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"I did it wrong, but i know it": Young children's metacognitive knowledge expressions during peer interactions in math activities

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ABSTRACT

Previous studies reveal that children's metacognitive skills make massive progress during the early childhood period. We believe that examining how children use metacognitive skills in the learning process is crucial for shaping future learning experiences. This case study explores how children's metacognitive knowledge emerges through peer interactions in mathematical measurement activities. Sixteen activities based on the dimensions of mathematical measurement skills of length, area, weight, and volume were applied and video recorded. We systematically observed two 5-year-old children in these activities for 10 weeks. A framework of analysis was developed from the results of previous research on children's metacognition. Children's metacognitive knowledge was analyzed in mathematical statements and other variables were also extracted. Using qualitative analysis, this study indicates how children's mathematical thinking skills are reflected in their expressions of metacognitive knowledge during peer interactions. Difficulties in assessing and measuring children's metacognition are also discussed.

1. Introduction

As a recent approach in mathematics education, instead of directly teaching the basic information, a new focus on cognitive skills such as strategy formation, planning, controlling, and evaluating has led researchers to the concept of metacognition, which constitutes a more complex level of cognitive development. Although studies confirm that metacognitive skills develop at a very basic level in early childhood (Flavell, 2000; Veenman, Van Hout-Wolters & Afflerbach, 2006), encouraging children to think, reflect on their thoughts, and evaluate learning processes significantly supports metacognitive development (Flavell, 2000; Kuhn & Dean, 2004; Whitebread et al., 2009). Research indicates that with the advancement of age, children's levels of awareness of cognitive capacities, information processing strategies, and variables that affect their performance increase (Schneider & Pressley, 1989; Veenman & Spaans, 2005). As these types of skills are evolving, major mathematical skills such as problem identification or prediction provide the basis for skills that require higher levels of effort, such as planning, self-evaluation, or self-regulation (Carr, Alexander & Folds-Bennett, 1994). Therefore, researchers have identified the use of metacognitive skills as a prerequisite for successful learning.

In recent years, various studies have been carried out on the mathematical thinking skills and metacognition of children (e.g., Baten, Praet & Desoete, 2017; Nelson & Fyfe, 2019; Schneider & Artelt, 2010). In these studies, children were observed during activities that they participated in with their peers and important results on children's metacognitive development were presented, but how children's metacognitive skills were shaped during social interaction was not clearly explained. Examining how children's

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metacognitive skills are shaped by peer interaction in the mathematical learning process is important in terms of creating effective interventions for subsequent learning. This paper presents the findings of a qualitative study of the metacognitive knowledge revealed by two children during peer interactions in math activities.

2. Theoretical framework

2.1. Metacognition and metacognitive knowledge

Metacognition can be defined as individuals becoming aware of their cognitive processes, products, and capabilities and monitoring those processes effectively and arranging them accordingly (Flavell, 1979). Although various conceptualizations of metacognition have been suggested over time, there is a consensus that metacognition generally consists of two dimensions: knowledge and regulation (Brown, 1987; Schraw, 1998). Metacognitive knowledge is the knowledge that a person acquires about his/her strengths and weaknesses as a result of internal beliefs and external interactions that may affect his/her cognitive processes (Flavell, 1979). This is an individual's general knowledge of cognitive processes and a personal perspective on the cognitive abilities of oneself and others. The basis of the distinction between cognitive and metacognitive knowledge is how knowledge is used rather than a fundamental difference. Metacognitive activity precedes or follows cognitive activity. Thus, they are closely related and interdependent (Wells & Purdon, 1999).

Metacognitive knowledge is generally addressed in terms of persons, tasks, and strategies. Knowledge of persons refers to general knowledge about how human beings learn and process information, as well as individual knowledge of one's own learning processes. This information may occur in the form of comparing one's own abilities (*intra-individual*) with the abilities of other individuals (*inter-individual*) or it may include generalizations about learning and learners (*universal*). The individual variable results from differences in cognitive abilities. Individuals may have knowledge of whether they will succeed or fail in a particular cognitive process. This is related to an individual's knowledge of their weaknesses and strengths (Flavell, 1979). In addition to general self-knowledge, metacognitive knowledge also includes beliefs about the individual's own motivational processes. Although not included in most cognitive models, studies have shown that there is a connection among an individual's motivational beliefs, cognition, and learning (Pintrich & Schrauben, 1992; Pintrich & Schunk, 2002).

Knowledge of tasks is knowledge about the content, difficulty, or requirements of a task to be done or currently being done. In terms of mathematical thinking skills, an individual with advanced knowledge of tasks will have awareness of his/her beliefs about mathematics and opinions about mathematical tasks; thus, he/she will be able to determine task strategies with less effort (Garofalo & Lester, 1985). An individual's beliefs about the value of the task, emotional reactions to the task, and attributions based on previous success or failure in similar tasks may affect metacognitive performance (Efklides & Vlachopoulos, 2012; Pintrich, 2002).

Strategy knowledge encompasses a great deal of information about identifying the purpose and goals of a task and determining which strategy is most efficient (Flavell, 1979). An individual with advanced knowledge of strategies will be able to determine why a strategy is more appropriate in the current task process (Whitebread et al., 2009). Strategy knowledge also describes an individual's knowledge of specific strategies learned through experience or education and how each strategy can be applied to different tasks (Flavell, 1979). These strategies encourage learners to think critically about their own learning attitudes and approaches to learning tasks, rather than relying solely on teacher-led instruction or explanation (Carr, 2010).

Metacognitive regulation includes the abilities to plan, monitor, and evaluate cognitive activities, followed by subsequent control that includes regulating cognitive processes, such as adjusting task goals, distributing study time, and selecting cognitive strategies (Fernandez-Duque, Baird & Posner, 2000; (Flavell, 1992); Livingston, 2003). It also helps to monitor one's level of knowledge, which assists in discovering any deficiencies and employing different strategies to overcome them (Driessen, 2014). The distinction between metacognitive knowledge and regulation can be expressed as follows: metacognitive knowledge refers to the awareness of one's performance in certain cognitive tasks, while metacognitive regulation is a more general cognitive phenomenon that involves the monitoring and control of other cognitive processes, which may or may not include explicit higher-level cognitive states of the subject (Fernandez-Duque et al., 2000).

2.2. Metacognitive knowledge and peer interaction

Peer interaction involves a process of social construction that especially occurs when groups of individuals engage in activities with common goals or aims (Moshman, 2008; Schraw & Moshman, 1995). Kuhn and Dean (2004) asserted that social interactions can help children interiorize processes of providing elaborations and explanations, which have been associated with improved learning outcomes. From a sociocultural perspective, peer interactions can provide experiences within the learner's zone of proximal development. This perspective shows us that mental capacity initially emerges at the interpersonal level and develops over time through the internalization and transformation of mental processes represented by tools and signs (Vygotsky, 1978; Vygotsky & Luria, 1994). Vygotsky highlighted the importance of language as a cognitive tool, used in early social interactions within the development of higher cognitive functions. Kuvalja, Basilio, Verma and Whitebread (2013) also discussed the importance of self-directed language, inner speech, and private gestures along with these cognitive tools in the development of early metacognition and concluded that researchers should address the use of both verbal and nonverbal semiotic systems and their interplay in the development of metacognition.

Schraw and Moshman (1995) argued that peer interactions and collaborative works could be more effective in developing metacognitive knowledge, skills, and awareness compared to direct instruction. According to them, the individual constructions of cognition and peer interaction are not exclusive paths in the development of metacognition; they are interrelated. These authors also emphasized a need for future research to explore the interactive role of these factors in the emergence of metacognitive theories.

2.3. Metacognition in mathematics

Metacognition, and especially metacognitive knowledge, grows through the development of a strong conceptual knowledge base and domain-specific strategies (Carr & Biddlecomb, 1998; Schneider, 2008). In other words, it is closely related to the learner's domain knowledge (Pressley, Borkwski & Schneider, 1989). Vo, Li, Kornell, Pouget and Cantlon (2014) provided evidence that metacognitive abilities develop in tandem with domain-specific changes in young children's knowledge. They also found that children's metacognitive ability in only the numerical domain predicted their school-based mathematics knowledge. Many studies have also noted that mathematics is an important domain in the development of metacognition ((Cornoldi and Lucangeli, 1997) Schneider & Artelt, 2010; (Van der Stel et al., 2010).

Studies have pointed out a significant relationship between metacognition and mathematical problem-solving skills (e.g., Pennequin, Soler, Nanty & Fontaine, 2010; Pugalee, 2001). During problem-solving stages, the ability to monitor and regulate oneself, or the ability to maintain executive control, clearly influences problem-solving skills (Lester, Lester & Garofalo, 1982). Carr et al. (1994) emphasized that even young children can understand when and how to use different strategies in mathematical problem-solving processes. Metacognitive knowledge enables children to organize their learning processes in mathematics activities more competently by establishing their conceptual knowledge acquisition. It helps learners identify a problem that needs to be solved and know how to achieve a goal (Kuzle, 2013). By participating in appropriate educational activities, children begin to realize that there are new situations that are worthwhile and waiting to be discovered (Kuhn & Dean, 2004).

2.3.1. Mathematical measurement activities for extensive observations

It is noteworthy that applied research in preschool mathematics education mainly focuses on teaching concepts, numbers and operations, and geometry or pattern-making skills. We can say that the measurement skills expected to develop in the early childhood period do not get considerable attention. Measurement, a comprehensive topic, is pertinent in many dimensions of mathematics. Children are mostly engaged with the locations, distances, or shapes of objects in measurement activities, thereby improving their geometry and spatial skills; at the same time, number and operation skills are reinforced while expressing measured objects in units (Clements & Sarama, 2009; Szilagyi, Clements & Sarama, 2013). Measurement activities include various aspects of mathematics and provide an opportunity for children to merge different mathematical skills (NCTM, 2000). In addition to that, these activities can easily be integrated into practical investigations because children can explore measurement concepts by experimenting with objects and discussing their findings (Murphy, 2004). Therefore, we believe that measurement activities are effective in understanding how children's mathematical thinking skills are reflected in their metacognitive knowledge expressions.

3. Aim

Previous research has emphasized the value of evidence regarding young children, because metacognition continues to be a complex construct that challenges our ability to understand how it can potentially emerge in peer interactions (Fox & Riconscente, 2008; Kuvalja et al., 2013; Schraw & Moshman, 1995). We believe it is valuable to present evidence for young children's metacognitive knowledge during peer interactions. In this context, the current study relies on the notion that metacognition develops individually but is formed through social interactions (Salonen, Vauras & Efklides, 2005; Schraw & Moshman, 1995). Considering measurement as an umbrella structure within mathematical skills, we assume that measurement activities are functional for revealing children's metacognitive knowledge. Thus, we are guided by two broad questions: First, what kind of metacognitive knowledge expressions do children reveal during peer interactions in measurement activities? Second, what are the differences in metacognitive knowledge expressions among children?

4. Materials and methods

4.1. Research design

The research was conducted within the framework of a case study design. Case studies investigate the circumstances related to one or numerous cases with a holistic approach to illustrate how these factors are affected by the pertinent case or how they influence the relevant case, or to disclose the adjustments that occur in a case (Yin, 2003). The case in this study was determined as the metacognitive knowledge of young children and the factors concerning the relevant case were established as peer interactions in mathematical measurement activities.

4.2. Participants

In this study, we selected a class from a kindergarten in Istanbul via easily accessible case sampling. The classroom consisted of 18 children, with nine boys and nine girls, from the age group of 60–68 months. First, we obtained detailed information about each child's mathematical skills by meeting with the classroom teacher. In addition, classroom forms containing the teacher's past observations of the children's development were examined. Based on this information, we identified two children as participants: a child who was observed to have various difficulties in math activities and a child who was observed to have high skill levels in math activities. The

Table 1
Participants' biographies.

	Educational background, mathematical competence, and family background
Dennis	Dennis, 64 months old, has been attending kindergarten in the same teacher's class for two years. His-parents are university graduates. He often states that he loves mathematics and that his mother is a mathematics teacher. His-peers often ask him for help and want to work with him in group activities. He often uses mathematical concepts in activities and can easily overcome mathematical challenges.
Michael	Michael, 62 months old, has been attending kindergarten in the same teacher's class for two years. His-mother is a high school graduate and his father is a graduate of higher education. He may experience various difficulties in math activities such as counting, expressing numbers with symbols, and operations. However, he is willing to participate in math activities and follows the rules of activities. He is a child who stands out with his positive social relationships.

aim here is to consider the possibility that children's mathematical competencies may affect their social interactions in terms of metacognitive aspects (e.g., teaching someone to do something or asking for help). Both of the selected children are boys and we have not considered differences by gender. Pseudonyms are used for the participating children (Table 1).

4.3. Procedures

4.3.1. Preparation stage

Since we aimed to implement activities that would allow peer interactions, we decided to develop outcomes and indicators that would form the basis of the activities. In the preparation stage, the cognitive development domain outcomes and indicators in the Preschool Education Program published by the Turkish Ministry of National Education (2013) were examined, and the outcomes and indicators based on children's measurement skills were selected to prepare the research activities. From the cognitive development domain in the program, the study included four outcomes and indicators without any alterations; two outcomes and their indicators were adjusted. Moreover, an outcome and its four indicators covering all measurement dimensions were added. One outcome per each dimension of measurement skills (length, area, weight, and volume) and 29 indicators were included. These outcomes were shared with a preschool education specialist and a mathematics education specialist. Taking their expert opinions into account, 13 outcomes and 60 indicators were organized and finalized. After, four activities were prepared for each measurement dimension. The activities allow peer interactions and they are ordered from easy to difficult. For all activities, two academics specializing in mathematics education in preschool and one academic with expertise in early childhood education were consulted. Essential adjustments were made and the activities were finalized Table 2.

Before the implementation, to minimize possible instances of shyness or reluctance to participate in the activities, the researcher who would carry out the activities introduced herself and announced that they would be performing various mathematical activities together. The content of the study was explained to the parents of all children in the classroom, it was stated that video recordings would be made, and a signed consent form was obtained from all parents. The researcher participated in routine classroom activities for two half-days and joined the children's games. She then explained to the children that she would do activities with them in their classrooms for a while and that she would record the activities on video. Before each activity, the children were told that they would perform a math activity, they were reminded that they would be video-recorded, and they were asked whether they would like to participate in the activity. Each activity was carried out after obtaining the verbal consent of the children. A pilot application was carried out with the children and the implementation started after three half-days.

Table 2
Information on the activities.

Order	Activity name	Measurement dimension	Aim
1.	Animal Shelters	Length	To have children select objects of appropriate length for the task
2.	Measurement Detectives	Length	To have children compare different lengths
3.	Measuring Boxes	Length	To have children measure lengths with non-standard units
4.	Measuring Rods	Length	To have children measure lengths with non-standard units and create a measuring rod with a non-standard unit
5.	How to Cover It?	Area	To have children cover a surface
6.	Making a Puzzle	Area	To have children divide a surface into pieces
7.	Shapes on Our Desk	Area	To have children cover a surface with non-standard units
8.	Which Book Do You Think?	Area	To have children cover a surface with non-standard units and compare units
9.	Bags	Weight	To have children compare different weights
10.	Measuring by Pulling	Weight	To have children compare different weights and divide weights into pieces
11.	Weights on My Arms	Weight	To have children compare the weights of objects of different sizes
12.	Market Place	Weight	To have children measure weights with non-standard units
13.	Let's Fill the Boxes	Volume	To have children fill an empty object with appropriate non-standard units
14.	Which One Should I Fill?	Volume/Capacity	To have children choose an appropriate object to fill with a specific non-standard unit
15.	Cylinders	Volume	To have children compare the volumes of objects
16.	Our New Year's Party	Volume/Capacity	To have children compare the capacities of objects and measure capacities with non-standard units

Table 3
Example of Michael's interactions in the "Measuring rods" activity.

Activity Type: Length measurement Activity Name: Measuring Rods Context	Who	Speech	Actions
The children create measuring rods with cardboard sticks. Each child makes unit lines on the measuring rod with objects such as paper clips, sticks, straws, or buttons chosen from the mathematics center and then places the required numbers on the unit lines (Fig. 1).	Another child Michael	"This is not how five is, you did it wrong." "I already know five. I did it wrong, but I know it."	Points to the number five on Michael's ruler.

4.3.2. Implementation stage and data collection

Sixteen activities prepared based on measurement skills were implemented two days a week. The researcher administered one activity a day. All activities were completed within approximately 10 weeks excluding official holidays or days of special institutional events. During the implementations, the entire process was video-recorded. Three cameras, shooting from different angles, were located in different parts of the classroom. Additionally, another researcher was invited to the classroom and asked to complete a semi-structured observation form including sections about the date, the type and name of the activity, the description of the learning environment, the observation period, and the main events of the activity. The independent researcher was introduced to the children and it was explained that she would take various notes about what was done during the activities. She participated in the whole process, including the warm-up and pilot studies. The role of the independent researcher here was unattended observation.

4.4. Data analysis

To examine the metacognitive knowledge of these young children, the observation forms and 28 h of video recording transcripts of 16 activities were analyzed by the researchers. The data analysis was mapped out according to but not limited by the framework that Whitebread et al. (2009, 2010) suggested for the metacognitive knowledge that young children are capable of:

1 Knowledge of persons – Expressing his/her own strengths or difficulties and others' strengths or difficulties in learning and academic working skills; talking about general ideas about learning.
2 Knowledge of tasks – Making comparisons between tasks that reveal similarities and differences; making judgements about the level of difficulty of cognitive tasks or rating tasks on the basis of pre-established criteria or previous knowledge.
3 Knowledge of strategies – Defining, explaining, or teaching others how to do or learn something; explaining procedures involved in a particular task; and evaluating the effectiveness of one or more strategies in relation to the context or the cognitive task.

In addition to this framework, we reviewed the results of previous research conducted with young children. These studies illustrate that young children have metacognitive knowledge that includes knowledge of tasks and knowledge of strategies as well as knowledge of persons (e.g., Larkin, 2006; Marulis, Palincsar, Berhenke & Whitebread, 2016). As mentioned earlier, knowledge of tasks refers to what learners need to know about tasks in order to accomplish them successfully and cognitive strategies are steps used by learners to learn new information and apply it to specific learning tasks (Vaidya, 1999; Wenden, 1991). In the present research, we prepared activities that included certain mathematical tasks/goals so that the children's cognitive strategies could be considered in a task-specific way. Therefore, we decided to investigate both of these variables together. Based on these frameworks, we devised a coding system that included: (a) knowledge of self, (b) knowledge of others, (c) knowledge of universal properties of cognition, and (d) knowledge of task-specific strategies. These four metacognitive knowledge variables were used as category codes to code the children's interactions.

Fig. 1. Measuring rod of Michael.

Table 4
Example of Michael's interactions in the "Making a puzzle" activity.

Activity type: Area measurement Activity name: Making a puzzle Context	Who	Speech	Actions
The children draw a picture of their choice on the backs of papers with various geometric shapes on the front side. Then they cut their pictures into pieces by following the lines of the shapes on the front. Next, they are asked to combine these pieces to complete their pictures again.	Another child	"You got it all mixed up!"	
	Michael	"Oh! I forgot to paste it, silly me."	Tries to fix the pieces by sticking them.

Table 5
Example of Dennis's interactions in the "Let's fill the boxes" activity.

Activity type: Volume measurement Activity name: Let's fill the boxes Context	Who	Speech	Actions
The measuring boxes that children used in previous activities are filled with wooden cubes this time. Dennis spontaneously starts explaining the task to his friends.	Dennis	"Now we'll measure the inside of it, not the surface."	Points to the inside of the box.
		"It's all mathematics, right?"	
		"I can understand when it's mathematics."	Confidently raises his arms up.

Table 6
Example of the children's interactions in the "Shapes on our desk" activity.

Activity type: Area measurement Activity name: Shapes on our desk Context	Who	Speech	Actions
Children are divided into groups; they place models of various geometric shapes on a large square base on their desks (Fig. 2).	Dennis	"She [another child] liked the green desk; look, she covered it quickly."	Points to the green desk. Then leaves the triangular piece and takes a square piece.
	Michael	"You figured it out, good for you."	Points to the square piece.

4.5. Validity and reliability

It is recommended that data be encoded by one or more different field experts to minimize possible coding errors and compare datasets by cross-checking (Gibbs, 2018). The data of a randomly selected activity were analyzed by an independent researcher. The level of agreement was %84.2. Then, all coded data were examined, the results were discussed together, and a consensus was reached. Afterwards, the data of the remaining 15 activities were reviewed. In order to reach data saturation, 16 activities were carried out. Classroom observations and video recordings were used to ensure diversity in terms of data sources in the study. Employing such variation (Creswell, 2009) is intended to minimize the data loss that may occur during activity processes and to ensure validity.

Fig. 2. "Shapes on our desk" activity.

Table 7
Example of Dennis's interactions in the "Cylinders" activity.

Activity type: Volume measurement Activity name: Cylinders Context	Who	Speech	Actions
Two cardboard cylinders of equal volume are presented to the children. The yellow one has a thin and long structure, while the pink one has a fat and short structure.	Teacher	"Let's guess. Which cylinder do you think will hold more beans?"	
	Children	"Pink!"	*In unison*
	Dennis	"No, they're wrong. It's only fat."	

Table 8
Example of Michael's interactions in the "Measuring by pulling" activity.

Activity type: Weight measurement Activity name: Measuring by pulling Context	Who	Speech	Actions
The children compare weights by tying strings to objects in the classroom and pulling them.	Michael		Observes how his friends tie the string.
		"Teacher, you didn't think about it, but I found it, right?"	Finds an easier way to tie the string.
			Eagerly shows this to the teacher.

5. Results and discussion

5.1. Metacognitive knowledge expressions

In this section, the metacognitive knowledge expressions revealed by the children are presented and discussed within the categories determined for the analysis of the data. Table 2

5.1.1. Knowledge of self

Examples of the children's expressions in this category are presented in Tables 3–5.

While creating his measuring rod, Michael symmetrically inverted the "5" symbol, and upon his friend's notice, he stated that he was aware of both the misdrawing and the correct form. Michael was able to detect his mistake and stated that he knew how to write the symbol for the number. Being aware of one's own inadequacy is important metacognitive knowledge. In order to be able to engage in more advanced cognitive activities such as self-monitoring and self-control, one must first have knowledge of one's own competencies and inadequacies (Efklides & Vlachopoulos, 2012; Livingston, 1996).

During this activity, no instruction was given on how the children would assemble the pieces they cut. However, Michael stated that he forgot to paste his pieces after his friend's warning. In this case, it seems that Michael had developed his own cognitive strategy but forgot about it after a while. As Flavell, Beach and Chinsky (1966) pointed out, preschool children have difficulty remembering some details even from a recent event, and their cognitive procedures and strategies are not yet sufficiently developed compared to those of adults. Although children have moderately deficient skills in memory processes such as recalling, they can articulate their weaknesses in learning processes, which is considered metacognitive knowledge of self (Whitebread et al., 2009). As also revealed in the previous example, Michael seemed to have metacognitive knowledge, which included identifying his own weaknesses in academic working skills.

Dennis expressed differences by comparing the volume measurement activity with previous area measurement activities. He revealed metacognitive knowledge by stating that he understood mathematics based on the comparison. In addition, confidently raising his arms can be considered a motivational expression. Whitebread et al. (2009) stated that young children may use nonverbal

Table 9
Example of the children's interactions in the "Bags" activity.

Activity type: Weight measurement Activity name: Bags Context	Who	Speech	Actions
The children give various objects to a blindfolded friend and ask the friend to compare their weights by holding the objects. One of the children moves around the classroom to find objects. Dennis warns his friend about finding light objects.	Teacher	"Why do you want her to find light things?"	
	Dennis	"No, teacher; if both are light she can't decide."	Makes a confused facial expression.
Afterwards, Michael, as the blindfolded child, is given a stack of papers in one hand and napkins in the other. Michael quickly says that the napkins are lighter.	Michael	"It isn't difficult to understand; it is soft and light."	

Table 10
Another example of Dennis's interactions in the "Cylinders" activity.

Activity type: Volume measurement Activity name: Cylinders Context	Who	Speech	Actions
The children fill cylinders with beans and then pour these beans into a jug and mark the level of filled content with a pencil.	Dennis	"Anyone can fill this but can't count because there are too many beans. One wouldn't know that many numbers."	

Table 11
Example of Dennis's interactions in the "How to cover it?" activity.

Activity type: Area measurement Activity name: How to cover it? Context	Who	Task speech	Strategy speech	Actions
During the activity, children are divided into groups and asked to cover the surfaces of their tables with a large piece of linoleum. As the linoleum hangs off the sides of the table, Dennis begins to cut the linoleum.	Another child	"Don't cut it, don't, the teacher said it has its borders."	"If we cut, there will be less to cover the table."	
	Dennis	"There is one more thing, teacher, if the sides stay like this, if they hang, it doesn't fit."		Points to the linoleum hanging off the table. Continues to cut the parts hanging off the table.

Table 12
Example of Dennis's interactions in the "Shapes on our desk" activity.

Activity type: Area measurement Activity name: Shapes on our desk Context	Who	Task speech	Strategy speech	Actions
A group of children realize that they need a rectangular model while covering the square-shaped surfaces on their desks. However, they only have triangular pieces.	Another child	"We need to cover them all. We couldn't."		
	Dennis		"Yes. Here's how."	Holds two triangles.
			"What happens when triangles come together? A square. A giant square."	Connects the triangular pieces.

Table 13
Example of Michael's interactions in the "Which book do you think?" activity.

Activity type: Area measurement Activity name: Which book do you think? Context	Who	Task speech	Strategy speech	Actions
This activity requires the children to conceal the cover of a book they chose from the book's center with self-stick mini post-it papers. While a child covers her book in a way that everyone can see to set an example (Fig. 3), the post-it papers overflows the page because she has left space between the papers.	Another child	"Only the surface and perimeter."		
	Michael		"So, you need to cut and put like this."	Places his hand next to a post-it so that there is no space in between.

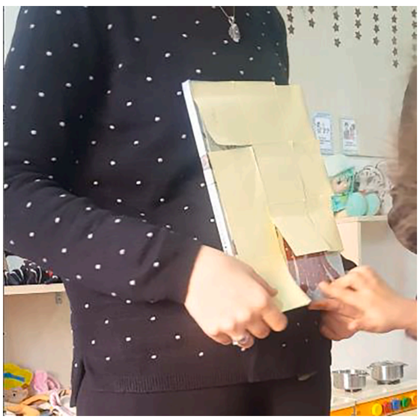

Fig. 3. Sample work of the activity.

Table 14
Example of Dennis's interactions in the "Measuring boxes" activity.

Activity type: Length measurement Activity name: Measuring boxes Context	Who	Task speech	Strategy speech	Actions
Each child tries to find an object that is two straws long. However, the children need to find the object using only one straw.	Dennis	"We need to find the length of two straws, but how?"		
	Another child		"Look, the straw is here. I put my finger. Then I turn the straw to the other side" (Fig. 4).	Places the straw end to end.
	Dennis		"I found it! It's like two of a straw. If you have two straws. If we add them end to end, that makes one."	

Fig. 4. Sample work of sliding the unit.

gestures as a strategy to support their own cognitive activities.

5.1.2. *Knowledge of others*

Examples of the children's expressions in this category are presented in Tables 6–8.

As seen in the small group activity described in Table 6, Dennis observed that a child in another group finished the task quickly and, based on that, he assessed his friend's comprehension ability. He then continued his own work and corrected a mistake that he had made. Immediately afterwards, Michael noticed this arrangement and praised Dennis. In this example, it is seen that the children had knowledge of the cognitive activities of others in the working process and reflected on it in group interactions. An example of Dennis's expressions regarding the knowledge of others is presented in Table 7.

The above example is important in terms of illustrating the relationship between metacognitive knowledge of others' cognitive processes and mathematical thinking skills. This example clearly shows that Dennis has acquired the skill of volume conservation. It is known that volume conservation skills usually develop between the ages of 10 and 12 years (Berk, 1997; Wubbena, 2013). However, some studies have noted that children in the pre-procedural period may have also developed conservation skills (Peter, 2014; Watanabe, 2017). When these expressions of Dennis are examined, we see that by taking conservation principles into account in the cognitive process he could make comparisons to identify the differences between the cylinders. In this way, he expressed his knowledge

Table 15
Example of the children's interactions in the "Measuring rods" activity.

Activity type: Length measurement Activity name: Measuring rods Context	Who	Task speech	Strategy speech	Actions
In this activity, the points to consider when measuring length by adding pieces of fabric are discussed.	Dennis	"We need to place them side by side. What's more, we shouldn't stick them everywhere like this."		Points out that the pieces should be on a line.
	Michael	"But we have to attach them. This is how it should be."		Attaches the pieces of paper end to end.

of the inadequacy of others in cognitive processes..

Previous studies showed that children could express their strengths by comparing other children's academic competencies with their own (Larkin, 2006; Whitebread & Coltman, 2010) or with the teacher's, as seen in this example. In addition, Michael very eagerly rushed toward his teacher to demonstrate that his technique worked. We can say that this comparison provided an emotional/motivational contribution to his working performance. Efklides, Samara and Petropoulou (1999) emphasized that the metacognitive knowledge base contains information about how, when, and where to use motivational strategies, including attributions regarding sources of task difficulty as well as the necessary effort and social support.

5.1.3. Knowledge of universal properties of cognition

Examples of the children's expressions in this category are presented in Tables 9 and 10.

In the examples above, the children appear to make statements of metacognitive knowledge that assess whether it is cognitively difficult or easy to understand or do something. According to the findings from the 16 activities, the children did not make any statements about general cognitive structures in activities based on length and area measurement skills; they only made such statements in activities based on weight and volume measurement skills. Previous studies revealed that young children convey the universality of cognition or share general ideas about learning (Whitebread & Coltman, 2010; Whitebread et al., 2009). This finding can be explained by the participating children's lack of sufficient experience of knowledge to share general ideas about learning.

5.1.4. Knowledge of task-specific strategies

Examples of the children's expressions in this category are presented in Tables 11–15.

Here a child stated that they should not cut the linoleum more than necessary. The child reminded Dennis that the surface needed to be completely covered, as a requirement of the task. Immediately afterwards, Dennis emphasized that the units should not overflow from the surface and he continued to cut the parts hanging off the table. Dennis knew that there were multiple task criteria to be considered. He chose the appropriate strategy and explained why. This is a clear example of his knowledge of a strategy appropriate for the task.

In this activity, the children identified a problem based on their current task knowledge. As a solution to the problem, Dennis suggested creating a rectangle by combining two triangles. Dennis knew the geometry content that a rectangle will be formed when two equilateral triangles are merged. A part-whole relationship is the basis for area conservation and preschool children succeed to a great extent in understanding part-whole relationships (Kospentaris, Spyrou & Lappas, 2011). Dennis thus showed knowledge of strategies that included instructing others how to do something by offering this solution to the problem. Children develop unique strategies while creating solutions to problem situations or overcoming tasks that make sense to them (Larkin, 2006). According to these findings, Dennis benefited from cognitive strategies to find clarifications to problem situations that he encountered and to reach the specified goals.

In this example, Michael is seen to propose a strategy that can be implemented based on the task criteria specified by another child. It is observed that children can develop metacognitive strategies based on the expressions of others through interactive activities. It is an important finding that children who may need help in the performance process, especially those like Michael, can reveal metacognitive knowledge through peer interactions. A similar example is given below.

Here, Dennis tried to define a strategy for the task criterion. He then observed the strategy that his friend used and explained to the others why it worked. Children experience various difficulties in measuring length by sliding or copying a unit (Godfrey & O'Connor, 1995; Kamii & Clark, 1997). Even though the children had intuitively spotted the correct object, they began to develop several strategies when asked to prove that it was two straws long. In this case, Dennis's experience was scaffolded and supported by his social interactions with others. Based on this experience, Dennis developed and demonstrated a task-specific strategy.

In this example, both children indicated what needed attention in the task and behaviorally expressed the strategies that should be implemented based on the task criteria. Whitebread et al. (2009, 2010) stated that children's metacognitive regulation can also be revealed through behavioral expressions. However, metacognitive knowledge is generally explained by referring to verbalizations. As seen in this example, the limited vocabulary of young children causes them to resort to behavioral expressions to express their metacognitive knowledge.

5.2. Individual differences in metacognitive knowledge

Both Dennis and Michael made various statements about the identified dimensions of metacognitive knowledge in activities that allowed for peer interactions. The statements of the children about their knowledge of persons are found to be mostly related to their own cognitive processes. Studies support this finding; for example, Frith and Happe (1999); Rochat (2003) and Blackemore and Choudhury (2006) concluded that young children are aware of others' cognitive processes or working skills, but their statements mostly include self-competency or limitation. However, it was revealed that Dennis did not make any statements about weakness in his own performance during the activities. As in the following findings, it was also revealed that he did not make any statements about his own working weakness among the statements that included task and strategy information. There may be multiple reasons for this. First, Dennis may think that he does not have any inadequacies in mathematical thinking skills in the activities performed. Second, he may be aware of his inadequacies but may not express any verbal or nonverbal explanations of them. However, it is clear that this situation needs to be investigated more deeply. Unlike Dennis, Michael made statements about weaknesses in his own performance in most of the activities. Before the implementations, it was observed by the class teacher that Michael lacked mathematical competence. This may indicate that Michael has awareness of his own performance.

Although the children were asked some questions to have them reflect on their own thoughts, especially during the evaluation process at the end of the activities, Dennis was more hesitant to express his own cognitive processes compared to Michael. It should also be noted that Michael frequently displayed thinking-aloud behaviors during the activities. Although we investigated behavioral expressions in depth, especially through video recordings, it is obvious that verbal indicators are clearer and more interpretable. Bannert and Mengelkamp (2008) argued that thinking aloud does not affect learning performances relative to the control condition; however, metacognitive promptings like thinking aloud could be supportive for students who lack metacognitive competence. The two children's differences in thinking aloud could have arisen from further thinking. Related to this behavior, self-talk or private speech is recognized as a valuable strategy for learning (Mead & Winsler, 2015), and it is also dependent upon maturation, becoming gradually internalized (Berk, 2014). Dennis could have reached a stage involving more internal monolog than Michael. This inner speech is silent and is used to direct behavior or thoughts. When this stage is reached, one can engage in higher mental functions (Gholami, Salehi, Azizi & Fazli, 2016).

Identifying children's competencies and inadequacies is a building block for future learning outcomes. Although children's academic achievement can be determined with standardized measurement tools, it is important for them to be aware of their own strengths and weaknesses in order to organize their further learning independently (Flavell, 1979). As another finding, we can say that metacognitive tasks and strategies were supported by peer interactions for both children. The children organized their own performances with the strategies or directions of others. They also made statements that included evaluations of the performance of others. This finding is similar to the results of research revealing the importance of peer interactions for supporting metacognitive skills (Larkin, 2006; (Smith and Mancy, 2018)

6. Conclusion

The present study has aimed to describe how children's metacognitive knowledge emerges through peer interactions in math activities using qualitative data analysis. It contributes to the current knowledge by providing evidence that children's metacognitive knowledge could be formed by peer interactions. The results of the research show that when children have opportunities to interact with their peers, they reveal metacognitive knowledge of self, others, universal properties of cognition, and task-specific strategies.

In this study, we investigated children's metacognitive knowledge through mathematical task-based activities. Methodologically, which techniques are more appropriate for measuring metacognition, including task-based, are still debated (Azevedo, 2020; Kelemen, Frost & Weaver, 2000). There is widespread consensus that the development of metacognitive knowledge begins domain-specifically and that it becomes more flexible and transcendent with experience (Borkowski, Chan & Muthukrishna, 2000; Butterfield, Albertson & Johnston, 1995). Therefore, we believe that it is important to include task-based implementations in practices with young children. Additionally, we examined the metacognitive knowledge of two children with 10 weeks of observation. While choosing the participants, we considered the children's developmental characteristics in mathematics activities. This was based on past observations of the classroom teacher, not on mathematical achievement as determined by standardized assessment tools. For future studies, we suggest an in-depth examination of children's metacognitive skills according to more explicit measurement results.

As researchers and teachers, we should seek ways to provide metacognitive experiences for young children. It is important to support children's awareness of and ability to express both their strong and weak academic working skills. This helps children allocate their resources and use strategies more effectively. In this way, children can determine the factors that influence their performances and, accordingly, they can determine the requirements of the tasks and develop appropriate strategies.

CRediT authorship contribution statement

Ebru Aydın: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Visualization. **Çağlayan Dinçer:** Supervision, Writing – review & editing.

Data Availability

The data that has been used is confidential.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.tsc.2022.101104.

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