Unbiased Polarised Parton Distribution Functions and their Uncertainties

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DOI: `http://dx.doi.org/10.3204/DESY-PROC-2012-02/273`

We present preliminary results on the determination of spin-dependent, or polarised, Parton Distribution Functions (PDFs) from all relevant inclusive polarised DIS data. The analysis is performed within the NNPDF approach, which provides a faithful and statistically sound representation of PDFs and their uncertainties. We describe how the NNPDF methodology has been extended to the polarised case, and compare our results with other recent polarised parton sets. We show that polarised PDF uncertainties can be sizeably underestimated in standard determinations, most notably for the gluon.

The interest in spin-dependent, or polarised, Parton Distribution Functions (PDFs) of the nucleon is mainly motivated by the desire to understand its spin structure in terms of its quark and gluon parton substructure. It largely originates from the first EMC results [1], originally interpreted as an indication that quark and anti-quark intrinsic angular momenta only contribute a small fraction of the full nucleon spin. A faithful knowledge of polarised PDFs is also an essential ingredient for exploring QCD beyond the helicity-averaged case and for studying the phenomenology of spin-dependent processes.

Polarised PDFs have been investigated with increasing precision in recent years. On one hand, several experiments have contributed a large amount of data for a variety of processes, mainly inclusive polarised deep-inelastic scattering (DIS) but also proton-proton collisions and semi-inclusive reactions. On the theoretical side, the interest has been ultimately focussed on the global reconstruction of PDFs, together with their uncertainties. At least four groups have constructed such polarised PDF sets recently: BB [2], AAC [3], LSS [4] and DSSV [5]. These sets slightly differ in the choice of datasets, PDF parametrisation and details of the QCD analysis (such as the treatment of higher-twist corrections). Nevertheless, they are all based on simple functional forms of the momentum fraction dependence of the PDFs at the refence scale (typically, a power-like behavior is assumed both at large and small momentum fraction) and on the Hessian approach for the estimate of uncertainties. Two main shortcomings are known to affect this methodology. The first one concerns how to propagate errors consistently from data to fitted parameters and then to observables: this is usually done by assuming Gaussian linear error propagation, which is not always adequate, in particular in those kinematical regions where few data are available. The second one consists in assessing the theoretical bias introduced by a fixed functional parton parametrisation. This is particularly delicate for polarised PDFs, owing to the quantity and the quality of the data, which are respectively less abundant and less accurate than their unpolarised counterparts.

In order to overcome these difficulties, in recent years the NNPDF collaboration has developed a new approach to parton fitting (see, for example, [6, 7, 8, 9, 10] and references therein). This new technique, designed to provide a faithful representation of PDFs and their uncertainties, is based on robust set of statistical tools, including Monte Carlo and Neural Network methods.

In the NNPDF approach, experimental data are sampled by generating an ensemble of Monte Carlo replicas with data probability distribution; individual replicas are allowed to fluctuate in such a way that the mean value, standard deviation and correlation computed over Monte Carlo ensemble reproduce the experimental values, provided the sample is sufficiently large. Fitting an ensemble of parton distributions automatically propagates statistical fluctuations to the PDFs and then to observables. Hence, expectation values and uncertainties of PDFs (or of any observable) are obtained by considering their Monte Carlo integrals over the ensemble of replicas. Furthermore, in this approach neural networks are used as unbiased interpolants for PDF parametrisation. Since they provide functions depending on a large number of parameters, they are very flexible tools: this flexibility allows one to reduce the bias associated to the choice of some fixed functional form.

The NNPDF approach has been succesfully applied to the determination of unpolarised PDFs and these NNPDF sets are routinely used by Tevatron and LHC collaborations for data analysis and data-theory comparisons. We will present here some preliminary results obtained by extending the NNPDF approach to the determination of a set of polarised PDFs. After illustrating the main features of our analysis, we will compare our results to those obtained by other collaborations. Specifically, we will see that the uncertainty on some polarised PDFs, most notably on the gluon PDF, are rather larger than previously estimated.

The first NNPDF analysis of polarised PDFs, `NNPDFpol1.0` henceforth, is based on a comprehensive set of polarised DIS data. We exclude from our analysis data points with $Q^2 \leq Q^2_{\rm cut} = 1$ GeV^2, since below such energy scale perturbative QCD cannot be considered reliable. We also impose $W^2 \geq W^2_{\rm cut} = 6.25$ GeV^2 for the squared invariant mass $W^2 = Q^2(1-x)/x$, according to the study presented in Ref. [11]. This choice removes the dependence of results on possible dynamical higher-twist effects, which we do not include even though we do include target-mass corrections. The dataset used in the `NNPDFpol1.0` analysis is shown, after kinematic cuts, in Fig. 1.

The experimental data used in this fit do not allow a full separation of individual flavour and anti-flavour parton densities. Hence, we define, for each light flavour q, the net amount of quark-antiquark spin density

$$\Delta q(x,Q^2) = q^{\uparrow\uparrow}(x,Q^2) + \bar{q}^{\uparrow\uparrow}(x,Q^2) - q^{\uparrow\downarrow}(x,Q^2) + \bar{q}^{\uparrow\downarrow}(x,Q^2) \ ,$$

where the superscript $\uparrow\uparrow$ ($\uparrow\downarrow$) denotes that the parton spin is parallel (antiparallel) to the proton spin. We parametrise PDFs at the scale $Q_0^2 = 1$ GeV^2 by choosing, besides the gluon density $\Delta g(x,Q_0^2) \equiv g^{\uparrow\uparrow} - g^{\uparrow\downarrow}$, the following three linear combinations of light quarks: the flavour-singlet

$$\Delta\Sigma(x,Q_0^2) \equiv \Delta u(x,Q_0^2) + \Delta d(x,Q_0^2) + \Delta s(x,Q_0^2) \ ,$$

the non-singlet triplet and the non-singlet octet

$$\Delta T_3(x,Q_0^2) \equiv \Delta u(x,Q_0^2) - \Delta d(x,Q_0^2) \ , \qquad \Delta T_8(x,Q_0^2) \equiv \Delta u(x,Q_0^2) + \Delta d(x,Q_0^2) - 2\Delta s(x,Q_0^2) \ .$$

Each of these four combinations is parametrised by a neural network, with a total number of $\mathcal{O}(200)$ parameters, to be compared to $\mathcal{O}(10-20)$ used in other existing fits.

A fast and accurate evaluation of polarised parton distributions, as required by the fitting, is achieved with the `FastKernel` method [12]. The accuracy of polarised PDF evolution has been shown to be $\mathcal{O}(10^{-5})$ comparing with the `HOPPET` code.

Theoretical constraints are taken into account during the fitting procedure. We have imposed positivity of physical cross-sections, which implies that the polarised structure function g_1 is bounded by its unpolarised counterparts F_1, so that $|g_1(x,Q^2)| \le F_1(x,Q^2)$ [13]. For consistency, the unpolarised structure functions have been computed from the recent NNPDF2.1 unpolarised PDF determination [10]. We have also used SU(3) symmetry to relate the first moments

$$a_3 \equiv \int_0^1 dx \Delta T_3(x,Q_0^2) \ , \qquad a_8 \equiv \int_0^1 dx \Delta T_8(x,Q_0^2) \ , \tag{1}$$

to the determination of a_3 and a_8 from baryon decay constants (allowing for large uncertainties). We have also performed a fit in which we have relaxed the first constraint in Eq. 1 and we have considered a_3 as a fit parameter, in which case we have found $a_3 = 1.21 \pm 0.08$ to be compared with the global average from experimental measurement of β-decay, $g_A = 1.2701 \pm 0.0025$ [14]. This result provides a consistency check of the fitting procedure and validates the Bjorken sum rule with an accuracy of about 10%.

We show preliminary results for the `NNPDFpol1.0` set at initial scale $Q_0^2 = 1$ GeV^2 together with DSSV08 [5] and BB10 [2] determinations (Fig. 2). In general, we can see that all PDFs show larger error bands than previously estimated, in particular at very small- or high-x values, where no DIS data are available. This is especially the case for the polarised gluon PDF, which cannot be constrained by the available DIS data. We also notice that, at least for the non-singlet triplet, the `NNPDFpol1.0` analysis seems to agree better with DSSV08 than with BB10.

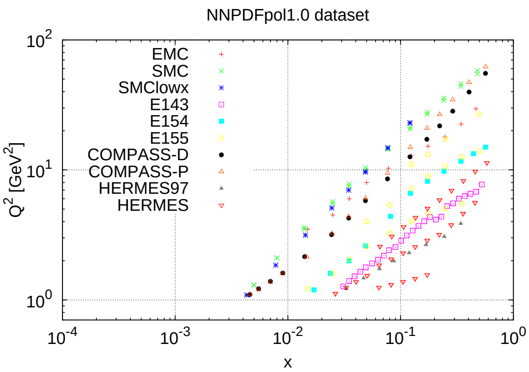

Figure 1: Experimental dataset after kinematic cuts for the `NNPDFpol1.0` analysis.

Finally, we compute the first momenta of polarised singlet and gluon PDFs

$$\Delta\Sigma(Q^2) \equiv \int_0^1 dx \Delta\Sigma(x,Q^2) \ , \qquad \Delta g(Q^2) \equiv \int_0^1 dx \Delta g(x,Q^2)$$

	NNPDFpol1.0	DSSV08 [5]	BB10 [2]	LSS10 [4]	AAC08 [3]
$\Delta\Sigma(Q^2)$	0.31 ± 0.10	0.37 ± 0.04	0.19 ± 0.08	0.21 ± 0.03	0.24 ± 0.07
$\Delta g(Q^2)$	-0.2 ± 1.4	-0.06 ± 0.18	0.46 ± 0.43	0.32 ± 0.19	0.63 ± 0.19

Table 1: The first momenta of the singlet and gluon polarised PDFs at the scale $Q^2 = 4$ GeV^2 in the $\overline{\text{MS}}$ scheme.

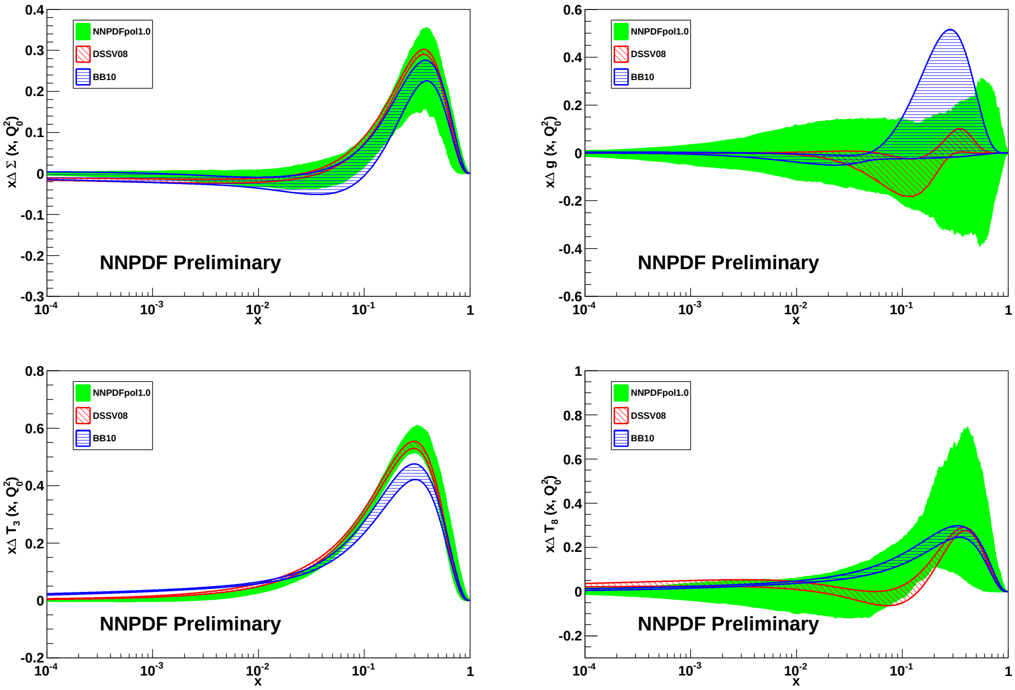

Figure 2: The `NNPDFpol1.0` parton set at the initial evolution scale $Q_0^2 = 1\ \text{GeV}^2$ compared to DSSV08 [5] and BB10 [2] determinations. Uncertainties on `NNPDFpol1.0` parton distributions are computed at 68% confidence level (see Ref. [9] for details).

at the scale $Q^2 = 4\,\text{GeV}^2$ and compare with the results obtained by other collaborations (Tab. 1). Again, we notice the uncertainties of our results: the error on the singlet momentum is between two and four times larger than that from other collaborations, while the error on gluon momentum is almost one order of magnitude larger.

More precise determinations of polarised PDFs will have to resort to data coming from other processes but DIS, such as open charm and jet production in fixed target experiments or inclusive jet and W boson production in proton-proton collisions. We plan to extend our analysis to these data in the near future, and also to use our PDF set to determine the strong coupling constant α_s.

Acknowledgements

The research of J. R. has been supported by a Marie Curie Intra–European Fellowship of the European Community's 7th Framework Programme under contract number PIEF-GA-2010-272515.

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