REPUTATION AUTO-CORRELATION: IMPLICATIONS FOR SIMULATION

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ABSTRACT

Previous work has established that the distribution of daily reputation scores is best modelled by a bi-partite pair of exponential distributions. Simulations developed from that distributional model did not account for auto-correlations in the data. We now extend the bi-partite model in two ways. Candidate auto-correlation methods are assessed in order to incorporate the auto-correlation structure of the data in a simulation. Negative reputational shocks are then modelled using a chi-square distribution, so that they can then adequately model runs of successive days of either positive or negative sentiment. Auto-correlation goodness-of-fit tests show that the optimal auto-correlation model uses the fitted auto-regression components of the original data, and that goodness-of-fit can be improved by inflating them by about 1%. This optimised model is successful in at least 88% of simulations where auto-correlation in the original data does not extend beyond 10 lags. In other cases (mainly due to severe reputational shock), 80% success can be expected. Examples of shock simulations for large corporate organisations are shown, and the implications for reputational analysis are discussed.

INTRODUCTION

Technological advances since 2014 have made it possible to measure reputation in a comprehensive and objective way. On a daily basis, the output of that measurement process is a single number (the *score*) that represents the reputation of a target organisation on the day of measurement. However, reputation has to be established over an extended period, using an accumulation of multiple successive daily reputation scores. Therefore, an objective view of a target's reputation is encapsulated in the statistical properties of the set of scores. In this paper we consider reputation in the context of large corporate organisations, with emphasis on those that have experience a severe reputational shock in the past decade: Volkswagen ('Dieselgate' emissions in 2015), Boeing (the *737 MAX* aircraft crashes in 2018 and 2019) and HSBC (illegal Swiss bank accounts first reported in 2015).

Previous work (Mitic (2017)) has established that the distribution of daily reputation scores is unusual. With measurements in the range $(-1, 1)$, there is a marked clustering close to zero, indicating that the majority of scores indicate near neutral reputation. The optimal model is a bi-partite pair of exponential distributions (the *BiExponential* distribution). That distribution, and any other, is independent of the order of the daily scores. Investigations show that reputation scores can manifest significant auto-correlations. In particular, sequences of successive negative reputation are frequent. A plausible reason is that agents (people, the press etc.) are encouraged to express negative sentiment if they see prior negative sentiment. The correlogram for Volkswagen in Figure 1 illustrates the point. Significant auto-correlations persist up to 26 lags. The date range for the data used to compile it span the period of the 'Dieselgate' emissions scandal in September 2015.

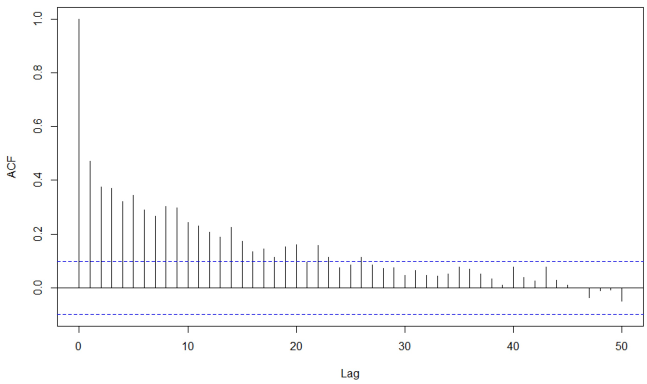

Figure 1: Volkswagen Correlogram, 30/06/2015 to 30/07/2016

The purpose of this paper is therefore to develop a reputation score distribution that incorporates an appropriate auto-correlation structure. Reputational scores can then be better simulated in order to study the effect of potential reputational events.

LITERATURE REVIEW

This literature review deals with two distinct aspects of reputation. First, its measurement, and second, issues related to auto-correlation.

Reputation Measurement Review

Three modes of reputation measurement are currently apparent. The first chronologically is by survey. The survey technique was pioneered by George Gallup with the founding of the Gallup organization in 1935. Surveys are appropriate for gathering statistics on single issues, and problems with them are now well known. Durant (1954) listed them as dependency of the outcome on the nature of the survey (location, respondents, questionnaire design and administration), cost, bias, and reliability of results. The survey technique has now morphed into online reviews, blogs and comments on platforms such as Twitter. The *Consumers Organisation* (https://www.which.co.uk/) is currently prominent in the UK for product reviews.

A more sophisticated survey method, *Reptrak*, specifically geared to reputation measurement, was developed in the 1990s (Fombrun (2015)). *Reptrak* is a software tool for tracking and analysing stake-holder perceptions. Its basis is to consider reputation as an intangible property rather than a quantity that can be measured directly. 23 such 'intangibles' are combined into 7 latent variables which are regarded as *observables*, using structural equation modelling (SEM). Overall, subjectivity is not avoided, and assessment can only be reasonably made twice yearly.

An alternative indirect approach is 'event association' (Perry and de Fontnouvelle (2005) and Fiordelisi (2013)). A proxy has to be used for reputation, and share price is readily available for corporates (although not for other organisations or individuals). 'Event association' proceeds by first identifying, subjectively, reputational events. The proxy is then projected forward in time from the point of a reputational event, and is compared to the actual proxy value at the forward projection point. The difference between the projected and actual values constitutes the 'abnormal return', which is used as a measure of reputation. In using a proxy in this way, an assumption is required that the proxy is a valid reputation measure. That is not always true for share price, which is subject to many other factors.

Direct reputation became possible during the past 10 years with widespread use of the internet. An outline of the 'sentiment mining' technique may be found in (alva Group (2021)) and (Mitic (2017)). Since 'sentiment mining' forms the basis of the reputation time series used in this research, a more detailed summary of the process follows in the section on *Reputation Measurement*. The method is more objective than others provided that the data feeds used can cover as many as possible relevant data sources, and that a history of measurement has been accumulated. Once sourced, contents are scored using sentiment analysis (see, for example, Lui (2015)). The idiosyncrasies of written language are difficult to analyse, and is therefore a source of error.

Auto-correlation Review

Press (1969) presents an account of early research on auto-correlation, including work on the distribution of the serial correlation coefficient (Anderson (1942)), and the standard statistical test for auto-correlation (Durbin-Watson (1951)). Auto-correlations in reputational time series have not hitherto been studied, although they have been noted in many other contexts. Fishman and Kiviat (1967) provide an early reference to the effect of induced auto-correlation in generic time series simulated by stochastic processes. They consider induced auto-correlation using the power spectral density as the Fourier transform of the auto-correlation function (ACF) of the time series. More specifically, negative auto-correlations in financial time series have been noted many times since the 1960s (Fama (1965) and Schwartz and Whitcomb (1997)). Early research in other contexts include Leith (1973) (weather and climate), Sayers et al (1981) (biomedical), and (later for the context) Wang et al (2014) (social networks).

Auto-correlation and Simulation

The basis of the copula method (namely separation of the analysis of dependence from the properties of marginals) for generating auto-correlated time series is discussed in, for example, Durante and Sempi (2010). Since copulas are used in this paper, discussion of copula usage in the context of reputational time series is deferred until the section on *Theoretical Models*.

In the Davis-Harte (1987) method, an auto-correlated time series is be generated using a stationary Gaussian Process. However, that method is only applicable if there is non-negative auto-correlation, and that either the auto-correlation sequence is convex and decreasing, or zero beyond some particular lag. That is not always the case with reputation time series. The *ARTA* ('Auto-Regressive To Anything') method of Cario and Nelson (1996) uses a Gaussian Process to transform a process with a known auto-correlation structure such that any desired marginal distribution results. *Cholesky decomposition* is a commonly-used way to generate a correlated time series from a non-correlated time series (See, for example, *https://en.wikipedia.org/wiki/Cholesky_decomposition*). However, the required positive-definite conditions cannot be guaranteed for reputational time series. In practice, some reputational auto-correlation intensities were under-estimated using *Cholesky decomposition*. ARIMA modelling (see, for example, Hyndman and Khandakar (2008)) requires much pre-conditioning, as in *https://otexts.com/fpp2/arima-r.html*). As an alternative, we have used the *auto.arima* function in the R package *forecast*, which selects a best-fit model from a range of viable alternatives using the minimum AIC criterion.

Reputation Measurement

Details of the reputation measurement process used to derive the data used in this study can be found in (Mitic (2017)). The *Reputation Score* originates by data mining *content* from as many sources (social and 'traditional' media) as possible, targeted upon a particular organisation or person. After rejecting irrelevancies, *Contents* are analysed using Natural Language Processing (*NLP*), thereby deriving a *sentiment score* for each. A weighted average of those *sentiment scores* is calculated using weights derived from the prominence of the content's source and mode of transmission. That constitutes a reputation score for a single day, for a target organisation or individual. It should be noted that a daily reputation score is built from a base of zero sentiment. This may explain reputational distribution properties (discussed below). *Reputation* is a long term (at least 6 months) time series of single day reputation scores.

Reputational Distribution

Prior work in (Mitic (2021)) showed that the *Bi-Exponential* distribution was a best-fit for all data sets considered. The *Bi-Exponential* density, $f_B()$, is shown in Equation 1. In that equation, c is the normalising constant determined from $c\int_{-1}^{m} e^{b(x-m)}dx + c\int_{m}^{1} e^{-a(x-m)}dx = 1$.

$$f_B(x,m,a,b) = \begin{cases} ce^{b(x-m)}; & x \in (-1,m] \\ ce^{-a(x-m)}; & x \in [m,1) \end{cases} \qquad (1)$$

The corresponding *BiExp* distribution function is denoted by F_B.

$$F_B(x,m,a,b) = \int_{-1}^{x} f_B(z,m,a,b)dz \qquad (2)$$

Reputational *Bi-Exponential* densities show major density concentrations near m, the empirical modal value. These originate from averaging of *NLP* calculations, leading to cancellation of positive and negative sentiments for individual contents. Contents that express extreme sentiment are rare.

Reputational Auto-correlation

Observations of auto-correlation in reputation data reveal two basic patterns. Some show no evidence of significant auto-correlations, indicating that the reputation on any one day is independent of the reputation on any other. Other cases have a very long 'memory'. Auto-correlations persist for between 3 and 4 weeks, especially in cases of severe reputational shock. One case (Nationwide Building Society) had a periodic ACF in the years 2015-16, with a period of approximately 1 week. We speculate that Nationwide generated very positive news weekly in that period. Any simulation needs to be able to replicate all those cases.

Reputational Shocks

Very few prolonged negative reputational shocks have been observed. For those that have, the following features are apparent:

- a rapid drop in the daily reputation score (1-5 days);
- a short period near the minimum score (2-14 days);
- a long relaxation period where reputation climbs slowly to or near to its pre-shock level;
- periodic 1-day 'after shocks', due to reiterations of previous negative sentiment.

Positive shocks follow a completely different pattern. They comprise isolated spikes of positive sentiment that last one, two, or three days only. They arrive in response to short lived positive sentiment related to particular events. Examples are positive reports of excellent trading results in the financial press, or very positive product reviews on social media.

THEORETICAL MODELS

The aim of the elements in this section is to generate a random sample that preserves both the auto-correlation, and the distribution of the original data. With both components, a more reliable simulation can be made than with any one of them. Care must be taken if random elements, other than sampling from the *BiExp* distribution, are introduced. Any random perturbation of a simulated time series that expresses an accurate auto-correlation structure always degrades the auto-correlation structure, often completely.

Initial Data Preparation

The common basis of two autocorrelation models used here (*Copula* and *Cholesky*), is an estimation of the *Bi-Exp* parameters m, a, b from Equation 1, followed by a calculation of the auto-correlations, a copula simulation, and then a separate shock simulation. The auto-correlations are generated by producing a matrix, M of lagged data. For a time series of length N with L lags, M and its (Pearson) correlation matrix C are given in Equation 3. Other methods use the original data series $\{x_t\}$.

$$\begin{cases} M = \begin{pmatrix} x_1 & x_2 & ... & x_N \\ x_2 & x_3 & ... & x_{N+1} \\ ... & ... & ... & ... \\ x_L & x_{L+1} & ... & x_{N+L-1} \end{pmatrix} \\ C = corr(M) \end{cases} \tag{3}$$

Auto-correlation Copula Model

Proposition 5.1 in Durante and Sempi (2010) shows that a copula is preserved under any transformation that generates any distribution. This proposition establishes a generic method for generating a random sample of any distribution from a random sample of any other. The copula is generated starting with a multi-variate normal (MVN) distribution, using any convenient generator. The *R* MVN function to generate n random samples of takes arguments n, C, and parameter m from Equation 1. The function Φ^{-1} transforms the multi-variate normal vectors to uniformly-distributed vectors. They, in turn, are transformed to *BiExp* distributions X using the *BiExp* distribution function.

$$\begin{aligned} z[i] = MVN(n, C, m); i = 1..L \\ u[i] = \Phi^{-1}(z[i]) \\ X[i] = F_B(u[i], m, a, b) \end{aligned} \tag{4}$$

The penultimate stage of the copula process is to recreate the temporal sequence that was used to formulate (n-by-N) matrix M, by interleaving the column vectors X row-wise. So if $Y[r] = \{X[1,r], X[2,r], ..., X[n,r]\}$ represents the r^{th} row of X, the interleaved sequence Y' is defined by the following 'interleave' operator Ξ.

$$Y' = \Xi(Y) = Y[1] \cup Y[2] \cup ... \cup Y[n] \tag{5}$$

The final stage is to select a random starting point, J, in the sequence Y' such that a sequence of n consecutive entries can be drawn. Donating this sequence by Y'_n, with a 'random select' operator Ψ, the random sample that preserves both the auto-correlation and the distribution of the original data is given in Equation 6.

$$Y'_n = \Psi(Y) = \left\{Y'[j]\right\}_{j=J}^{J+n-1} \tag{6}$$

Auto-correlation Cholesky Model

The equations below show a sequence for generating an auto-correlated simulation using a Cholesky decomposition. The starting point is the correlation matrix C from equation 3. U is its Cholesky decomposition: a proxy for the square root of C. b is a random *BiExp* vector, with dimension equal to the column dimension of U. The operators Ξ and Ψ are defined in the *Copula* section.

$$\begin{aligned} C &= UU^* \\ X &= (Ub)^T \\ Y &= \Xi(X) \\ Y' &= \Psi(Y) \end{aligned} \tag{7}$$

Auto-correlation AR Model

In the AR model, the simulation $\{x_t)\}$ proceeds using estimated auto-regressive parameters p_i in Equation 8, with standard deviation and mean parameters equal to the standard deviation and mean of the original data. To compensate for an underestimate of the auto-regressive parameters, those estimates are inflated by a small constant factor λ (i.e. $p_i \to \lambda p_i$). A value of $\lambda = 1.01$, empirically conditioned on the cases listed in the Results section, proved to be suitable. The auto-regression used is therefore as in Equation 8. This model produces surprisingly good results given its simplicity.

$$x_t = \mu + \lambda \sum_{i=1}^{p} p_i x_{t-i} + \epsilon_t \tag{8}$$

Auto-correlation ARTA Model

The ARTA model is an extended AR (auto-regression) process, based on a transformation of a distribution with a known correlation structure. 'TA' means 'To Anything'. Let $\{z_t\}$ be a standardised Gaussian AR(p) process with auto-correlations $r_1, r_2, ..., r_p$. Then, with the Normal distribution function Φ, a transformed auto-regressive process with a *BiExp* distribution, $\{y_t\}$, can be generated. The next stage is to (implicitly) define a function, ρ, that maps each auto-correlation r_i to a target auto-correlation value ρ_i.

$$\begin{aligned} y_t &= F_B^{-1}(\Phi(z_t)) \\ \rho_i &= \rho(r_i); \quad i = 1..p \end{aligned} \tag{9}$$

The function ρ is a search algorithm. It is shown in Cario and Nelson (1996) that any such algorithm that approximates each r_i to sufficient accuracy, preserves the target distribution F_B.

Auto-correlation ARIMA Model

ARIMA modelling shows that both auto-regressive (AR) and moving average (MA) components are important in reputational time series. With AR, MA and differencing parameters p, q and d respectively, plus a constant μ and error term ϵ_t, the ARIMA model used is given in 10. The same λ-factor that was used for the AR model was applied in the ARIMA model, although its effect was small.

$$x_t = \mu + \lambda \sum_{i=1}^{p} p_i x_{t-i} + \sum_{i=1}^{q} q_i \epsilon_{t-i} + \epsilon_t \tag{10}$$

Shock Models

A negative shock profile can be modelled using a *chi-square* density with an appropriate degrees of freedom (DoF) parameter that reflects the duration of the shock, d, from inception on day t_0 to the end of the relaxation period. Equation 11 shows the shock density, s, as a function of the day number, t, the DoF parameter, k, the shock duration, d, and the shock inception day, t_0. The factor 10 is a scaling so that the chi-squared density applies for a period in excess of a 365-day year.

$$\begin{cases} k = d/10 + 2 \\ s(t, t_0, k) = -\frac{1}{2^{(k/2)}\Gamma(k/2)} \left(\frac{t-t_0}{10}\right)^{(k/2-1)} e^{\left(\frac{t-t_0}{20}\right)} \end{cases} \tag{11}$$

The post-shock period is augmented by appropriate number of single-day shocks, with intensity between -I and $-I/2$, applied on random days between the middle of the relaxation period and the last day.

Other densities were considered as a shock model. A *LogNormal* density is a viable alternative, and gives similar results provided that its μ and σ parameters are both near to 1. Others were rejected because they their densities were too concentrated near the peak depression, and failed to model a long relaxation period. They include the *Beta* and *Gamma* densities, and by implication other fat-tailed distributions.

Having thus defined the shock density, a superposition of the shock density with a reputational data series, $R(t)$, can be made. Multiple shocks $s_1, s_2, ...$ can be added to the superposition, although more than two have not been observed to date. Therefore, a 'shocked' data series with n shocks, $S(t)$, may be expressed as in Equation 12.

$$S(t) = R(t) + \sum_{i=1}^{n} s_i(t, t_0, k) \tag{12}$$

A positive shock profile can be modelled in a similar way to Stage 4 of the negative shock profile. Positive 1- or 2-day shocks are superposed on a 'no-shock' simulated profile at random intervals, determined empirically.

RESULTS

Data

The data series used in this study were selected because they are rare examples of severe reputational shocks. Specifically, the shock model is based on observations of the Volkswagen 'Dieselgate' shock, the Boeing '737 MAX' shocks in October 2018 and March 2019 (Federal Aviation Authority (2020)), and the HSBC tax evasion reports in February 2015 (see, for example, *https://www.bbc.co.uk/news/business-31248913*). Some sector competitors have been added for comparison. The 'shocked' data series exhibit considerable auto-correlation, whereas the sector competitors more resemble random data.

ACF Significance Tests

The accuracy of the auto-correlation in the generated samples was measured by comparing ACFs for the sample and original time series. A t-test was used to compare heights of corresponding significant ACF components, up to 50 lags. Table shows the results. For each of the organisations considered, 25 simulations were generated. The table records the percentage of goodness-of-fit (GoF) passes in the simulations for each organisation (column **Org**) for each auto-correlation method (**Cop** and **Chol** are Copula and Cholesky respectively). A 'pass' in this test indicates that the difference between the simulation and original ACFs was small.

Table 1: ACF Significance Tests: Percentage of GoF passes

Org	**Cop**	**Chol**	**ARTA**	**ARIMA**	**AR**
VW	64.0	87.6	75.6	77.6	96.8
BMW	98.8	99.2	94.4	99.6	100.0
Mercedes	100.0	100.0	100.0	100.0	100.0
Boeing	31.6	98.4	44.4	50.0	80.8
Airbus	86.4	76.4	93.6	88.0	88.0
HSBC	60.8	93.6	70.8	74.4	91.6
Barclays	96.0	96.0	90.0	94.0	97.6
N'wide	82.8	98.0	88.0	88.4	97.6

In most cases, the percentage of passes was high. The AR (simple auto-regressive) model was the best performer. HSBC, Boeing and Volkswagen with *Copula* simulation were anomalies, but the reason is unclear. Although Nationwide/Cholesky scored well using the t-test, the periodic profile produced was irregular in many simulations. Cholesky simulations tended to be too heavily biased to either positive or negative sentiment, depending on the overall sentiment trend. The Nationwide/Copula simulations were able to reproduce

the periodic pattern more successfully. The *ARTA* simulation was mostly satisfactory, but ACFs showed a marked periodicity in almost all cases, which was absent in the original data. Not all Nationwide/ARTA simulations displayed the periodicity that exists in the Nationwide data. *ARIMA* simulations struggled to produce satisfactory auto-correlations for the original time series where auto-correlations persisted for more than 10 lags. The combination Nationwide/ARIMA produced a further anomaly. Although the pass rate is high, the periodic auto-correlation pattern was absent in some simulations, and tended to diminish in intensity with increasing lag in others.
Introducing the λ-factor into the AR and ARIMA models (Equations 8 and 10 respectively) had a mixed effect. It provided a significant boost in the success rate for the VW/AR combination, but only a small boost for the Boeing/AR combination. There was no significant boost for the VW/ARIMA and Boeing/ARIMA combinations.

Shock Simulation illustrations

We first show a series of simulation illustrations for Volkswagen, illustrating the 'Dieselgate' negative shock. *AR* simulation is used, since that method, in general, produces acceptable simulated ACF results (as in Table). Figures 2 and 3 show two views of the Volkswagen shock simulation, superimposed upon the original data. The sharp discontinuity following 'Dieselgate' inception on day 80 is apparent. Organisations tend to have their own distinct cumulative reputation profiles, and the shock appears as a marked discontinuity in the profile gradient.

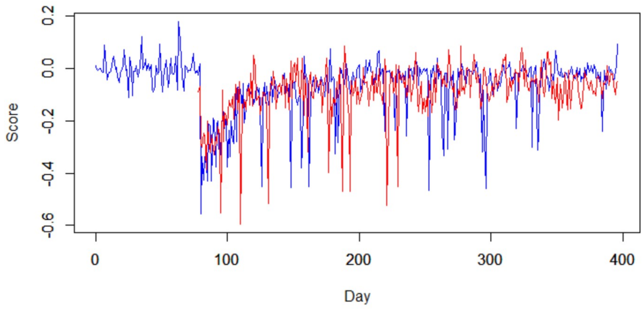

Figure 2: Volkswagen Reputation Shock Simulation: original data in blue, with 1 independent shock simulation in red

Projection Simulation illustrations

In order to explore possibilities for predicting future reputation, AR simulations based on the first 300 days of available Volkswagen data were developed, projected for the next 5 days (nominally a 'working week'), and

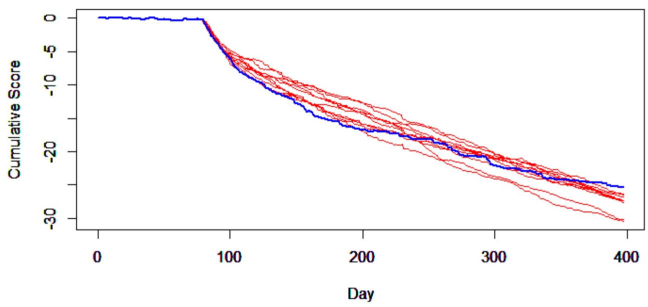

Figure 3: Volkswagen Cumulative Reputation Shock Simulation: original data in blue, 10 simulations in red

compared with the original data. The AR method produces deterministic projections, so small random perturbations were added to them so as to provide a small degree of variation without disturbing the auto-correlation structure too much. Figure 4 shows two sets of projections for Volkswagen. The first (in red) shows a view of Volkswagen's cumulative reputation if no attempt is made to mitigate to the consequences of 'Dieselgate'. Mitigation could be achieved, for example, by admitting liability, compensating customers, or strengthening operating procedures. The second (in green) shows what might happen if those measures are applied. In the former case, cumulative reputation continues to drop. In the latter case, cumulative reputation drops less. It does not reverse direction without a more significant boost. The simulations illustrate the important point that a poor reputation is very difficult to reverse.

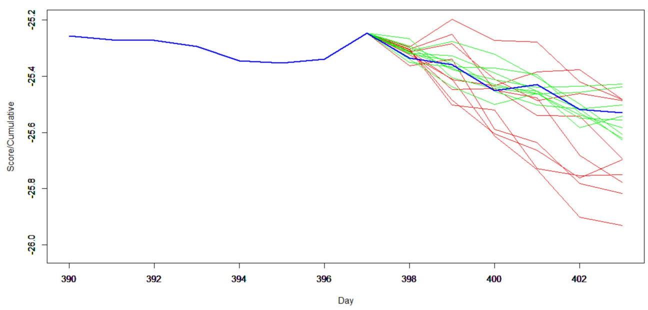

Figure 4: Volkswagen Projection Cumulative Simulation: original data in blue, 10 negative sentiment simulations in red, and 10 positive sentiment simulations in green

ACF Simulation illustrations

ACF simulations also follow the ACF profile of the original data closely. Figure 5 shows three independent VW ACF simulations. Some underestimation of correlations for lag 1-25 is apparent on some simulations. The black horizontal lines show, approximately, 95% significance levels. Significant auto-correlations lie above the upper bound or below the lower bound.

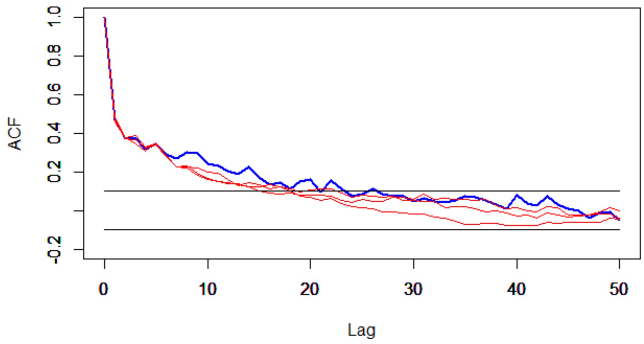

Figure 5: Volkswagen Auto-correlation Simulations: original data in blue, with 3 independent simulations in red

As comparisons, ACF simulations for BMW (Figure 6) and Nationwide (Figure 7) are also shown. The BMW data shows no significant auto-correlation, and the Nationwide data shows the curious periodicity referred to in the discussion of reputation distribution. In both cases, the simulations follow the profile of the original data.

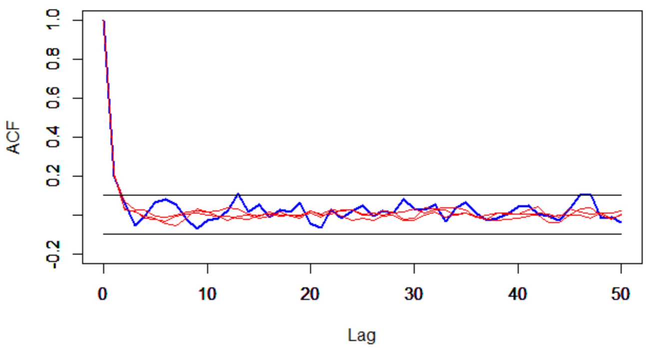

Figure 6: BMW Auto-correlation Simulations: original data in blue, with 3 independent simulations in red

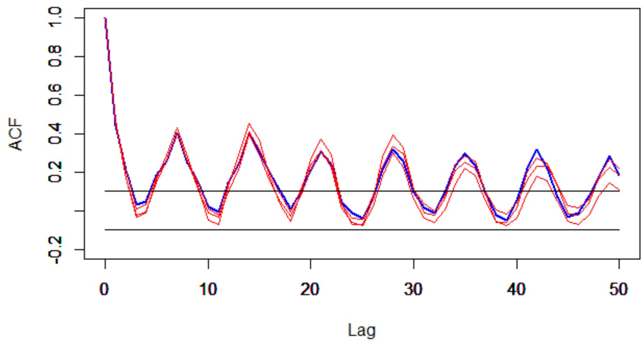

Figure 7: Nationwide Auto-correlation Simulations: original data in blue, with 3 independent simulations in red

DISCUSSION

It became evident in generating simulations, that sampling from the *BiExp* (or any other appropriate) distribution was not sufficient for preserving the auto-correlation structure of the original data. Four methods to generate an appropriate auto-correlation structure were considered. Three of them were 'established' (Cholesky, AR and ARIMA), and two of them were 'bespoke' (ARTA and Copula). Comparison of the ACF statistic successes in multiple runs of each method (Table), shows that the AR method's performance is, overall, the best of the five. Given that the ARIMA method is an extension of AR, it is surprising that the ARIMA method was not successful at reproducing the autocorrelation structure in cases where the significant autocorrelations persist for more than 10 lags. That was a problem common to all the simulation methods considered. Additionally, it proved to be difficult to reproduce the periodic ACF structure for Nationwide. A more general problem was that auto-correlations tended to be underestimated rather than overestimated.

Despite these misgivings, it has been possible to simulate shocks that closely resemble the few that have been observed. This ability is useful for practitioners in two ways. First, in generating scenarios to show what could happen in the event of a shock of specified intensity and duration. Second, to investigate what would have happened had a shock not occurred. It would be particularly valuable in decision-making. For example, there may be a reputational effect associated with bringing a new product to market, or in changing the nature of an existing product, or in dealings with another organisation. Reputational analysis, done in this way, provides a valuable addition to the insights apparent in balance sheet data.

Applications of auto-correlated simulations in other contexts are apparent. Of those mentioned in the literature review, simulated time series for financial instruments are important for risk mitigation and medical time series are key in drug trials. Uses in social networks are largely unexplored. In all cases, three points should be noted. First, the simulation depends on an underlying distribution (*BiExponential* in the case of reputation data), and an appropriate distribution should be fitted. Second, raw data should be normalised by a linear scaling to the range [-1,1]. Third, it should be noted that reputational time series are constructed by daily accumulations, from zero, of sentiment scores. As such, they might correspond more closely to first differences of time series from other domains. Further work in reputational analysis would also be useful to inject variation into the deterministic AR (and other) simulation, such that projection auto-correlation structures are not impaired.

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