Elzaki decomposition method for approximate solution of a one-dimensional heat model with axial symmetry

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***Abstract*—This paper considers the application of the Elzaki Decomposition Method (EDM) for approximate solution of a one-dimensional heat model with axial symmetry. By the proposed EADM, the series solutions of the sampled cases are obtained with ease and high level of accuracy as regards less computational time. These results, therefore, show the effectiveness of the proposed method.**

Keywords—Differential models; Approximate solution; Adomian Decomposition; Elzaki transform

I. INTRODUCTION

Differential models and their applications have been noted as building blocks in sciences and engineering. Though obtaining their exact or analytical solutions appears complicated and tedious in most cases [1-7]. In this work, a source-less heat model describing one-dimensional unsteady thermal processes with axial symmetry will be considered. This is mostly represented in the form of:

$$\begin{cases} \dfrac{\partial \Psi}{\partial t} = \dfrac{\beta}{\gamma}\dfrac{\partial}{\partial \gamma}\left(\gamma \dfrac{\partial \Psi}{\partial \gamma}\right) \\ \Psi(\gamma,0) = h(\gamma) \end{cases} \quad (1.1)$$

where $\beta > 0,$ controls the speed and spatial scale of the process; t, and γ are time and spatial parameters respectively. The temperature (of the body) at point γ and time t is denoted by $\Psi(t,\gamma)$. Numerical methods are being sought for approximate solutions of similar models [8-20].

Here, Laplace Decomposition Method (EADM) is employed for approximate solution of a one-dimensional heat model with axial symmetry.

II. ELZAKI ADOMIAN DECOMPOSITION METHOD [3]

Definition 2.1: Integral Transform:

Let $f(t),\ a<t<b$, be a given function, then the general integral transform of $f(t)$ is defined as:

$$I\left[f(t)\right] = \int_a^b f(t)\eta(s,t)\,dt \quad (2.1)$$

where $\eta(s,t)$ signifies the transformation kernel, depending on the differential types of kernels. This kernel tells the specific nature of the corresponding integral transform, such as Laplace transform, Elzaki transform, Fourier transform, Sumudu transform, and so on. In this paper, Elzaki transform will be considered as follows:

Definition 2.2: Elzaki Transform: Let A be a class of function such that

$$A = \left\{u(t): \left|u(t)\right| < M\exp\left(\left|t\right|k_j\right),\ \text{for}\ M, k_{1,}k_2 > 0\right\} \quad (2.2)$$

then, the Elzaki transform of $u(t)$ associated to A is presented as:

$$\begin{aligned} E\left[u(t)\right] &= T(v) \\ &= v\int_0^\infty u(t)\exp\left(-\frac{t}{v}\right)dt. \end{aligned} \quad (2.3)$$

The basic properties of the Elzaki Transform are presented as follows:

$$\begin{cases} E[1] = v^2,\ E[t] = v^3 \\ E\left[u^{(m)}(t)\right] = \dfrac{1}{v^m}T(x,v) - \displaystyle\sum_{k=0}^{m-1} v^{2-m-k}u^{(k)}(0),\ m \geq 1 \\ E\left[t^n\right] = n!v^{n+2} \Rightarrow \dfrac{1}{n!}E\left[t^n\right] = v^{n+2}. \end{cases} \quad (2.4)$$

978-1-7281-7177-7/20/$31.00 ©2020 IEEE
DOI 10.1109/MACISE49704.2020.00061

A. Elzaki transform and Adomian Decomposition

The Elzaki transform and Adomian Decomposition (EADM) is a combination of both the Elzaki approach and decomposition method (the Adomian decomposition method) to obtain solutions of differential and algebraic models. Consider the general first order non-linear partial differential equation of the form:

$$\begin{cases} Dh(x,t) + Rh(x,t) + Nh(x,t) = g(x,t) \\ g(x,0) = g_1^*,\; h = h(x,t) \end{cases} \quad (2.5)$$

where, D and R are linear operator (differential), are the remaining part of the differential operator, N is the non-linear part of the differential operator, and g is the non-homogenous part of the differential operator. Therefore, the Elzaki transform of (2.5) is taken as follows:

$$\begin{cases} E[Dh] + E[Rh] + E[Nh] = E[g] \\ \Rightarrow E[Dh] = E[g] - E[Rh] - E[Nh] \end{cases}$$

$$\begin{cases} \therefore \quad \frac{1}{v}T(x,v) - vh = E[g] - E[Rh+Nh] \\ \;\Rightarrow\; T(x,v) = v^2 h + vE[g] - vE[Rh+Nh]. \end{cases} \quad (2.6)$$

Thus,

$$T(x,v) = G(x,t) - vE[Rh+Nh], \quad (2.7)$$

where $G(x,t)$ results from the initial condition and source term when used.

Inverse Elzaki transformation of (2.7) gives:

$$E^{-1}T(x,v) = E^{-1}[G(x,t)] - E^{-1}\{vE[Rh+Nh]\}$$

$$\Rightarrow h = E^{-1}[G(x,t)] - E^{-1}\{vE[Rh+Nh]\}. \quad (2.8)$$

By ADM and A_m as Adomian polynomial, the series solution and the nonlinear term are defined as

$$\left\{ h = \sum_{n=0}^{\infty} h_n, \quad Nh = \sum_{n=0}^{\infty} A_m. \right. \quad (2.9)$$

Hence, (2.8) becomes

$$\sum_{m=0}^{\infty} h_n = g_1^* - E^{-1}\left[v\left(R\sum_{m=0}^{\infty} h_n + \sum_{m=0}^{\infty} A_n \right)\right]. \quad (2.10)$$

By comparing the terms in (2.11) we have:

$$\begin{cases} h_o = g_1^* \\ h_{n+1} = -E^{-1}\{vE[Rh_n + A_n]\}. \end{cases} \quad (2.11)$$

III. Numerical Applications

Suppose (1.1) is defined with known initial conditions for cases I and II as follows:

Case I: Consider the IVP of the form:

$$\begin{cases} \frac{\partial \Psi}{\partial t} = \frac{\beta}{\gamma}\frac{\partial}{\partial \gamma}\left(\gamma \frac{\partial \Psi}{\partial \gamma}\right) \\ \Psi(\gamma,0) = 2 + \gamma^2. \end{cases} \quad (3.1)$$

Case II: Consider the IVP of the form:

$$\begin{cases} \frac{\partial \Psi}{\partial t} = \frac{\beta}{\gamma}\frac{\partial}{\partial \gamma}\left(\gamma \frac{\partial \Psi}{\partial \gamma}\right) \\ \Psi(\gamma,0) = 1 + 2\gamma^2. \end{cases} \quad (3.2)$$

Then, by the proposed procedure in section 2, we have the following solutions for cases I and II respectively:

$$\Psi(\gamma,t) = 2 + (\gamma^2 + 4\beta t), \quad (3.3)$$

$$\Psi(\gamma,t) = 1 + 2(\gamma^2 + 4\beta t). \quad (3.4)$$

IV. Conclusion

The application of the Elzaki Decomposition Method (EADM) for approximate solution of the one-dimensional heat model with axial symmetry has been successfully considered in the present work. The solutions were obtained easily by the proposed method, even with less computational time. Thus, it is remarked for effectiveness and therefore recommended for higher order models.

Acknowledgment

The support from Covenant University is highly acknowledged.

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