Chapter 5
Using Fractio Quest in the Mathematics Classroom:
Insights Into Methodological Issues From a Study in Italy

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ABSTRACT

This work offers some insights coming from the use of the application Fractio Quest, which was developed in the Project Gamifying CLIL within a Mathematics Context, within the mathematics classroom. The app was conceptualized in principle for a CLIL approach to fraction learning. Focus is put on two complementary dimensions of usage: a pilot experiment conducted in Italy in 2020, which involved a grade 7 class and its mathematics teacher in using a preliminary English version of the app, and the development of supplementary materials for teachers. The chapter first discusses the challenges posed for teachers and students when the app is used in a foreign language and explores the language/content interplay fostered by the activity. Then, reflections on methodological aspects considered for the design of supplementary materials for teaching are presented. These aspects are framed into the CLIL framework and by following research trends in mathematics education.

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INTRODUCTION

This chapter discusses preliminary insights coming from classroom implementation of the application Fractio Quest, which was developed in the Project "Gamifying CLIL within a Mathematics Context" (CLIL is the acronym for Content Language Integrated Learning). The project is an interdisciplinary project oriented towards an integrated teaching and learning of mathematics and language. It aims to explore the potential of *game-based learning* to support students' numeracy, and linguistic and digital literacy skills, with particular attention to those students for whom the language of classroom instruction is not their main (native) language. The application introduces a narrative to which six different games are referred. The games are intended to provide students with new ways of exploring and engaging with fundamental aspects of fractions, a specific topic of mathematics with which many students experience difficulties (e.g., Pitkethly & Hunting, 1996). In mathematics education literature, these difficulties have been investigated and researchers have highlighted how they can often be related to the multiple interpretations and representations of fractions (Marmur et al., 2020) and how persistent they might be in students, still in secondary school.

In this chapter, we investigate the teaching and learning of fractions in a gamified context, where the language used to propose the mathematical content is not the first language of the learners.

Even though some research investigates the learning of fractions through applications (e.g., Riconscente, 2013), there still is a lack of research encompassing and explicitly addressing conceptual fraction learning, especially in CLIL environments. Our work aims to fill this gap by focusing on methodological issues arisen in the context of the aforementioned project.

The next sections frame these aspects into the CLIL framework and according to main research trends in mathematics education which have investigated the teaching and learning of fractions.

THEORETICAL BACKGROUND

Wide interest in the learning of fraction is shared among researchers in mathematics education and mathematics teachers. From an epistemological point of view, fraction is one possible representation and one crucial meaning of rational numbers among others (Kieren, 1976). Students commonly experience rational numbers in the real world as they deal with measurements or sales receipts, but they usually encounter fractions at school after being mainly acquainted with natural (whole) numbers, and this might create inconsistencies and misconceptions. Whole-number schemes can interfere with the learning of fractions to the extent to which children can apply whole number strategies to fraction problems: they sometimes view fractions as two separate whole numbers and deal with them accordingly (Hart, 1989) and this problematic can be tackled differently (Sun, 2019). More generally, the different ways in which rational numbers can be represented are a considerable source of misconceptions among students, together with the fact that many rules that are consistent within the natural number system are no longer applicable to rational numbers (like the existence of a successor for each natural number). Hart (1980) studied secondary school students at work with fractions and highlighted that, even though most of them could name the fraction which is represented by a region and dominated the initial ideas of the subject, more than a half of the students did not work on fractions as numbers, for example with dimensions expressed as fractions in a geometric problem that asks to calculate the surface of a figure. Other studies in mathematics education pointed out a gap between the use of signs and symbols in mathematics learning and the understanding of the associated concepts, often taking years for learners to grasp the functioning of the number representation system (e.g., Duval, 2006). As noted by Marmur et al. (2020) even the historical development of fractions (which existed for centuries as ideas connected to symbolic representations, before being fully legitimated as numbers) points out an ancient difficulty with the concept, which is not restricted to learners but touches upon teachers too (e.g., Osana & Royea, 2011).

Interestingly, this discourse also speaks directly to the problematic couple of procedural and conceptual in mathematics, which is particularly relevant in the context of fractions. In fact, learners capable of performing procedures to operate with fractions are often not able to provide explanations for their meaning. Argumentation processes are entangled with comprehension but also require the learners to use language in meaningful ways. Moreover, among the many representations of fractions (mainly visual/symbolic), we find natural language, which in turn requires text comprehension, therefore calling for specific attention from a didactical point of view. The same way in which a fraction is read (e.g., in English, ¾ is voiced "three fourths") involves a particular way of treating the numerator (cardinal numeral) and the denominator (ordinal numeral), as well as a different directionality (up-down instead of left-right) mediated by a specific symbol, the fraction bar. As highlighted by Bartolini-Bussi et al. (2014), this is not the same in every language or culture, with strong influences on cognition.

Various linguistic forms can be used to represent fractions, accordingly to their multiple features. The related polysemy in scholastic mathematical practice can create difficulties in fraction learning (Zazkis, 1998). Fandiño-Pinilla (2007) has highlighted that even in everyday language there are many uses of fractions, not necessarily made explicit, for instance for referring to time ("a quarter to ten") or for describing a slope ("a 10% rise"), often far from a scholastic idea of fractions.

In this chapter, the discussion on the relationships between language and fractions as they are treated in mathematics education literature is enriched by insights coming from a CLIL approach, a dual-focused educational approach in which an additional language is used for the integrated teaching and learning of content and language with the objective of promoting both content and language mastery to pre-defined levels (Marsh et al., 2010). Across Europe, the CLIL approach has emerged in the 90s and has been promoted since then, with considerable differences in terms of how, when, and why the integration between language and content is achieved (Marsh, 2012).

The key characteristics of CLIL pedagogies are described by Coyle (2007) using 4 dimensions, or 4Cs: Content, Cognition, Communication, and Culture. This framework leads to a synthesis of good practice based on appropriate content (meaningful, new, relevant); incorporation of intercultural understanding (where culture applies to a wide spectrum of forms of diverse interpretation); processing (personalized, peer-driven, and supported); and progression (sequences of learning scaffolded in relation to content and language, and the thinking demands required for progression in each). It emphasizes the relevance of the development of cognitive skills, creative learning, and collaborative social interaction. Accordingly, here disciplinary content knowledge does not imply the accumulation of knowledge but rather the creative construction of knowledge through generation and production of ideas. In addition, the use of language through communication is understood as social, cultural, and personal at once. Language and its functions play a critical role because knowledge and culture are embedded in language and the use of language is necessary to access knowledge. This also means that CLIL requires learners to use language for different purposes, that is, to learn the language itself, to learn the content, to operate in tasks and activities and to connect thinking skills with language learning. Language therefore acquires a socio-cultural character that recalls the Vygotskyan approach to mediation and to language as a tool for learning (Vygotsky, 1978).

OBJECTIVES

Inspired by this approach, the chapter wants to propose ideas for working on the synergy of language and mathematics, while valuing mathematically relevant concepts and cognitive processes. Didactical reflections are developed in the rest of the chapter, unfolding the discourse along two complementary threads. First, a pilot experiment conducted in Italy in 2020, which engaged a grade 7 class and its mathematics teacher in the use of a preliminary English version of the app Fractio Quest, is presented. We particularly discuss some of the challenges posed for teachers and students when the app is used in a foreign language within the classroom, and the language/content interplay fostered by the activity. Secondly, reflections on methodological aspects considered for the design of supplementary materials for teaching are offered, with specific focus on the development of such materials to support a CLIL approach to fraction learning.

USING FRACTIO QUEST IN THE MATHEMATICS CLASSROOM

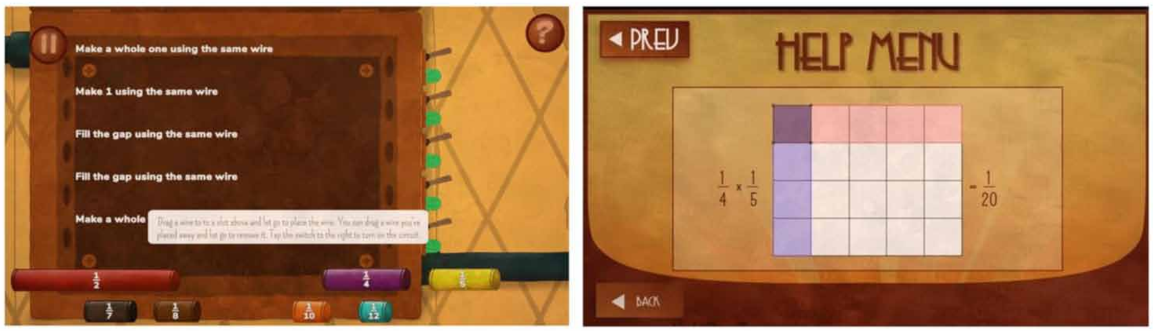
Fractio Quest

The application Fractio Quest introduces the learners to a journey with fractions. Each of the six games that are enclosed in the application focuses on a specific aspect (e.g., equivalent fractions, operations with fractions, conversions between fractions and decimals) and is now available in six different languages (Dutch, English, French, German, Italian and Spanish). The gamified activities are introduced and immersed in a narrative, which unfolds through dialogues between the main characters (Captain Taylor, River the helper, and evil Sky Lord Quinn). The dialogues are offered to users both in written and audio form, to help them develop their knowledge of the language in terms of grammar, vocabulary and listening. At the end of each game, a final worksheet, containing true/false, drag and drop and multiple-choice questions, is available. In this chapter the focus is not on the application itself, neither on the application nor on its design, nevertheless, some features, which are relevant for the rest of the discussion, are highlighted.

The games use multiple representations of fractions, to work with fractions in different registers. For example, in game 1, different fractions are represented with pieces of wires of different lengths, so that fractions can be compared, and equivalent fractions can be easily detected (Figure 1a).

In game 3 the multiplication between fractions is proposed through linear models, but also through the area model (Figure 1b), which may complement the traditional ("cancel-and-multiply") procedure with which the operation is usually carried out in the classroom. For each game, a help menu (with essential information to deal with the mathematical content or representation) and a journal (a log in which the students can answer the mathematical questions asked by the characters at the end of each level) complement the features of the application, providing the user with some guidance and the possibility to track her understanding. It is important to note that the games are not intended to substitute, but to complement, the teachers' lesson, and are so developed to trigger, and stimulate learning in a playful environment.

Figure 1. a) Fractions represented as wires to be assembled in game 1; b) the area model used to represent multiplication between fractions, in the first version of the game used in the pilot.



a *b*

Pilot Experiment

After the first (beta) version of the app was available, a pilot experiment was conducted in Italy with a class of 29 grade 7-students (from Scuola Internazionale "E. Agnelli" in Torino). The mathematics teacher of the class, S., agreed to collaborate with the researchers (the authors) in piloting the use of the app within the classroom. A teacher interview was conducted before the start of the experiment. The classroom-based intervention (Stylianides & Stylianides, 2013) consisted of 3 sessions (planned to be held in 2020 on February 17, 20 and 27) of 1 hour and 50 minutes each, during the regular mathematics lessons. Each session was designed to approach two out of the six games available in the application. Unfortunately, due to school closure because of the COVID outbreak, only two of the three sessions did take place, and a final interview with the teacher did not happen.

The purpose of the pilot experiment was twofold: on the one hand, gaining feedback on the application from students and teacher to better study potentialities and limitations; on the other hand, investigating how to use the application inside the classroom, gaining insights into students' and teacher's needs.

Teacher Interview

Before the intervention started, a semi-structured interview with the teacher involved in the pilot was conducted, to gain insights into her expectations about the project, the current school policies about Game-Based Learning (GBL) and CLIL, and her perception about possibilities of integrating the application in her teaching practice.

S. is an experienced mathematics teacher who collaborates with our research group. She is graduated in mathematics and teaches mathematics at upper secondary school and mathematics and science at lower secondary school. Before the interview, S. had the possibility to access the first version of the application and had time to explore and envision how to propose it to her students during her curricular mathematics lessons.

During the interview, S. described the GBL policy of her school: an additional hour of "mathematics experimentation" established every week. S. was trying to use a different methodology during that hour, including problem solving in groups or collective playing of games.

Concerning CLIL policy, the school was proposing one out of the two curricular hours (lessons) of geography in English (only for classes that choose this option). S. reported that soon this would have been also done for science, even if CLIL policies are restricted to English language.

S. expressed high expectations about the project, believing that this approach could be *captivating* for the students. She supposed that it could be very positive for her students. They have been studying English since primary school, so it probably will not be very difficult for the students to tackle, but at the same time they must be somehow prepared to this new way of working. She also anticipated that probably for some students the different language could constitute an issue. At the same time, she noticed that the games are very intuitive, so S. expected that some students would have been dealing with the games by understanding the mathematical ideas, and then would have realised that they should have improved their English lexicon, or vice versa. She was particularly interested in observing the interactions between the two subjects, that is for her the core idea of a CLIL methodology.

Concerning the use of the games within the classroom, S. planned with the researchers (the authors) to have couple of students working on the same tablet during the two-hour lesson, then to discuss collectively the issues that arose from their game play. She was keen on orchestrating the activities this way, as it is how she often structures her lessons. S. also mentioned that she is used to keep the pace of the students, following their problems and questions, even though the class is numerous (29 students). For her, this is the best approach, since giving restricted time with the anxiety of finishing everything according to a specific timetable on some occasions can be counterproductive. S. especially wanted to take advantage of peer interactions in each couple and in-between students more generally in the collective discussions.

The interview pointed out S.'s great interest concerning the potentiality of the activities, in line with the current policies that the school was progressively establishing concerning the integration of new methodologies in classroom practice and the shift towards specific content approached in a new language. Despite this interview was highly contextualized, it is true that many schools (in Italy) try to implement new methodologies in their policies, and this is promising for the project's focus on providing teachers with means and instruments to do so. Moreover, the intervention appeared to be perfectly integrating the methodologies adopted by the teacher in her usual classroom orchestration. It might be then interesting to study in the future different ways in which teachers envision the same (or similar) activities to take place. All these elements were important to set the ground for the intervention, which is briefly introduced in the following section.

Methodology of the Intervention

The students worked in group (pairs) each of which had a tablet (provided by the school) with the application installed in the English version and ready to use.

The meetings started with a collective introduction to a game, then each group played separately the game, at their own pace. At the end, after about 45 minutes, the teacher opened space for a class discussion, led with the support of the researchers.

The students were provided with a two-page glossary and worksheets The glossary contained words that were expected to be unknown to students (for example, "storehouse" and "engine pipes"), with English alternatives and Italian translations. We also provided the students with a text containing the initial part of the story that introduces each game, the list of questions posed inside the game and a space to keep track of their discoveries. The class discussions were video recorded, and a focus group was chosen based on the advice of the teacher.

Problems and Opportunities: The Role of Representations

The intervention gave interesting insights concerning the applications (technical, or in terms of design), its use (related cognition and pedagogy) and of possible classroom implementations (methodology). One issue that lies at the intersection between all these aspects and is particularly relevant concerning fraction learning (considering the theoretical background of this study) is that of the representations of fractions used inside the application. We propose and analyse here two micro-episodes from the classroom that illustrate problems and opportunities concerning some representations teachers and students engaged with.

Micro-Episode 1

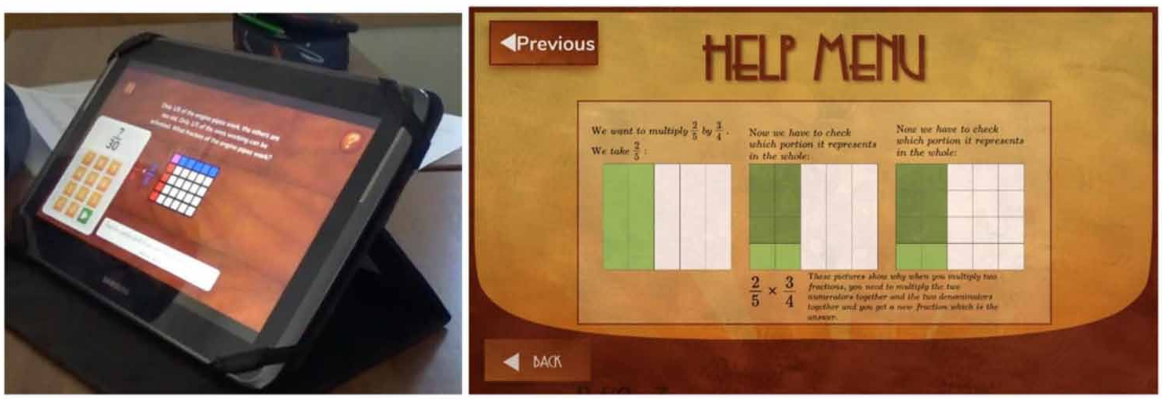
In the last level of game 1, two instruments were available: one to cut in half, another to double the wires (that were representing fractions, as in Figure 1a). Clicking on a wire representing 1/3 and then on the first instrument you obtain a new wire representing 1/6 ($1/3 \div 2$), while clicking on the second instrument you obtain a new wire of length 2/3 ($1/3 \cdot 2$).

The teacher observed that many students were not able to address this new task, which required to use such instruments to then represent a given fraction with a specific number of wires of the same length. She then proposes an example at the blackboard for the class to discuss: “How to have a ⅔ wire using 6 wires of the same length?”. The difficulty created by this part of the game and the question asked by the teacher, which was prompted by the students’ interactions with the digital environment, are interesting for many reasons. First, from a mathematical point of view, we can think of the question as a division ($2/3 \div 6 = 1/9$) which is moving forward the focus on equivalent fractions and creating opportunities for a less procedural approach to fractions learning. Secondly, most of the students did not understand how to proceed because they did not look for more information about the instruments (in the help menu), showing once again how language was fundamental for understanding the mathematical request. This prompted then the research team to modify the design of this level and the given instruction. As a last point, other students seemed to show misunderstandings about 1/3 being the half of 1/6 (while it is the double of 1/6): this delicate point was highlighted by playing the game, and was useful for the teacher, who immediately planned an intervention to resume the students’ ideas around this; at the same time, the application seems to foster conceptual work moving towards a more relational view of fractions.

Micro-Episode 2

The focus group is dealing with the following task of game 3: “Only 1/6 of the engine pipes work, the others are too old. Only 1/5 of the ones working can be activated. What fraction of the engine pipes work?” with the corresponding representation (see Figure 2a). The students are asked to read the question and insert in the calculator the numerator of the fraction that solves the problem (the denominator is already given, 30).

Figure 2. a) The task of game 3 faced by the focus group; b) the help menu added to illustrate the representation.



a *b*

As the researcher (R) approaches and asks what they are doing, the two girls (A & V) start translating in Italian the question.

R: Only one sixth of the total works, right?
A: Only one sixth can be activated (*pause*) ...
R: Only one sixth can be activated
A: It's a third…
V: So… one sixth works, so a sixth of thirty is five… one!
A: Can't it be one thirtieth?
R: Why?
A: Because, well, six times five is thirty
V: Because, divided by six, there are six sixths, only one sixth works, a sixth of thirty is five, only a fifth can be activated (*indicates with the right hand his left hand, open*), a fifth of five is one (*makes one with her thumb*), and then, one (*A enters 1 into the keypad*).
A: Hm, and does the drawing there help you in any way?
A: Yes, because it colours it for us (*points to the purple square*)
V: However, it confused me a bit, because of the five (*points to the red squares*), that is, the one sixth is here (*points to the blue squares*) ... and the fives are out of the one sixth.
A: The fives are out of the sixth?
V: That is, the fact that there is the purple square showing me how many there are, helps me, however, the sixth is here, I don't understand what these four are for (*pointing to the four red quares*)

In the brief interaction with the researcher, the girls then articulate their difficulties saying that it would be better if the red squares were blue and vice versa. The researcher observes they must understand what fraction is represented in the different colours, not count the squares of each colour. The girls then assert they have understood and correctly connect each fraction to its representation.

Aside, V tells the researcher:

V: What was not clear to me is that one fifth was one fifth of the result of a sixth, but there was one fifth of the total, a fifth of 30, that's what confused me.

In the following, A approaches the task by simply counting the number of purple squares, while V tries to interpret the task using the given representation, but she is often overruled by A, who has found an easy key to the solution, and does not want to engage in this more difficult way of reasoning.

In the first part of the transcript, we see that the students are able to reason on the task, and correctly perform the calculations needed to solve it, but are confused by the representation, as it seems not to match with their own way of representing the situation. V expressed her disappointment because she expected one fraction to be "inside" the other, while "the fives are out of the one sixth" and "there was one fifth of the total".

This points to the importance of representations and the possible conflicts that could emerge. At the same time, this small episode reveals opportunities for engaging with new ways of representing fractions and their potentiality for the classroom activity.

From the point of view of design, the help menu has then been revised, with clearer explanations; the denominator has been removed, so that the solution was not simply given by counting the purple squares and the students could engage deeply with the task. In conclusion, it was clear from the pilot experiment that the students were meaningfully engaged with different kinds of representations, and this created opportunities for the teacher to deepen interesting concepts with the class, and for the students to be challenged in their conceptual understanding, but more attention and more explanations were needed in structuring the materials.

Development of Supplementary Materials

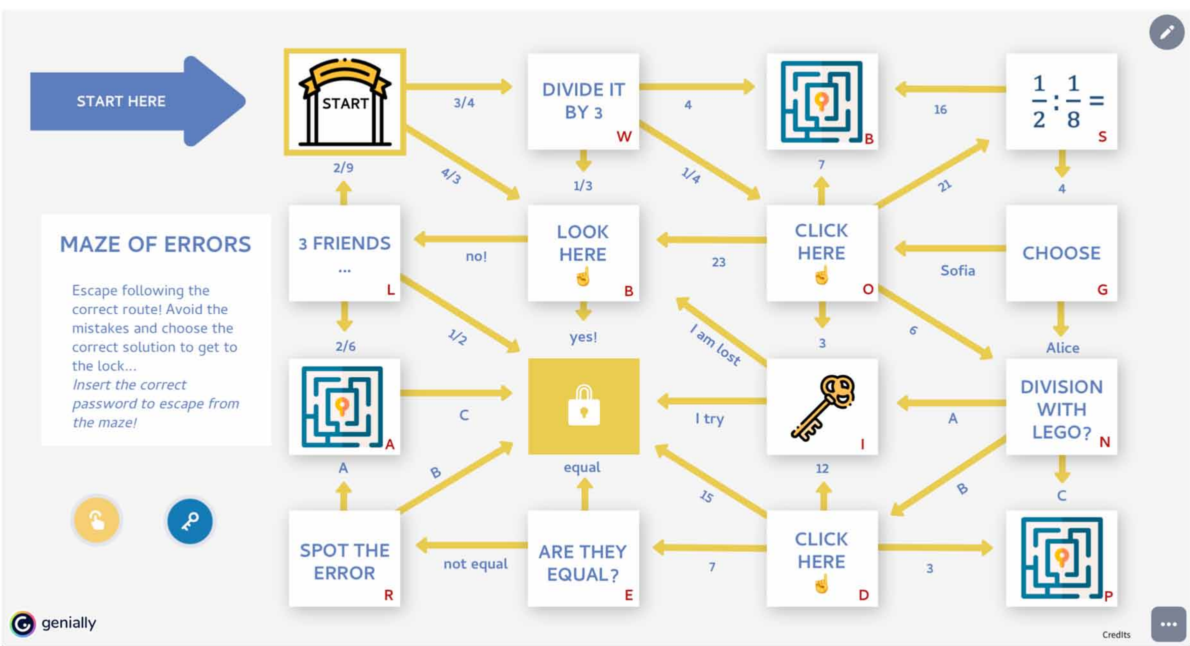
As a further step in the project, a set of supplementary materials was developed, to support teachers with a CLIL approach to fraction learning. In this section, we enrich our previous methodological considerations by focusing on the design of such materials, discussing an example of activity offered as a supplementary material for teaching. In our specific case, language is meant to be implicated in the app both in and for mathematics. The core idea that guided the design was to create upon the synergy of language and mathematics while keeping an eye on what is mathematically relevant and the cognitive processes that we wanted to be triggered. Communicating mathematics is relevant, along with working on explanations and arguments in mathematics, and solving mathematical problems and questions. A standpoint for fostering the delicate interrelations between language and mathematics is exactly argumentation. Indeed, being able to explain thoughts is crucial for mathematical thinking and understanding and far more important when we want to share mathematical ideas with others (especially if this happens in another language).

For the design of materials, we have also considered the 4 Cs framework already mentioned in the theoretical section, together with four different elements that we see as relevant in mathematics education, as they might offer students significant learning experiences. These elements are identified according to the kind of cognitive process stimulated (the first two) or the material activity associated (the last two). They are respectively: Arguing, Problem Solving, and the Use of Manipulatives or Digital Resources. When the rationale of an activity was centred on one of the four elements, we were able to construct the corresponding supplementary material exploiting ideas related to that specific aspect. In the following section we offer an example of developed material, which we find significant for its focus on errors (e.g., Kerslake, 1986).

An Example of Supplementary Material: The Maze of Errors

Using main results from mathematics education literature on fractions, a series of tasks embedded into a maze structure were developed, which the students can navigate opening an online page. The resource (see Figure 3; https://view.genial.ly/6121117070ad990dbae8fdda/interactive-content-maze-of-errors) has been developed using an online platform and consists of different blocks each containing a task or information about how to proceed in the maze, and a series of arrows connecting blocks that are close to each other. The users must solve the task of the chosen block and select the correct answers to be able to find the password for unlocking the lock. The password is a word constructed solely by the corresponding letters got by the solved blocks, in the correct order.

Figure 3. Screenshot of the initial page of the resource "Maze of errors".

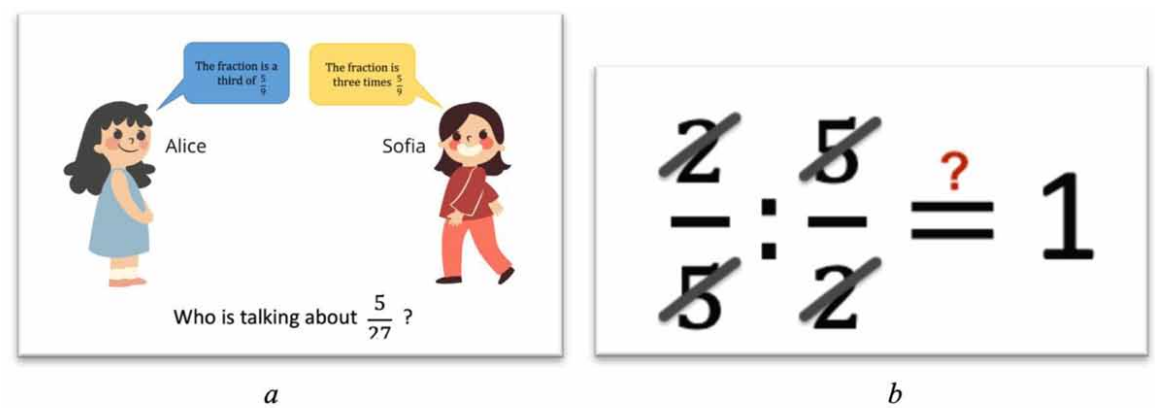


The activity is enclosed in game 4 bundle of activities that refers mainly (but not solely) to the division between fractions. Some examples of tasks are here discussed, to unfold and describe how the strict relationships between language and mathematics was exploited inside the activities.

Argumentation: Who Is Right?

Explaining your thinking is often a necessary step to evaluate a situation and then taking a decision: it is the case of the task presented in Figure 4a, where the student must evaluate two different sentences and establish which of the two refers to a specific fraction. The task uses a specific mathematical terminology to express two inverse process (a third, three times) that are expresses by two operations that are one the inverse of the other (division and multiplication). It has been already discussed (in the case of doubling and halving) that this might represent an obstacle for the students that confuse the two operations, especially when fractions are involved. This task aims at triggering process of argumentation in a contest of decision making in a playful environment: the teacher can use this same task in the classroom to have the students explain their thoughts and compare their strategies. Moreover, Pothier and Sawada (1983) have shown that algorithmic halving is a powerful strategy in partitioning processes but can prevent the children to deal with fractions with odd denominators, so it is important for students to deal with other kind of strategies to consolidate their knowledge on operation with rational numbers.

Figure 4. (a)-(b) Two tasks of the resource "Maze of errors".



Without Words: Typical Errors in Performing Divisions

In the case of fraction division, the typical approach consists of remembering a particular rule, which differs from the one the students are acquainted with whole number division (e.g., invert and multiply; Sun, 2019). One error that might arise from this procedural approach is that of forgetting one passage, so the students sometimes multiply without inverting the second fraction or divide after having flipped numerator and denominator of the dividend. The task presented in Figure 4b asks the students to evaluate a division where such errors have been committed and assess if the given result is correct. This allows for having the students reasoning on their own practices, looking for mistakes that hide conceptual misunderstandings and evaluating a situation where possibly have made a mistake (without putting too much pressure on the subject). In this case no words are used, so that the focus is entirely on making sense of the given symbolic relationship.

CONCLUSIVE REMARKS

In this chapter some methodological challenges and proposals for working inside the classroom with an approach in line with a CLIL approach were discussed. In the first part this was tackled by presenting insights from a pilot study conducted in Italy in which a first version of the application Fractio Quest was used in the classroom. The teacher's interview revealed a huge interest in developing such kind of intervention, which are in line with current policies and methodological innovations strategies that are slowly entering the schools. Methodological reflections have been discussed starting from micro-episodes from the classroom that highlighted challenges and opportunities to this approach, particularly concerning the use of representation in the application. Then, an example of supplementary material for teachers has been briefly described to highlight the ways in which the synergy between mathematics and language has been exploited through the design, drawing on literature in mathematics education. These materials have not been tested in the classroom yet, therefore this chapter mainly focuses on methodological issues regarding activity design. The materials will first be tested with groups of teachers and then used for classroom implementation.

For the sake of space, the engagement of the students has not been discussed but is of course an important aspect since the activities were embedded in game-based learning environment. Even though evidence has not been collected, the teacher and the researchers noted an increasing engagement of the students, especially concerning those games that stimulated creativity and explorations. It was also noted that the students at first were mainly relying on the glossary they have been given, while in the second meeting they were already trying to guess more in their reading. In other words, reading comprehension was more guided by their mathematical investigation, which was interpreted as an increasing fluency with the new language and a positive synergy between content and language in the activity. These aspects also need further investigation and will possibly constitute a new line of research.

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ADDITIONAL READING

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KEY TERMS AND DEFINITIONS

CLIL: Acronym for Content Language Integrated Learning.

Content Language Integrated Learning: Dual-focused educational approach in which an additional language is used for the learning and teaching of content and language with the objective of promoting both content and language mastery to predefinite level (Maljers et al., 2010).

Fraction Literacy: Mathematical skills and knowledge about fractions.

Game-Based Learning: Learning that is facilitated by the use of a game taking advantage of its playful engagement (Whitton, 2012).

Mathematical Representation: Any modality (numerical, graphical, verbal, symbolic, …) in which a mathematical concept or idea can be displayed and expressed. Representations are pervasive and crucial in the teaching and learning of mathematics.