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A transient resistance/capacitance network-based model for heat spreading in substrate stacks having multiple anisotropic layers

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A Transient Resistance/Capacitance Network-Based Model for Heat Spreading in Substrate Stacks having Multiple Anisotropic Layers

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Abstract

Heat spreading from local, time-dependent heat sources in electronic packages results in the propagation of temperature non-uniformities through the stack of material layers attached to the chip. Available models either predict the chip temperatures only in the steady-state or a single substrate for transient analysis, without the ability to predict the transient response in compound substrates having multiple anisotropic layers. We develop a transient resistance/capacitance network-based modeling approach capable of predicting the spatiotemporal temperature fields for this chip-on-stack geometry, accounting for in-plane heat spreading, through-plane heat conduction, and the effective convection resistance boundary conditions. The transient heat spreading resistance for an anisotropic substrate has been formulated by converting it to an effective isotropic substrate for which there is an available half-space solution. The transient heat spreading model for a step heat input to a single substrate is subsequently extended to any arbitrary transient heat input using a Fourier series and extending the half-space formulation from a step heat input to sinusoidal heat input. For obtaining the transient spreading resistance for heat inputs into multiple stacked substrates, a method has been outlined to convert the multi-substrate stack to a single substrate with effective isotropic properties by properly accounting for the in-plane and through-plane thermal conductivities, and the heat capacity. The estimates from the present model are validated with direct comparison to a finite-volume numerical model for three-dimensional heat conduction. Case studies are presented to demonstrate the capabilities of the proposed network-based model and compare its estimates with the numerical conduction solution. In the presence of a step heat input, the results demonstrate that the model accurately captures the transient

temperature rise across the multi-substrate stack comprising layers with different anisotropic properties. For a case where the rectangular stack is exposed to a sinusoidally varying heat input the model is able to capture the general trends in the transient temperature fields in the plane where the heat source is applied to the multi-substrate stack. In summary, the developed resistance/capacitance network-based transient model offers a low-computational-cost method to predict the spatiotemporal temperature distribution over an arbitrary transient heat source interfacing a multi-layer stack of substrates.

1 Introduction

Electronic packages typically comprise time-dependent heat sources attached to a composite substrate that results in temperature non-uniformities. The spatiotemporal temperature gradients arising from these transient heat inputs can strongly affect the device reliability. The device performance is largely influenced by heat conduction and spreading inside the multiple packaging layers, which determines the chip temperatures and gradients in the plane of these heat sources. Hence, it is critical to develop compact transient models to predict the chip temperatures within these electronic packages to facilitate the design of reliable packaging technologies.

There have been extensive previous investigations focusing on the development of models to compute the steady-state and transient temperature fields for conduction heat spreading from local heat sources, as reviewed by Razavi and Muzychka [1]. Lee et al. [2] proposed closed-form steady-state formulations for spreading resistances in an axisymmetric chip-on-substrate geometry, subjected to a wide range of boundary conditions. In a sequence of studies, Muzychka et al. developed multi-dimensional, series-solution-based analytical models for steady-state spreading resistance in a Cartesian domain for single and compound (i.e., multi-layer) isotropic substrates [3], and anisotropic substrates [4], later incorporating interfacial resistances between the layers in compound substrates [5]. Bagnall et al. [6] proposed an analytical series-solution-based model for estimating the steady-state spreading resistance for a complex multi-layered structure, accounting for anisotropy (viz., different in-plane versus through-plane conductivity), multiple heat sources, and application of convection boundary condition on the same face as the heat source.

Alternatively, several studies have focused on transient modeling of a chip-on-substrate geometry. In this architecture, heat from a local source on a substrate is spread out by conduction and then dissipated to a reference sink temperature with an effective convection coefficient. Yovanovich et al. [7,8] proposed a half-space integral formulation to estimate the temperature fields and transient spreading resistance for an isotropic substrate subjected to a step heat input. These multi-dimensional temperature solutions have been transformed into area-averaged and centroid-based transient spreading resistance models for arbitrarily shaped iso-flux heat inputs [9]. Yazawa et al. [10] developed a reduced-order model by extending the transient formulations reported in [9] to account for transient heat spreading and conduction in a heat spreader under a step heat input. Sadeghi et al. [11] proposed unified relations for accurately estimating the transient spreading resistance over a broad spectrum of heat source shapes under both isoflux and isothermal conditions. Recently, Sudhakar et al. [12] outlined an analytical series-solution-based model for computing the three-dimensional spatiotemporal temperature fields in a substrate subjected to arbitrarily located, time-varying heat inputs.

The thermal models developed in past literature are either capable of only estimating the steady-state chip temperatures for a multi-substrate stack, or for purposes of transient analysis, consider only a single substrate. The present study introduces a compact, transient thermal network-based modeling approach in a chip-on-stack geometry capable of estimating the spatiotemporal evolution of temperature fields in the plane of the heat source for multi-layer substrates where each layer has anisotropic properties. First, the transient thermal network for a general chip-on-stack architecture is developed. The transient heat spreading resistance for any given substrate with an anisotropic thermal conductivity has been formulated based on the pre-existing framework [9] for an isotropic substrate. The transient heat spreading model for a step heat input to a single substrate is subsequently employed to develop the formulation for any arbitrary time-dependent heat input. Lastly in the model development, an approach for considering a multi-substrate stack is described. Case studies are presented to demonstrate the capabilities of the proposed thermal model and to compare its estimates with a finite-volume numerical solution.

2 Model Development

The multi-substrate stack described schematically in Figure 1 comprises M heat sources placed arbitrarily underneath a stack of N substrates. Any spatially non-uniform heat input to a multi-substrate stack can be effectively represented as a combination of multiple superposed spatially uniform heat inputs. Hence, in this study we will only consider spatially uniform heat inputs. Each of the heat sources has a time-varying input heat load $Q_i(\tau)$ distributed uniformly over the heat source area, where the index i corresponds to a particular heat source. The remaining area surrounding the heat sources is insulated (as are the side surfaces of the substrates); a convective boundary condition with a uniform effective heat transfer coefficient h at a reference temperature T_∞ is considered as the boundary condition at the top of the substrate stack. The total power generated by the heat sources gets spread laterally and conducted across the stack before being dissipated from the top of the spreader. The transient resistance/capacitance network (see Fig. 1(b)) comprises representative elements denoting the different mechanisms of heat transfer in the multi-substrate stack (namely, the transient in-plane heat spreading, transient through-plane heat conduction, and the convection boundary condition). As time progresses, each of the substrates stores energy and conducts heat across its thickness. The resistance and capacitance corresponding to a particular substrate share a common temperature node, which denotes the average temperature at the base of a given substrate. The transient heat conduction through the j^{th} substrate is represented by a capacitance (C_j) and a resistance ($R_{cond,j}$), as:

$$
\begin{aligned}
&C_j = \rho_j c_j t_j A_j;\\
&R_{cond,j} = \frac{t_j}{k_{z,j} A_j}\\
&A_j = ab
\end{aligned}
\tag{1}
$$

The transient resistance arising from the heat spreading in the entire stack from the i^{th} source is denoted by $R_{sp,i}(\tau)$. This transient spreading resistance can be estimated for any given point in the plane of the heat source. In this study, we estimate the transient spreading resistance for the centroid of the heat input area. Consequently, this will correspond to the maximum transient spreading resistance and will yield the maximum temperature for a given heat source. Details of the approach behind the estimation of the transient spreading resistance are described in the subsections that follow.

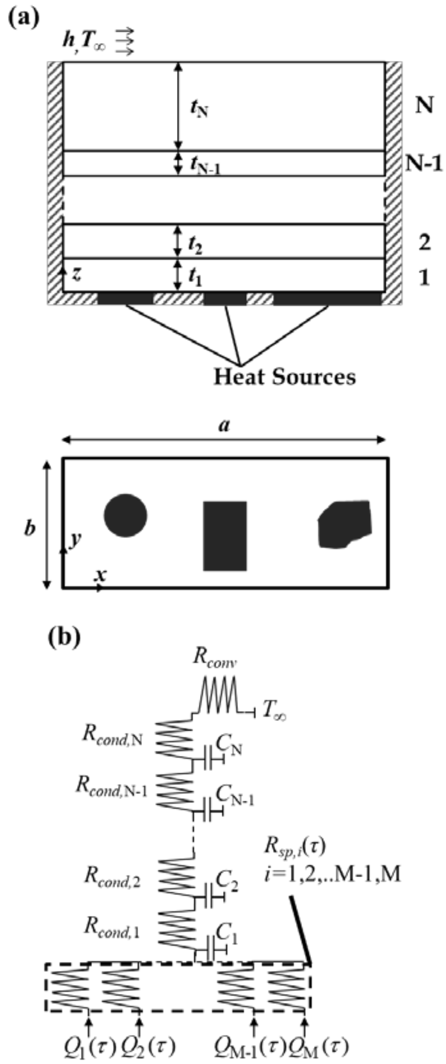

Fig. 1. (a) Schematic drawings of the cross-section of a general multi-substrate stack (top) subjected to discrete heat sources (bottom). (b) Transient network depicting the in-plane spreading resistances corresponding to each heat source $R_{sp,i}(\tau)$, through-plane conduction resistances $R_{cond,j}$ and capacitances C_j of each substrate, and the boundary convection resistance R_{conv} (where i denotes a given heat source and j denotes a particular substrate).

2.1 Transient Spreading Resistance for a Step Heat Input to a Single Substrate.

2.1.1 Isotropic Substrate. The resistance due to transient heat spreading for an arbitrarily located and randomly shaped step heat input, at a uniform heat flux, on an isotropic single substrate (see Fig. 2(a)), can be quantified using the half-space formulation proposed by Yovanovich [7], and can be represented as:

$$\frac{R_{sp}(\tau)}{R_{sp,ss}} = F\left(\frac{\alpha}{A_S}\right) \tag{2}$$

The steady-state spreading resistance ($R_{sp,ss}$) is estimated from an exact Fourier series based analytical solution of the temperature fields in a given stack, subjected to a given boundary condition [1]; as noted above, this study considers an effective convective boundary condition at the top of the stack. The exact nature of the function F depends on the shape of the heat input; details of the nature of the function for different shapes are available in Refs. [7,8]. For heat inputs having the shape of a circle or a regular polygon, and a given source area A_S, the transient spreading resistance is obtained as:

$$\frac{R_{sp}(\tau)}{R_{sp,ss}} = \left\{\sqrt{\frac{4\alpha\tau}{A_S}}\left(1-\exp\left(\frac{-A_S}{4\pi\alpha\tau}\right)\right)\right\} + \left\{\mathrm{erfc}\left(\sqrt{\frac{A_S}{4\pi\alpha\tau}}\right)\right\} \tag{3}$$

2.1.2 Anisotropic Substrate. The governing energy equation and boundary conditions for transient heat conduction in a substrate having different in-plane and through-plane conductivities (see Fig. 2(b)), subject to a given heat input at the base and a convective boundary condition at the top of the substrate, are given by:

$$\begin{aligned}
&\rho c\frac{\partial T}{\partial \tau} = k_{xy}\frac{\partial^2 T}{\partial x^2} + k_{xy}\frac{\partial^2 T}{\partial y^2} + k_z\frac{\partial^2 T}{\partial z^2}\\
&z = 0 : -k_z\frac{\partial T}{\partial z} = q''(\tau)\\
&z = t : -k_z\frac{\partial T}{\partial z} = h\left(T - T_\infty\right)
\end{aligned} \tag{4}$$

To obtain the resistance due to transient heat spreading in an anisotropic substrate for an arbitrarily located and randomly shaped step heat input (at a uniform heat flux), we find its effective isotropic diffusivity. Hence, we introduce the following mathematical transformations, similar to Ref. [4].

$$\begin{aligned}
&\rho_{eff}c_{eff} = \frac{\rho c k_{eff}}{k_{xy}}\\
&k_{eff} = \sqrt{k_{xy}k_z}\\
&\varepsilon = z\sqrt{\frac{k_{xy}}{k_z}}
\end{aligned} \tag{5}$$

The transformed governing equation and boundary conditions, rewritten in terms of the transformed through-plane coordinate variable ε, results in:

$$
\begin{aligned}
&\rho_{eff} c_{eff} \frac{\partial T}{\partial \tau} = k_{eff} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial \varepsilon^2} \right) \\
&\varepsilon = 0 : -k_{eff} \frac{\partial T}{\partial \varepsilon} = q^{''}(\tau) \\
&\varepsilon = \tilde{t} : -k_{eff} \frac{\partial T}{\partial \varepsilon} = h\left(T - T_{\infty}\right) \\
&\tilde{t} = t \sqrt{\frac{k_{xy}}{k_z}}
\end{aligned}
\tag{6}
$$

where $\tilde{t}$ is the transformed substrate thickness. The effective isotropic diffusivity is estimated as:

$$
\alpha_{eff} = \frac{k_{eff}}{\rho_{eff} c_{eff}}
$$

(7)

The steady-state spreading resistance ($R_{sp,ss}$) and the transient spreading resistance $R_{sp}(\tau)$, for the anisotropic substrate, can be obtained by substituting this effective isotropic diffusivity into Eq. (3). In the presence of multiple heat sources, the spreading resistance, following the formulation for an isotropic substrate above, can be computed using the principle of superposition, similar to the approach outlined in Ref. [3].

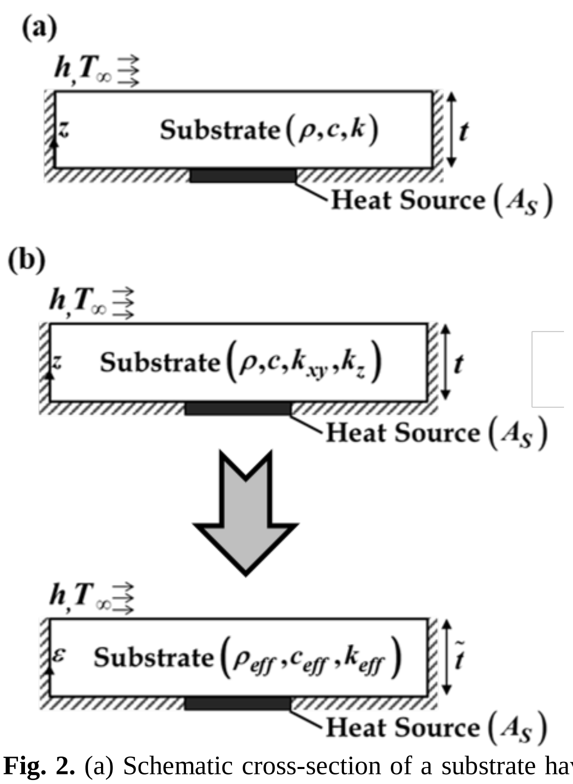

Fig. 2. (a) Schematic cross-section of a substrate having an isotropic conductivity subjected to a transient heat source having an area A_S. (b) Schematic cross-section depicting the transformation of a substrate having anisotropic conductivities to an analogous isotropic substrate with effective thermophysical properties.

2.2 Transient Spreading Resistance for any Arbitrary Transient Heat Input to a Single Substrate.

Any transient heat input $Q(\tau)$ to a substrate can be represented as a combination of sinusoidal heat inputs at multiple frequencies, as obtained from a Fourier series (where the summation of infinite terms may be truncated to a summation of a finite number of terms n beyond which the contribution of leading order terms becomes negligible), as:

$$Q(\tau) = \sum_{v=1}^{\infty} Q_v \sin(\omega_v \tau) \approx \sum_{v=1}^{n} Q_v \sin(\omega_v \tau) \tag{8}$$

Each constituent sinusoidal heat input $Q_v \sin(\omega_v \tau)$ will result in a change in temperature corresponding to a transient heat spreading resistance:

$$\begin{aligned} \Delta T_{sp}(\tau) &\approx \sum_{v=1}^{n} \Delta T_{sp,v}(\tau) \\ \Delta T_{sp,v}(\tau) &= R_{sp,v}(\tau) Q_v \sin(\omega_v \tau) \end{aligned} \tag{9}$$

The net transient spreading resistance attributable to all sinusoidal inputs can be subsequently computed as:

$$R_{sp}(\tau) = \frac{\Delta T_{sp}(\tau)}{Q(\tau)} \approx \frac{\sum_{v=1}^{n} \Delta T_{sp,v}(\tau)}{Q(\tau)} \tag{10}$$

To compute the new transient spreading resistance for arbitrary heat input to a single substrate, the spreading resistance for a sinusoidal heat input with a given amplitude must be first estimated. Then, all these estimated amplitudes can be summed and divided by the net heat load to estimate the new transient spreading resistance.

The transient spreading resistance for a sinusoidal heat input can be estimated following a method similar to the isotropic half-space formulation as outlined by Yovanovich [9]. To account for an anisotropic substrate property in estimation of the transient spreading resistance, the approach represented by Eq. (4) - Eq. (7) can again be used to obtain the effective isotropic diffusivity. The present solution approach employs the principle of superposition to evaluate the instantaneous temperature change at the centroid (see Fig. 3) of the heat source. The heat source provides a heat flux which is spatially uniform and sinusoidally varying in time $q''\sin(\omega\tau)$ over the heat input area A_S and an adiabatic boundary condition for all the points outside the heated area. The boundary conditions at points far away from the heat source have a temperature identical to the initial temperature (T_{init}) at any given time τ. The resulting governing equation for transient heat conduction in spherical coordinates, with the boundary conditions and the initial conditions, is given by:

$$\begin{aligned}
&\frac{\partial\theta}{\partial\tau} = \frac{\alpha_{eff}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right); \theta = T - T_{init} \\
&\tau = 0\text{: } \theta = 0 \\
&r = 0\text{: } -k_{eff}\frac{\partial\theta}{\partial r} = q^{"}\sin\left(\omega\tau\right); q^{"} = \frac{Q}{A_S} \\
&r \to \infty : \theta = 0
\end{aligned} \tag{11}$$

To solve Eq. (11), the temperature difference $\theta(r,\tau)$ with respect to the initial temperature is decomposed into two components.

$$\theta(r,\tau) = \theta_1(r,\tau) + \theta_2(r,\tau) \tag{12}$$

The governing differential equation with the respective boundary conditions and the initial conditions for the first decomposed component $\theta_1(r,\tau)$ is:

$$\begin{aligned}
&\frac{\partial\theta_1}{\partial\tau} = \frac{\alpha_{eff}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta_1}{\partial r}\right) \\
&\tau = 0\text{: } \theta_1 = \frac{-q^{"}dA_S\left\{\exp(-mr)\sin(mr)\right\}}{2\pi k_{eff}r} \\
&r = 0\text{: } -k_{eff}\frac{\partial\theta_1}{\partial r} = q^{"}\sin\left(\omega\tau\right) \\
&r \to \infty\text{: } \theta_1 = 0
\end{aligned} \tag{13}$$

where m = $\sqrt{\dfrac{\omega}{2\alpha_{eff}}}$. The complementary equation set for the second decomposed component $\theta_2(r,\tau)$ is given by:

$$\begin{aligned}
&\frac{\partial\theta_2}{\partial\tau} = \frac{\alpha_{eff}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta_2}{\partial r}\right) \\
&\tau = 0\text{: } \theta_2 = -\theta_1 \\
&r = 0\text{: } -k_{eff}\frac{\partial\theta_2}{\partial r} = 0 \\
&r \to \infty\text{: } \theta_2 = 0
\end{aligned} \tag{14}$$

The governing equations, boundary conditions, and initial conditions for the first decomposed component $\theta_1(r,\tau)$, is of a form following Stokes second problem [13]; the solution is thus:

$$\theta_1(r,\tau) = \frac{q^{"}dA_S\left\{\exp(-mr)\sin(\omega\tau - mr)\right\}}{2\pi k_{eff}r} \tag{15}$$

The governing differential equation for the second decomposed component $\theta_2(r,\tau)$ has homogeneous boundary conditions. Hence, the second decomposed component $\theta_2(r,\tau)$ can be evaluated by applying the separation of variables approach, as:

$$\begin{aligned} \theta_2(r,\tau) &= \sum_{g=1}^{\infty} \frac{C_g \sin(v_g r)\exp(-v_g{}^2 \alpha_{eff} \tau)}{r}; v_g = gm \\ \theta_2(r,0) &= -\theta_1(r,0) = \frac{q^{"} dA_S \left\{\exp(-mr)\sin(mr)\right\}}{2\pi k_{eff} r} \end{aligned} \tag{16}$$

The coefficients C_g can be estimated by Fourier series expansion, with the initial condition of $\theta_1(r,\tau)$ specified by Eq. (13). Now, the net temperature difference $\theta(r,\tau)$ can be computed by summing $\theta_1(r,\tau)$ and $\theta_2(r,\tau)$. The net temperature difference at the centroid of the given arbitrary shaped heat source can be computed by integrating the temperature change $\theta(r,\tau)$ over the entire area of heat input:

$$\theta_{sp}(0,\tau) = \int_{dA_S} \theta(r,\tau) dA_S \tag{17}$$

Equation (17) can be employed to compute the net temperature rise due to heat spreading from any arbitrary shape following a similar approach as developed for a step heat input in Refs. [7,8]. The net temperature changes at the centroid of the heat source due to the decomposed components $\theta_1(r,\tau)$ and $\theta_2(r,\tau)$ are termed $\theta_{sp,1}(0,\tau)$ and $\theta_{sp,2}(0,\tau)$, respectively. For heat inputs having the shape of a circle or a regular polygon, and a given area $A_S = \pi p^2$, the transient spreading resistance can be obtained as:

$$\begin{aligned} \theta_{sp}(0,\tau) &= \int_0^{2\pi}\int_0^{p} \theta(r,\tau) r dr d\theta \\ &= \theta_{sp,1}(0,\tau) + \theta_{sp,2}(0,\tau) \\ &= \int_0^{2\pi}\int_0^{p} \theta_1(r,\tau) r dr d\theta + \int_0^{2\pi}\int_0^{p} \theta_2(r,\tau) r dr d\theta \\ \theta_{sp,1}(0,\tau) &= \frac{q^{"}}{2k_{eff} m}\left\{\sin(\omega\tau) - \cos(\omega\tau) + \left\{\exp(-pm)\left(\sin(pm-\omega\tau) + \cos(pm-\omega\tau)\right)\right\}\right\} \\ \theta_{sp,2}(0,\tau) &= \sum_{g=1}^{\infty} \frac{2\pi C_g}{k_g dA_S} \exp(-k_g{}^2 \alpha\tau)\left\{1-\cos(k_g p)\right\} \end{aligned} \tag{18}$$

The transient spreading resistance $R_{sp}(\tau)$ due to the sinusoidal heat input $Q\sin(\omega\tau)$ can then be computed as:

$$R_{sp}(\tau) = \frac{\theta_{sp}(0,\tau)}{Q\sin(\omega\tau)} \tag{19}$$

The above formulation is obtained for a heat source whose centroid coincides with the centroid of the base of the effective isotropic half-space domain. In the case of a step heat input over the same area, when the heat source centroid does not coincide with the centroid of the base of the effective isotropic half-space domain, the transient spreading resistance $R_{sp,step,skew}(\tau)$ is related to its counterpart $R_{sp,step,center}(\tau)$ for a centered heat input as:

$$\frac{R_{sp,step,skew}(\tau)}{R_{sp,step,center}(\tau)} = \frac{R_{sp,ss,step,skew}}{R_{sp,ss,step,center}} \tag{20}$$

Hence, the transient spreading resistance $R_{sp,sin,skew}(\tau)$ arising from the non-centered sinusoidal heat input Q $\sin(\omega\tau)$ may be estimated as:

$$R_{sp,sin,skew}(\tau) = R_{sp,sin,center}(\tau)\frac{R_{sp,ss,step,skew}}{R_{sp,ss,step,center}} \tag{21}$$

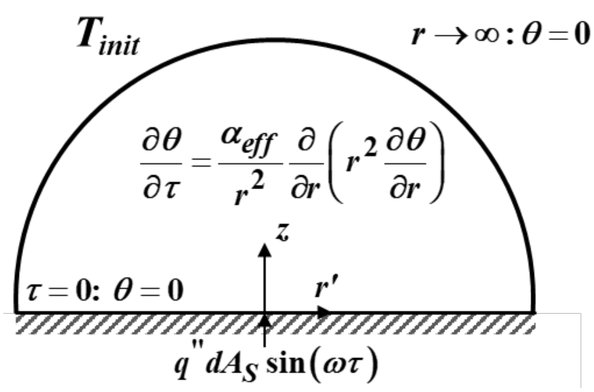

Fig. 3. Schematic of the half-space domain having an effective isotropic diffusivity, subjected to a spatially uniform and sinusoidally varying (in time) heat flux $q''\sin(\omega\tau)$ at the origin (r' denotes the radial coordinate in the plane of the heat source) and an adiabatic boundary condition for all the points outside the heated area. The governing differential equation for the domain is written in terms of the temperature difference (θ) with respect to the initial temperature (T_{init}). The boundary conditions at points far away from the heat source have a temperature identical to the initial temperature (T_{init}) at any given time τ.

2.3 Transient Spreading Resistance for any Arbitrary Transient Heat Input to a Multi-Substrate Stack.

To obtain the resistance due to transient heat spreading in a multi-substrate stack for arbitrary transient heat inputs, we evaluate the effective isotropic diffusivity of the entire stack. In practical applications, the resistances due to through-plane conduction across the substrates, computed based on the total cross-sectional area, substrate thickness, and the conductivity, are orders of magnitude lower than the spreading resistance and the convection resistance. Hence, for purposes of estimating the effective properties of the multi-substrate stack, the original transient network (as shown in Fig. 1(b)) can be reduced into the network shown in Fig. 4(a), which assumes the conduction resistances are much smaller than the spreading resistances ($R_{cond,j}$ <<

$R_{sp,i}$) and the convection resistance ($R_{cond,j}$ << R_{conv}). This network can then be simplified to represent through-plane conduction with a single effective resistance and capacitance (see Fig 4(b)) obtained as:

$$
\begin{aligned}
R_{eff} &= \sum_{j=1}^{N} R_{cond,j} \\
C_{eff} &= \sum_{j=1}^{N} C_j
\end{aligned}
\tag{22}
$$

The effective isotropic conductivity and the capacity (and thereby diffusivity) of the multi-substrate stack can then be estimated as:

$$
\begin{aligned}
k_{eff} &= \frac{\sum_{j=1}^{N} t_j}{R_{eff} A_j} \\
\rho_{eff} c_{eff} &= \frac{C_{eff}}{\sum_{j=1}^{N} A_j t_j}
\end{aligned}
\tag{23}
$$

Comparing the through-plane conduction resistances for the multi-substrate stack to the convection resistance yields the effective Biot number (*Bi*). The proposed formulation of the effective diffusivity for a multi-substrate stack is applicable when the through-plan conduction resistance is negligible relative to the in-plane spreading resistance and the convection resistance. The resistance arising from the convection from the top of the stack of substrates to the coolant is obtained, as:

$$R_{conv} = \frac{1}{hA_n} \tag{24}$$

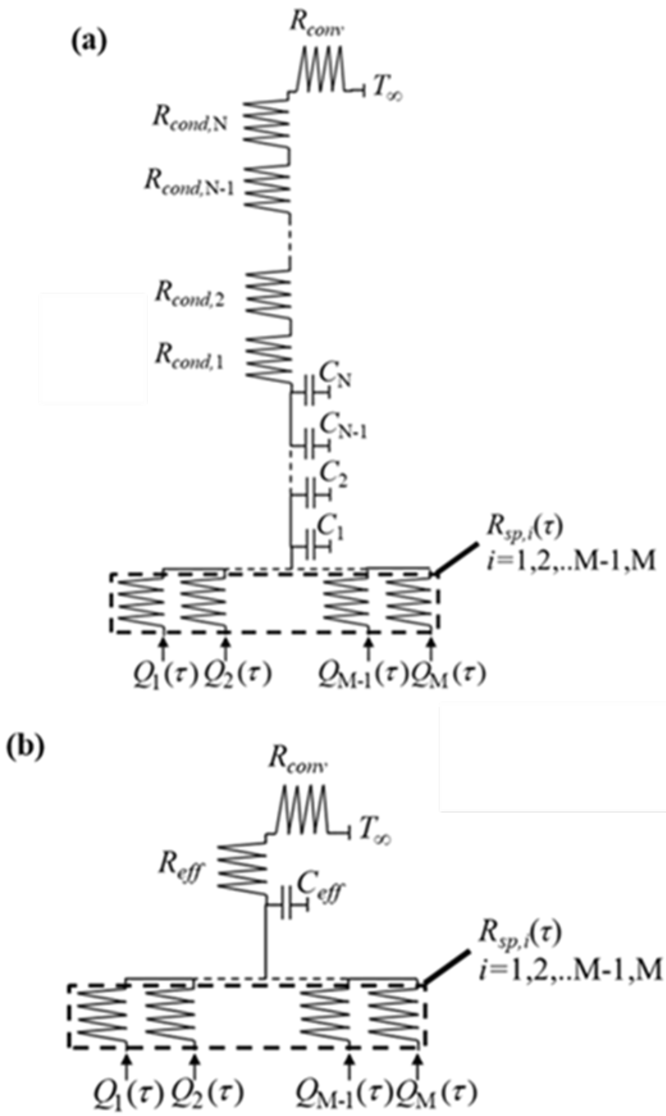

Fig. 4. (a) Reduced transient network representing the multi-substrate stack in the limit of negligible conduction resistances relative to the spreading and convection resistances ($R_{cond,j} << R_{sp,i}$) and ($R_{cond,j} << R_{conv}$) (b) The equivalent network of the multi-substrate stack for estimating the effective isotropic diffusivity in calculating the transient spreading resistance.

2.4 Solution Methodology

All the elements of the transient resistance/capacitance network (see Fig. 1(b)), corresponding to the multi-substrate stack are computed using the formulations described in the previous sections. Conservation of energy at each node of the network leads to the governing equation set:

$$
\begin{aligned}
&\frac{\theta_{max,i}-\theta_1}{R_{sp,i}(\tau)} = Q_i(\tau) \\
&C_1\left(\frac{d\theta_1}{d\tau}\right) = \frac{\theta_1-\theta_2}{R_{cond,1}} - \frac{\theta_{max,i}-\theta_1}{R_{sp,i}(\tau)} \;\; \forall\; i=1,2,3,...\text{M};\; j=1 \\
&C_j\left(\frac{d\theta_j}{d\tau}\right) = \frac{\theta_j-\theta_{j+1}}{R_{cond,j}} - \frac{\theta_{j-1}-\theta_j}{R_{cond,j-1}} \;\; \forall\; j=2,3,...\text{N-1} \\
&C_\text{N}\left(\frac{d\theta_\text{N}}{d\tau}\right) = \frac{\theta_\text{N}}{R_{cond,\text{N}}+R_{conv}} - \frac{\theta_{\text{N}-1}-\theta_\text{N}}{R_{cond,\text{N-1}}} \\
&\tau = 0:\;\; \theta_j = 0
\end{aligned}
\tag{24}
$$

where $\theta_{max,i}$ denotes the maximum temperature for the i^{th} heat source and θ_j is the temperature difference (T_j -T_∞) at the base of substrate j. In the transient network, this temperature difference occurs at the intersection of the conduction resistance $R_{cond,j}$ and the capacitance C_j. This set of coupled ordinary differential equations is solved using a finite difference scheme implemented in MATLAB [14]. The derivative of the temperature change with change in time, in the left-hand side of Eq. (26) is discretized using a backward finite difference scheme, as:

$$
\frac{d\theta_j}{d\tau} = \frac{\theta_j(\tau)-\theta_j(\tau-\Delta\tau)}{\Delta\tau} \;\; \forall\; \tau > 0 \tag{25}
$$

Substituting Eq. (27) in Eq. (26) converts the latter into a given set of algebraic equations which can be solved simultaneously at a given time τ to yield the temperatures at that time step, as the temperatures at the previous time step are known. The maximum temperature for the i^{th} heat source can be subsequently computed as:

$$
T_{max,i}(\tau) = \theta_{max,i}(\tau) + T_{init} \tag{26}
$$

The maximum resistance for the i^{th} heat source can be estimated, as:

$$
R_{max,i}(\tau) = \frac{\theta_{max,i}(\tau)}{Q_i(\tau)} \tag{27}
$$

For comparison in the following results section, this maximum resistance is also computed using the numerical conduction model implemented in the commercial software ANSYS Fluent [15]. A metric E is developed to map the deviation between the predictions from the present model and the finite-volume numerical model as a function of the non-dimensional time $\tau^* = \frac{\alpha_{eff}\,\tau}{A_S}$, as:

$$
E(\tau^*) = \frac{\left|R_{max,i,pm}(\tau^*) - R_{max,i,nm}(\tau^*)\right|}{R_{max,i,nm}(\tau^*)} \tag{28}
$$

3 Results and Discussion

The predictions of the developed model are demonstrated, and their accuracy evaluated, for step function and sinusoidal heat inputs for a stack of three rectangular substrates. For these cases, the rectangular stack comprises three anisotropic substrates with the thermophysical properties as given by Table 1, where ρ, c, k_{xy}, and k_z stand for the density, specific heat capacity, in-plane conductivity, and through-plane conductivity of the respective substrates. The length and width of each substrate are a = 150 mm and b = 60 mm. The substrate thicknesses are t_1 = 800 µm, t_2 = 400 µm, and t_3 = 600 µm. The multi-substrate stack is initially at a uniform temperature of 300 K. A convective boundary condition with a heat transfer coefficient 12 W/m^{2}K and an ambient temperature of 300 K is imposed on the top face. The faces coinciding with the plane of the heat source and the side walls are insulated (except for the source itself).

Table 1 Thermophysical properties of the substrates chosen for the case studies

Substrate Index (j)	ρ (kg/m^3)	c (J/kgK)	k_{xy} (W/mK)	k_z (W/mK)
1	2000	840	20	3
2	2200	710	1.3	1.3
3	1738	1020	159	159

3.1 Response of Multi-Substrate Stack to a Central Step Heat Input.

The transient model is demonstrated for a case where a given multi-substrate stack is subjected to a centrally located step heat input (see Fig. 5). For this case, starting at τ = 0 s, the stack is subjected to a heat input of 2 W each over a 4 cm^2 square area (d = 2 cm) at the location identified in Fig. 5a. The centroidal coordinates of the heat source are x_c = 75 mm and y_c = 30 mm. While obtaining the solution, the model is run at time steps of 0.01 s till τ = 2000 s.

Fig. 5c shows the model-predicted (dots) temporal evolution of the maximum temperature of the single heat source, which occurs at the center of the heat input area. The maximum temperature of the heat input area increases until ~ τ = 1500 s, after which it is within ~ 0.2% of the constant temperature that is reached at steady state. At steady state, the total power input to the multi-substrate stack gets dissipated by convection from the top of the stack at the same rate. Overall, this temperature response is characteristic of a step heat input to a stack-up interfacing a heat sink. Comparing the model prediction to the numerical simulations (solid line in Fig. 5c) reveals an excellent match. Fig. 5d shows the evolution of the deviation of the maximum non-dimensional resistance between the model and numerical simulation (as quantified by the metric E), as a function of the non-dimensional time τ^*. The deviation is largest at the start of the step input, and reduces with an increase in non-dimensional time, finally converging to a constant value that indicates the percentage error in the steady state prediction of only 0.2% for this case. This transient error reduces with time as the temperature gradients in the domain reduce and the half-space propagation becomes an increasingly more accurate representation of the actual transient temperature distribution. These results illustrate the capability of the proposed model to accurately capture the transient response of the complex, multi-substrate stack having layers with differing and anisotropic properties, when subjected to a step heat input.

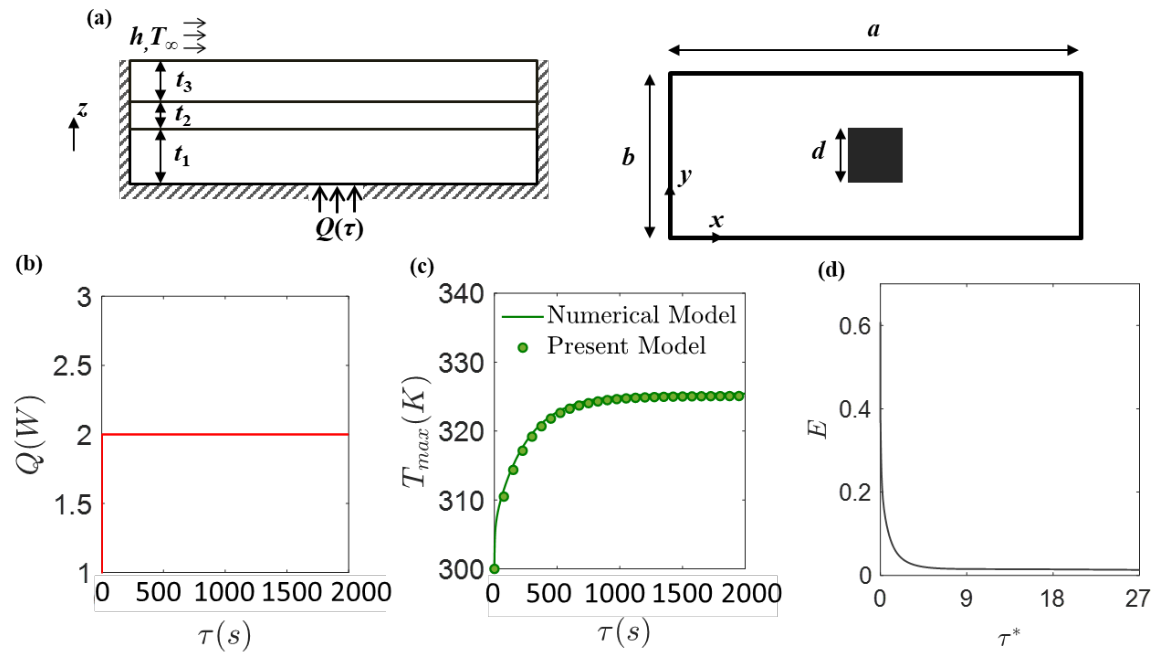

Fig. 5. (a) Sectional view of the multi-substrate stack (left) with a bottom-up view of the surface (right) with the heat source having a (b) step heat input profile. (c) Comparison of the maximum heat source temperature obtained from the present network-based model and the numerical simulations. (d) Non-dimensional metric E quantifying the deviation in the prediction of the maximum resistance obtained from the present model and the numerical simulations.

3.2 Response of Multi-Substrate Stack to an Off-Center Transient Heat Input.

The transient model is also demonstrated for a case (see Fig. 6) where the multi-substrate stack is subjected to a sinusoidal transient heat input, as depicted by Fig. 6b. Starting at $\tau = 0$ s, the stack is subjected to the specified heat input over a 4 cm^2 square area (d = 2 cm) at the off-center location identified in Fig. 6a. The centroidal coordinates of the heat source are x_c = 130 mm and y_c = 20 mm. While obtaining the solution, the model is run at time steps of 0.01 s, till τ = 1000 s.

Fig. 6c shows the temporal evolution of the maximum source temperature at the center of the heat input area as a dash-dot line. The maximum temperature of the heat source displays an oscillating periodic behavior following the sinusoidal heat input. The amplitude of the oscillations, as well as the mean, increase during the initial simulation time up till ~ τ = 750 s. After τ = 750 s, a steady time-periodic response is reached, and the amplitude of the oscillations in the maximum temperature become constant. Similar to the step input case, the time-averaged total power input to the multi-substrate stack and that dissipated by convection become balanced during this steady time-periodic response. Fig. 6d shows the evolution of the deviation of the maximum non-dimensional resistance as compared to the numerical simulation as a function of the non-dimensional time. With an increase in non-dimensional time τ^*, the deviation of the maximum non-dimensional resistance displays a periodic behavior with decreasing amplitude, and finally, the amplitude of oscillation attains a constant value. The slight deviation in the mean temperature (τ > 750) is attributed to the compounded effect of the off-center location of heat source and the assumption behind the linear scaling of the transient spreading resistance with the steady-state spreading resistances, as given by Eq. 20 and Eq. 21. While the mean temperature and trend in oscillations are well-predicted, the model is less accurately in capturing the peak amplitudes of the transient response of the temperatures, arising from the half-space formulation (superposition of point heat loads) of the sinusoidal heat input. However, it is worthwhile to note that there are three orders of magnitude reduction in the computational time from ~12 hr for the finite volume numerical simulation to ~0.02 hr for the present model, with both implemented on the same server node. These results illustrate the capability of the proposed model to capture the general transient behavior in the heat source plane of the multi-substrate stack when subjected to a sinusoidal heat input.

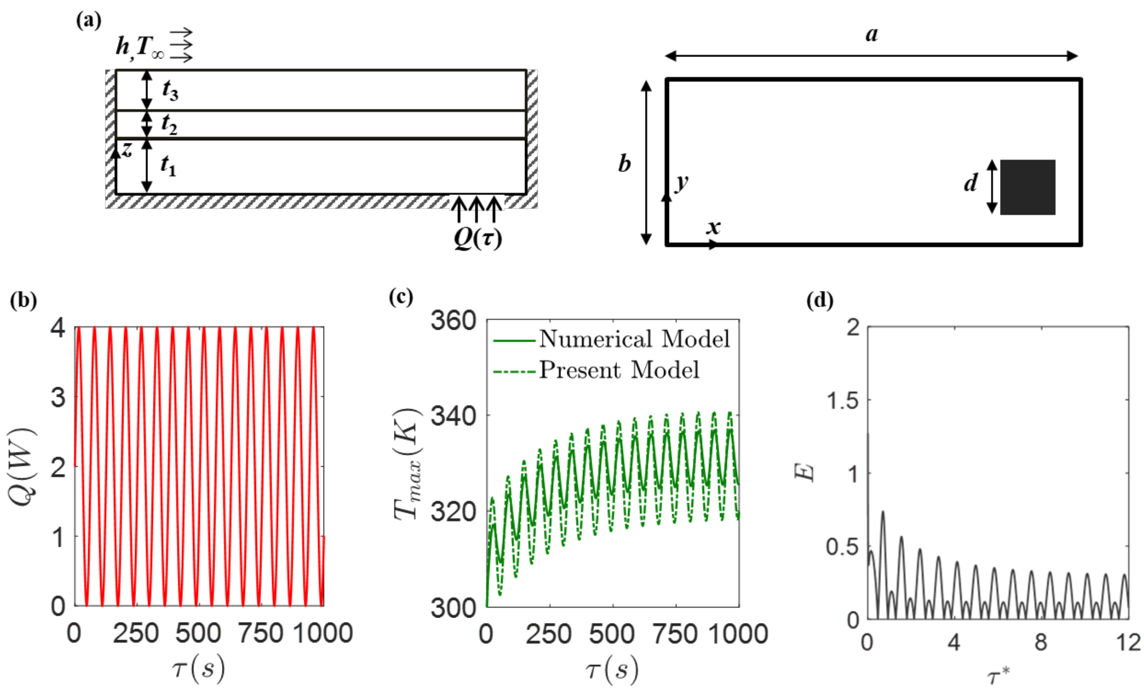

Fig. 6. (a) Sectional view of the multi-substrate stack with the plan-view location and (b) transient heat input profile for the heat source. (c) Comparison of the maximum heat source temperature obtained from the present model and the numerical simulations. (d) Non-dimensional metric *E* quantifying the deviation in the estimate of the maximum resistance obtained from the present model and the numerical simulations.

4 Conclusions

A low-computational-cost resistance/capacitance network-based modeling approach is developed to predict the spatiotemporal temperatures over an arbitrary located and shaped transient heat source attached to a multi-layer stack of substrates, where each substrate layer can have different anisotropic properties. The network considers all salient transport mechanisms of in-plane heat spreading, through-plane heat conduction across the layers, and an effective convection boundary condition. As a foundational building block, the transient model is first constructed for any single substrate subjected to a step heat input, accounting for anisotropic properties by obtaining the effective isotropic diffusivity representation. Based on the formulation for a step heat input response, this transient model is extended to accept arbitrary transient heat inputs using Fourier series. For heat inputs to multiple stacked substrates, the effective isotropic diffusivity is estimated for developing the model, by converting the multi-substrate stack to an equivalent single-substrate domain. The deviation between the temporal predictions of the maximum heat source temperature from the present model and finite volume simulations is evaluated for cases of a step heat input and sinusoidal heat input to a complex substrate stack-up. Results obtained from the proposed model indicate good accuracy in estimation of die temperatures relative to the finite volume conduction simulations at a significantly reduced computational cost.

While the proposed model is evaluated for only for a couple of specific cases in this study for demonstration purposes, it may be generally applicable for other material stackups and power maps, provided that the assumption of a dominant spreading resistance in the network holds. The resistance/capacitance network-based modeling approach can be particularly useful to minimize the time for co-design of stacked and heterogenous architectures for which low-computational-cost thermal models are needed to co-optimize along with other electrical and mechanical performance objectives. Also, resistance/capacitance network-based models are well-suited for the needs of thermal control systems in electronic devices [16], where the framework can be used to construct semi-empirical models based on available device test data. In summary, the analysis method developed in this work can be broadly useful in quantifying the heat source temperatures for conduction and spreading into a multi-substrate stack having a wide range of form factors and thermophysical properties under a variety of boundary conditions.

Nomenclature

A	cross-sectional area
a	length of rectangular substrate
Bi	Biot number
b	width of rectangular substrate
C	thermal capacitance
c	specific heat capacity
d	side length of square heat source
E	non-dimensional error metric
h	heat transfer coefficient
k	thermal conductivity
M	number of heat sources
N	number of substrates
Q	heat input
q''	heat flux
R	thermal resistance
r	radial coordinate direction
r'	radial coordinate in the plane of heat source
T	temperature
T_{init}	initial temperature
T_∞	ambient reference temperature
t	thickness of substrate
$\tilde{t}$	transformed thickness of substrate
$\tilde{t}\,v_g$	component of eigen value for estimating $\theta_2(r,\tau)$

x,y,z Cartesian coordinate directions

Greek Symbols

α thermal diffusivity
ϵ transformed coordinate in through-plane direction
ρ density
τ time
τ^* non-dimensional time
θ temperature rise from initial condition
ω frequency

Subscripts

c centroid
center centered heat input
cond conduction
conv convection
eff effective isotropic representation
i index for a heat source
init initial
j index for a substrate
max maximum
nm numerical model
pm present model
S heat source
sin sinusoidal heat input
skew non-centered heat input
sp spreading
ss steady-state
step step heat input
v summation variable
g summation variable
xy in-plane
z through-plane

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