ship, or even after the shipment has been delivered. Second, a long history of loose contracting practices have essentially made any agreed upon no-show or overbooking penalties unenforceable by either party. As a result of these two features, shippers readily abandon bookings because they have found cheaper rates (on the spot market, for example) or faster delivery with a different carrier. In response, carriers can often pledge double the number of their available TEUs to shippers in anticipation of these cancellations. Then, if overbooking ensues, carriers can freely choose to transport the cargo of the shippers whose agreed upon rates were highest, and "roll" the remaining cargo, which is the industry term for cargo that has been rescheduled due to overbooking. [67] concisely summarizes the catastrophic effects these practices have had on maritime container shipping:

> *"At its heart, the system is driven by uncertainty: shippers face no penalty for booking a certain volume of cargo on a ship, and then failing to send it; and carriers face little retribution for leaving containers on the dock that they have committed to shipping. The two behaviors, and the apparent disregard for the impact on the other party, fuels an unwillingness to take steps to change a system that clearly hurts both parties in the long run. Carriers end up running ships with plenty of empty space, and shippers have to scramble to make alternative arrangements to get their cargo to its destination on time when containers are left on the dock."*

Echoing this notion, Tom Smart, vice president of MOL (America), noted that between 17 and 52 percent of promised cargo never materializes, saying "There is no penalty for this fall down...I just have to manage it." [66]. According to [89], the overall effects of no-show shippers and rolled cargo amounts to a staggering $30-40 billion annually, which highlights the glaring need for a remedy to this issue.

Solution efforts: past and present. As one would naturally suspect, there have been multiple attempts to fix the overbooking and no-show issues that have plagued the container shipping industry for many years. In what follows, we summarize the most notable efforts, and discuss their efficacy with regards to shoring up trust between shippers and carriers.

- Penalty fees: In 2011, Maersk Line added two-sided fees to their contracts with shippers, which included monetary penalties for both no-shows and rolled cargo. These attempts proved unsuccessful, as there was no third party system to enforce the fees when shipping agreements were broken by either party. Later, in 2016, Hapag-Lloyd announced a $40 booking cancellation fee that was met with push back from shippers who questioned the lack of penalties on the carrier side for rolled cargo [11].
- A blockchain-based approach: More recently, in 2018, Hong-Kong-based start-up 300cubits proposed a blockchain-based approach to better enforce contracts between liners

and shippers, which are often times agreed upon just by email or verbal affirmations. To do so, they created TEU tokens (a play on the container unit of measurement), as well as an Etherium-based ecosystem to facilitate the use of their digital currency. The company launched an Initial Coin Offering (ICO) during April and May, 2018, and raised over $1.4 million in total. They subsequently released one million TEU token into the ecosystem, whose intended use was for deposits by both carriers and shippers that would serve to dissuade reneging by either party. More specifically, each booking request issued by shippers was to be accompanied by a TEU token deposit. If the carrier accepted the given booking, they would match the shipper's deposit via their own TEU token deposit of equal magnitude, with all of the details secured within an Etherium-based smart contract. These blockchain-enabled smart contracts were automatically enforced based on the outcome of the agreement, thus eliminating the need for third-party banks and the possibility of contract breaches. If both shipper and carrier lived up to their side of the agreement, then both would receive their deposit back in full. However, if either party reneged, then their deposit would be lost to the more trustworthy party,

In late 2019, 300cubits shut down its TEU token deposit system due to a lack of transaction volume, citing a "lack of clarity in regulation regimes surrounding digital currencies" as the reason. Nonetheless, they note that their two-sided deposit-based booking mechanism was generally successful across the relatively small number of transaction supervised by the 300cubits platform.[8]

- A digital matching platform: Founded in 2014, the New York Shipping Exchange (NYSHEX) was created to restore trust between shippers and carriers via two-way digital contracts, which are governed by NYSHEX through a handbook of rules created

[8] https://www.300cubits.tech/wp-content/uploads/2019/09/Booking_Deposit_Module_Announcement_20190930.pdf

by a committee of shippers and carriers. Using NYSHEX's digital platform, inquiring shippers can negotiate contracts with shippers that are intended to provide more predictable (albeit not necessarily lower) rates than the spot market, with built in negotiable penalty fees for no-shows or cancellation on the shippers side, and cargo rolling on the carriers side. NYSHEX initially experienced slow growth due the fact that most shippers are rooted in their old ways. However, through 2019 they experienced steady growth, and also reported that 96.4% of its contracts were fulfilled [91].

Research agenda. Motivated by the relative success of 300cubits and NYSHEK in mitigating no-shows and cargo rolling, we present a theoretical examination of the extent to which two-sided customized contracts between shippers and carriers have the potential to mitigate these two pain points of the industry. Delaying a full description until Section 2.1.1, we consider a booking mechanism that combines the honesty inducing token-based deposit system of 300cubits with the flexibility of NYSHEX's customizable contracts. Our intent is not to model the exact practices of any particular company, but instead to distill the essential elements of their booking problem and key contracting practices into a framework from which insights can be drawn. To achieve this goal, we focus on a carrier's sequential online decision problem regarding whether to accept or reject each incoming booking request, and ask the following research questions:

1. Can we develop simple and easy-to-implement booking policies for the carrier that come with robust performance guarantees?
2. Do the proposed policies encourage and reward (i.e., not roll the cargo of) reliable shippers, deemed to be those that make a strong commitment to deliver cargo?

2.1.1 Problem Formulation

We consider a cargo reservation system that has a finite collection of available service slots on one of its liners. To set the stage for an eventual formal description of our booking problem, we first provide a high-level account of how we model the process through which inquiring shippers acquire service slots. We concurrently supplement this discussion with an informal description of how the liner earns profit throughout this booking process.

We consider a finite horizon booking window, over which potential shippers arrive sequentially, each interested in acquiring one of the available service slots provided by the liner. Upon arrival, each shipper offers a deposit to secure a slot, from which the liner infers a show-up probability, i.e., the likelihood that the given shipper will need the service slot come time for the carrier's liner to depart. There are many observable factors influencing this need, e.g., the magnitude of their deposit relative to the spot market price, historical reliability of the shipper, freight route origin/destination, type of product to be shipped, time of the year etc ..., from which accurate forecasting models can be built. [74] and [109] develop such forecasting models for predicting no-show and cancellation rates in the airline and container shipping industries. After observing each shipper's deposit and corresponding show-up probability, the liner must make an irrevocable decision regarding whether to accept or reject the shipper's offer. The deposits of the accepted offers are frozen by a third escrow system (potentially through a smart contract) until the end of the booking window, while rejected shippers leave the system. We do not impose any limits on the number of deposits that the liner can accept, which as explained at the end of this section, is necessary for the development of any policy that achieves a non-trivial competitive ratio.

At the end of the booking window, each shipper whose offer was accepted either shows up to claim a slot or reneges according to its respective show-up probability. For each no-show, the liner keeps the deposit of the corresponding shipper. Among the shippers who show up to claim a slot, the liner allocates the available slots to the shippers whose deposits are largest. We refer to this process as the "Deposit-Ordered" (DO) slot allocation mechanism, which, as discussed at the end of Section 2.2.2, is in fact the profit-optimal way for the liner to assign its service slots slots if more shippers show-up than there are slots. Any shipper that receives a slot gets its deposit back, and pays the service fee for use of the service slot. On the other hand, shippers who show up but do not receive a slot get their deposit back, along with a reimbursement equal to their deposit. This reimbursement can be viewed as an overbooking penalty paid by the liner. The liner's goal is to design an online policy that governs the accept/reject decision for each arriving shipper so as to maximize its expected profit. In what follows, we first provide more details regarding how the show-probabilities linked to each deposit should be interpreted, and then proceed to formalize each ingredient of the liner's booking problem.

Preliminaries. We let $m \in \mathbb{Z}_+$ denote the number of service slots available to the liner. We consider a $T \in \mathbb{Z}_+$ period booking window, where during each period $t \in [T]$, a single shipper arrives with probability 1.[9] Abusing notation slightly we index the shippers through $[T]$, i.e., shipper $t \in [T]$ is the shipper who arrived in period t. The shipper arriving in period t is characterized by a "deposit tuple" (d_t, p_t), where d_t is the offered deposit and p_t is the estimated show-up probability. Furthermore, we let $Y = \{Y_t\}_{t\in[T]}$, where $Y_t \sim \text{Bernoulli}(p_t)$ represents the random variable capturing whether or not shipper t shows up at the end of the horizon. We assume that the shippers' offered deposits are capped at the service fee, which in turn allows us to normalize the service fee to 1, while also assuming that each deposit $d_t \in [0, 1]$. Since the carrier must match the deposit of each accepted shipper, it is natural to place a cap on the maximal deposit allowed so as not to tie up too much of the

[9]We model the arrival of shippers using a Bernoulli arrival process merely for ease of notation, noting that it is not difficult to see that all of our results go though for an arbitrary sequential arrival process.

carrier's cash in deposits. [10] Beyond this assumption, we place no further restrictions on the sequence of observed deposit tuples, and hence our results are agnostic to the particular model that the liner uses to estimate the show-up probabilities (i.e., it could be a blackbox machine learning model). Formally, we assume that the sequence of observed deposit tuples is chosen adversarially from the set $\mathcal{S} = \{(d_1, p_t), \ldots, (d_T, p_T) : (d_t, p_t) \in [0,1]^2, T \in \mathbb{Z}_+\}$, which naturally gives way to the most general and most challenging version of the liner's sequential booking problem.

Summary of key events. With this notation in-hand, we proceed to formalize the sequence of events that characterize our problem of interest. For the remainder of the paper, we work under an arbitrary fixed sequence of deposit tuples $(d_1, p_t), \ldots, (d_T, p_T) \in \mathcal{S}$.

1. At the start of the first period, the liner opens m service slots.

2. In period $t \in [T]$, the shipper associated with deposit tuple (d_t, p_t) arrives, and the liner must decide to accept or reject the shipper. After observing the full deposit sequence, let $\mathcal{A}_\pi \subseteq [T]$ denote the shippers whose deposit was accepted under an arbitrary accept/reject policy π.

3. In period $T + 1$ (at the end of the booking window), the liner observes $\{Y_t\}_{t \in \mathcal{A}_\pi}$, i.e. which shippers show up to claim a slot. Let $T_{\text{show}} = \{t \in \mathcal{A}_\pi : Y_t = 1\}$ denote the set of shippers whose deposit was accepted, and who showed up to claim a slot.

4. The liner allocates a service slots according to the DO allocation mechanism; shippers $t \in T_{\text{show}}$ whose deposit is among the m largest across $\{d_t\}_{t \in T_{\text{show}}}$ receive a service slot. The liner's profit from each shipper $t \in \mathcal{A}_\pi$ falls into one of the following three cases.

[10]The rate for a single TEU can range from $2,000-20,000

- Case 1 - no-shows: For each shipper $t \in \mathcal{A}_\pi \setminus T_{\text{show}}$, the liner keeps the shipper's deposit, and hence earns a profit of d_t.
- Case 2 - show + slot: For each shipper $t \in T_{\text{show}}$ that receives a slot, the liner earns the service fee of 1.
- Case 3 - show + no slot: For each shipper $t \in T_{\text{show}}$ that does not receive a slot, the liner pays a reimbursement of d_t.

The liner's profit function. It turns out that our future analysis will only require a formal account of the liner's profit for a fixed acceptance set $\mathcal{A} \subseteq [T]$. For this purpose, let $\rho : [T] \to [T]$ denote a bijection that maps each shipper $t \in [T]$ to its relative rank among all T deposits, where we use the convention that smaller rankings correspond to larger deposits. In other words, shipper $t \in [T]$ has the $\rho(t)$-th largest deposit among $\{d_t\}_{t \in [T]}$, where ties can be broken arbitrarily. Furthermore, let $R_t(\mathcal{A}, Y; m)$ denote the random profit earned from shipper $t \in \mathcal{A}$, given that there are m available slots. The following claim, whose proof can be found in Appendix B.1, provides an explicit expression for computing the expected profit earned from shipper $t \in \mathcal{A}$.

Claim 2.1. *For acceptance set $\mathcal{A} \subseteq [T]$, shipper $t \in \mathcal{A}$, and $m \in \mathbb{Z}_+$ we have*

$$\mathbb{E}\left[R_t(\mathcal{A}, Y; m)\right] = d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau < m\right] \cdot \left(p_t \cdot (1 + d_t)\right),$$

where the expectation is taken with respect to $\{Y_t\}_{t \in \mathcal{A}}$.

Letting $\mathcal{R}(\mathcal{A}; m)$ denote the total expected profit that the liner derives from acceptance set $\mathcal{A}$ when there are m available slots, it is straightforward to see that $\mathcal{R}(\mathcal{A}; m) = \mathbb{E}[\sum_{t \in \mathcal{A}} R_t(\mathcal{A}, Y; m)]$. When viewed in its explicit form, the liner's profit function elucidates the new technical hurdles that arise in relation to the liner's sequential booking problem.

Most notably, in our setting, the total overbooking penalty is a function of the *specific* shippers whose cargo is rolled, rather than just the number of shippers who must be turned away, as is typically the case in standard overbooking models [35, 99].

The competitive ratio. Let $\text{OPT} = \max_{\mathcal{A} \subseteq [T]} \mathcal{R}(\mathcal{A}; m)$ denote the best case profit garnered by a clairvoyant who has full access to the sequence of deposit tuples $(d_1, p_1), \ldots, (d_T, p_T)$. The dependence of OPT on m is not made explicit since this relationship is not important for future analysis. We note that OPT is clearly an upper bound on the best-case expected profit earned by the liner, since the liner's policy must make the accept/reject decisions in an online fashion. We say that any such online policy π, leading to acceptance set $\mathcal{A}_\pi \subseteq [T]$, is α-competitive if $\mathcal{R}(\mathcal{A}_\pi; m)/\text{OPT} \geq \alpha$, for some $\alpha \in [0, 1]$. Since we work under an arbitrary sequence of deposit tuples, an α-competitive policy can be understood to garner an expected profit of at least $\alpha \cdot \text{OPT}$ for any sequence of deposit tuples chosen from $\mathcal{S}$. We conclude this section by providing a simple two-period setting, which demonstrates that if we impose any sort of booking limit, then one cannot develop general policies with a non-zero competitive ratio. This example motivates our assumption that there are no limits on the number of deposits that the liner can accept.

Example 2.1. Consider a setting with $T = 2$ and $m = 1$. Furthermore, we assume that the liner cannot overbook, and thus it can only accept a single deposit. For arbitrarily small $\epsilon > 0$, let the first period deposit tuple be $(d_1, p_1) = (\epsilon, \epsilon)$. If the liner chooses a policy that accepts this deposit offer, then we assume that the second period deposit tuple is $(d_2, p_2) = (1, 1)$. In this case, the liner earns an expected profit of $\epsilon + (1 - \epsilon) \cdot \epsilon$, while $\text{OPT} = 1$, which is achieved by rejecting the first period offer, and accepting the second period offer. If, on the hand, the liner rejects the first period deposit offer, then we assume that the second period deposit tuple is $(d_2, p_2) = (0, 0)$. In this case, the liner earns nothing, while $\text{OPT} = \epsilon + (1 - \epsilon) \cdot \epsilon$, since a clairvoyant would accept the first offer. In the former scenario, we get a competitive ratio of $O(\epsilon)$, and in the latter scenario, we get a competitive ratio of 0.

2.1.2 Contributions

From an algorithmic perspective, our main result centers around the development of perhaps the simplest booking policy imaginable, in that it requires essentially no computation to implement. We refer to this policy as the "Threshold One-Half" (TOH) policy. Delaying a formal description of the this policy until Section 2.2, we note that one particularly intriguing feature of the TOH policy is that it reserves the m service slots for so-called reliable shippers, deemed to be those with show-up probabilities that exceed $\frac{1}{2}$. We ultimately show that the TOH policy is $\frac{1}{6}$-competitive through an amalgam of simple claims from which we can (i) easily analyze the worst-case deposit tuple sequence for the TOH policy and (ii) find overlap between the decisions made the TOH policy and the optimal booking policy. In Section 2.2.5, we present an extensive array of computation experiments, in which the TOH policy is benchmarked against the optimal clairvoyant policy using a handful of performance metrics. Quite remarkably, we find that the TOH policy far exceeds its worst-case guarantee of $\frac{1}{6} \cdot \text{OPT}$, earning, on average, near optimal profits across all test cases. Furthermore, with regards the secondary performance metrics, we observe that, on average, the TOH matches the optimal policy in terms of its usage capacity (i.e., fraction of the m slots allocated), while rolling slightly less cargo.

From a managerial perspective, we note that in settings where higher deposits imply higher show-up probabilities, the TOH policy has appealing structure. Such settings arise in practice when each shipper's show-up likelihood is highly influenced by the spot market price, i.e., the shipper reneges only if the spot market price dips to a level that makes it more profitable for the shipper to back out of their agreement and acquire its TEUs through the spot market.

In this case, higher deposits naturally beget high show-probabilities, since bigger swings in the spot market are needed to offset higher deposits. In this case, reliable customers will have the highest deposit among all those accepted, since they will be the only shippers with show-up probabilities exceeding $\frac{1}{2}$. Furthermore, as we have yet to reveal, the TOH policy accepts at most m reliable customers, and so any reliable shipper who shows up to claim a service slot will be allocated one, which also implies that only unreliable shippers will ever have their cargo rolled. In sum, the TOH policy is simple and easy to implement, comes with robust worst case performance guarantees, and it rewards honest and reliable shippers by guaranteeing them a service slot if their booking request is accepted, while only rolling the cargo of shippers who cannot make a strong commitment to use of the carrier's service.

2.2 The Threshold One-Half Booking Policy

The entirety of what follows, up to and including Section 2.2.4, is devoted to formalizing the TOH policy and showing that it is $\frac{1}{6}$-competitive. Section 2.2.5 then details a collection of numerical experiments aimed at measuring the efficacy of the TOH policy via a handful of performance metrics. All proofs are located in Appendix B.1.

Reliable and unreliable shippers. The TOH policy will critically depend on the following partitioning of the shippers. Specifically, we label all shippers $t \in [T]$ with show-up probability $p_t \geq \frac{1}{2}$ as "reliable" (abbreviated as "rel" throughout), and any shipper $t \in [T]$ with show-probability $p_t < \frac{1}{2}$ as "unreliable" (abbreviated throughout as "unrel"). Furthermore, let $T_{\text{rel}} = \{t \in [T] : p_t \geq \frac{1}{2}\}$ and $T_{\text{unrel}} = [T] \setminus T_{\text{rel}}$ respectively denote the sets of reliable and unreliable shippers for our fixed deposit sequence $(d_1, p_1), \ldots, (d_T, p_T)$. Finally, we let

$$\begin{aligned} \mathcal{R}^*_{\text{rel}} &= \max_{\mathcal{A} \subseteq T_{\text{rel}}} \mathcal{R}(\mathcal{A}; m) \\ \mathcal{R}^*_{\text{unrel}} &= \max_{\mathcal{A} \subseteq T_{\text{unrel}}} \mathcal{R}(\mathcal{A}; m) \end{aligned}$$

respectively denote the maximum expected profit that can be extracted from reliable and unreliable shippers by a clairvoyant that has full access to the deposit tuple sequence. Both $\mathcal{R}^*_{\text{rel}}$ and $\mathcal{R}^*_{\text{unrel}}$ depend on m, however, once again, making this relationship explicit serves no purpose in our future analysis.

Main theorem. Surprisingly, this exceedingly simple and intuitive policy can be shown to be $\Omega(1)$-competitive, as stated in the following theorem. We devote the remainder of this section to establishing this result.

Theorem 2.1. *For any $m \in \mathbb{Z}_+$, we have that*

$$\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{6} \cdot \text{OPT}.$$

The above theorem not only constitutes what we feel is a meaningful addition to the extensive literature concerning the development of overbooking policies in classical revenue management settings, but also complements the growing body of work related to the now-famous prophet inequality in optimal stopping [58, 82]. To the best of our knowledge, only [32] considers a version of this classical optimal stopping problem with aspects of overbooking, however, the notions of overbooking in this earlier work only mildly resemble those considered here.

2.2.1 High-Level Outline

We go-about proving Theorem 2.1 through a sequence of three steps, which are summarized below.

[58, 82]. To the best of our knowledge, only [32] considers a version of this classical optimal stopping problem with aspects of overbooking, however, the notions of overbooking in this earlier work only mildly resemble those considered here.

Step 1: The Reliable-First slot allocation mechanism (Section 2.2.2). In this first step, we consider an alternative slot allocation mechanism in lieu of the DO slot allocation mechanism that assigns the slots to the shippers with the largest deposits. Specifically, we consider a mechanism that first gives away the available slots to the reliable shippers, irrespective of their deposit sizes, before then allocating slots to unreliable shippers in decreasing order of deposit size. We aptly refer to this alternative mechanism as the the Reliable-First (RF) slot allocation mechanism, and show that under the RF mechanism, the expected profit of the liner is no larger than that earned under the DO slot allocation mechanism, yet, its dynamics turn out to be simpler to analyze.

Step 2: Competing against $\mathcal{R}^*_{\text{rel}}$ (Section 2.2.3). Recalling that $\mathcal{A}_{\text{TOH}}$ is the acceptance set returned by the TOH policy, in this second step, we prove that $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}$. We do so by first showing that the profit contribution of unreliable shippers is always non-negative, and hence in an effort to simplify the analysis, we can focus exclusively on the reliable shippers, and show that $\mathcal{R}(\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}; m) \geq \frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}$. This latter bound is then established by relating the worst case profit of $\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}$ to a simple upper bound on $\mathcal{R}^*_{\text{rel}}$.

Step 3: Competing against $\mathcal{R}^*_{\text{unrel}}$ (Section 2.2.4). In this third step, we show that $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot \mathcal{R}^*_{\text{unrel}}$. The key insight with regards to establishing this bound is showing that, if the TOH policy were to retroactively reject all reliable shippers that it had accepted, then from just the unreliable shippers alone, it would garner an expected profit of $\mathcal{R}^*_{\text{unrel}}$. In reality, however, the TOH policy may in fact accept up to m reliable shippers, which "steal away" profit from the unreliable shippers by potentially claiming service slots. Fortunately, we are able to show that this potential profit loss is minimal, bounding it in such a way that leads to the aforementioned performance guarantee that we seek in this step.

The final steps. In what follows, we show that the two performance bounds derived in Steps 2 and 3, namely $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}$ and $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot \mathcal{R}^*_{\text{unrel}}$, are all that is needed to prove Theorem 2.1, Having established these bounds, we see that

$$
\begin{aligned}
\frac{\mathcal{R}(\mathcal{A}_{\text{TOH}}; m)}{\text{OPT}} &\geq \frac{\max\left\{\frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}, \frac{1}{2} \cdot \mathcal{R}^*_{\text{unrel}}\right\}}{\mathcal{R}^*_{\text{rel}} + \mathcal{R}^*_{\text{unrel}}} \\
&\geq \min_{\mathcal{R}^*_{\text{rel}}, \mathcal{R}^*_{\text{unrel}} \geq 0} \frac{\max\left\{\frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}, \frac{1}{2} \cdot \mathcal{R}^*_{\text{unrel}}\right\}}{\mathcal{R}^*_{\text{rel}} + \mathcal{R}^*_{\text{unrel}}} \\
&= \frac{1}{6},
\end{aligned}
$$

where the first inequality follows from Claim 2.2 presented below, and the final equality follows by noting that the minimum is achieved when $\mathcal{R}^*_{\text{unrel}} = \frac{1}{2} \cdot \mathcal{R}^*_{\text{rel}}$. Consequently, in the worst case, the TOH policy earns a $\frac{1}{6}$-th fraction of OPT, as is stated in Theorem 2.1.

Claim 2.2. $\mathcal{R}^*_{\text{rel}} + \mathcal{R}^*_{\text{unrel}} \geq \text{OPT}$.

2.2.2 Step 1: The Reliable-First slot allocation mechanism

The expected profit function $\mathcal{R}(\mathcal{A}; m)$ is defined with respect to the DO slot allocation mechanism in which the liner assigns the available slots to the shippers that show up, and have the largest deposits. In this section, we detail an alternative slot allocation mechanism, referred to as the "Reliable-First" (RF) slot allocation mechanism, which we detail in full shortly. On the one hand, we show that the RF mechanism garners strictly worse profit than the DO mechanism, but on the other hand, its use dramatically simplifies our subsequent analysis.

The RF mechanism. First, we note that it is only necessary for us to define the RF mechanism in relation to acceptance sets that arise from the TOH policy, which accept at most m reliable customers. This alternative slot allocation mechanism works as follows.

- The slots are first allocated to the reliable shippers who show up, again noting that under the TOH policy there can be at most m.
- The remaining slots are then allocated to the unreliable shippers who show up according to the DO mechanism, i.e., in decreasing order of deposit size.

The RF profit function. Under the newly defined RF slot allocation mechanism, let $\hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m)$ denote the random profit earned from shipper $t \in \mathcal{A}_{\mathrm{TOH}}$. Furthermore, let $\hat{\mathcal{R}}(\mathcal{A}_{\mathrm{TOH}}; m) = \mathbb{E}[\sum_{t \in \mathcal{A}_{TOH}} \hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m)]$ denote the expected profit earned by the TOH policy under the RF slot allocation mechanism. The following claim, whose proof can be found in Appendix B.1, is the analogue of Claim 2.1, adapted to fit the RF mechanism.

Claim 2.3. *For shipper $t \in \mathcal{A}_{\mathrm{TOH}}$, we have*

$$\mathbb{E}\left[\hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m)\right] = p_t + (1 - p_t) \cdot d_t$$

if $t \in T_{\text{rel}}$. *On the other hand, if* $t \in T_{\text{unrel}}$, *we have*

$$\mathbb{E}\left[\hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] = d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} Y_\tau + \sum_{\substack{\tau \in T_{\text{unrel}}: \\ \rho(\tau) < \rho(t)}} Y_\tau < m\right] \cdot (p_t \cdot (1 + d_t)),$$

where all expectations are taken with respect to $\{Y_t\}_{t \in \mathcal{A}_{\text{TOH}}}$.

To conclude Step 1, we present the following lemma, which states that the expected profit earned by the TOH policy is never increased by adopting the RF mechanism. We note that it is not difficult to adapt the proof of Lemma 2.1 to show that there is in fact no alternative slot allocation mechanism that can outperform the DO mechanism in terms of profit, however this stronger result is not needed to prove Theorem 2.1.

Lemma 2.1. $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m)$

2.2.3 Step 2: Competing against $\mathcal{R}^*_{\text{rel}}$

In this section, we establish our first of two lower bounds on the profit earned by the TOH policy. Namely, we show that $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}$ via the following two lemmas.

Lemma 2.2. $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot |\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|$.

Lemma 2.3. $\mathcal{R}^*_{\text{rel}} \leq \min\{|T_{\text{rel}}|, 2m\}$.

We establish Lemma 2.2 by focusing on the RF slot allocation mechanism, where reliable shippers are prioritized. Under the TOH policy, the number of accepted reliable shippers is $|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|$, who, by definition, must all have show-up probability at least $\frac{1}{2}$. Consequently, under the RF mechanism, the expected profit earned from the reliable shippers alone is at least $\frac{1}{2} \cdot |\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|$. From here, the result is finalized by showing that one can ignore the unreliable shippers at no profit loss, which can be deduced from simple arguments that consider the expression for $\mathbb{E}[R_t(\mathcal{A}_{\text{TOH}}, Y; m)]$ given in Claim 2.1 when $p_t < \frac{1}{2}$. Lemma 2.3, which is the more surprising of the two, provides an upper bound on the optimal profit earned from reliable shippers by a clairvoyant who has full access to the deposit sequence. The proof of this result begins with the explicit expression for $\mathcal{R}^*_{\text{rel}}$ given in Claim 2.1, and then proceeds to establish a sequence of upper bounds on this optimal profit that successively lead to simpler and simpler expressions, culminating in the final bound, which involves only infinite sums of binomial random variables all with success probabilities of $\frac{1}{2}$. From here, we finalize the proof of Lemma 2.3 by proving one final bound on these infinite sums of binomial random variables.

We conclude by noting that Lemmas 2.2 and 2.3 can easily be combined to show the desired bound of $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}$. To see this, observe that if $|T_{\text{rel}}| \leq m$, then $|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}| = |T_{\text{rel}}|$, since the TOH policy accepts the first m reliable shippers. As such, in this case, we get that $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot |T_{\text{rel}}| \geq \frac{1}{2} \cdot \mathcal{R}^*_{\text{rel}}$. On the other hand, if $|T_{\text{rel}}| > m$, then $|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}| = m$, and we have that $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot m \geq \frac{1}{4} \cdot \mathcal{R}^*_{\text{rel}}$.

2.2.4 Step 3: Competing against $\mathcal{R}^*_{\text{unrel}}$

In this section, we show how our TOH policy competes against $\mathcal{R}^*_{\text{unrel}} = \max_{\mathcal{A} \subseteq T_{\text{unrel}}} \mathcal{R}(A; m)$; the optimal expected profit that can be derived from the unreliable shippers alone. To do so, we will heavily rely on the following dynamic-programming-based approach through which the liner's profit can be computed under the RF slot allocation mechanism. This alternative recursive way to compute the liner's profit provides a simpler template for proving the various results of this section.

The recursive profit function. For ease of notation in the remainder of this section, we assume the following re-indexing of each acceptance set $\mathcal{A} \subseteq [T]$. Namely, the unreliable shippers $t \in \mathcal{A} \cap T_{\text{unrel}}$ are given indices $1, \dots, |\mathcal{A} \cap T_{\text{unrel}}|$ in decreasing order of their respective deposit sizes (breaking ties arbitrarily), while the reliable shippers $t \in \mathcal{A} \cap T_{\text{rel}}$ can be arbitrarily assigned indices $|\mathcal{A} \cap T_{\text{unrel}}|, \dots, |\mathcal{A}|$. Given this re-indexing, we propose a dynamic program that can be used to compute the liner's expected profit when the acceptance set consists exclusively of unreliable shippers.

For an arbitrary unreliable acceptance set $\mathcal{A} \subseteq T_{\text{unrel}}$, we introduce value functions $V(t, k; \mathcal{A})$, which represent the expected profit garnered from shippers indexed $t, \dots, |\mathcal{A}|$ given that shippers $1, \dots, t-1$ consumed $m - k$ slots, under acceptance set $\mathcal{A}$. The value functions are valid under either of the proposed slot allocation mechanisms, since both the RF mechanism and the true mechanisms act identically when the acceptance set consists only of unreliable shippers. Formally, the recursion can be expressed as

$$V(t, k; \mathcal{A}) = (1 - p_t) \cdot (d_t + V(t+1, k; \mathcal{A})) + \begin{cases} p_t \cdot (1 + V(t+1, k-1; \mathcal{A})), & \text{if } k > 0 \\ p_t \cdot (-d_t + V(t+1, 0; \mathcal{A})), & \text{if } k = 0, \end{cases} \tag{2.1}$$

with base cases $V(|\mathcal{A}| + 1, \cdot; \mathcal{A}) = 0$. The above recursion can be interpreted as follows. If shipper $t \in \mathcal{A}$ is a no-show, which occurs with probability $1 - p_t$, then the liner collects the shipper's deposit of d_t, and we move to considering shipper $t + 1$, still with k slots remaining. If, on the other hand, shipper t shows, which happens with probability p_t, then we must consider two cases depending on whether k is no-zero. If $k > 0$, then the shipper will acquire a slot, since the recursion processes the shippers from highest to lowest deposit. As such, the liner collects the service fee and we move to shipper $t + 1$ with one fewer slot. If $k = 0$, there are no remaining slots, so in this case, the liner must pay a reimbursement of d_t. The following claim, formalizes the above discussion.

Claim 2.4. *For any unreliable acceptance set $\mathcal{A} \subseteq T_{\text{unrel}}$, shipper $t \in \mathcal{A}$ and $k \in [m]_0$, we have*

$$\mathbb{E}\left[\sum_{\tau=t}^{|\mathcal{A}|} R_\tau(\mathcal{A}, Y; k)\right] = V(t, k; \mathcal{A}),$$

where the expectation is with respect to $\{Y_\tau\}_{\tau \in \{t, t+1, \ldots, |\mathcal{A}|\}}$

Properties of the value functions. Below, we present two claims regarding the newly developed value functions presented in (2.1). The first shows that the value functions are always non-negative, while the second provides a universal upper bound on the marginal value of a single slot.

Claim 2.5. *For any acceptance set $\mathcal{A} \subseteq T_{\text{unrel}}$, shipper $t \in \mathcal{A}$, and $k \in [m]_0$,* [11] *we have $V(t, k; \mathcal{A}) \geq 0$.*

Claim 2.6. *For any acceptance set $\mathcal{A} \subseteq T_{\text{unrel}}$, shipper $t \in \mathcal{A}$, and $k \in [m]_0$, we have*

$$V(t, k; \mathcal{A}) - V(t, k-1; \mathcal{A}) \leq 1 + d_t.$$

The above claim is instrumental in establishing the following lemma, which states that if all shippers are unreliable, then it is optimal to accept each one. We will critically rely on this lemma for the development of the performance bound that we seek in this section.

Lemma 2.4. *Recalling that $\mathcal{R}^*_{\text{unrel}} = \max_{\mathcal{A} \subseteq T_{\text{unrel}}} \mathcal{R}(\mathcal{A}; m)$, we have*

$$V(1, m; T_{\text{unrel}}) = \mathcal{R}^*_{\text{unrel}}.$$

[11] $[m]_0 = \{0, 1, \ldots, m\}$

The performance bound. To finalize the proof of Theorem 2.1, we show the second performance bound for the TOH policy, which is formally provided in the following lemma.

Lemma 2.5. *Recalling that* $\mathcal{R}^*_{\text{unrel}} = \max_{\mathcal{A} \subseteq T_{\text{unrel}}} \mathcal{R}(\mathcal{A}; m)$*, we have*

$$\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot \mathcal{R}^*_{\text{unrel}}.$$

We prove the above lemma by using the intuition provided in the summary of Step 3 given in Section 2.2.1. Namely, via Lemma 2.4, we know that if the TOH policy were modified so that it accepted no reliable shippers, then it would earn an expected profit of precisely $\mathcal{R}^*_{\text{unrel}}$. However, in order to compete against $\mathcal{R}^*_{\text{rel}}$, the TOH needs to accept up to m reliable shippers. More specifically, the liner extracts profit from the $\min\{m, |T_{\text{rel}}|\}$ reliable shippers accepted by the TOH policy, but in turn, these shippers diminish the total profit contribution from the accepted unreliable shippers by consuming available slots. The key to establishing Lemma 2.5, is to show that the profit gain from the accepted reliable shippers "makes up", in a sufficient way, for the profit they draw away from the accepted set of unreliable shippers.

2.2.5 Numerical Experiments

In this section, we present the details of an extensive set of numerical experiments that were conducted to demonstrate the practical potential of the TOH policy, and in particular, show that its performance far exceeds its worst case theoretical guarantees.

Instance generator. We generate instances of the liner's booking problem with $T \in \{10, 20\}$ periods and $m \in \{1, 3, 5\}$ service slots. We note that there is nothing fundamentally easier about these "smaller" instances. In fact, it is likely most difficult to compete against OPT when m is small, since in this case, the effects of overbooking kick-in quickly, and it is these effects that make the booking problem difficult. We generate the period $t \in [T]$ deposit using one of the three distribution:

- Uni: The deposit d_t in uniformly sampled from the interval $[0, 1]$.
- High-Dep: We generate d_t from the distribution

  $$f(d_t) = \begin{cases} 2d_t, & \text{if } d_t \in [0,1] \\ 0, & \text{o.w.,} \end{cases}$$

  which captures setting where higher deposits are more likely.
- Low-Dep: We generate d_t from the distribution

  $$f(d_t) = \begin{cases} 2 - 2d_t, & \text{if } d_t \in [0,1] \\ 0, & \text{o.w.,} \end{cases}$$

  which captures setting where lower deposits are more likely.

Additionally, we consider the following three show-up probability functions, which we use to map a deposit to its corresponding show-up probability.

- Rand: The show-probability p_t is sampled from the interval $[0, 1]$.
- Concave-Inc: The show probability is modeled as the following increasing concave function of the deposit $p_t(d_t) = \log(1 + d_t)/\log(2)$.
- Convex-Inc: The show probability is modeled as the following increasing convex function of the deposit $p_t(d_t) = d_t^2$.

A single test case is characterized by a parameter configuration $(T, m, \mathcal{D}, \mathcal{P}) \in \{10, 20\} \times \{1, 3, 5\} \times \{\text{Uni, High-Dep, Low-Dep}\} \times \{\text{Rand, Concave-Inc, Convex-Inc}\}$, where $\mathcal{D}$ signifies our choice for the manner in which we generate the deposit in each period and $\mathcal{P}$ is the given show-up probability function. For each of the 54 test cases, we generate 10 streams of deposit tuples accordingly.

Performance metrics and results. For each of the 540 problem instances generated as described above, we carry out the TOH policy, and concurrently also compute the optimal clairvoyant acceptance set $\mathcal{A}^* = \arg\max_{\mathcal{A} \subseteq [T]} \mathcal{R}(\mathcal{A}; m)$ via brute force enumeration over all acceptance sets. Additionally, for both $\mathcal{A}_{\text{TOH}}$ and $\mathcal{A}^*$, we use 10,000 trials of Monte Carlo simulation to estimate the expected fraction of the m service slots ultimately allocated to shippers, and the expected number of shippers whose cargo gets rolled. First and foremost, the results of our experiments reveal that the TOH policy garners an expected profit that is, on average, within 5% of OPT. Furthermore, averaged over all test cases, the TOH policy allocates 90% of the m service slots, which is only 1% lower than the average usage percentage of $\mathcal{A}^*$. Finally, we see that the TOH policy rolls only 1.18 shippers on average, which is approximately 10% lower than the expected number of rolled shippers under $\mathcal{A}^*$. A more fine-grained picture of our results are presented in Appendix B.2.

2.3 Concluding Remarks

We conclude this paper by offering numerous directions for future research, which concern both the specific online booking problem we have considered, as well as various practical extensions.

Improving the competitive ratio. One question for future work that arises naturally is whether more sophisticated techniques can be applied to achieve a competitive ratio that exceeds $\frac{1}{6}$ in the current setting that we study. Along these lines, one could pursue the development of policies with improved competitive ratios under more structured instances of our problem. For example, can the the performance of the TOH policy (or other policies) be improved when the show-probabilities are determined by one of the functions outlined in Section 2.2.5?

Multi-slot booking requests. In practice, inquiring shippers can request multiple service slots from a carrier. In the current framework, we can capture multi-slot requests by assuming that the shipper offers a separate deposit for each slot it is interested in. An intriguing direction for future work could extend our framework to allow shippers to place a single deposit to simultaneously secure multiple slots. Moreover, given such a model, it is natural to wonder whether one can continue to show that simple booking policies have the potential to be $\Omega(1)$-competitive. A further extension of this multi-slot booking problem could make use of the following feature of NYSHEK's digital platform. Namely, within NYSHEK's digital platform, if a shipper has booked 100 TEUs, but realizes it only needs 80, then it can sell back the unused 20 TEUs to other shippers. Incorporating this "sell back" feature within a multi-slot extension of our original framework could represent a fruitful direction for future work.

Dynamic pricing of service slots. Another practical extension of our work would be to allow the carrier to dynamically price the service slots in each period. In this case, the posted prices take the place of the normalized service fee, and could also influence the arrival process of the shippers.

Chapter 3

Blockchain-Enabled Deep-Tier Supply Chain Finance

For many supply chains, deep-tier suppliers, due to their small sizes and lack of access to capital, are most vulnerable to disruptions. We study the use of advance payment (AP) as a financing instrument in a multitier supply chain to mitigate the supply disruption risk and compare the traditional system (with limited visibility) with the blockchain-enabled system (with perfect visibility). The main goal of this paper is to shed light on how blockchain adoption impacts agents' operational and financial decisions as well as profit levels in a multitier supply chain. Traditionally, because of the limited visibility in the deep-tiers, powerful downstream manufacturers' financing schemes offered to their immediate upstream suppliers are not effective in instilling capital into the deep-tiers. Advancements in blockchain technology improve the supply chain visibility and enable the manufacturer to better devise deep-tier financing to improve supply chain resilience. We develop a three-tier supply chain model and take a game-theoretic approach to compare how blockchain-enabled deep-tier financing schemes affect a financially constrained supply chain's optimal risk-mitigation and financial strategies. We find that although improved visibility via blockchain adoption can help the manufacturer make informed supply chain financing decision, whether it can benefit all supply chain members depends on the financing schemes in use. Blockchain-enabled delegate financing increases risk-mitigation investments and benefits all three tiers of the supply chain only when the tier-2 is severely capital-constrained with the working capital below a threshold. Because delegate financing endows the intermediary tier-1 supplier with a leverage over the manufacturer, the inefficiency inhibits an all-win outcome when the tier-2 is not severely capital-constrained. Blockchain-enabled cross-tier *direct financing* exhibits a compelling performance as it always leads to win-win-win outcomes (and thus ubiquitously implementable) regardless of the supplier's working capital profile. Our insights help firms assess opportunities and challenges associated with enhancing supply chain visibility via blockchain adoption.

3.1 Introduction

Many global supply chains share the characteristic of large, powerful brands supplied by multiple tiers of suppliers, many of whom are small and medium-sized enterprises (SMEs) who are capital strapped and lack access to capital markets. Those SMEs are vulnerable to disruptions of all sorts: production stoppage due to quality issues, input material/labor shortages, and natural disasters. Their ability to recover from those disruptions and provide reliable supply to downstream buyers is largely dependent on the resource they can draw, i.e., their working capital. A buyer typically only has a direct relationship with the tier-1 supplier and has limited visibility (visibility barrier) of the deep-tiers in its supply chain [65]. Due to the lack of visibility into deep-tier supply chains and related transactions, downstream buyers have limited information about the financial status of deep-tier suppliers and thus cannot optimally devise cross-tier direct financing strategies. Therefore, downstream buyers-initiated supply chain financing (SCF) schemes are limited to helping immediate upstream suppliers, and the buyers at the best hope their tier-1 suppliers can in turn help the tier-2 supplier through SCF. Successful implementation of deep-tier SCF requires the collaboration of many parties in the supply chain. Such multilateral collaboration has been challenging because (i) tier-1 suppliers are reluctant to share their upstream suppliers' information with downstream buyers for fear of eroding their margins; (ii) tier-2 suppliers are not comfortable sharing sensitive financial information (e.g., bank account balance in our setting) due to concerns over both security and privacy, as well as the asymmetry in monetizing data in an equitable manner.

Recent advancement of blockchain technology has propelled a growth of blockchain platforms aimed at improving the information flow and financial flow in supply chains. The key advantage of blockchain technology is that it can prevent information leakage to unintended parties while allowing supply chain participants (e.g., downstream buyers) to verify transaction attributes (e.g., deep-tier supplier's working capital level). This is achieved by combining a distributed ledger with zero-knowledge proof cryptography, which allows an agent to verify that some piece of information is true (e.g., supplier's bank account balance), without full access to all background information (e.g., supplier identity and associated detailed transaction records). For example, Skuchain has developed patented "Zero Knowledge Collaboration" technology that enables a company's data remaining encrypted to the parties.[12] Meanwhile, supply chain firms are still able to plan and collaborate with one another as though they had full information about one another even when that sensitive information stays hidden. With Skuchain's platform, algorithms (e.g., smart contract) can be applied to data on the distributed blockchain ledger without requiring that data be revealed to any party.

[12] https://www.skuchain.com/zk-collaboration/

Those third-party FinTech firms verify the data accuracy uploaded onto the blockchain and ensure privacy-preserving data sharing within supply chains. For more technical details, we refer the reader to a white paper published by the World Economic Forum on how to protect supply chain data on the blockchain using zero-knowledge and other technologies [34]. In addition, blockchain technology can also help reduce paperwork and facilitate safer and easier sharing of necessary supply chain transaction information. This improved information sharing makes it feasible for the supply chain to adopt a wide range of financing instruments, including the advance payment (AP) scheme, the focus of this paper [38].

Given these advantages of blockchain technology, industry pioneers have been exploring its applications in deep-tier SCF. Samsung Electronics is a recent example, who has established a KRW 500 billion ($450 million) fund to enable tier-1 suppliers to borrow the money they need to pay the tier-2 suppliers (a form of *delegate financing*, the focus of §3.5). The fund is to be created with Hana Bank, Shinhan Bank, and KB Kookmin Bank, according to the Korean business news website Pulse [90]. The new arrangements allow tier-1 suppliers to borrow a sum of money equal to their monthly payment to a tier-2 supplier. The money can be borrowed interest-free for up to a year, extendable to two years. The scheme is intended to enable the tier-1 suppliers to make payment to tier-2 suppliers within 30 days of delivery receipt at no financing cost to tier-1 suppliers.

One further solution was announced by Foxconn, one of the largest electronics manufacturers and an important supplier of Apple. Foxconn launched a Blockchain-powered system aimed at providing much-needed working capital directly to small upstream suppliers [62] (i.e., *direct financing* as we discuss in §3.6). Foxconn's supply chain consists of many tiers, where a considerable number of upstream suppliers, especially, tier-2 and above, are SMEs. They face severe capital constraints and lack the credit history to take bank loans, leading to their vulnerability to various business disruptions (i.e., natural disasters, machine failure, delayed payment from customers, etc). For big downstream manufacturers like Foxconn, ensuring the financial health of all players in its supply chain is critical to minimizing disruptions to their supply chains. The new system intends to reach all suppliers in Foxconn's network and offers viable financial support to needed parties.

Motivated by the above practical observations, we propose a deep-tier supply chain finance model focusing on the aspect of blockchain technology that enables visibility into the financial status of the deep-tier supplier. We start with the case with limited visibility and then examine the impacts of blockchain-enabled perfect visibility. We ask the following three main research questions in a multitier supply chain: (i) What are the optimal structures of SCF contracts and corresponding risk-mitigation measures without cross-tier visibility? (ii) How does the blockchain-enabled cross-tier visibility impact the SCF and risk-mitigation decisions? Can visibility benefit all supply chain members? (iii) Which type of blockchain-enabled deep-tier financing, delegate financing versus direct financing, can create a higher value for the supply chain?

We consider a three-tier supply chain with one downstream manufacturer, one tier-1 supplier, and one tier-2 supplier. The tier-2 supplier is an unreliable SME with fixed production capacity, and is subject to business disruptions (ranging from natural disasters to production line breakdowns and delayed customer payments). However, the tier-2 supplier's reliability (probability of successful production and delivery) can be improved through production investment – a form of *proactive* risk-mitigation measure, which drives the tier-2's need for capital. Unfortunately, the tier-2 supplier, operating in an environment with only rudimentary banking systems, lacks the credit history required to seek bank loans and has to rely on financing arrangements from supply chain partners. The credit-worthy tier-1 supplier, although is also capital-constrained, can secure loans from the bank for working capital needs, or by accepting the manufacturer's SCF arrangement, i.e., the advance payment (AP) contract that specifies the wholesale price and the prepayment amount. The tier-1 supplier has an emergency sourcing channel (at a higher cost) in case the tier-2 supplier fails to deliver – a form of *reactive* risk-mitigation measure. Our analysis yields three main sets of insights, which we summarize as follows.

First, under traditional deep-tier financing (without blockchain), the manufacturer's AP contract offer will be accepted by the tier-1 supplier only if the tier-2 supplier is severely capital-constrained (i.e., working capital level lower than a threshold); otherwise, the tier-1 supplier will take bank financing (BF). The manufacturer, with only the probability distribution information about the tier-2 supplier's working capital, cannot precisely predict the tier-1's financing decision in response to her AP contract offer. We show that in this case although the manufacturer can directly control the tier-1's reactive risk-mitigation decision (via setting wholesale price), she is unaware of how effective her AP contract is in helping improve tier-2's proactive risk-mitigation investment. Thus, the coordination of the two risk-mitigation measures suffers from the limited cross-tier visibility.

Second, improved visibility via blockchain allows the manufacturer to correctly anticipate the suppliers' response to her offer. However, her ability to influence the suppliers' risk mitigation effort is affected by the type of financing scheme in use, because different financing schemes imply different incentives on supply chain members. We find that *delegate financing* (e.g., the Samsung example) where the manufacturer delegates her financial support to the tier-2 supplier via the tier-1 supplier can benefit all three parties *only when* the tier-2 is severely capital-constrained, and blockchain will be adopted in this case. This is because the manufacturer with limited deep-tier visibility will make an AP contract offer targeting an "average" tier-2 supplier and a severely capital-constrained tier-2 will end up not receiving sufficient resources and under-investing in proactive risk mitigation. Gaining visibility allows the manufacturer to correct her offer and improve the supply chain reliability significantly to benefit all supply chain members. However, the manufacturer's correction for not severely constrained tier-2 can result in reducing her support for one risk-mitigation and at least one supply chain member receiving less support than that in the limited visibility case; blockchain will be rejected in this case.

Cross-tier direct financing (e.g., Foxconn's blockchain platform) where the manufacturer skips the intermediary tier-1 and finances the tier-2 directly delivers a compelling performance: it always leads to win-win-win outcomes (and thus ubiquitously implementable) regardless of the supplier's working capital profile. The fundamental reason behind this powerful result is that the wholesale price and the AP interest rate under direct financing play distinct roles. Specifically, the AP interest rate plays the role of incentivizing the tier-2 to make a (manufacturer's) desired level of reliability investment. The wholesale price, on the other hand, plays the role of affecting the tier-1's reactive risk-mitigation decision and deciding the profit division between the manufacturer and the tier-1. By contrast, the roles of wholesale price and AP interest rate under delegate financing cannot be completely decoupled, because the tier-1's incentive to offer AP contract to the tier-2 is affected by both the manufacturer offered wholesale price and AP interest rate. The manufacturer always needs to adjust both contract parameters to indirectly influence the proactive risk-mitigation investment. Finally, comparing cross-tier direct financing and delegate financing, we show that the former allows the manufacturer to improve the coordination of proactive and reactive risk-mitigation better than the latter, and the manufacturer can adjust the wholesale price of direct financing to make all three parties better-off than delegate financing.

The remainder of this paper is organized as follows. We position our paper in the literature in Section 3.2. We describe our model setups and assumptions in Section 3.3. In Section 3.4, we present a benchmark case of the deep-tier SCF without blockchain and analyze the interaction among three tiers of participants with limited visibility. In Section 3.5, we analyze the blockchain-enabled delegate financing (with perfect visibility) and discuss the profit changes for each supply chain member. We extend the model by allowing the cross-tier direct financing in Section 3.6. We conclude with a summary of main insights and discussions of future research in Section 3.7. Proofs of all results and supplemental materials are provided in the appendix.

3.2 Literature Review

Broadly speaking, our research builds on the supplier disruption management literature and supply chain finance literature. The supplier disruption management literature mainly studies firms' sourcing strategies and supplier competition in the presence of supply risks, e.g., [2, 6, 12, 18, 60, 96]. By contrast, we focus on financing strategies, instead of sourcing strategies, to mitigate supplier's disruption risk. The second is the supply chain finance literature (mostly, trade credit), e.g., [16, 17, 18, 45, 53, 54, 107, 108]. Closest to our research is the literature stream focusing on managing and financing capital-constrained suppliers, e.g., [26, 29, 30, 52], in particular, pre-shipment financing schemes (e.g., purchase order financing, buyer direct/intermediated financing, advance payment, etc) [79, 101]. [95] attempt to understand the relative efficiency of purchase order financing and buyer direct financing under the supplier's endogenous effort and the manufacturer's private information. [41] find that in a consignment selling environment with debt seniority choice, buyer direct financing weakly improves the expected payoffs of both the retailer and the supplier. Our paper complements the aforementioned works by serving as the first attempt to model the financing problem in a multitier (three-tier) supply chain and to examine the implications of blockchain-enabled supply chain visibility for related financing strategies.

Our model of deep-tier SCF without blockchain technology differentiates from the above works in that the buyer does not have the visibility of the tier-2 suppliers (and thus their working capital level). Our paper adds value to the aforementioned literature by focusing on: (i) the type of supply disruption risk that can be improved via capital investment; and (ii) the visibility issues in a multitier supply chain setting, and we highlight the optimally designed advance payment contracts play a crucial role for the profitability of multiple tiers in the supply chain.

Lastly, our paper contributes to the emerging literature stream on the economics of blockchain technology, e.g., [22, 23, 24, 43, 77, 85, 88, 93, 100, 105]. We refer the reader to [3] for a review of operations management implications of blockchain technology, and to [51] for a discussion of FinTech innovations for supply chains. [19] show how the proposed blockchain-enabled verifiability of physical goods transactions can be leveraged by high-quality firms to signal their operational capabilities through their upstream inventory orders, thereby finance their supply chain operations more efficiently. [20] and [36] study entrepreneurial financing problems in initial coin offerings (ICOs) where crypto-tokens are issued on existing blockchain platforms. [25] and [28] study the value of blockchain-enabled traceability in various supply chain settings. Different from the above studies and motivated by rising blockchain adoption cases in supply chain finance, we focus on financing activities within a multitier supply chain and quantify the value of blockchain-enabled visibility (of the deep-tier supplier's financial status) in such a setting.

3.3 Model Framework

A stylized three-tier supply chain consists of one downstream manufacturer M (she), one tier-1 supplier S_1 (he), and one tier-2 supplier S_2 (it). The tier-2 supplier has a fixed production capacity (normalized to 1) that is subject to supply disruption. Let $\mathcal{P}(y) \in [0, 1)$ denote the *production reliability* if the proactive risk mitigation investment is y.

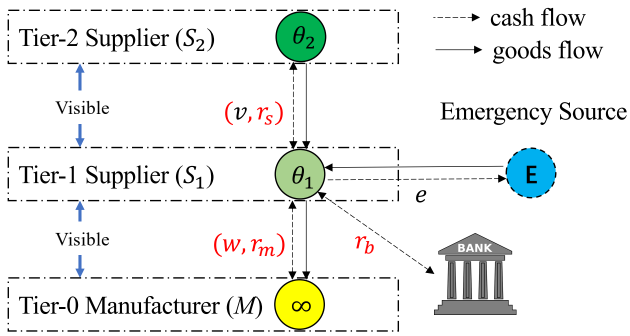


Figure 3.1: The Three-tier Supply Chain Structure

Figure 3.1 illustrates the structure of the three-tier supply chain. We assume the tier-1 supplier's procurement price v is exogenous, e.g., [1, 83]. This assumption reflects the fact that the supplier at this level tends to offer more standard/commoditized goods, where the prices are typically determined by the market and thus remain fixed over a period of time. The tier-1 supplier can reactively mitigate the tier-2's delivery-failure risk via an alternative emergency source **E** that can provide immediate and reliable supply but at cost e, $v < e < 1$, e.g., [95]. Let θ_1 and θ_2 denote the working capital of the tier-1 and the tier-2 suppliers, respectively. The tier-1 supplier has access to the capital market via short-term bank loans. The actual bank loan interest rate is based on the standard competitive pricing equation (the risk-free interest rate is normalized to 0). However, the tier-2 lacks the credit record to take bank loans or operates in an environment of rudimentary banking systems where credit systems are not well established and SMEs are underserved. The manufacturer in our model has sufficient capital, e.g., [41, 101]. Our main results and insights continue to hold when the manufacturer is also capital-constrained but can take loans from a competitive capital market. Assuming the tier-2 and the downstream supply chain players have unequal access to the capital market allows us to focus on the challenge faced by the deep-tier supplier.

We normalize the manufacturer's unit retail price to 1 and assume a constant demand (also normalized to 1); both are common assumptions in the supply disruption literature, e.g., [94, 95].

In order to improve the overall supply chain reliability, a downstream buyer (resp., the manufacturer, the tier-1) is willing to offer financial help to its immediate upstream supplier (resp., tier-1, tier-2) via *advance payment* (AP). That is, a buyer can prepay a proportion of the invoice right after placing an order to its immediate supplier (at time 0) to improve the latter's cash position. After the product is delivered, the supplier in return will pay an associated interest at the regular payment time (at time 2). We summarize the sequence of events as follows with the detailed timeline in Figure 3.2.

(1) **At Time** 0: The manufacturer proposes a tier-0 AP contract (w, r_m) with wholesale price w and AP interest rate r_m. The tier-1 supplier can (i) accept the entire contract, (ii) only accept the wholesale price contract, or (iii) reject the contract (no business transaction for all three parties). If the tier-1 supplier accepts the entire contract and requests the AP amount $B_m \geq 0$, the manufacturer prepays B_m and will pay $(w - (1 + r_m)B_m)$ (resp., 0) upon tier-1's successful delivery (resp., delivery failure). The tier-0 AP is also referred to as *manufacturer financing* (MF). If only accepting the wholesale price contract, the tier-1 will decide whether to take a bank loan, which is referred to as *bank financing* (BF). Note that the AP contract does not restrict the tier-1 supplier from taking bank loans. However, our results from Proposition 3.1 imply that it is never optimal for the tier-1 supplier to use dual financing.

Next, the tier-1 supplier offers the tier-1 AP contract (v, r_s) to the tier-2 supplier. The tier-2 supplier either accepts or rejects the contract. If it accepts the AP contract, it can request an AP, B_s, from tier-1 to bring its working capital to $\theta_2 + B_s$. The tier-2 supplier then decides its reliability investment, y, and $y \leq (\theta_2 + B_s)$. Both tier-0 and tier-1 AP interest rates should be non-negative, i.e., $r_m, r_s \geq 0$, otherwise, the borrower can earn free money from AP.

(2) **At Time** 1: The tier-2 production uncertainty is realized. If the tier-2 fails to deliver order, the tier-1 supplier needs to decide whether to use emergency source **E** to fulfill the manufacturer's order; if needed, the tier-1 can use (risk-free) BF to fund his emergency sourcing.

(3) **At Time** 2: If the tier-2 supplier successfully delivers the order, the tier-1 supplier will receive the corresponding payment from the manufacturer and then use it to pay to the tier-2 supplier. If the tier-2 supplier fails to deliver, it will receive no payment from the tier-1 supplier (under wholesale contract) or repay advanced payment and interest $(1 + r_s)B_s$ to the tier-1 supplier to the largest extent possible (under the AP contract). If the tier-1 supplier invokes emergency source **E** at time 1, he will receive corresponding payment from the manufacturer (under wholesale contract) or repay advanced payment and interest $(1 + r_m)B_m$ to the manufacturer to the largest extent possible (under the AP contract). Under BF, the tier-1 must repay the bank loan to the largest extent possible.

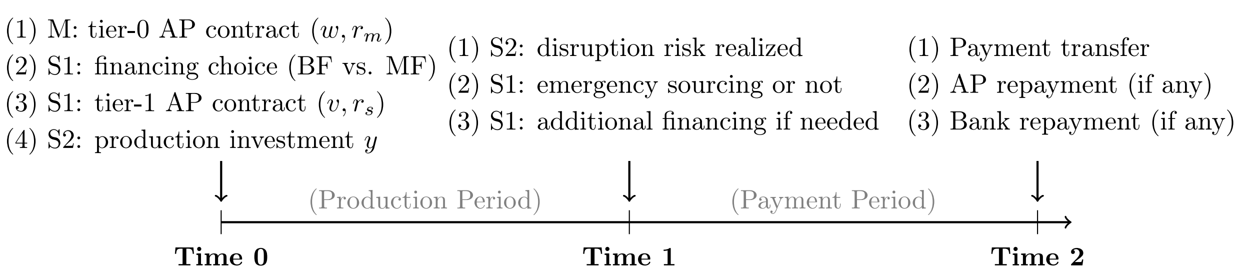


Figure 3.2: Sequence of Events in Deep-Tier Financing

3.4 Traditional Deep-Tier Financing with Limited Visibility

This section investigates the benchmark case of a traditional SCF system (without blockchain) where each tier of the supply chain has perfect information of its direct supplier's financial condition but imperfect information of its deep-tier supplier. Specifically, we assume that the manufacturer (resp. the tier-1) knows the tier-1's (tier-2's) working capital level, but the manufacturer only knows that the tier-2 supplier's working capital θ_2 follows a probability distribution (details to be specified in §3.4.3). The adjacent-tier visibility assumption is plausible and commonly accepted in the SCF literature because trading partners can often derive the capital level of their direct suppliers or customers by analyzing historical transactions, e.g., [53, 54, 55]. The cross-tier visibility, however, is hard to obtain due to the lack of access to non-trading partners' transaction data.

We derive the subgame perfect equilibrium using backward induction. We start with the tier-2 supplier's production investment decision (§3.4.1), whose best response function will feed into the tier-1 supplier's AP contract design and the risk-mitigation decision (§3.4.2). Finally, we characterize the manufacturer's AP contract design (§3.4.3).

3.4.1 Tier-2 Supplier's Problem

We first examine the tier-2 supplier's reliability investment decision without capital constraint: $\max_{y\geq 0} \mathcal{P}(y)v - y$. Because $\mathcal{P}(y)$ is concavely increasing in y, there exists a unique optimal tier-2 investment level c, which is derived from $v\mathcal{P}'(c) = 1$. Note that c is the maximal amount of capital the tier-2 supplier is willing to invest in reliability improvement without capital constraint. The subsequent analysis will focus on the more interesting case where the tier-2 is in need of capital to improve its reliability, i.e., $\theta_2 \in [0, c)$.

Given the tier-1 supplier's AP contract offer (v, r_s), the tier-2 supplier has two decisions: (i) whether to accept the tier-1's AP contract, and (ii) how much to invest in improving production reliability. The tier-2's decisions can be formulated as $\pi_2 = \max\{\pi_{2r}, \pi_{2a}\}$, where π_{2r} and π_{2a} respectively represent the tier-2's expected terminal cash level by rejecting and accepting the tier-1's AP offer. Specifically, if the tier-2 supplier rejects the AP contract, it cannot invest more than θ_2 into reliability improvement. The tier-2 supplier receives payment v from the tier-1 supplier upon the successful delivery, and zero otherwise. Hence, π_{2r} is expressed as: $\pi_{2r} = \max_{0 \le y \le \theta_2} \mathcal{P}(y)v + \theta_2 - y$.

If the tier-2 supplier accepts the AP contract, then, because of the positive AP interest rate, the tier-2 would request the AP amount that it will indeed use for reliability investment, i.e., $B_s = (y - \theta_2)^+$. Hence, π_{2a} is expressed as: $\pi_{2a} = \max_{y > \theta_2} \mathcal{P}(y)[v - (1 + r_s)(y - \theta_2)]^+$. It is straightforward that π_{2r} is concave and increases in investment level $y \in [0, \theta_2]$ since $\theta_2 < c$, whereas π_{2a} is concave in y and achieves the maximum at $z^*(r_s|\theta_2)$, which solves the first-order condition (FOC):

$$\mathcal{P}'(y)[v - (1 + r_s)(y - \theta_2)] - \mathcal{P}(y)(1 + r_s) = 0. \tag{3.1}$$

The interior optimal investment level $z^*(r_s|\theta_2)$ balances the marginal revenue increase from improved production reliability and the marginal cost of AP interest, and it decreases in r_s but increases in θ_2. Note that it is possible to have $z^*(r_s|\theta_2) > c$, which holds if and only if $r_s < \frac{1}{c - \theta_2 + \mathcal{P}(c)v} - 1$. That means when the tier-1 supplier offers a relatively low AP interest rate, the tier-2 supplier is willing to invest more than that without capital constraint, by taking advantage of the limited liability (downside protection).

Lemma 3.1. *Given the tier-1 AP interest rate $r_s > 0$, the tier-2 supplier's optimal investment level is $y^*(r_s|\theta_2) = \max\{z^*(r_s|\theta_2), \theta_2\}$, and the corresponding AP amount is $B_s^* = y^*(r_s|\theta_2) - \theta_2$.*

Lemma 3.1 shows that the tier-2 supplier accepts the AP if its working capital is below threshold $z^*(r_s|\theta_2)$, otherwise it self-finances its production investment. Given θ_2, the threshold $z^*(r_s|\theta_2)$ decreases in the tier-1 AP interest rate r_s. Hence, the tier-1 supplier can indirectly use r_s to influence the tier-2 supplier's investment decision and thus the production reliability.

3.4.2 Tier-1 Supplier's Problem

Anticipating the tier-2 supplier's best response to the tier-1 AP contract (Lemma 3.1), the tier-1 supplier needs to decide: At time 0, (i) his financing strategy, i.e., whether to accept the tier-0 AP contract (w, r_m) or use BF; (ii) his tier-1 AP contract offered (v, r_s) to the tier-2; at time 1, (iii) whether to use the reactive risk mitigation $\mathbf{E}$ if a disruption happens. We use the following tie-breaking rules: If the tier-1 supplier is indifferent between MF and BF, MF is used.

Following backward induction, we start with decision (iii), the time-1 reactive risk mitigation decision. The tier-1's optimal policy is straightforward: Use $\mathbf{E}$ as long as the tier-0 wholesale price is greater than the emergency sourcing cost, i.e., $w \geq e$. Notice that when the tier-1 supplier falls short of capital for emergency sourcing, he will borrow from the bank as needed at the risk-free rate 0 (because full repayment is guaranteed with the tier-1's wholesale revenue w). For conciseness, we denote the high wholesale price (i.e., $w \geq e$) case as the "proactive+reactive risk mitigation ($\mathbb{PR}$) case", and the low wholesale price case as the pure "proactive risk mitigation ($\mathbb{P}$) case". We use superscript $\mathbb{I} \in \{\mathbb{P}, \mathbb{PR}\}$ to represent the two cases respectively.

Under BF, decision (ii) is to decide the optimal tier-1 AP interest rate r_s to offer to the tier-2 supplier and the bank loan amount. Since any excessive borrowing will generate extra financial cost, the tier-1 supplier should borrow a loan just enough to cover tier-2's advancement payment requirement B_s (recall that the tier-1 will borrow from the bank for emergency sourcing if such a need arises at time 1). Hence, $B_b := (B_s^* - \theta_1)^+ = (y^*(r_s|\theta_2) - \theta_2 - \theta_1)^+$, where $y^*(r_s|\theta_2)$ is the tier-2's best-response reliability investment level. The bank interest rate r_b on loan amount B_b is obtained from the competitive lending equation.

The tier-1 supplier's time-2 terminal cash level is the accumulation of his initial capital level θ_1, the borrowing amount B_b, and the realized operational profit $(w - v)$ or $(w - e)^+$, subtract the time-0 AP to the tier-2 B_s^*, the time-2 bank loan repayment $(1 + r_b)B_b$, and plus the tier-2's repayment $(1 + r_s)B_s^*$ if she succeeds in production. Let $\tilde{\pi}_{1S}^b$ (resp., $\tilde{\pi}_{1F}^b$) denote his terminal cash level when the tier-2 supplier succeeds (resp., fails) to deliver under BF. Then, we have $\tilde{\pi}_{1S}^b = [\theta_1 + B_b + (w - v) - B_s^* - (1 + r_b)B_b + (1 + r_s)B_s^*]^+$, $\tilde{\pi}_{1F}^b = [\theta_1 + B_b + (w - e)^+ - B_s^* - (1 + r_b)B_b]^+$.

According to Lemma 3.1, for any θ_2, FOC (3.1) defines a one-to-one mapping between $z^*(r_s|\theta_2)$ and r_s, and $y^*(r_s|\theta_2) = \max\{z^*(r_s|\theta_2), \theta_2\}$. Hence, there is a one-to-one mapping between $r_s \in \left[0, \frac{\mathcal{P}'(\theta_2)}{\mathcal{P}(\theta_2)}v - 1\right]$ and tier-2 supplier's investment decision $y \in [\theta_2, z^*(0|\theta_2))$. The tier-1 supplier's interest rate decision r_s can be equivalently formulated as the tier-1 supplier deciding the tier-2 supplier's investment decision y, and we denote $r_s = z^{*-1}(y|\theta_2)$ where $y \geq \theta_2$. Then the tier-1 supplier's problem can be formulated as follows,

$$\max_{y \geq \theta_2} \pi_1^b(y|w) = \mathcal{P}(y)\tilde{\pi}_{1S}^b + (1 - \mathcal{P}(y))\tilde{\pi}_{1F}^b. \tag{3.2}$$

We now include the MF as a financing option for the tier-1 supplier in addition to BF. We first consider the tier-1's AP contract offer r_s and the risk-mitigation decision under MF, and then compare his expected payoffs between the two types of financing to decide his optimal financing policy. The tier-1 supplier's time-1 risk-mitigation decision should follow the same policy as that under BF: Use **E** if and only if $w \geq e$. This is because taking AP from the manufacturer at time 0 for emergency sourcing that may or may not happen at time 1 is more costly than taking a risk-free bank loan time 0 as needed. The tier-1's AP interest rate decision r_s, similar to the treatment under BF, can be equivalently formulated as deciding the tier-2 supplier's investment level y, which is formulated below:

$$\max_{y \geq \theta_2} \pi_1^m(y|w, r_m) = \mathcal{P}(y)\tilde{\pi}_{1S}^m + (1 - \mathcal{P}(y))\tilde{\pi}_{1F}^m, \tag{3.3}$$

where $\tilde{\pi}_{1S}^m$ (resp., $\tilde{\pi}_{1F}^m$) denote his terminal cash level when the tier-2 supplier succeeds (resp., fails) to deliver under BF, which take following forms, $\tilde{\pi}_{1S}^m = [\theta_1 + B_m + (w - v) - B_s^* - (1 + r_m)B_m + (1 + r_s)B_s^*]^+$, $\tilde{\pi}_{1F}^m = [\theta_1 + B_m + (w - e)^+ - B_s^* - (1 + r_m)B_m]^+$.

Let $y^{b*}(w|\theta_2, e)$ and $y^{m*}(w|\theta_2, e)$ denote the optimal target investment level of the tier-1 under BF and MF solving problem (3.2) and (3.3), respectively. We can show that they exist and each of them is unique (details in the appendix). The tier-1 supplier joins in the business and accepts at least the wholesale price only when the optimal terminal cash level is greater than the reservation option, i.e., rejecting to get zero profit, for any $\theta_2 \in [0, c)$. We should have $\pi_1^b(y^{b*}(w|\theta_2, e)|w) \geq \theta_1, \ \forall \theta_2 \in [0, c)$. It leads to a necessary condition that the manufacturer has to offer a wholesale price high enough to guarantee the tier-1 supplier's positive profit, i.e., $w \geq \underline{w}$, where $\underline{w} := v + y^{b*}(c|0, e) > v$. As the financing issue with capital-constrained suppliers is the focus of this paper, we will focus on the case where the two suppliers' total working capital is constrained to the extent that $\theta_1 + \theta_2 \leq y^{b*}(\underline{w}|c, e)$, where $y^{b*}(\underline{w}|c, e) > c$ by monotonicity. Let $\theta_1^{\max} := y^{b*}(\underline{w}|c, e) - c$ denote the upper bound of the tier-1 supplier's working capital, we only consider the suppliers' working capital space $(\theta_1, \theta_2) \in \mathcal{W} := [0, \theta_1^{\max}] \times [0, c)$ in the remainder of the paper.

At time 0, the tier-1 supplier compares his expected maximal terminal cash level under MF and BF, i.e., $\pi_1^{b*}(w)$ and $\pi_1^{m*}(w, r_m)$, to decide his optimal financing policy. Essentially, the tier-1 supplier's financing choice boils down to a price competition between two financing schemes. Note that MF and BF have different interest rate pricing mechanisms: a constant interest rate in MF, whereas a variable rate in BF based on the competitive lending equation. It implies that a higher working capital level at the tier-2 results in a lower BF interest rate and increases the attractiveness of BF to the tier-1. Proposition 3.1 characterizes the tier-1 financing choice based upon the tier-2 supplier's working capital θ_2.

Proposition 3.1. *For any $w \geq \underline{w}$ and $r_m > 0$, there exists a unique threshold $\bar{\theta}_2(w, r_m) \geq 0$, such that the tier-1 supplier prefers MF if and only if $\theta_2 \leq \bar{\theta}_2(w, r_m)$. Moreover, the threshold $\bar{\theta}_2(w, r_m)$ decreases in w and r_m.*

For notation convenience, we simply denote $\bar{\theta}_2(w, r_m)$ as $\bar{\theta}_2$, and refer to it as the *BF threshold* of the tier-2 working capital level. Proposition 3.1 states that the tier-1 supplier uses MF only when the tier-2 supplier is severely capital-constrained. The reason is that the actual bank loan interest rate, $r_b = \frac{1}{\mathcal{P}(y)} - 1$ (from the competitive lending equation), decreases in the tier-2 supplier's working capital. When $\theta_2 > \bar{\theta}_2$, the bank loan interest rate falls below r_m. It is intuitive that the financing threshold $\bar{\theta}_2(w, r_m)$ decreases in r_m – MF becomes less appealing when its interest rate r_m increases. Interestingly, $\bar{\theta}_2(w, r_m)$ decreases in the wholesale price w. This is because increasing w increases the tier-1's incentive to help the tier-2 invest in reliability; a more reliable supply chain enjoys a lower BF interest rate and favors BF more. Lastly, we also find that the dual financing strategy is never optimal. This is because the financial costs of BF and MF is not convexly increasing in the borrow amount. Separately borrowing from two financiers induces a higher total cost, and thus, dual financing with two lenders is always dominated by sole financing.

3.4.3 Manufacturer's Problem

Anticipating the tier-1 supplier's time-1 risk-mitigation decision rule and time-0 financing decision rule (Proposition 3.1), the manufacturer decides her wholesale price w and AP interest rate r_m to maximize her expected payoff. To model the manufacturer's limited visibility into the tier-2 supplier, we assume the manufacturer does not have precise information about tier-2's working capital level θ_2 and assigns a probability distribution $F(\theta_2)$ over $[0, c)$.

Let $\tilde{\pi}_{0S}^{k\mathbb{I}}$ (resp., $\tilde{\pi}_{0F}^{k\mathbb{I}}$) denote the manufacturer's time-2 payoff when the tier-1 supplier adopts financing scheme $k \in \{b, m\}$ and risk-mitigation decision $\mathbb{I} \in \{\mathbb{P}, \mathbb{PR}\}$ and the tier-2 supplier succeeds (resp., fails) delivery. We summarize $\tilde{\pi}_{0S}^{k\mathbb{I}}$ and $\tilde{\pi}_{0F}^{k\mathbb{I}}$ in Table 3.1. Specifically, if the tier-2 supplier successfully delivers the order, the manufacturer earns a selling profit of $1 - w$ and an AP interest payment $r_m B_m$ from the tier-1 if the latter chooses MF. If the tier-2 supplier fails to deliver, then the manufacturer will not earn the selling profit of $1 - w$ unless the tier-1 resorts to emergency sourcing. If the tier-1 takes MF, then the manufacturer incurs advance payment B_m but only receives a late payment $\min\{(1 + r_m)B_m, w - e\}$ if tier-1 resorts to the emergency source.

Table 3.1: Manufacturer's Time-2 Payoff $\tilde{\pi}_{0S}^{k\mathbb{I}}$ and $\tilde{\pi}_{0F}^{k\mathbb{I}}$ (Note $B_m = (y - \theta_1 - \theta_2)^+$)

Scenario	Probability	BF ($k = b$)		MF ($k = m$)	
		$\mathbb{P} : w < e$	$\mathbb{PR} : w \geq e$	$\mathbb{P} : w < e$	$\mathbb{PR} : w \geq e$
S (Success)	$\mathcal{P}(y)$	$1 - w$	$1 - w$	$1 - w + r_m B_m$	$1 - w + r_m B_m$
F (Failure)	$1 - \mathcal{P}(y)$	0	$1 - w$	$-B_m$	$1 - w - B_m + \min\{(1 + r_m)B_m, w - e\}$

If the manufacturer knows the working capital profile (θ_1, θ_2), she can precisely anticipate the tier-1's financing-decision response, k, to her AP contract (w, r_m) offering. Her expected payoff can be represented by:

$$\pi_0^{k\mathbb{I}}(w, r_m|e, \theta_1, \theta_2) = \mathcal{P}(y^{k*})\tilde{\pi}_{0S}^{k\mathbb{I}}(w, r_m|e, \theta_1, \theta_2) + (1 - \mathcal{P}(y^{k*}))\tilde{\pi}_{0F}^{k\mathbb{I}}(w, r_m|e, \theta_1, \theta_2), \quad (3.4)$$

where $k = m$ if $\theta_2 \leq \bar{\theta}_2$, and $k = b$ if $\theta_2 > \bar{\theta}_2$ (by Proposition 3.1), and $\mathbb{I} = \mathbb{P}$ if $\hat{w} < e$ and $\mathbb{I} = \mathbb{PR}$ if $\hat{w} \geq e$. However, the manufacturer, with only the probability distribution information about θ_2, cannot precisely predict the tier-1's financing decision in response to her AP contract offer (w, r_m). We assume the manufacturer takes expectation of $\pi_0^{k\mathbb{I}}(w, r_m|e, \theta_1, \theta_2)$ over the probability distribution of θ_2. Hence, her expected payoff is given by:

$$\pi_0^{\mathbb{I}}(w, r_m|e, \theta_1) = \mathbb{E}_{\theta_2}\left[\pi_0^{m\mathbb{I}}(w, r_m|e, \theta_1, \theta_2) \cdot \mathbf{1}_{\{\theta_2 \leq \bar{\theta}_2\}} + \pi_0^{b\mathbb{I}}(w, r_m|e, \theta_1, \theta_2) \cdot \mathbf{1}_{\{\theta_2 > \bar{\theta}_2\}}\right], \ \mathbb{I} \in \{\mathbb{P}, \mathbb{PR}\}. \quad (3.5)$$

The manufacturer sets (w, r_m) to maximize her expected payoff. Although the manufacturer cannot predict the tier-1 supplier's financing decision and AP interest rate r_s to the tier-2 supplier (both decisions would require knowledge about the tier-2 working capital level θ_2), she can precisely influence the tier-1's time-1 risk-mitigation decision. Theorem 3.1 identifies the threshold of e and characterizes the optimal tier-0 AP contract.

Theorem 3.1. *There exists a unique threshold $\bar{e}(\theta_1)$, such that the manufacturer's optimal tier-0 AP contract (w^*, r_m^*) takes the following form:*

$$(w^*, r_m^*) = \begin{cases} (e, r_m^{\mathbb{PR}}), & \text{if } e \leq \bar{e}(\theta_1); \\ (w^{\mathbb{P}}, r_m^{\mathbb{P}}), & \text{if } e > \bar{e}(\theta_1). \end{cases} \quad (3.6)$$

Theorem 3.1 identifies a threshold $\bar{e}(\theta_1)$ for the emergency sourcing cost e, below which $\mathbf{E}$ is relatively cheap and the manufacturer will offer a AP contract $(e, r_m^{\mathbb{PR}})$ to incorporate the reactive risk-mitigation measure in case of the tier-2's disruption. The intuition for $w^* = e$ in the “proactive+reactive" risk mitigation case is that once the reactive measure providing perfect reliability is used, continuing to increase the wholesale price to improve the proactive measure is unnecessary under both BF and MF. Corollary 3.1 presents the sensitivity analysis of the optimal tier-0 AP contract parameters.

Corollary 3.1. *(i) $w^{\mathbb{P}} < \bar{e}(\theta_1)$. $w^{\mathbb{P}}$ and $r_m^{\mathbb{P}}$ are independent of e, but decrease in θ_1.*

(ii) $r_m^{\mathbb{PR}}$ increases in e and decreases in θ_1.

(iii) $\bar{e}(\theta_1)$ decreases in θ_1.

Corollary 3.1(i) and (ii) show that contract terms decrease in the tier-1 supplier's working capital. This is because when the tier-1 supplier is less capital-constrained, the wholesale price's positive effect on stimulating the tier-1 supplier to improve the reliability (by reducing the tier-1 AP interest rate) decreases. In this case, the manufacturer prefers a lower wholesale price. As for the interest rate of MF, a less capital-constrained tier-1 supplier is less risky since he is willing to increase the financing of the tier-2 supplier. As a result, the manufacturer offers a lower interest rate. Corollary 3.1(iii) implies that as the tier-1's capital level increases the manufacturer is less willing to match the wholesale price to e to induce emergency sourcing. This is because the manufacturer expects the tier-1 to use his own capital as AP to the tier-2 to help improve reliability and the resulting more reliable supply chain has less need for reactive risk mitigation.

If we feed the equilibrium characterized by Theorem 3.1 to Proposition 3.1, we can depict the equilibrium financing arrangement and risk-mitigation strategy in the (e, θ_2) space (depicted in Figure 3.3). The manufacturer encourages reactive risk mitigation when emergency sourcing

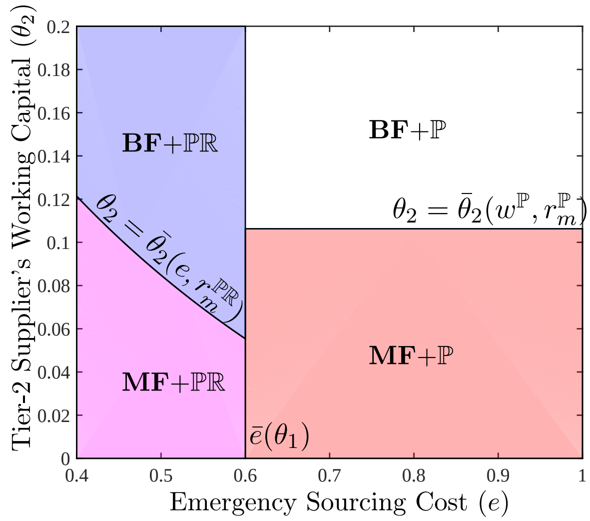


Figure 3.3: (Color Online) Tier-1 Supplier's Financing and Emergency Sourcing Regions

is not too costly. In this case, the manufacturer matches the wholesale price to the emergency sourcing cost to incentivize the reactive risk mitigation, but at the same time raises the interest rate to reduce the echelon's incentive of the proactive risk mitigation. The manufacturer will switch to the pure proactive risk-mitigation strategy when the emergency source exceeds threshold $\bar{e}(\theta_1)$. Combined with tier-1's financing decision of accepting MF when the tier-2 is severely capital-constrained ($\theta_2 < \bar{\theta}_2$), each of the risk-mitigation region is further divided into MF and BF regions. A more interesting finding is that the threshold $\bar{\theta}_2$ (above which BF is used) decreases in the emergency sourcing cost when the “proactive+reactive" risk mitigation is used. The reason is that as the emergency sourcing cost increases in this region, the manufacturer has to invest more to maintain the reactive risk mitigation (i.e., the wholesale price increases), which in turn reduces the manufacturer's incentive of using proactive risk mitigation. As a result, the manufacturer is willing to help finance the supply chain only when the tier-2 supplier is severely capital-constrained.

3.5 Blockchain-Enabled Delegate Financing

In this section, we discuss the deep-tier financing case where blockchain technology is adopted for improving supply chain visibility. Deep-tier financing via blockchain technology can provide an immutable, censorship-resistant, single source of truth for all supply chain transactions, thus producing a desired level of transparency into the working capital flow at all tiers of the supply chain. In addition, the privacy-preserving mechanism of blockchain enables that business-sensitive information is selectively shared and verified immutably on-chain, making it feasible for multilateral collaboration. In the example of blockchain-based Marco Polo Network, all supply chain transactions, and steps are stored on a digital ledger and processed on the Marco Polo Platform, which is directly integrated with the supplier's ERP and the supplier's bank to guarantee real-time visibility of working capital finance.[13] For technical details of such implementation, we refer the readers to a FinTech company, Plaid, which provides lenders with access to borrowers' bank data to make informed loan decisions via its Asset API.[14]

In our model setup, blockchain technology enables the manufacturer to gain necessary visibility of the tier-2's financial status (working capital level θ_2) even without direct business transactions. As a result, the manufacturer can precisely predict the tier-1's response (financing choice and AP interest rate offered to the tier-2) to her tier-0 AP contract, and thus can more effectively delegate her financial support to the tier-1 supplier to alleviate the tier-2's capital constraint and thus mitigate the supply disruption risk. Given blockchain's powerful technical potential and opportunities for deep-tier supply chain financing, two important questions must be addressed: (1) Whether is it in the best interest of a supply chain participant to adopt blockchain? The answer to this question is dependent on the answer

[13]https://www.marcopolonetwork.com/working-capital-finance/
[14]https://plaid.com/docs/assets/

to the second question; (2) Whether and how to adjust supply chain contracts to incentivize blockchain adoption? Supply chain visibility can be achieved via blockchain adoption when all parties agree to adopt blockchain. This section studies voluntary blockchain adoption that facilitates delegate financing, under which the downstream supply chain members take advantage of their dominance status relative to their respective upstream suppliers and push blockchain adoption by offering a new AP contract for blockchain adoption while honoring their exiting AP contract offer for no blockchain adoption. We also consider the model where the manufacturer requires mandatory blockchain adoption in the supply chain. Interested readers can refer to Appendix C for more detailed discussions.

Next, we introduce the model setup for delegate financing and use $\hat{\cdot}$ to denote the corresponding notations. The sequence of events is described as follows. First, the manufacturer offers a two-menu tier-0 AP contract consisting of a "blockchain menu" and a "traditional menu." If the tier-1 agrees to adopt blockchain, the "blockchain menu," denoted by $(\hat{w}^1(\theta_1, \theta_2), \hat{r}^1_m(\theta_1, \theta_2))$, will be implemented based on the tier-2's working capital level θ_2 revealed by the blockchain (note that θ_1 is known to the manufacturer regardless of the blockchain adoption). Essentially, the blockchain menu is a smart contract that specifies the contract term contingent on the tier-2 working capital level and can be automatically implemented once blockchain is adopted in the supply chain. If the tier-1 chooses not to adopt blockchain, the manufacturer honors the "traditional menu," and implements the equilibrium of §3.4 (i.e., (w^*, r^*_m)) which is a function of θ_1 (because θ_2 is not revealed in this case). Second, if the tier-1 agrees to adopt blockchain, he offers a two-menu tier-1 AP contract, a "blockchain menu" (denoted by $\hat{r}^1_s(\theta_2)$) and a "traditional menu" ($r^*_s(\theta_2)$, the equilibrium offer in §3.4). The "blockchain menu" menu must be more attractive than the "traditional menu" to the tier-2 to ensure the tier-2's voluntary adoption of blockchain. Note that the tier-1 knows θ_2 without blockchain. If the tier-1 decides not to adopt blockchain, he will offer one menu, the same one as in §3.4 to the tier-2. Lastly, the tier-2 supplier responds to the tier-1's contract offer by the blockchain adoption choice and the reliability investment decision (same analysis as in §3.4).

We formulate the above supply chain interaction as a sequential Stackelberg game and solve it through backward induction.

3.5.1 Tier-2 and Tier-1's Problem

Given a tier-1 AP contract, the tier-2's reliability investment decision is the same as what we have characterized in Lemma 3.1. When two tier-1 AP contracts, r_s^*) and $\hat{r}_s^1$, are presented, the tier-2 will choose the one with a lower interest rate. Given a tier-0 AP contract, $(\hat{w}_m^1, \hat{r}_m^1)$ or (w_m^*, r_m^*), the tier-1's problem is similar to that in §3.4.2. A small difference is that if the tier-1 agrees to adopt blockchain, he will fulfill his commitment to the manufacturer that the tier-2 will adopt blockchain by offering the tier-2 a blockchain menu with an AP interest rate strictly lower than that of the traditional menu, i.e., $\hat{r}_s^1 < r_s^*$, to steer the tier-2 to choose blockchain adoption.

Let $\hat{\pi}_i^*(w^*, r_m^*, \theta_1, \theta_2)$ and $\hat{\pi}_i^*(\hat{w}^1, \hat{r}_m^1, \theta_1, \theta_2)$ represent the tier-i's maximal profits if rejecting and accepting blockchain adoption, respectively, $i = 0, 1$. We assume that the tier-i supplier will adopt blockchain if and only if the blockchain brings a higher profit, i.e., $\hat{\pi}_i^*(\hat{w}^1, \hat{r}_m^1, \theta_1, \theta_2) > \hat{\pi}_i^*(w^*, r_m^*, \theta_1, \theta_2), i = 1, 2$.

3.5.2 Manufacturer's Problem

From the manufacturer's perspective, the fundamental incentive for her to adopt blockchain is that the blockchain-revealed tier-2 working capital θ_2 enables her to efficiently influence the tier-1 and tier-2 suppliers' risk-mitigation practice and improve her profit. Hence, blockchain adoption requires strict win-win-win for all three parties. Whether this ambitious goal can be achieved for each supply chain working capital profile (θ_1, θ_2) is unclear. The manufacturer needs to solve the following problem for any given (θ_1, θ_2) to identify (1) the parameter region where blockchain-enabled delegate financing is viable, and (2) the blockchain menu that enables the adoption.

$$\max_{\hat{w}^1, \hat{r}_m^1} \quad \hat{\pi}_0(\hat{w}^1, \hat{r}_m^1 | e, \theta_1, \theta_2) \tag{3.7}$$

$$\text{s.t.} \quad \hat{\pi}_i^*(\hat{w}^1, \hat{r}_m^1, \theta_1, \theta_2) > \pi_i^*(w^*, r_m^*, \theta_1, \theta_2), \forall i = 0, 1, 2. \tag{3.8}$$

The manufacturer, with perfect visibility of the tier-2's working capital level θ_2, is able to predict (i) the tier-1's bank loan interest rate $\hat{r}_b^*(w^1, \theta_2)$ under BF (equivalently, $\hat{y}^{b*}(w^1, \theta_2)$) and (ii) the tier-1 AP interest rate to the tier-2 supplier $\hat{r}_s^*$ under MF (equivalently, $\hat{y}^{m*}(w^1, r_m^1, \theta_1, \theta_2)$). Consequently, the expected payoff that the manufacturer optimizes is (3.4), which we repeat below:

$$\hat{\pi}_0^{k\mathbb{I}}(\hat{w}^1, \hat{r}_m^1 | e, \theta_1, \theta_2) = \mathcal{P}(\hat{y}^{k*}) \hat{\pi}_{0S}^{k\mathbb{I}}(\hat{w}^1, \hat{r}_m^1 | e, \theta_1, \theta_2) + (1 - \mathcal{P}(\hat{y}^{k*})) \hat{\pi}_{0F}^{k\mathbb{I}}(\hat{w}^1, \hat{r}_m^1 | e, \theta_1, \theta_2), \tag{3.9}$$

with additional participation constraints as specified in (3.8), where $k = m$ if $\theta_2 \leq \bar{\theta}_2(\hat{w}^1, \hat{r}_m^1)$ and $k = b$ if $\theta_2 > \bar{\theta}_2(\hat{w}^1, \hat{r}_m^1)$, and $\mathbb{I} = \mathbb{P}$ if $\hat{w} < e$ and $\mathbb{I} = \mathbb{PR}$ if $\hat{w} \geq e$.

3.5.3 Equilibrium Characterization

We present the optimal blockchain menu that the manufacturer should offer and contracts and conditions under which the tier-1 and tier-2 suppliers accept the blockchain menu in Theorem 3.2 (see Figure 3.4 for illustration).

Theorem 3.2. *(i) The optimal blockchain menu takes the following form: for any given* θ_1*, there exists a threshold of the tier-2's working capital* $\bar{\hat{\theta}}_2(\theta_1|e) \in [0, c)$ *such that for*

$\theta_2 \leq \bar{\hat{\theta}}_2(\theta_1|e)$,

$$(\hat{w}^{1*}, \hat{r}_m^{1*}) = \begin{cases} \left(e, \hat{r}_m^{1\mathbb{PR}*}\right), & \text{if } e \leq \bar{\bar{e}}(\theta_1, \theta_2); \\ \left(\hat{w}^{1\mathbb{P}*}, \hat{r}_m^{1\mathbb{P}*}\right), & \text{otherwise,} \end{cases} \tag{3.10}$$

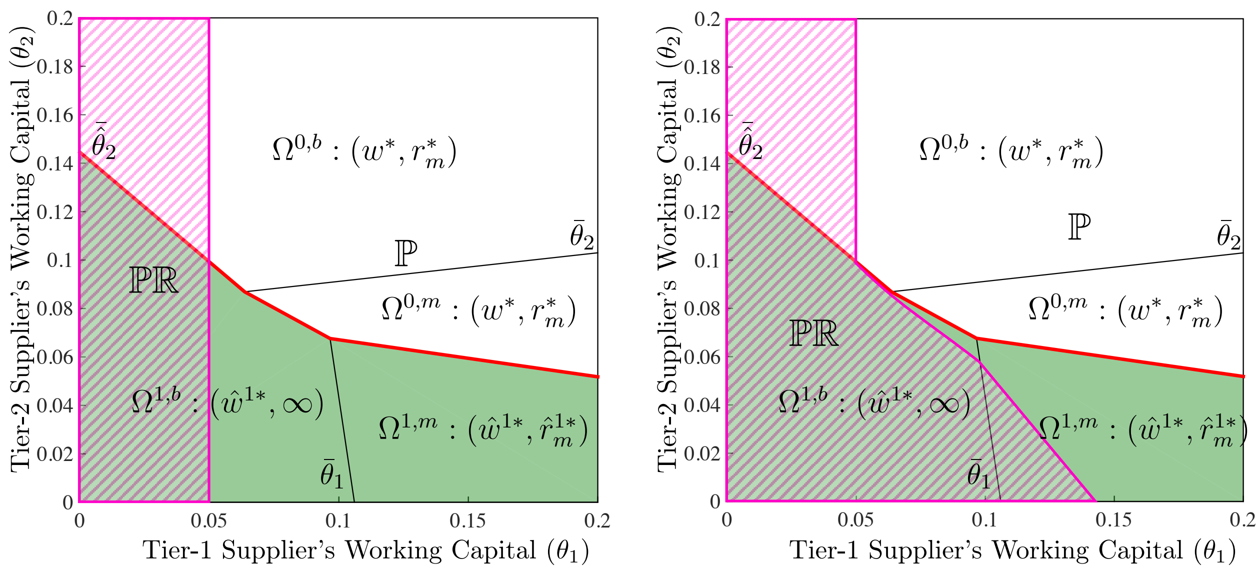
for $\theta_2 > \bar{\hat{\theta}}_2(\theta_1|e)$,

$$(\hat{w}^{1*}, \hat{r}_m^{1*}) = (w^*, r_m^*), \tag{3.11}$$

where $\bar{\bar{e}}(\theta_1, \theta_2)$ is the threshold of the emergency sourcing cost below which the reactive risk-mitigation measure is implemented. Moreover, $\hat{r}_m^{1\mathbb{I}*} = \infty$, $\mathbb{I} \in \{\mathbb{P}, \mathbb{PR}\}$ if $\theta_1 \leq \bar{\theta}_1(\theta_2)$, indicating BF is adopted.

(ii) The tier-1 and tier-2 suppliers adopt the blockchain menu if and only if $\theta_2 \leq \bar{\hat{\theta}}_2(\theta_1|e)$.

The key result of Theorem 3.2 is that the blockchain can be adopted only when the tier-2 is severely capital-constrained with the working capital below a threshold $\bar{\hat{\theta}}_2(\theta_1|e)$. The shaded area in Figure 3.4 represents the blockchain adoption region in the (θ_1, θ_2) space, where subregion $\Omega^{1,b}$ (resp. $\Omega^{1,m}$) is where the tier-1's takes BF (resp. MF) in equilibrium. $\Omega^{1,b} \cup \Omega^{1,m}$ is the region the visibility of the tier-2 working capital level θ_2 enables the manufacturer to tailor the tier-0 AP contract to improve the supply chain reliability through a more efficient proactive and reactive risk-mitigation combination and the at the same time increase profit for each supply chain member. However, when the tier-2 is not severely capital-constrained ($\theta_2 > \bar{\hat{\theta}}_2(\theta_1|e)$), the information of θ_2 cannot help the manufacturer achieve reliability improvement and profit improvement for all simultaneously through delegate financing. The unshaded area in Figure 3.4 represents (θ_1, θ_2) settings where the supply chain will not adopt blockchain and will use the traditional menu; subregion $\Omega^{0,b}$ (resp. $\Omega^{0,m}$) is where the tier-1's takes BF (resp. MF) in equilibrium.



(a) Without Blockchain (b) With Blockchain

Figure 3.4: (Color Online) Divisions of Working Capital Space for Blockchain Adoption, Financing Choice, And Risk Mitigation

Table 3.2: Impact of Blockchain Adoption on Supply Chain in Different Regions

Regions	Wholesale Price	Tier-0 Interest Rate	Risk-Mitigation Measures
$\Omega^{1,b}$	$\hat{w}^{1*} \geq w^*$ (↑)	$\hat{r}_m^{1*} > r_b^* \geq r_m^*$ (↑)	$\mathbb{P}\uparrow$, $\mathbb{PR}\uparrow$
$\Omega^{1,m}$	$\hat{w}^{1*} \geq w^*$ (↑)	$\hat{r}_m^{1*} \leq r_b^*$ (↑,→,↓)	$\mathbb{P}\uparrow$, $\mathbb{PR}\uparrow$

Note: "→", "↑", and "↓" represent "same", "increase", and "decrease" compared with the benchmark case in §3.4.

To see the driving forces behind these results, we start by examining how the manufacturer adjusts her AP contract offering to optimally utilize the blockchain-enabled visibility in the blockchain adoption region. Table 3.2 presents a comparison between the blockchain menu $(\hat{w}^1, \hat{r}^1)$ and the traditional menu (w_m^*, r_m^*) and the resulting risk-mitigating measures in the blockchain adoption region. In Region $\Omega^{1,b}$, where both tiers of suppliers are severely capital-constrained $(\theta_2 \leq \bar{\bar{\theta}}_2, \theta_1 \leq \bar{\theta}_1(\theta_2))$, the manufacturer finds delegate financing an expensive means to help improve the tier-2's reliability. She will set the tier-0 AP interest rate $\hat{r}_m^{1*}$ to be above the anticipated bank interest rate $\hat{r}_b^*$ to let the tier-1 rely on BF to finance tier-1 AP offering to improve the tier-2's reliability. To ensure that the bank's interest rate is reasonably low and the tier-1 has sufficient incentive to offer an attractive AP rate $\hat{r}_s^{1*}$, the manufacturer sets wholesale price $\hat{w}_m^{1*}$ to be above that of the traditional menu, w_m^*. In Region $\Omega^{1,m}$, where the tier-2 is severely capital-constrained but the tier-1 is not, the manufacturer it economical to use her delegate financing to influence the proactive risk-mitigation investment. Although the manufacturer will offer a tier-0 AP rate $\hat{r}_m^{1*}$ lower than the anticipated bank rate $\hat{r}_b^*$, $\hat{r}_m^{1*}$ is not necessarily lower than the traditional menu's tier-0 AP rate r_m^*. This because the manufacturer will also set wholesale price $\hat{w}^{1*}$ to be higher than traditional menu's w^* to incentivize the tier-1 to offer an attractive tier-1 AP rate $\hat{r}_s^{1*}$ to the tier-2.

In both $\Omega^{1,b}$ and $\Omega^{1,m}$, the blockchain menu offers a wholesale price higher than that of the traditional menu to help improve the proactive risk mitigation. However, whether the increased wholesale price is sufficiently high to support the reactive risk mitigation depends on the comparison of the emergency sourcing cost e and threshold $\hat{\bar{e}}(\theta_1, \theta_2)$. The visibility of θ_2 enables the manufacturer to revise the threshold from $\bar{e}(\theta_1)$ that depends on the tier-1's capital level to $\hat{\bar{e}}(\theta_1, \theta_2)$ that depends on both suppliers' capital levels. Corollary 3.2 below states that the threshold decreases as a supplier's working capital increases, because more capital can support more reliability investment and hence a less need for emergency sourcing. More interestingly, Corollary 3.2 also confirms that reactive risk mitigation also expands in the blockchain adoption region, i.e., $\hat{\bar{e}}(\theta_1, \theta_2) > \bar{e}(\theta_1)$.

Corollary 3.2. *In* $\Omega^{1,b} \cup \Omega^{1,m}$, $\hat{\bar{e}}(\theta_1, \theta_2)$ *decreases in* θ_1 *and* θ_2 *and* $\hat{\bar{e}}(\theta_1, \theta_2) > \bar{e}(\theta_1)$.

Next, let us discuss the reason that blockchain will not be adopted when the tier-2's working capital is not severely constrained, i.e., when $\theta_2 > \hat{\bar{\bar{\theta}}}_2$. In this case, the traditional menu (w^*, r_m^*) sets the proactive and reactive risk-mitigation to such levels that the manufacturer finds the only way to adjust the risk-mitigation combination to improve her profit will require a blockchain menu with a lower wholesale price and a higher AP interest rate than those of the traditional menu. Such adjustments will result in a lower profit for the tier-1. Essentially, without blockchain, the visibility barrier limits the manufacturer's ability to influence the supply chain risk-mitigation decisions and leaves the tier-1 in more control of such decisions to maximize his profit. Voluntary blockchain adoption protects the existing profit allocation within the supply chain. When the tier-2's working capital is not severely constrained, any delegate financing based on revealed θ_2 would leave at least one supply chain party worse-off than that without blockchain. In such a case, the manufacturer would simply offer a blockchain menu identical to the traditional menu (w^*, r_m^*), which leads to the tier-1's rejection of blockchain adoption.

In summary, improved visibility via the blockchain-enabled information channel can benefit all three parties by allowing the manufacturer to deploy improved reactive and proactive risk mitigation measures with full supply chain information. However, this is only achieved when the tier-2's working capital is severely constrained (Regions $\Omega^{1,b}$ and $\Omega^{1,m}$ of Figure 3.4). Such limitation is mainly driven by the intermediation of the tier-1 under delegate financing. Luckily, as we will show in the next section, the power of blockchain technology can be further enhanced if the blockchain can provide a direct financing channel from the manufacturer to the tier-2.

3.6 Blockchain-Enabled Cross-Tier Direct Financing

Blockchain's privacy-preserving connection and information sharing open up opportunities for financial transactions between supply chain members that are not immediate buyers and sellers. Cross-tier direct financing is one such practice facilitated by Foxconn's blockchain platform. Under cross-tier direct financing, the manufacturer knows that the tier-2 supplier's working capital level θ_2 can extend a loan offer to the tier-2 directly without offering an AP contract to tier-1 to finance tier-2 supplier indirectly. This section first studies cross-tier *direct* *financing* (§§3.6.1-3.6.2), and then compare it with the blockchain-enabled (adjacent-tier) *delegate financing* (§3.6.3). Hereafter, for brevity, we use the notation $\check{\cdot}$ to represent this case.

The sequence of events is as follows. At time 0, the manufacturer offers two menus of contracts to the tier-1, a traditional menu and a blockchain menu. The traditional menu offers the same contract derived by §3.4. Under the blockchain menu, the manufacturer offers cross-tier AP contract with interest rate $\check{r}^2_m(\theta_1, \theta_2)$ to the tier-2 and a simple wholesale price contract $\check{w}(\theta_1, \theta_2)$ to the tier-1, because the tier-2 supplier no longer needs a tier-1 AP contract to finance its reliability investment.[15] The tier-1 will choose the menu that gives him a higher profit. If the tier-1 agrees to adopt blockchain, the cross-tier direct financing contract $(\check{w}(\theta_1, \theta_2), \check{r}^2_m(\theta_1, \theta_2)$ will be executed as a smart contract, and the tier-1 will offer wholesale price contract v to the tier-2. If the tier-1 rejects blockchain adoption, the manufacturer will have no direct access to the tier-2 and the supply chain operates under the traditional menu, the same way as that in §3.4.

3.6.1 Tier-2 and Tier-1's Problem

The tier-2's reliability investment decision logic under direct financing is the same as that under delegate financing that is characterized by Lemma 3.1. That is, its optimal investment level is determined by the interest rate it receives, regardless it is offered by the manufacturer or the tier-1. The tier-2 always prefers the offer with a lower interest rate.

The tie-1's expected profit under direct financing with only wholesale price ($\check{w}^1$) is,

$$\check{\pi}^{\mathbb{I}}_1(\check{w}^1 | \check{r}^2_m, \theta_1, \theta_2) = \mathcal{P}(y)(\check{w}^1 - v) + (1 - \mathcal{P}(y))(\check{w}^1 - e)^+, \quad (3.12)$$

[15]We have checked the financing competition between the manufacturer and tier-1 and found the manufacturer always wins because her capital cost is lower.

where the second term represents that the tier-1 will invoke emergency sourcing upon tier-2's delivery failure if $\check{w}^1 \geq e$.

Let $\check{\pi}_i^*(w^*, r_m^*, \theta_1, \theta_2)$ and $\check{\pi}_i^*(\check{w}^1, \check{r}_m^1, \theta_1, \theta_2)$ represent the tier-i's maximal profits if rejecting and accepting blockchain adoption, respectively, $i = 0, 1$. We assume that the tier-i supplier will adopt blockchain if and only if the blockchain brings a higher profit, i.e., $\check{\pi}_i^*(\check{w}^1, \check{r}_m^1, \theta_1, \theta_2) > \check{\pi}_i^*(w^*, r_m^*, \theta_1, \theta_2), i = 1, 2$.

3.6.2 Manufacturer's Problem under Cross-Tier Direct Financing

We first present the manufacturer's profit function under cross-tier direct financing. Let $\check{\pi}_{0S}^{\mathbb{I}}$ and $\check{\pi}_{0F}^{\mathbb{I}}$ denote the manufacturer's profit with tier-2's delivery success and failure, respectively, for $\mathbb{I} = \mathbb{P}, \mathbb{PR}$; and their expressions are summarized in Table 3.3.

Table 3.3: Manufacturer's Payoff under Cross-Tier Direct Financing ($\check{B}_m = (y - \theta_2)^+$)

Scenario	Probability	Cross-Tier Direct Financing	
		$\mathbb{P}: \ \check{w}^1 < e$	$\mathbb{PR}: \ \check{w}^1 \geq e$
S	$\mathcal{P}(\check{y})$	$1 - \check{w}^1 + \check{r}_m^2 \check{B}_m$	$1 - \check{w}^1 + \check{r}_m^2 \check{B}_m$
F	$1 - \mathcal{P}(\check{y})$	$-\check{B}_s$	$1 - \check{w}^1 - B_m$

The main difference between direct financing (Table 3.3) and delegate financing (Table 3.1) is that the manufacturer's financial payoff under direct financing comes from the tier-2 whereas her financial payoff under delegate financing comes from the tier-1. Let $\check{\pi}_0^{\mathbb{I}}$ denote the manufacturer's expected payoff under direct financing. The supply chain's adoption of the blockchain-enabled cross-tier direct financing requires a strict win-win-win outcome for all three parties. Thus, the manufacturer needs to solve the following problem for any given (θ_1, θ_2) to identify both the parameter region and the optimal blockchain menu that enable the adoption:

$$\max_{\check{w}^1, \check{r}_m^1} \quad \check{\pi}_0^{\mathbb{I}}(\check{w}^1, \check{r}_m^2 | \theta_1, \theta_2, e) = \mathcal{P}(\check{y})\check{\pi}_{0S}^{\mathbb{I}} + (1 - \mathcal{P}(\check{y}))\check{\pi}_{0F}^{\mathbb{I}} \tag{3.13}$$

$$\text{s.t.} \quad \hat{\pi}_i^*(\check{w}^1, \check{r}_m^2, \theta_1, \theta_2) > \pi_i^*(w^*, r_m^*, \theta_1, \theta_2), \forall i = 0, 1, 2. \tag{3.14}$$

Theorem 3.3. *(i) The optimal blockchain menu for cross-tier direct financing takes the following form: There exists a threshold $\bar{\check{e}}(\theta_2)$ decreasing in θ_2, such that*

(a) the manufacturer offers a loan interest rate $\check{r}_m^{2*}(\theta_2) \leq r_s^*$ to the tier-2;

(b) the manufacturer offers the following wholesale price to the tier-1:
$\check{w}^{1*} = \bar{\check{w}}^1(\theta_1, \theta_2) \leq w^*$ if $e > \bar{\check{e}}(\theta_2)$; $\check{w}^{1*} = e$, otherwise, where $\bar{\check{e}}(\theta_2) \leq \bar{\hat{e}}(\theta_1, \theta_2)$.

(ii) The tier-1 and tier-2 will always adopt the blockchain menu.

(iii) The tier-2 reliability investment under the blockchain menu is higher than that under the traditional menu, $\check{y}^*(\theta_2) \geq y^*(\theta_2)$.

Theorem 3.3 presents a compelling case for blockchain-enabled cross-tier direct financing: it always results in win-win-win outcomes. The fundamental reason behind this powerful result is that the wholesale price and the AP interest rate under direct financing play distinct roles. Specifically, the AP interest rate $\hat{r}_m^{2*}$ plays the role of incentivizing the tier-2 to make a (manufacturer's) desired level of reliability investment. Once $\check{r}_m^{2*}$ is set, the tier-2's reliability investment level is determined. The wholesale price $\check{w}^{1*}$, on the other hand, plays the role of affecting the tier-1's reactive risk-mitigation decision and deciding the profit division between the manufacturer and the tier-1. By contrast, the roles of wholesale price and AP interest rate under delegate financing cannot be completely decoupled, because the tier-1's incentive to offer AP contract to the tier-2 is affected by both the manufacturer offered wholesale price and AP interest rate. The manufacturer always needs to adjust both contract parameters to indirectly influence the proactive risk-mitigation investment.

Theorem 3.3 (i)(a) and (iii) state that the manufacturer will offer an AP rate $\check{r}_m^{2*}$ strictly lower than that under the traditional menu to induce the tier-2 to invest more in reliability. Two reasons contribute to this result: (i) unlike the tier-1 who has to borrow to finance the tier-2, the manufacturer has ample capital to finance the tier-2, and (ii) the manufacturer under direct financing no longer needs to raise the wholesale price to incentivize the tier-1 to help the tier-2, her profit margin from receiving a successful tier-2's delivery is higher (see discussion below). Theorem 3.3 (i)(b) states that the manufacturer will support reactive risk-mitigation by setting $\check{w}^{1*} = e$ when emergency sourcing cost e is below the threshold $\bar{\check{e}}$; otherwise, she will set the wholesale price below that of the traditional menu. The tier-1 is willing to accept a wholesale price lower than that of the traditional menu because he no longer holds the responsibility of financing the tier-2 and saves financing cost. The emergency sourcing cost threshold $\bar{\check{e}}$ depends on the tier-2's working capital θ_2, suggesting that direct-financing reveals the fundamental economic factor that decides the supply chain risk-mitigation measures is the tier-2's working capital level: it drives both the optimal proactive risk-mitigation level $\check{y}$ and the conditions for supporting reactive risk mitigation. Intuitively, the higher is the tier-2's working capital, the less is the manufacturer willing to support reactive risk mitigation. Thus, based on θ_2, the manufacturer can devise the most efficient risk-mitigation strategy to increase the supply chain total profit and use wholesale price to allocate sufficient profit to the tier-1 (the party she by passes) to ensure the latter's blockchain adoption.

3.6.3 Discussion: Can Cross-Tier Direct Financing Dominate Delegate Financing?

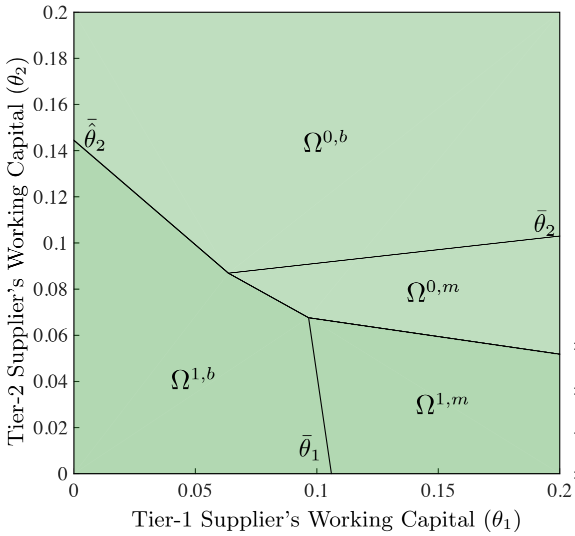
Theorem 3.3 suggests that cross-tier direct financing is more powerful than direct financing to convince the supply chain to adopt blockchain: cross-tier direct financing can bring win-win-win outcomes in (θ_1, θ_2) region where delegate financing cannot. A natural question arises: suppose the supply chain already adopted blockchain and operated under delegate financing, will the supply chain have an incentive to switch to cross-tier direct financing? Theorem 3.4 compares the supply chain members' profits between the two financing schemes (see illustration in Figure 3.5).

Theorem 3.4. *From delegate financing to direct financing:*

(*i*) *The manufacturer and the tier-2 supplier are weakly better-off;*

(*ii*) *The tier-1 supplier is worse-off in regions $\Omega^{1,b}$ and $\Omega^{1,m}$.*

Theorem 3.4 states that the tier-1 is the only party in the supply chain who will be against the switch from delegate financing to direct financing when the tier-2 is severely capital-constrained (in Region $\Omega^{1,b} \cup \Omega^{1,m}$). Recall in the discussion of Theorem 3.2, Region $\Omega^{1,b} \cup \Omega^{1,m}$ is where the manufacturer will raise the wholesale price to the tier-1 so that the latter would provide attractive AP contract to tier-2. This is the region where delegate financing gives the tier-1 significant lever over the manufacturer. Discussion of Theorem 3.3 shows that under delegate financing, the tier-1 has little lever over the manufacturer; the manufacturer can offer the lowest possible wholesale price enough to make the tier-1 slightly better-off than no-blockchain. Therefore, even though direct financing can improve both the manufacturer's and the tier-2's profits, the tier-1 will resist the switch from delegate financing to direct financing.



Regions	M's Profit	S_1's Profit	S_2's Profit	Chain's Profit
$\Omega^{1,b} \cup \Omega^{1,m}$	§6≥§5	§5≥§6	§6≥§5	§6≥§5
$\Omega^{0,b} \cup \Omega^{0,m}$	§6>§5	§6≥§5	§6>§5	§6>§5

Figure 3.5: (Color Online) Division of Working Capital Space in Equilibrium and Profit Comparisons

Convincing the tier-1 to switch to direct financing would require some supply chain member(s) to give up part of their profit to compensate the tier-1's loss due to relinquishing his leverage in the supply chain. Theorem 3.5 shows that direct financing is flexible enough to facilitate such profit transfer.

Theorem 3.5. *Under cross-tier direct financing with any (θ_1, θ_2, e), there exists a wholesale price threshold $\bar{\bar{w}}^{1*\prime}(\theta_1, \theta_2) \geq \bar{\bar{w}}^{1*}(\theta_1, \theta_2)$ such that when the manufacturer offers $\bar{\bar{w}}^{1*\prime}$, all three parties are weakly better-off compared to delegate financing.*

Compared to delegate financing, direct financing improves the supply chain reliability significantly. The decoupling of the roles of wholesale price and AP interest rate allows the manufacturer to raise the wholesale price to the tier-1 to increase his profit to be higher than his profit under delegate financing. Doing so will reduce the manufacturer's share of the supply chain profit but will not change the supply chain total profit.

3.7 Conclusion

Many deep-tier suppliers, due to their small sizes and lack of access to capital, are vulnerable to disruptions that impair their ability to fulfill supply chain orders. Downstream buyer-lead financing schemes such as advance payment (AP) intended to help the deep-tiers improve operational reliability are often ineffective because of the lack of visibility into the deep-tier's real needs for capital. Recent advancement in blockchain technology (e.g., zero-knowledge proof cryptography) has made secure, privacy-protection information sharing across the supply chain possible and propelled the development of blockchain-enabled supply chain financing based on deep-tier visibility. This paper studies how blockchain-enabled visibility affects the supply chain risk-mitigation effort.

For a serial three-tier supply chain, we compare three types of deep-tier AP financing schemes: traditional financing with limited visibility, blockchain-enabled delegate financing, and blockchain-enabled cross-tier direct financing. We first reveal that limited deep-tier visibility hampers the downstream manufacturer's ability to provide appropriate financial incentives to support the right mix of proactive and reactive risk-mitigation deployment. The manufacturer's limited visibility empowers the intermediary supply chain member, the tier-1 supplier in our model, to have more influence over the supply chain risk-mitigation decision. Next, we show that although blockchain adoption offers the manufacturer the needed visibility to address the above inefficiency, the solution scope and achievable efficiency level vary by the adopted deep-tier financing scheme. Delegate financing can bring win-win-win outcomes to all supply chain members (compared to the traditional financing) only when the tier-2 supplier is severely capital-constrained. This is because the lack of visibility of the severe capital constraint results in risk-mitigation under-investment, and visibility calls for increased manufacturer's investment that benefits everyone in the supply chain. However, supply chains with less-capital-constrained tier-2 suppliers will reject blockchain adoption, because the revealed visibility will suggest a re-balancing of the proactive and reactive risk-mitigation mix (e.g., more for one measure and less for the other measure) that result in a lower profit for some members of the supply chain.

Cross-tier direct financing, on the other hand, offers a more efficient approach to act upon the revealed deep-tier information and brings win-win-win outcomes for all scenarios of supplier working capital. This is because direct financing allows the manufacturer's instilled capital to be fully utilized by the tier-2 and significantly increases the supply chain proactive risk-mitigation investment whereas under delegate financing (with or without blockchain) the tier-1 offers a more expensive financing to the tier-2. Moreover, the manufacturer can use the wholesale price to adjust the profit distribution between her and the tier-1 supplier, the ability that she does not have under delegate financing where the wholesale price and AP interest rate must be used jointly to incentivize the tier-1 supplier. Finally, we show that if the manufacturer is willing to give up some of her profit share to the tier-1 supplier, direct financing can bring Pareto improvement over delegate financing.

Our research can be extended in several directions to address other open questions regarding deep-tier SCF. First, our work demonstrates research opportunities to integrate different SCF tools with the blockchain technology. Various SCF tools, such as factoring and reverse factoring, could be incorporated as possible future research venues. Second, this paper assumes that only tier-2 faces disruption risks. It will be interesting to explore how SCF tools help improve risk-mitigation investment when multiple tiers of suppliers face disruption risks. Finally, it will be interesting to explore other application potentials of blockchain technology in addition to our current focus on enabling cross-tier supply chain visibility. For example, blockchain adoption could help resolve the trust and commitment issues in various SCF activities.

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Appendix A

Managing Operations of a Hog Farm Facing Volatile Markets: Inventory and Selling Strategies

A.1 Summary of Key Notations

Table A1: Key Notations and Expressions

Category	Expression	Description
Global	q	Committed quantity for each period.
	α	Price discount for under-weight hogs $0 \leq \alpha \leq 1$.
Physical State	S_t^1	$s_t^1 \in \mathbb{R}^+$, the number of under-weight hogs in Pool 1 at period t, $0 \leq t \leq T$.
	S_t^2	$S_t^2 \in \mathbb{R}^+$, the number of regular-weight hogs in Pool 2 at period t, $0 \leq t \leq T$.
Exo. Info.	P_t^1	OM price when hogs are under-weight.
	P_t^2	OM price when hogs are regular-weight, $p_t^2 > p_t^1$.
	P_t^C	Contract price determined on the week of delivery.
	C_t^H	Holding cost, including the cost of yardage, feeding cost, labor cost etc.
	C_t^P	Penalty cost for the contractual shortage.
	$\mathbf{\Pi}_t$	$\mathbf{\Pi}_t = [\Pi_t^O, \Pi_t^M, \Pi_t^F]'$ are the factor prices in each market: OM, MM, and FM.
Random Path	W_t^1	The number of refilled hogs in under-weight pool, which is distributed according to F_t^1.
	W_t^2	The number of refilled hogs regular-weight pool, which is distributed according to F_t^2.
Action	x_t^1	The amount of under-weight hogs sold to the OM.
	x_t^2	The amount of regular-weight hogs sold to the OM.
	y_t	The amount of contractual shortage fulfilled by under-weight hogs.
	z_t	The amount of under-weight hogs held to the next period.

A.2 Data Calibration

A.2.1 Factor Prices

The factor price vector, $\mathbf{\Pi}_t \doteq [\Pi_t^O, \Pi_t^M, \Pi_t^F]'$, consists of livestock and pork prices, Π_t^O and Π_t^M, and prices of agricultural foodstuff used to produce animal feed, Π_t^F. Both the Chicago

Mercantile Exchange and USDA (United States Department of Agriculture) [104] publish them.

The livestock price, Π_t^O, is the daily negotiated average net price of hogs in the spot market as reported in the National Daily Slaughtered Swine Report (LM_HG201 from [102])[16], where the net price is the total amount paid by a packer to a producer per hundred pounds. It is the price at which the slaughterhouse sells to the downstream packers. The pork price, Π_t^M, is obtained from the National Daily Negotiated Pork Report (LM_PK602). We use the carcass cutout value from the report (measured per carcass hundred pounds) as the meat market price.

The fodder market price is obtained using the corn and soybean meal prices reported in SJ_GR850 and GX_GR117 reports from USDA. The former reports the 8-week corn price determined each Friday as the average of all daily reported high and low bids or offers per bushel for Corn, US Number 2 Yellow, Omaha location. The last reports the 8-week soybean meal price determined each Friday as the average of all daily reported high and low bids or offers per ton for 48% Soybean Meal Rail, Decatur location. We standardize these two prices' measure unit to per pound and sum them up as one fodder price for our analysis.

We obtain the weekly factor prices to match the farm's decision epoch by taking the average of the available daily prices in a week.

A.2.2 Fitting OTC Prices

In this section, we fit the OTC price evolution using a time-series model.

[16]The report excludes Saturdays and Sundays and six holidays (New Year's Day, Memorial Day, Independence Day, Labor Day, Thanksgiving Day, and Christmas Day).

The most general vector Auto-Regressive Integrated Moving Average model, i.e., VARIMA$(p,d,q)\times(P,D,Q)_m$, where p is the order (number of time lags) of the autoregressive model, d is the degree of differencing (the number of times the data have had past values subtracted), q is the order of the moving-average model; m refers to the number of periods in each season, and the uppercase (P,D,Q) refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model [86, 110].

The vector of OTC prices, $\boldsymbol{P}_t = [P_t^C,\, P_t^1,\, P_t^2,\, C_t^H,\, C_t^P]'$, evolves according to the following seasonal VARIMA$(p,d,q)\times(P,D,Q)_m$ (vector-ARIMA) model:

$$\begin{aligned} &\left(\boldsymbol{I}-\sum_{i=1}^{P}\boldsymbol{\Phi}_i B^{i\cdot m}\right)\left(\boldsymbol{I}-\sum_{j=1}^{p}\boldsymbol{\phi}_j B^j\right)(1-B^m)^D(1-B)^d\boldsymbol{P}_t \\ =\;& \boldsymbol{\epsilon}_0+\boldsymbol{\delta}_t+\boldsymbol{\Psi}\boldsymbol{\Pi}_t+\left(\boldsymbol{I}+\sum_{i=1}^{Q}\boldsymbol{\Theta}_i B^{i\cdot m}\right)\left(\boldsymbol{I}+\sum_{j=1}^{q}\boldsymbol{\theta}_j B^j\right)\boldsymbol{\epsilon}_t, \end{aligned} \tag{A1}$$

where $\left(\boldsymbol{I}-\sum_{i=1}^{P}\boldsymbol{\Phi}_i B^{i\cdot m}\right)$ are the *seasonal autoregressive* components of order P, with a (5×5) identity matrix $\boldsymbol{I}$, and a (5×5) coefficient matrix $\boldsymbol{\Phi}_i$; $\left(\boldsymbol{I}-\sum_{j=1}^{p}\boldsymbol{\phi}_j B^j\right)$ are the ordinary *autoregressive* components of order p, with (5×5) coefficient matrix $\boldsymbol{\phi}_j$; B^τ is the back-shift operator with time lag τ, so that $B^\tau\boldsymbol{P}_t = \boldsymbol{P}_{t-\tau}$; $\boldsymbol{\epsilon}_0$ is a five-dimensional constant vector; $\boldsymbol{\delta}_t$ represents a vector of linear time trend; $\boldsymbol{\Pi}_t$ represents a vector of *factor prices* (e.g., pork and fodder prices on the open market); $\boldsymbol{\Psi}$ is the coefficient matrix of exogenous regressors; $\left(\boldsymbol{I}+\sum_{i=1}^{Q}\boldsymbol{\Theta}_i B^{i\cdot m}\right)$ are the *seasonal moving average* components of order Q, with (5×5) coefficient matrix $\boldsymbol{\Theta}_i$; $\left(\boldsymbol{I}+\sum_{j=1}^{q}\boldsymbol{\theta}_j B^j\right)$ are the ordinary *moving average* components of order q, with (5×5) coefficient matrix $\boldsymbol{\theta}_j$; and $\boldsymbol{\epsilon}_t$ is a sequence of five-dimensional independently and identically distributed (iid) multivariate normal random noises with mean zero and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$[17].

[17] We will fit the distribution using our data later.

A.2.3 Fitting Factor Prices

As mentioned in §1.8.1, we need to forecast the factor price first in order to forecast OTC prices. In this section, we estimate the model for factor prices evolution.

We find that the best fit model for factor prices is VARIMA(1,1,1). Specifically,

$$(\boldsymbol{I} - \boldsymbol{\phi}_1 B)(\boldsymbol{\Pi}_{t+1} - \boldsymbol{\Pi}_t) = \boldsymbol{\epsilon}_0 + (\boldsymbol{I} + \boldsymbol{\theta}_1 B)\boldsymbol{\epsilon}_{t+1}, \tag{A2}$$

where $\boldsymbol{e}_{t+1}$ follows a three-dimensional independent and identically distributed normal distribution with mean zero and covariance matrix $\boldsymbol{\Sigma_e}$. In Equation (A2), the coefficients we need to estimate from data are the AR coefficients ($\boldsymbol{\phi}_1$), the vector of constants ($\boldsymbol{\epsilon}_0$), and the MA coefficients ($\boldsymbol{\theta}_1$).

The estimated model (A2) has the following parameters summarized in Table A2. ACF for the residuals of factor prices model is shown in Figure A1. From Figure A1, we find that the residuals are stationary given that ACF of the residuals is within confidence bounds.

Table A2: Estimation of of Factor Prices Using Model VARIMA(1,1,1)

Dep. Var. Regressor	Π_t^O	Π_t^M	Π_t^F
$\boldsymbol{\epsilon}_0$	−0.050	−0.137	0.002
	(0.361)	(0.277)	(0.010)
$\boldsymbol{\theta}_1$	0.621***	0.367**	0.889***
	(0.089)	(0.162)	(0.135)
$\boldsymbol{\phi}_1$	0.525***	0.398**	−0.701***
	(0.097)	(0.184)	(0.185)

Note: Standard errors are given in parentheses under coefficients. * indicates p-value at the 10% level; ** at the 5% level; and ***, at the 1% level.

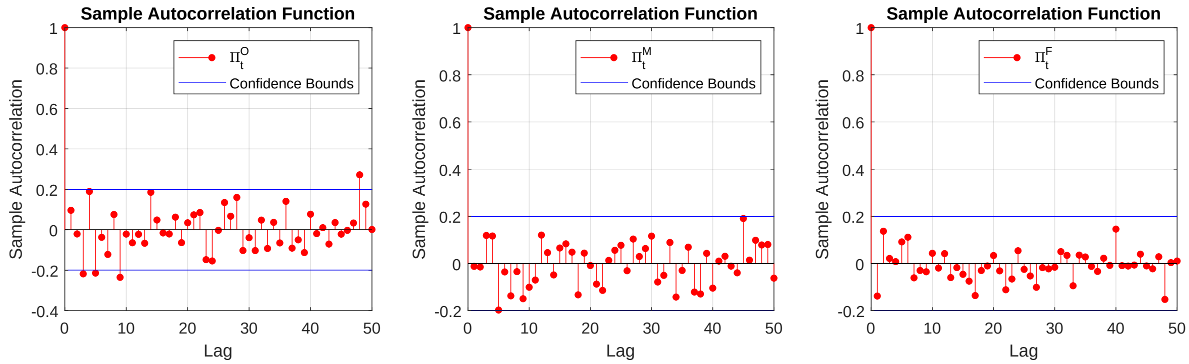


Figure A1: ACF of the Residuals of Factor Prices Using VARIMA(1,1,1) Model.

A.3 Proofs

We present the proofs of the Lemmas and Propositions in Appendix A.3. **Proof of Lemma 1.2**:

Proof. The proof follows that of [46]. In particular, we use an inductive argument to show that $V_t(\boldsymbol{s}_t, \boldsymbol{p}_t)$ is concave in $\boldsymbol{s}_t$ for each $\boldsymbol{p}_t$ and that $v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t)$ is concave in $(\boldsymbol{y}_t,\boldsymbol{z}_t)$ for each $(\boldsymbol{s}_t,\, \boldsymbol{p}_t)$, $t = 1, 2, \ldots, T$.

In the last period, the action-state pair function $v_T(\boldsymbol{y}_T, \boldsymbol{z}_T; \boldsymbol{s}_T, \boldsymbol{p}_T) = r_T(\boldsymbol{y}_T, \boldsymbol{z}_T; \boldsymbol{s}_T, \boldsymbol{p}_t)$ is linear, which is concave in $(\boldsymbol{y}_T, \boldsymbol{x}_T)$ for any state $(\boldsymbol{s}_T, \boldsymbol{p}_T)$. Since the conditions of concavity preservation under maximization of [75] hold at the last period, we find that the value function $V_T(\boldsymbol{s}_T, \boldsymbol{p}_T)$ is concave in $\boldsymbol{s}_T$ for each $\boldsymbol{p}_T$.

Now, suppose at period $t+1$, $t = 1, \cdots, T-1$, the value function V_{t+1} is concave in $\boldsymbol{s}_{t+1}$ for each $\boldsymbol{p}_{t+1}$ and the action-state pair function v_{t+1} is concave in the action $\boldsymbol{a}_{t+1}$ for any state

$\boldsymbol{s}_{t+1}$. Then at period t, the action-state pair function can be rewritten as follows,

$$
\begin{aligned}
& v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t) \\
= \; & p_t^C \left(y_t^2 + (1-\alpha)\, y_t^1\right) + \sum_{i=1}^{2} p_t^i \left(s_t^i - y_t^i - z_t^i\right) - \sum_{i=1}^{2} c_t^H \, z_t^i - c_t^P \left(q - y_t^1 - y_t^2\right)^+ \\
& + \beta \, \mathbb{E}_t V_{t+1} \left(\left[W_{t+1}^1, W_{t+1}^2 + z_t^1 + z_t^2\right], \boldsymbol{P}_{t+1} \right).
\end{aligned}
$$

The first line is the linear combination of $(\boldsymbol{y}_t, \boldsymbol{z}_t)$, which is concave. The second line is also concave because concavity preserves under expectation.

To prove concavity of the value function $V_t(\boldsymbol{s}_t, \boldsymbol{p}_t)$, we also need to check the conditions of concavity preservation under maximization of [75]. 1) Given the the state $(\boldsymbol{s}_t, \boldsymbol{p}_t)$, the feasible action space is $\{(\boldsymbol{y}_t, \boldsymbol{z}_t) | \boldsymbol{y}_t + \boldsymbol{z}_t \leq \boldsymbol{s}_t\}$, which is a convex set. 2) For any $\boldsymbol{s}_t$, the feasible set is nonempty because $\boldsymbol{y}_t = \boldsymbol{z}_t = \boldsymbol{0}$ is always a feasible action. Hence, the conditions hold and $V_t(\boldsymbol{s}_t, \boldsymbol{p}_t)$ is concave in $\boldsymbol{s}_t$ for any $\boldsymbol{p}_t$, which completes the proof. □

Proof of Lemma 1.3:

Proof. From Lemma 1.2, the value function $v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \cdot)$ is concave in $\boldsymbol{z}_t$, $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^i}, i = 1, 2$ is decreasing in z_t^i, hence there exist an upper bound and a lower bound. In addition, from equations 1.5a and 1.5b, $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^i} \in [\underline{p}_{t+1}^H, \bar{p}_{t+1}^H], i = 1, 2$. $\underline{z}_t^1$ exists if $(1-\alpha) p_t^C + c_t^P \in [\underline{p}_{t+1}^H - c_t^H, \bar{p}_{t+1}^H - c_t^H]$; else if $(1-\alpha) p_t^C + c_t^P < \beta \underline{p}_{t+1}^H - c_t^H$, then the firm holds all under-weight hogs, i.e., $\underline{z}_t^1 = \infty$; else if $(1-\alpha) p_t^C + c_t^P > \beta \bar{p}_{t+1}^H - c_t^H$, then $\underline{z}_t^1 = 0$. Note that $z_t^2 = 0$ from Assumption 1.1. □

Proof of Lemma 1.4:

Proof. Analogous to the proof of Lemma 1.3, the value function $v_t(\boldsymbol{y}_t, \boldsymbol{z}_t; \cdot)$ is concave in $\boldsymbol{z}_t$ from Lemma 1.2, hence $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^i}, i = 1, 2$ is decreasing in z_t^i, hence there exist an upper bound and a lower bound. From equations 1.5a and 1.5b, $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^i} \in [\underline{p}_{t+1}^H, \bar{p}_{t+1}^H], i = 1, 2$. If $p_t^2 < \beta \bar{p}_{t+1}^H - c_t^H$, there exists $\bar{z}_t^2$; If $p_t^2 \geq \beta \bar{p}_{t+1}^H - c_t^H$, then $\bar{z}_t^2 = 0$. $\bar{z}_t^1$ exists if $p_t^1 \in \left[\beta \underline{p}_{t+1}^H - c_t^H, \beta \bar{p}_{t+1}^H - c_t^H\right]$; else if $p_t^1 < \beta \underline{p}_{t+1}^H - c_t^H$, then $\bar{z}_t^1 = \infty$. Else if $p_t^1 > \beta \bar{p}_{t+1}^H - c_t^H$, then $\bar{z}_t^1 = 0$. Note that, if $(1-\alpha) p_t^C + c_t^P > p_t^1$, $\bar{z}_t^1 > \underline{z}_t^1$; if $(1-\alpha) p_t^C + c_t^P < p_t^1$, $\bar{z}_t^1 < \underline{z}_t^1$. □

Proof of Proposition 1.1:

Proof. For the notational convenience, let $\boldsymbol{z}_t := [z_t^1, z_t^2]$ define the quantity vector of two types of hogs holding to the next period at any period t, i.e., $z_t^i = s_t^i - x_t^i - y_t^i$, $i = 1, 2$. Then the sub-problem at any period $t = 1, \cdots, T-1$ can be formulated as follows,

$$\begin{aligned} V_t(\boldsymbol{s}_t, \boldsymbol{p}_t) \quad = \quad & \max_{\boldsymbol{y}_t, \boldsymbol{x}_t, \boldsymbol{z}_t} \; p_t^C \left(y_t^2 + (1-\alpha)\, y_t^1\right) + \sum_{i=1}^{2} p_t^i\, x_t^i - \sum_{i=1}^{2} c_t^H z_t^i - c_t^P \left(q - y_t^1 - y_t^2\right) \\ & + \beta\, \mathbb{E}_t V_{t+1} \left(\left[w_{t+1}^1, w_{t+1}^2 + \sum_{i=1}^{2} \left(s_t^i - x_t^i - y_t^i\right) \right], \boldsymbol{p}_{t+1} \right), \\ \text{s.t.} \quad & \boldsymbol{x}_t + \boldsymbol{y}_t + \boldsymbol{z}_t = \boldsymbol{s}_t, \\ & y_t^1 + y_t^2 \leq q, \\ & \boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t \geq 0. \end{aligned}$$

Lemma A.1 (Optimal Policy of the Last Period)**.** *At the last period T, the optimal end-up strategy is,*

1. *when the open market dominates $p_T^2 > p_T^C + c_T^P$, 1) if $p_T^1 > (1-\alpha) p_T^C + c_T^P$, the optimal strategy is ${y_T^1}^* = {y_T^2}^* = 0$, ${x_T^1}^* = s_T^1$, and ${x_T^2}^* = s_T^2$. 2) Otherwise, the optimal strategy is ${y_T^1}^* = s_T^1 \wedge q$, ${x_T^1}^* = (s_t^1 - q)^+$, ${y_T^2}^* = 0$, and ${x_T^2}^* = s_T^2$.*

2. *when the contract market dominates* $p_T^C + c_T^P \geq p_T^2$*, 1) if* $p_T^1 > (1-\alpha)p_T^C + c_T^P$*, the optimal strategy is* ${y_T^1}^* = 0$, ${x_T^1}^* = s_T^1$, ${y_T^2}^* = s_T^2 \wedge q$*, and* ${x_T^2}^* = (s_T^2 - q)^+$*. 2) Else if* $0 \leq (1-\alpha)p_T^C + c_T^P - p_T^1 \leq p_T^C + c_T^P - p_T^2$*, the optimal solution is* ${y_T^1}^* = s_T^1 \wedge (q - s_T^2)^+$, ${x_T^1}^* = s_T^1 - {y_T^1}^*$, ${y_T^2}^* = s_T^2 \wedge q$*, and* ${x_T^2}^* = (s_T^2 - q)^+$*. 3) Else, the optimal strategy is* ${y_T^1}^* = s_T^1 \wedge q$, ${x_T^1}^* = (s_T^1 - q)^+$, ${y_T^2}^* = s_T^2 \wedge (q - s_T^1)^+$*, and* ${x_T^2}^* = (s_T^2 - {y_T^2}^*)^+$*.*

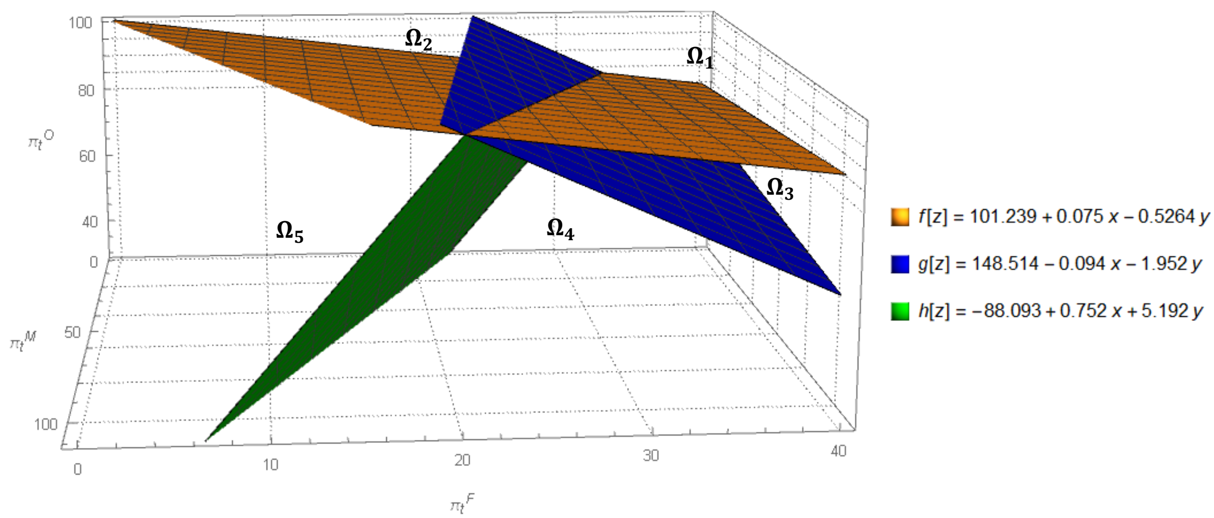


Figure A2: (Color Online) Optimal Decisions for the Last Period

Figure A2 provides insights for the optimal policy of the last period numerically using the data from the Maschhoffs. We normalized the factor prices to the unit of per hog, i.e., the fodder price π^F represents each hog's feeding cost. The price range for the open market is from \$38 to \$89 per hundred pounds, the price range for the pork market is from \$60 to \$105 per hundred pounds, and the price range for the fodder market is from \$21 to \$26 per bush. Each region denotes the decision area (corresponding to optimal decisions summarized in Table 1.4) for the hog farm depending on the factor prices. Regions 4 is more commonly observed, meaning that the decision maker will use the regular-weight hogs to fulfill the contract first, then use the under-weight hogs at most of the time. Under this region, only excess hogs sold to the open market. As is shown in region 5, the decision maker will use the under-weight hog to fulfill the contract first when the pork market and the fodder market price are relatively low. Region 5 expands when the open market price increases. Note that the decision maker is more likely to sell the hogs to the open market only under the extreme case where the open market price is very high. Both types of hogs will go to the open market when three prices are very high. These five regions reflect the current practice where the hog farm does not consider the value of holding.

Lemma A.1 can be proved as follows. Since there is no benefit for holding, the last period sub-problem is formulated as follows,

$$\begin{aligned}\max_{\boldsymbol{y}_T,\boldsymbol{x}_T} \quad & r_T(\boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t),\\ \text{s.t.} \quad & \boldsymbol{x}_T + \boldsymbol{y}_T = \boldsymbol{s}_T,\\ & y_T^1 + y_T^2 \leq q,\\ & \boldsymbol{x}_T, \boldsymbol{y}_T \geq 0.\end{aligned}$$

By introducing Lagrangian multipliers to each constraint, the Lagrangian function denoted by $\mathcal{L}_T : \mathbb{R}^4 \times \mathbb{R}^7 \to \mathbb{R}$ for the problem shown above takes the following form,

$$\begin{aligned}\mathcal{L}_T(\boldsymbol{x}_T, \boldsymbol{y}_T, \boldsymbol{\lambda}_T) : \;=\; & p_T^C \left(y_T^2 + (1-\alpha)\, y_T^1\right) + \sum_{i=1}^{2} p_T^i\, x_T^i - c_T^P \left(q - y_T^1 - y_T^2\right) + \lambda_0(s_T^1 - x_T^1 - y_T^1)\\ & + \lambda_1(s_T^2 - x_T^2 - y_T^2) + \lambda_2(q - y_T^1 - y_T^2) + \lambda_3 x_T^1 + \lambda_4 x_T^2 + \lambda_5 y_T^1 + \lambda_6 y_T^2.\end{aligned}$$

where $\boldsymbol{\lambda}_T = [\lambda_T^0, \lambda_T^1, \lambda_T^2, \lambda_T^3, \lambda_T^4, \lambda_T^5, \lambda_T^6]$ is the Lagrangian multiplier vector. It is easy to verify that the objective function r_T is jointly concave in (x_T, y_T) and the constraints are linear, we can find the KKT conditions are sufficient and necessary for the optimal solution as follows.

$$\frac{\partial \mathcal{L}_T}{\partial x_T^1} = p_T^1 - \lambda_0 + \lambda_3 = 0, \tag{A3a}$$

$$\frac{\partial \mathcal{L}_T}{\partial x_T^2} = p_T^2 - \lambda_1 + \lambda_4 = 0, \tag{A3b}$$

$$\frac{\partial \mathcal{L}_T}{\partial y_T^1} = (1-\alpha)p_T^C + c_T^P - \lambda_0 - \lambda_2 + \lambda_5 = 0, \tag{A3c}$$

$$\frac{\partial \mathcal{L}_T}{\partial y_T^2} = p_T^C + c_T^P - \lambda_1 - \lambda_2 + \lambda_6 = 0, \tag{A3d}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_0} = s_T^1 - x_T^1 - y_T^1 = 0, \tag{A3e}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_1} = s_T^2 - x_T^2 - y_T^2 = 0, \tag{A3f}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_2} = q - y_T^1 - y_T^2 \geq 0,\ \ \lambda_2 \geq 0,\ \ \lambda_2(q - y_T^1 - y_T^2) = 0 \tag{A3g}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_3} = x_T^1 \geq 0, \lambda_3 \geq 0,\ \ \lambda_3 x_T^1 = 0 \tag{A3h}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_4} = x_T^2 \geq 0, \lambda_4 \geq 0,\ \ \lambda_4 x_T^2 = 0 \tag{A3i}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_5} = y_T^1 \geq 0, \lambda_5 \geq 0,\ \ \lambda_5 y_T^1 = 0 \tag{A3j}$$

$$\frac{\partial \mathcal{L}_T}{\partial \lambda_6} = y_T^2 \geq 0, \lambda_6 \geq 0,\ \ \lambda_6 y_T^2 = 0. \tag{A3k}$$

We then discussing different cases indicated by the complementary slackness where each Lagrangian multiplier is zero or strongly positive.

From Equations (A3a) and (A3c), and Equations (A3b) and (A3d) we have $\lambda_2 + \lambda_4 - \lambda_6 = p_T^C + c_T^P - p_T^2$, and $\lambda_2 + \lambda_3 - \lambda_5 = (1-\alpha)p_T^C + c_T^P - p_T^1$.

Case 1a: If $\lambda_2 + \lambda_4 - \lambda_6 = p_T^C + c_T^P - p_T^2 < 0$, $\lambda_2 + \lambda_3 - \lambda_5 = (1-\alpha)p_T^C + c_t^P - p_t^1 < 0$, we have $\lambda_5 > 0$ and $\lambda_6 > 0$, hence, ${y_T^1}^* = {y_T^2}^* = 0$. From Equations (A3e) and (A3f), ${x_T^1}^* = s_T^1$, and ${x_T^2}^* = s_T^2$.

Case 1b: If $\lambda_2 + \lambda_4 - \lambda_6 = p_T^C + c_T^P - p_T^2 < 0$, $\lambda_2 + \lambda_3 - \lambda_5 = (1-\alpha)p_T^C + c_T^P - p_T^1 > 0$, we have $\lambda_6 > 0$, hence, ${y_T^2}^* = 0$, and ${x_T^2}^* = s_T^2$. Since $p_T^1 < (1-\alpha)p_T^C + c_T^P$, selling to contract will be more lucrative, hence, ${y_T^1}^* = s_T^1 \wedge q$, ${x_T^1}^* = (s_t^1 - q)^+$.

Case 2a: If $\lambda_2 + \lambda_4 - \lambda_6 = p_T^C + c_T^P - p_T^2 > 0$, $\lambda_2 + \lambda_3 - \lambda_5 = (1-\alpha)p_T^C + c_T^P - p_T^1 < 0$, we have $\lambda_5 > 0$, hence, ${y_T^1}^* = 0$, ${x_T^1}^* = s_T^1$. Since $p_T^C + c_T^P > p_T^2$, i.e., it is more lucrative to sell regular-weight hogs to contracts, hence, we have ${y_T^2}^* = s_T^2 \wedge q$, and ${x_T^2}^* = (s_T^2 - q)^+$.

Case 2b: If $0 \leq (1-\alpha)p_T^C + c_T^P - p_T^1 \leq p_T^C + c_T^P - p_T^2$, the marginal value for a regular-weight hog to fulfill the contract is higher than that for a under-weight one. Hence, fulfilling the contract using regular-weight hogs first will be more profitable. Therefore, the optimal solution is ${y_T^2}^* = s_T^2 \wedge q$, and ${x_T^2}^* = (s_T^2 - q)^+$, ${y_T^1}^* = s_T^1 \wedge (q - s_T^2)^+$, ${x_T^1}^* = s_T^1 - {y_T^1}^*$.

Case 2c: If $0 \leq p_T^C + c_T^P - p_T^2 \leq (1-\alpha)p_T^C + c_T^P - p_T^1$, the marginal value for a under-weight hog to fulfill the contract is higher than that for a regular-weight one. Hence, fulfilling the contract using under-weight hogs first will be more profitable. Therefore, the optimal solution is ${y_T^1}^* = s_T^1 \wedge q$, ${x_T^1}^* = (s_T^1 - q)^+$, ${y_T^2}^* = s_T^2 \wedge (q - s_T^1)^+$, and ${x_T^2}^* = (s_T^2 - {y_T^2}^*)^+$. We finished the proof.

Step 1: Preservation

From the Proof of Lemma 1.2, the following statements hold.

1. $V_t([w_t^1, w_t^2 + z_{t-1}^1 + z_{t-1}^2], \boldsymbol{p}_t)$ is concave in $\boldsymbol{z}_{t-1}$.

2. $v_t(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t; \boldsymbol{s}_t, \boldsymbol{p}_t)$ is concave in $(\boldsymbol{x}_t,\ \boldsymbol{y}_t,\ \boldsymbol{z}_t)$ for each given $(\boldsymbol{s}_t, \boldsymbol{p}_t)$.

Step 2: Attainment

Let $L_t : \mathbb{R}^6 \times \mathbb{R}^9 \to \mathbb{R}$ denote the Lagrangian, which is defined by:

$$\begin{aligned}
\mathcal{L}_t(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) : \quad = \quad & v_t(\boldsymbol{s}_t, \boldsymbol{p}_t; \boldsymbol{y}_t, \boldsymbol{x}_t) + \lambda_0 \left(s_t^1 - (x_t^1 + y_t^1 + z_t^1)\right) + \lambda_1 \left(s_t^2 - (x_t^2 + y_t^2 + z_t^2)\right) \\
& + \lambda_2 \left(q - (y_t^1 + y_t^2)\right) + \lambda_3 \left(s_t^1 - (x_t^1 + y_t^1)\right) + \lambda_4 \left(s_t^2 - (x_t^2 + y_t^2)\right) + \lambda_5 x_t^1 \\
& + \lambda_6 x_t^2 + \lambda_7 y_t^1 + \lambda_8 y_t^2,
\end{aligned}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^9$ is the Lagrangian multiplier vector. From Step 1, we know that $v_t(\boldsymbol{s}_t, \boldsymbol{p}_t; \boldsymbol{x}_t, \boldsymbol{y}_t)$ is concave in $(\boldsymbol{x}_t,\ \boldsymbol{y}_t,\ \boldsymbol{z}_t)$ for given $(\boldsymbol{s}_t, \boldsymbol{p}_t)$, V_{t+1} is concave in $\boldsymbol{z}_t$. Since the constraints are linear, we can find the optimal solutions by applying sufficient and necessary KKT conditions. Hence, we proceed to find solutions $(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda})$ to the following set of equations:

$$\frac{\partial L}{\partial x_t^1}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = p_t^1 - \lambda_0 - \lambda_3 + \lambda_5 = 0, \tag{A4a}$$

$$\frac{\partial L}{\partial x_t^2}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = p_t^2 - \lambda_1 - \lambda_4 + \lambda_6 = 0, \tag{A4b}$$

$$\frac{\partial L}{\partial y_t^1}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = (1-\alpha)p_t^C + +c_t^P - \lambda_0 - \lambda_2 - \lambda_3 + \lambda_7 = 0, \tag{A4c}$$

$$\frac{\partial L}{\partial y_t^2}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = p_t^C + c_t^P - \lambda_1 - \lambda_2 - \lambda_4 + \lambda_8 = 0, \tag{A4d}$$

$$\frac{\partial L}{\partial z_t^1}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = -c_t^H + \frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^1} - \lambda_0 = 0, \tag{A4e}$$

$$\frac{\partial L}{\partial z_t^2}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = -c_t^H + \frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^2} - \lambda_1 = 0, \tag{A4f}$$

$$\frac{\partial L}{\partial \lambda_0}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = s_t^1 - (x_t^1 + y_t^1 + z_t^1) = 0, \tag{A4g}$$

$$\frac{\partial L}{\partial \lambda_1}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = s_t^2 - (x_t^2 + y_t^2 + z_t^2) = 0, \tag{A4h}$$

$$\frac{\partial L}{\partial \lambda_2}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = q - (y_t^1 + y_t^2) \geq 0,\ \ \lambda_2 \geq 0,\ \ \lambda_2\left[q - (y_t^1 + y_t^2)\right] = 0, \tag{A4i}$$

$$\frac{\partial L}{\partial \lambda_3}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = s_t^1 - (x_t^1 + y_t^1) \geq 0,\ \ \lambda_3 \geq 0,\ \ \lambda_3\left[s_t^1 - (x_t^1 + y_t^1)\right] = 0, \tag{A4j}$$

$$\frac{\partial L}{\partial \lambda_4}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = s_t^2 - (x_t^2 + y_t^2) \geq 0,\ \ \lambda_4 \geq 0,\ \ \lambda_4\left[s_t^2 - (x_t^2 + y_t^2)\right] = 0, \tag{A4k}$$

$$\frac{\partial L}{\partial \lambda_5}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = x_t^1 \geq 0,\ \ \lambda_5 \geq 0,\ \ \lambda_5 x_t^1 = 0, \tag{A4l}$$

$$\frac{\partial L}{\partial \lambda_6}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = x_t^2 \geq 0,\ \ \lambda_6 \geq 0,\ \ \lambda_6 x_t^2 = 0, \tag{A4m}$$

$$\frac{\partial L}{\partial \lambda_7}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = y_t^1 \geq 0,\ \ \lambda_7 \geq 0,\ \ \lambda_7 y_t^1 = 0, \tag{A4n}$$

$$\frac{\partial L}{\partial \lambda_8}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t, \boldsymbol{\lambda}) = y_t^2 \geq 0,\ \ \lambda_8 \geq 0,\ \ \lambda_8 y_t^2 = 0. \tag{A4o}$$

From Equations (A4b) and (A4f) we know that $-\lambda_4 + \lambda_6 = -c_t^H - p_t^2 + \frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^2}$, and let the solution to the equation denote as $\bar{z}_t^2$. Since $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^2} \in \left[\underline{p}_{t+1}^H, \bar{p}_{t+1}^H\right]$ (proved later), $\bar{z}_t^2$ exists iff $p_t^2 < \bar{p}_{t+1}^H - c_t^H$; if $p_t^2 > \bar{p}_{t+1}^H - c_t^H$, $\bar{z}_t^2 = 0$;

From Equations (A4d) and (A4f) we know that $-\lambda_2 - \lambda_4 + \lambda_8 = -c_t^H - p_t^C - c_t^P + \frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^2}$, and let the solution to the equation denote as $\underline{z}_t^2$. Since $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^1} \in \left[\underline{p}_{t+1}^H, \bar{p}_{t+1}^H\right]$, $\underline{z}_t^2 = 0$ if $c_t^H + p_t^C + c_t^P > \bar{p}_{t+1}^H$;

From Equations (A4a) and (A4e) we know that $-\lambda_3 + \lambda_5 = -c_t^H - p_t^1 + \frac{\partial V_{t+1}}{\partial \mathbb{E}_t z_t^1}$, and let the solution to the equation denote as $\bar{z}_t^1$. Since $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^1} \in [\underline{p}_{t+1}^H, \bar{p}_{t+1}^H]$, $\bar{z}_t^1$ exists if $p_t^1 \in [\underline{p}_{t+1}^H - c_t^H, \bar{p}_{t+1}^H - c_t^H]$; else if $p_t^1 < (\underline{p}_{t+1}^H - c_t^H) \wedge (\bar{p}_{t+1}^H - c_t^H)$, $\bar{z}_t^1 = \infty$; else if $p_t^1 > (\underline{p}_{t+1}^H - c_t^H) \vee (\bar{p}_{t+1}^H - c_t^H)$, $\bar{z}_t^1 = 0$;

From Equations (A4c) and (A4e) we know that $-\lambda_2 - \lambda_3 + \lambda_7 = -c_t^H - (1-\alpha)p_t^C - c_t^P + \frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^1}$, and let the solution to the equation denote as $\underline{z}_t^1$. Since $\frac{\partial \mathbb{E}_t V_{t+1}}{\partial z_t^1} \in [\underline{p}_{t+1}^H, \bar{p}_{t+1}^H]$, $\underline{z}_t^1$ exists from Lemma 1.4.

From Equations (A4a) and (A4c), and Equations (A4b) and (A4d) we have $\lambda_6 + \lambda_2 - \lambda_8 = p_t^C + c_t^P - p_t^2$, and $\lambda_5 + \lambda_2 - \lambda_7 = (1-\alpha)p_t^C + c_t^P - p_t^1$.

Case 1a: If $\lambda_6 + \lambda_2 - \lambda_8 = p_t^C + c_t^P - p_t^2 < 0$, $\lambda_5 + \lambda_2 - \lambda_7 = (1-\alpha)p_t^C + c_t^P - p_t^1 < 0$. In this case, $p_t^2 > p_t^C + c_t^P$, from Lemma 1.4, $\bar{z}_t^2 = 0$. Since $\lambda_7 > 0$ and $\lambda_8 > 0$, we have $y_t^1 = y_t^2 = 0$. Therefore, $x_t^2 = s_t^2$. Since $p_t^1 > (1-\alpha)p_t^C + c_t^P$, $z_t = \bar{z}_t^1$, we have $x_t^1 = (s_t^1 - \bar{z}_t^1)^+$. Other parameters can be obtained accordingly. See Figure 1.4 for illustration.

Case 1b: If $\lambda_6 + \lambda_2 - \lambda_8 = p_t^C + c_t^P - p_t^2 < 0$, $\lambda_5 + \lambda_2 - \lambda_7 = (1-\alpha)p_t^C + c_t^P - p_t^1 > 0$. In this case $p_t^2 > p_t^C + c_t^P$, hence $\bar{z}_t^2 = 0$, $x_t^2 = s_t^2$. We still have $\lambda_8 > 0$, hence, $y_t^2 = 0$. Since $1-\alpha)p_t^C + c_t^P > p_t^1$, which means the marginal value of selling an under-weight hog to comtracts profits more than selling them to OM, then there exist $\bar{z}_t^1$ and $\underline{z}_t^1$, $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge q$,

$x_t^1 = (s_t^1 - \bar{z}_t^1 - q)^+$, $y_t^2 = 0$. Other parameters can be solved accordingly. See Figure 1.5 for illustration.

Case 2a: If $\lambda_6 + \lambda_2 - \lambda_8 = p_t^C + c_t^P - p_t^2 > 0$, $\lambda_5 + \lambda_2 - \lambda_7 = (1-\alpha)p_t^C + c_t^P - p_t^1 < 0$. In this case $p_t^2 < p_t^C + c_t^P$, there exists $\bar{z}_t^2$. Hence, $y_t^2 = s_t^2 \wedge q$, and $x_t^2 = (s_t^2 - q - \bar{z}^2)^+$. Since $\lambda_7 > 0$, we have $y_t^1 = 0$, $x_t^1 = (s_t^1 - \bar{z}_t^1)^+$. Other parameters can be solved accordingly. See Figure 1.6 for illustration.

Case 2b: If $0 \leq (1-\alpha)p_t^C + c_t^P - p_t^1 \leq p_t^C + c_t^P - p_t^2$, this means the marginal value difference between fulfilling the contract and selling to the OM for a regular-weight hog is greater than that of an under-weight hog. Moreover, after rearranging the terms, we have $0 \leq (1-\alpha)p_t^C + c_t^P + p_t^2 \leq p_t^C + c_t^P + p_t^1$, this means using one regular-weight hog to fulfill the contract and one under-weight hog to sell to the open market is more profitable than using one under-weight hog to sell to the contract and one regular-weight hog to sell on the open market. Therefore, the farm will use the regular-weight hog to fulfill the contract first, then the under-weight hog (if there is a contractual shortfall.) Since $p_t^C + c_t^P > p_t^2$, there exists $\bar{z}_t^2$, so $y_t^2 = s_t^2 \wedge q$, and $x_t^2 = (s_t^2 - q - \bar{z}_t^2)^+$. Since $p_t^1 \leq (1-\alpha)p_t^C + c_t^P$, there exist $\bar{z}_t^1, \underline{z}_t^1$, so $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge (q - s_t^2)^+$, $x_t^1 = (s_t^1 - y_t^1 - \bar{z}_t^1)^+$.Other parameters can be solved accordingly. See Figure 1.7 for illustration.

Case 2c: If $0 \leq p_t^C + c_t^P - p_t^2 \leq (1-\alpha)p_t^C + c_t^P - p_t^1$, this means the marginal value difference between fulfilling the contract and selling to the OM for an under-weight hog is greater than that of an regular-weight hog. Moreover, after rearranging the terms, we have $0 \leq p_t^C + c_t^P + p_t^1 \leq (1-\alpha)p_t^C + c_t^P + p_t^2$, this means using one under-weight hog to fulfill the contract and one under-weight hog to sell to the open market is more profitable than using one regular-weight hog to sell to the contract and one regular-weight hog to sell on the open market. Therefore, the farm will use the under-weight hog to fulfill the contract first, then the regular-weight hog (if there is a contractual shortfall.) Since $p_t^C + c_t^P > p_t^2$, there exists $\bar{z}_t^2$, so $y_t^2 = s_t^2 \wedge (q - (s_t^1 - \underline{z}_t^1)^+)^+$, and $x_t^2 = (s_t^2 - y_t^2 - \bar{z}_t^2)^+$. Since $p_t^1 \leq (1-\alpha)p_t^C + c_t^P$, there exist $\bar{z}_t^1, \underline{z}_t^1$, so $y_t^1 = (s_t^1 - \underline{z}_t^1)^+ \wedge q$, $x_t^1 = (s_t^1 - q - \bar{z}_t^1)^+$. Other parameters can be solved accordingly. See Figure 1.8 for illustration.

Therefore, by Step 1 (preservation) and Step 2 (attainment), we complete the proof. □

Proof of Proposition 1.2:

Proof. We formulate a two-period subproblem. The second stage is to maximize the second period expected profit given the forecast $\hat{\boldsymbol{P}}_{t+1}$,

$$\max_{\boldsymbol{y},\boldsymbol{z}} \mathbb{E}_t \left[\hat{P}_{t+1}^C (y^2 + (1-\alpha)y^1) + \sum_{i=1}^{2} \hat{P}_{t+1}^i (S_{t+1}^i - y^i - z^i) - \sum_{i=1}^{2} \hat{C}_{t+1}^H z^i - \hat{C}_{t+1}^P (q - y^1 - y^2) \right]$$

$$\text{s.t.} \quad y^i + z^i \leq S_{t+1}^i \quad \forall i = 1, 2$$
$$y^1 + y^2 \leq q.$$

We then summarize the optimal solution, the value function, and the marginal revenue of a unit of stock as follows for each cases:

- Case 1a: The optimal solution set is, $y^i = z^i = 0$, the value function is, $\sum_{i=1}^2 \hat{P}_{t+1}^i S_{t+1}^i - \hat{C}_{t+1}^P q$, and $\frac{\partial \mathbb{E}_t[V_{t+1}]}{\partial S_{t+1}^2} = \hat{P}_{t+1}^2$.
- Case 1b: The optimal solution set is, $y^1 = S_{t+1}^1 \wedge q$, $y^2 = z^i = 0$, the value function is, $\hat{P}_{t+1}^C (1-\alpha)(S_{t+1}^1 \wedge q) + \hat{P}_{t+1}^1 (S_{t+1}^1 - q)^+ + \hat{P}_{t+1}^2 S_{t+1}^2 - \hat{C}_{t+1}^P (q - S_{t+1}^1)^+$, and $\frac{\partial \mathbb{E}_t[V_{t+1}]}{\partial S_{t+1}^2} = \hat{P}_{t+1}^2$.
- Case 2a: The optimal solution set is, $y^2 = S_{t+1}^2 \wedge q$, $y^1 = z^i = 0$, the value function is, $\hat{P}_{t+1}^C (S_{t+1}^2 \wedge q) + \hat{P}_{t+1}^2 (S_{t+1}^2 - q)^+ + \hat{P}_{t+1}^1 S_{t+1}^1 - \hat{C}_{t+1}^P (q - S_{t+1}^2)^+$, and $\frac{\partial \mathbb{E}_t[V_{t+1}]}{\partial S_{t+1}^2} = \mathbf{1}_{\{S_{t+1}^2 < q\}} (\hat{P}_{t+1}^C + \hat{C}_{t+1}^P) + \mathbf{1}_{\{S_{t+1}^2 \geq q\}} \hat{P}_{t+1}^2$.

- Case 2b: The optimal solution set is, $y^1 = S^1_{t+1} \wedge (q - S^2_{t+1})^+$, $y^2 = S^2_{t+1} \wedge q$, $z^i = 0$, the value function is, $\hat{P}^C_{t+1}[(S^2_{t+1} \wedge q) + (1-\alpha)(S^1_{t+1} \wedge (q - S^2_{t+1})^+)] + \hat{P}^2_{t+1}(S^2_{t+1} - q)^+ + \hat{P}^1_{t+1}[S^1_{t+1} - (q - S^2_{t+1})^+]^+ - \hat{C}^P_{t+1}(q - S^1_{t+1} - S^2_{t+1})^+$, and $\frac{\partial \mathbb{E}_t[V_{t+1}]}{\partial S^2_{t+1}} = \mathbf{1}_{\{S^1_{t+1}+S^2_{t+1}<q\}}(\hat{P}^C_{t+1} + \hat{C}^P_{t+1}) + \mathbf{1}_{\{S^2_{t+1}<q\leq S^1_{t+1}+S^2_{t+1}\}}(\alpha \hat{P}^C_{t+1} + \hat{P}^1_{t+1}) + \mathbf{1}_{\{S^2_{t+1}\geq q\}}\hat{P}^2_{t+1}$.

- Case 2c: The optimal solution set is, $y^1 = S^1_{t+1} \wedge q$, $y^2 = S^2_{t+1} \wedge (q - S^1_{t+1})^+$, $z^i = 0$, the value function is, $\hat{P}^C_{t+1}[(S^2_{t+1} \wedge (q - S^1_{t+1})^+) + (1-\alpha)(S^1_{t+1} \wedge q)] + \hat{P}^1_{t+1}(S^1_{t+1} - q)^+ + \hat{P}^2_{t+1}[S^2_{t+1} - (q - S^1_{t+1})^+]^+ - \hat{C}^P_{t+1}(q - S^1_{t+1} - S^2_{t+1})^+$, and $\frac{\partial \mathbb{E}_t[V_{t+1}]}{\partial S^2_{t+1}} = \mathbf{1}_{\{S^1_{t+1}+S^2_{t+1}<q\}}(\hat{P}^C_{t+1} + \hat{C}^P_{t+1}) + \mathbf{1}_{\{S^2_{t+1}\geq q\}}\hat{P}^2_{t+1}$.

Combined with the first period-problem, we have that $\boldsymbol{y}^{OL}_t$ and $\boldsymbol{z}^{OL}_t$ have the same forms revealed in Proposition 1.1. And the break-even thresholds have following cases,

- If the forecast price of next period $\hat{\boldsymbol{P}}_{t+1}$ is realized in Case 1a and 1b, the farmer sells regular-weight hogs to OM. No need hold for shortage, and we have $\beta(\hat{P}^C_{t+1} + \hat{C}^P_{t+1}) - c^H_t - (1-\alpha)p^C_t - c^P_t \leq 0$ and $\beta(\hat{P}^C_{t+1} + \hat{C}^P_{t+1}) - c^H_t - p^i_t \leq 0$. Then $\underline{z}^i_t = \bar{z}^i_t = 0$.

- If the forecast price of next period $\hat{\boldsymbol{P}}_{t+1}$ is realized in Case 2a, we have $G_{t+1}(\xi) = G^{\mathrm{I}}_{t+1}$.

- If the forecast price of next period $\hat{\boldsymbol{P}}_{t+1}$ is realized in Case 2b, we have $\underline{z}^1_t$ and $\bar{z}^i_t$ solve the following FOCs respectively,

$$\mathbb{P}_w(W^1_t + W^2_t < q - z)\beta(\hat{P}^C_{t+1} + \hat{C}^P_{t+1}) + \mathbb{P}_w(W^2_t < q - z \leq W^1_t + W^2_t)\beta(\alpha\hat{P}^C_{t+1} + \hat{P}^1_{t+1})$$
$$+\mathbb{P}_w(W^2_t \geq q - z)\beta\hat{P}^2_{t+1} = c^H_t + (1-\alpha)p^C_t + c^P_t,$$
$$\mathbb{P}_w(W^1_t + W^2_t < q - z)\beta(\hat{P}^C_{t+1} + \hat{C}^P_{t+1}) + \mathbb{P}_w(W^2_t < q - z \leq W^1_t + W^2_t)\beta(\alpha\hat{P}^C_{t+1} + \hat{P}^1_{t+1})$$
$$+\mathbb{P}_w(W^2_t \geq q - z)\beta\hat{P}^2_{t+1} = c^H_t + p^i_t,$$

Therefore $G^{\mathrm{II}}_{t+1} \leq G_{t+1}(\xi) \leq G^{\mathrm{I}}_{t+1}$.

- If the forecast price of next period $\hat{\boldsymbol{P}}_{t+1}$ is realized in Case 2c, we have $G_{t+1}(\xi) = G^{\mathrm{II}}_{t+1}$.

□

Proof of Proposition 1.3:

Proof. By Lemma 1.5, the optimal supply decision exists and unique when the FOC holds. We have the the value function's first-order partial derivative with regard to u_t as follows,

$$\frac{\partial v_t}{\partial u_t} = \beta \frac{\partial \mathbb{E}_t V_{t+1}(\boldsymbol{S}_{t+1}, \boldsymbol{u}_{t+1}, \boldsymbol{P}_{t+1})}{\partial u_t} - c_t = \beta^{\tau} \frac{\partial \mathbb{E}_t V_{t+\tau}(\boldsymbol{S}_{t+\tau}, \boldsymbol{u}_{t+\tau}, \boldsymbol{P}_{t+\tau})}{\partial u_t} - c_t.$$

We find that without the expectation of the $t+\tau$ based on the forecast from the current period, the first-order partial derivative of the value function is as follows,

$$\begin{aligned} & \frac{\partial V_{t+\tau}(\boldsymbol{S}_{t+\tau}, \boldsymbol{u}_{t+\tau}, \boldsymbol{P}_{t+\tau})}{\partial u_t} \\ = & \left[(1-\alpha)p^C_{t+\tau} + c^P_{t+\tau} - p^1_{t+\tau}\right] \bigtriangledown_{u_t} y^1_{t+\tau} + (p^C_{t+\tau} + c^P_{t+\tau} - p^2_{t+\tau}) \bigtriangledown_{u_t} y^2_{t+\tau} \\ & - \sum_{i=1}^{2} (p^i_{t+\tau} + c^H_{t+\tau}) \bigtriangledown_{u_t} z^i_{t+\tau} + \sum_{i=1}^{2} p^i_{t+\tau} \gamma^i_{t+\tau} + \beta \frac{\partial \mathbb{E}_{t+\tau} V_{t+\tau+1}(\boldsymbol{S}_{t+\tau+1}, \boldsymbol{u}_{t+\tau+1}, \boldsymbol{P}_{t+\tau+1})}{\partial u_t}, \end{aligned} \tag{A5}$$

where $\bigtriangledown_a b = \frac{\partial b}{\partial a}$ for any notations of a and b. It is intractable because the last term causes the "curse of dimensionality". □

Proof of Proposition 1.5:

Proof. We introduce the new subscript $x_{t,T}$ to the variables, which represents the t-th period in a T-period time horizon. Note that the problem with $T \to \infty$ is the infinite horizon analogue of the original problem. The logic of this proof follows the fashion of [69]. We first show some important structural properties of the original problem including the montonicities of value functions, marginal values, and inventory/selling policies. Then we prove the asymptotic convergence of those functions and parameters. Last but not least, we check the conditions such that the optimal structural policy preserves in the infinite horizon.

Step 1: Structural properties of a finite horizon $T \in \mathbb{Z}^*$.

We keep the value function (on states) as $V_{t,T}(\boldsymbol{s}_{t,T}, \boldsymbol{p}_{t,T})$, and decompose (rearrange) actions and states in the objective function (1.4b) by introducing the profit function on actions as follows

$$\begin{aligned} K_{t,T}(\boldsymbol{y}_{t,T}, \boldsymbol{z}_{t,T} | \boldsymbol{p}_{t,T}) \;=\; & p^C_{t,T}\left(y^2_{t,T} + (1-\alpha)\, y^1_{t,T}\right) - \sum_{i=1}^{2} p^i_{t,T}\,\left(y^i_{t,T} + z^i_{t,T}\right) - \sum_{i=1}^{2} c^H_{t,T} z^i_{t,T} \\ & - c^P_{t,T}\left(q - y^1_{t,T} - y^2_{t,T}\right) + \beta\, \mathbb{E}_{t,T} V_{t+1,T}(\boldsymbol{S}_{t+1,T}, \boldsymbol{P}_{t+1,T}). \end{aligned} \tag{A6}$$

The dynamic program (1.4) can be rewritten as

$$V_{t,T}(\boldsymbol{s}_{t,T}, \boldsymbol{p}_{t,T}) = \max_{\boldsymbol{y}_{t,T}, \boldsymbol{z}_{t,T}} K_{t,T}(\boldsymbol{y}_{t,T}, \boldsymbol{z}_{t,T} | \boldsymbol{p}_{t,T}) + \sum_{i=1}^{2} p^i_{t,T} s^i_{t,T}, \tag{A7a}$$

$$\text{s.t.} \quad y^1_{t,T} + y^2_{t,T} \leq q, \tag{A7b}$$

$$y^i_{t,T} + z^i_{t,T} \leq s^i_{t,T}, \quad \text{for any} \quad i = 1, 2. \tag{A7c}$$

Note that formally this is a special case of the cash-balance model; thus, we can establish by backward induction the following group of results.

1. $K_{t,T}(\boldsymbol{y}_{t,T}, \boldsymbol{z}_{t,T} | \boldsymbol{p}_{t,T})$ is monotone in $\boldsymbol{y}_{t,T}$ with the direction depending on $\boldsymbol{p}_{t,T}$ (see Proposition 1.1 for details), and concave in $\boldsymbol{z}_{t,T}$.

2. The optimal stock level for the under weight hogs is

$$z_{t,T}^{1*} = \begin{cases} s_{t,T}^1, & s_{t,T}^1 < \underline{z}_{t,T}^1 \\ \underline{z}_{t,T}^1, & \underline{z}_{t,T}^1 \leq s_{t,T}^1 < \underline{z}_{t,T}^1 + y_{t,T}^{1*} \\ s_{t,T}^1 - y_{t,T}^{1*}, & \underline{z}_{t,T}^1 + y_{t,T}^{1*} \leq s_{t,T}^1 < \bar{z}_{t,T}^1 + y_{t,T}^{1*} \\ \bar{z}_{t,T}^1, & s_{t,T}^1 \geq \bar{z}_{t,T}^1 + y_{t,T}^{1*} \end{cases}$$

where $y_{t,T}^{1*}$ differs in 5 cases, and $\underline{z}_{t,T}^1$, $\bar{z}_{t,T}^1$ are determined implicitly as solutions to

$$\begin{aligned} \frac{\partial K_{t,T}}{\partial z_{t,T}^1} &= (1-\alpha)p_{t,T}^C + c_{t,T}^P - p_{t,T}^1, \\ \frac{\partial K_{t,T}}{\partial z_{t,T}^1} &= 0. \end{aligned}$$

3. The optimal stock level for the regular weight hogs is

$$z_{t,T}^{2*} = \begin{cases} 0, & s_{t,T}^2 = y_{t,T}^{2*} \\ s_{t,T}^2 - y_{t,T}^{2*}, & y_{t,T}^{2*} \leq s_{t,T}^2 < \bar{z}_{t,T}^2 + y_{t,T}^{2*} \\ \bar{z}_{t,T}^2, & s_{t,T}^2 \geq \bar{z}_{t,T}^2 + y_{t,T}^{2*} \end{cases}$$

where $y_{t,T}^{2*}$ differs in 5 cases, and $\bar{z}_{t,T}^2$ is determined implicitly as the solution to

$$\frac{\partial K_{t,T}}{\partial z_{t,T}^2} = 0.$$

4. $V_{t,T}(\boldsymbol{s}_{t,T}|\boldsymbol{p}_{t,T})$ is concave in $\boldsymbol{s}_{t,T}$ since the marginal value of yield decreases, taking the following forms, respectively for under and regular weight hogs,

$$\frac{\partial V_{t,T}}{\partial s^1_{t,T}} = \begin{cases} \dfrac{\partial K_{t,T}(s^1_{t,T})}{\partial z^1_{t,T}} + p^1_{t,T}, & s^1_{t,T} < \underline{z}^1_{t,T} \\ (1-\alpha)p^C_{t,T} + c^P_{t,T}, & \underline{z}^1_{t,T} \le s^1_{t,T} < \underline{z}^1_{t,T} + y^{1*}_{t,T} \\ \dfrac{\partial K_{t,T}(s^1_{t,T} - y^{1*}_{t,T})}{\partial z^1_{t,T}} + p^1_{t,T}, & \underline{z}^1_{t,T} + y^{1*}_{t,T} \le s^1_{t,T} < \bar{z}^1_{t,T} + y^{1*}_{t,T} \\ p^1_{t,T}, & s^1_{t,T} \ge \bar{z}^1_{t,T} + y^{1*}_{t,T} \end{cases}$$

$$\frac{\partial V_{t,T}}{\partial s^2_{t,T}} = \begin{cases} p^C_{t,T} + c^P_{t,T}, & s^2_{t,T} = y^{2*}_{t,T} \\ \dfrac{\partial K_{t,T}(s^2_{t,T} - y^{2*}_{t,T})}{\partial z^2_{t,T}} + p^2_{t,T}, & y^{2*}_{t,T} \le s^2_{t,T} < \bar{z}^2_{t,T} + y^{2*}_{t,T} \\ p^2_{t,T}, & s^2_{t,T} \ge \bar{z}^2_{t,T} + y^{2*}_{t,T} \end{cases}$$

Note that 4 establishes the necessary “nesting" policy.

Recall that all inventory will become regular weight next period, the future marginal value of holding an under-weight or regular weight hog is the same,

$$
\begin{aligned}
& \frac{\partial K_{t,T}}{\partial z^2_{t,T}} \\
= & -p^i_{t,T} - c^H_{t,T} + \beta \left[\int_0^{y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}} p^C_{t+1,T} + c^P_{t+1,T} dF^2_{t+1,T}(w^2) \right. \\
& + \int_{y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}}^{z^{2*}_{t+1,T} + y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}} \frac{\partial V_{t,T}(S^2_{t+1,T} - y^2_{t+1,T})}{\partial s^2_{t+1,T}} dF^2_{t+1,T}(w^2) \\
& \left. + \int_{z^{2*}_{t+1,T} + y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}}^{\infty} p^2_{t+1,T} dF^2_{t+1,T}(w^2) \right] \\
= & -p^i_{t,T} - c^H_{t,T} + \beta [(p^C_{t+1,T} + c^P_{t+1,T}) F^2_{t+1,T}(y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}) \\
& + p^2_{t+1,T} \bar{F}^2_{t+1,T}(z^{2*}_{t+1,T} + y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}) \\
& \left. + \int_{y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}}^{z^{2*}_{t+1,T} + y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T}} \frac{\partial V_{t,T}(S^2_{t+1,T} - y^2_{t+1,T})}{\partial s^2_{t+1,T}} + p^2_{t+1,T} dF^2_{t+1,T}(w^2) \right].
\end{aligned}
$$

Let $p^F_{t,T} := (p^C_{t,T} + c^P_{t,T})$, $p^S_{t,T} := p^2_{t,T}$, and $\Delta p_{t,T} := p^F_{t,T} - p^S_{t,T}$, representing the option value of the regular weight hogs from the long term contract, from the open market, and their gap, then the $p^F_{t,T} \leq p^S_{t,T}$ will lead to a trivial case where the hog producer will not fulfill the contract and the marginal value easily converges. We only discuss the non-trivial case with $p^F_{t,T} > p^S_{t,T}$ where the marginal value of holding is bounded by following inequalities

$$
\begin{aligned}
& -p^i_{t,T} - c^H_{t,T} + \beta [\mathbb{E}_t p^S_{t+1,T} + \mathbb{E}_t \Delta p_{t+1,T} F^2_{t+1,T}(y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T})] \leq \frac{\partial K_{t,T}}{\partial z^2_{t,T}} \\
\leq & -p^i_{t,T} - c^H_{t,T} + \beta [\mathbb{E}_t p^S_{t+1,T} + \mathbb{E}_t \Delta p_{t+1,T} F^2_{t+1,T}(z^{2*}_{t+1,T} + y^{2*}_{t+1,T} - z^1_{t,T} - z^2_{t,T})] \qquad \text{(A8)}
\end{aligned}
$$

Follow the proof from [69], we can obtain the monotonicity results which establish the existence of attainable upper and lower limits on the policy and cost functions. (The shorthand $\frac{\partial V_{t,T}}{\partial s^2_{t,T}} \uparrow$ is used to mean "the function $\frac{\partial K_{t,T}}{\partial z^2_{t,T}}$ is everywhere (weakly) decreased." $F^2_{t,T} \uparrow$ means "the distribution is replaced by one which is stochastically larger." All results remain true if all arrows are inverted.)

Lemma A.2. *For any $T \in \mathbb{Z}^*$ and $t = 1, \cdots, T$,*

1. $\frac{\partial V_{t+j,T}}{\partial s^2_{t+j,T}} \uparrow \Rightarrow \frac{\partial K_{t,T}}{\partial z^i_{t,T}} \uparrow, \bar{z}^i_{t,T} \uparrow, \underline{z}^1_{t,T} \uparrow, \frac{\partial V_{t,T}}{\partial s^2_{t,T}} \uparrow.$
2. $F^2_{t+j,T} \uparrow \Rightarrow \frac{\partial K_{t,T}}{\partial z^i_{t,T}} \uparrow, \bar{z}^i_{t,T} \uparrow, \underline{z}^1_{t,T} \uparrow, \frac{\partial V_{t,T}}{\partial s^2_{t,T}} \uparrow.$
3. $\frac{\partial K_{t+j,T}}{\partial z^i_{t+j,T}} \uparrow \Rightarrow \bar{z}^i_{t,T} \uparrow, \underline{z}^1_{t,T} \uparrow, \frac{\partial V_{t-1,T}}{\partial s^2_{t-1,T}} \uparrow, \frac{\partial K_{t-1,T}}{\partial z^i_{t-1,T}} \uparrow.$

For convenience, call the corresponding terminal conditions F and S, respectively, that is

$$\frac{\partial V^F_{T,T}}{\partial s^2_{T,T}} = p^F_{T,T}, \quad \text{and} \quad \frac{\partial V^S_{T,T}}{\partial s^2_{T,T}} = p^S_{T,T}.$$

Lemma A.3. *For any admissible ending condition whatever,*

1. $\frac{\partial V^S_{t,T}}{\partial s^2_{t,T}} \leq \frac{\partial V_{t,T}}{\partial s^2_{t,T}} \leq \frac{\partial V^F_{t,T}}{\partial s^2_{t,T}};$
2. $\frac{\partial K^S_{t,T}}{\partial z^2_{t,T}} \leq \frac{\partial K_{t,T}}{\partial z^2_{t,T}} \leq \frac{\partial K^F_{t,T}}{\partial z^2_{t,T}};$
3. $\bar{z}^{iS}_{t,T} \leq \bar{z}^i_{t,T} \leq \bar{z}^{iF}_{t,T}$ *for any* $i = 1, 2;$
4. $\underline{z}^{1S}_{t,T} \leq \underline{z}^1_{t,T} \leq \underline{z}^{1F}_{t,T}.$

Step 2: Preliminary convergence results. We derive preliminary convergence results in preparation for developing the infinite horizon model. First, we obtain that the “fulfilling" and "selling" case of the problesm are themselves nested as T increases. If $t \leq T$,

$$\frac{\partial K^S_{t,T+1}(\boldsymbol{z})}{\partial z^i_{t,T+1}} \geq \frac{\partial K^S_{t,T}(\boldsymbol{z})}{\partial z^i_{t,T}} \quad \forall i = 1,2;$$
$$\bar{z}^{iS}_{t,T+1} \geq \bar{z}^{iS}_{t,T} \quad \forall i = 1,2; \quad \underline{z}^{1S}_{t,T+1} \geq \underline{z}^{1S}_{t,T}$$
$$\frac{\partial V^S_{t,T+1}(s^2)}{\partial s^2_{t,T+1}} \geq \frac{\partial V^S_{t,T}(s^2)}{\partial s^2_{t,T}};$$

And these results hold for the "fulfilling" scenario with all inequalities reversed. Next, we obtained that any "selling" case bounds any "fulfilling" case, that is, for any T_1, T_2,

$$\frac{\partial K^S_{t,T_1}(\boldsymbol{z})}{\partial z^i_{t,T_1}} \leq \frac{\partial K^F_{t,T_2}(\boldsymbol{z})}{\partial z^i_{t,T_2}} \quad \forall i = 1,2;$$
$$\bar{z}^{iS}_{t,T_1} \leq \bar{z}^{iF}_{t,T_2} \quad \forall i = 1,2; \quad \underline{z}^{1S}_{t,T_1} \leq \underline{z}^{1F}_{t,T_2}$$
$$\frac{\partial V^S_{t,T_1}(s^2)}{\partial s^2_{t,T_1}} \leq \frac{\partial V^F_{t,T_2}(s^2)}{\partial s^2_{t,T_2}};$$

Since a uniformly bounded montonic sequence of functions possesses a limiting function, we define

$$\lim_{T\to\infty} \frac{\partial K^S_{t,T}(\boldsymbol{z})}{\partial z^1_{t,T}} = \frac{\partial K^S_t(\boldsymbol{z})}{\partial z^1_t}$$

and so forth. Then combining the two results above, the limiting functions may also be seen to be nested as in

Lemma A.4. *For any $T > t$,*

1. $\frac{\partial K^S_{t,T}}{\partial z^i_{t,T}} \leq \frac{\partial K^S_t}{\partial z^i_t} \leq \frac{\partial K^F_t}{\partial z^i_t} \leq \frac{\partial K^F_{t,T}}{\partial z^i_{t,T}}$ *for any $i = 1,2$;*
2. $\frac{\partial V^S_{t,T}}{\partial s^2_{t,T}} \leq \frac{\partial V^S_t}{\partial s^2_t} \leq \frac{\partial V^F_t}{\partial s^2_t} \leq \frac{\partial V^F_{t,T}}{\partial s^2_{t,T}}$*;*
3. $\bar{z}^{iS}_{t,T} \leq \bar{z}^{iS}_t \leq \bar{z}^{iF}_t \leq \bar{z}^{iF}_{t,T}$ *for any $i = 1,2$;*

4. $\underline{z}_{t,T}^{1S} \leq \underline{z}_t^{1S} \leq \underline{z}_t^{1F} \leq \underline{z}_{t,T}^{1F}$.

Step 3: The infinite horizon model.

The final step examines conditions under which the infinite horizon problem is well defined. We show in particular that if the selling and fulfilling cases converge to each other as the horizon increases, then there is a unique set of solutions to the infinite horizon equation. We define

$$\Delta\frac{\partial K_t}{\partial z_t^i} \equiv \frac{\partial K_t^F}{\partial z_t^i} - \frac{\partial K_t^S}{\partial z_t^i},$$

and $\Delta\frac{\partial V_t}{\partial s_t^2}$, $\Delta\bar{z}_t^i$, $\Delta\underline{z}_t^1$, similarly, and all ≥ 0. Then, we have

Lemma A.5. *1.* $0 \leq \Delta\frac{\partial V_t}{\partial s_t^2} \leq \Delta\frac{\partial K_t}{\partial z_t^i}$;

2. $0 \leq \Delta\frac{\partial K_t}{\partial z_t^i} = \int_{y_{t+1}^{2*}-z_t^1-z_t^2}^{\bar{z}_{t+1}^2+y_{t+1}^{2*}-z_t^1-z_t^2} \frac{\partial G_{t+1}(\boldsymbol{y}_{t+1}, z_{t+1}^1, s_{t+1}^2-y_{t+1}^2)}{\partial z_{t+1}^2} dF_{t+1}^2(w^2)$;

3. $0 \leq \Delta\bar{z}_t^i = \frac{\partial^2 r_t}{(\partial z_t^i)^2}\frac{\partial G_t(\boldsymbol{y}_t, \bar{z}_t^1, \bar{z}_t^2)}{\partial z_t^i} = 0$;

4. $0 \leq \Delta\underline{z}_t^1 = \frac{\partial^2 r_t}{(\partial z_t^i)^2}\frac{\partial G_t(\boldsymbol{y}_t, \underline{z}_t^1, 0)}{\partial z_t^1} = 0$,

Therefore, we can obtain the same marginal value functions as finite horizon as long as $\bar{z}_2^t = 0$ for any t. □

Appendix B

New Class of Revenue Management Problems with Overbooking and No-Shows: Shoring up Trust between Shippers and Carriers in Maritime Container Shipping

B.1 Proofs

Proof of Claim 2.1

To begin, we have

$$\begin{aligned}
&\mathbb{E}\left[R_t(\mathcal{A}, Y; m)\right] \\
&\quad = \mathbb{E}\left[R_t(\mathcal{A}, Y; m) \;\middle|\; \sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau < m\right] \cdot \Pr\left[\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau < m\right] \\
&\qquad + \mathbb{E}\left[R_t(\mathcal{A}, Y; m) \;\middle|\; \sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau \geq m\right] \cdot \Pr\left[\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau \geq m\right] \\
&\quad = \Pr\left[\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau < m\right] \cdot \left(p_t + (1 - p_t) \cdot d_t\right) \\
&\qquad + \Pr\left[\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau \geq m\right] \cdot \left(-p_t d_t + (1 - p_t) \cdot d_t\right).
\end{aligned}$$

To understand the above equality, note that if $\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau < m$, then if shipper t shows up, it will be among the m largest deposits across those corresponding to shippers who show up. Shipper t shows up with probability p_t, in which case it is allocated a slot and hence pays the service fee of 1. If shipper t is a no-show, which occurs with probability $1 - p_t$, then the liner keeps its deposit of d_t. On the other hand, in the event that $\sum_{\tau \in \mathcal{A}: \rho(\tau) < \rho(t)} Y_\tau \geq m$, all available slots will have already been allocated to higher ranked shippers. As such, if shipper t shows up, the liner must pay a reimbursement of d_t. If shipper t is a no-show, we are once again in the case where the liner keeps its deposit.

From here, observing that

$$\Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau \geq m\right] = 1 - \Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau < m\right],$$

we get

$$\begin{aligned}
&\mathbb{E}\left[R_t(\mathcal{A}, Y; m)\right] \\
&\quad = \Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau < m\right] \cdot (p_t + (1-p_t)\cdot d_t) \\
&\qquad\qquad + \left(1 - \Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau < m\right]\right) \cdot (-p_t d_t + (1-p_t)\cdot d_t) \\
&\quad = \Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau < m\right] \cdot p_t - \left(1 - \Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau < m\right]\right) \cdot p_t d_t + (1-p_t)\cdot d_t \\
&\quad\; = d_t \cdot (1-2p_t) + \Pr\left[\sum_{\tau\in\mathcal{A}:\rho(\tau)<\rho(t)} Y_\tau < m\right] \cdot (p_t \cdot (1+d_t)),
\end{aligned}$$

where the two equalities follow by basic algebraic manipulations.

B.1.1 Proof of Claim 2.2

To begin, define

$$\begin{aligned}
\mathcal{A}^* &= \arg\max_{\mathcal{A}\subseteq[T]} \mathcal{R}(\mathcal{A}; m) \\
\mathcal{A}^*_{\text{rel}} &= \arg\max_{\mathcal{A}\subseteq T_{\text{rel}}} \mathcal{R}(\mathcal{A}; m) && \text{(B1)} \\
\mathcal{A}^*_{\text{unrel}} &= \arg\max_{\mathcal{A}\subseteq T_{\text{unrel}}} \mathcal{R}(\mathcal{A}; m) && \text{(B2)}
\end{aligned}$$

to respectively denote the optimal acceptance sets with regards to the full set of shippers, just the reliable shippers, and just the unreliable shippers. We have

$$
\begin{aligned}
\mathcal{R}^*_{\text{rel}} + \mathcal{R}^*_{\text{unrel}} &= \mathbb{E}\left[\sum_{t\in\mathcal{A}^*_{\text{rel}}} R_t(\mathcal{A}^*_{\text{rel}}, Y; m)\right] + \mathbb{E}\left[\sum_{t\in\mathcal{A}^*_{\text{unrel}}} R_t(\mathcal{A}^*_{\text{unrel}}, Y; m)\right] \\
&\geq \mathbb{E}\left[\sum_{t\in\mathcal{A}^*\cap T_{\text{rel}}} R_t(\mathcal{A}^*\cap T_{\text{rel}}, Y; m)\right] + \mathbb{E}\left[\sum_{t\in\mathcal{A}^*\cap T_{\text{unrel}}} R_t(\mathcal{A}^*\cap T_{\text{unrel}}, Y; m)\right] \\
&\geq \mathbb{E}\left[\sum_{t\in\mathcal{A}^*\cap T_{\text{rel}}} R_t(\mathcal{A}^*, Y; m)\right] + \mathbb{E}\left[\sum_{t\in\mathcal{A}^*\cap T_{\text{unrel}}} R_t(\mathcal{A}^*, Y; m)\right] \\
&= \mathbb{E}\left[\sum_{t\in\mathcal{A}^*} R_t(\mathcal{A}^*, Y; m)\right] \\
&= \text{OPT.}
\end{aligned}
$$

The first inequality follows from observing that $\mathcal{A}^*\cap T_{\text{rel}}$ and $\mathcal{A}^*\cap T_{\text{unrel}}$ are feasible to problems (B1) and (B2) respectively. The second inequality follows from an iterative application of the following claim.

Claim B.1. *For any acceptance set $\mathcal{A}\subset[T]$ and shippers $t\in\mathcal{A}$ and $t^+\in[T]\setminus A$, we have*

$$
\mathbb{E}\left[R_t(\mathcal{A}, Y; m)\right] \geq \mathbb{E}\left[R_t(\mathcal{A}\cup\{t^+\}, Y; m)\right]
$$

Proof. Using the expression provide in Claim 2.1 for the expected profit garnered from shipper t, we have

$$\begin{aligned}
\mathbb{E}\left[R_t(\mathcal{A} \cup \{t^+\}, Y; m)\right] &= d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A} \cup \{t^+\} : \rho(\tau) < \rho(t)} Y_\tau < m\right] \cdot (p_t \cdot (1 + d_t)) \\
&= d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A} : \rho(\tau) < \rho(t)} Y_\tau + \mathbf{1}_{\rho(t^+) < \rho(t)} \cdot Y_{t^+} < m\right] \cdot (p_t \cdot (1 + d_t)) \\
&\leq d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A} : \rho(\tau) < \rho(t)} Y_\tau < m\right] \cdot (p_t \cdot (1 + d_t)) \\
&= \mathbb{E}\left[R_t(\mathcal{A}, Y; m)\right],
\end{aligned}$$

where the lone inequality follows since we clearly have that

$$\Pr\left[\sum_{\tau \in \mathcal{A} : \rho(\tau) < \rho(t)} Y_\tau + \mathbf{1}_{\rho(t^+) < \rho(t)} \cdot Y_{t^+} < m\right] \leq \Pr\left[\sum_{\tau \in \mathcal{A} : \rho(\tau) < \rho(t)} Y_\tau < m\right].$$

□

Proof of Claim 2.3

We consider the two cases outlined in the claim statement, which consider whether $t \in \mathcal{A}_{\text{TOH}}$ is reliable or unreliable.

- If $t \in T_{\text{rel}}$, then we begin by recalling that our TOH policy accepts at most m reliable shippers. Moreover, since the RF mechanism prioritizes the reliable shippers in terms of slot allocation, shipper t is guaranteed to get a slot if it shows up. Hence we have that

$$\mathbb{E}\left[\hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] = p_t + (1 - p_t) \cdot d_t.$$

- Next, we consider the other scenario in which $t \in T_{\text{unrel}}$. Recall that our TOH policy accepts all unreliable shippers, and so $\mathcal{A}_{\text{TOH}} \cap T_{\text{unrel}} = T_{\text{unrel}}$. As such, following much of the same logic as the proof of Claim 2.1, we have

$$\begin{aligned}
&\mathbb{E}\left[\hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] \\
&= \Pr\left[\sum_{\tau \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} Y_\tau + \sum_{\substack{\tau \in T_{\text{unrel}}: \\ \rho(\tau) < \rho(t)}} Y_\tau < m\right] \cdot (p_t + (1 - p_t) \cdot d_t) \\
&\qquad + \left(1 - \Pr\left[\sum_{\tau \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} Y_\tau + \sum_{\substack{\tau \in T_{\text{unrel}}: \\ \rho(\tau) < \rho(t)}} Y_\tau < m\right]\right) \cdot (-p_t d_t + (1 - p_t) \cdot d_t) \\
&= d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} Y_\tau + \sum_{\substack{\tau \in T_{\text{unrel}}: \\ \rho(\tau) < \rho(t)}} Y_\tau < m\right] \cdot (p_t \cdot (1 + d_t)) .
\end{aligned}$$

Proof of Lemma 2.1

We prove the lemma by arguing that under an arbitrary realization of $\{Y_t\}_{t \in \mathcal{A}_{\text{TOH}}}$, the expected profit earned by the RF mechanism is no larger than that earned by the DO mechanism. For this arbitrary realization of the show-ups, let $T_{\text{show}} = \{t \in \mathcal{A}_{\text{TOH}} : Y_t = 1\}$ and $T_{\text{noshow}} = \mathcal{A}_{\text{TOH}} \setminus T_{\text{show}}$. Clearly, from all shippers $t \in T_{\text{noshow}}$, both mechanisms garner the same profit of d_t. Furthermore, both mechanisms allocate $\min\{|T_{\text{show}}|, m\}$ slots. Consequently, the only difference in terms of the profit earned under the two mechanism arises from the $(|T_{\text{show}}| - m)^+$ shippers who showed up, but did not receive a slot. In this case, recall that the liner must reimburse each shipper's deposit. It is easy to see that the total reimbursement owed by the liner is minimized under the DO mechanism due to the fact that the slots are first given away to the shippers with the largest deposits.

Proof of Lemma 2.2

We set about showing that $\hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m) \geq \frac{1}{2} \cdot |\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|$, which is enough to establish the lemma due Lemma 2.1, which states that $\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) \geq \hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m)$. To do so, we first prove the following intermediate claim, which states that the unreliable shippers contribute non-negative profit towards $\hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m)$. The proof of this claim can be found at the end of this section.

Claim B.2. $\hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m) \geq \hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}; m)$

From Claim B.2, we see that

$$\begin{aligned}
\hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m) &\geq \hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}; m) \\
&= \sum_{t \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} p_t + (1 - p_t) \cdot d_t \\
&\geq \frac{1}{2} \cdot |\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|.
\end{aligned}$$

The first equality follows from Claim 2.3, while the final inequality results from observing that, for any reliable shipper, the expression $p_t + (1 - p_t) \cdot d_t$ is minimized at $p_t = \frac{1}{2}$ and $d_t = 0$.

Proof of Claim B.2. First, note that for any unreliable shipper $t \in T_{\text{unrel}}$, we have

$$\begin{aligned}
\mathbb{E}\left[\hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] &= d_t \cdot (1 - 2p_t) + \Pr\left[\sum_{\tau \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} Y_\tau + \sum_{\substack{\tau \in T_{\text{unrel}}: \\ \rho(\tau) < \rho(t)}} Y_\tau < m\right] \cdot (p_t \cdot (1 + d_t)) \\
&\geq d_t \cdot (1 - 2p_t) \\
&\geq 0.
\end{aligned}$$

The equality follows from Claim 2.3, and the second inequality follows from the fact that $p_t < \frac{1}{2}$ for unreliable shippers. From here, we have

$$\begin{aligned}
\hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m) &= \mathbb{E}\left[\sum_{t\in\mathcal{A}_{\text{TOH}}\cap T_{\text{rel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] + \mathbb{E}\left[\sum_{t\in T_{\text{unrel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] \\
&\geq \mathbb{E}\left[\sum_{t\in\mathcal{A}_{\text{TOH}}\cap T_{\text{rel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m)\right] \\
&= \hat{R}(\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}; m),
\end{aligned}$$

where the second equality follows from noting that, under the RF mechanism, the profit of the reliable shippers is unaffected by whether or not the unreliable shippers show up.

Proof of Lemma 2.3

Let $\mathcal{A}^*_{\text{rel}} = \arg\max_{\mathcal{A}\subseteq T_{\text{rel}}} \mathcal{R}(\mathcal{A}; m)$, and note that

$$\begin{aligned}
\mathcal{R}^*_{\text{rel}} &= \mathcal{R}(\mathcal{A}^*_{\text{rel}}; m) \\
&= \mathbb{E}\left[\sum_{t\in\mathcal{A}^*_{\text{rel}}} R_t(\mathcal{A}^*_{\text{rel}}, Y; m)\right] \\
&= \sum_{t\in\mathcal{A}^*_{\text{rel}}} \left(d_t \cdot \underbrace{(1-2p_t)}_{\leq 0,\text{ since } p_t \geq \frac{1}{2}} + \Pr\left[\sum_{\tau\in\mathcal{A}^*_{\text{rel}}:\rho(\tau)<\rho(t)} Y_\tau < m\right] \cdot (p_t \cdot (1+d_t)) \right) \\
&\leq \sum_{t\in\mathcal{A}^*_{\text{rel}}} \Pr\left[\sum_{\tau\in\mathcal{A}^*_{\text{rel}}:\rho(\tau)<\rho(t)} Y_\tau < m\right] \cdot (p_t \cdot (1+d_t) + d_t \cdot (1-2p_t)) \\
&= \sum_{t\in\mathcal{A}^*_{\text{rel}}} \Pr\left[\sum_{\tau\in\mathcal{A}^*_{\text{rel}}:\rho(\tau)<\rho(t)} Y_\tau < m\right] \cdot \underbrace{(p_t + d_t - p_t d_t)}_{\leq 1 \text{ for any } d_t, p_t \in [0,1]} \\
&\leq \underbrace{\sum_{t\in\mathcal{A}^*_{\text{rel}}} \Pr\left[\sum_{\tau\in\mathcal{A}^*_{\text{rel}}:\rho(\tau)<\rho(t)} Y_\tau < m\right]}_{\text{(term 1)}}.
\end{aligned}$$

We now establish both upper bounds.

- First, we establish that $\mathcal{R}^*_{\text{rel}} \leq |T_{\text{rel}}|$, by noting that

$$(\text{term 1}) = \sum_{t \in \mathcal{A}^*_{\text{rel}}} \Pr\left[\sum_{\tau \in \mathcal{A}^*_{\text{rel}}: \rho(\tau) < \rho(t)} Y_\tau < m\right] \leq |\mathcal{A}^*_{\text{rel}}| \leq |T_{\text{rel}}|.$$

- The second and more complex bound of $\mathcal{R}^*_{\text{rel}} \leq 2m$ is established next. We have

$$\begin{aligned}
(\text{term 1}) &= \sum_{t \in \mathcal{A}^*_{\text{rel}}} \Pr\left[\sum_{\tau \in \mathcal{A}^*_{\text{rel}}: \rho(\tau) < \rho(t)} Y_\tau < m\right] \\
&\leq \sum_{t \in \mathcal{A}^*_{\text{rel}}} \Pr\left[\sum_{\tau \in \mathcal{A}^*_{\text{rel}}: \rho(\tau) < \rho(t)} \text{Bernoulli}(1/2) < m\right] \\
&= \sum_{n=0}^{|\mathcal{A}^*_{\text{rel}}|-1} \Pr\left[\text{Binomial}(n, 1/2) < m\right] \\
&\leq \sum_{n=0}^{\infty} \Pr\left[\text{Binomial}(n, 1/2) < m\right] \\
&= \sum_{k=0}^{m-1} \sum_{n=k}^{\infty} \Pr\left[\text{Binomial}(n, 1/2) = k\right] \\
&= 2m.
\end{aligned}$$

  The first inequality comes from observation that $\Pr\left[\sum_{\tau \in \mathcal{A}^*_{\text{rel}}: \rho(\tau) < \rho(t)} Y_\tau < m\right]$ is maximized when $Y_\tau \sim \text{Bernoulli}(1/2)$, as this is the smallest show-up probability for any reliable shipper. The final inequality uses the following claim, whose proof is found at the end of this section.

  Claim B.3. *For any $k \in \mathbb{Z}_+$, we have*

$$\sum_{n=k}^{\infty} \Pr\left[\text{Binomial}(n, 1/2) = k\right] = 2.$$

Proof of Claim B.3. Observe that

$$\begin{aligned}
\sum_{n=k}^{\infty} \Pr\left[\text{Binomial}(n, 1/2) = k\right] &= \sum_{n=k}^{\infty} \binom{n}{k} \cdot \left(\frac{1}{2}\right)^{n} \\
&= \left(\frac{1}{2}\right)^{k} \cdot \left(\sum_{n=k}^{\infty} \binom{n}{k} \cdot \left(\frac{1}{2}\right)^{n-k}\right) \\
&= \left(\frac{1}{2}\right)^{k} \cdot \left(\sum_{n=k}^{\infty} \binom{n}{n-k} \cdot \left(\frac{1}{2}\right)^{n-k}\right) \\
&= \left(\frac{1}{2}\right)^{k} \cdot \left(\sum_{n=k}^{\infty} (-1)^{n-k} \cdot \binom{-k-1}{n-k} \cdot \left(\frac{1}{2}\right)^{n-k}\right) \\
&= \left(\frac{1}{2}\right)^{k} \cdot \left(\sum_{\ell=0}^{\infty} (-1)^{\ell} \cdot \binom{-k-1}{\ell} \cdot \left(\frac{1}{2}\right)^{\ell}\right) \text{ (letting } \ell = n-k) \\
&= \left(\frac{1}{2}\right)^{k} \cdot \left(\sum_{\ell=0}^{\infty} \binom{-k-1}{\ell} \cdot \left(-\frac{1}{2}\right)^{\ell}\right) \\
&= \left(\frac{1}{2}\right)^{k} \cdot \left(1 - \frac{1}{2}\right)^{-k-1} \\
&= 2.
\end{aligned}$$

The fourth equality uses the identity $\binom{n}{k} = (-1)^k \cdot \binom{k-n-1}{k}$ ([59]), while the second to last equality uses the identity $\sum_{k=0}^{\infty} \binom{n}{k} \cdot x^k = (1+x)^n$.

Proof of Claim 2.4

We prove the result via induction over t. The base case of $t = |\mathcal{A}| + 1$ holds trivially, since $V(|\mathcal{A}| + 1, \cdot; \mathcal{A}) = 0$. Moving to the general case of $t \leq |\mathcal{A}|$, we have

$$\begin{aligned}
&\mathbb{E}\left[\sum_{\tau=t}^{|\mathcal{A}|} R_\tau(\mathcal{A}, Y; k)\right] \\
&= \mathbb{E}\left[\sum_{\tau=t}^{|\mathcal{A}|} R_\tau(\mathcal{A}, Y; k) \;\middle|\; Y_t = 1\right] \cdot \Pr\left[Y_t = 1\right] + \mathbb{E}\left[\sum_{\tau=t}^{|\mathcal{A}|} R_\tau(\mathcal{A}, Y; k) \;\middle|\; Y_t = 0\right] \cdot \Pr\left[Y_t = 0\right] \\
&= \mathbb{E}\left[R_t(\mathcal{A}, Y; k) + \sum_{\tau=t+1}^{|\mathcal{A}|} R_\tau\left(\mathcal{A}, Y; (k-1)^+\right) \;\middle|\; Y_t = 1\right] \cdot p_t + \\
&\qquad \mathbb{E}\left[R_t(\mathcal{A}, Y; k) + \sum_{\tau=t+1}^{|\mathcal{A}|} R_\tau(\mathcal{A}, Y; k) \;\middle|\; Y_t = 0\right] \cdot (1 - p_t) \\
&= \left(\mathbb{E}\left[R_t(\mathcal{A}, Y; k) \mid Y_t = 1\right] + V(t+1, (k-1)^+; \mathcal{A})\right) \cdot p_t + \\
&\qquad \left(\mathbb{E}\left[R_t(\mathcal{A}, Y; k) \mid Y_t = 0\right] + V(t+1, k; \mathcal{A})\right) \cdot (1 - p_t), \qquad \text{(B3)}
\end{aligned}$$

where the final equality follows by the induction hypothesis. From here, we consider two cases bases on whether k is non-zero:

- If $k = 0$, and hence there are no remaining service slots, we have $\mathbb{E}[R_t(\mathcal{A}, Y; k) \mid Y_t = 1] = -d_t$ and $\mathbb{E}[R_t(\mathcal{A}, Y; k) \mid Y_t = 0] = d_t$. Consequently, we get

$$\begin{aligned}
\text{(B3)} \quad &= \quad (-d_t + V(t+1, 0; \mathcal{A})) \cdot p_t + (d_t + V(t+1, 0; \mathcal{A})) \cdot (1 - p_t) \\
&= \quad V(t, 0; \mathcal{A}).
\end{aligned}$$

- If $k > 0$, we have $\mathbb{E}[R_t(\mathcal{A}, Y; k) \mid Y_t = 1] = 1$ and $\mathbb{E}[R_t(\mathcal{A}, Y; k) \mid Y_t = 0] = d_t$, and so

$$\begin{aligned}
(\text{B3}) &= (1 + V(t+1, k-1; \mathcal{A})) \cdot p_t + (d_t + V(t+1, 0; \mathcal{A})) \cdot (1 - p_t) \\
&= V(t, k; \mathcal{A}).
\end{aligned}$$

Proof of Claim 2.5

We establish that $V(t, k; \mathcal{A}) \geq 0$ via induction over t. The base case of $t = |\mathcal{A}| + 1$ holds trivially, since $V(|\mathcal{A}| + 1, \cdot; \mathcal{A}) = 0$. Next, we move to showing the result for general $t \leq |\mathcal{A}|$, which is established by considering two cases based on whether k is non-zero.

- If $k = 0$, then from (2.1), we have

$$\begin{aligned}
V(t, 0; \mathcal{A}) &= d_t \cdot (1 - 2p_t) + V(t+1, 0; \mathcal{A}) \\
&\geq V(t+1, 0; \mathcal{A}) \\
&\geq 0,
\end{aligned}$$

  The first inequality follows from the fact that $p_t < \frac{1}{2}$ for unreliable shippers, while the second inequality uses the induction hypothesis.

- If $k > 0$, then from (2.1), we have

$$\begin{aligned}
V(t, k; \mathcal{A}) &= (1 - p_t) \cdot (d_t + V(t+1, k; \mathcal{A})) + p_t \cdot (1 + V(t+1, k-1; \mathcal{A})) \\
&\geq 0,
\end{aligned}$$

  where the inequality follows by the induction hypothesis, along with the fact that $d_t, p_t \in [0, 1]$.

Proof of Claim 2.6

We establish that $V(t,k;\mathcal{A}) - V(t,k-1;\mathcal{A}) \leq 1 + d_t$ via induction over t. The base case of $t = |\mathcal{A}| + 1$ again holds trivially, since $V(|\mathcal{A}|+1,\cdot;\mathcal{A}) = 0$. Next, we move to showing the result for general $t \leq |\mathcal{A}|$, which is established by considering the following two cases.

- If $k = 1$, then via (2.1) we have that

$$\begin{aligned} V(t,1;\mathcal{A}) - V(t,0;\mathcal{A}) &= p_t \cdot (1+d_t) + p_t \cdot (V(t+1,0;\mathcal{A}) - V(t+1,0;\mathcal{A})) + \\ &\qquad (1-p_t) \cdot (V(t+1,1;\mathcal{A}) - V(t+1,0;\mathcal{A})) \\ &= p_t \cdot (1+d_t) + (1-p_t) \cdot (V(t+1,1;\mathcal{A}) - V(t+1,0;\mathcal{A})) \\ &\leq p_t \cdot (1+d_t) + (1-p_t) \cdot (1+d_{t+1}) \\ &\leq 1 + d_t, \end{aligned}$$

  where the second to last inequality uses the induction hypothesis, and the last inequality uses the fact that the shippers are index in decreasing order of deposit size.

- If $k > 1$, then via (2.1) we have that

$$\begin{aligned} V(t,k;\mathcal{A}) - V(t,k-1;\mathcal{A}) &= p_t \cdot (V(t+1,k-1;\mathcal{A}) - V(t+1,k-2;\mathcal{A})) + \\ &\qquad (1-p_t) \cdot (V(t+1,1;\mathcal{A}) - V(t+1,0;\mathcal{A})) \\ &\leq p_t \cdot (1+d_{t+1}) + (1-p_t) \cdot (1+d_{t+1}) \\ &= 1 + d_{t+1} \\ &\leq 1 + d_t. \end{aligned}$$

Proof of Lemma 2.4

To begin, we note that for any unreliable acceptance set $\mathcal{A} \subseteq T_{\text{unrel}}$, we have that $V(1, m; \mathcal{A}) = \mathcal{R}(\mathcal{A}; m)$ by Claim 2.4, and hence we can alternatively express $\mathcal{R}^*_{\text{unrel}}$ as $\mathcal{R}^*_{\text{unrel}} = \max_{\mathcal{A} \subseteq T_{\text{unrel}}} V(1, m; \mathcal{A})$. As such, to prove the lemma, it is sufficient to prove that $V(1, m; T_{\text{unrel}}) \geq V(1, m; \mathcal{A})$ for any $\mathcal{A} \subseteq T_{\text{unrel}}$. To do so, we assume by way of contradiction that there exists $\mathcal{A}^* \subset T_{\text{unrel}}$ satisfying $V(1, m; \mathcal{A}^*) > V(1, m; T_{\text{unrel}})$, and use the following claim to establish the contradiction, whose proof is presented below.

Claim B.4. *For any $\mathcal{A} \subset T_{\text{unrel}}$, shipper $t \in \mathcal{A}$, $k \in [m]_0$ we have*

$$V(t, k; \mathcal{A} \cup \{t^+\}) \geq V(t, k; \mathcal{A}).$$

for arbitrary shipper $t^+ \in T_{\text{unrel}} \setminus \mathcal{A}$.

Before proving the claim, we note that as a direct consequence, we can iteratively add the shippers $t^+ \in T_{\text{unrel}} \setminus \mathcal{A}^*$ to $\mathcal{A}^*$ until the latter set becomes T_{unrel}, and our expected profit can only improve. Consequently we get that $V(1, m; T_{\text{unrel}}) \geq V(1, m; \mathcal{A}^*)$, which gives the intended contradiction.

Proof of Claim B.4. We prove the result via induction over $t \in \mathcal{A}$. To begin, we assume without loss of generality that deposit d_{t^+} is sandwiched by the deposits of shippers $t^*, t^* + 1 \in [|\mathcal{A}| + 1]$. In other words, the ordering of the deposits satisfies

$$d_1 \geq \ldots \geq d_{t^*} \geq d_{t^+} \geq d_{t^*+1} \geq \ldots \geq d_{|\mathcal{A}|+1} = -\infty,$$

where shipper $|\mathcal{A}| + 1$ represents a dummy shipper with a deposit of negative infinity. Our base cases for the induction argument will be all shippers $t \in \{t^*, t^* + 1, \ldots, |\mathcal{A}| + 1\}$. For shippers $t \in \{t^* + 1, \ldots, |\mathcal{A}| + 1\}$, we clearly have that $V(t, k; \mathcal{A} \cup \{t^+\}) = V(t, k; \mathcal{A})$, since shipper t^+ will not appear in any of these value functions. Next, we consider the remaining base case of $t = t^*$, where for ease of notation, we let $\mathcal{A}^+ = \mathcal{A} \cup \{t^+\}$. We have

$$V(t^*, k; \mathcal{A}) = (1 - p_t) \cdot (d_t + V(t^* + 1, k; \mathcal{A})) + \begin{cases} p_t \cdot (1 + V(t^* + 1, k - 1; \mathcal{A})), & \text{if } k > 0 \\ p_t \cdot (-d_t + V(t^* + 1, 0; \mathcal{A})), & \text{if } k = 0, \end{cases}$$

and

$$V(t^*, k; \mathcal{A}^+) = (1 - p_t) \cdot \big(d_t + V(t^+, k; \mathcal{A}^+)\big) + \begin{cases} p_t \cdot (1 + V(t^+, k - 1; \mathcal{A}^+)), & \text{if } k > 0 \\ p_t \cdot (-d_t + V(t^+, 0; \mathcal{A}^+)), & \text{if } k = 0. \end{cases}$$

Hence, comparing the above recursions, we see that establishing $V(t, k; \mathcal{A}^+) \geq V(t^*, k; \mathcal{A})$ requires just showing that $V(t^+, k; \mathcal{A}^+) \geq V(t^* + 1, k; \mathcal{A})$ for any $k \in [m]_0$:

- If $k = 0$, we have that

$$\begin{aligned} V(t^+, 0; \mathcal{A}^+) &= d_{t^+} \cdot (1 - 2p_{t^+}) + V\left(t^* + 1, 0; \mathcal{A}^+\right) \\ &\geq V\left(t^* + 1, 0; \mathcal{A}^+\right) \\ &= V\left(t^* + 1, 0; \mathcal{A}\right) \end{aligned}$$

,

where the inequality follows since $p_{t^+} < \frac{1}{2}$, and the last equality uses the induction hypothesis for $t \in \{t^* + 1, \ldots, |\mathcal{A}| + 1\}$.

- If $k > 0$, we have that

$$\begin{aligned}
V(t^+, 0; \mathcal{A}^+) &= p_{t^+} + d_{t^+} \cdot (1 - p_{t^+}) + V\left(t^* + 1, k; \mathcal{A}^+\right) - \\
&\quad p_{t^+} \cdot \left(V\left(t^* + 1, k; \mathcal{A}^+\right) - V\left(t^* + 1, k-1; \mathcal{A}^+\right)\right) \\
&\geq p_{t^+} + d_{t^+} \cdot (1 - p_{t^+}) - p_{t^+} \cdot (1 + d_{t^*+1}) + V\left(t^* + 1, k; \mathcal{A}^+\right) \\
&= \underbrace{d_{t^+} \cdot (1 - p_{t^+}) - p_{t^+} d_{t^*+1}}_{\geq 0 \text{ since } d_{t^+} \geq d_{t^*+1}, p_{t^+} < \frac{1}{2}} + V\left(t^* + 1, k; \mathcal{A}^+\right) \\
&\geq V\left(t^* + 1, k; \mathcal{A}^+\right) \\
&= V\left(t^* + 1, k; \mathcal{A}\right),
\end{aligned}$$

  where the first inequality uses Claim 2.6, and the last equality uses the induction hypothesis for $t \in \{t^* + 1, \ldots, |\mathcal{A}| + 1\}$.

We now prove the result for $t \in \{1, \ldots, t^* - 1\}$, having just established that $V(t, k; \mathcal{A}^+) \geq V(t, k; \mathcal{A})$ for $t \in \{t^*, t^* + 1, \ldots, |\mathcal{A}| + 1\}$. To do so, we consider two cases based on whether k is non-zero.

- If $k = 0$, then from (2.1), we have

$$\begin{aligned}
V(t, 0; \mathcal{A}^+) &= d_t \cdot (1 - 2p_t) + V(t + 1, 0; \mathcal{A}^+) \\
&\geq d_t \cdot (1 - 2p_t) + V(t + 1, 0; \mathcal{A}) \\
&= V(t + 1, 0; \mathcal{A}).
\end{aligned}$$

  The inequality follows from the induction hypothesis.

- If $k > 0$, then from (2.1), we have

$$\begin{aligned} V(t,k;\mathcal{A}^+) &= (1-p_t)\cdot\big(d_t + V(t+1,k;\mathcal{A}^+)\big) + p_t\cdot\big(1+V(t+1,k-1;\mathcal{A}^+)\big) \\ &\geq (1-p_t)\cdot(d_t + V(t+1,k;\mathcal{A})) + p_t\cdot(1+V(t+1,k-1;\mathcal{A})) \\ &= V(t,k;\mathcal{A}) \end{aligned}$$

  where the inequality follows by the induction hypothesis.

Proof of Lemma 2.5

For ease of notation, we use $\mathcal{Y}_{\text{rel}} = \sum_{\tau \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} Y_\tau$ to denote the random number of slots consumed by the reliable shippers under the RF mechanism. Next, we have

$$
\begin{aligned}
\mathcal{R}(\mathcal{A}_{\text{TOH}}; m) &\geq \hat{\mathcal{R}}(\mathcal{A}_{\text{TOH}}; m) \\
&= \sum_{k=0}^{|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|} \Pr\left[\mathcal{Y}_{\text{rel}} = k\right] \cdot \left(\mathbb{E}\left[\sum_{t \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\text{rel}} = k \right] \right. \\
&\quad \left. + \mathbb{E}\left[\sum_{t \in T_{\text{unrel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\text{rel}} = k \right] \right) \\
&= \sum_{k=0}^{|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|} \Pr\left[\mathcal{Y}_{\text{rel}} = k\right] \cdot \left(\mathbb{E}\left[\sum_{t \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\text{rel}} = k \right] \right. \\
&\quad \left. + \mathbb{E}\left[\sum_{t \in T_{\text{unrel}}} \hat{R}_t(T_{\text{unrel}}, Y; m-k) \right] \right) \\
&= \sum_{k=0}^{|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|} \Pr\left[\mathcal{Y}_{\text{rel}} = k\right] \cdot \left(\mathbb{E}\left[\sum_{t \in \mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\text{rel}} = k \right] \right. \\
&\quad \left. + V(1, m-k; T_{\text{unrel}}) \right) \text{ (by Claim 2.4)} \\
&\geq \underbrace{\sum_{k=0}^{|\mathcal{A}_{\text{TOH}} \cap T_{\text{rel}}|} \Pr\left[\mathcal{Y}_{\text{rel}} = k\right] \cdot \left(k + V(1, m-k; T_{\text{unrel}})\right)}_{\text{(term i)}}
\end{aligned}
$$

The second equality follows from the observation that, under the RF mechanism, we have

$$
\mathbb{E}\left[\sum_{t \in T_{\text{unrel}}} \hat{R}_t(\mathcal{A}_{\text{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\text{rel}} = k \right] = \mathbb{E}\left[\sum_{t \in T_{\text{unrel}}} \hat{R}_t(T_{\text{unrel}}, Y; m-k) \right],
$$

since the profit earned from unreliable shippers is only affected by the number of slots taken by the reliable shippers. The second inequality use the fact that

$$\mathbb{E}\left[\sum_{t\in\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}} \hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\mathrm{rel}} = k\right] \geq k, \tag{B4}$$

since in the event that $\mathcal{Y}_{\mathrm{rel}} = k$, the liner is assured to give away k slots, and then earn additional profit from the no-shows. From here, we have that

$$\begin{aligned}
\text{(term i)} \;&\geq\; \sum_{k=0}^{|\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}|} \Pr\left[\mathcal{Y}_{\mathrm{rel}} = k\right] \cdot (k + V(1, m; T_{\mathrm{unrel}}) - 2k) \\
&=\; V(1, m; T_{\mathrm{unrel}}) - \sum_{k=0}^{|\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}|} \Pr\left[\mathcal{Y}_{\mathrm{rel}} = k\right] \cdot k \\
&\geq\; V(1, m; T_{\mathrm{unrel}}) - \hat{\mathcal{R}}(\mathcal{A}_{\mathrm{TOH}}; m) \\
&\geq\; V(1, m; T_{\mathrm{unrel}}) - \mathcal{R}(\mathcal{A}_{\mathrm{TOH}}; m) \\
&=\; \mathcal{R}^*_{\mathrm{unrel}} - \mathcal{R}(\mathcal{A}_{\mathrm{TOH}}; m) \text{ (by Lemma 2.4).}
\end{aligned}$$

The first inequality follows from a simple iterative application of Claim 2.6, with the additional observation that $1 + d_t \leq 2$, since $d_t \in [0, 1]$. The second inequality follows because

$$\begin{aligned}
\hat{\mathcal{R}}(\mathcal{A}_{\mathrm{TOH}}; m) \;&=\; \mathbb{E}\left[\sum_{t\in\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}} \hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m)\right] \\
&\qquad + \sum_{k=0}^{|\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}|} \Pr\left[\mathcal{Y}_{\mathrm{rel}} = k\right] \cdot V(1, m-k; T_{\mathrm{unrel}}) \\
&\geq\; \mathbb{E}\left[\sum_{t\in\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}} \hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m)\right] \text{ (by Claim 2.5)} \\
&=\; \sum_{k=0}^{|\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}|} \Pr\left[\mathcal{Y}_{\mathrm{rel}} = k\right] \cdot \left(\mathbb{E}\left[\sum_{t\in\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}} \hat{R}_t(\mathcal{A}_{\mathrm{TOH}}, Y; m) \,\middle|\, \mathcal{Y}_{\mathrm{rel}} = k\right]\right. \\
&\geq\; \sum_{k=0}^{|\mathcal{A}_{\mathrm{TOH}}\cap T_{\mathrm{rel}}|} \Pr\left[\mathcal{Y}_{\mathrm{rel}} = k\right] \cdot k \text{ (by (B4)).}
\end{aligned}$$

B.2 Results of Numerical Experiments

Tables B1a, B1b, B1c show the results of our experiments, broken down by test case. In each table, the fourth column gives the percent optimality gap of the TOH policy, measured as $100 \times \frac{\text{OPT}-\mathcal{R}(\mathcal{A}_{\text{TOH}};m)}{\text{OPT}}$, averaged over the 10 instances generated for each test case. The fifth and sixth column show the expected fraction of the m slots allocated to shippers under the two policies, while columns seven and eight offer the expected number of rolled shippers across the two policies. When digesting these results, it is important to recall Lemma 2.4, which states that the TOH is optimal if all shippers are unreliable. This fact explains why the smallest optimality gaps arise for the test cases with $\mathcal{D}$ = Low-Dep and $\mathcal{P} \in$ {Concave-Inc, Convex-Inc}, since in these cases, low deposits are most likely and they lead to lower show-up probabilities. Finally, we note that our results persist across the test cases where $\mathcal{P}$ = Rand, and hence the TOH performs equally well on all three metrics without the assumption that the show-up probabilities are increasing in the deposit size.

T	m	$\mathcal{P}$	Avg. % Gap	Avg. % Usage $\mathcal{A}_{\text{TOH}}$	$\mathcal{A}^*$	Avg. rolled $\mathcal{A}_{\text{TOH}}$	$\mathcal{A}^*$
10	1	Concave-Inc	7.56	0.92	0.95	1.02	1.28
10	3	Concave-Inc	6.13	0.91	0.94	0.69	0.99
10	5	Concave-Inc	9.78	0.80	0.91	0.16	0.66
10	1	Convex-Inc	5.23	0.95	0.89	1.16	0.93
10	3	Convex-Inc	5.14	0.85	0.89	0.52	0.63
10	5	Convex-Inc	0	0.63	0.63	0.06	0.06
10	1	Rand	2.86	0.90	0.95	0.81	1.17
10	3	Rand	6.43	0.91	0.95	0.83	1.22
10	5	Rand	3.32	0.90	0.93	0.46	0.79
20	1	Concave-Inc	8.18	0.96	0.97	1.70	1.86
20	3	Concave-Inc	6.29	0.97	0.97	1.70	1.89
20	5	Concave-Inc	5.29	0.96	0.97	1.51	1.81
20	1	Convex-Inc	6.18	0.99	0.97	2.45	1.97
20	3	Convex-Inc	5.43	0.96	0.94	1.63	1.41
20	5	Convex-Inc	3.46	0.95	0.93	1.18	1.06
20	1	Rand	5.36	0.98	0.98	2.30	2.21
20	3	Rand	5.63	0.98	0.97	2.23	2.09
20	5	Rand	4.13	0.96	0.98	1.63	2.26

(a) Instances with $\mathcal{D}$ = Uni

T	m	$\mathcal{P}$	Avg. % Gap	Avg. % Usage $\mathcal{A}_{\text{TOH}}$	$\mathcal{A}^*$	Avg. rolled $\mathcal{A}_{\text{TOH}}$	$\mathcal{A}^*$
10	1	Concave-Inc	12.94	0.87	0.94	0.67	0.99
10	3	Concave-Inc	7.04	0.91	0.95	0.54	1.03
10	5	Concave-Inc	9.61	0.86	0.95	0.22	0.74
10	1	Convex-Inc	6.86	0.91	0.93	1.02	1.11
10	3	Convex-Inc	4.07	0.91	0.92	0.71	0.79
10	5	Convex-Inc	2.54	0.85	0.87	0.30	0.37
10	1	Rand	8.25	0.92	0.93	0.83	1.05
10	3	Rand	6.43	0.91	0.94	0.71	1.00
10	5	Rand	3.53	0.84	0.88	0.31	0.52
20	1	Concave-Inc	10.86	0.95	0.96	1.16	1.50
20	3	Concave-Inc	11.56	0.93	0.96	1.00	1.38
20	5	Concave-Inc	9.69	0.93	0.96	0.63	1.39
20	1	Convex-Inc	7.36	0.99	0.97	2.55	2.07
20	3	Convex-Inc	7.75	0.98	0.96	2.01	1.72
20	5	Convex-Inc	7.70	0.96	0.95	1.55	1.42
20	1	Rand	5.67	0.98	0.97	2.33	2.13
20	3	Rand	7.68	0.98	0.96	1.97	1.71
20	5	Rand	5.21	0.96	0.97	1.81	1.77

(b) Instances with $\mathcal{D}$ = High-Dep

T	m	$\mathcal{P}$	Avg. % Gap	Avg. % Usage $\mathcal{A}_{\text{TOH}}$	$\mathcal{A}^*$	Avg. rolled $\mathcal{A}_{\text{TOH}}$	$\mathcal{A}^*$
10	1	Rand	3.73	0.93	0.95	1.43	1.47
10	3	Rand	0.23	0.87	0.87	0.72	0.78
10	5	Rand	0.53	0.73	0.73	0.20	0.24
10	1	Concave-Inc	1.64	0.84	0.83	0.65	0.66
10	3	Concave-Inc	0	0.48	0.48	0.05	0.05
10	5	Concave-Inc	0	0.37	0.37	0	0
10	1	Convex-Inc	6.63	0.93	0.96	0.88	1.27
10	3	Convex-Inc	4.13	0.92	0.95	0.79	1.32
10	5	Convex-Inc	0.01	0.85	0.85	0.36	0.44
20	1	Rand	2.65	0.99	0.99	3.27	3.21
20	3	Rand	3.41	0.99	0.98	2.80	2.64
20	5	Rand	2.09	0.97	0.98	1.92	2.25
20	1	Concave-Inc	1.19	0.96	0.96	2.07	1.96
20	3	Concave-Inc	0	0.83	0.83	0.69	0.69
20	5	Concave-Inc	0	0.68	0.68	0.15	0.15
20	1	Convex-Inc	4.31	0.99	0.99	2.29	2.49
20	3	Convex-Inc	4.92	0.99	0.99	2.19	2.62
20	5	Convex-Inc	5.35	0.95	0.99	1.16	2.33

(c) Instances with $\mathcal{D}$ = Low-Dep

Table B1: Average performance of the TOH policy.

Appendix C

Blockchain-Enabled Deep-Tier Supply Chain Finance

C.1 Supplemental Materials

C.1.1 Summary of Notations

To keep track of the different components in our model, we summarize the notations throughout the paper in Table C1.

Table C1: Summary of Model Notations

Parameters	
c	the maximal amount of capital the tier-2 supplier is willing to invest in production.
v	exogenous unit procurement price for tier-1.
e	exogenous unit procurement cost of the tier-1 supplier for emergency product.
θ_2	working capital of tier-2 supplier S_2, $i = 1, 2$.
F	the CDF of θ_2.
θ_1	working capital of the tier-1 supplier S_1.
Decisions	
w	unit tier-0 wholesale price proposed by the manufacturer.
r_m	the tier-0 AP interest rate charged by the manufacturer.
r_s	the tier-1 AP interest rate charged by the tier-1 supplier.
r_b	actual interest rate of bank loan.
y	tier-2 supplier's investment level in production.
B_s	AP amount the tier-2 supplier borrowed from the tier-1 supplier.
B_m	AP amount the tier-1 supplier borrowed from the manufacturer.
Derived	
$\mathcal{P}(\cdot)$	reliability function on the invested capital.
π_0	expected payoff of the manufacturer at time 2
π_1	expected payoff of the tier-1 supplier at time 2
π_2	expected payoff of the tier-2 supplier at time 2
$\bar{\theta}_2$	financing threshold of the tier-2 supplier's working capital
$\bar{e}$	emergency sourcing threshold of the emergency sourcing cost
$\hat{\theta}_2$	contract threshold of the tier-2 supplier's working capital

C.1.2 Summary of Modeling Assumptions

We summarize additional standard assumptions in our model.

(A1) All three players in the deep-tier supply chain are risk-neutral, i.e., they maximize the expected pay-off.

(A2) The capital market is perfect to the tier-1 supplier, (no taxes, transaction costs, and bankruptcy costs); all bank loans are competitively priced (perfectly competitive banking sector).

(A3) The tier-1 supplier is credit-worthy and will repay the loan obligations (if any) to the extent possible, whereas the tier-2 supplier are not.

(A4) We use the following tie-breaking rules: If the tier-1 supplier is indifferent between manufacturer and bank financing, he uses MF.

C.1.3 Additional Results for §3.4

Lemma C.1. *Given any* $\theta_2 \in [0, c)$ *and* $\theta_1 \geq 0$*, the optimal tier-1 AP interest rate is* $r_s^* = z^{*-1}(y^{b*}(w|\theta_2, e), \theta_2)$*, where* $y^{b*}(w|\theta_2, e)$ *is the tier-2 investment level that solves the FOC of* (3.2)*.*

(i) r_s^* *is independent of* θ_1 *but decreases in the tier-0 wholesale price (bounded by emergency sourcing cost, same below);*

(ii) $y^{b*}(w|\theta_2, e)$ *is independent of* θ_1 *but increases in the tier-0 wholesale price.*

(iii) $\pi_1^{b*}(w) := \pi_1^b(y^{b*}(w|\theta_2, e)|w)$ *increases in the tier-0 wholesale price.*

Lemma C.1 reveals that in the competitive capital market environment, whether the tier-1 supplier is capital-constrained does not influence the operation decision, that is, the tier-1 supplier offers an AP interest rate to the tier-2 supplier to stimulate the same investment level $y^{b*}(w|\theta_2, e)$ as if there is no capital constraint. This investment level is independent of the tier-1 supplier's working capital because borrowing from the bank induces a risk-free expected interest rate. Using his own capital and borrowing from the bank have the same financial cost. We find $y^{b*}(w|\theta_2, e) > \theta_2$ always holds, so the tier-2 supplier must borrow from the tier-1 supplier to improve the reliability. If the tier-1 supplier needs to borrow from the bank, i.e., $y > \theta_1 + \theta_2$, it immediately follows from competitive lending equation that the bank loan interest rate on $B_b^* = y^{b*} - \theta_1 - \theta_2$ is $r_b^* = \frac{1}{\mathcal{P}(y^{b*})} - 1$.

Lemma C.1(i)-(iii) reveal the monotonocities of r_s^*, $y^{b*}(w|\theta_2, e)$, and $\pi_1^{b*}(w)$. The reason that $y^{b*}(w|\theta_2, e)$ increases in θ_2 is that a less capital-constrained tier-2 supplier has a low bankruptcy risk, which reduces the cost that the tier-1 supplier offers an AP. The reason that $y^{b*}(w|\theta_2, e)$ increases in the tier-0 wholesale price is two-fold. In the $\mathbb{P}$ case, where the tier-1 does not use **E**, increasing w increases the tier-1's margin when the tier-2's production succeeds, hence increases the tier-1's investment in reliability. In the $\mathbb{PR}$ case, where the tier-1 uses **E** when the tier-2 fails, increasing the emergency sourcing cost decreases the tier-1's margin when the tier-2's production fails, hence increases the tier-1's incentive for proactive risk mitigation.

The tier-1 supplier joins in the business and accepts at least the wholesale price only when the optimal terminal cash level is greater than the reservation option, i.e., rejecting to get zero profit, for any $\theta_2 \in [0, c)$. We should have $\pi_1^b(y^{b*}(w|\theta_2, e)|w) \geq \theta_1, \ \forall \theta_2 \in [0, c)$. It leads to a necessary condition that the manufacturer has to offer a wholesale price high enough to guarantee the tier-1 supplier's positive profit, i.e., $w \geq \underline{w}$, where $\underline{w} := v + y^{b*}(c|0, e) > v$. As the financing issue with capital-constrained suppliers is the focus of this paper, we will focus on the case where the two suppliers' total working capital is constrained to the extent that $\theta_1 + \theta_2 \leq y^{b*}(\underline{w}|c, e)$, where $y^{b*}(\underline{w}|c, e) > c$ by monotonicity. Let $\theta_1^{\max} := y^{b*}(\underline{w}|c, e) - c$

denote the upper bound of the tier-1 supplier's working capital, we only consider the suppliers' working capital space $(\theta_1, \theta_2) \in \mathcal{W} := [0, \theta_1^{\max}] \times [0, c)$ in the remainder of the paper.

Lemma C.2. *The tier-1 supplier's optimal AP interest rate is* $r_s^* = z^{*-1}(\max\{y^{m*}, \theta_1 + \theta_2\}, \theta_2)$*, where* y^{m*} *is the tier-2 investment level that solves the FOC of* (3.3).

(i) $y^{m*} := y^{m*}(w, r_m|\theta_1, \theta_2, e)$ *increases in* w*,* θ_1*, and* θ_2*, but decreases in* r_m*.*

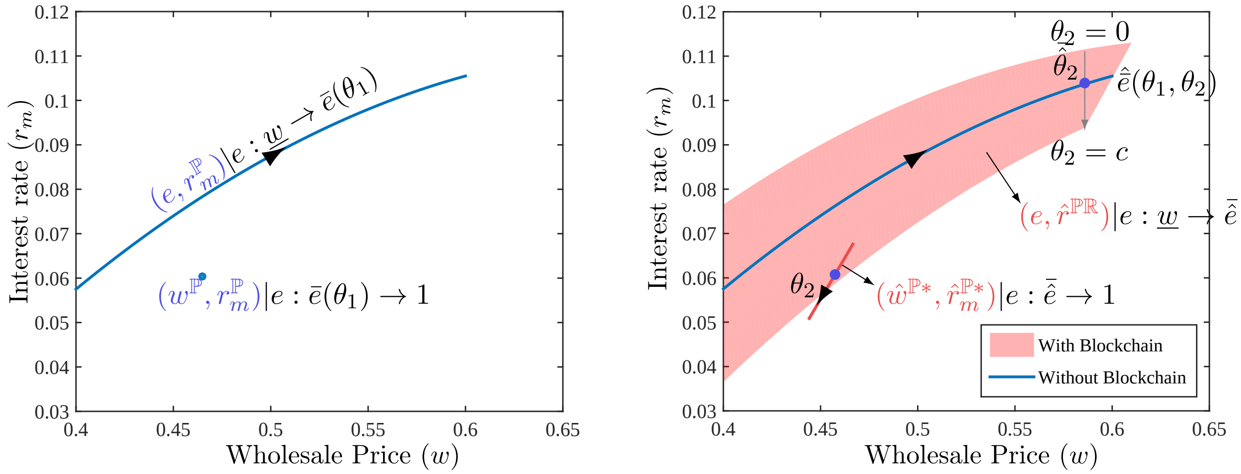
(ii) r_s^* *decreases in* w*,* θ_1*, and* θ_2*;*

(iii) $\pi_1^{m*}(w, r_m) := \pi_1^m(y^{m*}|w, r_m)$ *increases in* w*, but decreases in* r_m*.*

Lemma C.2 characterizes the optimal tier-1 AP interest rate r_s^*. By comparing Lemmas C.1 and C.2, we find the optimal decisions have different structures under BF and MF. Under BF, the tier-1 supplier has a single optimum $y^{b*}(w|\theta_2, e)$ no matter what is the echelon's capital. Under MF, it bears a similar structure as in Lemma 3.1, namely, whether the tier-1 supplier would use financing depends on the relationship between the echelon working capital level $\theta_1 + \theta_2$ and the optimal investment level y^{m*}. If the echelon is wealthy enough to cover the target investment y^{m*}, no financing happens, and the tier-1 supplier uses as much as the echelon capital to the production until $y^{b*}(w|\theta_2, e)$. We also find that $y^{b*}(w|\theta_2, e)$ is independent of θ_1, whereas $y^{m*}(w, r_m|\theta_1, \theta_2, e)$ increases in θ_1.

C.2 Delegate Financing with Mandatory Blockchain Adoption

Figure C1a plots the trajectory of the optimal tier-0 AP contract (w^*, r_m^*) as emergency sourcing cost e increases. The solid trajectory line, $(e, r_m^{\mathbb{P}})$, moving from lower-left to upper-right, corresponds to e increasing from $\underline{w}$ to $\bar{e}(\theta_1)$; the solid point, $(w^{\mathbb{P}}, r_m^{\mathbb{P}})$, corresponds



(a) Without Blockchain (b) With Blockchain

Figure C1: (Color Online) Trajectory of the Optimal Tier-0 AP Contract (as e increases from v to 1)

$e > \bar{e}(\theta_1)$. At $e = \bar{e}(\theta_1)$, the manufacturer is indifferent between using **E** (by offering $(e, r_m^{\mathbb{P}})$) and not using **E** (by offering $(w^{\mathbb{P}}, r_m^{\mathbb{P}})$). The fact that $(w^{\mathbb{P}}, r_m^{\mathbb{P}}) < (\bar{e}(\theta_1), r_m^{\mathbb{P}}(\bar{e}(\theta_1)))$ reflects the trade-off between using a high wholesale price ($w^* = e$) to guarantee a perfect reliability from emergency sourcing and using a low interest rate ($r_m^{\mathbb{P}}$) to encourage tier-2 production reliability investment. The implications of the optimal tier-0 AP contract (w^*, r_m^*) for the tier-1 supplier's financing and risk-mitigation decisions (Proposition 3.1) are illustrated in Figure 3.4b in the parameter space of (e, θ_2).

Figure C1b plots the trajectory of the optimal tier-0 AP contract $(\hat{w}^*, \hat{r}_m^*)$ as emergency sourcing cost e increases and for $\theta_2 \in [0, c)$. The shaded band, $(e, \hat{r}^{\mathbb{PR}})$, moving from lower-left to upper-right, corresponds to e increasing from v to $\bar{\hat{e}}(\theta_1, \theta_2)$, with the top (resp., bottom) boundary of the band corresponding to $\theta_2 = 0$ (resp., $\theta_2 = c$). The red line, $(\hat{w}^{\mathbb{P}*}, \hat{r}_m^{\mathbb{P}*})$, moving from up to down, corresponds to $\theta_2 : 0 \to c$. Overlaying the optimal tier-0 AP contract (w^*, r_m^*) trajectory without blockchain (from Figure C1a), we see how the limited visibility can lead the manufacturer to offer over-priced or under-priced AP contracts compared to that with perfect visibility.

To investigate the impact of the blockchain on performance measures of the supply chain, we incorporate a customized bundle of decisions for each region in Proposition C.1, i.e., (w^*) vs. $(\hat{w}^*)$ for Region I, (w^*, r_m^*) vs. $(\hat{w}^*, \hat{r}_m^*)$ for Region II, (w^*, r_m^*) vs. $(\hat{w}^*, \hat{r}_b^*)$ for Region III, and (w^*, r_b^*) vs. $(\hat{w}^*, \hat{r}_m^*)$ for Region IV. Next, we compare (w^*, r_m^*) and $(\hat{w}^*, \hat{r}_m^*)$, r_s^* and $\hat{r}_s^*$ (equivalently, y_s^* and $\hat{y}_s^*$).

Recall the visibility difference with and without blockchain, we define following expected equity functions at time 1 on the visible information for different parties. Without blockchain, the manufacturer's expected payoff at the equilibrium is defined as $\pi_m^*(\theta_1)$, whereas with blockchain it is defined as $\hat{\pi}_0^{1*}(\theta_1, \theta_2)$. For other parties, there is no information asymmetry between two different scenarios, and let $\pi_1^*(\theta_1, \theta_2)$ and $\hat{\pi}_1^*(\theta_1, \theta_2)$ denote the tier-1 supplier's expected cash at time 1 without and with blockchain, while $\pi_2^*(\theta_1, \theta_2)$ and $\hat{\pi}_2^*(\theta_1, \theta_2)$ denote the tier-2 supplier's expected cash at time 1 without and with blockchain.

We further compare the corresponding equilibrium outcomes in different scenarios i.e., comparing $\pi_i^*(\cdot|e, \theta_2, \theta_1)$ with $\hat{\pi}_i^*(\cdot|e, \theta_2, \theta_1)$, $i \in \{0, 1, 2\}$, to understand the impact of blockchain adoption in deep-tier SCF problem. The tier-2 supplier's expected payoff increases in its equilibrium investment level, but decreases in the AP interest rate offered by the tier-1 supplier, r_s. The tier-1 supplier's expected payoff increases in the wholesale price but decreases in the interest rate of tier-0 AP contract, and but the wholesale price is the main driving-force.

C.2.1 Manufacturer's Problem

We start with the manufacturer's interest rate decision $\hat{r}_m$. By Proposition 3.1, the manufacturer faces two choices: (i) issue a cheaper MF than BF to make the tier-1 supplier takes MF; (ii) issue a more expensive MF than BF to make the tier-1 supplier takes BF. A quick study of the manufacturer's time-2 payoff (Table 3.1) reveals that for any given wholesale price w, the manufacturer is indifferent for any $\hat{r}_m \in (r_b^*(w, \theta_2), \infty)$, because in this case the tier-1's takes the bank loan with interest rate $r_b^*(w, \theta_2)$ and the manufacturer's expected operational payoff and financial payoff are not affected by her interest rate $\hat{r}_m$. For exposition convenience, assume that when the manufacturer prefers the tier-1 supplier to use BF she offers $\hat{r}_m = \infty$. The following lemma characterizes the manufacturer's optimal interest rate $\hat{r}_m^*$ for any given wholesale price.

Theorem C.1. *Given the working capital profile (θ_1, θ_2) observed via blockchain, there exists a threshold $\bar{\hat{e}}(\theta_1, \theta_2)$, such that the optimal tier-0 AP contract $(\hat{w}^*, \hat{r}_m^*)$ takes the following form:*

$$(\hat{w}^*, \hat{r}_m^*) = \begin{cases} \left(e, \hat{r}^{\mathbb{PR}*}(e|e, \theta_2, \theta_1)\right), & \textit{if } e \leq \bar{\hat{e}}(\theta_1, \theta_2); \\ \left(\hat{w}^{\mathbb{P}*}, \hat{r}^{\mathbb{P}*}(\hat{w}^{\mathbb{P}*}|e, \theta_2, \theta_1)\right), & \textit{otherwise.} \end{cases} \tag{C1}$$

Theorem C.1 suggests that the manufacturer's AP contract offering shares a similar decision protocol as in Theorem 3.1, which critically depends on the value of the emergency sourcing cost e. Blockchain-enabled visibility into the tier-2 working capital level, however, allows the manufacturer to fine-tune her AP contract offer, notably in two places. First, the emergency sourcing cost threshold $\bar{\hat{e}}(\theta_1, \theta_2)$ is contingent on both θ_1 and θ_2, whereas the counterpart threshold $\bar{e}(\theta_1)$ (Theorem 3.1) without blockchain independent of θ_2. This can be directly observed by comparing the two plots in Figure C2, which further indicates that $\bar{\hat{e}}(\theta_1, \theta_2)$ decreases in θ_2, implying that when the tier-2 supplier is more capital-constrained, the manufacturer would rely more on the emergency sourcing as the production reliability improvement becomes less effective (more discussion follows in Corollary 3.2). Second, as Lemma 3.1 elucidated, the manufacturer precisely decides when she should let the bank finance the tier-1 (by offering $\hat{r}_m^* > r_b^*$) and when she should finance the tier-1 (by offering

$\hat{r}_m^* \leq r_b^*$) to instill more capital into the tier-2. Corollary 3.2 presents the sensitivity analysis of the optimal tier-0 AP contract parameters with respect to θ_1 and θ_2.

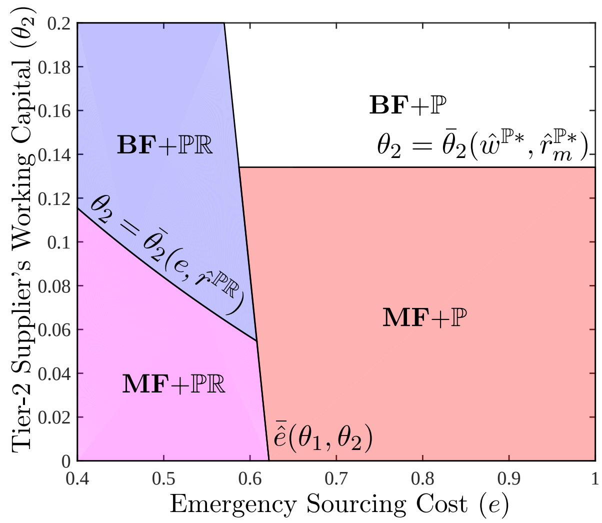


Figure C2: (Color Online) Tier-1 Supplier's Financing and Emergency Sourcing Regions with Mandatory Blockchain Adoption

Corollary 3.2 shows that θ_1 and θ_2 have the same directional effect on the manufacturer's optimal contract offering. This is because, from the manufacturer's perspective, it is the total working capital $\theta_1 + \theta_2$ between the tier-1 and tier-2 suppliers that determines the potential resource for proactive risk mitigation. Increasing either θ_1 and θ_2 will (weakly) increase the reliability investment (Lemmas C.1 and C.2) and decrease the tier-1 and tier-2 suppliers' bankruptcy risk. The manufacturer can lower wholesale price $\hat{w}^*$ to incentivize the tier-1 and lower interest rate $\hat{r}_m^*$ to increase her financing to a less risky supply chain (Corollary 3.2(ii)). For the same reason, she is less likely to use proactive risk mitigation (Corollary 3.2(i)).

C.2.2 The Impact of Blockchain-Enabled Visibility with Mandatory Blockchain

In this section, we compare the supply chain performances with and without blockchain adoption. We start with the manufacturer. For the no-blockchain case, let $\pi_m^*(e,\theta_1,\theta_2) \equiv \pi_0^{k\mathbb{I}}(w^*, r_m^*|e,\theta_1,\theta_2)$, where $\pi_0^{k\mathbb{I}}(w^*, r_m^*|e,\theta_1,\theta_2)$ is as defined in (3.4). Thus, $\pi_m^*(\theta_1,\theta_2)$ represents the manufacturer's actual expected profit by offering (w^*, r_m^*) without blockchain. It is straightforward to establish that the manufacturer is better-off with blockchain, i.e., $\pi_m^*(e,\theta_1,\theta_2) \leq \hat{\pi}_0^{1*}(e,\theta_1,\theta_2)$. This is because the manufacturer's decision based on imperfect information (probability distribution of θ_2) is intended to optimize (3.5), an estimation of the expected profit (3.4); this decision can never outperform her decision based on accurate information (exact value of θ_2) that optimizes the expected profit (3.4).

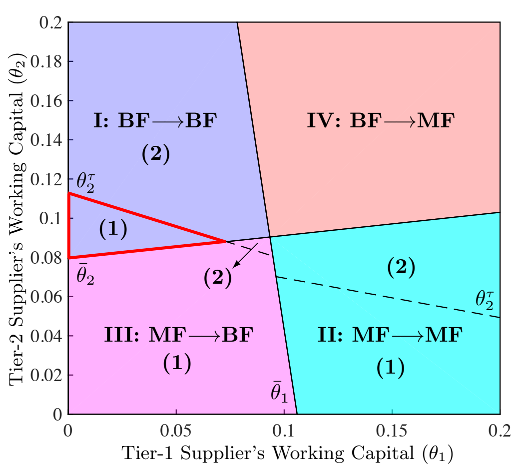
To compare the tier-1 supplier's performance with and without blockchain, we first recognize that blockchain adoption may result in changes in the tier-1's optimal financing choice. Recall that for the no-blockchain case, Proposition 3.1 establishes a tier-2 working capital threshold $\bar{\theta}_2(w, r_m)$ such that the tier-1 supplier chooses MF if and only if $\theta_2 < \bar{\theta}_2(w, r_m)$. For the blockchain case, Lemma C.3 establishes a tier-1 supplier working capital threshold $\bar{\theta}_1(\theta_2)$ such that the tier-1 chooses MF if and only if $\theta_1 \geq \bar{\theta}_1(\theta_2)$. We can divide the (θ_1, θ_2) space into four regions based on the impact of blockchain adoption on the tier-1 supplier's equilibrium financing choice.

For notation convenience, we denote $\bar{\theta}_1(\hat{w}^*, \theta_2)$, $\bar{\theta}_2(w^*, r_m^*)$, $r_b^*(\hat{w}^*, \hat{r}_m^*)$ as $\bar{\theta}_1$, $\bar{\theta}_2$, $\hat{r}_b^*$, respectively. Table C2 summarizes the equilibrium contract terms comparison (proof is provided in Appendix) and Figure C3 summarizes supply chain members' profit comparison without blockchain and with blockchain for Regions I-IV. We will build insights for the comparisons starting with Region I and Region II, where blockchain adoption does not change the tier-1 supplier's financing choice. Because the tier-1's expected profit under BF increases in wholesale price w (Lemma C.1(iii)) and his expected profit under MF increases w and decreases in r_m (Lemma C.2(iii)), we need to understand how blockchain adoption affects those contract parameters at equilibrium – compare w^* and $\hat{w}^*$ in Region I and III and compare (w^*, r_m^*) and $(\hat{w}^*, \hat{r}_m^*)$ in Region II.

Table C2: Equilibrium Contract Terms Comparison in Regions I-IV

Decisions	Tier-0 Wholesale Price	BF/MF Interest Rate	Tier-1 Interest Rate	Tier-2 Investment Level
Region I(1)	$\hat{w}^* \geq w^*$	$\hat{r}_b^* \leq r_b^*$	$\hat{r}_s^* \leq r_s^*$	$\hat{y}^* \geq y^*$
Region I(2)	$\hat{w}^* \leq w^*$	$\hat{r}_b^* \geq r_b^*$	$\hat{r}_s^* \geq r_s^*$	$\hat{y}^* \leq y^*$
Region II(1)	$\hat{w}^* \geq w^*$	$\hat{r}_m^* \geq r_m^*$	$\hat{r}_s^* \geq r_s^*$	$\hat{y}^* \leq y^*$
Region II(2)	$\hat{w}^* \leq w^*$	$\hat{r}_m^* \leq r_m^*$	$\hat{r}_s^* \leq r_s^*$	$\hat{y}^* \geq y^*$
Region III(1)	$\hat{w}^* \geq w^*$	$\hat{r}_b^* \leq r_m^*$ or $\hat{r}_b^* > r_m^*$	$\hat{r}_s^* \leq r_s^*$ or $\hat{r}_s^* > r_s^*$	$\hat{y}^* \leq y^*$ or $\hat{y}^* > y^*$
Region III(2)	$\hat{w}^* \leq w^*$	$\hat{r}_b^* \geq r_m^*$	$\hat{r}_s^* \geq r_s^*$	$\hat{y}^* \leq y^*$
Region IV	$\hat{w}^* \leq w^*$	$\hat{r}_m^* \leq r_b^*$ or $\hat{r}_m^* > r_b^*$	$\hat{r}_b^* \leq r_m^*$ or $\hat{r}_b^* > r_m^*$	$\hat{y}^* \leq y^*$ or $\hat{y}^* > y^*$

Note: For $\hat{w}^* \geq w^*$, equality holds when $e \leq \bar{e}(\theta_1)$; For $\hat{w}^* \leq w^*$, equality holds when $e \leq \bar{\bar{e}}(\theta_1, \theta_2)$



Decisions	M's Profit	S_1's Profit	S_2's Profit	Chain's Profit
Region I(1)	+	+	+	+
Region I(2) & III(2)	+	−	−	−
Region II(1)	+	+	−	+/−
Region II(2)	+	−	+	+/−
Region III(1)	+	+	+/−	+
Region IV	+	−	+/−	+/−

Figure C3: (Color Online) Division of Working Capital Space and Impact of Blockchain Adoption

Recall that without blockchain the manufacturer sets her contract (w^*, r_m^*) based on her probability distribution assessment of the tier-2's working capital θ_2. Lemma C.4 shows that the manufacturer, after gaining perfect visibility of θ_2, when committed to MF (resp., BF), if seeing the tier-2's working capital $\theta_2 = \bar{\bar{\theta}}_2(\theta_1, e)$ (represented by the dashed line in Figure C3), would offer the exact same contract as she would without perfect visibility. That is, $(w^*, r_m^*) = (\hat{w}^*, \hat{r}_m^*)|_{\theta_2=\bar{\bar{\theta}}_2}$ if the manufacturer commits to MF ($\theta_1 \geq \bar{\theta}_1$), $w^* = \hat{w}^*|_{\theta_2=\bar{\bar{\theta}}_2}$ if the manufacturer commits to BF ($\theta_1 < \bar{\theta}_1$). Because $\hat{w}^*$ and $\hat{r}_m^*$ decreases in θ_2 (Corollary 3.2), $\bar{\bar{\theta}}_2(\theta_1, e)$ divides Region I and Region II each into two subregions (as presented in Figure C3 and Table C2). In subregion (1) where $\theta_2 \in [0, \bar{\bar{\theta}}_2)$, perfect visibility leads to higher wholesale price $\hat{w}^*$ and higher interest rate $\hat{r}_m^*$ (resp., lower bank loan interest rate $\hat{r}_b^*$) under MF (resp., under BF). In subregion (2) where $\theta_2 \in (\bar{\bar{\theta}}_2, c)$, perfect visibility leads to lower wholesale price $\hat{w}^*$ and lower interest rate $\hat{r}_m^*$ (resp., higher bank loan interest rate $\hat{r}_b^*$) under MF (resp., under BF). Moreover, we find for the tier-1 supplier, the wholesale price change is the main driving-force when both contract term moving to the same direction. Therefore, Lemma C.4 implies that the tier-1 supplier is better-off (resp., worse-off) with blockchain adoption in subregion (1) (resp., subregion (2)) of Region I and Region II (see Figure C3).

The implications of changes in the tier-0 AP contract for the tier-1's AP interest rate r_s to the tier-2 supplier is straightforward. The tier-1 supplier will increase (resp., decrease) r_s when his own loan interest rate increases (resp., decreases), regardless of whether he finances from the bank or the manufacturer. Consequently, perfect visibility increases (resp., decreases) proactive risk mitigation if the tier-1's loan interest rate deceases (resp., increases) (see the last column of Table C2). Because the tier-2 supplier's expected payoff increases in its reliability investment level, it is better-off with blockchain adoption in Region I(1) and Region 1(2) and worse-off in Region I(2) and Region II(1) (see Figure C3). In both Region I and II, the blockchain adoption preserves the financing choice, but the manufacturer adjust

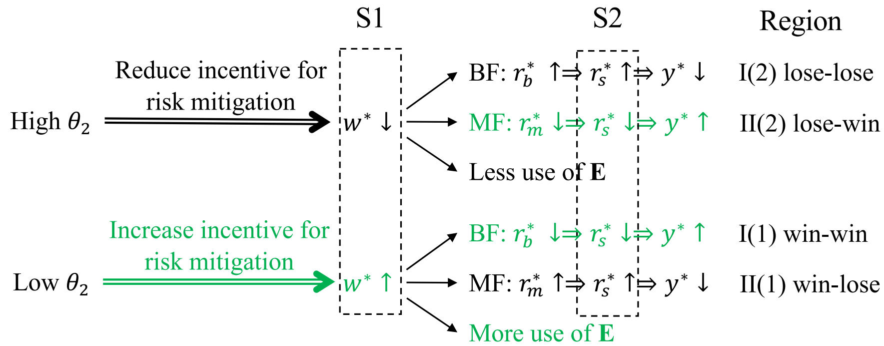


Figure C4: Impact of Blockchain Adoption on Risk Mitigation When the Finance Choice Keeps the Same

the contract terms based upon the observed tier-2 information. We summarize the impact of blockchain adoption on manufacturer's risk mitigation incentives in Region I and II as Figure C4.

In those regions, tier-1 supplier's profits are driven by the wholesale price offered by manufacturer: the profit decreases if the wholesale price decreases. How the manufacturer adjusts wholesale price depends on the value of θ_2. Recall that without visibility, the manufacturer's decision is equivalent to targeting a specific $\bar{\hat{\theta}}_2$. This value divides each region into two subregions, one subregion has θ_2 higher than $\bar{\hat{\theta}}_2$ and the other subregion is below $\bar{\hat{\theta}}_2$.

When the manufacturer finds the actual θ_2 is lower than this value (subregions I(1) and II(1)), the manufacturer realizes the she did not provide sufficient incentive for risk mitigation under limited visibility. She will raise the wholesale price to increase the tier-1's incentive for risk mitigation, which has two-folded implications. First, increasing w (weakly) increases the reactive risk mitigation. Second, how increasing w affects the tier-1's incentive for proactive risk mitigation depends on which financing source the tier-1 uses. Under BF (Region I(1), increasing w decreases the bank's loan interest rate, which, in turn decreases tier-1 interest rate offer to tier-2, and increases reliability investment. This subregion increases both reactive and proactive risk mitigations. Under MF (Region II(1)), increasing w is accompanied by

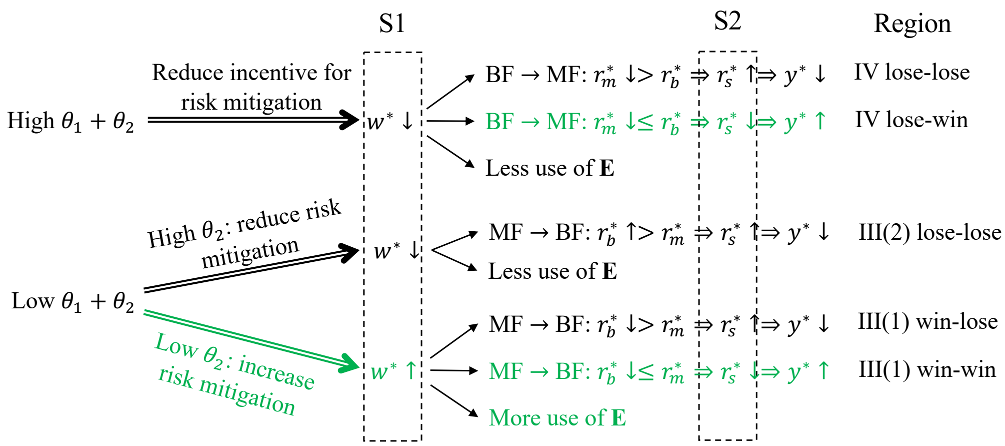


Figure C5: Impact of Blockchain Adoption on Risk Mitigation When the Financing Choice Changes

increasing manufacturer's interest rate, which in turn increases the tier-1 interest rate, and decrease tier-2's reliability investment. In this sub-region, the tier-1 increase uses of **E** but decreases reliability investment.

If θ_2 turns out to be high (subregion I(2) and II(2)), then the manufacturer finds she over-incentivizes the risk-mitigation effort under limited visibility. She will reduce that incentive by reducing wholesale price, which reduces the reactive risk mitigation and the tier-1 will be worse-off. If the tier-1 uses BF, reducing of w translates into increasing bank interest rate which reduces proactive risk mitigation, and tier-2 is worse-off as well. In this region, the tier-1 both use less of **E** and invest less on reliability improvement. If tier-1 uses the MF, reducing w is accompanied by reducing r_m, which increases tier-2's reliability investment. In this region, the tier-1 reduces the reactive but increase proactive risk mitigation.

In both Region III and IV, the blockchain adoption changes the financing choice. We summarize the impact of blockchain adoption on manufacturer's risk mitigation incentives in these regions as Figure C5.

Region IV shows that the manufacturer, after observing an echelon that is not capital-constrained visibility, will adjust her AP contract offer to induce the tier-1 supplier to switch from BF to MF. Lemma C.4(i) states that $\bar{\hat{\bar{\theta}}}_2 < \bar{\theta}_2$. It implies that in Region IV, switching the tier-1 from BF to MF is achieved by the manufacturer's lowering her wholesale price and interest rate, i.e., $\hat{w}^* \leq w^*$, $\hat{r}_m^* \leq r_m^*$. Had the tier-1 supplier chosen MF at (w^*, r_m^*) without blockchain, he would find himself worse-off after blockchain adoption because his expected payoff under MF decreases in the wholesale price, and the reactive risk mitigation reduces. However, the change of financing price from BF to MF is not determined even though the manufacturer offers a cheaper interest rate. Thus, blockchain adoption's impact on the proactive risk mitigation is indeterminate.

In Region III, the manufacturer, after observing a severely capital-constrained echelon, will adjust her AP contract offer to induce the tier-1 supplier to switch from MF to BF. Unlike Region IV, this region is divided into two subregions by $\bar{\hat{\bar{\theta}}}_2$, implying that the manufacturer will set $\hat{r}_m^* = \infty$ to induce the tier-1 supplier to choose BF, but will increase wholesale price ($\hat{w}^* \geq w^*$) in Region III(1) where $\theta_2 < \bar{\hat{\bar{\theta}}}_2$, and decrease wholesale price ($\hat{w}^* \leq w^*$) in Region III(2) where $\theta_2 > \bar{\hat{\bar{\theta}}}_2$. Using the reasoning similar to that for Region IV, the tier-1 supplier is worse-off with blockchain adoption in Region III(2): had the tier-1 supplier chosen BF at (w^*, r_m^*) without blockchain, he would find himself worse-off after blockchain adoption because of the wholesale price decrease. The fact that the tier-1 chooses MF over BF without blockchain suggests that he earns a higher profit under MF and will be worse-off with blockchain adoption. Moreover, the manufacturer's interest rate without blockchain, r_m^*, is lower than the bank loan interest rate $r_b^*(w^*)$, and the wholesale price decrease leads to a higher bank loan interest rate $\hat{r}_b^*(w^*)$. Consequently, the proactive risk mitigation decreases and the tier-2 supplier is worse-off with blockchain adoption in Region III(2) (see Figure C3).

The use of the reactive risk mitigation is determined by the threshold of emergency sourcing cost, i.e., $\bar{e}(\theta_1)$ without blockchain and $\bar{\bar{e}}(\theta_1, \theta_2)$ with blockchain. If the manufacturer observes a more severely capital-constrained tier-2 supplier (e.g., $\theta_2 \leq \theta_2^\tau$) than her expectation without visibility, she will increase the emergency cost threshold based on Corollary 3.2(i) that the threshold decreases in θ_2. The intuition behind is that if the blockchain incentivizes the manufacturer to offer a higher wholesale price because the tier-2 supplier is capital-constrained and not reliable, it will encourage the tier-1 supplier to use the emergency source more to mitigate the disruption risk reactively.

As we discussed above, improved visibility always benefits the manufacturer, who can incentivize both reactive and proactive risk mitigation with full supply chain information. This result is consistent with practical observations that it is the downstream powerful manufacturer who advocates (and even requires) the adoption of such blockchain platform for deep-tier financing (e.g., Foxconn and Samsung examples mentioned earlier). However, the manufacturer's gain can be at the cost of the tier-1, the tier-2, and even the entire supply chain. The tier-1 supplier is worse-off when the tier-2 supplier is less capital-constrained, because the manufacturer uses the information to adjust her AP contract offer to lower the wholesale price offered to the tier-1 relying more on tier-2's own capital for reliability investment. For this reason, the tier-2 supplier can be worse-off when it is not severely capital-constrained. When the tier-2 is severely capital-constrained but the tier-1 is not, the tier-2 supplier can be worse-off because the manufacturer raises her interest rate to the tier-1 supplier to cope with the tier-2 bankruptcy risk.

The "win-win-win" situation happens in Region I(1) (as highlighted in Figure C3) where the tier-1 supplier is severely capital-constrained but the tier-2 supplier's working capital falls into a certain medium range. Recall that in Region I(1), blockchain adoption would not change the tier-1 supplier's financing choice (keep using BF). Under BF, perfect visibility leads to higher wholesale price $\hat{w}^*$ and lower bank loan interest rate $\hat{r}_b^*$, thus benefits the tier-1 supplier, who in turn would decrease his AP interest rate offered to the tier-2 and thus benefits the tier-2 as well. Such virtuous cycle would easily collapse if either the tier-1 or the tier-2 supplier becomes less capital-constrained.

What drive the total supply chain profit are the two risk-mitigation measures: proactive (reliability investment) and reactive (use of **E**). When the reliability investment increases and the supply chain is more likely to use **E** when tier-2 production fails, the supply chain benefits. Region I(1) is one such region. However, in Region I(2), Region III(2), and Region IV, reliability and use of emergency sourcing reduce, and the supply chain is worse-off.

C.3 Proofs of Statements

We present all the proofs of the Lemmas, Propositions, and Theorems of Chapter 3 in this Appendix.

Proof of Lemma 3.1:

Proof. The first-order derivatives of π_{2r} and π_{2a} in the investment level y are:

$$\frac{\partial \pi_{2r}}{\partial y} = \mathcal{P}'(y)v - 1 \geq 0, \quad \text{and} \quad \frac{\partial \pi_{2a}}{\partial y} = \mathcal{P}'(y)[v - (1+r_s)(y-\theta_2)] - \mathcal{P}(y)(1+r_s),$$

since $y \leq c$ and $\mathcal{P}'(c)v - 1 = 0$. The second-order derivatives of π_{2a} in the investment level y is:

$$\frac{\partial^2 \pi_{2a}}{\partial y^2} = \mathcal{P}''(y)[v - (1+r_s)(y-\theta_2)] - 2\mathcal{P}'(y)(1+r_s) < 0,$$

since $\mathcal{P}''(y) < 0$ and $\mathcal{P}'(y) > 0$. Hence π_2 is increasing in the first segment and concave in the second segment. We can show that π_2 is unimodal by the continuity at $y = \theta_2$. Let $z^*(r_s|\theta_2)$ solving $\mathcal{P}'(y)[v - (1 + r_s)(y - \theta_2)] - \mathcal{P}(y)(1 + r_s) = 0$ denote the optimum of the second segment. Since π_2 is continuous and $\frac{\partial \pi_{2r}}{\partial y} > \frac{\partial \pi_{2a}}{\partial y}$ at $y = \theta_2$, the optimal investment level $y^* = \theta_2$ if $z^*(r_s|\theta_2) < \theta_2$, but $y^* = z^*(r_s|\theta_2)$ otherwise. □

Proof of Proposition 3.1:

Proof. We first proof Lemma C.1 and Lemma C.2 in Appendix B.3. When $y \leq \theta_1 + \theta_2$, the tier-1 supplier's problem can be formulated as follows for any k:

$$\max_{y \leq \theta_1 + \theta_2} \pi_1^k(r_s|w, r_m) = \mathcal{P}(y)[w - v + (1 + r_s)(y - \theta_2)] + \theta_1 + \theta_2 - y + (1 - \mathcal{P}(y))(w - e)^+.$$

We have the first-order derivative of π_1 in y as:

$$\frac{\partial \pi_1}{\partial y} = \mathcal{P}'(y)\min(w, e) + \mathcal{P}(y)\partial_y r_s(y - \theta_2) - 1,$$

And the second-order derivative is,

$$\frac{\partial \pi_1}{\partial y} = \mathcal{P}''(y)\min(w, e) + \mathcal{P}(y)\partial_y r_s + \mathcal{P}'(y)\partial_y r_s(y - \theta_2) - \mathcal{P}(y)\partial^2_{y^2} r_s < 0,$$

Hence π_1 is concave in y. Let y^{b*} denote the solution of the following FOC,

$$\mathcal{P}'(y)\min(w, e) + \mathcal{P}(y)(y - \theta_2)\partial_y r_s - 1 = 0, \tag{C2}$$

We have y^{b*} is the interior maximizer under no-financing.

When $y > \theta_1 + \theta_2$, the tier-1 supplier's problem can be formulated as follows for any k :

$$\pi_1^k(r_s|w, r_k) = \begin{cases} \mathcal{P}(y)[w - v + (1+r_s)(y-\theta_2)] + (1-\mathcal{P}(y))(w-e)^+, & \text{if } k = b; \\ \mathcal{P}(y)[w - v + (1+r_s)B_s - (1+r_m)B_m] + (1-\mathcal{P}(y))(w-e-(1+r_m)B_m)^+, & \text{if } k = m. \end{cases}$$

The first-order derivatives of π_1^b and π_1^m in y are:

$$\begin{aligned} \frac{\partial \pi_1^b}{\partial y} &= \mathcal{P}'(y)\min(w, e) + \mathcal{P}(y)(y-\theta_2)\frac{\partial r_s}{\partial y} - 1, \\ \frac{\partial \pi_1^m}{\partial y} &= \mathcal{P}'(y)[w - (1+r_m)(y-\theta_1-\theta_2)] + \mathcal{P}(y)\left[(y-\theta_2)\frac{\partial r_s}{\partial y} - (1+r_m)\right]. \end{aligned}$$

The corresponding second-order derivatives are both negative. Hence, π_1^b and π_1^m are concave in y. The FOC of the BF case is the same as the no financing case, i.e., equation (C2). Let y^{m*} denote the solution of the following FOC for MF,

$$\mathcal{P}'(y)\left[w - (1+r_m)(y-\theta_1-\theta_2)\right] + \mathcal{P}(y)\left[(y-\theta_2)\partial_y r_s - (1+r_m)\right] = 0. \tag{C3}$$

The optimal tier-1 interest rate and the tier-2 supplier's response solving (KF) and $\mathcal{P}'(y)[v - (1+r_s)(y-\theta_2)] - \mathcal{P}(y)(1+r_s) = 0$ for $k = b, m$. Since $\frac{\partial \pi_1^b}{\partial y} \geq \frac{\partial \pi_1^m}{\partial y}$, we have that fixing $(\theta_1, \theta_2, w, r_m)$, $y_b \geq y_m$.

To prove Proposition 3.1, we need to show that $r_m \leq r_b^*$ is the sufficient and necessary condition of $\pi_1^m(y^{m*}|w, r_m) \geq \pi_1^b(y^{b*}|w, r_m)$.

- Sufficiency: By $r_m \leq r_b^*$, we have $\pi_1^b(y^{b*}|w, r_m) \leq \pi_1^m(y^{b*}|w, r_m) \leq \pi_1^m(y^{m*}|w, r_m)$.
- Necessity: Suppose $r_m > r_b^*$, we have $\pi_1^m(y^{m*}|w, r_m) < \pi_1^m(y^{m*}|w, r_b^*) = \pi_1^b(y^{m*}|w, r_b^*) \leq \pi_1^b(y^{b*}|w, r_b^*) = \pi_1^b(y^{b*}|w, r_m)$.

To find the financing threshold of θ_2, we compare the first-order derivatives of π_1^b and π_1^m on θ_2 as follows,

$$\begin{aligned}
\frac{\partial \pi_1^b}{\partial \theta_2} &= \mathcal{P}'(y)[\min(w,e) - v + (1+r_s)(y-\theta_2)]\frac{\partial y}{\partial \theta_2} \\
&\quad + \mathcal{P}(y)\left[(1+r_s)\left(\frac{\partial y}{\partial \theta_2} - 1\right) + (y-\theta_2)\frac{\partial r_2}{\partial \theta_2})\right] > 0, \\
\frac{\partial \pi_1^m}{\partial \theta_2} &= \mathcal{P}'(y)[w - v + (1+r_m)\theta_1 + (r_s - r_m)(y-\theta_2)]\frac{\partial y}{\partial \theta_2} \\
&\quad + \mathcal{P}(y)\left[(r_s - r_m)\left(\frac{\partial y}{\partial \theta_2} - 1\right) + (y-\theta_2)\frac{\partial r_2}{\partial \theta_2})\right] > 0.
\end{aligned}$$

We have $\frac{\partial \pi_1^b}{\partial \theta_2} \geq \frac{\partial \pi_1^m}{\partial \theta_2}$ since $\theta_1 < y - \theta_2$, that is, the tier-1 supplier's expected cash position increases in the tier-2's initial capital under both financing schemes, but faster under BF. In order to prove the existence of the threshold, we only need to show the $\pi_1^{b*} \leq \pi_1^{m*}$ when θ_2 is small, but $\pi_1^{b*} \geq \pi_1^{m*}$ when θ_2 is large.

Let $\theta_2 = 0$, the optimal investment level under BF y^{b*} solves $\mathcal{P}'(y)w + \mathcal{P}(y)y\frac{\partial r_s}{\partial y} = 1$, which is strictly less than c. Hence, for any $r_m \leq \frac{1}{\mathcal{P}(y^{b*}(0|e,e))} = r_b$, we have $\pi_1^{b*} \leq \pi_1^{m*}$. Similarly, when $\theta_2 = c$, the optimal investment level under BF is $y^{b*}(c|v) = c$. Hence, for any $r_m \leq \frac{1}{\mathcal{P}(c)} = r_b$, we have $\pi_1^{b*} \geq \pi_1^{m*}$. Therefore, for any $r_m \in \left[\frac{1}{\mathcal{P}(c)}, \frac{1}{\mathcal{P}(y^{b*}(0|e,e))}\right]$, there exists a $\bar{\theta}_2 \in [0, c)$ such that $\pi_1^{b*} = \pi_1^{m*}$ and $\pi_1^{b*} \leq \pi_1^{m*}$ iff $\theta_2 \leq \bar{\theta}_2$. We nominate this θ_2 as $\bar{\theta}_2(w, r_m)$.

The monotonicity of r_m is trivial. To show the monotonicity of w is equivalent to show that y^{b*} increases in w. It can be proved by the fact that the second-order partial derivative of π_1^b with regard to y and w is, $\frac{\partial^2 \pi_1^b}{\partial y \partial w} = \mathcal{P}'(y) > 0$. Hence, y^{b*} increases in w. □

Proof Theorem 3.1 and Corollary 3.1:

Proof. The manufacturer's expected payoff function of the tier-0 AP contract (w, r_m), given the tier-2 supplier with working capital θ_2, takes the following forms when the tier-1 supplier using BF and MF, respectively.

$$\pi_0^b(w, r_m|e, \theta_1, \theta_2) = \begin{cases} \mathcal{P}(y^{b*})(1-w), & \text{if } w < e; \\ 1-w, & \text{otherwise.} \end{cases} \tag{C4}$$

$$\pi_0^m(w, r_m|e, \theta_1, \theta_2) = \begin{cases} \mathcal{P}(y^{m*})[1-w+(1+r_m)B_m] - B_m, & \text{if } w < e; \\ \mathcal{P}(y^{m*})[e-w+(1+r_m)B_m] + 1 - e - B_m, & \text{otherwise.} \end{cases} \tag{C5}$$

Let π_m^I denote the corresponding payoff function when the wholesale price results in case $\mathbb{I}$. Then, we have

$$\pi_0^{\mathbb{P}}(w, r_m|e, \theta_1) = \int_0^{\bar{\theta}_2} \mathcal{P}(y^{m*})[1-w+(1+r_m)B_m] - B_m dF(\theta_2) + \int_{\bar{\theta}_2}^{c} (1-w)\mathcal{P}(y^{b*})dF(\theta_2);$$
$$\pi_0^{\mathbb{PR}}(w, r_m|e, \theta_1) = \int_0^{\bar{\theta}_2} \mathcal{P}(y^{m*})[e-w+(1+r_m)B_m] + 1 - e - B_m dF(\theta_2) + (1-w)[1-F(\bar{\theta}_2)].$$

All two cases are concave in (w, r_m), since the Hessian matrix is negative-definite, but the entire function $\pi_0(w, r_m|e, \theta_1)$ is not because of discontinuity. The high price case has the following first-order derivative,

$$\frac{\partial \pi_0^{\mathbb{PR}}}{\partial w} = \int_0^{\bar{\theta}_2} \left[\mathcal{P}'(y^{m*})e + \mathcal{P}(y^{m*})(y^{m*} - \theta_2)\frac{\partial r_s}{\partial y} - 1\right] \frac{\partial y^{m*}}{\partial w} - \mathcal{P}(y^{m*})dF(\theta_2) - [1 - \bar{F}(\theta_2))] < 0,$$

since both terms is negative. Hence, the optimal wholesale price in this case is $w^* = e$. We has the following FOC for r_m

$$\int_0^{\bar{\theta}_2} [\mathcal{P}'(y^{m*})(1+r_m)B_m + \mathcal{P}(y^{m*})(1+r_m) - 1] \frac{\partial y^{m*}}{\partial r_m} + \mathcal{P}(y^{m*})B_m dF(\theta_2) = 0, \tag{C6}$$

The low price case has following FOCs, and let $(w^{\mathbb{P}}, r_m^{\mathbb{P}})$ denote the solution of them,

$$\int_0^{\bar{\theta}_2}\left[\mathcal{P}'(y^{m*})+\mathcal{P}(y^{m*})(y^{m*}-\theta_2)\frac{\partial r_s}{\partial y}-1\right]\frac{\partial y^{m*}}{\partial w}-\mathcal{P}(y^{m*})dF(\theta_2)$$
$$+\int_{\bar{\theta}_2}^{c}\mathcal{P}'(y^{b*})(1-w)\frac{\partial(y^{b*})}{\partial w}-\mathcal{P}(y^{b*})dF(\theta_2)=0, \tag{C7}$$

$$\int_0^{\bar{\theta}_2}\left[\mathcal{P}'(y^{m*})[1-w+(1+r_m)B_m]+\mathcal{P}(y^{m*})(1+r_m)-1\right]\frac{\partial y^{m*}}{\partial r_m}$$
$$+\mathcal{P}(y^{m*})B_m dF(\theta_2)+(1-w)\int_{\bar{\theta}_2}^{c}\mathcal{P}'(y^{b*})\frac{\partial y^{b*}}{\partial r_m}dF(\theta_2)=0, \tag{C8}$$

By fixing w, we have $r_m^{\mathbb{P}} < r_m^{\mathbb{P}}$ by comparing FOCs (C8) and (C6). Therefore, the candidate local maximizers for the manufacturer's expected payoff are $w^{\mathbb{P}}$ and e.

We get $\pi_0^{\mathbb{P}\mathbb{R}}(w, r_m|e, \theta_1) > \pi_0^{\mathbb{P}}(w, r_m|e, \theta_1)$ when fixing $(w, r_m|e, \theta_1)$. Recall the monotonicity of π_m^H, for each of them there exists a unique emergency sourcing cost denoted by $\bar{e}$, such that $\pi_m^H(e, r_m|e, \theta_1) = \pi_m^L(w^{\mathbb{P}}, r_m|e, \theta_1)$, where $\bar{e} > w^{\mathbb{P}}$.

We then prove the sensitivities (ii) - (iv).

(ii). For $\bar{e}(\theta_1)$'s monotonicity, we need to compare $\frac{\partial \pi_0^{\mathbb{P}}}{\partial \theta_1}$ at $(w^{\mathbb{P}}, r_m^{\mathbb{P}})$ with $\frac{\partial \pi_0^{\mathbb{P}\mathbb{R}}}{\partial \theta_1}$ at $(e, r_m^{\mathbb{P}})$. Their difference takes the following form,

$$\frac{\partial \pi_0^{\mathbb{P}}(w^{\mathbb{P}}, r_m^{\mathbb{P}})}{\partial \theta_1}-\frac{\partial \pi_0^{\mathbb{P}\mathbb{R}}(e, r_m^{\mathbb{P}})}{\partial \theta_1}=\int_0^{\bar{\theta}_2}\mathcal{P}'(y^{m*})(1-w^{\mathbb{P}})\frac{\partial y^{m*}}{\partial \theta_1}-\mathcal{P}\frac{\partial w^{\mathbb{P}}}{\partial \theta_1}dF(\theta_2)>0$$

Hence the manufacturer's optimal profits intercepts faster as θ_1 increases, which implies $\bar{e}(\theta_1)$ is a decreasing function.

(iii) For $w^{\mathbb{P}}$ and $r_m^{\mathbb{P}}$'s sensitivity. It is trivial that they are independent of e. For the sensitivity in θ_1, we find their second order partial derivatives take the following forms,

$$\frac{\partial^2 \pi_0^{\mathbb{P}}}{\partial w \partial \theta_1} = \int_0^{\bar{\theta}_2} \frac{\partial y^{m*}}{\partial \theta_1} \left\{ \frac{\partial y^{m*}}{\partial w} \left[\mathcal{P}''(y^{m*}) + \mathcal{P}(y^{m*}) \frac{\partial r_s}{\partial y} \right] - \mathcal{P}'(y^{m*}) \left[1 - \frac{\partial y^{m*}}{\partial w}(y^{m*} - \theta_1) \right] \right\} dF(\theta_2) < 0,$$

$$\begin{aligned} \frac{\partial^2 \pi_0^{\mathbb{P}}}{\partial r_m \partial \theta_1} &= \int_0^{\bar{\theta}_2} \frac{\partial y^{m*}}{\partial \theta_1} \left\{ \frac{\partial y^{m*}}{\partial r_m} [\mathcal{P}''(y^{m*})[1 - w + (1 + r_m) B_m] + 2\mathcal{P}'(y^{m*})(1 + r_m)] \right. \\ &\quad \left. - \mathcal{P}'(y^{m*}) B_m + \mathcal{P}(y^{m*}) \right\} - \mathcal{P}'(1 + r_m) - \mathcal{P}(y^{m*}) dF(\theta_2) < 0. \end{aligned}$$

Hence, both $w^{\mathbb{P}}$ and $r_m^{\mathbb{P}}$ decreases in θ_1. We can get similar result that $r_m^{\mathbb{P}}$ decreases in θ_1 by checking $\frac{\partial^2 \pi_0^{\mathbb{PR}}}{\partial r_m \partial \theta_1} < 0$.

Corollary 3.1 can be proved by analyzing the second order partial derivatives. □

Proof of Theorem 3.2:

Proof. There are several steps to prove this Theorem.

Step 1: financing choice when blockchain is adopted.

Lemma C.3. *Given (w, θ_2), there exists a unique threshold $\bar{\theta}_1(w, \theta_2)$ such that*

(i) if $\theta_1 \geq \bar{\theta}_1(w, \theta_2)$, the optimal tier-0 interest rate $\hat{r}_m^*$ takes the following form:

$$\hat{r}_m^*(w|e, \theta_2, \theta_1) = \begin{cases} \hat{r}_m^{\mathbb{P}*}, & \text{if } w < e; \\ \hat{r}_m^{\mathbb{PR}*}, & \text{if } w \geq e; \end{cases} \tag{C9}$$

(ii) otherwise, $\hat{r}_m^*(w|e, \theta_2, \theta_1) = \infty$*.*

Here, $\hat{r}_m^{\mathbb{I}*}$ *solves the KKT of* (3.9) *with respect to* $\hat{r}_m$ *with* $k = m$*,* $\mathbb{I} \in \{\mathbb{P}, \mathbb{PR}\}$*; and* $\hat{r}_m^{\mathbb{I}*} < r_b^*(w, \theta_2)$*.*

Proof: Given the manufacturer offers a wholesale price w to the tier-1 supplier, the manufacturer's expected profit takes the following forms when the tier-1 supplier using BF and MF, respectively:

$$\hat{\pi}_0^b(w, r_m|e, \theta_1, \theta_2) = \begin{cases} \mathcal{P}(y^{b*})(1-w), & \text{if } w < e; \\ \mathcal{P}(y^{b*})(e-w) + 1 - e, & \text{otherwise.} \end{cases} \tag{C10}$$

$$\hat{\pi}_0^m(w, r_m|e, \theta_1, \theta_2) = \begin{cases} \mathcal{P}(y^{m*})(1-w+(1+r_m)B_m) - B_m, & \text{if } w < e; \\ \mathcal{P}(y^{m*})(e-w+(1+r_m)B_m) + 1 - e - B_m, & \text{otherwise.} \end{cases} \tag{C11}$$

where the advance payment amount $B_m = (y - \theta_1 - \theta_2)^+$. It is trivial that under BF, $\hat{\pi}_0^b$ is independent of r_m, whereas under MF, it is concave in r_m for both $w < e$ and $w \geq e$. Hence, if the manufacturer wants the tier-1 supplier use BF, she will set $r_m = \infty$. The KKT conditions under MF when $w < e$ are

$$\begin{cases} \lambda(\hat{\pi}_1^{1*} - \pi_1^*) = 0; \\ \mu(\hat{y}^{1*} - y^*) = 0; \\ [\mathcal{P}'(y^{m*})[1-w+(1+r_m)B_m] + \mathcal{P}(y^{m*})(1+r_m) - 1]\dfrac{\partial y^{m*}}{\partial r_m} + \mathcal{P}(y^{m*})B_m + \lambda\dfrac{\partial \hat{\pi}_1^{1*}}{\partial r_m} + \mu\dfrac{\partial \hat{y}^{1*}}{\partial r_m} = 0; \\ [\mathcal{P}'(y^{m*})(1+r_m)B_m + \mathcal{P}(y^{m*})(1+r_m) - 1]\dfrac{\partial y^{m*}}{\partial r_m} + \mathcal{P}(y^{m*})B_m + \lambda\dfrac{\partial \hat{\pi}_1^{1*}}{\partial w} + \mu\dfrac{\partial \hat{y}^{1*}}{\partial w} = 0. \end{cases} \tag{C12}$$

and when $w \geq e$, the last equation is substituted with

$$[\mathcal{P}'(y^{m*})(1+r_m)B_m + \mathcal{P}(y^{m*})(1+r_m) - 1]\frac{\partial y^{m*}}{\partial r_m} + \mathcal{P}(y^{m*})B_m = 0. \tag{C13}$$

Let $\hat{r}_m^{\mathbb{P}*}$ (resp., $\hat{r}_m^{\mathbb{PR}*}$) denote the maximizer of when $w < e$ ($w \geq e$), which takes the solution of KKT conditions (C12) ((C13)) with $r_b^*(w, \theta_2)$.

The last step is to find the optimal r_m is to compare $\hat{\pi}_0^b(w, \infty|e, \theta_2, \theta_1)$ with $\hat{\pi}_0^m(w, \hat{r}_m^{\mathbb{I}*}|e, \theta_2, \theta_1)$. We find the former is independent of θ_1, since y^{b*} is independent of θ_1, whereas the latter increases in θ_1. Also we have when $\theta_1 = 0$, the former is greater but when $\theta_1 \to \infty$, the latter is greater. Therefore, there exists a threshold denoted by $\bar{\theta}_1(\theta_2)$ such that the former is greater when $\theta_1 \in [0, \bar{\theta}_1(\theta_2)]$ but the latter is greater when $\theta_1 \in [\bar{\theta}_1(\theta_2), \infty]$. We have $\bar{\theta}$ decrease in θ_2 by the second-order partial derivatives w.r.t. θ_1 and θ_2.

Proposition C.1. *From no-blockchain to blockchain adoption, in equilibrium, the tier-1 supplier:*

(i) keeps using BF in Region I: $\{(\theta_1, \theta_2) : \theta_1 < \bar{\theta}_1(\hat{w}^{1*}, \theta_2), \theta_2 > \bar{\theta}_2(w^*, r_m^{1*})\}$*;*

(ii) keeps using MF in Region II: $\{(\theta_1, \theta_2) : \theta_1 \geq \bar{\theta}_1(\hat{w}^{1*}, \theta_2), \theta_2 \leq \bar{\theta}_2(w^{1*}, r_m^{1*})\}$*;*

(iii) switches from MF to BF in Region III: $\{(\theta_1, \theta_2) : \theta_1 < \bar{\theta}_1(\hat{w}^{1*}, \theta_2), \theta_2 \leq \bar{\theta}_2(w^{1*}, r_m^{1*})\}$*;*

(iv) switches from BF to MF in Region IV: $\{(\theta_1, \theta_2) : \theta_1 \geq \bar{\theta}_1(\hat{w}^{1*}, \theta_2), \theta_2 > \bar{\theta}_2(w^{1*}, r_m^{1*})\}$*.*

Proof: The unique solution of KKT conditions (C12) and (C13) exists because of the concavity on (w, r_m) by testing the Hessian Matrix is negative semi-definite for both cases.

We find the manufacturer's expected payoff is supermodular in the interest rate $\hat{r}_m^1$ and the wholesale price $\hat{w}^1$ since the second-order derivatives on (w, r_m) is positive, so the interest rate under low price case is lower, i.e., $\hat{r}_m^{\mathbb{P}*} < \hat{r}_m^{\mathbb{PR}*}$. Similar as the results we find from Theorem 3.1, whether to choose high or low wholesale price depends on the value of emergency source. Recall the monotonicity of $\hat{\pi}_0^1(\hat{w}^1, \hat{r}_m^1|e, \theta_2, \theta_1)$, there exists a unique threshold emergency sourcing cost denoted by $\bar{\hat{e}}$, such that $\hat{\pi}_0^1(e, \hat{r}_m^1|e, \theta_2, \theta_1) = \hat{\pi}_0^1(\hat{w}^{\mathbb{P}*}, \hat{r}_m^1|e, \theta_2, \theta_1)$, where $\bar{\hat{e}} > \hat{w}^{\mathbb{P}*}$. We obtain that $\hat{\pi}_0^1(e, \hat{r}_m^1|e, \theta_2, \theta_1) \geq \hat{\pi}_0^1(\hat{w}^{\mathbb{P}*}, \hat{r}_m^1|e, \theta_2, \theta_1)$ only when $e \leq \bar{\hat{e}}$.

We also get the following monotonicity: Since $\hat{r}_b$ is independent of θ_1, but $\hat{r}_m^{\mathbb{I}}$ is decreasing in θ_1, we have $\bar{\theta}_2$ is increases in θ_1 for any $\mathbb{I} \in \{\mathbb{P}, \mathbb{PR}\}$. Others follow the same logic as Theorem 3.1.

Step 2: when the blockchain is adopted.

Lemma C.4. *For any θ_1, there exists a threshold of the tier-2's working capital $\bar{\theta}_2^{\mathcal{N}}(\theta_1|e) \in [0, c)$ where $\mathcal{N} \in \{I, II, III, IV\}$, such that the tier-1 adopts blockchain-enabled delegate financing if and only if $\theta_2 \leq \bar{\theta}_2^{\mathcal{N}}$. The optimal tier-0 AP contract $(w^*, r_m^*; \hat{w}^{1*}, \hat{r}_m^{1*})$ takes the following form:*

(i) *the optimal traditional menu is the same as the benchmark scenario in §3.4;*

(ii) *the optimal blockchain menu $(\hat{w}^{1*}, \hat{r}_m^{1*})$ is used when $\theta_2 \in [0, \bar{\theta}_2^{\mathcal{N}}(\theta_1|e))$.*

Proof: Before proving Theorem 3.2, we should show that there exists a $\bar{\hat{\theta}}_2 \in [0, c)$ such that $(\hat{w}^*, \hat{r}_m^*) = (w^*, r_m^*)$ (resp., $\hat{w}^* = w^*$) at $\theta_2 = \theta_2^{\tau}$, $\hat{w}^* \leq w^*, \hat{r}_m^* < r_m^*$ ($\hat{w}^* \leq w^*$) at $\theta_2 \in (\bar{\hat{\theta}}_2, c)$, and $\hat{w}^* \geq w^*, \hat{r}_m^* > r_m^*$ ($\hat{w}^* \geq w^*$) at $\theta_2 \in (0, \bar{\hat{\theta}}_2)$. Since the FOC without blockchain is the expectation form of the distribution of $\theta_2 \in [0, c)$, and the manufacturer's maximal payoff increases in θ_2, the thresholds for $\hat{w}^* = w^*$ and $\hat{r}_m^* = r_m^*$ is naturally exists in $[0, c)$. To avoid multiple definition when $w^* = e$ (and $\hat{w}^* = e$ for some θ_2) given $\theta_1 < \bar{\theta}_1$, we define $\bar{\hat{\theta}}_2(e, \theta_1) = \bar{\hat{e}}^{-1}(e|\theta_1)$, which is an inverse function of $\bar{\hat{e}}$ with regard to θ_2. Therefore, Proposition C.4(ii) naturally holds.

We then show Proposition C.4(i). We first discuss the emergency source thresholds with and without blockchain. Since $\bar{e}(\theta_1)$ is independent of θ_1 but $\bar{\hat{e}}(\theta_1, \theta_2)$ decreases in θ_2, we have for any θ_1 there exists a $\bar{\hat{\theta}}_2(\theta_1, \bar{e}(\theta_1)$ such that $\bar{e}(\theta_1) = \bar{\hat{e}}(\theta_1, \theta_2)$ at $\theta_2 = \bar{\hat{\theta}}_2(\theta_1, \bar{e}(\theta_1))$, $\bar{e}(\theta_1) < \bar{\hat{e}}(\theta_1, \theta_2)$ when $\theta_2 > \bar{\hat{\theta}}_2(\theta_1, \bar{e}(\theta_1))$, and $\bar{e}(\theta_1) > \bar{\hat{e}}(\theta_1, \theta_2)$ when $\theta_2 < \bar{\hat{\theta}}_2(\theta_1, \bar{e}(\theta_1))$. In order to prove the existence of $\bar{\hat{\theta}}_2$ in the whole range of $e \in [\underline{w}, 1)$, we consider two cases: 1) when $e \leq \bar{e}(\theta_1)$ so that $w^* = e$, and 2) when $e > \bar{e}(\theta_1)$ so that $w^* = w^{\mathbb{P}}$.

1) $e \leq \bar{e}(\theta_1)$ implies $w^* = e$ and $r_m^* = r_m^{\mathbb{P}}$. So there is no case for $\hat{w}^* \geq w^* = e$. Also we have $\hat{w}^* = \hat{w}^{\mathbb{P}*} < e$ and $\hat{r}$ if and only if $\theta_2 > \bar{\hat{e}}^{-1}(e|\theta_1)$, otherwise, $\hat{w}^* = e$ and $\hat{r}_m^* = \hat{r}_m^{\mathbb{PR}*}$. The only thing we left for 1) is to prove $\bar{\hat{\theta}}_2 \leq \bar{\hat{e}}^{-1}(e|\theta_1)$. We can prove it by contradiction. Suppose $\theta_2 > \bar{\hat{e}}^{-1}(e|\theta_1)$, it implies $\hat{r}_m^* = \hat{r}_m^{\mathbb{P}*} < \hat{r}_m^{\mathbb{PR}*} = r_m^*$. It is a contradictory.

2) $e > \bar{e}(\theta_1)$ implies $w^* = w^{\mathbb{P}}$ and $r_m^* = r_m^{\mathbb{P}}$. We have $\hat{w}^* = e > w^{\mathbb{P}}$ and $\hat{r}_m^* = \hat{r}_m^{\mathbb{PR}*} > r_m^{\mathbb{P}}$ if and only if $\theta_2 \leq \bar{\hat{e}}^{-1}(e|\theta_1)$. Therefore, $\bar{\hat{\theta}}_2$ can only exist in the range of $[\bar{\hat{e}}^{-1}(e|\theta_1), c]$, where $\hat{w}^* = \hat{w}^{\mathbb{P}*}$ and $\hat{r}_m^* = \hat{r}_m^{\mathbb{P}*}$. What we need to finish 2) is to show for any θ_2 such that $\hat{w}^{\mathbb{P}*} > w^{\mathbb{P}}$, we have $\hat{r}_m^{\mathbb{P}*} > r_m^{\mathbb{P}}$, and vice versa. Recall that $(w^{\mathbb{P}}, r_m^{\mathbb{P}})$ solves FOCs (C7) and (C8), whereas $(\hat{w}^{\mathbb{P}*}, \hat{r}_m^{\mathbb{P}*})$ solves KKTs (C12) and (C13). For any θ_2 such that $\hat{w}^{\mathbb{P}*} > w^{\mathbb{P}}$, we have the LHS of equations (C7) and (C8) is negative at $w = \hat{w}^{\mathbb{P}*}$, which imples $\hat{r}_m^{\mathbb{P}*}(\hat{w}^{\mathbb{P}*}) > \hat{r}_m^{\mathbb{P}*}(w^{\mathbb{P}}) = \hat{r}_m^{\mathbb{P}*}$. Similarly, we can prove for any θ_2 such that $\hat{w}^{\mathbb{P}*} < w^{\mathbb{P}}$, we have $\hat{r}_m^{\mathbb{P}*} < r_m^{\mathbb{P}}$.

There is another more convenient way to prove the existence of the threshold. From the KKT conditions (C12) and (C13), we can easily find that the Lagrangian multipliers λ and μ increase in (θ_1, θ_2). Since the constraints are binding when $\lambda^* = 0$ or $\mu^* = 0$, we can show that the IC constraints of the tier-1 and tier-2 suppliers are binding when (θ_1, θ_2) are large enough. Additionally, since we have when $\theta_1 = \theta_2 = 0$, $\hat{\pi}_1^{1*} > \hat{\pi}_i^*$, and when $\theta_1 = \theta_2 = c$,

$\hat{\pi}_1^{1*} = \hat{\pi}_i^*$, the thresholds of θ_2 as a function of θ_1 that is the smallest θ_2 letting at least one constraint is binding must exist. Meanwhile, the threshold $\bar{\bar{\theta}}_2$ is decreasing in θ_1.

Now we can prove Theorem 3.2 and Corollary 3.2 together. The proof for the manufacturer's problem (part (i)) is straightforward. By definition, for any $\theta_2 \in (0, c]$ we have,

$$\hat{\pi}_0^{1*}(\theta_1, \theta_2) = \max_{w, r_m}\{\hat{\pi}_0(w, r_m|e, \theta_2, \theta_1), \text{s.t.} \hat{\pi}_i^{1*} \geq \pi_i^*\},$$

$$\pi_0^*(\theta_1, \theta_2) = \pi_0(w^*, r_m^*|e, \theta_1, \theta_2),$$

where $(w^*, r_m^*) = \text{argmax}_{w, r_m}\left\{\int_0^c \pi_0(w, r_m|e, \theta_2, \theta_1) dF(\theta_2)\right\}$. Since the optimal solution without visibility, (w^*, r_m^*) is also a feasible solution with visibility, we have $\hat{\pi}_0^{1*}(\theta_1, \theta_2) \geq \pi_0^*(\theta_1, \theta_2)$.

We analyze the tier-2's (part (iii)) and the tier-1's problem (part (ii)) in order. For the tier-2 supplier, the optimal expected cash level has the following first-order derivative on the tier-1 interest rate r_s^*,

$$\frac{\partial \pi_2^*}{\partial r_s^*} = \mathcal{P}(y)[v - (1 + r_s)(y - \theta_2)]\frac{\partial y}{\partial r_s} - \mathcal{P}(y)\left[(y - \theta_2) + (1 + r_s)\frac{\partial y}{\partial r_s}\right] = -\mathcal{P}(y)(y - \theta_2) < 0.$$

Hence, we have that π_2^* is decreasing in r_s^* but increasing in y^*. Therefore, $\hat{\pi}_2^* > \pi_2^*$ if and only if $\hat{y}^* > y^*$.

For the tier-1 supplier, we need to consider BF and MF, respectively. We consider the $\mathbb{P}$ case with $e > \bar{\hat{e}}$ that only the proactive risk mitigation is used (and results also preserves for the $\mathbb{PR}$ case). The tier-1 supplier's optimal expected cash level has the following first-order derivatives on the tier-0 AP contract (w^*, r_m^*),

$$\frac{\partial \pi_1^*}{\partial w^*} = \mathcal{P}(y) > 0, \quad \text{and} \quad \frac{\partial \pi_1^*}{\partial r_m^*} = -\mathcal{P}(y)(y - \theta_1 - \theta_2) < 0.$$

Hence, we have that π_1^* is increasing in w^* but decreasing in r_m^*, and the y^* has the same monotonicity in w^* and r_m^* since the following second-order derivatives,

$$\frac{\partial^2 \pi_1^*}{\partial y^* \partial w^*} = \mathcal{P}'(y) > 0, \quad \text{and} \quad \frac{\partial^2 \pi_1^*}{\partial y^* \partial r_m^*} = -\mathcal{P}'(y)(y - \theta_1 - \theta_2) < 0.$$

After obtaining the monotoninicity, we need to find the change of decision variables with the adoption of blockchain. The second-order partial derivatives of $\hat{\pi}_0^*$ on $(\hat{w}^*, \theta_2)$ and $(\hat{r}_m^*, \theta_2)$ are as follows,

$$\frac{\partial^2 \hat{\pi}_0^*}{\partial \hat{w}^* \partial \theta_2} = -\mathcal{P}'(y)\frac{\partial y}{\partial \theta_2} < 0, \quad \text{and} \quad \frac{\partial^2 \hat{\pi}_0^*}{\partial \hat{r}_m^* \partial \theta_2} = -\mathcal{P}'(y)(y - \theta_1 - \theta_2)\frac{\partial y}{\partial \theta_2} + \mathcal{P}(y)\left(\frac{\partial y}{\partial \theta_2} - 1\right) < 0.$$

Hence, both $\hat{w}^*$ and $\hat{r}_m^*$ decrease in θ_2. Without blockchain, the optimal AP contract (w^*, r_m^*) is determined based on the manufacturer's expectation on θ_2. Hence we have $\lim_{\theta_2 \downarrow 0} \hat{w}^* > w^*$ and $\lim_{\theta_2 \uparrow c} \hat{w}^* < w^*$ and so as $\hat{r}_m^*$. We prove the existence of $\bar{\hat{\theta}}_2$. The proof of the existence of $\hat{\theta}_1^L$ can be shown by the fact that the $\bar{\hat{\theta}}_2$ decreases in θ_1, where as $\bar{\theta}_2(\hat{w}^{\mathbb{P}*}, \hat{r}_m^{\mathbb{P}*})$ increases in θ_1 since $(\hat{w}^*, \hat{r}_m^*)$ decrease in θ_1 but $\bar{\theta}_2(\hat{w}^{\mathbb{P}*}, \hat{r}_m^{\mathbb{P}*}) = \bar{\theta}_2$ decreases in $(\hat{w}^*, \hat{r}_m^*)$.

Combined with the financing region revealed by Proposition C.1. We conclude the change of optimal decisions in Table C2. Combined with the monotonicity, we trivially get that for case I(ii), we have $\hat{\pi}_1^* < \pi_1^*$ and $\hat{\pi}_2^* < \pi_2^*$; for case I(1), we have $\hat{\pi}_1^* > \pi_1^*$ and $\hat{\pi}_2^* > \pi_2^*$. The wholesale price and the interest rate has opposite effects on two suppliers. However, we find the tier-1 supplier is more sensitive on the wholesale price, whereas the tier-2 supplier is more sensitive on the interest rate. The former one can be proved by finding $\frac{\partial \pi_1^*}{\partial w^*}\frac{\partial w^*}{\partial \theta_2} \geq \frac{\partial \pi_1^*}{\partial r_m^*}\frac{\partial r_m^*}{\partial \theta_2}$. The second one can be proved in two methods: 1) by finding $\frac{\partial y^*}{\partial w^*}\frac{\partial w^*}{\partial \theta_2} \leq \frac{\partial y^*}{\partial r_m^*}\frac{\partial r_m^*}{\partial \theta_2}$, and 2) by directly showing $\frac{\partial y^*}{\partial \theta_2} \leq 0$. Therefore, for case (a)(i) & (b)(i), we have $\hat{\pi}_1^* > \pi_1^*$ but $\hat{\pi}_2^* < \pi_2^*$; for case (a)(ii), we have $\hat{\pi}_1^* < \pi_1^*$ but $\hat{\pi}_2^* > \pi_2^*$. □

Proof of Theorem 3.3, 3.4, and 3.5:

Proof. Under cross-tier direct financing, the tier-0 AP contract offered by the manufacturer to the tier-1 supplier will not affect the tier-2 supplier's decision any more, that is the tier-2 supplier's investment level y as well as the reliability is independent of $\breve{w}$. Since the reliability $\mathcal{P}(y)$ is independent of the wholesale price $\breve{w}$, so the expected payoff decreases in $\breve{w}$ in both two segments. Hence, the candidates of optimal wholesale price that satisfies tier-1's IC constraint denoted by $\bar{\breve{w}}^1$ or $\breve{w}^* = e$. That means, the manufacturer squeezes all profit from the tier-1 supplier, and the multitier SCF actually becomes a two-tier problem where the tier-2 supplier performs the same as the original problem.

If the manufacturer does not need the perfect reliability from the reactive risk mitigation, namely,$\breve{w} < e$, the wholesale price does not affect the total profit of the manufacturer and the tier-1. So the manufacturer's target investment level $\breve{y}^{\mathbb{P}*}$ and optimal wholesale price solves the following KKT conditions (C14),

$$\begin{cases} \lambda(\breve{\pi}_1^{1*} - \pi_1^*) = 0; \\ \mu(\breve{y}^{1*} - y^*) = 0; \\ -\mathcal{P}(y) > 0; \\ (1 - \breve{w} + v)\mathcal{P}'(y) + \mathcal{P}(y)\left[(y - \theta_2)\dfrac{\partial \breve{r}_m^2}{\partial y} - (1 + r_m)\right] = 0. \end{cases} \tag{C14}$$

where $\frac{\partial \breve{r}_m^2}{\partial y} = \frac{\mathcal{P}''(y)[v-(1+\breve{r}_m^2)(y-\theta_2)-2\mathcal{P}'(y)(1+\breve{r}_m^2)]}{y\mathcal{P}(y)+\mathcal{P}(y)}$ is the impact of a change in the investment level on the tier-1 interest rate, which is negative. The investment level derived by equation (C14) must be greater than that derived by equation (C2) and (C3) since $w \leq 1$. Additionally, the third condition shows that $\breve{w}^* = \bar{\breve{w}}^1$.

If the manufacturer needs the perfect reliability from the reactive risk mitigation, namely, $\breve{w}^* = e$, the manufacturer's target investment level $\breve{y}^{\mathbb{PR}*}$ solves the same KKT conditions with the last equation substituted by,

$$(1 - e + v)\mathcal{P}'(y) + \mathcal{P}(y)\left[(y - \theta_2)\frac{\partial \breve{r}_m^2}{\partial y} - (1 + r_m)\right] = 0 \tag{C15}$$

The investment levels derived by equation (C14) and (C15) are greater than that derived by equation (C2) and (C3) since $1 \geq w$. The investment level derived by equation (C2) and (C15) is less than that derived by equation (C3). Hence, we have $\breve{y}^{1*} > \hat{y}^*$ under both cases.

We have when $e = v$, the value of the objective function of the second candidate is larger than the first, and it decreases in e. While when $e = 1$, the first one must be larger. Hence, there exists a threshold emergency sourcing cost denoted by $\bar{\breve{e}}$ such that the manufacturer is indifferent between emergency sourcing or not, and sourcing outperforms only when $e \leq \bar{\breve{e}}$. The relationship between $\bar{\breve{e}}$ and $\bar{\hat{e}}$ can be shown by their definition and the fact that in the $\mathbb{P}$ case the investment level is greater under cross-tier direct financing than delegate finance. Hence, we have $\bar{\breve{e}} \leq \bar{\hat{e}}$.

We prove Theorem 3.4 for three players, respectively. For the manufacturer, we have $\breve{\pi}_0^* \geq \hat{\pi}_0^*$ for any w, θ_1, θ_2, since with cross-tier direct financing, the manufacturer also get profit from the tier-1 financing. For the tier-1 supplier, we have $\hat{\pi}_1^* \geq \breve{\pi}_1^* = \pi_1^* + \epsilon$ since the manufacturer squeezes all profit. For the tier-2, $\hat{\pi}_2^* \geq \breve{\pi}_2^*$ otherwise because of the monotonicity of π_2^* in the investment level y^*.

The prove of Theorem 3.5 is straightforward. Recall that changing the wholesale price will not change the manufacturer's interest rate to the tier-2, the tier-2's profit will not change. Then, the profit allocation between the manufacturer and the tier-1 depends on the wholesale price. The monotonicity of $\breve{\pi}_0^*$ and $\breve{\pi}_1^*$ in the $\breve{w}^*$ shows that the manufacturer can share more profit with the tier-1 by increasing the wholesale price. Since the total profit between the manufacturer and the tier-1 is higher than that under delegate financing because of higher chain's reliability, (i.e., $\breve{\pi}_{01}^* := \breve{\pi}_0^* + \breve{\pi}_1^* > \hat{\pi}_0^* + \hat{\pi}_1^* = \hat{\pi}_{01}^*$), where $\breve{\pi}_{01}^*$ defined as the total profit between M and S_1 is fixed when $\breve{r}_m^2$ is fixed. By knowing $\lim_{\hat{w}\to 1} \breve{\pi}_1^* > \breve{\pi}_{01}^*$ and $\lim_{\hat{w}\to v} \breve{\pi}_1^* < 0$, there must exist a threshold wholesale price denoted by $\bar{\breve{w}}^{1*\prime}$ such that $\breve{\pi}_1^* = \hat{\pi}_1^*$ and $\breve{\pi}_0^* > \hat{\pi}_0^*$. Then we can add an infinitesimal to $\bar{\breve{w}}^{1*\prime}$ that making the tier-1 is also strictly better under direct financing. $\square$