AN IMPROVED GREY WOLF WITH WHALE ALGORITHM FOR OPTIMIZATION FUCTIONS

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DEDICATION

I praise and thank the Most Gracious and Merciful, Allah (SWT)

Special thanks for my beloved mother.

To my dearest Naseem, Asif, Muhammad Ammar, and Hamza

For all postgraduate members, fellow friends and housemates.

This thesis is dedicated to all of you.

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ABSTRACT

The Grey Wolf Optimization (GWO) is a nature-inspired, meta-heuristic search optimization algorithm. It follows the social hierarchical structure of a wolf pack and their ability to hunt in packs. Since its inception in 2014, GWO is able to successfully solve several optimization problems and has shown better convergence than the Particle Swarm Optimization (PSO), Gravitational Search Algorithm (GSA), Differential Evolution (DE), and Evolutionary Programming (EP). Despite providing successful solutions to optimization problems, GWO has an inherent problem of poor exploration capability. The position-update equation in GWO mostly relies on the information provided by the previous solutions to generate new candidate solutions which result in poor exploration activity. Therefore, to overcome the problem of poor exploration in the GWO the exploration part of the Whale optimization algorithm (WOA) is integrated in it. The resultant Grey Wolf Whale Optimization Algorithm (GWWOA) offers better exploration ability and is able to solve the optimization problems to find the most optimal solution in search space. The performance of the proposed algorithm is tested and evaluated on five benchmarked unimodal and five multimodal functions. The simulation results show that the proposed GWWOA is able to find a fine balance between exploration and exploitation capabilities during convergence to global minima as compared to the standard GWO and WOA algorithms.



ABSTRAK

The Gray Wolf Optimization (GWO) adalah algoritma pengoptimuman carian metaheuristik yang diilhamkan oleh alam. Ia mengikuti struktur hierarki sosial dari serigala dan kemampuan mereka untuk memburu bungkus. Sejak ditubuhkan pada tahun 2014, GWO berjaya menyelesaikan beberapa masalah pengoptimuman dan telah menunjukkan penumpuan yang lebih baik daripada Particle Swarm Optimization (PSO), Gravitational Search Algorithm (GSA), Differential Evolution (DE), dan Evolutionary Programming (EP). Walaupun memberikan penyelesaian yang berjaya untuk masalah pengoptimuman, GWO mempunyai masalah kemampuan penjelajahan yang lemah. Persamaan kemas kini kedudukan di GWO kebanyakannya bergantung pada maklumat yang diberikan oleh penyelesaian sebelumnya untuk menghasilkan penyelesaian calon baru yang mengakibatkan aktiviti penerokaan yang buruk. Oleh itu, untuk mengatasi masalah penerokaan yang lemah di GWO, bahagian eksplorasi algoritma Paus mengoptimumkan (WOA) digabungkan di dalamnya. Algoritma Pengoptimuman Paus Gray Wolf yang dihasilkan (GWWOA) menawarkan kemampuan penerokaan yang lebih baik dan mampu menyelesaikan masalah pengoptimuman untuk mencari penyelesaian yang paling optimum di ruang carian. Prestasi algoritma yang dicadangkan diuji dan dinilai pada lima fungsi unimodal dan lima fungsi multimodal bertanda aras. Hasil simulasi menunjukkan bahawa GWWOA yang dicadangkan dapat menemui keseimbangan antara kemampuan eksplorasi dan eksploitasi semasa penumpuan ke minima global berbanding dengan algoritma GWO dan WOA standard.



TABLE OF CONTENTS

	TITL	E	i	
	DECI	LARATION	ii	
	DEDI	CATION	iii	
	ACKN	NOWLEDGEMENT	iv	
	ABST	RACT	v	
	ABSTRAK TABLE OF CONTENTS LIST OF TABLES LIST OF FIGURES			
	LIST	OF SYMBOLS AND ABBREVIATIONS	xii	
	LIST	OF PUBLICATION	xiii	
CHAPTER 1	INTR	ODUCTION	1	
	1.1	Research Background	1	
	1.2	Problem Statement	3	
	1.3	Research Objectives	4	
	1.4	Research Scope	4	
	1.5	Significance of the Research	5	
	1.6	Outlines of the Thesis	5	
CHAPTER 2	2 LITEI	RATURE REVIEW	7	
	2.1	Introduction	7	

	2.2	Backg	round of Metaheuristics Optimization Algorithm	7
	2.3	Grey V	Wolf Optimization (GWO)	8
		2.3.1	Hunt Behaviour of the Wolves	9
		2.3.2	Encircling the Prey	9
		2.3.3	GWO Technique	10
		2.3.4	Exploitation and Exploration in GWO	10
		2.3.5	The Standard Grey Wolf Optimization Algorithm	12
		2.3.6	Improve Grey Wolf Optimization Algorithm	14
		2.3.7	Hybrid Grey Wolf Optimization Algorithm	15
	2.4	Whale	Optimization Algorithm	18
		2.4.1	Basic Concepts	18
		2.4.2	The Standard Whale Optimization Algorithm	20
		2.4.3	Improving Whale Optimization Algorithm	21
		2.4.4	Hybrid Whale Optimization Algorithm	23
	2.5	Chapte	er Summary	24
CHAPTER 3	B RESE	ARCH	METHODOLOGY	25
	3.1	Introdu	uction	25
	3.2	Resear	rch Process	25
	3.3	The Pr	oposed Research Process	26
		3.3.1	Data Specification	28
		3.3.2	Stage 2: The Proposed Algorithm Implementation	28
		3.3.3	Stage 3: Result and Analysis	28
		3.3.4	Performance of the Proposed GWWOA	28
		3.3.5	Comparative Analysis	32
	3.4	The G	WWOA Algorithm	32
		3.4.1	Mathematical Model of GWWOA	35
		3.4.2	Encircle Prey	36
		3.4.3	Hunting	36
	3.5	Chapte	er Summary	39
CHAPTER 4 EXPERIMENTAL RESULT 4				40
	4.1	Introdu	action	40
	4.2	GWW	OA Algorithm	40
	4.3	Compa	arative Analysis	41

		4.3.1 Unimodal Benchmark Functions	41
		4.3.2 Multimodal Benchmark Functions	45
		4.3.3 Convergence Time	48
	4.4	Chapter Summary	50
CHAPTER 5	CONC	CLUSION AND FUTURE WORK	51
	5.1 Introduction		
	5.2	Objective Achievement	51
		5.2.1 Objective 1	51
		5.2.2 Objective 2	52
		5.2.3 Objective 3	52
	5.3	Research Contribution	52
	5.4	Future Direction	53
	5.5	Remarks	53
REFERENCES 54			
	VITA		63

ix

LIST OF TABLES

2.1	Standard GWO applications for Optimization Problems	13
2.2	Variants of improved GWO for different Optimization Problem	15
2.3	Hybrid Grey Wolf Optimization Algorithm Solves	17
	Different Optimization	
2.4	Limitation of GWO and Its Variants	17
3.1	Unimodal Functions	29
3.2	Performance Evaluation Metrics	29
3.3	Multimodal Functions	31
4.1	Performance Evaluation for Unimodal	41
4.2	Performance evaluation for Multimodal	45
4.3	Convergence time for proposed GWWOA algorithm on	50
	unimodal functions	
4.4	Convergence Time for proposed GWWOA algorithm on	50
	multimodal functions	

LIST OF FIGURES

2.1	Standard GWO Flow Diagram	11
2.2	The Grey Wolf Optimization Algorithm	12
2.3	The Whale Optimization Algorithm	19
2.4	The WOA Flow Diagram	20
3.1	Research Framework	26
3.2	Research Process	27
3.3	The Proposed GWWOA algorithm Flow Diagram-Stage 1	33
3.4	The Proposed GWWOA algorithm Flow Diagram-Stage 2	34
3.5	The proposed GWWOA algorithm Flow Diagram-Stage 3	35
3.6	The Proposed pseudo code of GWWOA algorithm	37
4.1	Comparison of the average case values of proposed GWWOA,	44
	GWO and WOA algorithms on unimodal functions	
4.2	Comparison of the best-case values of proposed GWWOA,	44
	GWO and WOA algorithms on unimodal functions	
4.3	Comparison of the average case values of proposed GWWOA,	48
	GWO and WOA algorithms on multimodal functions	
	Comparison of the best-case values of proposed GWWOA,	
4.4	GWO and WOA algorithms on multimodal functions	48

LIST OF SYMBOLS AND ABBREVIATIONS

GWWOA	-	Grey Wolf Whale Optimization Algorithm
GA	-	Genetic Algorithm
WOA	-	Whale Optimization Algorithm
HS	-	Harmony search
ES	-	Evolution Strategy
BBO	-	Biography-Based Optimization
DE	-	Differential Evolution
GSA	-	Gravitational Search Algorithm
GWO	-	Gray Wolf Optimization
AI	-	Artificial Intelligence
		Gravitational Search Algorithm Gray Wolf Optimization Artificial Intelligence

LIST OF PUBLICATION

Hafiz Maaz Asgher, Yana Mazwin Mohmad Hassim, Rozaida Ghazali, Muhammad Asif Saleem, "A Systematic Literature Review- SLR On Recent Advances And Variants Of Grey Wolf Optimization", International Journal of Scientific & Technology Research, ISSN(online)2277-8616, Vol. 9, issues 06, 2020 (pp.512-518)

Hafiz Maaz Asgher, Yana Mazwin Mohmad Hassim, Rozaida Ghazali, Muhammad Aamir, "Improved Grey Wolf Algorithm for Optimization Problems", Indonesian Journal of Electrical Engineering and Computer Science, ISSN:2502-4752, Vol. 22, No. 3, June 2021, pp.1573-1579



CHAPTER 1

INTRODUCTION

1.1 Research Background

Presently, capitalist societies require maximum benefit from a product with minimum cost for manufacture. Optimization provides with the most optimal cost of a particular problem in a search space (Yıldız *et al.*, 2008). The optimized process makes the most effective use to various specified set of parameters without violating any important contents. Optimization techniques play important role in engineering design, information science, economics management, operational research, and related areas (Seif & Ahmadi, 2015; Koupaei *et al.*, 2016).



In computer science and mathematics, a problem is said to be an optimization problem; if, it has many viable solutions and the optimal solution is required to be found among all the feasible solutions by applying the least possible cost (Olivella-Rosell *et al.*, 2018). For defining the optimization problem, two types of variables namely dependent variable and independent variable are used. The dependent variable shows the value representing the solution of the problem, whereas the independent variable is used for defining variable solution represented by the dependent variable (Aguirre-Cipe *et al.*, 2019). Depending on the nature of the independent variable used, the optimization problem is divided into two categories namely discrete optimization problem and continuous optimization problem (Nesterov *et al.*, 2018).

In the discrete optimization problem, an objective is represented by an integer value, graphical value or permutation value taken from a finite set of values (or sometimes from an infinite set of values). In the continuous optimization problem, the solution is found based on some constraints of the independent variable. In modern times, optimization search algorithms have been categorized into three types depending on the natural traits they follow to solve a problem. Evolutionary based, Physical based, and Swarm based algorithms. Evolutionary based algorithms are inspired by the laws of nature. Mostly, Evolutionary based algorithms start with the randomly generated populations and then following certain criterion of crossover, and mutation are able to find the fit solutions in the search space. Some of the examples of evolutionary based algorithms are Genetic Algorithm (GA) (Elsayed, et al., 2014; Kuo and Lin et al., 2013), Evolution strategy (ES) (Vicente et al., 2015), Biography-Based Optimization (BBO) (Xiangtao et al., 2011), and Differential Evolution (DE) (Wang et al., 2011; Zhou et al., 2016). The second kind of metaheuristics are physical algorithms that follow the basic principles of the physical based rule in the universe. Simulated Annealing (SA) (Selim & Alsultan, 1991), Central Force Optimization (CFO) (Formato & Engineers, 2014) and Black Hole (BH) (Hatamlou et al., 2013). Gravitation Search Algorithm (GSA) (Yadav et al., 2016), and Chemical Reaction Optimization (CRO) (Z. Li et al., 2014), are some of the physical based algorithms. The third kind of metaheuristics are swarm-based nature inspired algorithms that mimic the swarming behavior of animals. Here are some latest popularly used swarm algorithms i.e. Particle swarm Optimization (PSO) (Tang et al., 2015); (Jensi & Jiji, 2016); Zhan et al., 2011), Artificial Bee Colony (ABC) (Zhong, Li, & Zhong 2017; Yurtkuran & Emel, 2015), Cuckoo Search Algorithm (CSA) (Rakhshani & Rahati, 2017); Mlakar, Fister, & Fister, 2016), Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016), Firefly Algorithm (FA) (Gandomi et al., 2013), Ant Colony Optimization (ACO) (Chen, Zhou, & Luo, 2017), Bat Algorithm (BA) (Chakri et al., 2016), and Flower Pollination Algorithm (FPA) (Nabilet et al., 2016) etc.

The drawback of the standard GWO operation is its poor exploration capability at small randomization (Dhargupta *et al.*, 2020). The poor exploration may lead to skip the most optimal solution and even the present solution (Dhargupta *et al.*, 2020). The standard GWO is also poor in convergence rate and ultimately degrades the global solution quality (Long *et al.*, 2018). In order to overcome this problem, WOA is incorporated with GWO that improve the exploration capability, which in turn improves the global convergence results and ensures better quality solution.

This research study focuses on the nature inspired Grey Wolf Optimization (GWO) algorithm (Seyedali Mirjalili, *et al.*, 2014) GWO is a swarm intelligent algorithm that follows the basic principles of leadership hierarchy and hunting behavior of grey wolves in nature. Due to its simplicity, GWO has been widely utilized

to solve many practical optimization problems (Gölcük & Ozsoydan, 2020). GWO faces many challenging problems, it can easily get trapped in local optima because of imbalanced relationship between exploitation and exploration. Also, the position-update equation in GWO mostly relies on the information provided by the previous solutions to generate new candidate solutions which result in poor exploration activity (Heidari & Pahlavani, 2017a). Therefore, to overcome the problem of poor exploration in the GWO the exploration part of the Whale optimization algorithm (WOA) is integrated in it (Mirjalili & Lewis, 2016b). The resultant Grey Wolf Whale Optimization Algorithm (GWWOA) offers better exploration ability and is able to solve the optimization problems to find the most optimal solution in search space. The performance of the proposed algorithm is tested and evaluated on five benchmarked unimodal and five multimodal functions against GWO and WOA algorithms.

1.2 Problem Statement



Grey wolf optimization (GWO) is a metaheuristics algorithm follows the predatory behavior of wolves to search for optimal food source in packs and capture anything while keeping other wolves in their line of sight. In GWO, the alpha parameter or the alpha wolf leads the pack while other wolves are divided into beta and delta wings for finding optimal neighborhood positions. Despite providing optimal solutions better than GA and PSO, GWO unfortunately is not able to maintain the quality of the solution. This is due to its inability to ensure a fine balance between the exploration (finding new search space) and exploitation (finding solution in local search space) of the solution search space during grey wolf position update stage (Long *et al.*, 2018) (Heidari & Pahlavani, 2017a). This inherent imbalance in GWO's behavior not only fails to ensure proper exploration and exploitation of the search space; but it also leads the algorithm to converge to less optimal or sometimes unreliable solutions (Teng, Lv, & Guo, 2019). Therefore, to overcome the problem of poor exploration in Grey Wolf Optimization (GWO), the initial population for the GWO is initialized using the searching-prey mechanism of the Whale Optimization Algorithm (WOA). This ensures a well-balanced relationship between exploration and exploitation phase in the proposed Grey Wolf Whale Optimization Algorithm (GWWO) algorithm.

In the initial stage of the standard GWO it has the low convergence rate in the search space (Teng, Lv, & Guo, 2019). The position update equation perform well in exploitation and is not able to perform well during exploration (Long et al., 2018). In the grey wolf position change stage, the solution set generates the position based on the previous best position of alpha (α), beta (β), and delta (δ) wolves (Majeed & Patri, 2018). At this stage, the ratio of exploitation and exploration randomly chose individuals from the population by a component called randomization factor that plays the most important role in changing the Grey wolf position it is used in turn to control the solution search of the optimization problem to be solved. The major role of this factor is to check the availability of the most optimal solution in vicinities of the current solution (exploitation). Also it has to explore the solution search space to find new solution (exploration) is search of the most optimal solution (Heidari & Pahlavani, 2017a).

1.3 **Research Objectives**

AMINA Based on the problem identified in the previous section, following three objectives are proposed.

- (i) Improve GWO with WOA in terms of exploration of the standard GWWOA optimization algorithm.
- (ii) Balance the exploration and exploitation of the GWWOA algorithm.
- (iii) Evaluate performance of the improving algorithm.

1.4 **Research Scope**

For resolving the problem associated with standard GWO, the proposed algorithm solves the optimization problem.

- (i) Evaluation of the performance of the proposed GWWOA algorithm on standard benchmark optimization function.
- (ii) The criterion for performance evaluation on standard benchmark function has the best-case optimal solution, average case solution, worst case solution and standard deviation.

(iii) The whole experimentation is performed on MATLAB running over windows 10 operating system installed on core i7 CPU with 16GB RAM.

1.5 **Significance of the Research**

This research has introduced significant contribution in swarm intelligence by targeting one of its recently evolved techniques namely GWO. The proposed methodology and results associated with the proposed approach will be highly helpful for the academicians, professionals, industrialists, and the computer scientists by applying proposed GWWO algorithm for solving various types of the optimization problems where other optimization algorithm find difficulty in getting the desired solution.

In the proposed Grey Wolf Whale Optimization Algorithm (GWWO), A Grey Wolf Optimization (GWO) is integrated with the standard Whale Optimization Algorithm (WOA) algorithm to improve its convergence to global minima. The initial population for the GWO is initialized using the searching prey mechanism of WOA. This ensures a well-balanced relationship between exploration and exploitation phase Outlines of the Thesis in the proposed GWWO algorithm.

1.6

Chapter 1 gives a brief background of the optimization process. The problem statement, objective, scope and significance of this research activity. The remaining chapters are as follows:

Chapter 2 explores the basic conceptual terminologies and the operational procedures of different swarm intelligence algorithms used for solving various types of optimization problems. This chapter also gives a comprehensive literature review of different improved and hybrid versions of GWO algorithm.

Chapter 3 presents the research framework. This chapter shows the whole research activity that has been conducted. The proposed research framework comprises of three phases. Framework data selection, development of the proposed model, and data analysis.

Chapter 4 presents the result of the proposed model and evaluate the performance of the proposed model. The result of the proposed model is presented and brings into discussion n in this chapter.

Chapter 5 presents the conclusion of the whole thesis and discuss about the proposed GWWO model and the future directions of this research work.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter begins with explaining the optimization problems and its need in improving the search direction. Then swarm intelligent optimization algorithms like Grey Wolf Optimization (GWO) and Whale Optimization Algorithm (WOA) are discussed in detail. Then further down in the sections, different models of the GWO like standard GWO, improved GWO and hybrid GWO are discussed in detail. During the discussion, qualities, and shortcomings of recently introduced GWO and WOA are also taken into account. The discussion focuses on how to solve the different optimization problems using GWO. Finally, the chapter is concluded with details on the current and the possible new GWO variants emerging from the merger of current metaheuristics available.



2.2 Background of Metaheuristics Optimization Algorithm

More recently, artificial intelligent metaheuristics algorithms have been used to solve many optimization problems. There are three types of metaheuristics algorithms, i.e. Evolutionary based, Physical based, and Swarm based algorithms. Evolutionary based algorithms are inspired by the laws of nature. Mostly, evolutionary based algorithms start with the randomly generated populations and then following certain criterion of crossover, and mutation are able to find the fit solutions in the search space. Some of the examples of evolutionary based algorithms are Genetic Algorithm (GA) (Elsayed, *et al.*, 2014; Kuo & Lin, 2013), Evolution strategy (ES) (Vicente, 2015), Biography-

Based Optimization (BBO) (Xiangtao et al., 2011), and Differential Evolution (DE) (Wang et al., 2011; Zhou et al., 2016). The second kind of metaheuristics are physical algorithms that follow the basic principles of the physical based rule in the universe. Simulated Annealing (SA) (Selim & Alsultan, 1991), Central Force Optimization (CFO) (Formato & Engineers, 2014) and Black Hole (BH) (Hatamlou 2013). Gravitation Search Algorithm (GSA) (Yadav et al., 2016), and Chemical Reaction Optimization (CRO) (Z. Li et al., 2014), are some of the physical based algorithms. The third kind of metaheuristics are swarm-based nature inspired algorithms that mimic the swarming behavior of animals. Here are some latest popularly used swarm algorithms i.e. Particle swarm Optimization (PSO) (Tang et al., 2015); (Jensi & Jiji, 2016); Zhan et al., 2011), Artificial Bee Colony (ABC) (Zhong, Li, & Zhong, 2017; Yurtkuran & Emel, 2015), Cuckoo Search Algorithm (CSA) (Rakhshani & Rahati, 2017; Mlakar, Fister, & Fister, 2016), Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016), Firefly Algorithm (FA) (Gandomi et al., 2013), Ant Colony JNKU TUN AMINA Optimization (ACO) (Chen, Zhou, & Luo, 2017), Bat Algorithm (BA) (Chakri et al., 2016), and Flower Pollination Algorithm (FPA) (Nabil et al., 2016).

Grey Wolf Optimization (GWO) 2.3



Grey Wolf Optimization (GWO) is a swarm-based optimization algorithm developed by Seyed Ali in 2014 (Mirjalili, Mohammad, & Lewis, 2014). This algorithm is based on the leadership hierarchy and hunting mechanism of Grey wolves in nature. Grey wolves belong to the Canidae family. Grey wolf considers the top of the food chain and live-in packs. The pack is divided into different parts like alpha, beta, omega, and delta. The grey wolves pack follows leadership rule and hunt using social hierarchy of GWO. The social hierarchy of power and domination are related in each pack of grey wolves. The group leader is alpha who is the most dominant wolf and leads the whole pack in hunting, movement and feeding. If, the alpha dies or gets old than the second powerful or beta wolf becomes alpha to lead the pack. The GWO hunting behavior and the exploration and exploitation is discussed in the subsections.

Hunt Behaviour of the Wolves 2.3.1

A wolf can move to any place around the prey with alpha leading the pack. However, this social intelligence is not enough for grey wolves. During hunting the group hierarchy plays an important part during the hunting and the stability of a pack. To reproduce, social ranking, the three finest solutions are alpha, beta, and delta.

2.3.2 **Encircling the Prey**

Mathematical step of Grey wolf optimization has two points in a dimensional space and update the new position based on others. Equation (2.1) is used for update new position:

$$X(t + 1) = X(t) - B.D$$
 (2.1)

where X(t + 1) is the new position of the wolf, X(t) is is the current position, **B** is a coefficient matrix and D is a vector that depends on the location of the prey (X_n) .D is calculated as follows:

$$D = |C. X(t) - X(t)|$$
 (2.2)
 $M = 2.r_2$ (2.3)

where,

$$\boldsymbol{M} = 2.\,\boldsymbol{r}_2\tag{2.3}$$

 r_2 is a casually created vector from the interval [0, 1]. With these two equations, a solution can relocate around another solution. Since the equations uses vector, hence this can be applied to any number of dimensions.

The casual components in the beyond equation simulates various step sizes and movement speeds of grey wolves. The equation to define their values are as follows:

$$\boldsymbol{B} = 2\boldsymbol{a}.\boldsymbol{r}_1 - \boldsymbol{a} \tag{2.4}$$

a Vector where its value is linearly reduced from 2 to 0 during the run. r_1 is a randomly produced vector from the interval [0, 1]. The equation to update the parameter is as follow:

$$a=2-t\left(\frac{2}{T}\right) \tag{2.5}$$

where, t is the current iteration and T is the maximum number of epochs.

2.3.3 GWO Technique

The GWO architecture is one of the swarm intelligence algorithms, the optimization procedure with the solution of a random set, in every problem maintains with the help of vector and values of parameters. Every iteration gets the objective values of each solution to calculate the first step. Therefore, all the time one variable is saved in objective. When solving the problem in GWO the vector and variable are mentioned and key data is saved in memory, three vector and three variables. These vector and variable store the location and principal values, the previous value of Alpha, Beta and Delta wolves in the memory. These variables are updated when updating the previous position.

The GWO architecture update the solution using Equation (2.6) and (2.7). Now to compute these equations, the space b/w the resent clarification and alpha, beta and delta would calculate initial values using Equation 2.7. The involvement of alpha, beta, and delta to improve the position of the solution is then calculated by applying Equation (2.7). The objective value of the solution and their position, the main supervisory parameter of Grey Wolf Optimization (B, M and a) is updated earlier for position improvement.

$$Z_{1} = X_{\alpha} (t) - B_{1} D_{\alpha}$$

$$Z_{2} = X_{\beta} (t) - B_{2} D_{\beta}$$

$$Z_{3} = X_{\delta} (t) - B_{3} D_{\delta}$$
(2.6)

 $\mathbf{D}\alpha$, \mathbf{D}_{β} and \mathbf{D}_{δ} are calculating applying Equation (2.7).

D

$$D_{\alpha} = |M_{1} \cdot X_{\alpha} - Z|$$

$$D_{\beta} = |M_{2} \cdot X_{\beta} - Z|$$

$$D_{\delta} = |M_{3} \cdot X_{\delta} - Z|$$

$$(t+1) = (d_{1} + d_{2} + d_{3})/3$$

$$(2.7)$$

2.3.4 Exploitation and Exploration in GWO

Exploitation and exploration are two basic phases of problem solving through metaheuristics search (Črepinšek *et al.*, 2013). Exploitation and exploration are used to find new regions from previous positions to find out the solution in the search space.

The global convergence performance of any swarm-based algorithm is guaranteed through a fine balance between exploitation and exploration phases.

GWO belongs to the class of swarm-based algorithms and the key control exploitation parameter in GWO is *M*. The *M* is the random value in the $[-\alpha, \alpha]$ where *a* value gradually decreases from 2 to 0 thus switching from exploration to exploitation (Azlan *et al.*, 2019). In addition, the adjusting parameter for exploration is *B* in GWO. This parameter value also depends on a. The random value of the parameter *B* is in the range of [-2, 2]. Exploration is done after B> 1 or B < -1, while there is emphasis on exploitation when -1 < B < 1. As mentioned above better balance between exploitation and exploration is required to find out the exact search in global and local optimum using the algorithm. The standard GWO flow diagram and its algorithm are given in the Figures 2.1 and 2.2 respectively.

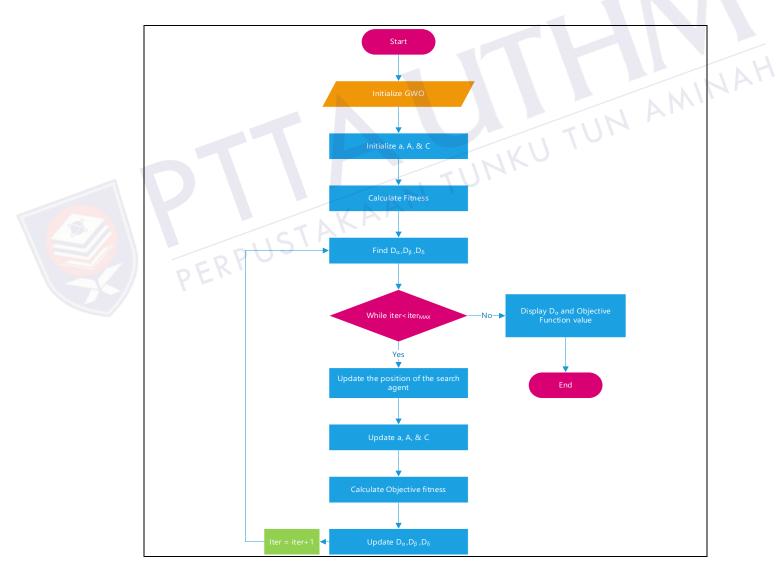


Figure 2.1: Standard GWO Flow Diagram

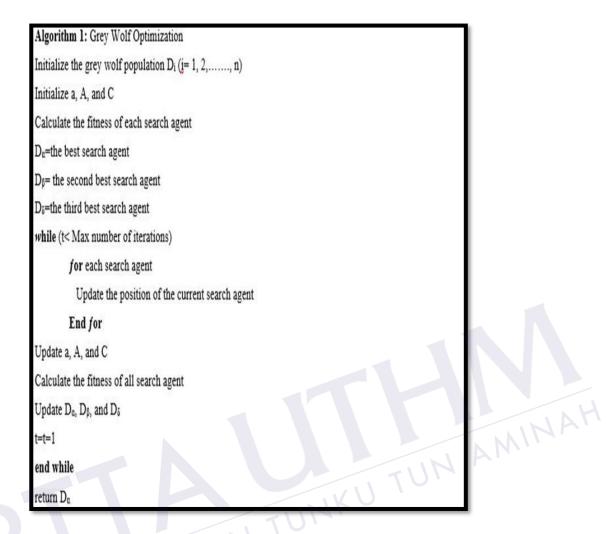


Figure 2.2: The Grey Wolf Optimization Algorithm (Seyedali Mirjalili, Mohammad, & Lewis, 2014)

2.3.5 The Standard Grey Wolf Optimization Algorithm

The standard GWO algorithm is applied to different optimization problems and is able to produce good results. GWO also uses search patterns to solve the optimization problem like a smart grid power system (Mahdad & Srairi, 2015). The efficiency of the GWO is also validated by performing strong tests on three combined heat and power dispatch (CHPD) problems; i.e. static economic dispatch, environmental economic dispatch, and dynamic economic dispatch. (Jayakumar *et al.*, 2016). Similarly, GWO is utilized for enhanced clustering mechanism in Vehicular ad hoc networks (VANET) and to overcome frequent topological chances due to moving nodes (Fahad *et al.*, 2018). Seeing its low computational cost, GWO was used to reduce the parameter sensitivity in fuzzy control systems (CSs) (Precup *et al.*, 2017).

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61

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