Reichenbach's 'Causal' Theory of Time: A Re-Assessment

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Abstract:

The paper proposes a re-assessment of Reichenbach's 'causal' theory of time. Reichenbach's version of the theory, first proposed in 1921, is interesting because it is one of the first attempts to construct a causal theory as a relational theory of time, which fully takes the results of the Special theory of relativity into account. The theory derives its name from the cone structure of Minkowski space-time, in particular the emission of light signals. At first Reichenbach defines an 'order' of time, a 'before-after' relationship between mechanical events. In his later work, he comes to the conclusion that the 'order' of time needs to be distinguished from the 'direction' of time. He therefore abandons the sole focus on light geometry and turns to Boltzmann's statistical version of thermodynamics. However, as Einstein pointed out, the emission and reception of light signals have thermodynamic aspects. When this is taken into account, Reichenbach's 'causal' theory turns out to be an entropic theory of time. It also emerges that Reichenbach discusses phase space and typicality arguments in support of his dynamic view of time. They provide a better understanding of the notion of entropy. This unifies his approach and helps to answer some of the standard objections against a causal theory of time.

Key Words:

Arrow of time; Boltzmann, branch structure; causal theory of time; entropy; Leibniz; phase space; Reichenbach; relational view of time; space-time; Second law of thermodynamics; typicality.

1. Introduction

In a series of publications, starting in 1921, Hans Reichenbach attempted to formulate what he characterized as a causal theory of time [1-6]. Reichenbach's efforts are important for several reasons: **1)** Historically, he was one of the first philosophers, after A. A. Robb (1914), to construct the causal theory as a *relational* theory of time, which fully took into account the results of the Special theory of relativity. This was an important attempt because the predominant view amongst physicists and philosophers alike at that time was that one of the main results of the Special theory, the relativity of simultaneity, implied an idealist view of time or even a block universe (Eddington, Einstein, Gödel, Jeans, Weyl) [7]. **2)** Reichenbach explicitly rejects Kant's idealist view of time and constructs his theory according to objective, physical parameters, especially the exchange of light signals. His first efforts are limited to what he called the linear 'order' of time: the 'before-after' relation between events. He explicitly excludes a consideration of the irreversible 'direction' of time. **3)** In his later work Reichenbach is at pains to show that 'time is not only *ordered* but also *unidirectional*' [4: 108; italics in original]. He draws a distinction between the order (passage) of time and the direction (arrow) of time, showing that their characterization requires the employment of different criteria. He introduces Boltzmann entropy to mark the arrow of time. He also employs the notions of branch structure and phase space in order to solve the paradox of the 'reversibility of micro-processes' and the 'irreversibility of macro-processes'. **4)** With his turn to entropy he abandons his exclusive focus on causal light signals for the characterization of time and thereby his original approach to define time in terms of order.

It is sometimes said that a causal theory attempts to explicate causal order in terms of physically possible causation and that causal theorists rely on a variant of Reichenbach's 'mark method'. As such the theory has faced major objections: 1) Reducing time to causation seems to 'be a case of explaining the obscure in terms of the obscurer' [8: 138]. 2) Extending the theory to general relativistic space-times also seems to be a problem because the General theory of relativity allows solutions with closed time-like curves, CTCs [8: 155].

This paper is an attempt to show that Reichenbach's theory can be entirely re-interpreted in terms of entropy. Reichenbach's mark method to characterize causation is a misleading distraction from the employment of light signals and light geometry. Light signals are carriers of information and crucially have thermodynamic aspects due to the irreversible nature of the emission process. Seen in this light Reichenbach's 'causal' theory is in fact an entropic theory of time order and consistent with his entropic theory of the arrow of time. My thesis is that this re-interpretation can avoid the well-known objections against the causal theory and present Reichenbach's theory as a unified approach to the problem of time.

The development of Reichenbach's general line of argument is as follows: objective time order is at first defined in terms of objective causal order; causal order is then defined in terms of light signals in Minkowski space-time; time direction is defined in terms of branch structure or phase space. But causality ultimately reduces to probabilistic events. That is, Reichenbach characterizes causality and time in terms of irreversible physical events and processes. At first, his notions of temporal order and temporal direction seem to have little in common until it is realized that both light signals and branch systems are subject to irreversible processes. Such processes can be spelt out in terms of phase space and typicality arguments, rather than simply as an entropic increase in disorder.

2. A Causal Theory

A causal theory aims to reduce temporal order to causal order, since it sees a close connection between these two types of order [4: §3]. Like other theorists, Reichenbach credits Leibniz with having formulated a causal theory of time.[1] But he holds that it was only after the discovery of the theory of relativity that a causal theory of time 'could be completed' [4: 25]. The theory of time is to be based on the causal relation: '*If* E_2 *is the effect of* E_1*, then* E_2 *is called later than* E_1*.*' [5: 136; italics in original]

Causal theorists refer to Leibniz's 'Initia Rerum Mathematicarum Metaphysica' (1715) as the key text, which introduces the causal theory of time.

> *If several states of things are supposed to exist, none of which involves the other, they are said to exist* ***at the same time.*** Thus we deny that those things which happened last year and those happening presently exist at the same time, since they involve opposite states of the same thing.
>
> *If one of two states that are not simultaneous involves the reason for the other, the former* is held to be the *earlier*, the *latter* to be the *later*. My earlier state involves the reason for the existence of my later state. (Quoted in [9: 151], bold and italics in original; cf. [5: 269]).

Leibniz's causal view derives from his belief in a deterministic universe and is grounded in his relational theory of time. If the universe is conceived as a causal network then the cause-effect relation determines successive events. But in his *Correspondence* with Clark (1715-16) Leibniz characterizes time more generally as the *order* of the succession of events [10]. Reichenbach agrees with Leibniz that space and time are not directly observable and must be inferred from spatial and temporal relations [2: 421]. Leibniz formulated his relational view in opposition to Newton's absolute notion of time. It was absolute and universal because Newton characterized time as a parameter over and above the occurrence of all physical events. Leibniz's relational view does not treat time as independent of physical events but grounded in them. The notion of order is of particular significance in this context for two reasons. First, Reichenbach draws a distinction between

[1] It is disputed amongst Leibniz scholars whether Leibniz's theory really qualifies as a causal theory of time [9].

the 'order' and the 'direction' of time. The order of time is merely a 'before-after' relationship between events, as they occur in classical mechanics. A linear order of events, like the motion of a ball, does not imply a direction of time. Both the forward and reverse motion are compatible with the laws of mechanics [4: 32]. Secondly, the order of successive events need not be causal, as Leibniz's general characterization shows. It may be regular (in a deterministic universe) or irregular (in a chaotic universe). Even a random succession of events – say the irregular blinking of a light in a dark universe – could indicate to an observer the passage of time. For this observer could count the number of blinks and thus realize that event **n** is earlier than event **n+1**. In order to measure the succession of events, however, the observer needs regular, preferably periodic events, such as the orbit of a planet around a centre. The 'regularity' of orbital periods served for a long time as a standard for clocks. Even though planetary orbits are 'regular' they are not causally linked. (Slight irregularities in the orbital motion of the planets motivated Newton to postulate 'absolute' time.) But it is not sufficient for one observer to notice the regularity; the regularity must also be invariant. That is, it must be the same irrespective of the perspective of the observer. The regularity of planetary motion is the same for every observer stationed around the globe.

Reichenbach writes in the context of the theory of relativity. The Special theory imposes an important constraint on the requirement of invariance, which is reflected in Reichenbach's account by the restriction of causal signals to light signals and null cones (Minkowski space-time). The time-like succession of events is the same for every observer. What is not invariant is the measured duration of events across two reference frames moving inertially with respect to each other. But Reichenbach recognizes, at least implicitly, that the focus on light signals is only a sufficient, not a necessary condition for the causal theory of time. In his *Axiomatization* [6: Ch. 1, §5] signal transmission, which he also calls 'causal chains', 'is not restricted to light signals until Axiom III'. Winnie, who also proposes a causal theory, makes this explicit:

> Intuitively, event e_1 is causally connectible to event e_2 just in case a signal (massive or massless) can be sent from e_1 to e_2 or conversely [11: 146, §III]; cf. [12: §3]; [13: §3].

For imagine, according to Reichenbach's Axioms of Time Order [6: §6], that a light signal and a sonar signal are emitted simultaneously from **P** to **P'** - which Reichenbach labels events (SP) and (SP') respectively – but the light signal is deflected whereas the sonar signal is detected at **P'**. Then it is known that event (SP) - the emission at **P** - is earlier than the arrival at (SP'), although the light signal is lost. Even though sound travels much more slowly than light, any regular and invariant signal may be used to establish an 'earlier-later'-relation.

So far we have only used the notion of the order of events, without relying on the notion of causation. Events can be ordered regularly, without being causally linked. How does

Reichenbach link temporal to causal order? In order to distinguish cause and effect, Reichenbach [4: §23; 5: §21] introduces his 'mark method'. He adds that the mark method uses a criterion of causal order, which makes no use of the direction of time and can therefore be employed to define temporal order.

> If E_1 is the cause of E_2, then small variations in E_1 (cause) are associated with small variations in E_2 (effect) but not vice versa.

According to Reichenbach [5: §43], it is the causal chain that transmits the marks. (Note that in the same paragraph Reichenbach excludes considerations of the arrow of time, a topic to which he returns in his later work, as discussed below.) Van Fraassen [14: 191, italics in original] gives a general definition:

> *E_2 is later than E_1* if and only if it is physically possible for there to be a chain $s_1, s_2, \ldots, s_k$ such that for each *i*, from 1 to *k-1*, s_i is a cause of s_{i+1}; and such that E_1 coincides with s_1 and E_2 with s_k.

At this point the first objection, mentioned above, arises. Despite its name, a causal theory cannot rely on ordinary causation. First, the notion of causation is not clear enough to serve as a basis for a causal theory of time. Causes can be direct or indirect, they can act instantaneously or with delay; hence they do not satisfy the criteria of regularity and invariance. Also, when the question arises whether a cause is 'instantaneous' or 'delayed', temporal notions are presupposed. Kant [15: B248-9] discusses the notion of simultaneous causation and observes, correctly, that even in such a case the cause is temporally prior to the effect. Van Fraassen [14: 193] uses as an example a chalk mark on a stone that 'is thrown across a creek'. It will still be present when the stone lands. But if the mark is made on the stone after landing, the mark will not be on the stone as it flies across the creek. The problem is that the mark method still employs temporal notions, similar to Kant's example of simultaneous causation.

Marks do not necessarily have causal efficacy. If the stone lands in the creek, it will be the stone not the chalk mark, which causes the water to splash. Nor do light beams always have causal effects on material substances. Radiation from the sun burns our skin but light rays do not displace objects.

However, Reichenbach notes that a cause does not just leave marks, a 'mark is the result of an intervention by means of an irreversible process' [4: 198]. A slight variation in the cause will change the effect but an interference with the effect will not change the cause. What is important is not that a cause leaves a mark, but that Reichenbach makes an inference from time to cause and from cause to irreversible, that is, thermodynamic processes. The transmission of general signals is a physical process, subject to thermodynamic effects. Although Reichenbach does not make this step, this must also apply to the trajectory of light signals. It opens the way to a re-interpretation of Reichenbach's causal theory as an entropic theory of time, given an appropriate characterization of entropy.

It also shows how Reichenbach's theory of causal order can avoid the first objection, namely that the notion of causation is obscure. The mark method presupposes temporal notions: on a common understanding a cause always precedes its effect. And Reichenbach characterizes the 'order' of events without explicitly referring to the notion of cause. I suggest that the mark method is a distraction if it is limited to a cause leaving a mark. Reichenbach's causal theory focuses on the exchange of light signals. If a light signal is sent from point P_1 to point P_2, all inertially moving observers will agree that the emission is earlier then the reception of the signal. The use of light signals is an indicator of temporal irreversibility, as Einstein observed long ago. In his response to Gödel, Einstein considers the emission of a signal from a point *A*, whose source is located in the past light cone of an observer at Here-Now, *P*, to a point *B* in the future light cone of the observer. According to Einstein this process is irreversible. On thermodynamic grounds he asserts that a *time-like* world line from *A* to *B*, through *P*, takes the form of an arrow, which sees *A* happen before *B*. This order of events would be the same for all *time-like* related observers. According to Einstein this process secures the

> …'one-sided (asymmetrical) character of time (…), i.e. there is no free choice for the direction of the arrow [16: 687].

Light signals are ideal candidates for the regularity and invariance of signals. They are not subject to the first standard objection against the causal theory. Reichenbach considers light signals to be physical signals, which leads to a *de facto* irreversibility between the emission and absorption of a light signal. The mark method is questionable but irreversible light signals do not suffer from the drawbacks of the notion of causation.

In *Axiomatization* Reichenbach refers to Einstein's light geometry and proposes to define 'earlier-later' relations by reference to signals, i.e. causal chains. 'Time is not a form of pure intuition. The physical world consists of causal chains', which give rise to topological and metrical axioms.

> We possess a time order only because the structure of the causal chains admits such an order. Time is the order type of causal chains [6: §3].

The causal chains themselves consist of the propagation of signals. The problem is that the linear order of events does not provide us with a direction (or arrow) of time. The 'singular nature of time' requires not only a characterization of linear order but also of its *directionality* as an additional property of time [5: §43]. In order to characterize the direction of time he notes that it

> …is possible to construct the causal net and its direction by a direct use of irreversible processes, which are applied in such a way that they do not presuppose a previous order, but supply order and direction [4: §23, p. 197].

Hence both the direction and order of time are to be characterized by irreversible processes in the universe. Irreversibility is to be derived from statistical mechanics.

As mentioned Reichenbach at first explicitly excludes the unidirectional nature from a consideration of time [5: 139] and concentrates on the 'order' of time. He distinguishes between linear order and the arrow of time. The focus on the irreversibility of light signal transmission opens the way to a consideration of thermodynamic aspects. Ultimately order is related to entropy since causal order is the result of interferences and interferences are irreversible. In his later work [4] he relies on Boltzmann's statistical version of the Second law to characterize the arrow of time.

3. Epistemological Questions

The debate in the 1970s about the causal theory was partly motivated by epistemological attitudes towards space-time, i.e. the relationship between theory and evidence [13]; [14, 17]. Earman's objections against the causal theory were based on realism about space-time. Earman held that what the General theory of relativity postulates as theoretically possible is also real, i.e. space-time is a basic spatio-temporal entity. If it is a temporally oriented space-time, 'there is no need for a causal theory, temporal betweenness falls out of temporal orientedness; we can have the latter without temporal order properties' [13: 80-1]. But from the point of view of a relational theory the bone of contention is whether space-time exists, as Earman explained.

> The scientific community sees time as an aspect of spacetime as a fundamental entity. Van Fraassen sees spacetime as an abstract theoretical construct, used to represent the relational structure of events that constitutes world history. He distinguishes between the actual structure of world history and space-time. This picture is not consistent with GR [13: 83].

Van Fraassen objected to Earman's 'hyperrealism' and contrasted it with his version of empiricism. He rejects the isomorphism between theory and reality, which motivates Earman's objections. Van Fraassen makes a distinction between the 'total relational structure of events that is world history' [14: 117] and the logical space, the model, used to represent that structure. Similarly, space-time is a model, a logical space, used to represent the succession of events. Space-time as such does not exist, for

> …time is a mathematical structure used to represent temporal relations among events; (…) space-time is similarly the mathematical structure used to represent spatio-temporal relations [14: 220].

The implication is that

> …the causal theory should say only that the structure of actual causal connections can be embedded in the relevant logical space [14: 228].

If the logical space is not isomorphic to the 'actual temporal structure of events', then from an empirical point of view the actual temporal structure becomes embeddable in a larger logical space. A logical space could be compared to the spectrum of all colours, from infrared to ultraviolet, in which visible light is embedded. A paint manufacturer may have a master chart of all colours, of which seasonal charts are issued each year. A seasonal chart will not exhaust all possible colour combinations, but will be embedded in the master chart. If time is treated as a logical space, not identical with the actual sequence of events, then its topological properties can be investigated: is time circular, cyclic or linear? Does time have a beginning and an end, a beginning and no end, no beginning and no end? [14: 117-21]; [18: Ch. 2] The permission of physically possible signals is required by the cone structure of Minkowski space-time, since it is a general theory of how space-time events are linked.

The realist postulation of a time-oriented space-time has its own drawback: Although this approach invests space-time with a conventional temporal direction, it does not invest it with a temporal arrow because a temporally orientable space-time is not the same as a temporally oriented space-time. The space-time model is time-orientable but this does not tell us what the actual time-orientation of the universe is, which is being modeled. Consequently, as even some proponents of this approach admit,

> …(t)emporal orientability is merely a necessary condition for defining the global arrow of time, but it does not provide a physical, nonarbitrary criterion for distinguishing between the two directions of time' [19: 2496]; cf. [20].

Reichenbach was tempted to avoid such ontological claims by insisting that what is theoretically possible – for instance closed causal curves – does not necessarily exist in reality. His starting point is not realism about space-time but spatio-temporal relations, which can be embedded in a space-time theory. This offers the advantage that only the relativistic theory of gravitation 'can reveal the physical structure into which space-time order relations can be embedded …' [5: 268, 285].

Reichenbach [6: §1] also demands of the axiomatic expositions that it must be logically consistent and physical axioms must reflect factual judgments. Thus the question of the empirical adequacy of the causal theory arises irrespective of epistemological differences about the ontology of space-time. If we stay within the framework, set by Reichenbach, and restrict attention to Minkowski space-time, then light signals provide a criterion for the irreversible passage of time. But null light cones do not provide a criterion for the global arrow of time. It will not be sufficient for a general causal theory of time to be restricted to Minkowski space-time. Reichenbach is therefore right to say that a consideration of the arrow or direction of time requires him to go beyond the Special theory of relativity and ground the arrow of time in different criteria. This policy is in line with his adoption of a relational view of time.

In his book on the *Direction of Time* [4: §§3, 11] Reichenbach still holds that time order is reducible to causal order, as a relation between physical events. This time order is 'invariant under the Lorentz transformations' but the 'cause-effect' relation is no longer treated as primitive. This means that Reichenbach regarded deterministic causation as a limiting case of probabilistic causation ($C \rightarrow E \leq 1$). The principle of causation cannot be formulated without the principle of a statistical distribution. Causal claims take the form of an implication ('If *C*, then *E*'). As causation, on Reichenbach's understanding, means irreversible interference, he arrives at thermodynamic considerations, not only for the 'cause-effect' relationship but also the direction of time. The 'problem of the direction of time' – the arrow of time – requires him to go beyond Einstein's Special relativity theory and consider time direction in terms of statistical mechanics. As statistical mechanics is a probabilistic theory, it also underlies the notion of probabilistic causation. The combination of causation and probability leads to a conditional view of causality [3: 715-6]; [4: 82]; cf. 21].

With this move Reichenbach effectively no longer *defines* time exclusively in terms of causal connectibility in Minkowski space-time. He *seems* to adopt a *different* solution for the direction of time. It is derived from the statistical understanding of the Second law of thermodynamics. However, if the irreversible emission of light signals is taken into account, his appeal to the Second law unifies his approach. Reichenbach's 'causal' theory turns out to be an entropic theory of time.

4. A Closed Universe?

The restriction to the space-time of the Special theory of relativity represents a severe limitation of the causal theory. Order is reduced to causal order in Minkowski space-time. But invariant, regular order need not be based on causal relations or light signals. The 'causal' theory abandons the generality of Leibniz's formulation of the relational theory in his correspondence with Clarke. On the other hand, this restriction avoids the first objection mentioned above, i.e. that the theory explains the obscure notion of time by the even more obscure notion of causality. The theory concentrates on light signals, which, as we have seen, are irreversible because they are subject to entropic effects. The *characterization* of a causal theory in terms of the structure of Minkowski space-time faces the challenge of not being in accordance with certain empirical or theoretical results of the General theory of relativity. As a relational theory, it is committed to empirical adequacy. The question then arises whether the causal theory can deal with the behaviour of light signals under gravitational effects. This is basically the second general objection. As the General theory predicts, light rays are deflected in the vicinity of strong gravitational fields.

The bending of light in gravitational fields is not an objection to Reichenbach's theory *per se*. Although Reichenbach [5: §22]; [6: §5] stipulates that light signals are the fastest signal between two points, he also accepts that signals can be combined into causal chains. The possibility of causal chains suggests that deflected light rays can be accommodated in the theory because emission, E_1, and absorption, E_2, are still clearly distinct phenomena. But the General theory also confronts the 'causal' view with the theoretical possibility of closed causal loops, which Reichenbach excludes on physical grounds [5: §§ 21, 43]; [6: §3]; cf. [11].

Earman's objections against the causal theory were based on the theoretical possibilities of closed causal loops [13: §VI]. He objected that the field equations of the General theory have 'solutions which do not possess a global time slice or cannot be partitioned by spacelike hypersurfaces.' And a causal theory does not apply to space-times, which are closed in their temporal aspects, i.e. for every $x \in \mathbf{M}$, manifold $\mathbf{M}= [C_{a-}(x) \cup C_{a+}(x)]$, where $C_{a-}(x)$ and $C_{a+}(x)$ stand for the causal past and future of 'x' respectively.

In order to deal with this objection, van Fraassen holds that 'arbitrary pairs of events are not causally connectible.' Only time-like connected events (within light cones) but not space-like connected events can be causally linked. Actual trajectories must not go 'all the way around time', which rules out signals travelling faster than light. If a causal curve in space-time 'is given by the function $f(t) = (x_t, y_t, z_t, t)$, for all t, then only its *proper* segments' – only continuous parts of the line, other parts being excluded – 'can be the paths of possible causal signals' [17: §III, italics in original]. Consequently van Fraassen does not grant 'that events are causally connectible *exactly if* the points in the mathematical space-time at which they are located are linked by a causal curve' [17: 94; italics in original].

Proponents of the causal theory sometimes grant that the theory is not well suited for cosmological models of time. Thus Winnie admits that if 'the General theory is considered "liberally" the causal theory is clearly false' [11: 196]. This is due to the fact that on such a 'liberal' reading GTR allows CTCs, closed time-like curves, as a solution. He refers to Gödel (1949): 'for every point event **e** in **M** there is a closed causal curve through **e**', from which Gödel concluded that there was no objective flow of time. Such chronology-violating space-times contain closed causal curves.

> (W)e can define the *chronology violating* region $\mathcal{V} \subset \mathbf{M}$ as including all points $p \in \boldsymbol{M}$ such that a CTC passes through p; in other words $\mathcal{V}$ is the region containing CTCs [12: §3, italics in original; cf. [22].

A closed causal curve would mean that cause and effect, past and future are no longer clearly separated. A cause becomes its effect, like the proverbial snake, which bites its own tail. If time is defined in terms of causal terms, closed causal curves would render time circular in the sense that the beginning would coincide with the end of time. A circular model, which differs from an incoherent cyclical model of time, is conceptually consistent even if empirically unconfirmed. For if temporal relations are reduced to causal relations, and the universe has a circular structure then it seems that globally a cause no longer precedes its effect. Or an additional instruction of which way to travel around the circle would have to be given. But this presupposes a notion of a global arrow of time. Here a distinction between local and global aspects becomes useful. On a local level, in a small section of the circular model, a cause still precedes its effect. But on a global level the universe would return to its 'beginning' in time. But how is its beginning to be

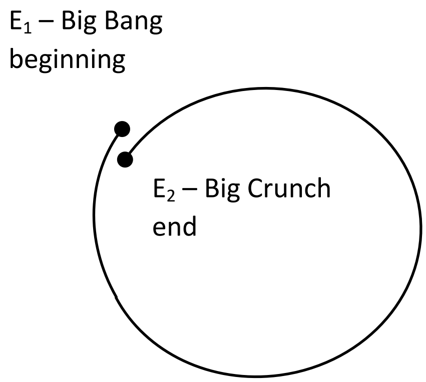


Figure Ia: An open circle, in which the beginning of the temporal universe would not coincide with its end. This scenario, as envisaged by Roger Penrose, means that the physical conditions at the Big Bang would be physically very different from the Big Crunch: the Big Crunch has much higher entropy than the Big Bang. Penrose calls this scenario the Weyl curvature hypothesis – it has considerable implications for the asymmetry of time.

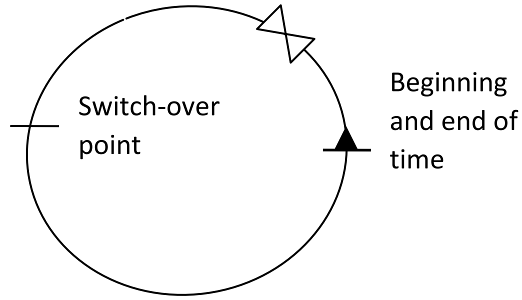


Figure Ib: In this scenario, in which the topology of the temporal universe is modeled as a closed circle, the Big Bang and the Big Crunch would be physically identical. It creates the two-time boundary problem: the symmetry of initial and final conditions implies a flipping of the arrow of time at the switch-over point. Light cones would begin to tip over before the switch-over point, leading to CTCs.

characterized? When talk is of circular time, it is important to distinguish the topology of an *open* from a *closed* circle. (Figure 1a, 1b)

The emphasis of a relational theory is on empirical reality. It is not obvious that the conceptual model of a closed circle fits cosmological conditions. True, the model of a circular universe can be envisaged. For the universe to have a closed circular structure, its beginning in the Big Bang would have to return to an end in a Big Crunch. That is, cosmological conditions at the end would have to be identical to cosmological conditions at the beginning. Then it seems that a cause can be both before and after the effect. But the universe still has a temporal dimension because the end of the universe is separated, on a relational view, from its original birth by all the events which happen between the two events. In this case, then, the temporal relation does not reduce to a causal dimension. The chronological topology does not reduce to the causal chronology of space-time, **M.**

The Big Crunch as the time-reverse of the Big Bang is not a realistic model. If gravitational effects are taken into account, the Big Crunch has much higher entropy, due to black hole formation, than the Big Bang [32: 436-40]; [33: 719-20, 728-9]. Although the Big Crunch is a theoretical possibility, evidence suggests that the universe will end in a Big Chill. If the end of the universe differs from the Big Bang, the universe will display an arrow of time. Cosmological data [23]; [24] show that the universe is expanding at an accelerated rate and will end in what the 19th century called the 'heat death'. The hot Big Bang and the dissipation of energy at the end mark a clear irreversible order. The universe would acquire an unmistakable arrow of time. The real universe seems to be better captured by FLRW models. (There are 3 standard FLRW cosmological models, depending on whether the parameter, Ω, the ratio of actual to critical mass density, is smaller than 1, equal to 1 or greater than 1). Beyond his dismissal of CTCs, Reichenbach's 'causal' theory seems to offer no systematic boundary between observed empirical phenomena and theoretical possibilities. But his reflections on the arrow of time go beyond a mere refusal to consider theoretical scenarios. It turns out that he relies on phase space arguments and hints at typicality arguments in support of his entropic theory of the arrow of time.

5. The Arrow of Time

In *Axiomatization* [6: §5, bold in original] Reichenbach declares that 'only signals are to be used for all metrical and topological functions: it is a physical process that travels from a real point **P** to another point **P'**.' Temporal order does not deliver an irreversible direction of time. So, it is no surprise that Reichenbach should turn to the notion of Boltzmann entropy to characterize the arrow of time.

Both Boltzmann and Eddington had done so before him. Eddington [25: 68] even declared the Second law a supreme law of nature. Eddington [26: Ch. V, p. 92] and later Wheeler [27] regarded temporal relations as inferences from observed entropic processes. Reichenbach, however, fully accepts the statistical nature of the Second law and adopts Boltzmann's solution to the reversibility objections. He sees Boltzmann's achievement in having combined the unidirectional nature of macro-time with the reversibility of micro-processes. It leads to the statistical nature of time direction. Reichenbach introduces the notion of branch systems, which undergo entropic processes. The universal increase in entropy is reflected in general trends in branch systems. That is, Reichenbach sees the notion of time – the global arrow of time – as an inference from the observation of branch systems.

> A statistical definition of time direction presupposes a plurality of systems which in their initial phases are not isolated, but acquire their initial improbable states through interaction with other systems, and from then on remain isolated for some time. That our universe, which is an isolated system, possesses a time direction is due not merely to the rise of its general entropy curve, but to the fact that it includes a plurality of branch systems of the kind described. The direction of time is supplied by the direction of entropy, because the latter direction is made manifest in the statistical behavior of a large number of separate systems, generated individually in the general drive to more and more probable states [4: 135].

Reichenbach [4: 111, fn2] is of course aware of Loschmidt's reversibility objection and Zermelo's recurrence objection. He argues that the reversibility objection cannot be met by reference to isolated systems. It requires a reference to a plurality of systems [4: 132, cf. 121]. The overwhelming majority of branch systems tend to occupy larger areas of phase space.

This approach has an effect on how Reichenbach interprets Liouville's theorem. (Figure II) Recall that Liouville's theorem in classical mechanics states that a volume element along a flow line preserves the classical distribution function $f(r,v)drdv$:

$$f(t+dt, r+dr, v+dv) = f(t,r,v) \ .$$

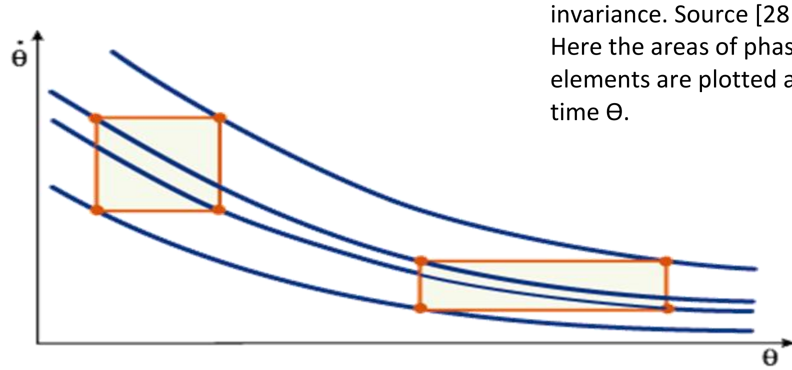
That is, if a volume of a bundle of trajectories is taken and let to evolve over time, then its volume element along a flow line remains invariant although its shape may change.

Yet, Liouville's theorem of volume invariance does not undermine the entropic theory of time. The theorem does not require that 'the domains keep their shapes.' In fact

> …if the initial position of the phase point is known within a small interval ε, we cannot conclude that this knowledge determines the position of the point for all times within a small interval δ of exactness which is a function of ε. In fact, with growing time we may expect to find the point at any distance larger than a given δ from the predicted place *P'*, if we define *P'* as the point on which P in the center of [a spherical domain] *A* is mapped [4: 93-4; italics in original].

An immediate consequence of this theorem is that even though the *volume* is preserved the *shape* of this phase space region is not preserved and this implies a dynamic evolution of the trajectories within this region. For two shapes cannot differ from each other without an evolution of the trajectories. It also implies that a *reversed* evolution of the trajectories will preserve the volume but not necessarily the shape and hence that reversed trajectories need not be invariant with respect to the shape of the phase space region.

Figure II: An illustration of Liouville's theorem: volume invariance but not shape invariance. Source [28: 206]. Here the areas of phase space elements are plotted against time Θ.



Let us assume that there are today two influential approaches to the explanation of the observable increase in entropy of macro-systems from time-symmetric microphysics: 1) the argument from the topology of phase space and 2) the postulation of a Past Hypothesis [29: 326]. Reichenbach does not explicitly postulate a Past Hypothesis but assumes that the entropy of the universe was lower in the past. He argues in terms of the geometry of phase space; and hints at typicality arguments.

How do these arguments establish a global arrow of time? On an *empirical* level it is now known that the universe expands at an accelerated rate. This is in line with Reichenbach's dismissal of CTCs as mere theoretical possibilities. But Reichenbach has more constructive arguments at his disposal, which take him beyond Boltzmann's notion of sectional time (i.e. entropy increases only in certain parts of the universe). On a more *theoretical* level he considers phase space and typicality arguments to assert that there is a global arrow of time. To these considerations Reichenbach could add the invariance of entropy in both the Special and the General Theory of relativity to support his theory of a dynamic notion of time.

Let us consider these arguments in turn.

6. Phase Space arguments

Reichenbach [4: 71, 78] adopts the standard characterization of phase space. He starts from a three-dimensional physical space, which is combined with the three dimensions of a velocity space, to form 'a six dimensional parameter space.' Traditionally, a classical system in mechanics is described in terms of a 6N-dimensional phase space, Γ, in which each individual particle has three position coordinates (x, y, z) and three momentum coordinates (since 'momentum = mass x velocity' is a vector quantity). Single particle systems or many particle systems are represented by a single point X, its micro-state, which moves around in phase space according to the deterministic laws of Hamiltonian mechanics. Reichenbach does not mention that the phase space usually comes endowed with a Lebesgue measure, μ, which roughly is a volume measure of the phase space, available to the systems. For Hamiltonian systems the Lebesgue measure is invariant under the dynamics; this statement is equivalent to Liouville's theorem. If energy conservation is taken into account, 'the motion of the system is confined to a (6N-1)-dimensional energy hypersurface Γ_E' which Reichenbach calls an 'energy surface'. It describes the phase space *available* to the evolution curves (or phase flow φ_t) [30: 471].[2]

When speaking of the phase space of a particular system, some care should be taken as to the precise meaning of this term. When Maxwell's Demon sits in a sealed container, with a small opening in a partition wall, which allows the Demon to separate the slow from the fast molecules, he is confined to a three-dimensional Euclidean coordinate space. When the partition is removed the gas molecules will spread through the whole available three-dimensional volume. Before the removal of the partition, the phase space, which the molecules occupied, was smaller than the total phase space available to them. This three-dimensional example suggests that a distinction should be drawn between the phase space *occupied* and the phase space *available* to the molecules; a distinction which Reichenbach accepts [4: 74-75]. When phase space (or the space of states) is conceived as the phase space of all the possible states, which a system could hypothetically occupy, the reversibility of the fundamental equations requires that this available phase space remain invariant [31: 336, 340-2]; [32: 407ff]; [33: 701]. The phase space must remain invariant if the trajectories are allowed, in theory, to return to their initial conditions. But asymmetry obviously implies some evolution: what changes is not the number of *possible* configurations but the number of *actual* configurations. That is, the volume of phase space, which the distributions actually occupy before reaching equilibrium, is smaller than the volume of phase space available to them. Given the expansion of the universe the phase space available to cosmological systems is obviously much larger than the volume of phase space currently occupied. Due to the difference between phase space occupied and phase space available, the system evolves, such that the equilibrium macro-state is larger than any other state. As a system, like the universe, undergoes expansion, it begins to occupy different volumes, due to the different configurations it can occupy – for instance $|\Gamma_{M_{equilibrium}}| \gg |\Gamma_{M_{initial}}|$ – and these different volumes allow the construction of

[2] Reichenbach [4: 57, 71] makes the distinction between a distribution and an arrangement. A distribution is given by the number of molecules in each cell. Arrangements are micro-states. A macro-state is a 'disjunct' of distributions. As usual a macro-state can be realized by various micro-states.

volume ratios – $|\Gamma_{M_i}| / |\Gamma_{M_{eq}}|$ – which become important for asymmetry arguments. Eddington regarded these volume differences as another criterion – apart from the traditional understanding of entropy – for a cosmological arrow of time [26: 67-8].

If the phase space argument is applied to the universe as a whole, the universe must be treated as a closed system. This means that the 'degrees of freedom that are available to the whole universe are described by the total phase space' [32: 407-11, 466-7]; [33:701]. However we face the same problem as with phase space in a classical system. The phase space of the universe remains invariant means that the trajectories are allowed in theory to return to their initial conditions. This is basically Loschmidt's reversibility objection. In Reichenbach's words, 'separation processes must be as frequent as mixing processes' [4: 110]. As seen from the current state, the entropy is expected to increase into the future, leading to a high entropy-state, like the Big Chill. But in a time-reverse direction we should expect entropy to be high in the past, which contradicts the evidence and raises the question of how the universe could move from high entropy in the past to lower entropy now. Therefore Reichenbach imposes a constraint on the initial condition – now characterized as the Past Hypothesis – which states that the entropy was low in the past.

Reichenbach assumes that the overwhelming majority of branch systems tend to occupy larger areas of phase space. He was not aware that phase space arguments face the problem of double standards. That is, initial conditions are favoured over final conditions. By assuming that the entropy of the universe was lower in the past and will be higher in the future [4: 131], he seems to commit what has become known as the 'double standard fallacy' [34]. One way of overcoming such double standards is to enhance phase space arguments by typicality arguments. Implicitly this is the strategy, which Reichenbach employs.

Although notions like 'fibrillation' and 'spreading' have an intuitive physical appeal, in the statistical-mechanical literature spreading usually refers to the realisability of the macro-state with respect to the available combinations of micro-states. Realisability describes the number of micro-states, which are compatible with or make up a given macro-state, as reflected in Boltzmann's definition of entropy, which Reichenbach adopts: $S = k_B logW$ (where W is thermodynamic probability). It is tempting to argue that the realisability of different configurations 'extracts a direction of time even though the molecular collisions which give rise to the diffusion of the gas are each time-reversible' [35: 75]; cf. [36]. For if a system has a greater degree of realisability available to its macro-states – a greater amount of spreading into the available phase space – this evolution could serve as a criterion for an arrow of time. As Reichenbach states, 'unordered states are highly probable', which 'means that they cover a very large area of the energy surface' [4: 75]. However, as it stands, the phase space argument is vulnerable to reversibility objections. As a corollary to a Past Hypothesis, the realisability or phase-space argument must assume that there is no future constraint, which acts on the current state, for instance in the sense of influencing the current state of entropy. Realisability is assumed to be greater towards the future than towards the past but from a statistical point of view realisability should be equally likely in both directions. The question arises whether realisability can be saved if realisability in one direction can be said to be more typical than realisability in the opposite direction. Phase space arguments require the postulation of asymmetric boundary conditions but here typicality arguments come to play their part. They show that phase space arguments, supported by typicality, provide more plausible accounts of asymmetry than the symmetry required by Gold universe models [37].

7. Typicality Arguments

Reichenbach implicitly uses the notion of *typicality* to deal with the obvious objection, as formulated by Popper [38] against Boltzmann that time is unidirectional and not subject to statistical fluctuations. This is evident from his discussion of branch structure: the direction of entropy is 'made manifest in the statistical behaviour of a large number of separate systems'. Furthermore, there is a parallelism of entropy increase in a vast majority of the branch systems [4: §§15-16]. Typicality arguments are concerned with the overwhelming majority of cases in a specified set. This implies the ratio of overwhelmingly many to a small number of divergent cases, hence a large number ratios, in particular volume ratios. According to one formulation of the typicality view [39] 'for any {micro-region} [Γ_{Mi}] the relative volume of the set of micro-states [x] in Γ_{Mi} for which the Second law is violated (…) goes to zero rapidly (exponentially) in the number of atoms and molecules in the system' (quoted in [40: 84]). Such a notion of typicality resembles the notion of weak *t*-invariance [41: 8], since a weakly *t*-invariant process, such as diffusion and heat conduction, allows for its (improbable) time-reverse without violating the 'laws of elementary processes'. But such processes are never observed in nature; hence their realisability is practically zero or would require practically infinite amounts of time. Reichenbach also appeals to a quasi-irreversible shuffling mechanism, whereby reversible micro-processes are turned into practically irreversible macro-processes [4: 70, §17-18]. He adds that the

> …time direction supplied by the ensemble of branch systems originates from a mixing of processes, in the same sense that the time direction of a diffusion process results from the mixing of molecules [4: 122].

Penrose also highlights the collisions between particles: 'by far the main contribution to the entropy comes from the random particle motions', which make a return to the original distribution atypical [32: 402]. The notion of typicality leads to a notion of factual irreversibility, which is perfectly in line with Reichenbach's commitment to a relational view of time. Note that it is the large *majority* of these systems which start in an improbable state of low entropy, due to interaction with other systems.

The realisability argument appeals to 'molecular collisions' to explain the evolution towards equilibrium. In the language of typicality, the realisability of more macro-states from fewer macro-states is more typical than the realisability of fewer macro-states from more macro-states because of the difference between occupied and available phase space. The increasing arrow of time can therefore be understood as a function of the ratio of occupied to available phase space. These occupied regions of phase space are regarded as statistically irreversible, whilst the equations of motion, which govern the trajectories, remain time-reversal invariant. [Cf. 20: 408] But these time-reversible equations of motion are compatible with the statistical irreversibility of the macro-systems.

One advantage of volume-ratio typicality arguments[3] is that they are not based on probabilities; hence the problem of the *t*-invariance of probabilities does not immediately arise. Typicality approaches do not start from the assumption of the temporal neutrality between initial and final conditions. They do not assume that all evolutions are equally probable. In fact, a majority of branch systems start in a low-entropy state. Hence the topology of the system changes over its thermodynamic evolution. According to typicality approaches the evolution to a fibrillated state is typical, whilst a return to a smooth, ordered state is atypical. Hence typicality approaches do not give rise to a demand for parity-of-reasoning: a quasi-identity of initial and final conditions is highly atypical, although it is not excluded [37]. The number of degrees of freedom in a fibrillated state is much greater than in a smooth state, and hence their return to a more ordered state requires 'perfect aiming', which, given the physical constraints on a system, make it extremely atypical. It would require a Loschmidt Demon, whose work in an expanding universe, according to Eddington, becomes very difficult, since Liouville's theorem does not guarantee his success.

Eddington argues that, if an expanding universe is taken into consideration, we are no longer forced to conclude 'that every possible configuration of atoms must repeat itself at some distant date'.

> In an expanding space any particular congruence becomes more and more improbable. The expansion of the universe creates new possibilities of distribution faster than the atoms can work through them, and there is no longer any likelihood of a particular distribution being repeated. If we continue shuffling a pack of cards we are bound sometime to bring them into their standard order – but not if the conditions are that every morning one more card is added to the pack [26: 68].

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[3] Generally speaking, a set is typical if it contains an 'overwhelming majority' of points in some specified sense. In classical statistical mechanics there is a 'natural' sense: namely sets of full phase-space volume. That is, one considers the Lebesgue-measure on phase-space, which is invariant (by Liouville's theorem) and which, when 'cut to the energy surface', can be normalized to a probability measure; then sets of volume close to one are considered typical' [42: 803]. Even if typicality claims are formally called 'probability measures', this does not make them probabilities in the philosophical sense [30: 475fn]: with which interpretation would they be associated? Typicality concerns volume ratios, not a measure of actual over possible cases.

A scenario which is theoretically possible, according to Loschmidt's reversibility objection, does not thereby become typical, statistically relevant behaviour in a dynamic universe.

8. Entropy

As we have seen, Reichenbach's considerations of the order and the arrow of time ultimately rely on the notion of entropy. Probing further into the question of entropy, another argument becomes available to an entropic theory of time. Physicists regard entropy as invariant across reference frames, even in the General theory of relativity. The increase in entropy in thermodynamic systems is certainly a regular process – based on the Second law of thermodynamics – but the crucial point is that entropy is frame-invariant. In fact, in thermodynamic systems, moving with velocity, *v*, several thermodynamic parameters remain invariant. Planck [43] has shown that both pressure, *p*, and entropy, *S*, are invariant relationships in relativistic thermodynamics. In a system undergoing a reversible and adiabatic process, the 'entropy of a body does not depend on the choice of reference system' [43: 14, my translation]. If two systems, 1 and 1', are in inertial or non-inertial motion with respect to each other, whose initial entropic states are S_1 and S'_1 and whose final entropic states are S_2 and S'_2 respectively, it follows that $S_1 = S'_1$ and $S_2 = S'_2$, and generally $S = S'$. This invariance can also be inferred from the definition of entropy in statistical mechanics: $S = klogN$. The number of micro-states, *N*, which correspond to a given macro-state, do not depend on the velocity of the reference frame, so that $S = S'$. More generally, the 'equations of thermodynamics are the same in curved spacetime as in flat spacetime; and the same in (relativistic) flat spacetime as in classical nonrelativistic thermodynamics' [44: 562]; cf. [45: 175].

If entropy is an invariant process, Reichenbach was right to regard it as an important criterion for the notion of time. As such he will face the objection that the Second law is an empirical, not a fundamental law [46] and that the unidirectional nature of time is not a matter of statistics [38]. It has also been doubted that typicality can explain the approach to equilibrium [40]. However, Reichenbach proposes an entropic theory of time, not an explanation of equilibrium. Yet Reichenbach seems to explain fundamental properties such as the order and direction of time by reference to non-fundamental thermodynamic properties. It would indeed be a mistake to *define* time and its properties by reference to a statistical notion of entropy. But Reichenbach's endorsement of relationism offers a way out. Entropy is not used to define time but it is offered as one of the numerous natural processes which allow a grounding of temporal notions. Decoherence, the expansion of the universe, measurement processes in quantum mechanics, transition processes in atomic systems and invariant properties in the theory of relativity (*c*, *ds*) all serve as *criteria* for the passage of time. Furthermore, the explication of entropy increase in terms of increasing disorder has been replaced by a more precise notion of phase space volume in conjunction with typicality arguments. 'So we have the following situation: if the asymmetrical treatment of the "initial" and "final" boundary conditions of the universe is a reflection of the fact that time passes *from* the initial *to* the final, then the entropy gradient, instead of explaining the direction of time, is explained by it' [47, italics in original].

9. Conclusion

I have argued that Reichenbach's 'causal' theory is, on closer inspection, an entropic theory of time. His account can meet the two major objections, which have been levelled at the 'causal' theory of time. 1) His characterization of causality in terms of the mark method is a misleading distraction and subject to the objections discussed above. The 'causal' theory really focuses on the exchange of light signals. They are subject to thermodynamic irreversibility. Even the mark method ultimately refers to irreversible interferences [4: Ch. IV]. The murkiness of the notion of causation is thereby avoided but at the price of a limitation to the cone structure of Minkowski space-time. As such it is, however, one candidate for a characterization of the 'before-after' relationship, with which Reichenbach starts his reflection on time. But Reichenbach goes beyond a 'past-future' relationship and considers the global arrow of time. 2) The second objection was that the General theory of relativity allows closed causal loops as solutions to the field equations. Reichenbach answers that theoretical possibilities are not the same as physical realisabilities. Even if a model of space-time is invested with temporal orientability, the model still needs to be tested against physical data. Astronomical data do not reveal causal loops. The universe is more likely to end in a Big Chill than a return to its initial conditions. If that is the case, a more constructive answer arises from his employment of phase space arguments, which should be enhanced by typicality arguments in order to avoid the double standard fallacy. An entropic theory of time remains controversial but it provides Reichenbach with a unified approach to his relational view of time. His concern was with empirical adequacy of the model through which he arrived at a notion of the factual irreversibility of time.

References

1. Reichenbach, H.: Bericht über eine Axiomatik der Einstein'schen Raum-Zeit-Lehre. Physikalische Zeitschrift 22, 683-6 (1921)
2. Reichenbach, H.: Bewegungslehre bei Newton, Leibniz und Huyghens. Kant-Studien 29, 416-28 (1924)
3. Reichenbach, H.: Das Kausalproblem in der Physik. Die Naturwissenschaften 19, 713-22 (1931)
4. Reichenbach, H.: The Direction of Time. Dover (1956)
5. Reichenbach, H.: The Philosophy of Space and Time. Dover (1958). Translation of Philosophie der Raum-Zeit-Lehre. De Gruyer, Berlin/Leipzig (1928)
6. Reichenbach, H.: Axiomatizaton of the Theory of Relativity. University of California Press (1969). Translation of Axiomatik der relativistischen Raum-Zeit-Lehre. Vieweg, Braunschweig (1924)
7. Weinert, F.: The Scientist as Philosopher. Springer, Berlin/Heidelberg/New York (2004)
8. Callender G.: What Makes Time Special. Oxford University Press, Oxford (2017)
9. Arthur, R. T.: Leibniz's Causal Theory of Time Revisited. The Leibniz Review 26, 151-72 (2016)
10. Leibniz, G. W.: Philosophical Writings. G. H. R. Parkinson ed., J. M. Dent & Sons, London (1973), 205-238
11. Winnie, J.: Causal Theories of Spacetime. In: J. Earman/C. Glymour/J. Stachel eds., Foundations of Spacetime Theories. University of Minnesota Press, 134-205 (1977)
12. Smeenk, Ch./Ch. Wüthrich: Time Travel and Time Machines. In: C. Callender ed., Oxford Handbook of Time. Oxford University Press, Oxford (2011): 577-630, Ch. 20
13. Earman, J.: Notes on the Causal Theory of Time. Synthese 24, 74-86 (1972)
14. Van Fraassen, B.: An Introduction to the Philosophy of Time and Space. Columbia University Press (²1985/1970)
15. Kant, I.: Kritik der reinen Vernunft (1781/²1787). English translation: Critique of Pure Reason, translated by N. Kemp Smith. The Macmillan Press, London (1976)
16. Einstein, A.: Reply to Criticisms. In A. P. Schilpp ed., Albert Einstein: Philosopher-Scientist. Open Court, La Salle (Ill.), Volume II, 665-88 (1949)
17. Van Fraassen, B.: Earman on the Causal Theory of Time. Synthese 24, 87-95 (1972)
18. Weinert, F.: The March of Time. Springer, Berlin/Heidelberg/New York (2013)
19. Castagnino, M./L. Lara/O. Lombardi: The Direction of Time: From the Global Arrow to the Local Arrow. International Journal of Theoretical Physics 42/10, 2487-2504 (2003)
20. Aiello, M./M. Castagnino/O. Lombardi: The Arrow of Time: From Universe Time-Asymmetry to Local Irreversible Processes. Foundations of Physics 38, 257-92(2008)
21. Weinert, F.: A Conditional View of Causality. In: F. Russo/J. Williamson eds., Causality and Probability in the Sciences. College Publications, Texts in Philosophy Series 5, London (2007), 415-37
22. Lobo, S.N.: 'Closed timelike curves and causality violation.'arXiv: 1008vl [gr-qc] (2010)
23. Krauss, L.M./R. J. Scherrer: The end of cosmology. Scientific American, 47-53 (March 2008)
24. Schmidt, B. P.: The Expanding and Accelerating Universe. In Cosmic Explosions. Springer Proceedings in Physics Volume 99, Part VIII, 511-23 (2005)
25. Eddington, A.S.: The Nature of the Physical World. Cambridge University Press, Cambridge (1932/1928)
26. Eddington, A. S.: New Pathways in Science. Cambridge University Press, Cambridge (1935)
27. Wheeler, J. A.: It from Bit (1989). In: At Home in the Universe. Springer, Heidelberg/Berlin/New York, 295-311(1996)

28. Stöckler, H. ed.: *Taschenbuch der Physik*. Verlag Harri Deutsch, Frankfurt a. M (2000)
29. Wallace, D.: The Emergent Universe. Oxford University Press, Oxford (2012)
30. Werndl, Ch.: Justifying typicality measures of Boltzmannian statistical mechanics and dynamical systems. Studies in History and Philosophy of Modern Physics 44, 470-9 (2013)
31. Carroll, S.: From Eternity to Here. OneWorld, Oxford (2010)
32. Penrose, R.: The Emperor's New Mind. Vintage, Oxford/New York (1989)
33. Penrose, R.: The Road to Reality. Vintage Books, London (2005)
34. Price, H.: Time's Arrow and Archimedes' Point. Oxford University Press, Oxford (1996)
35. Landsberg, P. T.: A Matter of Time. In P. T. Landsberg ed., The Enigma of Time. Adam Hilger Ltd., Bristol, 60-85 (1982)
36. Kupervasser, O./H. Nikolić/V. Zlatić: The Universal Arrow of Time. Foundations of Physics 42, 1165-1185 (2012)
37. Weinert, F.: Time-symmetric cosmological Gold models revisited. In: International Studies in the Philosophy of Science 30, 231-243 (2016)
38. Popper, K.: The Arrow of Time. *Nature* 177, 538 (1956)
39. Goldstein, Sh. /J. L. Lebowitz: On the (Boltzmann) Entropy of Non-equilibrium Systems. *Physica* D 193, 53-66 (2004)
40. Frigg, R.: Why Typicality Does Not Explain the Approach to Equilibrium. In M. Suárez ed., Probabilities, Causes and Propensities in Physics, Synthese Library: Springer, Heidelberg, 77-93 (2011)
41. Landsberg, P. T.: 'Introduction.' In P. T. Landsberg ed., *The Enigma of Time*. Bristol: Adam Hilger Ltd., 1-30 (1982)
42. Volchan, S. B.: Probability as typicality. Studies in History and Philosophy of Modern Physics 38, 801-14 (2007)
43. Planck, M.: 'Zur Dynamik bewegter Systeme.' Sitzungsberichte der königlichen Preußischen Akademie der Wissenschaften (1907), 542-70; Annalen der Physik 26, 1-34 (1908)
44. Misner, Ch. W./K.S. Thorne/J.A. Wheeler: Gravitation. W.H. Freeman and Company, New York (1973)
45. Schutz, B.: A First Course in General Relativity. Cambridge University Press, Cambridge (^{2}2009)
46. Earman, J.: An Attempt to Add a Little Direction to "the Problem of the Direction of Time". Philosophy of Science 41, 15-47 (1974)
47. Maudlin, T.: The Metaphysics within Physics. Oxford University Press, Oxford (2007)