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Support for Small Businesses Amid COVID-19

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Abstract
How should the government support small and medium-sized enterprises amid a pandemic crisis while balancing the trade-off between short-run stabilization and long-run allocative efficiency? We develop a two-sector equilibrium model featuring small businesses with private information on their likely future success and a screening contract. Businesses in the sector adversely affected by a pandemic can apply for government loans to stay afloat. A pro-allocation government sets a harsh default sanction to deter entrepreneurs with poorer projects, thereby improving long-run productivity at the cost of persistent unemployment, whereas a pro-stabilization government sets a lenient default sanction. Interest rate effective lower bound leads to involuntary unemployment in the other open sector and shifts the optimal default sanction to a lenient stance. The rise in firm markups exerts the opposite effect. A high creative destruction wedge polarizes the government's hawkish and dovish stances, and optimal default sanction is more lenient, exacerbating resource misallocation. The model illuminates how credit guarantees might be structured in future crises.

1 | INTRODUCTION

The combination of the Coronavirus pandemic and the policy measures of lockdown and quarantine in response has had a drastic effect on the cash flows and solvency of businesses in the many countries affected, in particular on small and medium-sized enterprises (SMEs) (see Fairlie 2020). If the country was to avert an economic collapse, with a large proportion of its SME population being forced to shut up shop, the need was to get external financial assistance to them quickly. Given the massive numbers of SMEs, for example, over 99% of all businesses in the European Union, accounting for over 65% of the workforce according to European Commission statistics, this could hardly be done, at least not quickly enough, directly from a government office; rather, it had to be done via the existing relationship between SMEs and their main bank.

If the banks themselves were to be left carrying the can for any significant share of the losses arising from non-performing loans (NPLs), then they would have wished to be extremely careful in monitoring, which, however, takes time and effort. Moreover, the banks might be more conservative in their own interests than would be socially or politically desirable. For example, Chodorow-Reich *et al.* (2022) show that in 2020Q1 and 2020Q2 in the USA, small firms had no net drawdown of bank credit lines, whereas large firms did. For all these reasons, many countries in Europe, as well as the USA, implemented credit guarantees for their loan support to SMEs. For example, in the UK, the government then decided that such emergency loans to SMEs, known as 'bounce-back loans' (BBLs), would henceforth be 100% guaranteed by the government—that is, they would not count as NPLs or cause losses to the banks. Similarly, in Germany, the credit guarantee covers from 90% to 100%, and in the USA, the government credit guarantee ranges from 95% to 100%. However, the Netherlands and Spain seem more cautious and strict, and the credit guarantee in the Netherlands can be as limited as 67.5%; see European Central Bank (ECB) (2020,b).

The government credit guarantees have triggered a series of public comments on the likely massive defaults on emergency loans, or 'soft default' in the form of future inflation.[1] One major concern is the likely misallocation of resources that will come from keeping alive businesses that are sub-par, even zombie companies. While there is empirical work such as Bachas *et al.* (2021) suggesting that lending supply to SMEs is responsive to government guarantees, the welfare and allocative effects of these programmes remain to be seen. Assuming that the government puts some weight on allocative efficiency, in order to raise productivity and output over future years, it will need to try to screen out unprofitable and less profitable potential borrowers. So, should the government take a lenient or harsh stance?

Our objective is to assess the normative issue of whether and what degree of screening would have been socially optimal against the backdrop of the state of the world *during* the pandemic. In particular, advanced economies have been characterized by rising markups and declining interest rates until very recently. (We do not focus on the post-pandemic period inflation where the central banks are raising rates.) Figure 1 plots the rising aggregate markup in the USA estimated by De Loecker *et al.* (2020) against the Federal funds rate and the 3-month Treasury bill secondary market rate since 1980. These trends are not US-specific. Del Negro *et al.* (2019) show that the world interest rate of safe and liquid assets has dropped significantly over the past three decades. Parallel to this trend, Díez *et al.* (2021) provide empirical evidence suggesting a decline in

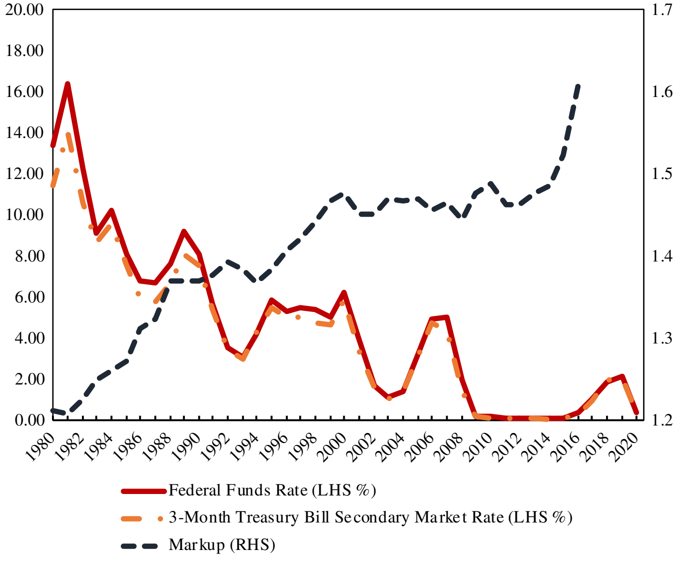


FIGURE 1 Aggregate markup and interest rates in the USA. Sources: The markup data are from the De Loecker *et al.* (2020) estimation. Data Federal funds rate and the 3-month Treasury bill secondary market rate are from the Board of Governors of the Federal Reserve System (USA), retrieved from FRED, Federal Reserve Bank of St Louis.

competition at the global level. We ask whether the optimal level of screening for the government loan support to SMEs might change when the competitiveness of the affected industry differs, and how the interest rate effective lower bound (ELB) might shift the toughness or leniency of optimal screening.

To this end, we develop a two-sector equilibrium model of small businesses with imperfect competition in the presence of a pandemic shock. The sectors of the economy locked down by the COVID-19 pandemic were mainly service sectors, such as hospitality and entertainment. In these cases the standard small enterprise will still normally have some degree of market power as a result of location, customer familiarity and reputation. The COVID-19 pandemic shock forces the adversely shocked sector to close while the other sector remains open. The rationale of modelling a two-sector economy rather than a single-sector economy is twofold: first, in terms of realism, the pandemic lockdown policy forced only certain sectors to close but kept key sectors open; second, a two-sector setup allows us to show that the aggregate demand externality between sectors affects the optimal level of screening in a non-trivial way, which particularly matters for the low interest rate environment during the pandemic. This channel would not be there if the model had only a single sector. The government provides BBLs to the adversely shocked sector to retain at least some key workers, so that these firms avoid large extra costs associated with matching and training a completely new workforce when production resumes in the future. Potential applicants for government loans have private information about their expected profitability. Those with lower profitability are likely to default on government loans. The government can choose to implement a default sanction to ameliorate such adverse selection. Although the context of the model is the pandemic crisis, the model could be applied in more general settings, namely, for an analysis of financial contracting in conjunction with an aggregate demand externality between sectors.

In the benchmark model, the default penalty—i.e. the default sanction—is modelled as a pecuniary deduction from the defaulter's residual income, which is in effect a personal guarantee pledged on future output.[2]This default penalty is used as a screening device, and if borrowers default, then they will lose the personal guarantee. While we acknowledge that in practice, banks provide loans to SME borrowers, and the loan support instrument that the government uses is credit guarantee, which affects banks' monitoring incentive, here by abstracting away banks' monitoring incentive problem and using the default sanction as the screening device in the benchmark model, we obtain analytic tractability. Nevertheless, we extend the benchmark model in Section VI to include the banking sector by considering its monitoring incentive à la Martinez-Miera and Repullo (2017), and we change the policy instrument from the default sanction to credit guarantee. We find that the extended model is isomorphic to the benchmark model, and that our results go through. In the extended model, it is up to the lender to pass through the effect of credit guarantees to the borrower through its monitoring effort, and we show that the credit guarantee is one implementation of the default sanction of the benchmark model. When the government sets 100% credit guarantee, the default sanction is shown to be zero, attracting those with potentially less profitable businesses that stay afloat going forward. As the government reduces credit guarantee, the default sanction becomes harsher. Essentially, increasing the default sanction is equivalent to lowering the degree of credit guarantees, and vice versa. The higher the default sanction, the lower the credit guarantee, so lenders have a stronger incentive to pursue defaulters and garnish their residual income.

If the government is pro-allocation, then we show analytically that the government can choose to be hawkish and implement a harsh default sanction to deter potentially less profitable entrepreneurs from applying for loans, but this pro-allocation policy leads to unemployment in the short run, and demand shortage is persistent. If the government is dovish and pro-stabilization, then we show that the government can choose to implement a lenient default sanction or even no sanctions (i.e. 100% credit guarantees) to fully stabilize short-run employment. In this case, demand shortage is short-lived, but the economy is shifted to a lower equilibrium in the future due to misallocation. Moreover, we develop an analytic measure 'stabilization proclivity' to characterize the conditions under which the government is likely to be more pro-allocation or pro-stabilization. In particular, we show that the market power of small businesses affects stabilization proclivity negatively. When the market power is high, stabilization proclivity is low, which signals a harsher stance, assuming that the government has the same discount factor as households. Indeed, as we show in our numerical solution, when competition increases, it causes a noticeable shift of the optimal default sanction to a lenient stance. This is because when the industry is more competitive, the labour income share accounts for a larger fraction of total output. So the social utility loss associated with unemployment commands a much higher weighting, hence the government leans towards employment stabilization.

The two-sector setup enables us to uncover the aggregate demand externality mechanism à la Guerrieri *et al.* (2022) to inform our optimal screening contract. Assuming that the sectors are complementary to each other, we show that the interest rate ELB causes *involuntary* unemployment even in the sector that is allowed to remain open during the pandemic crisis. Numerically, we demonstrate that the optimal default sanction is more lenient than the case when the economy is away from the ELB. This is because the complementarity of the goods sector means that the spread between the inter-temporal elasticity of the households' utility function and the intra-temporal elasticity is positive. With a negative supply shock, the interest rate decreases but the ELB prevents the interest rate from flexible adjustments, causing inefficiency in employment across sectors. To assess how sensitive the interest rate changes are to the changes of these two elasticities, we vary the inter-temporal elasticity and the intra-temporal elasticity individually while keeping the spread unchanged. We find that the unbounded interest rate changes are more sensitive to the intra-temporal elasticity. Typically, the interest rate dynamics should be more sensitive to the inter-temporal elasticity, but here it is the opposite. This is because the negative supply shock and goods complementarity cause the interest rate dynamics to reflect Keynesian demand shortage in Guerrieri *et al.* (2022).

Therefore, at the interest ELB, involuntary unemployment leads to a more lenient default sanction as optimal policy, which, in terms of implementation, corresponds to a more generous credit guarantee scheme. This has the opposite implication of congestion externalities on resource misallocation. To assess how the interaction with ELB and market power jointly affect involuntary unemployment, we vary both the price elasticity of demand and industry size. We find that the changes in the price elasticity on the optimal default sanction are asymmetric. When we decrease the price elasticity from 1.2 of our benchmark case to 1.05, which means that firm markups and rents increase, we find involuntary unemployment in the open sector increases across the whole spectrum of default sanction. In particular, when the default sanction is at the harsh stance that deters all inefficient projects, involuntary unemployment in the open sector increases to 7.5%. This is striking because the open sector is allowed to operate, and pandemic policies and the elasticity change are applied only in the closed sector. This reflects the aggregate demand externality due to complementarity across sectors in the interest rate ELB environment. This decrease in price elasticity pushes the optimal default sanction to be more lenient, and in our numerical simulations, the optimal default sanction becomes 4.37% more lenient than that of the benchmark parameter space.

In contrast, when we increase the price elasticity from 1.2 to 1.4, involuntary unemployment in the open sector still exists, but it is noticeably less across all spectrums of the default sanction. This would mean that the change in the leniency of the optimal default sanction is small (only 0.5% in our numerical simulation). Interestingly, the change of industry size almost has little effect on the open sector's involuntary unemployment even though it affects firm markup. This is because the industry size does not have a first-order effect on interest rate dynamics, whereas the elasticity parameters do.

Since the key trade-off on which we focus is between short-run employment stabilization and long-run allocative efficiency, which makes a link to creative destruction, we define a wedge to reflect the extent to which unproductive incumbents can be replaced promptly by productive entrants. More precisely, the wedge is the difference between the fraction of unproductive incumbents and the fraction of productive new entrants. The creative destruction wedge is a manifestation of weak property rights and/or high bureaucratic costs to set up businesses, which can deter creative destruction. We solve the closed-form solution to show how this wedge affects the two extreme cases of default sanctions, that is, the harsh default sanction that cares only about long-run allocative efficiency, and the lenient default sanction that cares only about short-run employment stabilization. We show that the former increases with the wedge, while the latter decreases with the wedge. This is because when the creative destruction wedge is large, unproductive incumbents cannot be replaced promptly by productive entrants. To fully stabilize short-term employment, the threshold of the lenient sanction has to decrease. And to remove resource misallocation in the long run, the threshold of the harsh sanction has to increase, since the lack of business dynamism implies that the borrowers' outside option is weak, and the government has to be more hawkish to keep out the inefficient ones. Therefore the lack of creative destruction (higher wedge) polarizes the two extreme cases even more.

As the wedge reflects business dynamism, to assess its quantitative effects, we need a dynamic setting. Thus in our numerical illustration, we adapt our stylized model to a simple dynamic model. We show that indeed, when we vary the parameter space to generate a wide range of the creative destruction wedge, the optimal policy leniency responds positively to the wedge. The higher the wedge, the more lenient the optimal default sanction. This implies that for countries where the business environment is less dynamic, the government rationally chooses a higher degree of forbearance, thereby exacerbating resource allocation. Furthermore, when the economy is at the ELB, the creative destruction wedge in the sector that is forced to close also spills over to the open sector, exacerbating its involuntary unemployment. This suggests that the ELB environment further exacerbates the distortion due to the creative destruction wedge. Nevertheless, the quantitative effect is small compared with that due to changes in the intra/inter-temporal elasticities and price elasticity.

Finally, we extend the model to consider banks and model explicitly how the government credit guarantee affects banks' monitoring incentives. We show that the quality of banks' monitoring is a decreasing function of credit guarantee, which is consistent with the empirical evidence in Lelarge *et al.* (2010). In equilibrium, as the credit guarantee becomes more generous, the aggregate lending also increases, in line with Bachas *et al.* (2021). Moreover, we show that lowering credit guarantees in the extended model is equivalent to increasing the default sanction in the benchmark model.

The remainder of the paper is structured as follows. In Section I, we review the extremely rapidly growing associated literature on the effects of the pandemic on the economy, especially in relation to the objectives of stabilization and allocative efficiency. Then in Section II, we set out a model, simplified as far as possible, in which we aim to explore the effects of introducing a screening contract into a government scheme for financing companies adversely affected by enforced closure, on the twin objectives of stabilization and allocative efficiency. Section III provides equilibrium characterization and analysis. Section IV solves for the optimal default sanction and its interaction with interest rate ELB. Section Vconducts comparative statics on the interaction between ELB, markup and creative destruction. Section VI is a model extension, and Section VII considers other numerical experiments. Section VIIIconcludes.

2 | RELATED LITERATURE

First and foremost, our paper relates to the group of COVID-19 literature on public liquidity provision (see, for example, Bachas *et al.* 2021; Chodorow-Reich *et al.* 2022; Elenev *et al.* 2020;

Kahn and Wagner 2020; Minoiu *et al.* 2021; Segura and Villacorta 2020; Philippon 2021). On the empirical front, Minoiu *et al.* (2021) document that the Main Street Lending Program in the USA increased banks' willingness to lend, and that participating banks were less likely to tighten lending standards. Chodorow-Reich *et al.* (2022) present a framework on bank liquidity provision across firms and develop testable predictions, and they find that the US-government-sponsored Paycheck Protection Program alleviated liquidity shortfalls to small firms at a cost to the government. Bachas *et al.* (2021) find that lending supply is highly sensitive to government credit guarantees. These empirical findings focus mostly on the effectiveness of government support on lending supply, but less is clear about its welfare trade-off, so our normative assessment complements these empirical analyses. In this sense, our paper is related to that of Gale (1991) who develops a theoretical framework à la Stiglitz and Weiss (1981) to analyse the quantitative effects of federal lending on credit allocation and economic efficiency. Gale (1991) shows that federal credit programmes are effective in the allocation of credit, but the estimated efficiency cost of the actual credit policy is high. The model maintains a simple direct link between credit and real activity in that an increase in credit leads to increased real activity, whereas our model incorporates the composition of real activity so that credit contractual terms change the marginal incentive on whether to become an entrepreneur or stay out as a worker. Therefore our contribution lies in the trade-off between unemployment and misallocation, and the interplay between sectors, with implications on the policy effectiveness in a low interest rate environment.

On the theoretical side of COVID-19 support programmes, Philippon (2021) analyses how government interventions can improve efficiency when the decentralized economy amid COVID-19 is distorted by wage rigidity. Our paper complements Philippon (2021) by focusing on the government's long-run and short-run trade-off, and differs in the following distinct ways. First, the key trade-off in our paper is between short-run employment stabilization and long-run allocative efficiency, whereas Philippon (2021) presents a one-period model and hence does not highlight the short-run and long-run trade-off. Second, the short-run unemployment in our paper is a direct result of the government's default sanction, but the unemployment in Philippon (2021) arises due to downward wage rigidity. Thus our paper can show analytically the effect of the government's screening policy on short-run employment stabilization. Third, Philippon (2021) assumes that private creditors can observe firms' project productivity, so that the government can implement optimal policy based on the government's observation of private creditors' actions. This is similar to the conditional loan bridge schemes considered in Elenev *et al.* (2020). Our paper takes a different view, that private creditors (e.g. banks) would not be able to monitor borrowers or acquire information. This is not just due to the scale and speed of the pandemic crisis, but is also because government credit guarantees would leave little incentive for banks to make efforts to monitor and acquire information.

Kahn and Wagner (2020) develop a theory to underpin the conditions under which direct provision of liquidity is preferable to the traditional distribution of liquidity, and vice versa. The main trade-off there is between externalities and informational advantages. In a similar spirit but with a different friction, Segura and Villacorta (2020) analyse different types of government interventions to support firms, and develop a pecking order between direct transfers and indirect support through guarantees to new loans or reductions in the capital requirement. The critical friction in their paper is the moral hazard due to the borrower's unobserved effort cost. Our paper differs from Kahn and Wagner (2020) and Segura and Villacorta (2020) in that we focus on setting the contractual terms of government loans to ameliorate adverse selection while endogenizing its impact on persistent unemployment, which is not present in Kahn and Wagner (2020) and Segura and Villacorta (2020). More specifically, in Segura and Villacorta (2020), the optimal policy is derived from the deployment of funding through the workings of financial intermediaries to reduce moral hazard. However, our paper focuses on adverse selection, and the optimal policy stems from the trade-off between allocation (reducing agency cost) and stabilization (reducing unemployment). Another distinction between our paper and these two papers is that we model multiple goods and sectors, offering an additional perspective on the aggregate demand externality and its interaction with optimal policy. Of course, there are other ways to design screening to address adverse selection issues. For example, if the government can exploit information in observables (e.g. employment at $t = 0$), then the loan support policy can better target more efficient borrowers. A recent paper by Wang and Wang (2021) shares a similar spirit to this idea. Wang and Wang (2021) show that the government can exploit the information dispersed within the private banking sector to improve policy targeting; this is because the demand for loans mirrors the borrower's funding deficit and reflects the net present value.

Relatedly, Elenev *et al.* (2020) build a structural model calibrated with the US data to evaluate three government policies aimed at short-circuiting the interplay between corporate defaults and banking fragility. They find that the government loan schemes in the USA are preventing the bulk of firm bankruptcies. Our paper differs from theirs in that we consider explicitly the agency problem between the government and the borrowing entrepreneurs or firms that possess private information on their expected profitability. Moreover, we consider the potential adverse effect of government loan schemes and guarantees that cause zombification and the drop in long-term productivity.

Therefore our paper contributes to the fast-growing COVID-19 literature by emphasizing and modelling explicitly the policy trade-off between short-run stabilization and long-run allocation, with the optimal policy provided given various features of the underlying economy. On stabilization, Barrero *et al.* (2020) estimate that that COVID-19 shock caused 3 new hires for every 10 layoffs, and that 32–42% of COVID-induced layoffs will be permanent, suggesting a slow absorption of labour into new jobs. Fairlie (2020) also provides timely and early evidence on the impacts of social distancing restrictions and demand shifts from COVID-19 on small businesses. In contrast, on allocation, Acharya *et al.* (2020) document the effect of cheap credit and zombification on firm markups and inflation. Jordà *et al.* (2020) provide long-run evidence on the economic costs of corporate debt booms and inefficient debt restructuring. Our paper contributes to these two groups of literature by presenting a model on how to balance the policy trade-off between them and search for the social optimum.

Sharing a similar spirit but with a different mechanism, Acharya *et al.* (2021) build a theory on zombie lending in a broader context. The authors find that aggressive unconventional policy can induce low-capitalized banks to supply credit to low-productivity firms and solve the optimal policy response. However, in Acharya *et al.* (2021), risk-shifting induced by regulatory forbearance is the primitive economic force pushing towards zombie lending, so the authors do not explicitly model asymmetric information or screening, which is the mechanism of this present paper.

In terms of the macro-financial environment, we introduce financial contracting in the presence of asymmetric information to the multi-sector economy along the lines of Guerrieri *et al.* (2022), who build a multi-sector infinite horizon model and show how a negative supply shock due to shutdown can translate into a demand shortage. Similar to Guerrieri *et al.* (2022), the pandemic shock of our model is modelled as a shutdown of the adversely affected sector while the other sector stays open, and we make use of the demand shortage and aggregate demand externality in the Guerrieri *et al.* (2022) framework. However, the main friction in our paper is the adverse selection stemming from the borrowing entrepreneurs' private information, which is not present in Guerrieri *et al.* (2022). Furthermore, the central question in Guerrieri *et al.* (2022) is under what conditions a negative supply shock can cause aggregate demand shortage, while in our paper, we are investigating specifically government loans and credit guarantees that support small businesses during a pandemic. This is a policy issue in which the trade-off between reducing unemployment and reducing the agency cost of adverse selection emerges naturally. The theoretical contribution of our paper is to combine financial contracting and insights from the industrial organization (see Shapiro 1989) with the aggregate demand externality of the macroeconomy.

Finally, given the information friction in our model and the use of screening, our model connects with the vast literature on private information and screening in financial contracting that follows a rich tradition; see Jensen and Meckling (1976), Fama (1978), Rothschild and Stiglitz (1978), Smith and Warner (1979), Stiglitz and Weiss (1981), Webb (1984) and Bester (1985) as classic examples, and more recently, Dubey and Geanakoplos (2002) and Lester *et al.* (2019). Nevertheless, the concern of our paper is not only on designing the financial contract to reduce the agency cost, but also about the unintended and undesirable consequences of reducing the agency cost in a macroeconomic setting. After all, the focus of our paper is on how to trade off the pandemic-induced stabilization issues as a result of reducing the agency cost against long-run productivity. We endogenize the social cost of reducing the agency costs as the near-term surge in unemployment and reduction in production, and bring to the forefront the policy trade-off between reducing near-term unemployment and increasing long-term productivity.

3 | THE MODEL

3.1 | Environment

The economy has three types of agents: entrepreneurs, workers and the government. It lasts for three periods ($t = 0, 1, 2$). Date $t = 0$ is when the pandemic occurs, date $t = 1$ refers to the immediate short run, and date $t = 2$ is interpreted as the long run. There are two sectors, I and J, that produce different goods. Each sector has N entrepreneurs who hire workers to carry out a project to produce sector-specific goods. Entrepreneurs in each sector choose how much to produce and take into account the impact on setting prices. There are Q workers in the economy, and $Q/2$ workers specialize in each sector. Each worker is endowed with 1 unit of labour supplied to an entrepreneur inelastically. Each entrepreneur's maximum production capacity is to employ $Q/2N$ workers. In the absence of the pandemic shock, full employment would be achieved in both sectors. Both entrepreneurs and workers consume goods from both sectors, and exhibit the same constant elasticity of substitution (CES) preferences for consumption goods. Note that we abstract away physical capital investment for production. The reason for this is that during crises, governments typically provide loans to small businesses to retain workers rather than to fund investment. Market loans for capital investment are relatively accessible since capital, unlike workers, can be pledged as collateral, which may obviate the need for government loans (see Gonzalez-Uribe and Wang 2020).

In times of pandemics—for example, COVID-19—sector I is adversely shocked and forced to close. The government provides BBLs to the adversely shocked sector to retain workers, so that firms can avoid large extra costs associated with matching and training a completely new workforce when production resumes in the future.[3] If the borrowing entrepreneurs default, then the government can choose to implement a default sanction. If the government sets zero default sanctions, then the government assumes all the cost of default and effectively provides full credit guarantees for the BBLs. Figure 2 outlines the flow of funds in the economy.

Figure 3 illustrates the timeline, and we divide each of the first two dates into two sub-periods. At the start of $t = 0$, entrepreneurs in sector I use their previous sales revenues to pay wages for the labour hired for production in period $t = 0$. Then an unanticipated short-lived pandemic shock occurs at the end of $t = 0$ and hits sector I. Sector I is forced to close, while sector J remains open. Thus at the end of $t = 0$, the entrepreneurs in sector I have not produced anything, and the economy's spending falls onto sector J at $t = 0$.

During the pandemic, the government provides loans for businesses to retain at least some key workers to avoid large additional training and matching costs in the future. The prior

FIGURE 2 Flow of funds.

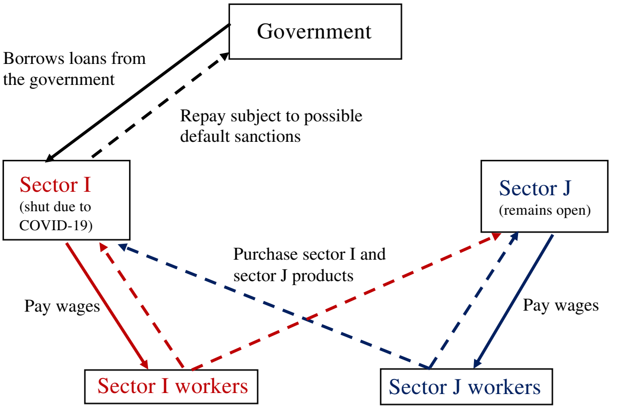
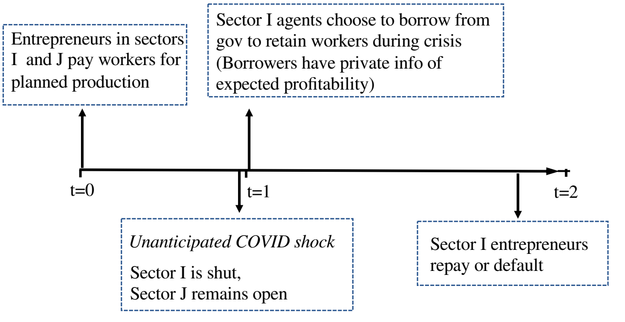


FIGURE 3 Timeline.



entrepreneurs can choose to borrow loans and continue operating their businesses, or choose to become workers in this sector. If they choose to borrow government loans, then they pay wages immediately at the start of $t = 1$ to retain key workers in this sector. The prior entrepreneurs have projects with different productivity levels, and they have private information on the project productivity. Those with lower profitability are more likely to default if their revenue is low at the end of date $t = 1$ when production is resumed.[4]

Given the scale and the speed of the pandemic, we assume that the government is unable to acquire information on the entrepreneurs' prospective profitability in time. However, the government may implement, should it wish, a sanction if the borrowing entrepreneurs default. As one might expect, if the government implements a lenient default sanction or no sanctions at all, then inefficient entrepreneurs with lower expected profitability may try to borrow. If the government implements a harsh default sanction, then it is possible to deter those with low profitability, but this policy may increase the unemployment rate. We will discuss the policy trade-off in more detail shortly, after we set up the model formally.

3.2 | Entrepreneurs

We first characterize the entrepreneurs' optimization in the absence of the pandemic shock, when the entrepreneurs simply solve a static profit maximization problem, while taking into account their price impact.

We set the following notation: W_E is the nominal wage paid by entrepreneurs in sector E, for $E \in \{\mathrm{I}, \mathrm{J}\}$; P_E is the price of goods in sector E; $q_E(n)$ is the quantity of goods produced by entrepreneur n in sector E; q_E is the total quantity of goods in sector E; $h_E(n)$ is the labour demand by entrepreneur n in sector E; and production technology is given as

$q_E(n) = \sigma\ h_E(n)$. We call σ the project productivity, and $\sigma = 1$ in the absence of the pandemic shock.

Formally, entrepreneurs maximize the profits of $\Pi_E(n)$ subject to the production technology as below. With a finite number of firms in the industry, competition is imperfect. Entrepreneur n takes into account the impact of their production plan on price setting. Therefore, in their objective function, the price P_E—i.e. $P_E\left(q_E(n) + \sum_{m\neq n} q_E(m)\right)$—is a function of production quantity. We have

$$\max_{q_E(n),h_E(n)} \Pi_E(n) = P_E\left(q_E(n) + \sum_{m\neq n} q_E(m)\right)(q_E(n)) - W_E\ h_E(n),$$

subject to

$$q_E(n) = \sigma\ h_E(n).$$

We introduce ε_E as the price elasticity of demand, and by definition, it is expressed as

$$\varepsilon_E = -\frac{P_E}{q_E}\frac{\partial q_E}{\partial P_E}.$$

As we assume CES preferences, ε_E is the same across goods, and we will drop the indexing E hereafter (so $\varepsilon_E = \varepsilon$). We focus on symmetric equilibria so that each entrepreneur sets $q_{Et}(n) = q_{Et}/N$. The optimality condition for entrepreneur n leads to

$$W_E = \sigma\left(1 - \frac{1}{N\varepsilon}\right)P_E,$$

where we assume the parameter space $\varepsilon > 1$. Substituting in $\sigma = 1$, it follows that $W_E = (1 - 1/N\varepsilon)P_E$. In the subsequent equilibrium characterization, we normalize the price of goods J to 1.

The profits of entrepreneur n are positive as long as the total number of businesses is not infinite or the price elasticity of demand is not infinite. In the symmetric equilibrium, entrepreneur n employs the labour of $h_E(n) = Q/2N$, so we can now drop the indexing for labour and denote it as h instead. The real profits of entrepreneur n, $\pi_E(n)$, can be expressed as $\pi_E(n) = \sigma h/N\varepsilon$. Entrepreneurs use profits to purchase goods from the two sectors. Let $\pi(E)$ be the total entrepreneurial profits in sector E, let p_I be the relative price of goods I, and let $c_\mathrm{I}(E), c_\mathrm{J}(E)$ be sector E entrepreneurs' consumption of the two goods, so $p_\mathrm{I}\ c_\mathrm{I}(E) + c_\mathrm{J}(E) = \pi_E$. Entrepreneurs' preferences for the two goods are assumed to be the same as those of the workers, to be specified in a subsection below.

Pandemic shock

We now turn to an unanticipated short-lived pandemic shock that occurs towards the end of $t = 0$. At the start of $t = 0$, entrepreneur n in sector I uses their previous sales revenues to pay wages for the labour hired for the planned production in $t = 0$. Then, after the pandemic shock, sector I is forced to close while sector J remains open. Thus sector I has no production materialized.

As said above, during the pandemic, the government provides loans for businesses to retain workers in order to avoid large additional training and matching costs in the future. The prior entrepreneurs totalling N differ in their expected profitability, and they can choose to borrow and continue business, or they can choose to be a worker instead. If they choose to borrow, then they pay wages immediately at the beginning of $t = 1$ to retain workers in this sector. To capture potential crowding out, a new set of workers with new ideas totalling a fraction ϕ of the industry cap can also borrow and become entrepreneurs. Since sector I was completely shut at $t = 0$, the industry gap at $t = 1$ is simply N. In the future at $t = 2$, we take a benchmark assumption that the industry gap is closed by new entries. Later, we will relax this assumption to capture the long-run scarring effect of the pandemic.

If this set of workers and the prior entrepreneurs choose to borrow the government's BBLs of F_1, then they use the money to retain workers immediately at the start of $t = 1$. Among the N prior entrepreneurs, a fraction of them totalling $(1-\alpha)N$ have inefficient projects of different productivity levels that are lower than the pre-pandemic level (i.e. $\sigma(n) < 1$). The rest of these prior entrepreneurs (αN) and the normal set of workers with new ideas (ϕN) are profitable; they have good projects of the same productivity as the pre-pandemic level. We assume $\phi + \alpha < 1$ for two reasons. First, this assumption reflects that the pandemic worsens the overall profitability of the adversely shocked sector in the short run. Second, this assumption is equivalent to $\phi < 1 - \alpha$, which also implies limited scope for creative destruction. We define the wedge between $1 - \alpha$ and ϕ as Δ, and $\Delta = (1-\alpha)/\phi$. So the limited scope for the creative destruction assumption is essentially $\Delta > 1$. The higher the creative destruction wedge, the less promptly unproductive incumbents can be replaced by productive entrants.

When the fraction of prior entrepreneurs with inefficient projects manages to obtain government loans to stay afloat, aggregate productivity will drop due to misallocation, that is, inefficient production. The issue at hand is to design a mechanism that can deter borrowers with inefficient projects while taking into account the impact on unemployment. Specifically, if the prior entrepreneurs decide not to borrow, then they simply become workers and receive wages. If they borrow at the start of $t = 1$ to stay afloat, then they can carry out a project that produces output as $\sigma(n)\, h_1$ once the shut-down restrictions are lifted. And $\sigma(n)$ is the project productivity of entrepreneur n, which represents their profitability. For the profitable entrepreneurs, their project productivity is equal to 1, the same as the pre-pandemic level. For the less profitable entrepreneurs, we assume that their project productivity $\sigma(n)$ follows a uniform distribution $\sigma(n) \sim \mathcal{U}(\sigma_B, \overline{\sigma})$, where

$$\overline{\sigma} = 1 - \frac{1}{N\varepsilon}, \quad \sigma_B = \frac{2N - 2/\varepsilon}{(1-\gamma)Q}. \tag{1}$$

The parametrization in equations (1) is not necessary for the main result but is used in the benchmark case for ease of analytical exposition. As we will show in the equilibrium characterization, the parameters σ_B and $\overline{\sigma}$ ensure the following. In the absence of sanctions, the entrepreneur with the lowest profitability among the inefficient entrepreneurs will try to borrow government loans to retain workers and stay afloat. However, after production resumes in the future, their low revenues are simply insufficient for loan repayment. The entrepreneur with the highest profitability among the inefficient entrepreneurs will try to borrow government loans, but after production resumes in the future, their revenues are just enough to repay the loan obligations (in our numerical simulations, we also vary $\overline{\sigma}$ and σ_B widely for robustness checks). However, as we will explain shortly, because they can divert a fraction of their funds due to an imperfect monitoring and verification technology, they will default on the government loans nevertheless.

Importantly, $\sigma(n)$ is the private information of the borrowing entrepreneurs. Different types of entrepreneurs borrow the same amount to retain workers, so *ex ante* they are indistinguishable. Furthermore, when the project productivity is realized at the end of $t = 1$ and the entrepreneur defaults, the government takes away only a fraction γ of the entrepreneurs' remaining funds.[5] We interpret γ as the quality of the government's verification or monitoring technology.

First, we characterize the default decision of entrepreneur n assuming zero sanctions from the government. When the project productivity is sufficiently high—so that if entrepreneur n were to declare default after production, then the amount of funds that the government garnishes would be higher than or equal to the loan obligations, i.e. $\gamma\ \sigma(n)\ h_1 \geq F_1$—the entrepreneur would choose to repay fully. When the project productivity is sufficiently low that the revenues are insufficient to repay loans—i.e. $\sigma(n)\ h_1 < F_1$—they are simply unable to repay the loans and have to default after production. However, there is an intermediate region for the project productivity where the entrepreneurs will default even when they have enough revenue. We call this type of default *strategic default*. With a relatively high project productivity, even if the revenues are large enough to repay the loans in full, but due to limited commitment the government takes away a relatively small amount of funds in the case of default such that $\gamma\ \sigma(n)\ h_1 < F_1 \leq \sigma(n)\ h_1$, the entrepreneur will nevertheless default strategically. Let $\mathbb{I}$ be the indicator for default; that is, $\mathbb{I} = 1$ means default, and $\mathbb{I} = 0$ means repayment. Lemma 1 summarizes the endogenous choice of default.

Lemma 1. (*Default decision*) *Assuming zero default sanctions, the decision to default is summarized by*

$$\mathbb{I} = \begin{cases} 0, & \text{if } F_1 \leq \gamma\ \sigma(n)\ h_1, \\ 1 \quad \text{(strategic)}, & \text{if } \gamma\ \sigma(n)\ h_1 < F_1 \leq \sigma(n)\ h_1, \\ 1, & \text{if } \sigma(n)\ h_1 < F_1. \end{cases}$$

Lemma 1 indicates that the higher the project productivity $\sigma(n)$, the lower the likelihood of default. To deter the inefficient entrepreneurs from borrowing and, in turn, reduce default, the government may implement a default sanction as a screening contract such that when the entrepreneurs default at the end of $t = 1$, a pecuniary deduction is taken from their residual income. Because the borrowing entrepreneurs have the outside option of becoming a worker earning wage income, the residual income of any defaulting entrepreneur is positive (no less than the outside option), and a pecuniary deduction is possible.

Let λ_1 be the default sanction. At the start of $t = 1$, entrepreneur n forms a conditional expectation of their proceeds and evaluates them against their outside option. Suppose that the borrowing entrepreneur n defaults. They can keep a fraction of the revenues totalling $(1-\gamma)\ \sigma(n)\ h_1$ as their residual income (as in Rampini and Viswanathan 2013). And due to the sanction, the total amount of money that they will receive at the end of $t = 1$ amounts to $(1-\gamma)\ \sigma(n)\ h_1 - \lambda_1$. However, had they chosen not to borrow and instead become a worker at the start of $t = 1$, then they would have received wages of $\mathbb{E}_1(W_{11} | \mathbb{I}_d = 1,\ \lambda_1)$, where $\mathbb{I}_d = 1$ indicates remaining as a worker. Therefore, if the incentive constraint

$$\underbrace{(1-\gamma)\ \sigma(n)\ h_1 - \lambda_1}_{\text{default}} \geq \underbrace{\mathbb{E}_1(W_{11} | \mathbb{I}_d = 1,\ \lambda_1)}_{\text{outside option}} \tag{2}$$

holds, then they will choose to apply for government loans and stay afloat. In the case of the equality sign, we assume that the entrepreneur chooses to borrow, and we call this entrepreneur the marginal entrepreneur.

The left-hand side of the incentive constraint (2) states the gains if the entrepreneur defaults, and the right-hand side is the value of her outside option. The outside option is the expected wage if she were to remain as a worker.[6]

As inequality (2) is conditional on her choosing to default, her benefits of repayment must be smaller or at most equal to her gains in the default case, so the following must hold:

$$\underbrace{(1-\gamma)\ \sigma(n)\ h_1 - \lambda_1}_{\text{default}} \geq \underbrace{\sigma(n)\ h_1 - F_1}_{\text{repay}}.$$

Note that as the government increases the sanction λ_1, the inefficient entrepreneurs with low profitability may be deterred from borrowing, reducing the aggregate default. This relationship is proved to be monotonic in Proposition 1 below, after we define the equilibrium.

3.3 | Government

The government choice variable is the default sanction λ_1, and the government commits to the sanction. The government provides loans of F_1 to each borrowing entrepreneur to retain workers and stay afloat,[7] where

$$F_1 = \frac{W_{I1}Q}{2N}.$$

At the end of $t = 1$, some borrowing entrepreneurs default. Let df_1 be the total amount of default, and let Λ_1 be the total money collected from sanctions. The government uses the money collected via sanctions plus any borrowing if needed to cover default as in

$$df_1 \leq B_1 + \Lambda_1.$$

If the money collected via default sanctions is insufficient to cover default—i.e. $df_1 > \Lambda_1$—then the government borrows money by issuing a one-shot undated consol of B_1 to the workers and entrepreneurs, and the government pays only the interest in future periods by raising an equivalent amount of taxation. As we show shortly, after defining the equilibrium, the agents in the economy have sufficient savings to lend to the government after the pandemic. Note that if the government chooses to implement zero default sanctions, then the government essentially provides full guarantees for its loan scheme. As the government increases the default sanctions, the credit guarantees decrease accordingly.

3.4 | Workers

Workers consume goods in both sectors. Let us label workers by $i \in Q$. Let $c_{It}(i)$ be the consumption of sector I goods, and let $c_{Jt}(i)$ be the consumption of sector J goods. Their preferences are represented by the utility function

$$\mathbb{E}_t \sum_{t=0}^{2} \beta^t \; U\left(c_{It}(i), c_{Jt}(i)\right), \tag{3}$$

where

$$U\left(c_{It}(i), c_{Jt}(i)\right) = \frac{1}{1-\delta}\left(\left(\frac{1}{2}\right)^{\nu} c_{It}(i)^{1-\nu} + \left(\frac{1}{2}\right)^{\nu} c_{Jt}(i)^{1-\nu}\right)^{(1-\delta)/(1-\nu)}. \tag{4}$$

The utility function satisfies $U' > 0$, $U'' < 0$, and it features CES $1/\nu$ ($\nu < 1$) between the two sectors' goods bundles and constant inter-temporal elasticity of substitution $1/\delta$.

In our benchmark case, we assume that workers have access to real, zero net supply, one-period bonds, paying interest rate r_t to share labour income risks. The approximate real-world mapping of this assumption often takes the form of the government's unemployment benefits and various types of tax transfers. Risk sharing provides analytical convenience without the loss of generality; moreover, it allows us to focus exclusively on the short-run and long-run trade-offs of the government's support loan scheme due to asymmetric information without introducing additional frictions. The policy question, therefore, is how much more the government should care about short-run stabilization in addition to providing unemployment benefits.

First, we characterize the workers' maximization. Due to the equal weighting of goods in CES preferences, exploiting symmetry, the relative goods price is 1 and wages are the same across sectors. Let $a_t(i)$ be the one-period bonds held by each worker, let $p_{\mathrm{I}t}$ be the relative price of goods I, and let r_t be the interest rate.[8] Each worker i maximizes function (3) subject to the budget constraint

$$p_{\mathrm{I}t}\ c_{\mathrm{I}t}(i) + c_{\mathrm{J}t}(i) + a_t(i) \le w_t\ h_t(i) + (1 + r_{t-1})\ a_{t-1}(i).$$

The optimality condition gives the Euler equation for consumption goods J. Let $U_{c_{\mathrm{J}}}$ be the partial derivative of U with respect to $c_{\mathrm{J}t}$. Given homothetic preferences, we have Gorman aggregation, so the individual's marginal rate of substitution between goods equals the relative price, which is a macro variable. Hence if the Euler equation holds individually, then it also holds for the aggregate consumption of the workers denoted as $c(W)$, as in

$$U_{c_{\mathrm{J}}}(c_{\mathrm{I}t}(W), c_{\mathrm{J}t}(W)) = \beta(1 + r_t)\ U_{c_{\mathrm{J}}}(c_{\mathrm{I}t+1}(W), c_{\mathrm{J}t+1}(W)).$$

In the absence of the pandemic, $c_{\mathrm{I}t}(W) = c_{\mathrm{I}t+1}(W)$ and $c_{\mathrm{J}t}(W) = c_{\mathrm{J}t+1}(W)$, so the interest rate is $r = 1/\beta - 1$.

When the pandemic shock hits at date $t = 0$, sector I shuts down, so $\boldsymbol{c}_{\mathrm{I}0} = 0$. The workers' consumption falls onto sector J, and

$$c_{\mathrm{J}0}(W) = \frac{Q}{2}\left(1 - \frac{1}{N\varepsilon}\right).$$

We define the natural interest rate in this context as the interest rate in the Euler equation in the hypothetical case of no private information, and enough profitable entrepreneurs borrow at the start of $t = 1$ to stay afloat. Thus the natural interest rate is the interest rate when the economy operates as if in full potential from date $t = 1$ onwards.

Let r_0^* be the natural interest rate for date $t = 0$, so

$$\begin{aligned}
1 + r_0^* &= \frac{1}{\beta}\ \frac{U_{c_{\mathrm{J}}}(0, c_{\mathrm{J}0})}{U^*_{c_{\mathrm{J}}}(c_{\mathrm{I}1}, c_{\mathrm{J}1})} \\
&= \frac{1}{\beta}\ \frac{U_{c_{\mathrm{J}}}(0, (1 - (1/N\varepsilon))Q/2)}{U_{c_{\mathrm{J}}}((1 - (1/N\varepsilon))Q/2, (1 - (1/N\varepsilon))Q/2)} \\
&= \frac{1}{\beta}\left(\frac{1}{2}\right)^{(\nu-\delta)/(1-\nu)}.
\end{aligned} \tag{5}$$

Lemma 2. *With complete markets, and given the pandemic shock,* $r_0^* < 1/\beta - 1$, *and the supply shock causes a demand shortage at date* $t = 0$ *if and only if*

$$\nu > \delta. \tag{6}$$

Lemma 2 follows directly from equation (5); it is an insight from Guerrieri *et al.* (2022). It states that when the two sectors' goods are complements, the supply shock due to the pandemic causes a demand shortage. In a single-sector economy, a negative supply shock typically increases the current-period marginal utility of consumption, which increases the natural interest rate. Because a negative supply shock causes a demand boom in a single sector at $t = 0$, the interest rate increases to equilibrate the economy. However, in our environment and also in the multi-sector case of Guerrieri *et al.* (2022), inequality (6) implies that the two goods are complements, and interestingly, the natural rate decreases. The reason is that a negative supply shock decreases the current-period marginal utility of consumption due to goods being complements. So the negative supply shock causes a demand shortage for goods in the unshocked sector, and the interest rate needs to decrease to equilibrate the economy. This characteristic, thanks to a multi-sector setup, will have interesting implications for designing the screening contract and setting the optimal default sanction.

If the interest rate is downward rigid, for example, at the ELB where the interest rate cannot adjust downwards sufficiently, then it leads to a decline in the demand for goods J, and hence involuntary unemployment in sector J. Indeed, akin to the *Drèze equilibrium*, when prices are downward rigid, the supply is rationed (Drèze 1975).

3.5 | Equilibrium

The two-sector equilibrium with imperfect competition is defined as an allocation with prices, given the screening contract λ_1 such that:

(i) entrepreneurs engage in imperfect competition and choose their actions simultaneously taking into account their price impact;
(ii) agents maximize subject to borrowing frictions, the incentive constraint and budget constraints;
(iii) goods markets, labour markets and loan markets clear, and expectations are rational—in particular, goods market clearing conditions are that the output of each sector is consumed by workers and the entrepreneurs of both sectors, that is,

$$c_{\mathrm{I}t}(W) + \sum_{E=\mathrm{I,J}} c_{\mathrm{I}t}(E) = q_{\mathrm{I}t}, \quad c_{\mathrm{J}t}(W) + \sum_{E=\mathrm{I,J}} c_{\mathrm{J}t}(E) = q_{\mathrm{J}t}.$$

4 | EQUILIBRIUM CHARACTERIZATION

With our equilibrium definition, first, Lemma 3 shows that the agents in the economy have sufficient savings to lend to the government after the pandemic. Due to the pandemic shock, there is zero production in sector I, and the economy is unable to spend all of its nominal income on goods J alone at the end of $t = 0$. Consequently, agents in the economy end up having extra money as savings at the end of $t = 0$, and carry it forward. As proved in the Appendix, this amount of savings is more than enough to invest in the government's one-shot issuance of the undated consol.

Lemma 3. *At the end of* $t = 1$, *agents in the economy have sufficient savings to finance the government borrowing of* B_1.

Proof. See the Appendix. ∎

Now let us suppose that the government cares only about resource allocation and sets the default sanction λ_1 sufficiently harshly to deter less profitable borrowers. We derive the harsh default sanction λ_{A1} so that it is a Nash equilibrium for the entrepreneurs with profitable projects to apply for loans, while those with inefficient projects become workers. Then we suppose that the government wants to ensure full employment by setting a lenient default sanction. We solve for such a lenient sanction λ_{B1} so that it is a Nash equilibrium for the profitable entrepreneurs along with a fraction of inefficient entrepreneurs to apply for loans, and the rest become workers, while ensuring that full employment is achieved from $t = 1$ onwards. To derive the expressions for the harsh and lenient default sanctions, we first need to check that monotonicity holds—put differently, that the harsher the sanction, the higher the project productivity of the marginal entrepreneur, and the lower the number of defaults.

Let M ($\leq N$) be the number of borrowing entrepreneurs. We define $u_{\text{I}1}(M)$ as the unemployment rate in sector I at date $t = 1$ as follows, and Proposition 1 proves monotonicity:

$$u_{\text{I}1}(M) = 1 - \frac{QM/2N}{(Q/2) + N - M}.$$

Proposition 1. (*Monotonicity*) *Whenever $df_1 > 0$ and $M \leq N$, an increase in the default sanction λ_1 leads to higher profitability of the marginal entrepreneur and fewer defaults.*

Proof. See the Appendix. ■

Given monotonicity, Theorem 1 below derives the harsh default sanction λ_{A1} and the lenient default sanction λ_{B1}. A harsh sanction indicates a pro-allocation government, and a lenient sanction indicates a pro-stabilization government.

Theorem 1. (*Screening, default and unemployment*)

(a) *Suppose that the government sets the sanction $\lambda_1 > \lambda_{A1}$, where*

$$\lambda_{A1} = \left(1 - \frac{1}{N\varepsilon}\right)\left((1-\gamma)\,\frac{Q}{2N} - (1 - u_{\text{I}}(\phi N + \alpha N))\right).$$

Then the $(\phi + \alpha)N$ profitable entrepreneurs will borrow, and all the inefficient entrepreneurs will stay out, and no one defaults.
The sector I short-run unemployment rate is $u_{\text{I}1}(\phi N + \alpha N)$. Full employment obtains only in the long run.

(b) *Suppose that the government sets the sanction as λ_{B1}, where*

$$\lambda_{B1} = (1-\gamma)\left(\frac{\phi}{1-\alpha}\,\overline{\sigma} + \frac{1-\phi-\alpha}{1-\alpha}\,\sigma_B\right)\frac{Q}{2N} - \left(1 - \frac{1}{N\varepsilon}\right)(1 - u_{\text{I}}(N-1)).$$

Then $(1 - \phi - \alpha)N$ inefficient entrepreneurs will borrow and then default, but sector I achieves full employment in both the short run and the long run.

(c) *With no sanctions, all the prior entrepreneurs will apply for loans, and a fraction of profitable applicants are crowded out.*

Proof. See the Appendix. ■

Theorem 1 states that when the default sanction is sufficiently harsh, the government can keep all the $(1 - \alpha)N$ inefficient projects out. In this scenario, the economy suffers persistent unemployment. However, if the government chooses a sufficiently lenient default sanction, then it can restore full employment immediately at date $t = 1$ and beyond. However, in this case, the entrepreneurs with low profitability stay afloat, harming aggregate productivity in sector I at both $t = 1$ and $t = 2$. Furthermore, if the government does not impose default sanctions at all, then all the $(1 - \alpha)N$ inefficient entrepreneurs will try to borrow. This scenario will cause some of the $(\phi + \alpha)N$ profitable entrepreneurs to be excluded, and further dampen aggregate productivity. In Theorem 1, the harsh default sanction $\lambda_1 > \lambda_{A1}$ and the lenient default sanction $\lambda_1 = \lambda_{B1}$ outline two extreme cases. It is possible that the optimal default sanction could be an intermediate case.

Furthermore, two interesting observations emerge when we rewrite the harsh sanction λ_{A1} and the lenient sanction λ_{B1} in terms of the creative destruction wedge Δ. Note that

$$\lambda_{A1} = \left(1 - \frac{1}{N\varepsilon}\right)\left((1-\gamma)\,\frac{Q}{2N} - \left(1 - u_{\mathrm{I}}\left(\frac{1-\alpha}{\Delta} + \alpha\right)N\right)\right)$$

and

$$\lambda_{B1} = (1-\gamma)\left(\frac{\overline{\sigma} - \sigma_B}{\Delta} + \sigma_B\right)\frac{Q}{2N} - \left(1 - \frac{1}{N\varepsilon}\right)(1 - u_{\mathrm{I}}(N-1))\,.$$

We can see that λ_{A1} increases with Δ, whereas λ_{B1} decreases with Δ. When the creative destruction wedge is large, unproductive incumbents cannot be replaced promptly by productive entrants. To fully stabilize short-term employment, the threshold of the lenient sanction has to decrease. To remove resource misallocation in the long run, the threshold of the harsh sanction has to increase, since the lack of business dynamism implies that the outside option is weak, and the government has to be more hawkish to keep out the inefficient ones. Therefore the lack of creative destruction (higher Δ) polarizes the two extreme cases even more.

Before moving on to solve for the optimal default sanction, Proposition III characterizes the interest rate dynamics via the aggregate Euler equations of these two polar cases.

Proposition 2. (*Sanctions, interest rates and demand shortage*) *Suppose that inequality* (6) *holds.*

(a) *If* $\lambda_1 > \lambda_{A1}$, $r_0 < 1/\beta - 1$ *and*

$$1 + r_0 = \frac{1}{\beta}\left(\frac{1}{(\phi+\alpha)^{1-\nu} + 1}\right)^{(\nu-\delta)/(1-\nu)},$$

then the interest rate remains below $1/\beta - 1$ *for* $t = 1$, *and demand shortage is persistent.*

(b) *If* $\lambda_1 = \lambda_{B1}$, *then* $r_0 < 1/\beta - 1$ *also holds, but from* $t = 1$ *onwards, the interest rate immediately returns to* $1/\beta - 1$. *Demand shortage is short-lived.*

Proof. See the Appendix. ∎

Proposition states that when the government is pro-allocation and sets a harsh default sanction, the interest rate goes down and remains low persistently. This harsh sanction stifles production in the short run, and the supply shortage leads to a demand shortage due to the complementarity of the two sectors. In equilibrium, the consumption for goods I increases gradually while the consumption for goods J remains the same. Given $\nu > \delta$, goods I and goods J are complements, so the marginal utility of consumption for goods J immediately drops before it increases gradually. Demand shortage is persistent. However, if the government is pro-stabilization and sets a lenient sanction, then the interest rate goes down only for date $t = 0$ due to the pandemic shock, and then it immediately returns to $1/\beta - 1$. Demand shortage is short-lived. The reason is that the cheap loan programme attracts enough entrepreneurs to stay afloat, and full employment is reached immediately, at the possible cost of lowering future productivity.

5 | OPTIMAL DEFAULT SANCTIONS

To solve for the optimal sanction, we first define the social welfare function and then allow the government to choose the number of borrowing entrepreneurs M, where $M \in [(\phi+\alpha)N, N]$, subject to the decentralized optimality conditions of the entrepreneurs and workers. We will show shortly that M has a one-to-one mapping with the default sanction λ_1. The M^* that maximizes the social welfare corresponds to the optimal default sanction λ_1^*.

5.1 | Analytics

We assume that the government assigns equal weights to every worker and every entrepreneur in the economy, so the social welfare function takes the form of the sum of the consumption utilities of all agents in the economy. Let $\boldsymbol{c}_{\mathrm{I}t}$ represent the aggregate consumption of goods I, and let $\boldsymbol{c}_{\mathrm{J}t}$ represent the aggregate consumption of goods J. Recall that workers and entrepreneurs all exhibit the same CES preferences for consumption goods, and their utility function is homogeneous of degree 1. Then the social welfare function at date $t = 1$ takes the form

$$V_1 = \mathbb{E}_1 \sum_{t=1}^{2} \beta^{t-1} \frac{1}{1-\delta} \left(\left(\frac{1}{2}\right)^{\nu} \boldsymbol{c}_{\mathrm{I}t}^{1-\nu} + \left(\frac{1}{2}\right)^{\nu} \boldsymbol{c}_{\mathrm{J}t}^{1-\nu} \right)^{(1-\delta)/(1-\nu)}. \tag{7}$$

Let $\sigma(n')$ denote the marginal entrepreneur's project productivity when the number of loan applicants is M. Conditional on M, there is no uncertainty from date $t = 1$ onwards, so we drop the expectation sign hereafter. Given that $\sigma(n)$ follows a uniform distribution, $\sigma(n) \sim \mathcal{U}(\sigma_B, \overline{\sigma})$, it follows that

$$\sigma(n') = \left(1 - \frac{M - (\alpha+\phi)N}{(1-\alpha)N}\right)\overline{\sigma} + \frac{M - (\alpha+\phi)N}{(1-\alpha)N}\,\sigma_B, \tag{8}$$

so the default sanction that corresponds to M is

$$\begin{aligned}\lambda_{M1} = (1-\gamma)\left(\left(1 - \frac{M - (\alpha+\phi)N}{(1-\alpha)N}\right)\overline{\sigma} + \frac{M - (\alpha+\phi)N}{(1-\alpha)N}\,\sigma_B\right)\frac{Q}{2N} \\ - \left(1 - \frac{1}{N\varepsilon}\right)(1 - u_{\mathrm{I}1}(M-1)).\end{aligned} \tag{9}$$

Thus M and λ_{M1} have a one-to-one mapping.

First, given M, we solve the aggregate consumption of the two goods by substituting in the decentralized optimality conditions of the entrepreneurs and the workers. Thus we can express the aggregate consumptions $\boldsymbol{c}_{\mathrm{I}t}$ and $\boldsymbol{c}_{\mathrm{J}t}$ as functions of exogenous parameters and the policy variable M, and hence the social welfare function can be re-expressed as functions of M with other exogenous parameters. What then remains to be done is to search for the M that maximizes the social welfare function. Given equation (9), which expresses the default sanction λ_M as a function of exogenous parameters that include the number of borrowing entrepreneurs M, as long as the optimal M is obtained, we can derive the optimal sanction.

At $t = 1$, there are $(\alpha+\phi)N$ profitable entrepreneurs borrowing, and the number of inefficient borrowers amounts to $M - (\alpha+\phi)N$. Thus the aggregate consumption of sector I goods in equilibrium is given as

$$\boldsymbol{c}_{\mathrm{I}1} = \frac{Q}{2N}\left(\frac{\sigma(n') + \overline{\sigma}}{2}(M - \phi N - \alpha N) + (\alpha+\phi)N\right), \tag{10}$$

where $\sigma(n')$ is given in equation (8).

At $t = 2$, the industry gap closes and the number of entries at $t = 2$ amounts to $N - M$. The aggregate consumption of goods I at $t = 2$ is thus expressed as

$$c_{\mathrm{I}2} = \frac{Q}{2N}\left(\frac{\sigma(n') + \overline{\sigma}}{2}\,(M - \phi N - \alpha N) + (\alpha + \phi)N + (N - M)\right). \tag{11}$$

The aggregate consumption of goods J remains $Q/2$ throughout when the interest rate is unconstrained from the ELB.

Therefore substituting equations (10) and (11), and $c_{\mathrm{J}t} = Q/2$, in the social welfare function (7), the social welfare function can be expressed as a function of exogenous parameters and the policy variable M. Before we solve for the optimal M^*, we first develop an analytic measure conditional on M, which we call 'stabilization proclivity' (SP) to characterize the government's policy stance: the higher the stabilization proclivity, the more likely the government will set a lenient sanction (i.e. being closer to λ_{B1}); the lower the stabilization proclivity, the more likely the government will set a harsh sanction (i.e. being closer to λ_{A1}). Formally, SP is expressed as

$$SP = \frac{1}{\beta}\left(\frac{2}{\sigma(n') + \overline{\sigma} + (\sigma_B - \overline{\sigma})(M - \phi N - \alpha N)/(1-\alpha)N} - 1\right)^{-1}.$$

To understand the economic intuition of our measure, let us use $U(c_{\mathrm{I}t}, c_{\mathrm{J}t})$ to denote the single-period utility, that is,

$$U(c_{\mathrm{I}t}, c_{\mathrm{J}t}) = \frac{1}{1-\delta}\left(\left(\frac{1}{2}\right)^{\nu} c_{\mathrm{I}t}^{1-\nu} + \left(\frac{1}{2}\right)^{\nu} c_{\mathrm{J}t}^{1-\nu}\right)^{(1-\delta)/(1-\nu)}.$$

Then the first-order derivative of $V_1(M)$ for M is expressed as

$$\frac{\partial V_1(M)}{\partial M} = \frac{\partial U(c_{\mathrm{I}1}, c_{\mathrm{J}1})}{\partial M} + \beta\,\frac{\partial U(c_{\mathrm{I}2}, c_{\mathrm{J}2})}{\partial M}. \tag{12}$$

As we show in Proposition IV below, the first term on the right-hand side of equation (12), the short-run social utility, increases with M, that is, $\partial U(c_{\mathrm{I}1}, c_{\mathrm{J}1})/\partial M > 0$, and the second term on the right-hand side, the long-run social utility, decreases with$M$, that is, $\partial \tilde{U}(c_{\mathrm{I}2}, c_{\mathrm{J}2})/\partial \tilde{M} < 0$. When the first term dominates the second term, the government may be more pro-stabilization, as it would prefer a larger M; when the second term dominates the first term, the government may be more pro-allocation. We develop the measure SP, borrowing the concept of marginal rate of substitution. The marginal rate of substitution between short-run and long-run social utility is

$$-\left(\frac{\partial \tilde{U}(c_{\mathrm{I}1}, c_{\mathrm{J}1})}{\partial \tilde{M}}\right) \Big/ \left(\beta\,\frac{\partial \tilde{U}(c_{\mathrm{I}2}, c_{\mathrm{J}2})}{\partial \tilde{M}}\right).$$

Let $q_{\mathrm{I}t}$ be the sector I production at date t, and as we will show in Proposition IV, the measure SP is derived from $-(\partial q_{\mathrm{I}1}/\partial \tilde{M})/(\beta\, \partial q_{\mathrm{I}2}/\partial \tilde{M})$, and it offers an indication of whether the government prefers to be more pro-allocation or more pro-stabilization.

Proposition 3. (*Stabilization proclivity*) *Given equations* (10) *and* (11), *and* $M \in [(\phi + \alpha)N, N]$, *the short-run social utility increases with* M, *and the long-run social utility decreases with* M. *That is,*

$$\frac{\partial U(c_{\mathrm{I}t}, c_{\mathrm{J}t})}{\partial M} > 0, \qquad \frac{\partial U(c_{\mathrm{I}2}, c_{\mathrm{J}2})}{\partial M} < 0,$$

and SP is derived from

$$-\frac{\partial q_{11}/\partial M}{\beta\,\partial q_{12}/\partial M}=\underbrace{\frac{1}{\beta}\left(\frac{2}{\sigma(n')+\overline{\sigma}+(\sigma_B-\overline{\sigma})(M-\phi N-\alpha N)/(1-\alpha)N}-1\right)^{-1}}_{SP}.$$

Proof. See the Appendix. ■

SP depends on exogenous parameters. By investigating how *SP* responds to changes in these parameters, Proposition 4 characterizes the government's policy stance on the trade-off between allocation and stabilization.

Proposition 4. (*Policy stance*) *Keeping all other parameters unchanged, we have the following.*

(a) *An improvement in the monitoring technology γ increases SP.*
(b) *An increase in ε or N decreases entrepreneurial rents and increases SP.*
(c) *An increase in σ_B increases SP.*
(d) *An increase in β decreases SP.*

Proof. See the Appendix. ■

When the monitoring/verification technology improves, the government tends to be pro-stabilization and set a more lenient default sanction. If the monitoring technology is poor, then the government tends to set a harsher sanction to deter inefficient entrepreneurs. If a government is myopic—e.g. the government is more impatient than the public—then the government discounts the future utilities more; consequently, the government will take a very pro-stabilization stance. For example, if the government cares about immediate re-elections, then it is likely to set a lenient default sanction or no sanctions at all, to promote short-term employment and sacrifice long-term productivity. The overall toll on social welfare could be substantial.

When the price elasticity of demand is high or the size of the entrepreneurial pool is large, the industry is more competitive, so entrepreneurial rents decrease and workers' wages increase. Consequently, the government tends to be more pro-stabilization. On the contrary, with declining competition and rising markup, the government tends to be pro-allocation. Furthermore, an increase in σ_B increases *SP*, and a decrease in σ_B decreases *SP*. The reason is that as the lowest profitability σ_B decreases, the overall quality of the prior entrepreneurs worsens. Consequently, the government tends to be less pro-stabilization and more pro-allocation.

5.2 | Analytics of the ELB

The interest rate ELB limits monetary policy and constrains it from falling to prop up demand in the unshocked sector. Goods J produced by the unshocked sector are complements to goods I that are produced by the adversely shocked sector, so the marginal utility of consumption for goods J decreases, leading to a demand shortage. The interest rate should decrease to equilibrate the economy, leaving the supply of goods J unconstrained. When the interest rate cannot adjust downwards, however, it causes an inward shift of the supply curve, and in turn, involuntary unemployment in sector J. This is an aggregate demand externality that the government would want to avoid. Therefore the social cost of a harsh default sanction may outweigh that of our benchmark. Proposition 5 formalizes our argument.

Proposition 5. (*ELB, default sanctions and involuntary unemployment*) *Suppose that the economy is at the ELB where the interest rate cannot go below* $1/\beta - 1$. *Then demand shortage leads to involuntary unemployment at sector* J.

Proof. See the Appendix. ■

Remark. At the interest rate ELB, the optimal default sanction may be more lenient than its counterpart when the interest rate is unconstrained.

To see why this is, let us observe in

$$\frac{\partial U(c_{It}, c_{Jt})}{\partial M} = \underbrace{\frac{\partial U(c_{It}, c_{Jt})}{\partial c_{It}} \frac{\partial c_{It}}{\partial M}}_{X} + \underbrace{\frac{\partial U(c_{It}, c_{Jt})}{\partial c_{Jt}} \frac{\partial c_{Jt}}{\partial M}}_{Y}$$

how the single-period utility of consumption $U(c_{It}, c_{Jt})$ changes with respect to the number of loan applicants M when full employment is not achieved in sector I.

As shown in Proposition 3, the increase of M, or equivalently the leniency of the default sanction, leads to an increase in the single-period utility via increasing the marginal utility of consuming goods I, that is, $X > 0$. At the ELB, the demand shortage causes the consumption of goods J to decrease, so the marginal utility of consuming goods I, $\partial U(c_{It}, c_{Jt})/\partial c_{It}$, is lower than its counterpart when the interest rate is unconstrained, thus X is lower than in the unconstrained case.

However, in our benchmark case, the increase of M causes no externality to sector J, so it does not change the single-period utility via the marginal utility of consuming goods J, that is, $Y = 0$. In contrast, at the ELB, as we have shown in Proposition 5, an increase in the consumption of goods I leads to an increase in the consumption of goods J, and it follows that the leniency of the default sanction increases the consumption of goods J, via the aggregate demand externality channel, that is, $\partial c_{Jt}/\partial M > 0$. Therefore an increase in M, or equivalently the leniency of the default sanction, leads to an increase in the single-period utility via increasing the marginal utility of consuming goods J. So at the ELB, it follows that $Y > 0$, countervailing the reduction in X. Overall, the increase of M, or equivalently the leniency of the default sanction, may contribute to a higher utility gain at the ELB than the economy away from the ELB, suggesting that the government may take a more lenient stance in the ultra-low interest rate environment.

6 | NUMERICAL ANALYSIS

In this section, we assign numerical values to deep parameters in Table 1 as the benchmark case, and we run numerical simulations to illustrate the optimal default sanctions with different features of the underlying economy.

The number of entrepreneurs in each sector is set to be 10 in normal times, and the total number of workers is 800. Thus each small business firm employs 40 workers in normal times. According to the UK government's 2021 national statistics, 5.5 million businesses are small, which means that they have fewer than 50 employees. Thus we believe that our benchmark employee number of 40 is a reasonable size for small businesses. Later, we also vary N widely to change the number of workers per firm for robustness checks. The price elasticity of demand is chosen to be 1.2, so that in normal times, the entrepreneur's profit is around 3.6 times the worker's wages. The CES ν is therefore calibrated internally as 0.83, and the inter-temporal elasticity of substitution $(1/\delta)$ is set to be 1.43, so that $\nu > \delta$ holds; that is, goods in sectors I and J are

TABLE 1 Parametrization

Discount factor	$\beta = 0.97$
Monitoring technology	$\gamma = 0.7$
Price elasticity of demand	$\varepsilon = 1.2$
Inter-temporal elasticity of substitution	$1/\delta = 1.43$
Fraction of new workers entering	$\phi = 0.1$
Fraction of profitable prior entrepreneurs	$\alpha = 0.5$
Number of entrepreneurs in each sector	$N = 10$
Total number of workers	$Q = 800$

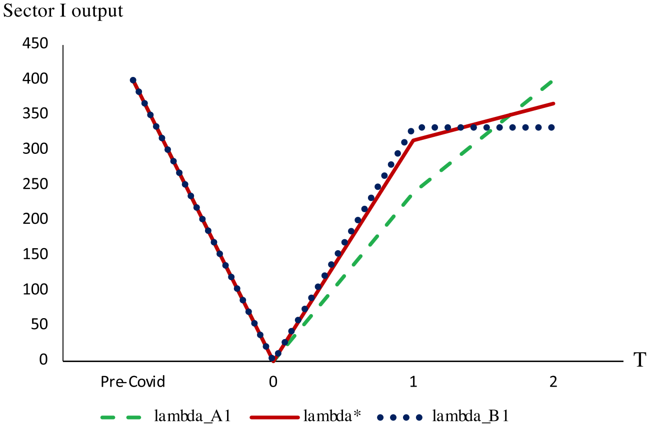


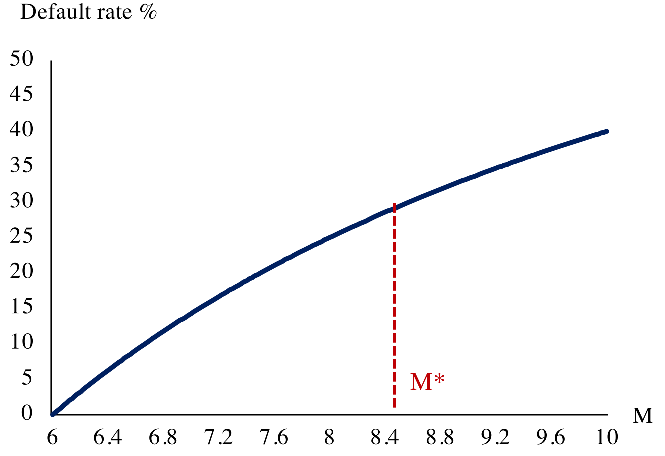
FIGURE 4 Output and default sanctions.

complements. In the subsequent sections, we vary ν and δ widely for robustness checks and also for assessing the sensitivity of interest rate changes to these parameters.

The fraction of new workers with new ideas as potential entrepreneurs in each period is set to be 0.1, and the fraction of profitable prior entrepreneurs is set to be 0.5, so that if we were to incorporate the model to an infinite setting where each period would correspond to one year, then it would take 5 years for sector I to reach full employment and fully restore production if the government sets a harsh default sanction to keep all inefficient projects out. In the second subsection of this section, we extend the model to a dynamic setting and explain why this reaches a 5-year target. Given the severity of the COVID-19 pandemic, we think 5 years is a plausible figure. For robustness checks, in the later sections we also vary this year number. The discount factor β is set to 0.97, which is close to a quarterly discount factor of 0.99, the same as in Ottonello and Winberry (2020). And finally, we set the monitoring technology γ to be 0.7. The parametrization in Table 1 and our assumption of the distribution of the inefficient entrepreneurs' project productivity as in equations (1) implies that the highest project productivity $\overline{\sigma}$ among the inefficient entrepreneurs is around 0.92, and that the lowest project productivity σ_B among them is around 0.076.

Figure 4 illustrates the responses of sector I output to the default sanction in the benchmark case. Its horizontal axis indicates the time that starts at $t = -1$, which corresponds to the pre-pandemic equilibrium. The dashed line corresponds to a pro-allocation government that sets the harsh default sanction λ_{A1}, the dotted line corresponds to a pro-stabilization government that sets the lenient default sanction λ_{B1}, and the solid line corresponds to the optimal default sanction that maximizes the social welfare. Let us observe the case of the pro-stabilization government. As the pandemic occurs at date $t = 0$, illustrated by the dotted line, the sector I output drops to zero before it goes back up at $t = 1$ at a lower level than the pre-pandemic equilibrium, and remains

FIGURE 5 Optimal default.



there ever since. This is because the pro-stabilization government uses a lenient default sanction to mop up the unemployment immediately after the pandemic passes.

Let us now turn to the dashed line in Figure 4, which indicates a pro-allocation government. Compared with the pro-stabilization government, the output does not rebound as much at $t = 1$, because of short-run unemployment. Output gradually increases and overtakes the pro-stabilization case only at $t = 2$. At $t = 2$, the economy bounces back to a long-run equilibrium at exactly the same pre-pandemic level. The reason is simple: the pro-allocation government uses a harsh default sanction to deter all the inefficient entrepreneurs so that the long-run productivity can be restored fully. The case of the optimal default sanction is illustrated by the solid line. It is intermediate and is a result of balancing the trade-off between short-term employment and long-run productivity. The optimal case does not suggest a lenient default sanction to mop up unemployment completely, and it also tolerates some productivity loss in the long run so that the short-run unemployment rate is lower. The implication for defaults, almost by definition, varies with the harshness of the sanction. As can be seen in Figure 5, the harsh default sanction that corresponds to $M = 6$ rules out default completely, and the lenient default sanction that corresponds to $M = 10$ leads to 40% of borrowers defaulting. The optimal default corresponding to M^* is again intermediate.[9]

Then we increase the number of entrepreneurs in each sector from the benchmark case of 10 to 40.[10] This parameter change increases industry competitiveness and reduces entrepreneurial rents. In turn, labour income share accounts for a larger fraction of total output. Let us turn to Figure 6. The vertical axis is the value of total social utility denoted as V, which is defined in equation (7). The horizontal axis is the number of borrowing agents, each of which corresponds to a default sanction. In particular, the number of borrowers $M = 6$ corresponds to the harsh default sanction λ_{A1}, and the number of overall borrowers $M = 10$ corresponds to the lenient default sanction λ_{B1}. The number of borrowers decreases monotonically with the harshness of the default sanction. As we can see in the second plot of Figure 6, the optimal default sanction becomes a more lenient stance as stabilization has become more pertinent.

We also increase the price elasticity of demand ε to 4 from the benchmark case 1.2, which decreases the entrepreneurial rents. As we show in Figure A3 in the Appendix, it causes a shift to a more lenient stance, since lower rents imply a higher wage, which worsens the utility loss associated with unemployment.

6.1 | Interest rate ELB and firm markup

In this subsection, we assess numerically the implication of the interest rate ELB for the optimal default sanction and its interaction with firm markups. As our numerical example illustrates in

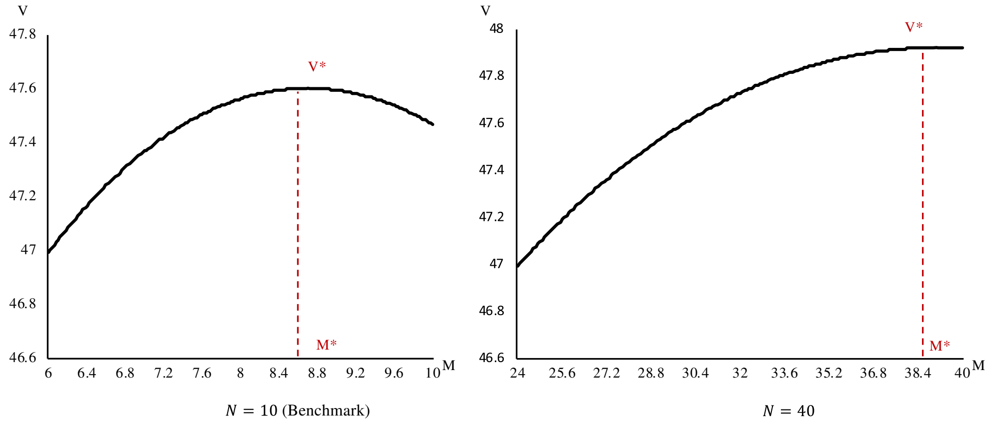


FIGURE 6 Social welfare, optimal sanctions and N.

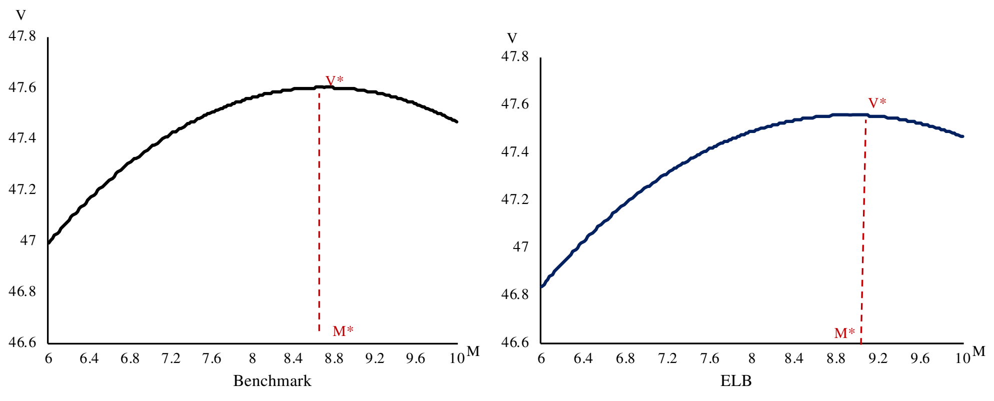


FIGURE 7 Social welfare, optimal sanctions and ELB.

Figure 7, at the ELB, the overall welfare is lower than the benchmark case due to involuntary unemployment in sector J, and the optimal default sanction turns out to be more lenient than the benchmark case. With our benchmark parameters, the unbounded interest rate would fall to −39%. This fall in the interest rate is stark because we model only two sectors in the economy, and 50% is forced to close during $t = 0$. As we see in Table 2, this means that the optimal default sanction becomes 24% more lenient than the case without the ELB. Furthermore, as shown in Guerrieri *et al.* (2022), market incompleteness relaxes the conditions for a negative supply shock to cause the demand shortage; therefore, when the government fails to provide unemployment benefits to insure workers against labour income risks, the ELB is likely to exacerbate the aggregate demand externality, and the optimal default sanction would turn out to be even more lenient.

In the benchmark parameter space, the large difference between the intra-temporal elasticity $1/\nu$ and the inter-temporal elasticity $1/\delta$ implies a large drop in the unbounded interest rate during the pandemic, which generates the noticeable shift in the leniency stance of the optimal default sanction. The unbounded interest rate reduction is key. To understand its sensitivity, in our comparative statics in Table 2, we reduce the spread between ν and δ to generate a modest fall in the unbounded interest rate. When we increase δ to 0.82 while keeping ν the same as the benchmark case, the difference between them decreases from 0.13 to 0.01. This produces the

TABLE 2 Interest Rate Changes, Intra/Inter-temporal Elasticities, Optimal Policies

ν	δ	$\nu - \delta$	Unbounded r_0	Change in leniency
0.83	0.7	0.13	−39%	24%
0.83	0.82	0.01	−2%	0.57%
0.71	0.7	0.01	−0.06%	0.51%

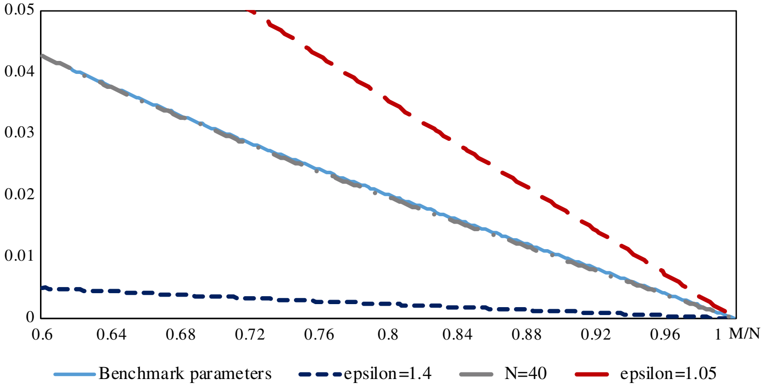


FIGURE 8 Sector J involuntary unemployment and firm markup. *Notes*: The horizontal axis is the number of borrowers that participate, normalized by industry size. The further to the right, the more lenient the default sanction. The vertical axis is the involuntary unemployment rate in sector J.

unbounded interest rate as −2% per annum. In equilibrium, the optimal default sanction becomes 0.57% more lenient.

Then we decrease ν to reduce the spread of $\nu - \delta$ to 0.01 as well, but compared with the previous case, the unbounded interest rate falls to only −0.06%, a larger change compared with the benchmark case. This is because the interest rate dynamics is more sensitive to the intra-temporal elasticity and less to the inter-temporal elasticity. This leads to the optimal default sanction being 0.51% more lenient than the benchmark case. Interestingly, the change in the optimal sanction in this case is not so different from the previous case where the unbounded interest rate falls more. This is because although the unbounded interest rate falls less in this case, which should imply a more modest change in leniency, the decrease in ν increases the price elasticity of demand, which reduces the firm profits, causing the change in leniency to increase.

The above numerical sensitivity analysis suggests that the price elasticity and firms' markup power would affect the leniency of the optimal default sanction in a low interest rate environment. Motivated by the stylized fact that we have documented pre-pandemic on the fall in interest rates and the rise in markup, we conduct the following experiment by changing the industry size and the price elasticity of demand such that firm markup changes in equilibrium. Figure 8 documents how involuntary unemployment in sector J changes accordingly. As can be seen, when the price elasticity ε decreases from 1.2 (benchmark) to 1.05, which means that firm markup increases and rents increase, involuntary unemployment in sector J increases across the whole spectrum of default sanction. In particular, when the default sanction is at the harsh stance that deters all inefficient projects, involuntary unemployment in sector J increases to 7.5%. This is striking because sector J is allowed to open and operate, and pandemic policies are applied only in sector I, and the parameter change is also applied only in sector I, rather than sector J. This reflects the aggregate demand externality due to complementarity between sector I and sector J in the interest rate ELB environment. This decrease in ε should push the optimal default sanction to be more lenient. In our numerical simulations, the optimal default sanction becomes 4.37% more lenient than the benchmark parameter space.

Similarly, when we increase the price elasticity from 1.2 to 1.4, the involuntary unemployment in sector J still exists, but it is noticeably less across all spectrums of the default sanction. This would mean that the change in leniency of the optimal default sanction is small (only 0.5% in our numerical simulation). Finally, the change in industry size almost has little effect on sector J involuntary unemployment because the industry size does not have a first-order effect on interest rate dynamics, whereas the elasticity parameters do.

One qualification of the above aggregate demand externality is that it has the opposite implication of congestion externalities commonly assumed in the literature of credit misallocation. As in the seminal contribution of Caballero *et al.* (2008), subsidized credit to poorly performing firms affects aggregate productivity and output not only because it attracts more inefficient firms to stay afloat, but also because of congestion externalities in input and output markets due to the presence of zombie firms, which reduce the productivity or profitability of healthy firms. For example, in Acharya *et al.* (2021), the authors model the productivity of profitable firms being negatively affected by the extent of zombie lending in the previous period, such that policies aimed at avoiding short-term recessions can be trapped into protracted low rates and excessive forbearance, possibility leading to permanent output losses. Therefore such congestion externalities would tilt the optimal default sanction in our model towards a harsher stance, exerting opposite effects as opposed to the aggregate demand externality.

6.2 | Creative destruction

The creative destruction wedge Δ that we have defined reflects the extent to which unproductive incumbents can be replaced promptly by productive entrants. Since the wedge reflects business dynamism, to capture this meaningfully, we modify the benchmark model slightly by introducing a simple dynamic dimension as follows.

During normal times in every period, a proportion ϕ of the N existing entrepreneurs find their technology outdated and exit. Meanwhile, an equal number of workers totalling ϕN ($< Q$) enter and become entrepreneurs. The economy is in the efficient steady state. When the pandemic shock forces sector I to close at the end of t, the normal set of workers with new ideas totalling ϕN, and the prior entrepreneurs totalling N, can choose to borrow government support loans and retain workers, as in the static case. The difference is now that it takes multiple periods for sector I to recover because each normal period has only a fraction ϕ of new entries. Figure 9 illustrates this point intuitively. With our benchmark parameters, the fraction of new workers entering in each period is set to be 0.1, and the fraction of profitable prior entrepreneurs is set to be 0.5, so that it would take 5 years for sector I to reach full employment and fully restore production if the government sets a harsh default sanction to keep all unprofitable projects out. Given the severity of the COVID-19 pandemic, we think 5 years is a plausible figure, and we also vary this figure for sensitivity analysis.

Now we vary the composition of α and ϕ, both of which matter for the wedge Δ and the number of years before sector I recovers. By varying the composition of the creative destruction wedge, we can obtain the corresponding change in optimal default sanction, and we define the

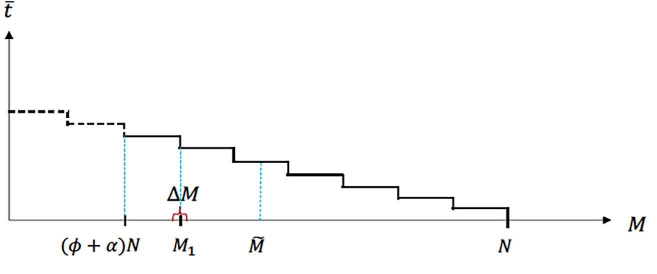


FIGURE 9 Time periods before sector I reaches full employment. *Notes*: The horizontal axis is the number of firms cumulatively in sector I. The vertical axis is the time periods before sector I reaches full employment.

changes as the normalized distance from the hawkish stance λ_A, that is,

$$\frac{M^* - (M|\lambda_A)}{(M|\lambda_B) - (M|\lambda_A)},$$

so the further the distance, the more lenient the optimal policy.

As Table 3 shows, the benchmark wedge with $\alpha = 0.5$ and $\phi = 0.1$ is equal to 5, and the number of years before sector I employment recovery is 5. The wedge turns out to be equal to the number of years before sector I employment recovery. In this case, the distance from λ_A is 7.5%. When we increase ϕ to 0.124 or increase α to 0.6, the wedges in both cases are equal to 4: the distance from λ_A becomes smaller compared with the benchmark case. When we decrease α or decrease ϕ to decrease the wedge, the distance from λ_A becomes larger compared with the benchmark case. This means that as the creative destruction wedge increases, the optimal policy becomes more lenient because higher wedges imply that the economy takes a longer time for productive ones to enter. Thus higher wedges lead to higher levels of resource misallocation.

Furthermore, we find that the optimal policy tends to be slightly more sensitive to the changes in α than to changes in ϕ. For example, keeping the wedge the same at 6, we can either reduce α

TABLE 3 Creative Destruction Wedge and Composition of α and ϕ

Years	α	ϕ	Wedge Δ	% from λ_A
4	0.5	0.124	4	6.67%
4	0.6	0.1	4	6.67%
5[a]	0.5[a]	0.1[a]	5	7.5%
6	0.4	0.1	6	10%
6	0.5	0.083	6	8%
6.5	0.35	0.1	6.5	11%
6.5	0.5	0.077	6.5	9%
7	0.3	0.1	7	11.7%
7	0.5	0.071	7	10%

Notes [a] Indicates benchmark value.

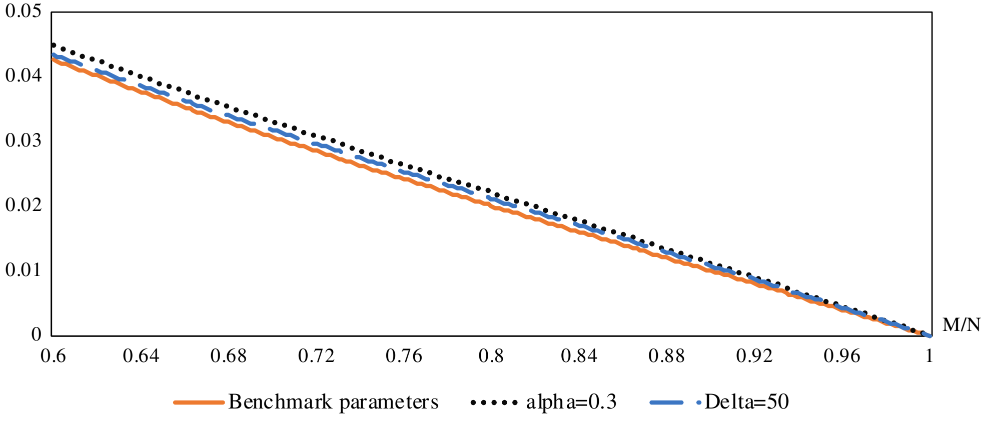


FIGURE 10 Sector J involuntary unemployment and creative destruction. *Notes*: The horizontal axis is the number of borrowers that participate, normalized by industry size. The further to the right, the more lenient the default sanction. The vertical axis is the involuntary unemployment rate in sector J.

or reduce ϕ, but the distance from λ_A by reducing α is 10%, whereas that by reducing ϕ is 8%. The pattern is the same for the wedges 6.5 and 7. This implies that if the driving force of the changes in the wedge is due to the crisis severity α, then the change in optimal policy is larger, and the resource misallocation due to a higher wedge is larger.

When the economy is at the ELB, the creative destruction wedge in sector I also affects sector J involuntary unemployment through the aggregate demand externality, although the effect is mild. For ease of exposition, we use the stylized setup to illustrate this point numerically. As Figure 10 shows, as the wedge increases to 50 from 5 due to the decrease in ϕ, sector J involuntary unemployment goes up in all spectrums of the default sanction. Similarly, when we decrease the proportion of profitable prior entrepreneurs from 0.5 to 0.3, which also leads the wedge to increase, we see an increase in sector J involuntary unemployment. Therefore the optimal default sanction becomes even more lenient. This exercise suggests that the ELB environment further exacerbates the resource misallocation due to the creative destruction wedge.

7 | EXTENSION: CREDIT GUARANTEE AS IMPLEMENTATION

So far, we have abstracted away banks' incentive problem for analytical tractability, and the policy instrument in the above benchmark model is the default sanction λ_1. One might reasonably question the micro-foundation of λ_1 and how it would interact with banks' incentives. As we know, in practice, government does not lend directly to SME borrowers, whereas banks do, and the government's policy instrument is the credit guarantee, which affects banks' monitoring incentives. Therefore, in this section, we extend the model to consider banks that provide loans to SMEs, rather than the government. The government's loan support policy instrument is credit guarantee g, rather than the default sanction λ_1. We show that the model with banks is isomorphic to the benchmark model, the credit guarantee provides a micro-foundation for the default sanction, and our results still go through.

Following Martinez-Miera and Repullo (2017), let us consider banks that are lenders but exert private monitoring efforts. Let $\eta(e)$ be a bank's monitoring technology, where e is the bank's monitoring effort, and $\eta(e)$ is monotonically increasing in e. In this paper, we take a short-cut by not estimating the functional form of $\eta(e)$, as the aim here is to illustrate the intuition. This is also the reason why we position banks with credit guarantees as a model extension, rather than as the benchmark main model. When banks monitor and pursue the defaulters, if there are remaining funds, then banks can take away a fraction $\eta(e)$ of the remaining funds, and defaulters divert away a fraction $1-\eta(e)$, as in the benchmark model. Thus a higher $\eta(e)$ corresponds to a more superior monitoring technology.

We further assume $\eta(0)=\gamma$ and $\eta(+\infty)=1$. This assumption implies that when banks do not exert monitoring effort, the defaulters can privately divert away a fraction $1-\gamma$ of their remaining funds, if any, and when banks exert an extremely high level of monitoring effort, their monitoring technology is near-perfect, and the amount of diverted funds approaches zero. What could affect banks' monitoring effort is the government's credit guarantee g. If $g = 100\%$, then the government fully makes up the loss of the defaulted amount to the banks; if $g = 0\%$, then the banks fully take on the loss of the defaulted amount.

Suppose that an entrepreneur announces default; then the bank monitors and pursues the defaulter's remaining funds. The bank's total proceeds from monitoring and pursuing the defaulters include the fraction $\eta(e)$ of remaining funds that the bank is able to take away, plus the money received from the government credit guarantee on the defaulted amount, so the bank earns $\eta(e)+g(1-\eta(e))$ per unit of funds. Meanwhile, the bank exerts effort cost $\iota e^2/2$ per unit of funds à la Martinez-Miera and Repullo (2017). Thus banks maximize the objective function $\eta(e)+g(1-\eta(e))-\iota e^2/2$ by choosing the monitoring effort e. The following proposition summarizes the main result.

Proposition 6. (*Credit guarantee and default sanction*) *In equilibrium, we have the following.*

(a) *Banks' monitoring technology is a decreasing function of credit guarantee.*
(b) *Credit guarantee has a one-to-one mapping with default sanction.*
(c) *When credit guarantee $g = 100\%$, monitoring effort $e = 0$, monitoring technology $\eta = \gamma$, and default sanction $\lambda_1 = 0$.*

Proof. See the Appendix. ∎

The intuition of Proposition 6 is as follows. As the government increases the level of credit guarantee, banks' marginal benefits of exerting monitoring effort decrease. Because banks exert effort privately, they choose the effort such that the marginal benefits of exerting effort equate to its marginal cost; in equilibrium, as the level of credit guarantee increases, banks exert less effort, which leads to an inferior monitoring technology. The inferior monitoring technology increases the marginal entrepreneur's benefits of borrowing, compared with her outside option of staying as a worker, thus attracting more inefficient entrepreneurs to borrow. Similarly, as the government reduces credit guarantees, the inefficient entrepreneur with low profitability may be deterred from borrowing, reducing the aggregate default. As we establish in the proof of Proposition 6, the credit guarantee in this enriched setup has a one-to-one mapping with the default sanction in the benchmark model. When the government provides 100% credit guarantee, banks' monitoring technology becomes γ of the benchmark model, and the default sanction λ_1 corresponds to zero; the lower the credit guarantee, the larger the default sanction becomes.

8 | OTHER NUMERICAL EXPERIMENTS

In this section, we vary other parameters to assess how the optimal sanction changes its stance with the figures displayed in the Appendix. Figure A1 displays the responses of the social utility to the number of loan applicants, or equivalently, the default sanction. The first plot is our benchmark case where the discount factor β is set to 0.97; the second plot sets $\beta = 0.9$; and the third subplot sets $\beta = 0.5$. The decrease in β can be interpreted as a more impatient or myopic government.

The first plot of Figure A1 shows that in our benchmark case, the optimal default sanction is intermediate; it is more lenient than the harsh default sanction λ_{A1}, but harsher than the lenient default sanction λ_{B1}. The second plot of Figure A1 assumes a lower discount factor, $\beta = 0.9$. The optimal default sanction is also intermediate, but compared with the case $\beta = 0.97$, it moves further away from the harsh default sanction λ_{A1}. The third plot of Figure A1 assumes an extremely low discount factor $\beta = 0.5$. This result suggests that if the government is myopic,—i.e. it cares more about the short-term gains than the long-run productivity—then it will choose a more lenient default sanction than our benchmark case, and attract more borrowers at $t = 1$ to reduce unemployment.

Next, we vary parameters to see how they alter the optimal default sanction. First, we increase γ from 0.7 to 0.95, which means an improvement in the lender monitoring technology. As the second plot of Figure A2 in the Appendix illustrates, the optimal default sanction becomes the most lenient stance of λ_{B1}. Indeed, a near-perfect monitoring technology could justify the government's most generous credit guarantees for its BBLs to SMEs. However, this scenario is unlikely to hold during the pandemic. Given the scale and the speed of the crisis, lenders are unlikely to carry out meticulous monitoring in haste. We then increase σ_B and ϕ to improve the overall quality of the inefficient entrepreneurial pool. This also results in a shift to a more lenient optimal default sanction (see Figures A4 and A5 in the Appendix), as the utility loss associated with future productivity slowdown becomes less severe.

Furthermore, we consider the effects of long-run scarring by considering a scarring function. Specifically, we do not need the industry gap to close at $t = 2$. If there is an industry gap at the end of $t = 1$ (i.e. $N > M$), then at the start of $t = 2$, $(N - M)/\kappa$ workers with new ideas could fill in the industry gap, where κ (≥ 1) is an increasing and concave function of short-run unemployment that reflects long-run scarring of the pandemic: a large κ could result in insufficient new entries in the long run, hence long-run structural unemployment is a possibility. We assume that the scarring function takes the form $\kappa = \ln(U_{\mathrm{I}}(M_1) + e)$, where $U_{\mathrm{I}}(M_1)$ is the short-run unemployment rate. This implies that the short-run unemployment leads to insufficient entries in the long run and structural unemployment, but if the short run obtains full employment, then the long-run scarring disappears and date $t = 2$ sees no structural unemployment. Figure A6 in the Appendix illustrates the comparative statics between our benchmark and the long-run scarring case. The overall welfare with long-run scarring is lower than the benchmark case; moreover, the optimal default sanction becomes more lenient, so the government will care more about short-run stabilization rather than allocative efficiency.

9 | CONCLUSION

This paper has assessed government loan support for small businesses during a pandemic from a normative angle. We apply adverse selection in financial contracting in a macroeconomic framework. A two-sector equilibrium model has been developed, featuring screening in the presence of a pandemic shock and aggregate demand externality. To characterize the government's policy stance, we have developed an analytic measure 'stabilization proclivity'. Numerically, we have demonstrated that the optimal default sanction is intermediate. The interest rate lower bound shifts the optimal default sanction to a more lenient stance due to the aggregate demand externality channel. Lower price elasticity of demand increases firm markups, exacerbating involuntary unemployment at the ELB, and the optimal default sanction becomes even more lenient. The creative destruction wedge polarizes the government's hawkish and lenient stances, and a higher creative destruction wedge implies a more lenient optimal default sanction, worsening long-run allocative efficiency. Although we study SMEs, our results on firm markups shed light on the macro-financial policies for the rise of superstar firms characterized by small labour shares and large profit shares (see Autor *et al.* 2020). Rents due to imperfect competition alter firms' incentive constraints. Using macro-financial policies to shift firms' incentive constraints should naturally consider profit shares, as we do in this paper.

One simplifying assumption that we have made is that both incumbents and new entrants can benefit from government guarantees. We acknowledge that policy interventions in response to the COVID-19 shock were skewed disproportionatel towards incumbent firms. If the guarantee scheme were available only for the incumbents of our model, then the implication would be as follows. First, when the government sets a harsh default sanction to deter all the inefficient ones from borrowing, the sector I short-run unemployment rate will be greater, the fall in interest rate is greater, and demand shortage is greater. With the issue of effective lower bound, this will cause further involuntary unemployment in sector J, pushing the optimal default sanction to a more lenient realm. Second, when the government sets a lenient default sanction to stabilize short-run employment, with the guarantee scheme available only to the incumbent, the lenient sanction has to be lower than the benchmark case. This means that more inefficient entrepreneurs have to be granted access to loan support, worsening allocative efficiency. Thus withholding the loan support from new entrants polarizes the government's hawkish and dovish stances.

While the aggregate demand externality that we highlight in this paper lends more support to a lenient policy stance, some caveats remain. On the one hand, when the credit support is scaled back in the future, it may cause banks to roll over legacy lending rather than recognizing potential losses (see the evergreening channel in Acharya *et al.* 2021). On the other hand, given the pressure from incumbents to keep them afloat (see Buera *et al.* 2013), it is unclear whether such credit support would be scaled back soon or become protracted. Therefore the ultimate costs due to resource misallocation might be larger than expected, which would lend more support to a harsher policy stance. We have kept the model simple, and in the future it would be useful to embed the model mechanism in a large quantitative setup to evaluate policy effectiveness.

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NOTES

1 See, for example, Morris *et al.* (2020), Treanor (2020), Elliott (2020) and Thomas (2021).

2 Some credit schemes encourage lenders to request personal guarantees (see, for example, Ono and Uesugi 2009; Mayordomo *et al.* (2021), which aligns incentives of the lender to screen borrowers. We thank an anonymous referee for raising this point.

3 Of course, in reality, output is produced also using equipment and property, and there are overhead payments to be made—for example, of local and national taxes, rents, minimal utility costs, etc.—if the business is not to default when it cannot produce. Some of these overhead costs were deferred during the pandemic, but not all. Our approach here, using a requirement to maintain at least some key workers, is a reasonable way of dealing with such continuing overhead costs within the context of the simplest possible model to analyse such circumstances. Indeed, governments have the objective to help firms to retain workers and stabilize employment when implementing loan support programmes; see, for example, the US Department of the Treasury's article on the US Paycheck Protection Program (https://home.treasury.gov/policy-issues/coronavirus/assistance-for-small-businesses/paycheck-protection-program, accessed 2 December 2022), as well as the UK HM Treasury's article on government-backed financial support schemes (https://www.gov.uk/government/news/government-backed-loans-help-thousands-of-businesses-to-protect-jobs-during-pandemic, accessed 4 January 2023).

4 We are aware that the pandemic can evolve over time and restrictions might be re-introduced if circumstances require. Our modelling assumption is that the very end of $t = 1$ is when the vaccination programme is expected to take effect, so that the economy could function with production resumed. Our model is stylized to keep the analytics tractable, so it does not incorporate the evolving dynamics of the pandemic, which would require a fully-fledged dynamic model with inputs of medical expertise on the evolving nature of the virus. Nevertheless, we believe that our mapping captures the substance of the pandemic episode in 2020–21.

5 That the lender obtains only a fraction of the remaining funds in the case of default is in line with a common assumption in the financial contracting literature (e.g. Lorenzoni 2008; Rampini and Viswanathan 2013; Dávila and Korinek 2018; Dávila 2020) that a financial contract is subject to a form of limited commitment. In Dávila (2020), this fraction is interpreted as the efficiency of the bankruptcy process.

6 Precisely, $E_1(W_{11} | \mathbb{I}_d = 1,\ \lambda_1)$ can be expressed as follows, as can be seen in the derivations in the Appendix:

$$\begin{aligned} E_1(W_1 | \mathbb{I}_d = 1,\ \lambda_1) &= (1 - u_1(M-1))\left(1 - \frac{1}{N\varepsilon}\right) \\ &= \frac{Q(M-1)/2N}{(Q/2) + N - M + 1}\left(1 - \frac{1}{N\varepsilon}\right), \end{aligned}$$

where M is the number of borrowing entrepreneurs, and $u_1(\cdot)$ is the unemployment at $t = 1$ as a function of borrowing entrepreneurs. As we can see from the above equation, if the entrepreneur is to remain as a worker, then the labour supply has to adjust by $u_1(M-1)$, which in turn affects the conditional expected wage.

[7] In practice, such timely and almost real-time government loan support involves the banking sector issuing inside money against an offsetting credit, with the government providing guarantees. Indeed, in response to the COVID-19 crisis, many governments worldwide have unveiled large-scale loan stimulus programmes through the banking system.

[8] The interest rate is in terms of goods J, rather than the real interest rate obtained by deflating the nominal interest rate by the expected inflation rate from the price index of the two types of goods. Since during the pandemic sector I goods are not traded, we cannot observe their price, nor can we measure such a price index. Indeed, as estimated in Cavallo (2020), the official CPI does not reflect the rapid changes in prices in various sectors due to COVID-19. So both for simplicity and in line with the most current developments, the interest rate in our context refers to the interest payment in terms of goods J.

[9] We assume that for the less profitable entrepreneurs, their type $\sigma(n)$ follows a uniform distribution. The assumption of the distribution would make a quantitative difference to the optimal policy. Suppose that the left tail of the distribution is more dense than the right tail of the distribution. Then the lenient default sanction λ_{B1} that stabilizes short-run employment will lead to greater loss in allocative efficiency, shifting the optimal policy to a harsher stance, and vice versa. Nevertheless, there are still quantitative effects on incentives by changing λ_1. Suppose that n^* is far in the left tail of the distribution. By increasing λ_1, this firm n^* incentive constraint will no longer be satisfied, thus it will be deterred from borrowing, but by decreasing λ_1 a tiny bit, the quantitative effect on the incentives is small because n^* is ready far in the left tail. Thus the quantitative effects of changing λ_1 would be asymmetric.

[10] This means that Q/N has decreased from 80 to 20. When we increase Q proportionally such that Q/N remains unchanged, the optimal default sanction will change as well. This means that individual levels of Q and N matter, not just their ratio. This is because individual levels of Q matter for the outside option in the incentive constraint (2) through the aggregate labour supply, and it also matters for the calculation of unemployment rates, whereas N, rather than Q, matters for markup.

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APPENDIX

Proof of Lemma 3.

Since $P_{\mathrm{I}(t=-1)} = P_{\mathrm{J}(t=-1)} = 1$, at the start of $t = 0$, workers in each sector get wages totalling $(1 - 1/(N\varepsilon))(Q/2)$, and entrepreneurs in each sector have profits from last period totalling $Q/(2N\varepsilon)$. They spend half of the sum on goods J at the end of $t = 1$. Consequently, at the start of $t = 1$, workers in each sector have extra money totalling $(1 - 1/(N\varepsilon))(Q/4)$, sector I entrepreneurs have extra money totalling $Q(4N\varepsilon)$, and sector J entrepreneurs have extra money totalling $Q/(4N\varepsilon)$. Therefore the extra sum of money due to no spending on goods I amounts to $Q/2$.

Since $\max(df_1) \le F_1 N$, that is, $\max(df_1) \le (1 - 1/(N\varepsilon))(Q/2)$, and $\Lambda_1 > 0$, given that $df_1 = B_1 + \Lambda_1$, it follows that

$$B_1 < \frac{Q}{2} - \frac{Q}{2N\varepsilon},$$

so the extra money that agents carry over from $t = 0$ is more than enough to invest in B_1. ■

Proof of Proposition 1.

Conditional on $df_1 > 0$, for the marginal entrepreneur n, the incentive constraint takes the equality sign and can be re-expressed as

$$\lambda_1 = (1 - \gamma)\, \sigma(n)\, h_1 - \mathbb{E}_1(W_{\mathrm{I}1} | \mathbb{I}_d = 1,\ \lambda_1). \tag{A1}$$

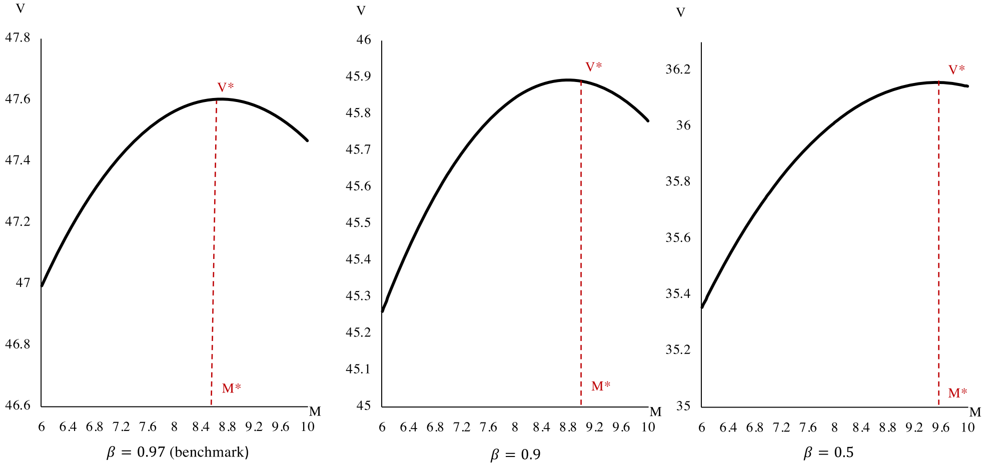

FIGURE A1 Social utility, optimal sanctions and β.

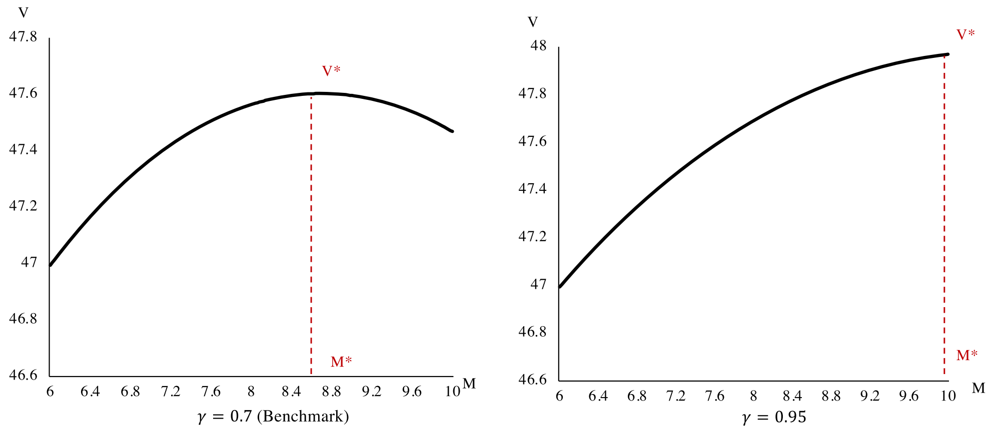

FIGURE A2 Social welfare, optimal sanctions and γ.

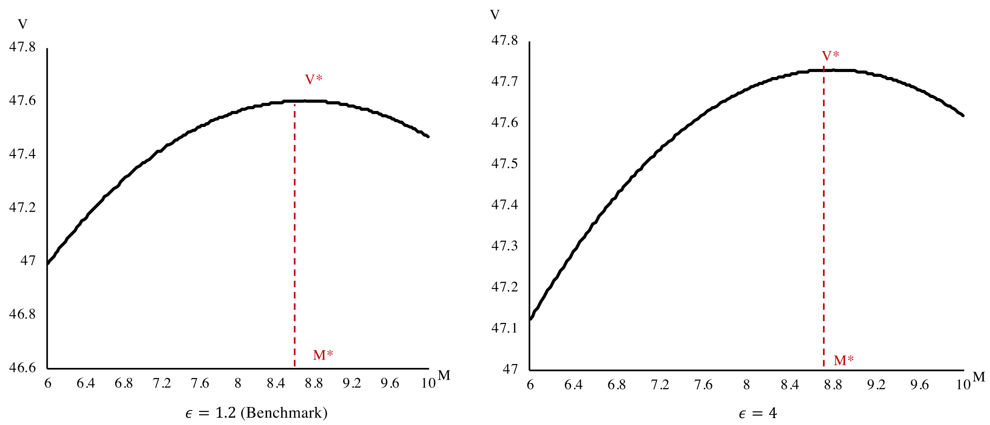

FIGURE A3 Social welfare, optimal sanctions and ε.

Suppose that M ($\leq N$) entrepreneurs-to-be apply for government loans. Of these, $(\phi + \alpha)N$ are profitable entrepreneurs, and $M - \phi N - \alpha N$ are inefficient ones. Given the uniform distribution assumption, it follows that

$$\sigma(n) = \left(1 - \frac{M - (\alpha + \phi)N}{(1-\alpha)N}\right)\overline{\sigma} + \frac{M - (\alpha + \phi)N}{(1-\alpha)N}\,\sigma_B. \tag{A2}$$

The expected wage conditional on deviation at $t = 1$ needs to be adjusted by the out-of-equilibrium unemployment rate, that is,

$$\begin{aligned} E_1(W_1 | \mathbb{I}_d = 1,\ \lambda_1) &= (1 - u_1(M-1))\left(1 - \frac{1}{N\varepsilon}\right) \\ &= \frac{Q(M-1)/2N}{(Q/2) + N - M + 1}\left(1 - \frac{1}{N\varepsilon}\right). \end{aligned}$$

We can see that $\partial E_1(W_1 | \mathbb{I}_d = 1,\ \lambda_1)/\partial M > 0$, and since $\partial M/\partial\sigma(n) < 0$ and given equation (A2), it follows that

$$\partial E_1(W_1 | \mathbb{I}_d = 1,\ \lambda_1)/\partial\sigma(n) < 0. \tag{A3}$$

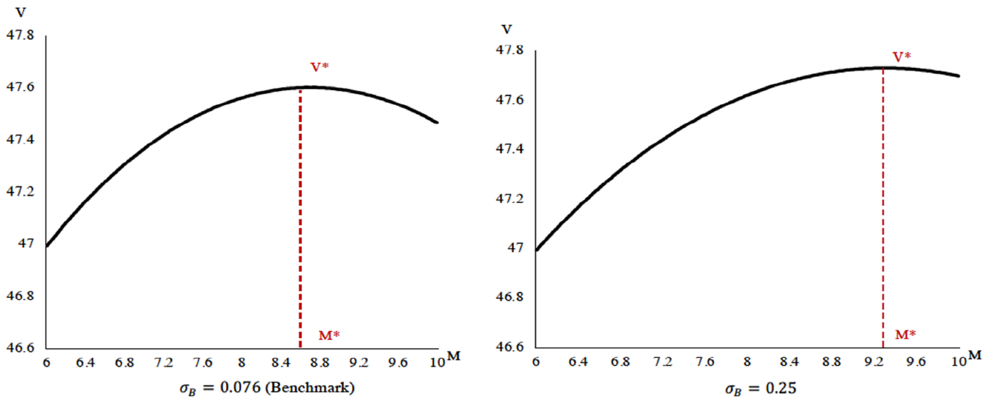

FIGURE A4 Social welfare, optimal sanctions and σ_B.

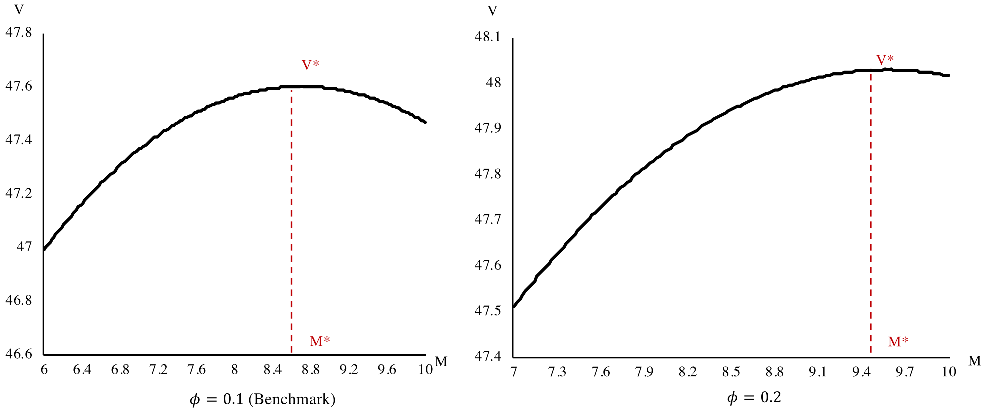

FIGURE A5 Social welfare, optimal sanctions and ϕ.

Given equation (A1) and inequality (A3), we can see that $\partial\sigma(n)/\partial\lambda_1 > 0$.

Thus when λ_1 increases, the number of borrowing entrepreneurs M decreases, and fewer entrepreneurs with bad projects borrow, so the number of defaulters decreases. ■

Proof of Theorem 1.

Part (a) Given $1 - 1/(N\varepsilon) = \overline{\sigma}$, $F_1 = \overline{\sigma}h_1$, the inefficient entrepreneur with the highest profitability is on the verge of strategic default in the absence of sanctions. Among the prior entrepreneurs, $(1-\alpha)N$ will default conditional on borrowing.

Now let us set $\lambda_1 > \lambda_{A1}$, where

$$\lambda_{A1} = \left(1 - \frac{1}{N\varepsilon}\right)\left((1-\gamma)\,\frac{Q}{2N} - (1 - u_{\mathrm{I}}(\phi N + \alpha N))\right).$$

Then for the entrepreneur with $\overline{\sigma}$, their benefits of default are smaller than their outside option. They are deterred from borrowing. By monotonicity, all the other inefficient entrepreneurs are deterred from borrowing as well. At date $t = 1$, the unemployment rate is $u_{\mathrm{I}1}(\phi N + \alpha N)$. In the benchmark assuming away scarring, full employment obtains in the long run.

Part (b) Since the government wants to obtain full employment at $t = 1$, it needs to attract $(1 - \phi - \alpha)N$ prior entrepreneurs to apply for loans. By our uniform distribution specification, the

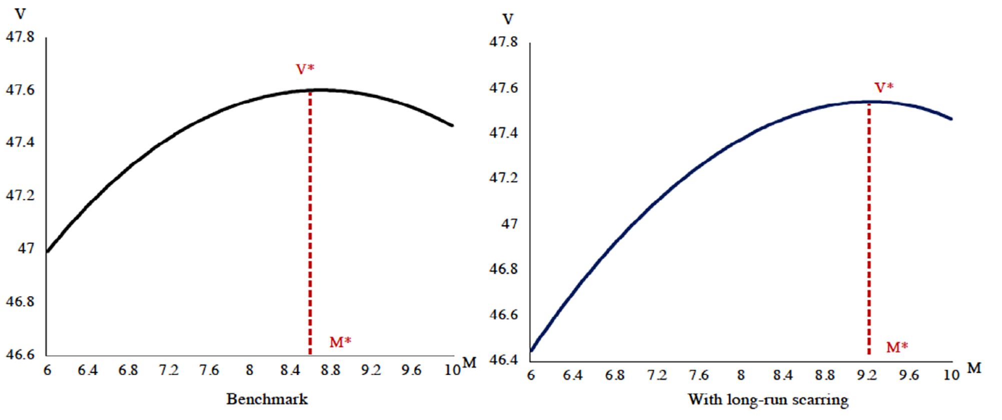

FIGURE A6 Social welfare, optimal sanctions and long-run scarring.

marginal entrepreneur's $\sigma(n)$ must satisfy

$$\frac{\overline{\sigma}-\sigma(n)}{(1-\phi-\alpha)N}=\frac{\overline{\sigma}-\sigma_B}{(1-\alpha)N},$$

which is equivalent to

$$\sigma(n)=\frac{\phi}{1-\alpha}\,\overline{\sigma}+\frac{1-\phi-\alpha}{1-\alpha}\,\sigma_B.$$

Suppose that λ_{B1} just satisfies the marginal entrepreneur's incentive constraint, that is,

$$(1-\gamma)\,\sigma(n)\,h_1-\lambda_{B1}=\left(1-\frac{1}{N\varepsilon}\right)(1-u_{\mathrm{I}}(N-1)).$$

Then

$$\lambda_{B1}=(1-\gamma)\left(\frac{\phi}{1-\alpha}\,\overline{\sigma}+\frac{1-\phi-\alpha}{1-\alpha}\,\sigma_B\right)\frac{Q}{2N}-\left(1-\frac{1}{N\varepsilon}\right)(1-u_{\mathrm{I}}(N-1)).$$

By monotonicity, the inefficient entrepreneurs with profitability higher than

$$\frac{\phi}{1-\alpha}\,\overline{\sigma}+\frac{1-\phi-\alpha}{1-\alpha}\,\sigma_B$$

borrow and default, and those with lower profitability stay out.
Part (c) Given the parameter

$$\sigma_B=\frac{2N-2/\varepsilon}{(1-\gamma)Q},$$

it follows that $(1-\gamma)\sigma_B h_1=W_{\mathrm{I}1}$ and $\sigma_B h_1<F_1$. Thus the inefficient entrepreneur with the lowest profitability will try to borrow at the start of $t=1$ in the absence of sanctions, and then default at the end of $t=1$.

Given that $\overline{\sigma}=1-1/(N\varepsilon)$, it follows that $\gamma\overline{\sigma}h_1<F_1=\overline{\sigma}h_1$. Thus the inefficient entrepreneur with the highest probability will try to borrow in the absence of sanctions, and then strategically default at the end of $t=1$. Among the prior entrepreneurs, conditional on getting the loans, a

fraction $1-\alpha$ of them will default. As the total number of applicants amounts to $(\phi+1)N$ and the industry size is N, some profitable applicants are crowded out. ■

Proof of Proposition 2.

Part (a) The following Euler equations for the workers' consumption hold for date t:

$$U_{c_\mathrm{J}}(c_{\mathrm{I}t}(W), c_{\mathrm{J}t}(W)) = \beta(1+r_t)\ U_{c_\mathrm{J}}(c_{\mathrm{I}t+1}(W), c_{\mathrm{J}t+1}(W)),$$

which is equivalent to

$$1+r_t = \frac{1}{\beta}\,\frac{U_{c_\mathrm{J}}(c_{\mathrm{I}t}(W), c_{\mathrm{J}t}(W))}{U_{c_\mathrm{J}}(c_{\mathrm{I}t+1}(W), c_{\mathrm{J}t+1}(W))}.$$

Given the CES preference (3), we have

$$1+r_t = \frac{1}{\beta}\,\frac{(\boldsymbol{c}_{\mathrm{I}t}(W)^{1-\nu} + \boldsymbol{c}_{\mathrm{J}t}(W)^{1-\nu})^{(\nu-\delta)/(1-\nu)}\ \boldsymbol{c}_{\mathrm{J}t}(W)^{-\nu}}{(\boldsymbol{c}_{\mathrm{I}t+1}(W)^{1-\nu} + \boldsymbol{c}_{\mathrm{J}t+1}(W)^{1-\nu})^{(\nu-\delta)/(1-\nu)}\ \boldsymbol{c}_{\mathrm{J}t+1}(W)^{-\nu}}. \tag{A4}$$

We know that $c_{\mathrm{I}0}(W)=0$ and $c_{\mathrm{J}0}(W) = (1-1/(N\varepsilon))\,(Q/2)$. Given $\lambda_1 > \lambda_{A1}$, at date $t=1$, only $(\alpha+\phi)N$ good entrepreneurs produce in sector I. It follows that

$$c_{\mathrm{I}1}(W) = \left(1-\frac{1}{N\varepsilon}\right)\frac{(\phi+\alpha)Q}{2} \quad \text{and} \quad c_{\mathrm{J}1}(W) = \left(1-\frac{1}{N\varepsilon}\right)\frac{Q}{2}.$$

Substituting these values into equation (A4), we obtain

$$1+r_0 = \frac{1}{\beta}\left(\frac{1}{(\phi+\alpha)^{1-\nu}+1}\right)^{(\nu-\delta)/(1-\nu)}. \tag{A5}$$

Therefore, given $\nu > \delta$, we have $1+r_0 < \frac{1}{\beta}$.

Moving onto $t=2$, as the industry gap closes, it follows that

$$1+r_1 = \frac{1}{\beta}\left(\frac{(\alpha+\phi)^{1-\nu}+1}{2}\right)^{(\nu-\delta)/(1-\nu)}, \tag{A6}$$

so we can see that $r_1 < (1/\beta)-1$. Low interest rate is persistent.

Part (b) Given $\lambda_1 = \lambda_{B1}$, full employment obtains from $t=1$ onwards. By logic similar to that in the proof of Proposition 2(a), we can show that $r_0 < (1/\beta)-1$. As production remains constant from $t=1$ onwards, it follows that $r_1 = (1/\beta)-1$. ■

Proof of Proposition 3.

First, we work out the partial derivative of the single-period utility as follows. For $t=1$,

$$\begin{aligned}\frac{\partial U(\boldsymbol{c}_{\mathrm{I}1}, \boldsymbol{c}_{\mathrm{J}1})}{\partial M} &= \frac{1}{1-\nu}\left(\left(\frac{1}{2}\right)^{\nu}\boldsymbol{c}_{\mathrm{I}1}^{1-\nu} + \left(\frac{1}{2}\right)^{\nu}\boldsymbol{c}_{\mathrm{J}1}^{1-\nu}\right)^{(\nu-\delta)/(1-\nu)}\left(\frac{1}{2}\right)^{\nu}(1-\nu)\boldsymbol{c}_{\mathrm{I}1}^{-\nu}\\ &\quad\times\frac{Q}{4N}\left(\sigma(n') + \overline{\sigma} + (\sigma_B - \overline{\sigma})\,\frac{M-\phi N-\alpha N}{(1-\alpha)N}\right) > 0.\end{aligned}$$

At date $t=2$, given equation (8),

$$\frac{\partial U(\boldsymbol{c}_{\mathrm{I}2}, \boldsymbol{c}_{\mathrm{J}2})}{\partial M} = \frac{1}{1-\nu}\left(\left(\frac{1}{2}\right)^{\nu} \boldsymbol{c}_{\mathrm{I}2}^{1-\nu} + \left(\frac{1}{2}\right)^{\nu} \boldsymbol{c}_{\mathrm{J}2}^{1-\nu}\right)^{(\nu-\delta)/(1-\nu)} \left(\frac{1}{2}\right)^{\nu} (1-\nu)\boldsymbol{c}_{\mathrm{I}2}^{-\nu}$$
$$\times\left(\frac{Q}{4N}\left(\sigma(n') + \overline{\sigma} + (\sigma_B - \overline{\sigma})\,\frac{\tilde{M} - \phi N - \alpha N}{(1-\alpha)N}\right) - \frac{Q}{2N}\right) < 0.$$

It is straightforward to derive that

$$-\frac{\partial q_{\mathrm{I}1}/\partial M}{\beta \partial q_{\mathrm{I}2}/\partial M} = \underbrace{\frac{1}{\beta}\left(\frac{2}{\sigma(n') + \overline{\sigma} + (\sigma_B - \overline{\sigma})(M - \phi N - \alpha N)/(1-\alpha)N} - 1\right)^{-1}}_{SP}.$$

■

Proof of Proposition 4.

Let

$$X = \frac{1}{2}\left(\sigma(n') + \overline{\sigma} + (\sigma_B - \overline{\sigma})\,\frac{M - \phi N - \alpha N}{(1-\alpha)N}\right).$$

Given equation (8), we can further simplify X as

$$X = \overline{\sigma} + \left(\frac{M}{(1-\alpha)N} - \frac{\alpha + \phi}{1-\alpha}\right)(\sigma_B - \overline{\sigma}).$$

For $M \in ((\phi + \alpha)N, N)$, we have

$$\frac{M}{(1-\alpha)N} - \frac{\alpha + \phi}{1-\alpha} > 0.$$

Given $\overline{\sigma}$, the lower σ_B is, the lower X is, and the smaller SP is.

Note that

$$\overline{\sigma} = 1 - \frac{1}{N\varepsilon}, \quad \sigma_B = \frac{2N - 2/\varepsilon}{(1-\gamma)Q},$$

so X can be simplified further as

$$X = 1 - \frac{1}{N\varepsilon} - \left(\frac{M}{(1-\alpha)N} - \frac{\alpha+\phi}{1-\alpha}\right)\left(1 - \frac{1}{N\varepsilon}\right)\left(1 - \frac{2N}{(1-\gamma)Q}\right).$$

Thus $\partial X/\partial\varepsilon > 0$, $\partial X/\partial\gamma > 0$ and $\partial X/\partial N > 0$.

Since

$$SP = \frac{1}{\beta((1/X) - 1)},$$

SP increases with X and decreases with β; that is, an increase in ε, γ, σ_B or N leads to an increase in SP, whereas an increase in β decreases SP. ■

Proof of Proposition 5.

As shown in the proof of Proposition III, the Euler equation gives

$$1 + r_t = \frac{1}{\beta}\,\frac{U_{c_{\mathrm{J}}}(\boldsymbol{c}_{\mathrm{I}t}, \boldsymbol{c}_{\mathrm{J}t})}{U_{c_{\mathrm{J}}}(\boldsymbol{c}_{\mathrm{I}t+1}, \boldsymbol{c}_{\mathrm{J}t+1})}.$$

Suppose that $\bar{t}$ is when sector J reaches full employment, so

$$1 + r_{\bar{t}-1} = \frac{1}{\beta} \frac{U_{c_J}(\boldsymbol{c}_{I\bar{t}-1}, \boldsymbol{c}_{J\bar{t}-1})}{U_{c_J}(\boldsymbol{c}_{I\bar{t}}, \boldsymbol{c}_{J\bar{t}})}.$$

Since at the ELB the interest rate is bound by $1/\beta - 1$, it follows that

$$1 = \frac{U_{c_J}(\boldsymbol{c}_{I\bar{t}-1}, \boldsymbol{c}_{J\bar{t}-1})}{U_{c_J}(\boldsymbol{c}_{I\bar{t}}, \boldsymbol{c}_{J\bar{t}})}. \tag{A7}$$

And because $\boldsymbol{c}_{I\bar{t}-1} < \boldsymbol{c}_{I\bar{t}}$, $\partial U_{c_J}(\boldsymbol{c}_{It}, \boldsymbol{c}_{Jt})/\partial c_{It} > 0$ and $\partial U_{c_J}(\boldsymbol{c}_{Jt}, \boldsymbol{c}_{Jt})/\partial c_{Jt} < 0$, for equation (A7) to hold, $\boldsymbol{c}_{J\bar{t}-1} < \boldsymbol{c}_{J\bar{t}}$ has to hold. Before $\bar{t}$, a decrease in the consumption of goods I corresponds to a decrease in consumption of goods J. Accordingly, solving backwards, it follows that $\boldsymbol{c}_{J0} < \boldsymbol{c}_{J1} < \cdots < \boldsymbol{c}_{J\bar{t}-1}$. Moreover, $\bar{c}_{J\bar{t}} = Q/2$, which corresponds to full employment in sector J, so for periods before $\bar{t}$, sector J suffers involuntary unemployment. ∎

Proof of Proposition 6.

Let $\eta(e)$ be a bank's monitoring technology, where e is the bank's monitoring effort. We assume that $\eta(e)$ is monotonically increasing in the monitoring effort e, and $\eta(0) = \gamma$ and $\eta(+\infty) = 1$. The government's credit guarantee is g. If $g = 100\%$, then the government fully makes up the loss of the defaulted amount to the banks; if $g = 0\%$, then the banks fully take on the loss of the defaulted amount.

When entrepreneurs announce default, banks monitor and pursue the defaulters' remaining funds. Banks' total proceeds from monitoring and pursuing the defaulters include the fraction $\eta(e)$ of remaining funds that banks are able to take away, plus the money received from the government credit guarantee on the defaulted amount, so banks earn $\eta(e) + g(1 - \eta(e))$ per unit of funds. Meanwhile, banks exert effort cost $\iota e^2/2$ per unit of funds. Thus banks maximize the objective function

$$\eta(e) + g\,(1 - \eta(e)) - \frac{\iota}{2}\, e^2.$$

Banks choose the monitoring effort e, and the optimality condition is

$$(1 - g)\, \eta'(e) = \iota e.$$

Total differentiating this optimality condition, it follows that

$$\frac{\partial g}{\partial e} = -\iota\, \frac{\eta'(e) - e\, \eta''(e)}{(\eta'(e))^2}.$$

Because $\eta'(e) > 0$ and $\eta''(e) < 0$, we obtain $\partial e/\partial g < 0$, and using the chain rule, $\partial \eta/\partial g < 0$. Thus the banks' monitoring technology is a decreasing function of the government's credit guarantee. Now we can write η as $\eta(g)$.

Next, we show that the default sanction λ_1 in our benchmark model is a decreasing function of the government credit guarantee.

Note that the marginal entrepreneur's participation constraint in this model extension is simply

$$\underbrace{(1 - \eta(g))\, \sigma(n)\, h_1}_{\text{default}} \geq \underbrace{\mathbb{E}_1(W_{I1} | \mathbb{I}_d = 1,\ \lambda_1)}_{\text{outside option}},$$

and recall the participation constraint in the benchmark case,

$$\underbrace{(1-\gamma)\,\sigma(n)\,h_1-\lambda_1}_{\text{default}} \geq \underbrace{\mathbb{E}_1(W_{\text{I}1}|\mathbb{I}_d=1,\ \lambda_1)}_{\text{outside option}}.$$

Equating the left-hand sides of the above participation constraints, we get

$$\lambda_1=(\eta(g)-\gamma)\,\sigma(n)\,h_1.$$

Using the chain rule, it follows that $\partial\lambda_1/\partial g<0$. And moreover, when $g=1$, $e=0$ and $\eta=\gamma$, and $\lambda_1=0$.

Other comparative statics ■