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Splitting recursion schemes into reversible and classical interacting threads

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Abstract. Given a simple recursive function, we show how to extract from it a reversible and an classical iterative part. Those parts can synchronously cooperate under a Producer/Consumer pattern in order to implement the original recursive function. The reversible producer is meant to run on reversible hardware. We also discuss how to extend the extraction to a more general compilation scheme.

1 Introduction

Our goal is to compile a class of recursive functions in a way that parts of the object code produced can leverage the promised green foot-print of truly reversible hardware. This work illustrates preliminary steps towards that goal. We focus on a basic class of recursive functions in order to demonstrate its feasibility.

Contributions. Let recF[p,b,h] be a recursive function defined in some programming formalism, where p is a predecessor function, h a step function, and b a base function. We show how to compile recF[p,b,h] into itFCls[b,h] and itFRev[p,pInv] such that:

$$recF[p,b,h] \simeq itFCls[b,h] \parallel itFRev[p,pInv]$$
, (1)

where: (i) "\sigma" stands for "equivalent to"; (ii) itFCls[b,h] is a classical for-loop that, starting from a value produced by b, iteratively applies h; (iii) itFRev[p, pInv] is a reversible code with two for-loops in it one iterating p, the other its inverse pInv; (iv) "||" is interpreted as an interaction between itFCls[b,h] and itFRev[p,pInv], according to a Producer/Consumer pattern, where itFRev[p, pInv] produces the values that itFCls[b,h] consumes to implement the initially given recursion recF[p,b,h]. In principle, itFRev[p,pInv] can drive a real reversible hardware to exploit its low energy consumption features.

In this work we limit the compilation scheme (1) to use: (i) a predecessor p such that the value p(x)-x is any constant Δ_p equal to, or smaller than, -1; (ii) recursion functions recF[p,b,h] whose condition identifying the base case

is x<=0 instead than the more standard x==0; this means that more than one base non positive value for recF[p,b,h] exists in the interval $[\Delta_p + 1,0]$. This slight generalization will require a careful management of the reversible behavior of itFRev[p,pInv] and its interaction with itFCls[b,h] in order to reconstruct recF[p,b,h].

Contents. Section 2 sets the stage to develop the main ideas about (1), restricting recF[p,b,h] to a recursive function that identifies its base case by means of the standard condition x==0; this ease the description of how itFRev[p,pInv] and itFCls[b,h] interact. Section 3 extends (1) to deal with recF[p,b,h] having x<= 0, and not x==0, to identify its base case(s); this impacts on how itFRev[p,pInv] must work. In both cases, the programming syntax we use can be interpreted into the reversible languages SRL [3,4] and RPP [5,6,4], up to minor syntactic details. Section 4 addresses future work.

Fig. 1. The recursive function recF.

```
/*** Assumption: the inital value of x is 3 */
                  // ==2
    x = p(x)
                  // ==1
    x = p(x)
                  // ==0
    x = p(x)
    y = b(x)
                  // ==b(p(p(g(3))))
                  // ==h(p(p(3))),b(p(p(3))))
    y = h(x,y)
    x = pInv(x)
                  // ==pInv(p(p(3))) ==p(p(3))
    y = h(x,y)
                  // ==h(p(p(3)),h(p(p(p(3))),b(p(p(p(3))))))
    x = pInv(x)
                  // ==pInv(p(g(3)))==p(3)
    y = h(x,y)
                  // ==h(p(3),h(p(p(3)))
10
11
                             h(p(p(3))),b(p(p(3)))))
    x = pInv(x)
                  // ==pInv(p(3))==3
12
                  // ==h(3,h(p(3),h(p(p(3)))
    y = h(x,y)
13
                   // h(p(p(3))),b(p(p(3))))))
```

Fig. 2. Iterative unfolding recF(3): the bottom-up part.

2 The driving idea

Let recF[p,b,h] in (1) have a structure as in Fig. 1 where b(x) is the base function, h(x,y) the step function, p(x) the predecessor x-1, and c(x) the condition x==0 to identify a unique base case.

Fig. 2 details out h(3,h(p(3),h(p(p(3)),h(p(p(p(3))),b(p(p(p(3)))))))), unfolding of recF(3). Every comment asserts a property of the values that x or y stores. Lines 2-4 unfold an iteration that computes p(p(p(3))), which eventually sets the value of x to 0. Line 5 starts the construction of the final value of recF(3) by applying the base case of recF, i.e. b(x). By definition, let pInv denote the inverse of p, i.e. pInv(p(z))==p(pInv(z))==z, for any z. Clearly, in our running example, the function pInv(x) is x+1. Lines 6-13 alternate h(x,y), whose result y, step by step, gets closer to the final value recF(3), and pInv(x), which produces a new value for x.

```
s = 0, e = 0, g = 0, w = 0
     w = w + x;
     for (i = 0; i \le w; i++)
                                    {
                (x > 0) \{ g++; \}
       if
       else if (x==0) \{ e++; \}
       else
                       { s++; }
       x = p(x);
    for (i = 0; i \le w; i++)
                                               {
       x = pInv(x);
10
       if
                (x > 0) \{ g - -; y = h(x,y); \}
11
       else if (x==0) { e--; y = b(x);
12
       else
                       { s--;
13
    w = w - x;
```

Fig. 3. Iterative itF equivalent to recF.

Let us call itF the code in Fig. 3. It implements recF by means of finite iterations only. Continuing with our running example, if we run itF here above starting with x==3, then x==0 holds at line 8, just after the first for-loop; after the second for-loop y==recF(3) holds at line 14.

The code of itF has two parts. Through lines 2–7 the variable g counts how many times x remains positive, the variable e how many it stays equal to 0, and the variable s how many it becomes negative. In this running example we notice that x never becomes negative, for the iteration at lines 3–7 is driven by the value of x which, initially, we can assume non negative, and which p(x) decreases of a single unity. We shall clarify the role of s later. Lines 9–13 undo what lines 2–7 do by executing pInv(x), g--, e--, s--, i.e. the inverses, in reversed order, of p(x), g++, e++, s++. So the correct values of x are available at lines 12, and

4

11, ready to be used as arguments of b(x) and h(x,y) to update y as in Fig. 3, according to the results we obtain by the recursive calls to recF.

```
s = 0, e = 0, g = 0, w = 0
   w = w + x;
   for (i=0; i<=w; i++)</pre>
             (x>0) { g++; } //number of times x is 'g'reater than 0
     else if (x==0) { e++; } //number of times x is 'e'qual to 0
                     { s++; } //number of times x is 's'maller than 0
     else
     x = p(x);
                              }
                                                                     {
   for (i=0; i<=w; i++)</pre>
     x = pInv(x);
10
             (x> 0) { g--; /* Value of x for h available here */ }
     else if (x==0) { e--; /* Value of x for b availabe here */}
                     { s--;
w = w - x;
```

Fig. 4. Reversible side of itF.

Now, let us focus on the main difference between Fig. 4 and Fig. 3.

Both x=b(x) and y=h(x,y) at lines 12, and 11 of **Fig. 3** are missing from lines 12, and 11 of **Fig. 4**. Dropping them let **Fig. 4** be the *reversible side* of itF; calling b(x) and h(x,y) in it generatesy, which is the result we need, so preventing the possibility to reset the value of every variable dealt with in **Fig. 4** to their initial value. This is why we also need a *classical side* of itF that generates y in collaboration with the *reversible side* in order to implement the initial recF correctly.

Fig. 5. Classical side of itF: the consumer itFCls.

The previous observations lead to Fig. 5 which defines the *classical side* itFCls of recF, and to Fig. 6 which defines the *reversible side* itFCRev of recF.

```
s = 0, e = 0, g = 0, w = 0;
    x = /* Inject here the value of x at line 2 of itFCls */
    w = w + x;
    for (i = 0; i<=w; i++)</pre>
              (x>0) \{ g++; \}
      if
      else if (x==0) { e++; }
                      { s++; }
      else
      x = p(x);
    /* itFCls probes here g which has the number of iterations */
    for (i = 0; i<=w; i++)</pre>
10
      x = pInv(x);
             (x>0) { g--; /* itFCls probes here the
      if
                                first argument value of h */ }
      else if (x==0) { e--; /* itFCls probes here the
14
                                 argument value of b
                                                            */ }
                                                               } }
      else
                      { s--;
16
    w = w - x;
```

Fig. 6. Reversible side of itF updated to be the producer itFRev of the values that the consumer itFCls needs.

So, here below we can illustrate how itFCls and itFRev synchronously interact, itFRev producing values, itFCls consuming them as arguments of b(x) and h(x,y).

Line 2 of itFCls is the starting point of the synchronous interaction between itFCls and itFRev; its comment:

```
/* Inject the current x at line 2 of itFRev to let it start */
```

describes what, in a fully implemented version of itFCls, we expect in that line of code. The comment says that itFCls injects (sends, puts) its input value x to line 2 of the reversible side itFRev (cf. Fig. 6). Once itFRev obtains that value at line 2, as outlined by:

```
/* Inject here the value of x from line 2 of itFCls */
```

its for-loop at lines 4–8 executes.

After line 2, itFCls stops at line 3. It waits for itFRev to produce the number of times that itFCls has to iterate line 7. Accordingly to:

```
/* Probe line 9 of itFRev to get the number of iterations to execute */
itFRev makes that value available in its variable g at line 9:
```

```
\slash * itFCls probes here g which has the number of iterations */ .
```

Once gotten the value in iterations, itFCls proceeds to line 5 and stops, waiting for itFRev to produce the argument of b which is eventually available for probing at line 14 of itFRev.

Once the argument becomes available b is applied, and itFCls enters its for -loop, stopping at line 7 at every iteration. The reason is that itFCls waits for line 12 in itFRev to produce the value of the first argument of h(x,y). This interleaved dialog between line 7 of itFCls and line 12 of itFRev lasts iterations times.

```
Fix recG(x) {
   if (x<=0) { b(x); }
   else { h(x,recG(p(x))); } }</pre>
```

Fig. 7. The generic structure of recG.

3 From recursion to iteration

We now generalize what we have seen in Section 2. Inside (1) we use recG of Fig. 7 instead than recF of Fig. 1. This requires to generalize Fig. 6.

From the introduction we recall that, given a *predecessor* p(x), we define $\Delta_p = p(x)-x$, which is a negative value. In this section Δ_p can be any *constant* k <=-1, not only k ==-1; this requires to consider the slightly more general *condition* x <=0 in recG. For example, let p(x) be x-2. The computation of recG(3) is h(3,h(p(3),h(p(p(3)),b(p(p(3)))))) which looks for the least n of iterated applications of p(x) such that p(...p(3)...) <=0; in our case we have 2==n <3.

Fig. 8 introduces it 6 which generalizes it F in Fig. 3.

The scheme itG iteratively implements any recursive function whose structure can be brought back to the one of recG. We remark that line 1 in Fig. 8 initializes ancillae s, e, g, and w, like Fig. 3 initializes the namesake variables of itF, but line 2 of itG has new ancillae z, predDivX, and predNotDivX.

We also assume an initial non negative value for x. The reason is twofold. Firstly, it keeps our discussion as simple as possible, with no need to use the absolute value of x to set the upper limit of every index i in the for-loops that occur in the code. Second, negative values of x would widen our discussion about what a classical recursive function on negative values is and about what its reversible equivalent iteration has to be; we see this as a very interesting subject connected to [1], which is much more oriented than us to optimization issues of recursively defined functions.

We start observing that line 3 of itG sets w to the initial value of x; the reason is that every for-loop, but the one at lines 10–12, has to last x+1 iterations, and x changes in the course of the computation; so, w stores the initial value of x and stays constant from line 4 through line 21. In fact it can change at lines 22–33. We will see why, but w is eventually reset to its initial value o at line 36.

```
s = 0, e = 0, g = 0, w = 0;
    z = 0, predDivX = 0, predNotDivX = 1;
    w = w + x; /* x is assumed to be the input */
    for (i = 0; i <= w; i++) {
      if (x > 0) \{ g++; \}
      else if (x == 0) \{ e++; \}
                        { s++; }
      else
                                   }
      x = p(x);
    for (i = 0; i < e; i++)</pre>
                                               {
10
      predDivX = predDivX + predNotDivX;
11
      predNotDivX = predDivX - predNotDivX; }
12
13
    for (j = 0; j < predDivX; j++)</pre>
                                                 {
14
      for (i = 0; i <= w; i++)
15
         x = pInv(x);
16
         if (x > 0) \{ g--; y = h(x,y); \}
17
         else if (x == 0) \{ e--; y = b(x);
18
                           { s--;
                                               }}}
19
         else
20
    for (j = 0; j < predNotDivX; j++)</pre>
                                                                    {
21
22
      for (i = 0; i <= w; i++)</pre>
                                                                   {
23
        x = pInv(x);
24
25
              (x > 0) \{ g--; 
                             x = p(x);
26
                             if (z < 0) {
27
                             else if (z == 0) \{ y = b(x); z++; \}
28
                                               {y = h(x,y);}
                             else
                                                                 }
29
                                                                  }
                             x = pInv(x);
30
         else if (x == 0) { e--;
                                               }
31
                                               }}
32
         else
                           { s--;
                                                                    }
33
    for (i = 0; i < predNotDivX; i++) {</pre>
34
      z--;
35
    w = w - x;
36
    /* y carries the output */
```

Fig. 8. The iterative function itG.

With the here above assumptions, given a non negative x, and in analogy to itF, the for-loop at lines 4–8 of itG iterates the application of p(x) as many times as w+1, i.e. the initial value of x plus 1. So, the value of x at line 9 is equal to $w+(w+1)*\Delta_p$ which cannot be positive. In particular, all the values that x assumes in the for-loop at lines 4–8 belong to the following interval:

$$I(\mathbf{w}) \triangleq [\mathbf{w} + (\mathbf{w} + \mathbf{1}) * \Delta_{\mathbf{p}}, \mathbf{w} + \mathbf{w} * \Delta_{\mathbf{p}}, \dots, \mathbf{w} + \Delta_{\mathbf{p}}, \mathbf{w}]$$
(2)

from the least to the greatest; the counters g, e, s say how many elements of I(x) are greater, equal or smaller than 0, respectively. Depending on 0 to belong to I(x) determines the behavior of the reminder part of itG, i.e. lines 10-36.

We distinguish two cases in order to illustrate them.

First case. Let $w \% \Delta_p == 0$, i.e. the integer value Δ_p divides with no reminder the initial value of x that we find in w. So, $0 \in I(x)$, which implies the following relations hold at line 9:

$$g = -\frac{w}{\Delta_p}$$
 $s = (w+1)-g-e$. (3)

Fig. 9. A possible replacement of lines 10–12 in Fig. 8.

Lines 10–12 execute exactly once, swapping predDivX and predNotDivX. As a remark, we could have well used the if-selection in Fig. 9 (a construct of RPP) in place of the for-loop at lines 10–12, but we opt for a more compact code.

Swapping predDivX and predNotDivX sets predDivX==1 and predNotDivX==0, computationally exploiting that Δ_p divides w with no reminder: the for-loop body at lines 15–19 becomes accessible, while lines 22–33, with for-loops among them, do not. Lines 15–19 are identical to lines 10–16 of itF in Fig. 4 which we already know to correctly apply b(x) and h(x,y) in order to simulate the recursive function we start from.

As a second case. Let $\sqrt[m]{\Delta_p} != 0$, i.e. the integer value Δ_p divides the initial value of x that w stores, but with some reminder. So, $0 \notin I(x)$, which imply:

$$e == 0$$
 $g == -\left\lfloor \frac{w}{\Delta_p} \right\rfloor$ $s == (w+1)-g-e$ (4)

hold at line 9. Lines 11–12 cannot execute, leaving predDivX and predNotDivX as they are: lines 22–33 become accessible and the for-loop at lines 15–19 does not. Line 22 increments w to balance the information loss that the rounding of g in (4) introduces; line 33 recovers the value of w when the outer for-loop starts. The if-selection at lines 25–32 identifies when to apply b(x), which must be followed by the required applications of h(x,y). We know that $0 \notin I(x)$, so x==0 can never hold. Clearly, s-- is executed until x>0. But the first time x>0 holds true we must compute b(p(x)), because the base function b(x) must be used the last time x assumes a negative value, not the first time it gets positive; lines 26–30 implement our needs. Whenever x>0 is true, the value of x is one step ahead the required one: we get one step back with line 26 and, if it is the first time we step back, i.e. z==0 holds, then we must execute line 28. If not, i.e. z!=0, we must apply the step function at line 29. Line 30, restores the right value of x. Finally, the for-loop at line 34 sets z to its initial value.

At this point, in order to obtain the fully reversible version of Fig. 8 we must think of replacing the calls to h(x,y) and b(x) at lines in 28, and 29 by means of actions that probe the value of x, in analogy to Fig. 6, lines 12 and 14. The full details are in [7] which we look as a playground with Java classes that implement Fig. 8 and Fig. 5 as synchronous and parallel threads, acting as a producer and a consumer.

4 Future work

We have shown that we can decompose every classical recursive function, based on a *predecessor* that decreases every of its input by a constant value, into reversible and classical components that cooperate to implement the original recursive functions under a Producer/Consumer pattern (see (1)).

Firstly, we plan to extend (1) to recursive functions recF based on predecessors p not limited to a constant Δ_p not greater than -1. A predecessor p should be at least such that:

- 1. Δ_p is not necessarily a constant. For example, $\Delta_p == -3$ on even arguments, and -2 on odd ones can be useful;
- 2. the predecessor can be an integer division x/k, for some given k>0, like in a dichotomic search, which has k==2.

Secondly, we aim at generalizing (1) to a compiler $[\cdot]$:

The domain of [] should be a class R of recursive functions built by means of standard composition schemes, starting from a class of predecessors p1, p2, ... each of which must have the corresponding inverse function p1Inv, p2Inv,

In these lines we want to explore interpretations of || more liberal than the essentially obvious synchronous Producer/Consumer that we implement in [7]. We shall very likely take advantage of parallel discrete events simulators as described in [8,9] in order to get rid of any explicit synchronization between the pairs of reversible-producer/classical-consumer that (5) would recursively generate when applied to an element in R.

We also plan to follow a more abstract line of research. The compilation scheme (5) recalls Girard's decomposition $A \to B \simeq !A \multimap B$ of a classical computation into a linear one that can erase/duplicate computational resources. Decomposing recF[p,b,h] in terms of itFCls[b,h] and itFRev[p,pInv] suggests that the relation between reversible and classical computations can be formalized by a linear isomorphism $A^n \multimap B^n$ between tensor products A^n , and B^n of A, and B, in analogy to [2]. Then we can think of recovering classical computations by some functor, say γ , whose purpose is, at least, to forget, or to inject replicas, of parts of A^n , and B^n in a way that $(\gamma A^n \to \gamma A^n) \uplus (\gamma A^n \leftarrow \gamma A^n)$ can be their type. The type says that we move from a reversible computation to a classical one by choosing which is input and which is output, so recovering the freedom to manage computational resources as we are used to when writing classical programs.

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