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Technical Paper

Development of genetic-based models for predicting the resilient modulus of cohesive pavement subgrade soils

Behnam Ghorbani [a], Arul Arulrajah [a], Guillermo Narsilio [b], Suksun Horpibulsuk [a,c,*], Myint Win Bo [d]

[a] Department of Civil and Construction Engineering, Swinburne University of Technology, Hawthorn, VIC 3122, Australia
[b] Department of Infrastructure Engineering, The University of Melbourne, Parkville, VIC 3010, Australia
[c] School of Civil Engineering and Center of Excellence in Innovation for Sustainable Infrastructure Development, Suranaree University of Technology, Nakhon Ratchasima, Thailand.
[d] Bo & Associates Inc., Mississauga, Ontario, Canada

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Abstract

The accurate determination of resilient modulus (M_r) of pavement subgrade soils is an important factor for the successful design of pavement system. The important soil property M_r is complex in nature as it is dependent on several influential factors, such as soil physical properties, applied stress conditions, and environmental conditions. The aim of this study is to explore the potential of an evolutionary algorithm, i.e., genetic algorithm (GA), and a hybrid intelligent approach combining neural network with GA (ANN-GA), to estimate the M_r of cohesive pavement subgrade soils. To achieve this aim, a reliable database containing the results of repeated load triaxial tests (RLT) and other index properties of subgrade soils was utilized. GA was employed to develop a precise equation for predicting M_r of subgrade soils. In addition, GA was used as a tool for determining the optimal values of the weights and the bias of the ANN-GA approach. The developed ANN-GA model was then transferred to a functional relationship for further application and analyses. Several validation and verification phases were conducted to examine the performance of the developed models. The results indicated that both GA and ANN-GA models could accurately predict the M_r of cohesive subgrade soils, and performed better than other models in the literature. Finally, a sensitivity analysis was conducted to evaluate the effect of the utilized parameters on M_r.

Keywords: Pavement subgrade; Resilient modulus; Genetic algorithm; Optimized neural network; Hybrid

1. Introduction

The successful design of a pavement system with various material properties is affected by the stiffness and strength of the pavement layers, i.e. surface, base, subbase, and subgrade. Among the pavement layers, the subgrade is considered the foundation of the pavement system that transfers applied loads to the ground. To characterize the subgrade material under different environmental and loading conditions, the resilient modulus (M_r) has been introduced as a fundamental material property that describes the inelastic behavior of material under traffic loading (AASHTO, 2003). Since pavement layers undergo repeated traffic loading, the subgrade soil experiences both recoverable and permanent strains with each load repetition. When the number of load repetitions is increased, plastic deformation

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* Corresponding author at: School of Civil Engineering and Center of Excellence in Innovation for Sustainable Infrastructure Development, Suranaree University of Technology, Nakhon Ratchasima, Thailand (S. Horpibulsuk).
E-mail addresses: bghorbani@swin.edu.au (B. Ghorbani), aarulrajah@swin.edu.au (A. Arulrajah), narsilio@unimelb.edu.au (G. Narsilio), suksun@g.sut.ac.th (S. Horpibulsuk), mwbo@boassociates.ca (M.W. Bo).

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decreases until it becomes all recoverable (Sadrossadat et al., 2016). M_r can therefore be defined as the ratio of the applied deviator stress to the recoverable strain (AASHTO, 2003).

The M_r of subgrade soils is typically determined using repeated load triaxial (RLT) tests on remoulded or undisturbed samples (AASHTO, 2003). In this test, varying combinations of confining and deviator stresses are applied to the specimen in various sequences to simulate the field conditions. Despite their accuracy and reliability, performing RLT tests requires sophisticated equipment and skilled personnel. In order to avoid performing time consuming and uneconomical RLT tests, several relationships have been proposed that formulate M_r in terms of stress states, physical properties, and strength tests parameters.

M_r mainly depends on stress states and physical properties of the subgrade soils. Generally, models obtained for the calculation of the resilient modulus of subgrade soils can be classified into two main groups: (I) Correlations with laboratory tests, and (II) in situ test results using constitutive equations. Recently, P-wave and S-wave measurements have also been used to estimate the resilient modulus of pavement materials (Schuettpelz et al., 2010). A summary of some of the available equations to estimate the M_r of cohesive soils are summarized in Table 1.

Correlations are typically developed by relating the M_r value to the results of laboratory tests, such as the California bearing ratio (CBR) and the unconfined compressive strength (UCS); in-situ tests, such as the cone penetration test (CPT) and the dynamic cone penetrometer (DCP); physical properties of the soil, like moisture content (w) and dry density (γ_d); stress states, such as confining pressure (σ_3) and deviator stress (σ_d); or a combination of these parameters. Constitutive models, on the other hand, are obtained by relating the M_r value to various stress invariants, i.e. bulk stress (θ), octahedral shear stress (τ_{oct}), and octahedral normal stress (σ_{oct}). Unknown parameters of the constitutive models can be related to the physical properties of the soil to include both physical properties and stress conditions.

According to the Kim (2004) equation in Table 1, regression coefficients for cohesive subgrade soils (A-6 soils) can be obtained using the following equations (Hanittinan, 2007):

$$k_1 = a_1 a_3^{a_2} + a_3\left(\frac{S_r}{100}\right)^{a_4} + a_5 q_u + a_6 PI + a_7(LL - w) + a_8(w_{opt} - w) + a_9(P_{200} - a_{10}) \quad (1)$$

$$k_2 = b_1 \sigma_3^{b_2} + b_3\left(\frac{S_r}{100}\right)^{b_4} + b_5 {q_u}^{b_6} + b_7 PI + b_8 LL \quad (2)$$

$$a_1 = a_{11} + a_{12}\left(\frac{w_{opt} - w}{w_{opt}}\right) \quad (3)$$

$$b_1 = b_{11} + b_{12}(w - w_{opt}) \quad (4)$$

where a_1 to a_{12} and b_1 to b_{12} are constants of Kim's model. A detailed description of Kim's regression coefficients can be found in Hanittinan (2007). As can be seen in Table 1, constitutive models such as those provided by Seed et al. (1967), Witczak and Uzan (1988), and Puppala et al. (1996) have a pre-defined structure, and unknown coefficients of these equations are typically obtained by performing a regression analysis. According to the Kim (2004) equation in Table 1, a second set of the regression analysis is required to relate the obtained coefficients, i.e., k_1 and k_2,

Table 1
Available prediction equations for M_r of subgrade soils.

Reference	Equation	Affecting variables	Applicability
Seed et al. (1967)	$M_r = k_1\left(\frac{\theta}{P_a}\right)^{k_2}$	θ, P_a, k_2	All soil types
Witczak and Uzan (1988)	$M_r = k_1 P_a\left(\frac{\theta}{P_a}\right)^{k_2}\left(\frac{\tau_{oct}}{P_a}\right)^{k_3}$	θ, P_a, τ_{oct}, k_2, k_3	All soil types
Puppala et al. (1996)	$M_R = k_1 P_a\left(\frac{\sigma_3}{P_a}\right)^{k_2}\left(\frac{\sigma_d}{P_a}\right)^{k_3}$	σ_3, σ_d, P_a, k_2, k_3	All soil types
Kim (2004)	$\frac{M_R}{P_a} = k_1\left[\frac{P_a\sigma_{oct}}{\tau^2_{oct}}\right]^{k_2}$	σ_{oct}, τ_{oct}, P_a, k_1, k_2	A-4 and A-6 soils
Carmichael and Stuart (1985)	$M_r = 37.431 - 0.4566PI - 0.6179w - 0.1424P_{200} + 0.1791\sigma_3 - 0.3248\sigma_d + 36.422CH + 17.097MH$	PI, w, P_{200}, σ_3, σ_d, MH, CH	Cohesive subgrade soils
AASHTO (2003)	$M_r(MPa) = 17.6(CBR)^{0.64}$	CBR	All soil types
Mohammad et al. (2007)	$M_R = \frac{165.5}{DCPI^{1.147}} + 0.0966\left(\frac{\gamma_d}{w}\right)$	$DCPI$, γ_d, w	A-4 , A-6, A-7–5, and A-7–6 soils
	$M_R = \frac{151.8}{DCPI^{1.096}}$	$DCPI$	
Mohammad et al. (2002)	$\frac{M_R}{\sigma_3^{0.55}} = \frac{1}{\sigma_1}\left(31.79q_c + 74.81\frac{f_s}{w}\right) + 4.08\frac{\gamma_d}{\gamma_w}$	σ_3, σ_d, q_c, f_s, γ_d, γ_w	Cohesive subgrade soils
Sadrossadat et al. (2018b)	$M_R = \sigma_3 + W_{opt} + (3P_{\#200} + q_u + \sigma_3 - 2\sigma_d - (5LL \times (W_c + S_r + 8\sigma_3 + \sigma_d - 8(PI - W_{opt} + 1.5)^4))/q_u)/W$	w_{opt}, P_{200}, q_u, σ_3, σ_d, LL, w, S_r	A-6 soils

θ: bulk stress ($[\sigma_1 + 2\sigma_3]$); P_a: atmospheric pressure (=101 kPa); σ_{oct}: octahedral normal stress ($[\sigma_1 + 2\sigma_3]/3$); τ_{oct}: octahedral shear stress ($[2^{0.5}(\sigma_1 - \sigma_3)/3$); σ_1: major principal stress ($[\sigma_3 + \sigma_d]$; σ_3: minor principal stress; σ_d: deviatoric stress ($[\sigma_1 - \sigma_3]$); PI: plasticity index; w: moisture content; P_{200}: percentage of soil particles passing through #200 sieve; CBR: California bearing ratio; DCPI: dynamic cone penetration index; γ_d: dry density; q_c: cone tip resistance; f_s: sleeve friction resistance; q_u: unconfined compressive strength; S_r: degree of saturation; k_1, k_2, k_3: regression coefficients; CH = 1 for CH soil; MH = 1 for MH soil.

to soil parameters as summarized in Eqs. (1)–(4). As can be seen, the procedure for finding the final model is complicated and time consuming. On the other hand, while regression based equations may perform well on the utilized datasets, they are not typically tested and validated on new datasets (Gandomi et al., 2013, Ghorbani et al., 2018). The complexity of the M_r factor as well as the importance of considering the nonlinear interaction between variables necessitates the use of more advanced techniques for the prediction of M_r of subgrade soils.

To overcome the limitations of the traditional modeling techniques, artificial intelligence (AI) methods have been employed by various researchers to solve complicated engineering problems (Ghorbani et al., 2018, Chen et al., 2019, Jayawardana et al., 2019, Ghorbani and Hasanzadehshooiili, 2018). Unlike traditional modeling techniques, artificial intelligence approaches are capable of determining the nonlinear relationship between variables in a model effectively, without considering any prior assumptions about the problem. Kim et al. (2014) and Hanittinan (2007) used ANNs, and Khoury and Maalouf (2018) employed a support vector machine method to predict the resilient modulus of subgrade soils. Sadrossadat et al. (2018b) used a variant of genetic programming, namely linear genetic programming (LGP), to directly estimate the M_r of cohesive subgrade soils. Amongst computation intelligence methods, artificial neural networks (ANNs) are the most widely used due to their inherent features, which include managing complex problems with large datasets, handling problems with multiple outputs, and predicting the unseen data effectively. However, ANNs come with disadvantages, such as a slow learning rate and getting stuck in local minima. Furthermore, ANNs have been referred to as black-box systems since they do not provide a distinct relationship between inputs and the output (Ziaee et al., 2015).

To improve the prediction capability of ANNs, evolutionary algorithms such as genetic algorithm (GA) and particle swarm optimization (PSO) have been applied to find the optimal values of the weights for the ANN. Evolutionary algorithms can aid ANNs in converging to the global minima, and hence improve the prediction performance of the network. In this regard, Mousavi et al. (2017) proposed a hybrid neural network and simulated annealing to predict the daily solar radiation. Alsarraf et al. (2019) applied the PSO-ANN technique to predict the exergetic performance of a building integrated photovoltaic/thermal system. Mosallanezhad and Moayedi (2017) investigated the potential of an integrated imperialist competitive algorithm ANN to estimate the pull-out resistance of screw piles. While evolutionary algorithms have been found efficient for solving engineering problems, the application of these methods in the field of pavement geotechnics has been limited to date. Few studies on the application of evolutionary algorithms and hybrid methods for providing formulations of resilient modulus of subgrade soils have been reported in the literature.

This paper proposes the application of GA as well as a hybrid ANN-GA approach to predict the M_r of cohesive subgrade soils. GA was employed to establish a precise equation to predict the M_r of subgrade soils. The hybrid ANN-GA model was developed using GA to determine the optimal values of weights and bias of the ANN-GA approach, which can result in a more robust model. To achieve this, a comprehensive and reliable set of data, including the results of RLT tests on cohesive subgrade soils was utilized to develop models. Several validation and verification study phases were involved in evaluating the performance of the proposed models. Furthermore, results were compared with available equations in the literature to verify the superiority of the proposed models. The developed ANN-GA model was also converted to a tractable formula for hand calculation and pre-design purposes to reduce the time and cost associated with performing RLT tests.

2. Methodology

2.1. Genetic algorithm (GA)

GA is a heuristic search and optimization algorithm introduced by Holland (1975). GA was inspired by Darwin's theory of evolution and imitates the process of natural evolution and selection. In addition, unlike conventional optimization methods, GA requires less information about the problem and is well suited for more complex problems. In GA, a population of individuals are

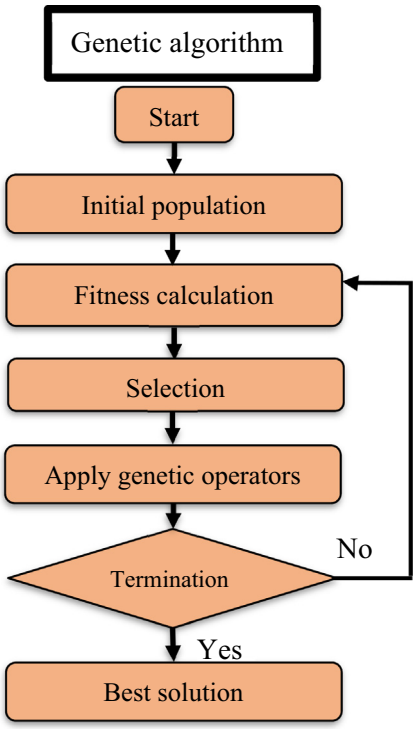

Fig. 1. Flowchart of GA for obtaining the optimal solution (Rani et al., 2013).

randomly generated to solve a problem. Each solution is encoded as a fixed-length binary string of 0 s and 1 s, known as a chromosome. A chromosome has several genes and total number of chromosomes indicate the population size (Holland, 1975). The superiority of one solution over other solutions is evaluated using a fitness function. Solutions are evolved in successive iterations (or generations) until a satisfactory criterion, i.e. the maximum number of generations or a predefined fitness value, is met. In the process of evolution, several operators, i.e., mutation and crossover, are used to modify solutions transferred to next generations (Muduli and Das, 2015). Fig. 1 summarizes steps taken in the GA for reaching the optimal solution.

2.2. Hybrid ANN-GA approach

ANNs are a branch of artificial intelligence techniques that aim to mimic the behavior of human brain and nervous system for solving complex problems. Amongst different variants of ANN, multi-layer perceptron (MLP) neural networks is the most widely used (Cybenko, 1989). A MLP neural networks consists of three distinct layers: the input layer, a hidden layer and an output layer. Each layer is connected to the subsequent layer through computing elements known as nodes. Except for the input nodes, which are fixed nodes, each node is a neuron or processing element with a nonlinear activation function. In a neuron, an input from previous layer is multiplied by a weight coefficient which connects two layers. The output of each neuron is then calculated by passing through a nonlinear activation function (Ziaee et al., 2015). Training of the network is done by adjusting the weights of the network so that the network's predicted values matches the target values in the datasets. To do so, a back propagation algorithm is typically used in which the calculated error is back propagated and the network weights are altered accordingly to minimize the prediction error (Ghorbani and Hasanzadehshooiili, 2018).

One of the advantages of GA over classic search methods is its ability to perform a global search and hence avoid the risk of trapping into a local minimum (Gandomi et al., 2013). Thus, a hybrid ANN-GA model uses GA as a powerful search algorithm to find the best parameters for the ANN. In the ANN-GA method, GA is employed to adjust the weights of the ANN in a way that the error of the GA-ANN model (i.e., fitness value) is minimized. A fitness function (i.e., RMSE) is considered to measure the fitness of each solution vector. In case of reaching the defined termination criterion, the simulation procedure stops and the results are represented. The termination condition is typically set to the maximum value until which the algorithm is iterated to find better solutions.

3. Database and model variables

The database used for development of ANN-GA model was collected from a study conducted by Hanittinan (2007), which reported on data collected from earlier studies by Kim (2004), Huang (2001), and Rodgers (2006). The database was composed of the physical soil properties, unconfined compressive strengths, and the results of 283 RLT tests performed on cohesive subgrade soils. Soils were mainly silt, clay, and silty clay, classified as A-6 based on the AASHTO soil classification code. Samples were collected from seven different locations and tests were performed in several US universities (Hanittinan, 2007). RLT tests were performed at a range of moisture contents from 4% below optimum moisture content to 3% above optimum moisture content, based on AASHTO designation T294-94 (Hanittinan, 2007).

Over the past decades, many studies have examined the factors affecting the M_r of subgrade soil. Results of laboratory studies indicate that the M_r of pavement materials is highly influenced by the stress state parameters, i.e., σ_3 and σ_d. For cohesive subgrade soils, increasing the σ_d at a constant σ_3 decreases the M_r value (Seed et al., 1967, Huang, 2001, Kim, 2004). It is known that increasing the σ_3 increases the M_r of soils (Kim et al., 2014, Sadrossadat et al., 2016). In addition to stress state parameters, several studies have been performed to study the influence of the physical properties of soil, its compaction characteristics, and environmental factors on the M_r. These factors should be considered in the model development for providing more comprehensive and robust models. Moisture content and degree of saturation (S_r) describe the changes in the soil environment and seasonal variations. Soil physical properties such as percentage of soil particles passing through #200 sieve (P_{200}), liquid limit (LL), and plasticity index (PI) are commonly used in the classification and identification of the soil type. Besides, UCS has been identified as a static strength parameter which is positively correlated with the M_r (Kim, 2004, Hanittinan, 2007). There are several equations in the literature which relate the M_r to strength parameters such as UCS and CBR. On the other hand, the compaction characteristics of soils is an important factor in the design of pavement layers. Herein, the optimum moisture content of the soil (w_{opt}), which is defined as the soil moisture content at its maximum dry density, is incorporated in the model development procedure.

In addition to the aforementioned factors, the viscous nature of cohesive soils can cause creep deformations under tapplied loads. The developed time dependent shear strains under sustained or repeated loads can reduce the M_r of cohesive soils (Viyanant et al., 2007). In saturated soil conditions, the application of load cycles generates excess pore water pressure in cohesive soils which can cause creep deformation (Holzer et al., 1973).

With an aim to incorporate most of the influential parameters and considering the available database, the model for prediction of M_r of cohesive subgrade soils is expressed as a function of the following parameters:

$$M_r = f(LL, PI, P_{200}, w_{opt}, S_r, w, q_u, \sigma_3, \sigma_d) \quad (5)$$

Table 2
Descriptive statistics of the variables in the database.

Parameter	LL	PI	P_{200} (%)	w_{opt} (%)	S_r (%)	w (%)	q_u (kPa)	σ_3 (kPa)	σ_d (kPa)	M_r (MPa)
Mean	30.83	11.17	77.74	14.01	81.13	13.90	324.87	20.95	41.45	65.34
Median	32.00	11.00	84.00	14.00	85.07	14.00	316.50	20.69	41.37	64.83
Mode	31.00	11.00	56.00	13.40	66.44	14.00	696.13	0.00	41.37	104.00
Standard deviation	4.31	1.72	15.48	2.03	14.23	2.84	178.70	16.36	18.12	31.22
Kurtosis	0.65	1.43	−1.40	0.97	−0.24	−0.61	−0.29	−1.41	−1.05	−0.14
Skewness	−1.10	−1.37	−0.15	−0.85	−0.77	−0.39	0.76	−0.03	−0.02	0.29
Range	16.20	6.10	44.00	8.40	57.08	11.47	607.18	41.40	60.21	167.54
Minimum	21.00	7.00	56.00	9.40	42.92	7.53	88.95	0.00	11.00	11.90
Maximum	37.20	13.10	100.00	17.80	100.00	19.00	696.13	41.40	71.21	179.44

To have a better understanding of the model variables, key descriptive statistics of the variables are summarized in Table 2.

4. Data pre-processing

Artificial intelligence techniques utilize data to determine the optimum model, which is the one that best describes the relationship between input and output parameters. One key issue in finding this relationship is known as overfitting, which needs to be avoided when seeking a model with better generalization. Overfitting occurs when the model has relatively small error on the trained dataset, while when a new set of data is introduced to the model, the error value becomes very high. To avoid overfitting, the dataset needs to be divided into two subsets: training data and testing data. The training data is used to construct the models, and the testing data is used to evaluate its performance on unseen data. Accordingly, the testing dataset is used to find a model with better generalization. It has been suggested by many researchers that between 15% and 30% of the data should be used to test the model performance (Gandomi et al., 2013). It is worth noting that while the results of the model on training data represents the ability of the model to learn the behavior of variables in the database, the test performance indices show the performance on the model on unseen data, which indicates its generalization capability. In this study, 80% and 20% of the datasets were used for training and testing the model, respectively.

The number of datasets in a database has a significant effect on the model performance in training stage of the network. Models with a larger number of datasets are more reliable and safer for further analysis. It is suggested that the minimum ratio of datasets in a database to the variables of the model should be more than five (Gandomi et al., 2013). In this study, this ratio is equivalent to 31, which is much higher than the required value.

5. Model performance assessment

In general, several important criteria need to be checked to evaluate the accuracy and generalization of the developed ANN-GA model. The correlation coefficient (R), mean absolute error (MAE), and round mean squared error (RMSE) are among essential statistical criteria that indicate the overall performance of the developed model (Ghorbani et al., 2018):

$$R^2 = \frac{\left(\sum_{i=1}^{n}(M_{rm}-\bar{M}_{rm})(M_{rp}-\bar{M}_{rp})\right)^2}{\sum_{i=1}^{n}(M_{rm}-\bar{M}_{rm})^2\sum_{i=1}^{n}(M_{rp}-\bar{M}_{rp})^2} \tag{6}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(M_{rp}-M_{rm})^2}{n}} \tag{7}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|M_{rp}-M_{rm}\right| \tag{8}$$

in which M_{rm} and M_{rp} are the measured and predicted values of the i^{th} output, $\bar{M}_{rm}$ and $\bar{M}_{rp}$ are the average values of the measured and predicted results, and n is the number of samples.

Smith (1986) suggests that there is a strong correlation between the predicted and measured values if the correlation coefficient $R^2 \geqslant 0.64$. Additionally, error values, i.e., the MAE and RMSE, should be minimum for both training and testing data.

6. Development of the GA model

In this section, GA is employed to find a precise equation for prediction of the M_r of subgrade soils. In this regard, a code is written in a MATLAB environment (Mathworks, 2017). The input and output values are normalized before model development to increase the capability of algorithm in finding the relationship between input variables and the output. The database is normalized to lie between 0 and 1 using the following equation:

$$X_n = \frac{X - X_{\min}}{X_{\max} - X_{\min}} \tag{9}$$

where X_{min}, X_{max}, and X_n are the minimum, maximum, and normalized values of the variable X, respectively. The denormalized value of the output can be calculated as:

$$X = X_n(X_{\max} - X_{\min}) + X_{\min} \tag{10}$$

The general form of the considered equation to predict the M_r of subgrade soils is given in Eq. (11):

$$M_r = w_1 LL^{a_1} + w_2 PI^{a_2} + w_3 P_{\#200}{}^{a_3} + w_4 W_{opt}{}^{a_4} + w_5 W_c{}^{a_5} + w_6 S_r{}^{a_6} + w_7 q_u{}^{a_7} + w_8 \sigma_3{}^{a_8} + w_9 \sigma_d{}^{a_9} + b \quad (11)$$

where w_1 to w_9 and a_1 to a_9 are the coefficients of the equation and b is the bias value. GA is employed as a tool to find the optimal values of the unknown coefficients. The RMSE function is considered as the fines function to evaluate solutions in each iteration.

Several parameters, including the population size (N_{pop}), crossover probability, mutation rate, and maximum number of iterations, affect the prediction capability of the GA approach. These values are typically obtained by a trial and error method or by using values recommended by other researchers. The combination of parameters considered to find the optimal model is summarized in Table 3. Several preliminary runs were performed to come up with a parameter setting to provide a robust model with high generalization capability. Crossover and mutation rates were selected based on some previously suggested values (Momeni et al., 2014, Khandelwal and Armaghani, 2016, Rostami et al., 2018). The proper determination of population size is dependent on the size and the complexity of the investigated problem. To investigate the effect of population size on the performance of the GA model, a parametric study was done as shown in Fig. 2. It was observed that model with $N_{pop} = 400$ had the lowest RMSE value. Considering different values for parameters resulted in $2 \times 5 \times 2 \times 2 = 40$ combinations of parameters. Also, 5 replications of each parameter combination were tested and evaluated. In total, 200 runs with different combinations of parameters were conducted. The best values of population size, mutation rate, and crossover probability for the developed GA model were 400, 30%, and 70%, respectively. The optimal coefficients of the developed equation based on GA optimization are summarized in Table 4.

Table 3
Parameter settings for development of GA model.

Parameter	Setting
Number of generation	500, 1000
Population size	50, 100, 200, 300, 400
Mutation rate (%)	10, 30
Crossover rate (%)	70, 95
Fitness function	RMSE

Considering one of the testing samples (i.e. $LL = 31$, $PI = 12$, $P_{200} = 56\%$, $w_{opt} = 13.4\%$, $w = 11.4\%$, $S_r = 66.44\%$, $q_u = 696.13$ kPa, $\sigma_3 = 41.37$ kPa, $\sigma_d = 40.76$-kPa, $M_r = 112.81$ MPa), the normalized values of LL, PI, P_{200}, w_{opt}, w, S_r, q_u, σ_3, and σ_d are equal to 0.617, 0.82, 0, 0.476, 0.337, 0.412, 1, 0.999, and 0.494, respectively. By substituting the obtained coefficient values in Eq. (11), M_r is calculated equal to 0.6076. Using Eq. (10), the denormalized value of M_r is calculated to be equal to 113.7 (MPa).

7. Hybrid ANN-GA model development

In order to develop the ANN-GA model for prediction of resilient modulus of subgrade soils, a code is written in a MATLAB environment (Mathworks, 2017). The procedure for the modeling of M_r using ANN-GA is illustrated in Fig. 3. The model includes nine input variables and M_r is the only output of the model.

In the first phase of constructing the model, the dataset should be normalized between −1 and 1 to facilitate model development using following equation (Mathworks, 2017):

$$X_n = \frac{X - X_{\min}}{X_{\max} - X_{\min}} - \frac{X_{\max} - X}{X_{\max} - X_{\min}} \quad (12)$$

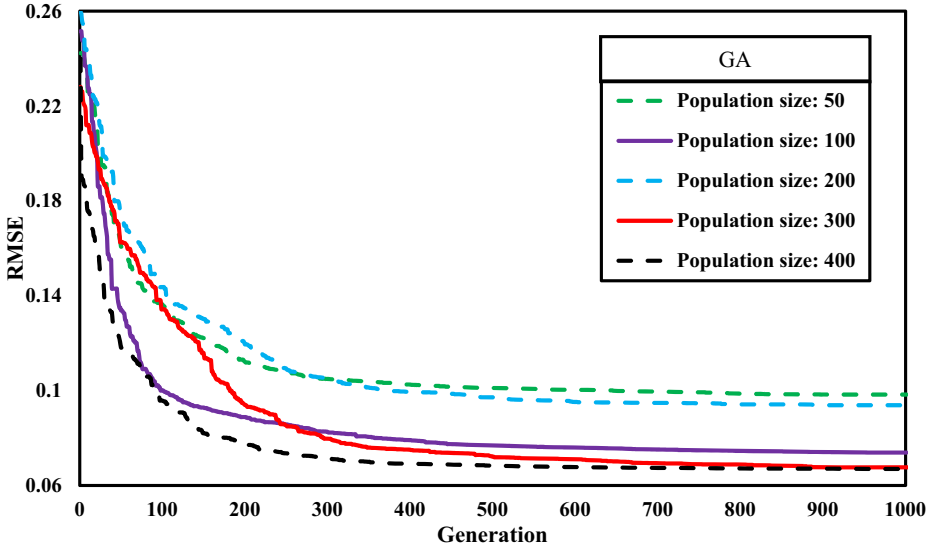

Fig. 2. Effect of the population size on development of the GA model.

Table 4
Optimal coefficients of the developed GA model.

Coefficients	Coefficient number									bias (b)
	1	2	3	4	5	6	7	8	9	
w_i	0.103	−0.204	−0.131	0.310	−0.085	−0.273	0.351	0.168	−0.292	0.726
a_i	5.999	0.023	0.961	3.096	0.045	0.862	0.640	0.723	0.148	

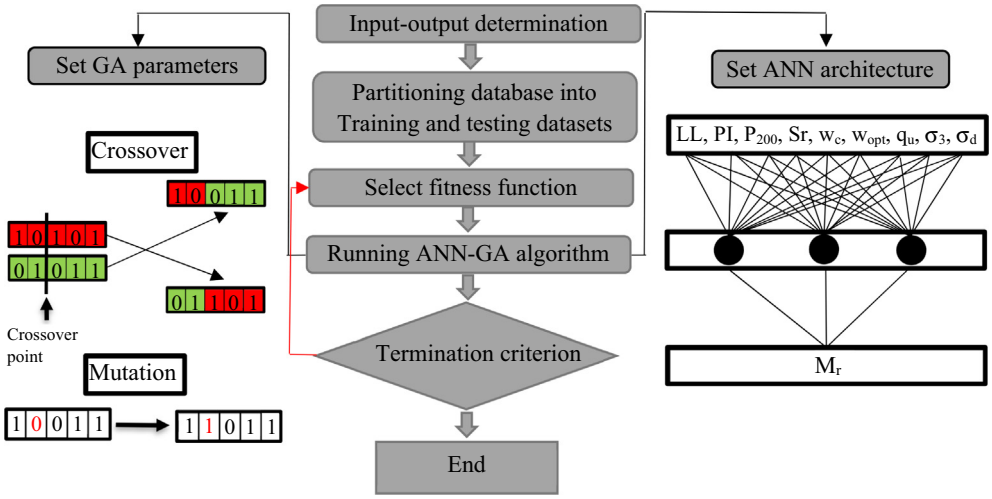

Fig. 3. Hybrid ANN-GA model development procedure.

Considering Eq. (13), the de-normalized value of the output can be calculated as:

$$X = 0.5(X_n + 1)(X_{\max} - X_{\min}) + X_{\min} \tag{13}$$

In the hybrid ANN-GA model development, there are several parameters in both the ANN and GA that need to be set. The precision of the ANN models increases as the number of nodes in the hidden layer increases. However, increasing the nodes would result in a more complicated model with many different parameters. Cybenko (1989) indicated that a single hidden layer neural network would provide satisfactory results for use in the approximation of nonlinear problems. Thus, in this study, only one hidden layer consisting of three nodes was considered to develop a simple model that can be represented as a tractable formulation rather than a complicated black-box. The activation function for the hidden layer nodes was the following Tangent-sigmoid (Mathworks, 2017):

$$f(x) = \frac{2}{1 + e^{-2x}} - 1 \tag{14}$$

The crossover and mutation rates were chosen to be equal to the values adopted for developing the GA model. In addition, a parametric study was performed by developing several ANN-GA models to find the optimal value of N_{pop}. While increasing population size usually increases the chance of obtaining better results, it decreases the speed of the model development. The population size was varied from 50 to 400 and the maximum number of generations was set to 500. Fig. 4 shows the effect of population size on the model performance based on the RMSE criteria. As evident, the model with $N_{pop} = 400$ had the lowest RMSE value. It is also evident that after about 300 generations, there is no significant change in the RMSE value. The hybrid ANN-GA model was run several times with $3 \times 5 \times 2 \times 2 = 60$ different combinations of the parameters and 5 replications for each combination, which resulted in a total of 300 runs. The parameter settings during the ANN-GA model development are summarized in Table 5.

The best ANN-GA model for predicting the resilient modulus of cohesive subgrade soils has the following parameters: Population size = 400; Mutation rate = 10%; Crossover rate = 70%. In order to transform the optimal ANN-GA model into a tractable formulation for further analysis, the following function is used (Ziaee et al., 2015):

$$h = f_{HO}\left(bias_h + \sum_{k=1}^{h} V_k f_{IH}\left(bias_{hk} + \sum_{i=1}^{m} w_{ik} x_i\right)\right) \tag{15}$$

where $bias_h$ is the hidden layer bias; V_k is the weight connection between neuron k of the hidden layer and the single output neuron; $bias_{hk}$ would be the bias at neuron k of the hidden layer ($k = 1, h$); w_{ik} denotes the weight connection between the input variable ($i = 1$, m) and neuron k of the hidden layer; x_i would be the ith input parameter; f_{HO} is the transfer function between the hidden layer and the output layer; and f_{IH} is the transfer function between the input and hidden layer.

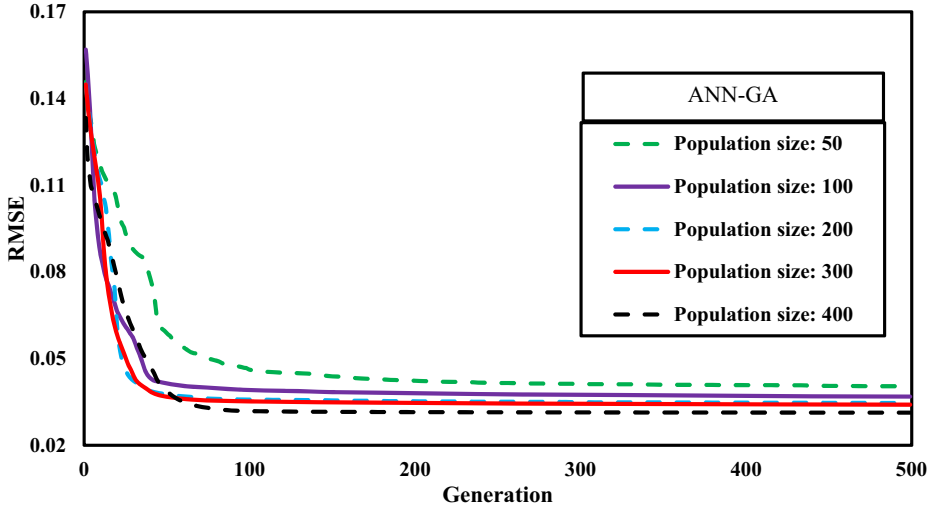

Fig. 4. Effect of the population size on development of the ANN-GA model.

Table 5
Parameter settings for development of ANN-GA model.

Parameter	Setting
Number of generation	100, 300, 500
Population size	50, 100, 200, 300, 400
Mutation rate (%)	10, 30
Crossover rate (%)	70, 95
Number of hidden layers	1
Number of hidden nodes	3
Activation function	Tangent-sigmoid
Fitness function	RMSE

After de-normalization of the output, the optimal ANN-GA model for predicting the M_r of cohesive subgrade with nine inputs (LL, PI, P_{200}, w_{opt}, w, S_r, q_u, σ_3, σ_d) can be expressed as:

$$(M_{r_i})_{ANN-GA} = 83.77(M_{r_i})_n + 11.9 \quad (16)$$

where,

$$(M_{r_i})_n = \sum_{k=1}^{3} V_k \tanh(A_k) + bias_h \quad (17)$$

$$A_k = w_{1k}LL_n + w_{2k}PI_n + w_{3k}P_{200_n} + w_{4k}W_{opt_n} + w_{5k}W_{c_n} + w_{6k}S_{r_n} + w_{7k}q_{u_n} + w_{8k}\sigma_{3_n} + w_{9k}\sigma_{d_n} + bias_k \quad (18)$$

where LL_n, PI_n, P_{200n}, w_{optn}, w_n, S_{rn}, q_{un}, σ_{3n}, and σ_{dn} are the normalized input values obtained from Eq. (12), and k is the number of hidden layer nodes (i.e., 3). The obtained values of bias and weights of the optimal model for input-hidden and hidden-output layers are summarized in Tables 6 and 7, respectively.

Considering the same testing sample used for calculating the output of the GA model (LL = 31, PI = 12, P_{200} = 56%, , w_{opt} = 13.4%, w = 11.4%, S_r = 66.44%, q_u = 696.13 kPa, σ_3 = 41.37 kPa, σ_d = 40.76 kPa, M_r = 112.81 MPa), the procedure for calculating the M_r is described as:

Table 6
Values of weights and bias for input-hidden layer.

Weight (w_{ik})	Neuron (k) Between Input and Hidden Layer		
	1	2	3
W_{1k}	0.519	0.171	−1.909
W_{2k}	0.003	−0.196	−1.203
W_{3k}	−0.252	0.125	0.640
W_{4k}	0.778	0.522	−0.867
W_{5k}	−0.541	−1.093	1.409
W_{6k}	−0.245	0.138	0.960
W_{7k}	0.141	0.103	0.921
W_{8k}	0.035	−0.363	−0.105
W_{9k}	−0.148	−0.067	0.703
$bias_k$	0.787	−1.563	−1.256

Table 7
Values of the weights and bias of the hidden-output layer.

Weight	Neuron (k) Between Hidden and Output Layer			$bias_h$
	1	2	3	
W_k	2.6376	−1.9416	1.6468	−2.4069

Step 1: Normalization of the input dataset. Using Eq. (12), the normalized values of LL, PI, P_{200}, w_{opt}, w, S_r, q_u, σ_3, and σ_d are 0.235, 0.64, −1, −0.048, −0.325, −0.176, 1, 0.99, −0.011, respectively.

Step 2: Calculation of the hidden nodes parameters. Using the values summarized in Table 6, the values of A_1 to A_3 are calculated: A_1 = 1.522; A_2 = −1.73; A_3 = −2.89.

Step 3: Prediction of the M_{rn}. The output of each hidden layer neuron is calculated by passing through an activation function (i.e., tansig). M_{rn} can then be calculated as the summation of output of each neuron which is multiplied to the hidden-output weights as given in Table 7.

Step 4: De-normalization of the M_r. Using Eq. (13), M_r is calculated in the range of datasets. The output of the ANN-GA model obtained is equal to 110.46 MPa.

8. Performance analysis of ANN-GA and GA models

In order to represent the capability of the obtained ANN-GA and GA models, the predicted versus measured values of M_r for training and testing datasets are depicted in Fig. 5(a) and (c), respectively. In addition, histograms of the errors obtained by each method for training and testing datasets is illustrated in Fig. 5(b) and (d) to have a general view of the frequency of the errors in different intervals.

As evident in these figures, the ANN-GA model has R^2 values of 0.97 for both training and testing datasets, and RMSE values are 5.5 and 5.2 for training and testing datasets, respectively. On the other hand, the R^2 of GA model for both training and testing datasets is equal to 0.87. The RMSE value for training and testing datasets are 11.1 and 11.3 for training and testing data, respectively. Considering the high R^2 values and low RMSE values, it can be concluded that the developed models are capable of predicting the M_r of subgrade soils with a high degree of accuracy. In addition, close values of R^2 and RMSE values for training and testing datasets indicate that overfitting is avoided (Ghorbani et al., 2018, Sadrossadat et al., 2018a). This means that the proposed models would have a satisfactory performance on unseen data and thus have a suitable generalization performance. It should be noted that while the functional structure of the GA model is simpler, the ANN-GA model outperforms the GA model with a high degree of accuracy.

9. Comparative study

To further examine the performance of the proposed models, their performance was compared with the LGP

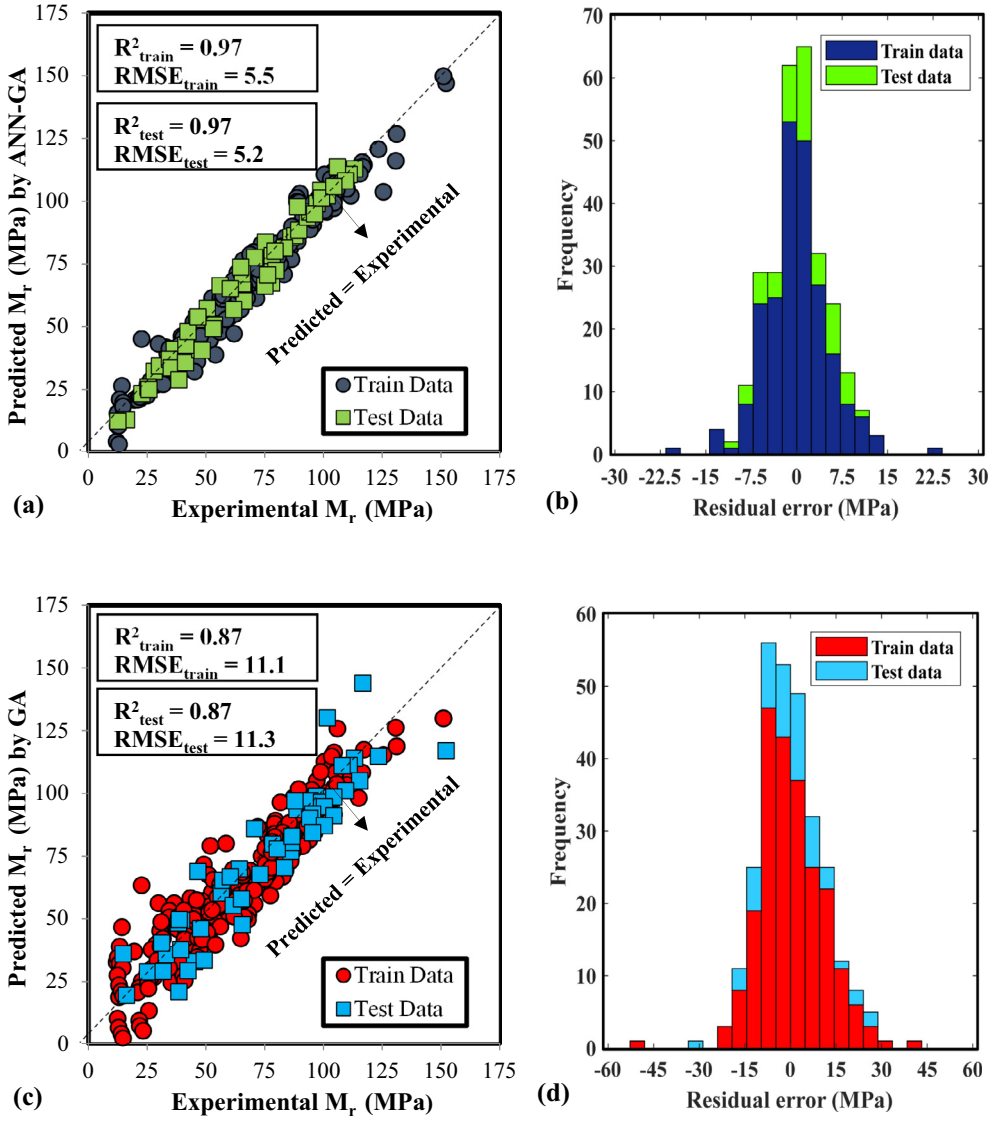

Fig. 5. (a) and (c) Measured versus predicted M_r values of the GA and ANN-GA models; (b) and (d) Histogram of errors obtained for training and testing data in GA and ANN-GA models.

model (Sadrossadat et al., 2018b) and the model proposed by Kim (2004). The Kim model was chosen for the purpose of comparison as it outperformed other existing regression based models in the literature (Kim, 2004). In this regard, testing datasets were used to evaluate the prediction capability of all methods. The results of comparative study are represented in Fig. 6. Residual error (RE) is calculated as the difference between the measured and predicted values of each model. As evident in Fig. 6, the ANN-GA approach outperforms the other three methods in terms of all indicators. The R^2 values for ANN-GA, GA, LGP, and Kim's model are 0.97, 0.87, 0.83, and 0.56, respectively. The max |RE| of the ANN-GA model is about 11, while this value for GA, LGP, and Kim's model are around 29, 33, and 62, respectively. Furthermore, it is evident that AI-based methods perform notably better than the regression based model proposed by Kim (2004).

While R, MAE and RMSE together give an overall perspective of the performance of each model, other criteria need to be satisfied on the testing datasets to ensure the external validation of a prediction model. In this regard, a ranking index (RI) is used in this study that incorporates

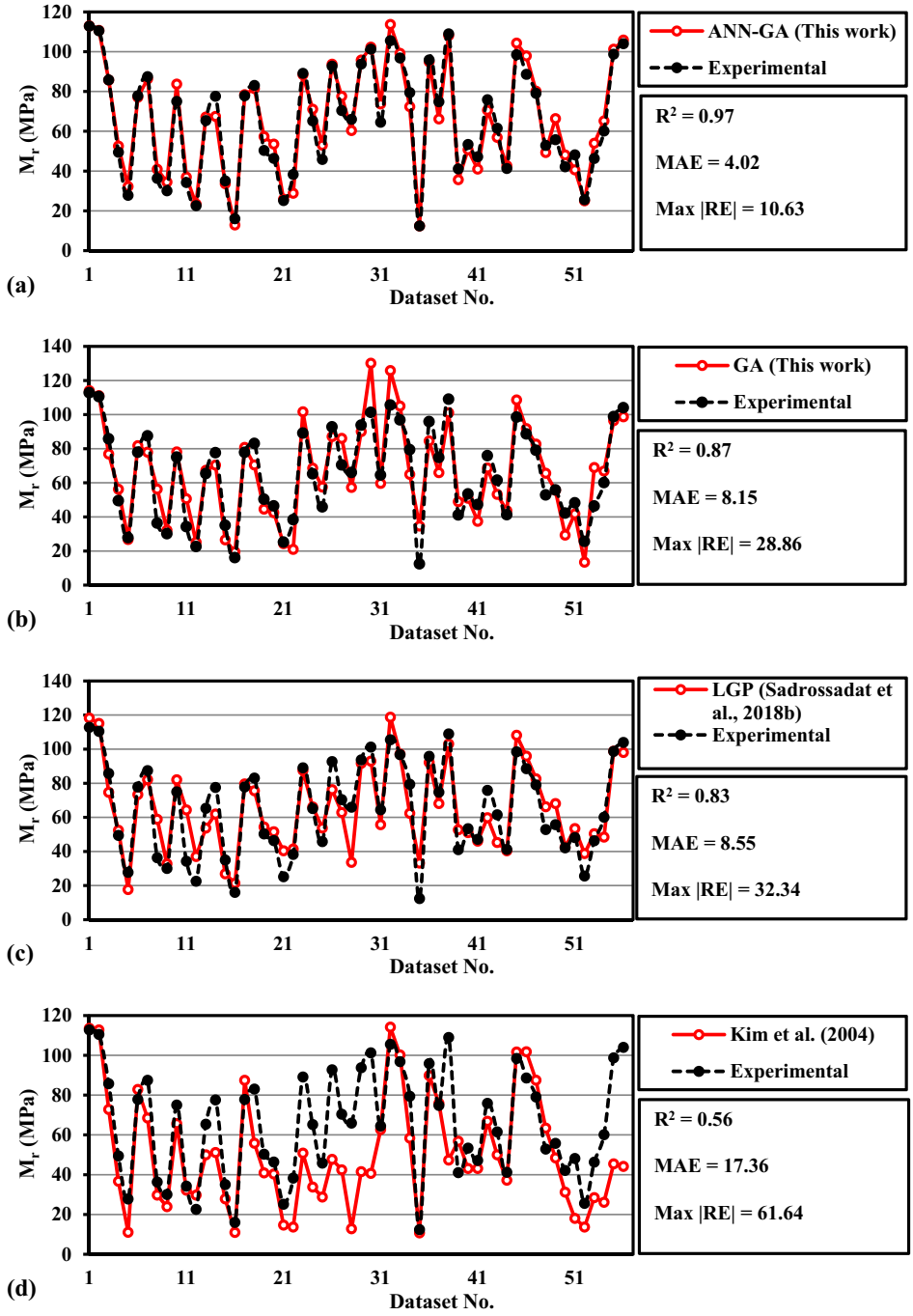

Fig.6. Measured values of M_r and those obtained by (a) ANN-GA (b) GA (c) LGP (Sadrossadat et al., 2018b) (d) Kim (Kim, 2004) method.

some other statistical criteria to compare the performance of the models (Abu-Farsakh and Titi, 2004, Ghorbani et al., 2018). This statistical procedure is based on the following four criteria:

1. The equation of best fine line of predicted (M_{rp}) versus measured (M_{rm}) resilient modulus ($M_{r\text{-}fit}/M_{rm}$), along with corresponding coefficient of determination (R^2). It is worth noting that model with ($M_{r\text{-}fit}/M_{rm}$) and R^2 closer to 1 has the best performance.
2. The arithmetic mean (μ) and standard deviation (σ) of M_{rp}/M_{rm}. It is suggested that a model with μcloser to 1 and σcloser to 0 has a better performance in prediction of the M_r.
3. The 50% cumulative probability ($P_{50\%}$) of M_{rp}/M_{rm}. To calculate the $P_{50\%}$, the values of M_{rp}/M_{rm} are arranged in an ascending order, and the cumulative probability is calculated using following equation:

$$P = \frac{i}{n+1} \tag{19}$$

The value of the $P_{50\%}$ for the optimal model should be close to one.

4. The coefficient of efficiency (E). This parameter evaluates how well each model describes the variance of the datasets. E can be calculated using following equations:

$$E = \frac{E_1 - E_2}{E_1} \tag{20}$$

$$E_1 = \sum_{i=1}^{n} \left(M_{rm} - \bar{M}_{rp}\right)^2 \tag{21}$$

$$E_2 = \sum_{i=1}^{n} \left(M_{rp} - M_{rm}\right)^2 \tag{22}$$

The overall performance of each model can be measured in terms of rank index (RI), which is the sum of the ranks of each sub criteria:

$$RI = R_1 + R_2 + R_3 + R_4 \tag{23}$$

where R_1, R_2, R_3, and R_4 are the ranks from each of the explained sub criteria. The model with the lowest RI has the best performance when predicting the M_r of cohesive subgrade soils. The best fit line of M_{rp}/M_{rm} for each inves-

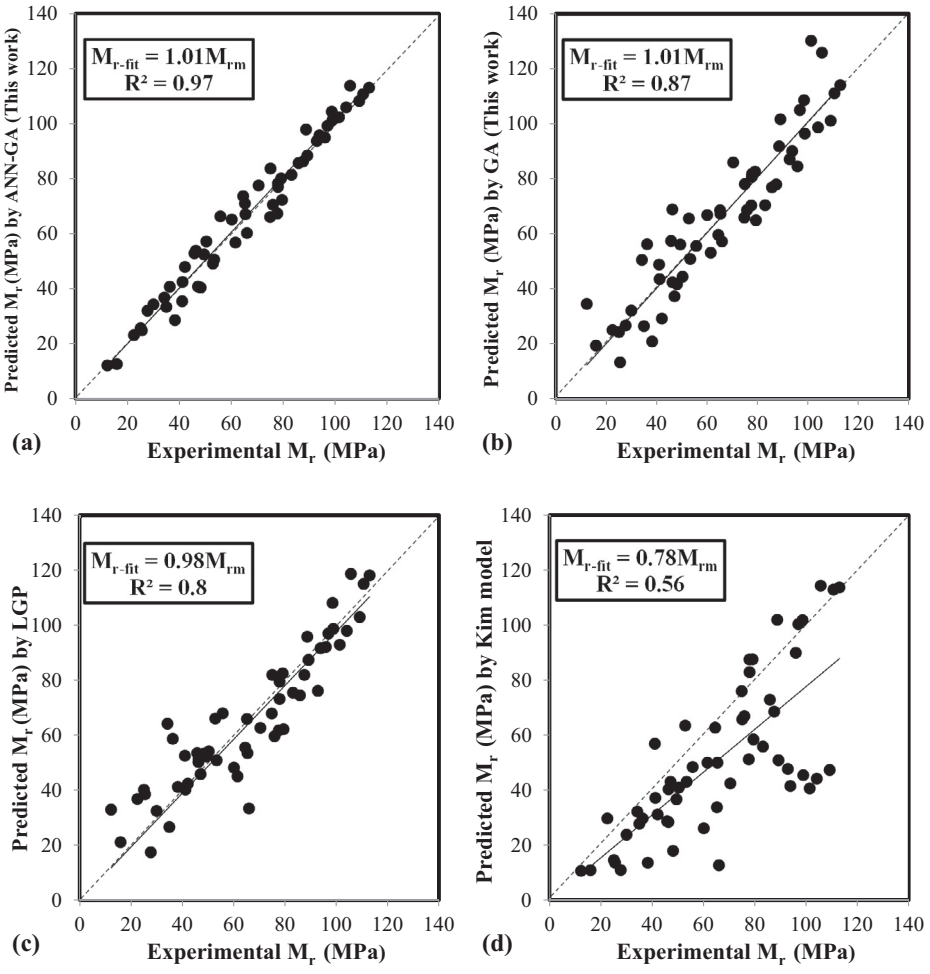

Fig. 7. Best fit line of M_{rp}/M_{rm} for (a) ANN-GA model (b) GA model (c) LGP model (d) Kim model.

Table 8
Results of statistical analysis of models.

Method	Overall performance			Arithmetic calculation of M_{rp}/M_{rm}			Cumulative probability			Coefficient of efficiency		Overall rank	
Model	M_{rfit}/M_{rm}	R^2	R_1	Mean	SD	R_2	M_{rp}/M_{rm} at P_{50}	R_3	E	R_4	RI	Final rank
ANN-GA	1.01	0.97	1	1.01	0.09	1	1	1	0.97	1	4	1
GA	1.01	0.87	2	1.04	0.30	2	1	1	0.85	2	8	2
Sadrossadat et al. (2018b)	0.98	0.83	3	1.06	0.33	3	1	1	0.82	3	11	3
Kim (2004)	0.78	0.56	4	0.77	0.26	4	0.8	4	0.45	4	16	4

tigated model is illustrated in Fig. 7. As evident, the ANN-GA model performs the best among all models, followed by GA, LGP and the Kim model. Therefore, ANN-GA model is ranked 1 (i.e. $R_1 = 1$) based on the first criterion. While AI-based models are highly capable of predicting the M_r, Kim's model tends to underestimate the M_r values. The results of the first criterion are summarized in the column corresponding to R_1 in Table 8.

Considering the cumulative probability criteria, the plots of M_{rp}/M_{rm} versus cumulative probability (%) are illustrated in Fig. 8. As is evident from this figure, the M_{rp}/M_{rm} values in more than 95% of the predicted values by the ANN-GA model are in the 0.8–1.2 range. The $P_{50\%}$ for the ANN-GA, ANN, LGP, and Kim's model are 1, 1, 1, and 0.8. A similar trend can be found in all other criteria, as is summarized in Table 8. It can be concluded that the ANN-GA approach is highly capable of predicting the M_r of subgrade soils by considering several validation criteria, followed by GA model and the LGP model. Generally, AI-based methods perform notably better than the regression based method by Kim (2004).

10. Sensitivity analysis

To measure the relative importance of each input variable and its contribution to the final model, a sensitivity analysis (SA) is performed. SA aims to measure the strength of the relationship between model inputs and the output variable. In this study, the Cosine amplitude method (CAM) is used as an indicator of significance of each input variable (Yang and Zhang, 1997, Majdi and Rezaei, 2013). Considering a set of n data samples in the common X-space, a data array X can be defined as:

$$X = \{X_1, X_2, ..., X_m\} \tag{24}$$

Each of the elements, x_i, in the data array X is a vector of lengths of m, that is:

$$X_i = \{X_{i1}, X_{i2}, ..., X_{im}\}_i \tag{25}$$

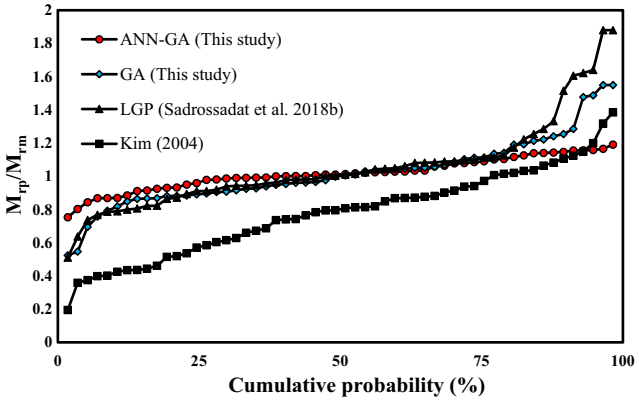

Fig. 8. Plots of M_{rp}/M_{rm} versus cumulative probability (%) for different models.

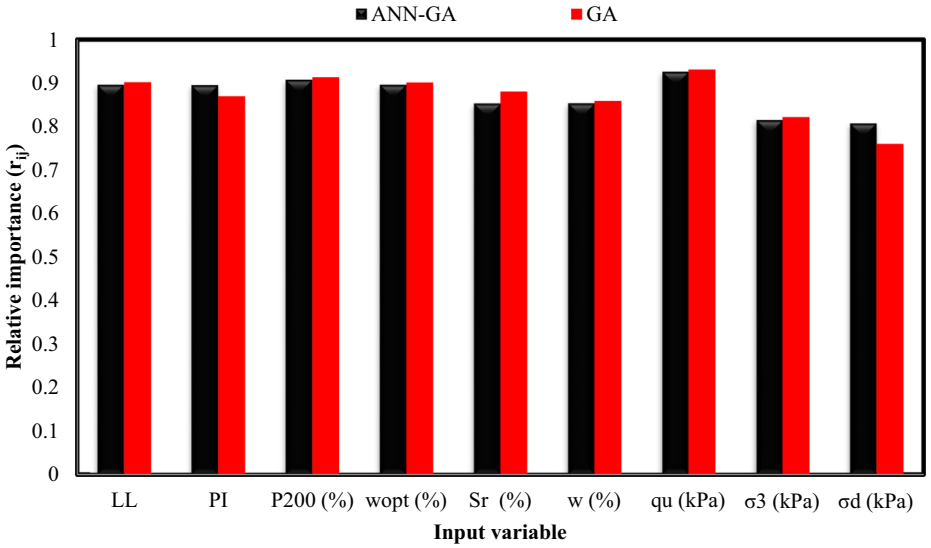

Fig. 9. Relative importance of the input variable in the developed models.

Each data sample can therefore be regarded as a point in m-dimensional space, where each point requires m coordinates for full description. Each element of relation, r_{ij} is the result of a pairwise comparison of two data samples, x_i and x_j. The strength of the relation between these two data pairs is in a 0 to 1 scale and is expressed by the following equation (Sadrossadat et al., 2016):

$$r_{ij} = \sum_{k=1}^{m} x_{ik}.x_{jk} / \sqrt{\sum_{k=1}^{m} x_{ik}^2 . \sum_{k=1}^{m} x_{jk}^2} \quad (26)$$

Fig. 9 describes the relative importance of each input variables in the developed models. The closer the r_{ij} to 1, the more impact corresponding variable has on the M_r value. As is evident, the r_{ij} values for all input variables of both methods are between 0.75 and 0.92, which indicates the significance of all input variables for model development. In other words, all input variables are approximately equally important in the prediction of the M_r of subgrade soils, and their importance for model development cannot be neglected. However, q_u, P_{200}, and $_{wopt}$ are the most influential parameters on the M_r. These results are in agreement with those of similar studies in the literature (Sadrossadat et al., 2016, Kim, 2004, Sadrossadat et al., 2018b).

11. Conclusions

In this study, the potential of two intelligent methods, i.e. GA and ANN-GA, was evaluated for prediction of M_r of cohesive subgrade soils. GA was used to develop an equation for prediction of M_r of subgrade soils. In addition, GA was utilized to enhance the predictive capability of the ANN by adjusting the weights and bias.

A comprehensive database was utilized for the development of the models. The nine input parameters used for model development were as follows: the liquid limit (LL), plastic index (PI), percentage of soil particles passing through #200 sieve (P_{200}), optimum moisture content (w_{opt}), degree of saturation (S_r), moisture content (w), unconfined compressive strength (q_u), confining stress (σ_3), and deviator stress (σ_d). The predicted performance of the developed models was compared with existing prediction equations in the literature. A ranking index was used to evaluate the external capability of the proposed model.

The results of this study indicate that both the GA and ANN-GA methods can be employed as efficient tools in predicting the M_r of cohesive soils. The R^2 of the predicted and measured values for the ANN-GA model and GA model was 0.97 and 0.87 for both training and testing datasets, which was superior to the available prediction equations. One of the main objectives of the present study was to clearly show that the ANN-GA model can be expressed as explicit formula which can be used for manual calculation purposes. Also, the shown prediction capability of the developed GA model clearly indicates that the evolutionary algorithms can be regarded as efficient tools for providing precise and simple equations. The results obtained by the ANN-GA and GA models outperformed other existing equations in the literature in terms of precision and accuracy for several validation criteria.

The obtained results of the sensitivity analysis indicated the importance of all input variables for predicting the M_r of cohesive subgrade soils. It should be noted that the capability of the AI-based methods is mostly limited to the range, number and statistical features of the database used for model developments. To resolve this, developed models can be enhanced by increasing the number of datasets. Nonetheless, the models provided in this study could be used to estimate the M_r of subgrade soils without conducting any tests. The proposed models are expected to be useful in the preliminary design stages or when the testing is not feasible.

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