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Can Price Collars Increase Insurance Loss Coverage? SSRN Electronic Journal
Preprint Site . ISSN 1556-5068. (Submitted)

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CAN PRICE COLLARS INCREASE INSURANCE LOSS COVERAGE?

BY INDRADEB CHATTERJEE, MINGJIE HAO, PRADIP TAPADAR
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ABSTRACT

Loss coverage, defined as expected population losses compensated by insurance, is a public policy criterion for comparing different risk classification regimes. Using a model with two risk-groups (high and low) and iso-elastic demand, we compare loss coverage under three alternative regulatory regimes: (i) full risk-classification (ii) pooling (iii) a price collar, whereby each insurer is permitted to set any premiums, subject to a maximum ratio of its highest and lowest prices for different risks. Outcomes depend on the comparative demand elasticities of low and high risks. If low-risk elasticity is sufficiently low compared with high-risk elasticity, pooling is optimal; and if it is sufficiently high, full risk classification is optimal. For an intermediate region where the elasticities are not too far apart, a price collar is optimal, but only if both elasticities are greater than one. We give extensions of these results for more than two risk-groups. We also outline how they can be applied to other demand functions using the construct of arc elasticity.

KEYWORDS

Insurance loss coverage; risk classification; price collar; partial community rating; pooling; iso-elastic demand; demand elasticity; arc elasticity.

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1 INTRODUCTION

Restrictions on risk classification are common in retail insurance markets, usually in the form of bans on the use of particular variables (e.g. gender, race, genetic tests). Restrictions of this type can align insurance practice with social norms which deprecate discrimination on particular variables. But the use of deprecated variables is not the only public policy issue with risk classification. Another concern is the equity of a wide dispersion in prices for different risks, irrespective of the particular underlying variables which it might reflect; and in particular, the problem of unaffordable insurance for higher risks. For this type of concern, piecemeal restriction of specific variables seems an indirect

and hard-to-calibrate form of regulation. Also, if insurers can use other variables which are correlated with the banned ones, the effect of the regulation can be partly undone, unless more prescriptive regulation requires estimation procedures which neutralise this effect (Pope and Sydnor (2011), Lindholm et al. (2021)).

A potentially more direct way of addressing equity concerns is to set an explicit limit on the dispersion of prices for different risks, via what we call a *price collar*. A price collar (sometimes called “partial community rating”) is a maximum ratio, say κ , for the highest and lowest prices an insurer may charge for different risks. A price collar restricts only an insurer’s relative prices, not the absolute level. It can be used either in addition to, or instead of, limits on specific variables. One example is the Affordable Care Act in the US, which permits differentiation by a factor of up to $1.5\times$ for tobacco use and $3\times$ for age (and no other factors except coverage tier, geography, and number of dependants).

In this paper, we abstract from the reference to specific variables and consider a price collar which limits the overall dispersion of an insurer’s prices. The collar regime, which can also be described as *partial risk classification*, represents a compromise approach, in between *full risk classification* (where prices are fully differentiated for individual risks, which we assume are fully observable via insurers’ underwriting procedures), and *pooling* (where all risk classification is banned and each insurer sets a single price, the same for all risks).

1.1 LOSS COVERAGE

A price collar can potentially ameliorate equity concerns, but policymakers also need to consider its efficiency consequences, in particular the adverse selection which may be induced. We take a nuanced view of this: we argue that whilst excessive adverse selection is a concern, a modest degree of adverse selection can actually increase efficiency, if it increases what we call “loss coverage”. This concept has been described elsewhere (Thomas (2008, 2009, 2017); Hao et al. (2016, 2018, 2019)), but it may remain unfamiliar to many readers; so we now review the rationale, in the specific context of a price collar.

The usual story about adverse selection runs as follows. Compared to fully risk-differentiated prices, the imposition of a price collar raises prices for low risks and lowers prices for high risks. Low risks buy less insurance, and high risks buy more (adverse selection). The weighted average price paid for insurance across the whole market rises. Also, because high risks are typically less numerous than low risks, the number of risks insured falls. This combination of a rise in average price and fall in demand is usually seen as a bad outcome, both for insurers and for society.

However, the social purpose of insurance is to compensate the population’s losses (not to sell “coverage” indiscriminately, with no regard for any prospect of loss). Insurance of one high risk contributes more in expectation to this objective than insurance of one low risk. This suggests that public policymakers might welcome increased purchasing by high risks, except for the usual story about adverse selection.

The usual story about adverse selection overlooks one point: with adverse selection, expected losses compensated by insurance – a quantity we term “loss coverage” – can be *higher* than with no adverse selection. The rise in weighted average price when a price collar is imposed reflects a shift in coverage towards higher risks. From a public policymaker’s viewpoint, this means that more of the “right” risks (i.e. those more likely to suffer loss) buy insurance. If this shift in coverage is large enough, it can more than offset the fall in numbers insured, so that loss coverage is increased. We argue that where adverse selection leads to higher loss coverage – that is, more risk being voluntarily transferred and more losses being compensated – then from society’s perspective, this should be seen as a good outcome from adverse selection. Formally, we define loss coverage as:

$$\text{Loss coverage} = \text{Expected losses compensated by insurance} \quad (1.1)$$

and then we argue that a policymaker should prefer risk classification schemes which generate higher loss coverage for the population as a whole (note: not necessarily the same as higher number of persons insured, irrespective of their individual risks).

Another way of putting this is that a public policymaker designing a risk classification scheme in the context of adverse selection faces a trade-off between insurance of the “right” risks (those more likely to suffer loss), and insurance of a larger number of risks. The optimal trade-off depends on demand elasticities of high and low risks. The concept of loss coverage quantifies this trade-off, and provides a metric for comparing the efficacy of different risk classification schemes in facilitating compensation for the losses of the population as a whole.¹

Previous work (Hao et al. (2016, 2018)) used models of demand elasticity for high and low risks to compare loss coverage for the polar cases: full risk classification (where prices fully reflect individual risks) and pooling (where all risk classification is banned). This led to results as sketched in Figure 1. In the left-hand region of the parameter space (i.e. low demand elasticity for low risks compared with high risks), pooling gives higher loss coverage; and in the right-hand region, full risk-classification gives higher loss coverage. It is natural to wonder if there might be an intermediate region where a price collar – a compromise between pooling and full risk-classification – gives higher loss coverage than either of the two polar cases. That possibility is the subject of this paper. To anticipate our main conclusion, the answer is yes, but only if demand elasticities for both high and low risks are greater than 1.

¹For a toy example illustrating the arithmetic of loss coverage for a small group of risks, see section 2 of Hao et al. (2018) or <https://blogs.kent.ac.uk/loss-coverage/improving-insurance-with-some-adverse-selection/>.

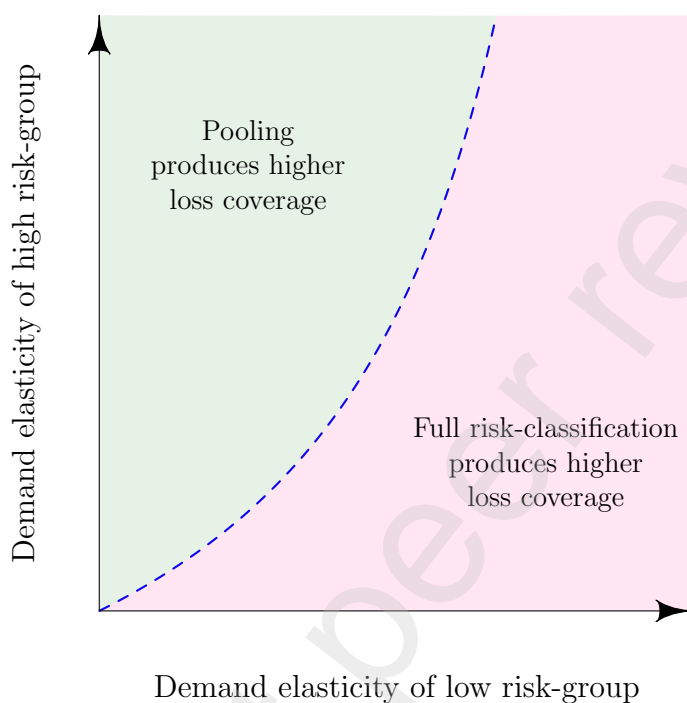


Figure 1: Regions where loss coverage is higher for pooling and full risk-classification.

1.2 LITERATURE REVIEW

This paper is related to recent literature on State-level health insurance exchanges established in the United States under the Affordable Care Act (and earlier legislation in some States), which all involve some form of price collar (Ericson and Starc (2015), Mahoney and Weyl (2017), Einav et al. (2019), Geruso et al. (2021)). These papers all focus on the specific context of US healthcare, and so incorporate many institutional constraints and details which we omit: a fixed width of collar (in contrast, our primary focus is on the effect of varying the collar), risk adjustment, premium subsidies for insurance purchase and penalties for non-purchase, etc. One of the conclusions is that demand elasticity on the healthcare exchanges for younger people (i.e. lower risks) appears empirically to be about twice that for older people (i.e. higher risks); and therefore that if subsidies were

differentiated by age (higher to younger people, rather than age-independent subsidies as under the Act), this could increase enrolment (Ericson and Starc (2015)), and represents a more effective and flexible instrument for this purpose than risk adjustment (Einav et al. (2019)). This is broadly consistent with our results: if demand elasticity is high for low risks compared with high risks, more differentiated prices tend to increase coverage (and loss coverage). However the details of our analysis differ, in that we assume the policy criterion is to maximise loss coverage (risk-weighted demand), not numbers insured (un-weighted demand).

To illustrate the difference, if each high risk has two times the expected losses of each low risk, then on our criterion, the policymaker is indifferent between coverage of one high risk or two low risks; but in the other papers just cited, the policymaker is indifferent between coverage of one high risk and one low risk. In the healthcare example, our objective amounts to increasing (expected) coverage of sickness, rather than coverage of persons irrespective of their probability of sickness. We do not say that prioritising coverage of sickness over coverage of persons is the only reasonable preference, but we do say that it is at least arguable.²

Einav et al. (2019) recognise the point when they note that their suggestion of a move from risk adjustment and uniform premium subsidies to less risk adjustment and more risk-differentiated subsidies, whilst increasing enrolment, could also make higher risks worse off; to address this, they also suggest adding a Pareto-type restriction on the policy change (i.e. the effective prices faced by high risks, after the move to less risk adjustment and more differentiated subsidies, must be no higher than before the change). The restriction ensures that the policy change increases enrolment for low risks, and also does not reduce enrolment for high risks; in our terms, this ensures that the policy change increases loss coverage. But it does not necessarily *maximise* loss coverage.

The contribution of this paper is to analyse market outcomes over the full parameter space for demand elasticities for high and low risk-groups (iso-elastic for each risk-group); and over the full feasible range for a price collar, from complete pooling to full risk-classification. The paper also represents a natural extension of Hao et al. (2018), which considered only the polar cases of full risk-classification and pooling, and not the intermediate possibilities under a price collar.

²Another reasonable policy objective might be to maximise utilitarian social welfare, defined as expected utility for a random member of the population behind a Rawlsian veil of ignorance that screens off knowledge of one's risk type (Hoy (2006)). But the practical difficulty with this is that utility functions are idiosyncratic to each individual, and unobservable by the policymaker. Furthermore, at least for same iso-elastic demand for all risk-groups, it can be shown that ranking risk-classification schemes by loss coverage will always give the same ordering as ranking by utilitarian social welfare (Hao et al. (2019)). In this sense, loss coverage can be thought of as an observable proxy measure for expected utility.

2 POLITICAL AND REGULATORY CONSTRAINTS

By “political constraints”, we mean certain general notions of fairness and proportionality, which are unlikely to be explicitly stated in insurance regulations, but may nevertheless constrain both the actions of regulators and insurers’ response to those actions. By “regulatory constraints”, we mean rules about risk-classification which are explicitly stated in insurance regulations (such as the rules imposing a price collar). This section discusses these two types of constraint.

2.1 POLITICAL CONSTRAINTS

The main role of “political constraints” in our analysis is to rule out premium regimes which may be technically feasible, but which we think are likely to be widely regarded as perverse, unreasonable, or otherwise publicly unacceptable. Here are some examples:

- (i) *Hyper-differentiation*. High risks are charged more than their true risks, and low risks are charged less than theirs. This gives a cross-subsidy between risk-groups, but for most contexts, it is in the “wrong” direction (i.e. a disadvantaged high risk-group is over-charged to subsidise an already fortunate low risk-group).
- (ii) *Hyper-pooling*. Low risks are charged more than high risks. This could be economically feasible, if demand elasticity for low risks is sufficiently low compared to that for high risks. Here, the cross-subsidy is in the “right” direction for equity, but too large.
- (iii) *Groupings which lead to unfair ordering of premiums*. If there are three risk-groups, say low, medium and high risks, then a regime which groups low and high risk-groups at one premium, and medium risks at another premium, could be economically feasible. But this might be politically unacceptable, if it leads to low risks being charged more than medium risks. (Stated differently, the objection is that this gives a similar result to hyper-pooling of low and medium risks.)

To summarise these considerations: if premiums are differentiated, the ordering of premiums should be the same as the ordering of risks, and the span of premiums should not exceed the span of the risks. To state these notions formally, we now introduce some notation for the rest of the paper:

- We assume a population consisting of n distinct risk-groups with probabilities of loss given by $\underline{\mu} = (\mu_1, \mu_2, \dots, \mu_n)$, which are ordered for convenience (μ_1 smallest, μ_n largest).
- The proportion of the population belonging to risk-group i is p_i , for $i = 1, 2, \dots, n$.

- Members of risk-group i are offered premium (per unit of loss) π_i . We call $\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$ a premium regime, or risk-classification regime.

The political constraints can then be encapsulated as:

Constraint 1 (Political). *Given risks $\underline{\mu}$, a politically acceptable premium regime $\underline{\pi}$ needs to satisfy:*

$$\mu_1 \leq \pi_1 \leq \pi_2 \leq \dots \leq \pi_n \leq \mu_n. \quad (2.1)$$

Other examples of politically unacceptable premium regimes might include those which lack face validity (e.g. combine risk-groups having no apparent similarities), or which disadvantage socially protected classes (e.g. a high premium for high risk may be less acceptable for disability than for dangerous sports). It is not possible to specify all the political considerations that might arise. But we think Constraint 1 above seems broadly applicable, and helps to rule out perverse interpretations of the price collar concept.

2.2 REGULATORY CONSTRAINTS

The price collar concept was outlined in the introduction and is formally specified as Constraint 2.

Constraint 2 (Price collar). *Given a prescribed price collar, κ , where $\kappa \geq 1$, any premium regime $\underline{\pi}$ needs to satisfy:*

$$\pi_H \leq \kappa \pi_L, \quad (2.2)$$

where $\pi_L = \min_i \pi_i$ and $\pi_H = \max_i \pi_i$.

The acceptable range for a price collar is limited by the prohibition on hyper-separation and hyper-pooling in political Constraint 1, which implies that:

$$1 \leq \kappa \leq \frac{\mu_n}{\mu_1}. \quad (2.3)$$

Note that the extremes $\kappa = 1$ and $\kappa = \mu_n/\mu_1$ correspond to pooling and full risk-classification respectively. We use the term *partial risk-classification* to refer all regimes where κ is set to an intermediate value.

A price collar can be circumvented if insurers can simply decline high risks (which amounts to quoting an infinite price above the collar for high risks). To be effective, it needs to be supported by *guaranteed issue*, that is an obligation on the insurer to accept any applicant at some price within the collar, as stated in Constraint 3.

Constraint 3 (Guaranteed issue). *Insurers are required to quote a price to all applicants. Nobody can be declined for insurance.*

Guaranteed issue might be unreasonable for types of insurance where there are a few exceptionally high probabilities of loss (e.g. term insurance for people with a terminal illness). This problem can be alleviated by a rule which permits a small fraction of higher prices, e.g. up to 1% of the prices charged by an insurer over a trailing three-year period are permitted to exceed the collar. But for simplicity, we assume guaranteed issue is required.

3 INSURANCE DEMAND AND EQUILIBRIUM

This section develops the theory of insurance demand and equilibrium under perfect competition, but subject to the political and regulatory constraints stated in Section 2.

3.1 INSURANCE DEMAND

Typical theories of insurance demand assume individuals know their own probabilities of loss and have a common utility function. Given an offered premium, individuals with the same probabilities of loss then all make the same purchasing decision. This does not correspond well to the observable reality of insurance markets, where individuals with similar probabilities of loss often appear to make different decisions, and substantial fractions of individuals do not purchase insurance at all.

We follow the different approach introduced in Hao et al. (2018, 2019); Chatterjee et al. (2021), which allows for heterogeneity in risk aversion across individuals with the same probabilities of loss, and hence generates the partial take-up of insurance that we observe in practice. In summary, our approach is based on the following assumptions.

- (i) Individuals know their probability of loss and their own risk aversion, and make purchasing decisions accordingly.
- (ii) Insurers can observe (via the usual underwriting procedures) individual's probabilities of loss and so correctly assign them to risk-groups, but cannot observe their individual risk aversion.
- (iii) All insurance is for one unit of cover, in a contract which is standardised across all insurers, who compete only on price. Insurers do not offer partial cover or other contract menus.
- (iv) Viewed by the insurer, the demand for insurance from risk-group i at premium π_i is then a function $d_i(\pi_i)$. The demand function represents the proportion of the

risk-group who buy insurance, such that $0 < d_i(\pi_i) < 1$. We assume that this is decreasing and continuous.

3.2 MARKET EQUILIBRIUM

To recap on notation: we have a population of n risk-groups, where members of risk-group i each have risk μ_i , are offered premium π_i , and collectively represent proportion p_i of the aggregate population. In a perfectly competitive insurance market, we then have:

$$\text{Premium income} = \sum_{i=1}^n p_i d_i(\pi_i) \pi_i. \quad (3.1)$$

$$\text{(Expected) insurance claim} = \sum_{i=1}^n p_i d_i(\pi_i) \mu_i. \quad (3.2)$$

$$\text{(Expected) profit : } E(\underline{\pi}) = \sum_{i=1}^n p_i d_i(\pi_i) (\pi_i - \mu_i). \quad (3.3)$$

$$\text{Market equilibrium} \Rightarrow E(\underline{\pi}) = 0. \quad (3.4)$$

Any candidate equilibrium premium regime $\underline{\pi}$ satisfying Equation 3.4 also needs to be stable in the following sense: it must be impossible for any single insurer to profitably disrupt the equilibrium by using a different regime.³ This property relies on a corollary of perfect competition: *no insurer has any market power*, because all risks choose the lowest offer in the market for their risk.

To see why this corollary is important, imagine that one insurer could set a collar of the same maximum permitted width κ as other insurers, but with a lower mid-point, and increase its market share of (profit-making) low risks *without* an equivalent increase in market share of (loss-making) high risks. This insurer would profitably disrupt an existing zero-profit regime. But this is impossible under perfect competition, because an insurer which sets lower prices than other insurers for both high and low risks will attract essentially *all* the market demand from both low risks and high risks; and therefore if the other insurers were previously zero-profit, the deviating insurer will make a loss.⁴

³Our concept of *different regime* is limited to *different set of prices* for insurance contracts, which are standardised for regulatory or institutional reasons, as in Akerlof (1970); we do not consider differentiation by contract design, as in Rothschild and Stiglitz (1976).

⁴Stating “no market power” differently: *brand elasticity* of demand for each insurer is infinite. Note that no contradiction arises from infinite *brand* demand elasticities combined with relatively low *product* demand elasticities, because they relate to different choice problems. Product elasticity relates to the *choice over products* in the consumer’s consumption bundle. Brand elasticity relates to the *choice over brands*, given that insurance is to be purchased. Alternatively, a weaker assumption could do the same work: if one insurer reduces prices for high and low risks by the same amount, it attracts the *same increase in market share* for high and low risks (as assumed in Mahoney and Weyl (2017)); that is, the *semi-logarithmic brand elasticity of demand* is uniform for high and low risks.

3.3 THE PRICE COLLAR EQUILIBRIUM

The equilibrium conditions stated in Section 3.2, combined with the political and regulatory constraints stated in Section 2, lead to the following solutions for different values of the price collar:

- (i) $\kappa = \mu_n/\mu_1$: full risk-classification, where all risk-groups are charged their fair premiums i.e. $\underline{\pi} = \underline{\mu}$, is a solution.
- (ii) $\kappa = 1$: pooling, where all risk-groups are charged the same premium π_0 , is a solution.⁵
- (iii) Intermediate values of κ : a tripartite solution of this form:
 - a super-group \mathcal{L} of low risk-groups all charged the same π_L (more than their fair premiums);
 - a super-group \mathcal{M} of “middle” risk-groups all charged their fair premiums;
 - a super-group \mathcal{H} of high risk-groups all charged the same π_H (less than their fair premiums);

where the grouping into three super-groups (hence “tripartite”) has the pattern shown in Figure 2.

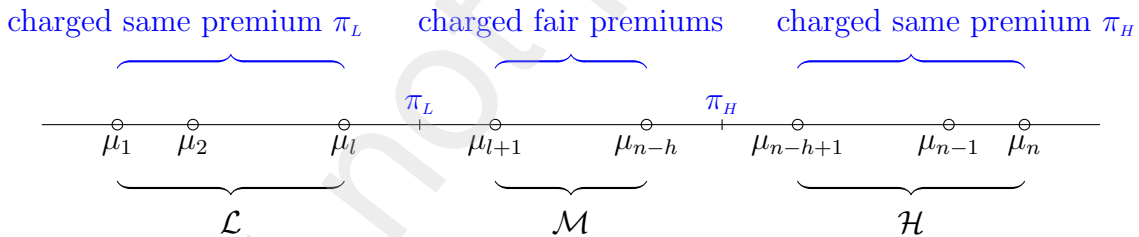


Figure 2: Price collar: tripartite solution.

For intuition on why the intermediate collar solution must take this tripartite form, first note that each insurer has to come up with its own collar limits, π_L and $\pi_H = \kappa \pi_L$. Then all risks lower than π_L have to be charged at least π_L ; and all risks higher than π_H

⁵Given continuous demand functions, the existence of at least one pooling equilibrium is ensured by the intermediate value theorem. Uniqueness is technically not guaranteed, but pertains for plausible combinations of demand elasticities; and if, exceptionally, there are multiple solutions, any besides the lowest will be eliminated by competition. The same elimination principle applies for any multiple solutions under the collar in the next point (for proof see Appendix A).

have to be charged at most π_H . Competition in pricing for low and high risks drives all insurers towards the same values for the collar limits π_L and π_H . Then note that inside the limits of the collar, the same competitive forces operate as if no collar applied, with the same outcome: if one insurer attempts to pool a higher and lower risk-group inside the collar, this can be destabilised by another insurer offering the lower risk-group in the putative pooling a fair premium. Hence competition drives all insurers to charge all “middle” risk-groups exactly their fair premiums.

A formal statement and proof of the tripartite solution is given as Theorem 1 in Appendix A).

3.4 ISO-ELASTIC INSURANCE DEMAND

So far, we have only needed insurance demand function, $d_i(\pi_i)$ for risk-group i , to be continuous and decreasing in π_i . We will now further assume differentiability, to define the (point price) elasticity of insurance demand as:

$$\epsilon_i(\pi_i) = -\frac{\partial \log d_i(\pi_i)}{\partial \log \pi_i} = -\frac{\frac{\partial d_i(\pi_i)}{d_i(\pi_i)}}{\frac{\partial \pi_i}{\pi_i}} = -\frac{\pi_i}{d_i(\pi_i)} \frac{\partial d_i(\pi_i)}{\partial \pi_i}, \quad (3.5)$$

which implies that insurance demand can also be expressed as:

$$d_i(\pi_i) = \tau_i \exp \left[-\int_{\mu_i}^{\pi_i} \epsilon_i(s) d \log s \right], \quad (3.6)$$

where $\tau_i = d_i(\mu_i)$ is the *fair-premium demand* for risk-group i . A tractable functional form for demand is iso-elastic, i.e. demand elasticity is a positive constant, λ_i :

$$\epsilon_i(\pi_i) = \lambda_i; \quad (3.7)$$

and by Equation 3.6, demand for risk-group i then takes the form:

$$d_i(\pi_i) = \tau_i \left(\frac{\mu_i}{\pi_i} \right)^{\lambda_i}, \quad (\text{subject to a cap of } 1). \quad (3.8)$$

Note that if the premium charged is sufficiently small, it is possible for a risk-group to be fully insured, i.e. $d_i(\pi_i) = 1$.

For iso-elastic demand, the equilibrium condition in Equation 3.4 takes the form:

$$E(\underline{\pi}) = \sum_{i=1}^n p_i \tau_i \left(\frac{\mu_i}{\pi_i} \right)^{\lambda_i} (\pi_i - \mu_i) = 0. \quad (3.9)$$

4 LOSS COVERAGE

In Section 1.1, loss coverage was defined as: expected losses compensated by insurance for the population as a whole, i.e.:

$$\text{Loss coverage: } C(\underline{\pi}) = \sum_{i=1}^n p_i d_i(\pi_i) \mu_i. \quad (4.1)$$

For iso-elastic demand, the expression for loss coverage takes the form:

$$C(\underline{\pi}) = \sum_{i=1}^n p_i \tau_i \left(\frac{\mu_i}{\pi_i} \right)^{\lambda_i} \mu_i. \quad (4.2)$$

We suggest that a regulator or policymaker wishes to calibrate the price collar κ to maximise loss coverage over all possible premiums regimes. This will ensure that voluntary purchases of insurance cover the largest possible fraction of the population's losses.⁶ Mathematically, the objective can be stated, in terms of premiums, as:

$$\max_{\kappa} C(\underline{\pi}), \text{ subject to } E(\underline{\pi}) = 0. \quad (4.3)$$

4.1 THE CASE OF TWO RISK-GROUPS

Consider two premium regimes: $\underline{\pi} = (\pi_1, \pi_2)$ and $\underline{\pi} + \underline{\Delta\pi} = (\pi_1 + \Delta\pi_1, \pi_2 + \Delta\pi_2)$, where both regimes satisfy the equilibrium condition in Equation 3.9, so that $E(\underline{\pi} + \underline{\Delta\pi}) = E(\underline{\pi}) = 0$. If $\underline{\Delta\pi}$ is "small", ignoring higher-order terms in the Taylor series expansion gives :

$$\Delta E = E(\underline{\pi} + \underline{\Delta\pi}) - E(\underline{\pi}) = E_1 \Delta\pi_1 + E_2 \Delta\pi_2, \text{ where } E_i = \frac{\partial E}{\partial \pi_i} \text{ for } i = 1, 2. \quad (4.4)$$

As $E(\underline{\pi} + \underline{\Delta\pi}) = E(\underline{\pi}) = 0$, and thus $\Delta E = 0$, the relationship between $\Delta\pi_1$ and $\Delta\pi_2$ can be expressed as:

$$\Delta\pi_2 = -\frac{E_1}{E_2} \Delta\pi_1. \quad (4.5)$$

⁶Compulsory purchase (a mandate) could ensure 100% coverage. But the reduction in liberty this involves is often unacceptable, except possibly for healthcare or third-party liability insurances (e.g. auto and employer third-party liability).

⁷Readers who are familiar with Lagrange multipliers and the Kuhn-Tucker theorem (please see Dixit (1990) for an exposition from an economic perspective) will realise that the constrained maximisation problem can be framed in terms of these optimisation approaches. However, instead of applying these methods mechanically, we provide the intermediate steps, so that the underlying economic interpretations are not overlooked.

To compare loss coverages under two equilibrium premium regimes: $\underline{\pi} = (\pi_1, \pi_2)$ and $\underline{\pi} + \underline{\Delta\pi} = (\pi_1 + \Delta\pi_1, \pi_2 + \Delta\pi_2)$, Taylor series expansion ignoring higher-order terms gives:

$$\Delta C = C_1 \Delta\pi_1 + C_2 \Delta\pi_2, \text{ where } C_i = \frac{\partial C}{\partial \pi_i} \text{ for } i = 1, 2. \quad (4.6)$$

$$= \left[C_1 - \frac{E_1}{E_2} C_2 \right] \Delta\pi_1 \quad (4.7)$$

For the specific case of iso-elastic demand functions, we have:

$$E_i = \frac{\partial E}{\partial \pi_i} = p_i \tau_i \lambda_i \left(\frac{\mu_i}{\pi_i} \right)^{\lambda_i+1} (1 - m_i), \text{ where } m_i = \frac{\pi_i}{\mu_i} \left(1 - \frac{1}{\lambda_i} \right); \quad (4.8)$$

$$C_i = \frac{\partial C}{\partial \pi_i} = -p_i \tau_i \lambda_i \left(\frac{\mu_i}{\pi_i} \right)^{\lambda_i+1}; \quad (4.9)$$

so that under iso-elastic demand for two risk-groups, the sensitivity of the loss coverage to small changes in the equilibrium premium regimes is given by:

$$\Delta C = T (m_2 - m_1) \Delta\pi_1, \text{ where } T = \frac{p_1 \tau_1 \lambda_1 \left(\frac{\mu_1}{\pi_1} \right)^{\lambda_1+1}}{1 - m_2}. \quad (4.10)$$

Note that the term T is always positive, because the political constraint requires $\pi_2 \leq \mu_2$, and thus $m_2 < 1$.

The sign of the term $(m_2 - m_1)$ depends on the elasticities λ_1 and λ_2 , and determines how C depends on the low-risk premium π_1 . Specifically, for given values of λ_1 and λ_2 , if the term $(m_2 - m_1)$ is positive or negative over the whole politically feasible range of π_1 , this tells us that loss coverage is maximised for pooling or full risk-classification respectively; and if it changes sign over the range of π_1 , there will be a turning point for loss coverage. A further interpretation of $(m_2 - m_1)$ will be given in Section 5.3 below.

The result is illustrated in Figure 3. A formal statement as Theorem 2 and a proof are given in Appendix B.

Figure 3 shows which risk-classification scheme – pooled, full or partial – gives highest loss coverage in each region of the (λ_1, λ_2) -plane. There are two large regions, \mathcal{P} (Pooled, left, green) and \mathcal{F} (Full risk-classification, right, red); and two smaller intermediate regions, \mathcal{I}_L (Intermediate, low elasticity, yellow) and \mathcal{I}_H (Intermediate, high elasticity, orange). The large green region (pooling best) and red region (full best) regions are each divided into into three sub-regions, delineated by the vertical and horizontal lines $\lambda_1 = 1$ and $\lambda_2 = 1$.

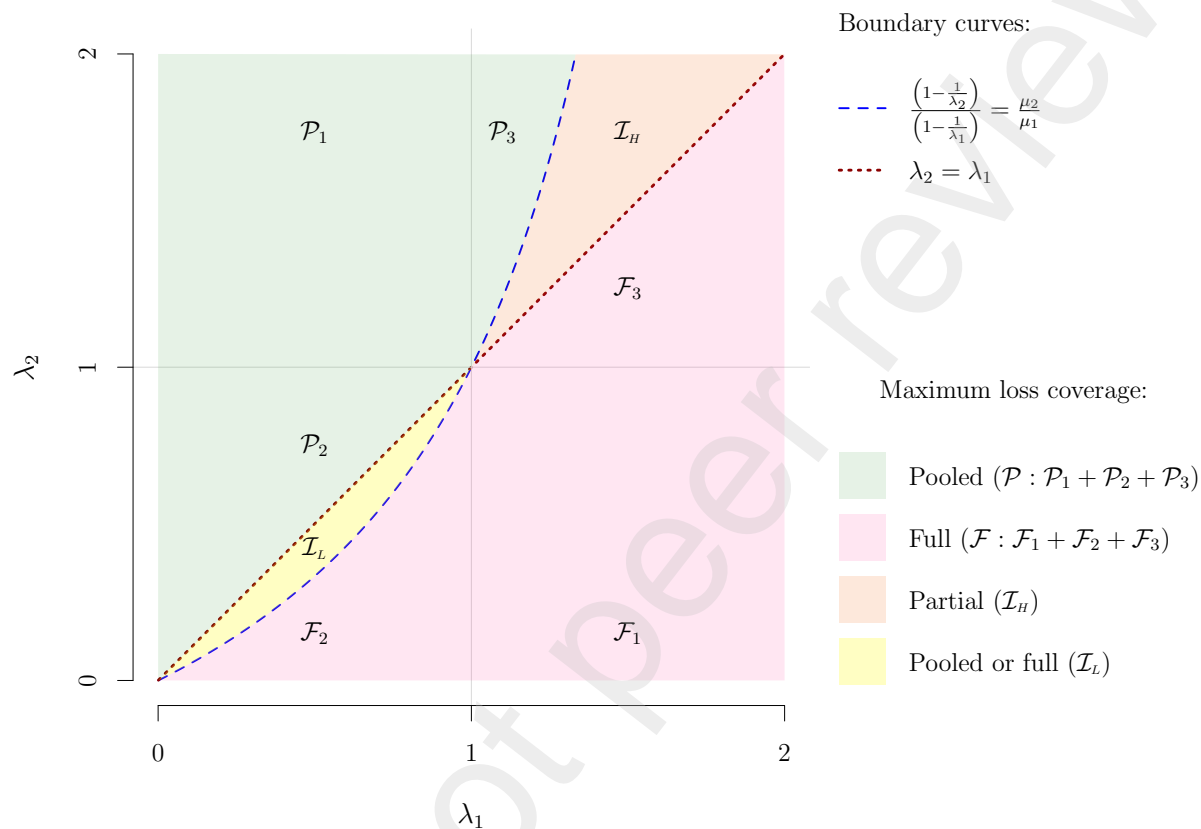


Figure 3: Regions of (λ_1, λ_2) -plane where pooling, full or partial risk-classification give highest loss coverage. (Basis: $\mu_2/\mu_1 = 2$, but similar appearance for any $\mu_2 > \mu_1$.)

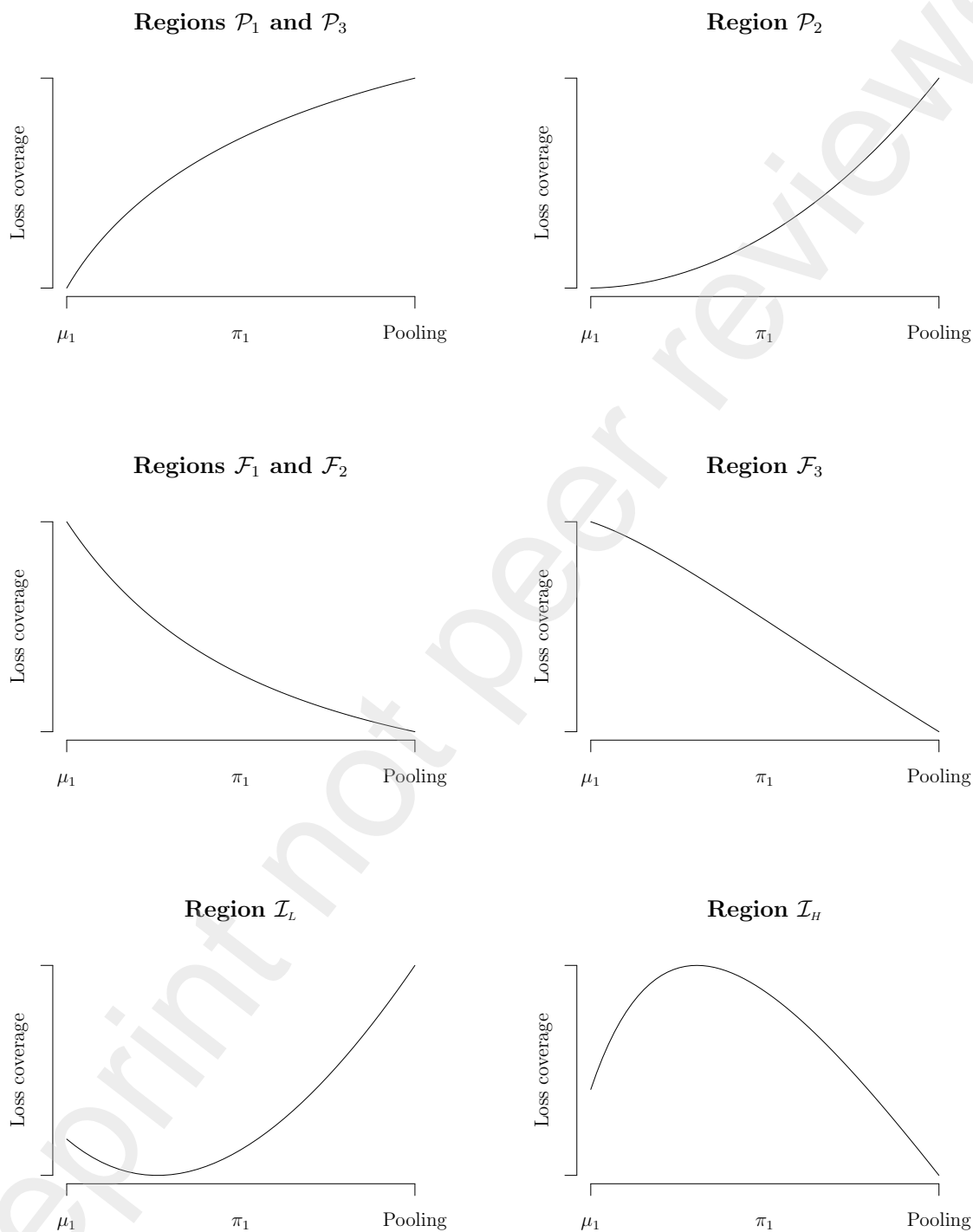


Figure 4: Slope and convexity of loss coverage–premium plots

5 INTUITION AND INTERPRETATION

5.1 SLOPE AND CONVEXITY OF LOSS COVERAGE–PREMIUM PLOTS

For graphical intuition into the general pattern of Figure 3, look at Figure 4, where the six panels show representative plots of equilibrium loss coverage against π_1 for all the labelled regions in Figure 3. In each panel, increasing π_1 along the x-axis corresponds to reducing the price collar from its maximum of $\kappa = \mu_2/\mu_1$ (which permits full risk-classification) to its minimum of $\kappa = 1$ (which enforces pooling). Then note that the slope and convexity of the various plots account for the pattern of results illustrated in Figure 4. In particular:

- (i) For pooling to be optimal, the plot needs to have a maximum at its right extreme: regions \mathcal{P}_1 , \mathcal{P}_2 & \mathcal{P}_3 (first row of Figure 4).
- (ii) For full risk-classification to be optimal, the plot needs to have a maximum at its left extreme: regions \mathcal{F}_1 , \mathcal{F}_2 & \mathcal{F}_3 (second row of Figure 4).
- (iii) For partial risk-classification to be optimal, the plot needs to have an interior maximum (right panel of third row of Figure 4). This is true only in the upper-right intermediate region \mathcal{I}_H , where both demands are elastic, with low risk elasticity moderately lower than high-risk elasticity.
- (iv) For partial risk-classification to be worst, the plot needs to have an interior minimum (left panel of third row of Figure 4). This is true only in the lower-left intermediate region \mathcal{I}_L , where both demands are inelastic, with low risk elasticity moderately higher than high-risk elasticity.
- (v) In more detail: in Figure 3, somewhere inside the lower-left intermediate region \mathcal{I}_L there is a line of equality, along which pooling and full risk-classification give equal loss coverage. The exact position of this line is not shown, because it depends on the population proportions and fair-premium demands. Elsewhere in region \mathcal{I}_L , pooling gives highest loss coverage to the left of the line of equality, and full risk-classification to the right.
- (vi) The initially surprising feature that partial risk-classification can never be optimal in the lower-left region is explained graphically by the concavity of the plots for all the sub-regions \mathcal{P}_2 , \mathcal{I}_L and \mathcal{F}_2 in Figure 4. A deeper explanation of this feature will be given in Sub-section 5.3 below.

5.2 REVENUE ELASTICITIES: UPPER LEFT AND LOWER RIGHT RECTANGLES OF FIGURE 3

The outcomes in the upper left and lower right rectangles (Regions \mathcal{P}_1 and \mathcal{F}_1 in Figure 3) can be quickly understood by recalling the concept of *revenue elasticity*, the percentage change in revenue for a small percentage change in price, defined formally as

$$\text{revenue elasticity} = -\frac{\partial \log (d_i(\pi_i) \times \pi_i)}{\partial \log \pi_i} = \text{demand elasticity} - 1. \quad (5.1)$$

Revenue elasticity < 0 indicates that when prices rise, revenue rises; and vice versa for revenue elasticity > 0 . This immediately leads to the following intuitions.

- (i) For the ‘upper left rectangle’ region \mathcal{P}_1 (i.e. $\lambda_1 < 1, \lambda_2 > 1$): the first elasticity ($\lambda_1 < 1$) implies that revenue moves in the same direction as price in the low risk-group, and the second ($\lambda_2 > 1$) implies that revenue moves in the opposite direction to price in the high risk-group. Therefore on any small movement towards pooling, (i.e. $\pi_1 \uparrow, \pi_2 \downarrow$), both revenues increase; and so aggregate revenue in equilibrium (and hence loss coverage) must be maximised by pooling.
- (ii) The inverse argument applies for the ‘lower right rectangle’ region \mathcal{F}_1 (i.e. $\lambda_1 > 1, \lambda_2 < 1$): any small movement *away* from pooling increases revenue from both risk-groups, and so aggregate revenue in equilibrium (and hence loss coverage) must be maximised by full risk-classification.

5.3 FUNDING RATIOS: LOWER-LEFT AND UPPER-RIGHT SQUARES OF FIGURE 3

In these regions, on any move towards or away from pooling, revenue moves in opposite directions in the two risk-groups, so examination of the signs of revenue elasticities is not sufficient. Instead, we need to use the constructs of marginal revenue and marginal cost to define a *funding ratio* for each risk-group.

The total revenue and total cost for risk-group i are $TR_i(\pi_i) = \pi_i d_i(\pi_i)$ and $TC_i(\pi_i) = \mu_i d_i(\pi_i)$. The marginal revenue and marginal cost are:

$$MR_i(\pi_i) = \frac{\partial TR_i(\pi_i)}{\partial d_i(\pi_i)} = \frac{\partial [\pi_i d_i(\pi_i)]}{\partial d_i(\pi_i)} = \pi_i + d_i(\pi_i) \frac{\partial \pi_i}{\partial d_i(\pi_i)} = \pi_i \left(1 - \frac{1}{\epsilon_i(\pi_i)} \right); \text{ and} \quad (5.2)$$

$$MC_i(\pi_i) = \frac{\partial TC_i(\pi_i)}{\partial d_i(\pi_i)} = \mu_i. \quad (5.3)$$

The *funding ratio*, say m_i for risk-group i , can then be defined as:

$$m_i = \frac{MR_i(\pi_i)}{MC_i(\pi_i)} = \frac{\pi_i}{\mu_i} \left(1 - \frac{1}{\epsilon_i(\pi_i)} \right) \quad (5.4)$$

$$= \frac{\pi_i}{\mu_i} \left(1 - \frac{1}{\lambda_i} \right), \text{ for iso-elastic demand.} \quad (5.5)$$

Funding ratios for each risk-group previously arose in Equations 4.8 and 4.10 in Section 4.1 (where they were labelled as m_i , but not named), and were seen to determine the slope of the loss coverage-premium plot: $\Delta C/\Delta\pi_1 = T(m_2 - m_1)$.

The funding ratio for a risk-group measures the extent to which a small increase in the risk-group's cost (i.e. its demand, multiplied by its risk rate μ) is *funded* by a corresponding increase in its revenue (i.e. its demand, multiplied by its premium rate π).

Note that for inelastic demand, revenue rises when price rises, i.e. when demand (and hence cost) falls. So because revenue and cost are moving in opposite directions, the funding ratio must be negative; and vice versa for elastic demand. By inspection of Equation 5.4, we can see that the funding ratio has a feasible range of $(-\infty, \frac{\pi_i}{\mu_i})$. It can be further interpreted as follows:

- (1) A positive funding ratio (corresponding to elastic demand, i.e. $\lambda_i > 1$) says that when the risk-group's demand (and hence cost) *increases*, its revenue also increases. So *increasing* demand from the risk-group (i.e. reducing price) tends to make an efficient contribution to loss coverage (aggregate cost in equilibrium): the increased cost from the risk-group partly "pays for itself", and so requires only a *partial* subsidy from the the other risk-group.
- (2) Conversely, a negative funding ratio (corresponding to inelastic demand, i.e. $0 < \lambda_i < 1$) says that when the risk-group's demand (and hence cost) *decreases*, its revenue increases. So *reducing* demand from the risk-group (ie. increasing price) tends to make an efficient contribution to loss coverage (aggregate cost in equilibrium): the reduced cost from the risk-group is set against more (possibly much more) revenue, creating a surplus which can then be applied in equilibrium to subsidise a *larger* increase in cost from the other risk-group.
- (3) When we make a small reduction in the price collar κ , premiums rise for low risks and fall for high risks, and vice versa for the corresponding demands. The change in loss coverage (i.e. aggregate risk-weighted demand in equilibrium) then depends on the *difference* in funding ratios across the two risk-groups. It helps to shift demand towards the risk-group with higher funding ratio (point (1) above), and vice versa (point (2) above). When we move towards pooling (i.e. $\pi_1 \uparrow, \pi_2 \downarrow$), we shift demand towards the higher risk-group and away from the lower risk-group, and hence loss coverage increases if $m_2 > m_1$. This explains the role of the $(m_2 - m_1)$ term in determining the gradient of the loss coverage-premium plot: per Equation 4.10, $\Delta C/\Delta\pi_1 = T(m_2 - m_1)$.

We can now use funding ratios to understand outcomes in the lower-left and upper-right squares of Figure 3.

5.3.1 A turning point for loss coverage

A turning point for loss coverage is obtained by setting the equilibrium premiums to the pair, say (π_1^*, π_2^*) , that equalises the funding ratios across the two risk-groups:

$$m_1 = m_2 \Rightarrow \frac{\pi_1}{\mu_1} \left(1 - \frac{1}{\lambda_1}\right) = \frac{\pi_2}{\mu_2} \left(1 - \frac{1}{\lambda_2}\right). \quad (5.6)$$

The rationale for equalisation to give a turning point is as follows. In any equilibrium, aggregate profits on low risks must be equal and opposite to aggregate profits on high risks. Then suppose we make an infinitesimal change in the two premiums – either towards one another, or away from one another – such that the changes in cost in the two groups are equal and opposite. Equal funding ratios then imply that the changes in revenue in the two risk-groups are also equal and opposite. So aggregate costs and aggregate revenues are both unchanged; and therefore equilibrium and loss coverage are unchanged.⁸

What elasticity parameters does $m_1 = m_2$ require? Observe that $\frac{\mu_1}{\pi_1} \leq 1$ and $\frac{\mu_2}{\pi_2} \geq 1$. Then by inspection of Equation 5.6, when both $\lambda_i < 1$, we need $\lambda_2 < \lambda_1$; this implies that in region \mathcal{I}_L , the turning point lies *below* the 45-degree line. Conversely, when both $\lambda_i > 1$, we need $\lambda_2 > \lambda_1$; this implies that in region \mathcal{I}_H , the turning point lies *above* the 45-degree line.

5.3.2 Minimum or maximum?

By inspection of Equation 5.6, when both $\lambda_i < 1$, both funding ratios are negative. It follows that the the turning point for loss coverage in the lower left square in Figure 3 must be a minimum. This is because when we reduce a risk-group's premium, its demand rises, *and its funding ratio becomes "less negative"*. and vice versa when we increase a risk-group's premium. So if we start from the turning point (π_1^*, π_2^*) , then whichever way we move the premiums to give a new equilibrium – either towards one another, or away from one another – the funding ratio becomes *less negative* in the risk-group with *rising* demand, and *more negative* in the risk-group with *falling* demand. This is a favourable combination: the “improving” ratio is applied to the group with rising demand, and vice versa. So loss coverage will always increase, implying that the turning point (π_1^*, π_2^*) from which we started must be a minimum.

Conversely, when both $\lambda_i > 1$, both funding ratios are positive. The argument just given is then inverted, so the turning point for loss coverage in region \mathcal{I}_H must be a maximum.

⁸Practical application of this argument is restricted to the intermediate regions \mathcal{I}_L and \mathcal{I}_H , in which turning points for loss coverage occur within the politically acceptable range of premiums. “Technical” turning points also occur in other regions, but the required premiums are politically unacceptable. For example, in region \mathcal{P}_2 , there is a minimum point for loss coverage with hyper-differentiated premiums (i.e. $\pi_1 < \mu_1$ and $\pi_2 > \mu_2$); this can be envisaged in Figure 4 by mentally extending the panel for region \mathcal{P}_2 to the left.

5.3.3 Another insight: which risk-group's premium does it help to reduce?

Another insight can be obtained by noting that increasing demand from a risk-group (i.e. reducing its premium) helps loss coverage only if the funding ratio for that demand is sufficiently favourable (compared with the funding ratio applicable to the corresponding decreased demand from the other risk-group).

When both demands are inelastic (lower-left square in Figure 3), increasing a risk-group's demand *improves* its funding ratio (i.e. makes it “less negative”). So there is no trade-off: as a rough insight, it helps to reduce the premium for whichever risk-group has higher elasticity, to the fullest extent that the political constraint permits. So “pooling best” will tend to apply for $\lambda_2 > \lambda_1$, i.e. above the 45-degree line; and “full best” for $\lambda_2 < \lambda_1$, i.e. below the 45-degree line. This rough insight is then modified by noting that because $\mu_2 > \mu_1$, high-risk demand has a higher weighting in loss coverage than low-risk demand; and hence the “pooling best” region extends a little below the 45-degree line into region \mathcal{I}_L .

Conversely, when both demands are elastic (upper-right square in Figure 3), increasing a risk-group's demand *worsens* its funding ratio. So in this case, there is a trade-off between increasing a risk-group's demand and improving the funding ratio applicable to that demand. If the ratio of high-risk to low-risk elasticities is sufficiently high (i.e. region \mathcal{P}_3), it helps to reduce the premium for high risks to the fullest extent that the political constraint permits; in other words, “pooling best”. If the ratio of elasticities is only modestly above 1 (i.e. region \mathcal{I}_H), it still pays to reduce the premium for high risks, but only so far; in other words, “partial best”.

6 EXTENSIONS AND DISCUSSION

6.1 MORE THAN TWO RISK-GROUPS

For more than two risk-groups, we can generalise Theorem 2, using:

- (i) the known tripartite structure of the stable equilibrium premium regime under a price collar, as illustrated in Figure 2 and formalised in Theorem 1, and
- (ii) a heuristic assumption that all risk-groups in the “low” super-group \mathcal{L} have the same iso-elastic demand elasticity λ_L , and all risk-groups in the “high” super-group \mathcal{H} have the same iso-elastic demand elasticity λ_H . This is based on the premise that risk-groups with broadly similar risks are likely to have broadly similar elasticities.

Intuitively, the key insight for more than two risk-groups is that because all risk-groups in the “middle” super-group \mathcal{M} always pay their fair premiums, they do not contribute to the cross-subsidies which determine equilibrium (and hence loss coverage). So for the purpose of determining how loss coverage changes when the collar changes, the “middle”

super-group \mathcal{M} can be completely disregarded. Therefore the analysis of loss coverage for two risk-groups in Equation 4.10 can be re-stated in terms of the two super-groups \mathcal{L} and \mathcal{H} , with the super-group elasticities λ_L and λ_H in place of λ_1 and λ_2 , and other parameters also set to their super-group values, i.e.:

$$\Delta C = T (m_H - m_L) \Delta \pi_L, \quad (6.1)$$

$$\text{where } T = \frac{p_L \tau_L \lambda_L \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L+1}}{1 - m_H}; \text{ and } m_i = \frac{\pi_i}{\mu_i} \left(1 - \frac{1}{\lambda_i}\right); \quad i = L, H; \quad (6.2)$$

where for the “low” and “high” super-groups \mathcal{L} and \mathcal{H} :

- μ_L and μ_H are the respective pooled equilibrium premiums;
- π_L and π_H are the corresponding premiums;
- p_L and p_H are the proportions of the population belonging to the super-groups;
- τ_L and τ_H can be interpreted as the ‘fair-premium demand’ when all risk-groups in the respective super-groups are pooled and charged the same pooled premium.

As T is always positive, the same inferences as before can then be made. A formal derivation of Equation 6.1 is given in Appendix C.

It is possible that when the width κ of the collar changes, the constituent risk-groups in the super-groups \mathcal{L} , \mathcal{M} and \mathcal{H} may slightly change. This is not problematic, for the following reasons:

- Starting from pooling (i.e. $\kappa = 1$), all risk-groups with risks less than the pooled premium belong to the super-group \mathcal{L} having the same demand elasticity λ_L ; while the remaining risk-groups with higher risks belong to the super-group \mathcal{H} having the same demand elasticity λ_H .
- As the price collar, κ , is increased, more and more risk-groups from the super-groups \mathcal{L} and \mathcal{H} join \mathcal{M} . However, the remaining risk-groups in \mathcal{L} and \mathcal{H} continue to have the same demand elasticities λ_L and λ_H respectively. So, any change in the compositions of \mathcal{L} , \mathcal{M} and \mathcal{H} would not affect the underlying demand elasticities of the super-groups \mathcal{L} and \mathcal{H} .
- Risk-groups at the upper end of \mathcal{L} or lower end of \mathcal{H} , which are on the threshold of moving to \mathcal{M} , are *already* paying very close to their fair premiums. So their contribution to loss coverage is almost unchanged when they cross the threshold and transfer into \mathcal{M} .

- So provided insurance demands are “well-behaved” functions of premiums, which is indeed the case for iso-elastic demand functions, the loss coverage would be a continuous function of the price collar. Hence the conclusions would remain unaffected if the constituents of \mathcal{L} , \mathcal{M} and \mathcal{H} change when the price collar changes.

6.2 OTHER DEMAND FUNCTIONS

To apply the results for non-iso-elastic demand functions, we can use the construct of *arc elasticity* (Hao et al. (2018), Vazquez (1995)) between two prices a and b , defined as:

$$\text{arc elasticity: } \eta_i(a, b) = \frac{\int_a^b \epsilon_i(s) d \log s}{\int_a^b d \log s}. \quad (6.3)$$

Arc elasticity can be interpreted as the *average* value of (point) demand elasticity over the price logarithmic arc from price a to price b . The political constraints on prices imply feasible arcs for π_i to be (μ_i, π_0) or (π_0, μ_i) depending on whether or not μ_i is less than π_0 . If we re-define λ_i as the corresponding arc elasticity, i.e.:

$$\lambda_i = \eta_i(\mu_i, \pi_i) = \frac{\int_{\pi_i}^{\mu_i} \epsilon_i(s) d \log s}{\int_{\pi_i}^{\mu_i} d \log s}, \quad (6.4)$$

we can then apply the techniques presented in previous sections using these λ_i -s, to compare various price collar regimes.

Note that if we are evaluating only a small change in the risk-classification scheme (i.e. a small changes in the high-risk and low-risk premiums), then only short arcs encompassing the proposed premium changes need to be considered. Furthermore, if elasticity is either less than 1 or greater than 1 for a risk-group throughout its relevant arc, then so is the arc elasticity. This allows quick inferences to be made without calculating arc elasticities, if one is prepared to assume either “elastic demand” or “inelastic demand” for a risk-group throughout its relevant arc.

6.3 DISCUSSION

A common regime of risk-classification in retail insurance markets is “full, but with bans on particular variables (e.g. gender, age, genetic tests, etc)”. Conceptually, this “full with restrictions” risk-classification is not quite the same as our price collar. But if insurers are unable to fully compensate for the banned variables by using correlated variables, “full with restrictions” seems likely to lead to a similar outcome to a price collar: low risks are charged a bit more than their fair premiums, and high risks a bit less. Our result in Section 5.3 then seems rather striking: partial risk-classification can never be optimal for loss coverage if both demand elasticities (or arc elasticities over the relevant arcs) are less than 1. The reason was highlighted in Section 5.3.3: for inelastic demand, increasing a

risk-group's demand (i.e. reducing its premium) *improves* its funding ratio. So there is no trade-off between demand and funding ratio: it helps to increase demand (i.e. reduce premium) for whichever risk-group which has higher elasticity (but adjusted to reflect $\mu_2 > \mu_1$), to the fullest possible extent that the political constraint permits.

However, this result says nothing about the quantum of the difference in loss coverage between any particular partial regime and the optimal regime (be it full or partial risk-classification). Partial risk-classification is not a single third option, but rather a continuum of options between full risk-classification and pooling. If the particular partial regime enforced by a price collar is "close" to whichever of full or pooling is optimal, the reduction in loss coverage compared to the optimum will be small.

7 CONCLUSIONS

Loss coverage, defined as expected population losses compensated by insurance, is a public policy criterion for comparing different risk classification regimes. Using a model with two risk-groups (high and low) and iso-elastic demand, we compared loss coverage under three alternative regulatory regimes: (i) full risk classification (ii) full pooling (iii) a price collar, whereby each insurer is permitted to set any premiums, subject to a maximum ratio of its highest and lowest prices for different risks. Outcomes depend on the comparative demand elasticities of low and high risks. If low-risk elasticity is sufficiently low compared with high-risk elasticity, pooling is optimal; and if it is sufficiently high, full risk classification is optimal. For an intermediate region where the elasticities are not too far apart, a price collar is optimal, but only if both elasticities are greater than one.

For more than two risk-groups, the results can be extended via the insight that equilibrium always involves a tripartite arrangement of risk-groups into three super-groups: a super-group \mathcal{L} of low risk-groups all charged the same low premium, a super-group \mathcal{H} of high risk-groups all charged the same high premium, and a middle super-group \mathcal{M} all charged their actuarially fair premiums.

For non-iso-elastic demand functions, the results can be extended using the construct of arc elasticity of demand, which can be thought of as the average value of point elasticity over the logarithmic arc between two prices.

DECLARATIONS OF INTEREST

None.

ACKNOWLEDGEMENTS

We thank Angus Macdonald for useful comments. Indradeb Chatterjee and MingJie Hao thank Radfall Charitable Trust for research funding.

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APPENDICES

APPENDIX A THEOREM 1

Theorem 1. *If there are n risk-groups, with risks $\mu_1 < \mu_2 < \dots < \mu_n$ with a price collar of κ , where $1 \leq \kappa \leq \mu_n/\mu_1$, there exists a stable equilibrium premium regime $\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$, such that:*

$$\pi_i = \begin{cases} \pi_L & \text{if } \mu_i < \pi_L; \\ \mu_i & \text{if } \pi_L \leq \mu_i \leq \pi_H; \\ \pi_H & \text{if } \mu_i > \pi_H. \end{cases} \quad (\text{A.1})$$

where $\pi_L = \min_i \pi_i$, $\pi_H = \max_i \pi_i$ and $\pi_H = \kappa \pi_L$.

Proof. We will prove the theorem using the following steps:

1. An equilibrium premium regime with the structure proposed in Equation A.1 exists.
2. If there are multiple equilibrium premium regimes with the same proposed structure, the regime with the smallest π_L is stable among all such regimes.
3. Given π_L and $\pi_H = \kappa \pi_L$, the premium regime with the proposed structure cannot be destabilised by any other equilibrium premium regime with the same π_L and π_H but having a different structure.
4. Given a compulsory price collar κ , the premium regime with the proposed structure cannot be destabilised by any other premium regime based on a voluntary smaller price collar.

Proof of step 1. Given a price collar κ , define the expected profit from setting the lowest premium $\underline{\pi}$, where $\mu_1 \leq \pi_L \leq \mu_n/\kappa$, as follows:

$$e_\kappa(\pi_L) = E(\underline{\pi}) = \sum_{i=1}^n p_i d_i(\pi_i) (\pi_i - \mu_i); \text{ where } \pi_i = \begin{cases} \pi_L & \text{if } \mu_i < \pi_L; \\ \mu_i & \text{if } \pi_L \leq \mu_i \leq \kappa \pi_L; \\ \kappa \pi_L & \text{if } \mu_i > \kappa \pi_L. \end{cases} \quad (\text{A.2})$$

If $\pi_L = \mu_1$, as $\kappa \mu_1 \leq \mu_n$, expected profit cannot be positive, i.e.: $e_\kappa(\mu_1) \leq 0$.

If $\pi_L = \mu_n/\kappa$, as $\mu_1 \leq \mu_n/\kappa$, expected profit cannot be negative, i.e.: $e_\kappa(\mu_n/\kappa) \geq 0$.

Assuming continuity of the demand functions $d_i(\pi_i)$ for all risk-groups, $e_\kappa(x)$ is also a continuous function. So, by the intermediate value theorem, there exists a value π_L , such that $\mu_1 \leq \pi_L \leq \mu_n/\kappa$, for which $e_\kappa(\pi_L) = 0$. This proves the existence of an equilibrium premium regime as outlined in the theorem. \square

Proof of step 2. If there are multiple solutions to the equation, $e_\kappa(\pi_L) = 0$, the premium regime based on the smallest of these roots cannot be destabilised by premium regimes based on any other solutions of $e_\kappa(\pi_L) = 0$. To show this, suppose if possible there are two premium regimes:

$$\begin{aligned} \underline{\pi} &= (\pi_1, \pi_2, \dots, \pi_n), \text{ with } \pi_H = \kappa \pi_L, \text{ where } \pi_L = \min_i \pi_i \text{ and } \pi_H = \max_i \pi_i; \\ \underline{\hat{\pi}} &= (\hat{\pi}_1, \hat{\pi}_2, \dots, \hat{\pi}_n), \text{ with } \hat{\pi}_H = \kappa \hat{\pi}_L, \text{ where } \hat{\pi}_L = \min_i \hat{\pi}_i \text{ and } \hat{\pi}_H = \max_i \hat{\pi}_i; \end{aligned}$$

with $\pi_L < \hat{\pi}_L$ (and consequently $\pi_H < \hat{\pi}_H$), such as shown in Figure 5.

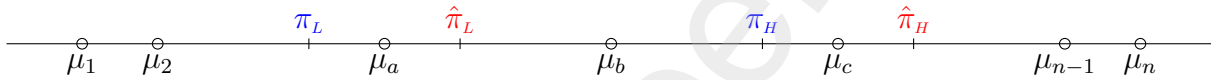


Figure 5: Theorem 1: Proof of Step 2.

- $\underline{\hat{\pi}}$ involves higher premiums for all risk-groups with $\mu_i < \hat{\pi}_L$, and higher premiums for all risk-groups with $\mu_i > \pi_H$, and the same (actuarially fair) premiums to all risk-groups in between.
- So all risks are either not attracted to $\underline{\hat{\pi}}$ (because it charges more), or are indifferent (because it charges the same); and in the latter case, the premium regime $\underline{\pi}$ would also be indifferent to the loss of these risk-groups, because they generate no profit or loss under $\underline{\pi}$.

Therefore any higher solution $e_\kappa(\pi_L) = 0$ cannot destabilise the lowest solution. \square

Proof of step 3. Any alternative regime $\underline{\hat{\pi}}$ with the same π_L and π_H and a different structure needs to charge some risks more, and/or some risks less.

- Risks which are charged less will be attracted by $\underline{\hat{\pi}}$, and generate a smaller profit contribution (or larger loss) than they do under $\underline{\pi}$.
- To achieve zero profit then requires that this deficit be made up by charging other risks more.
- But no risks will be prepared to pay more, because they will prefer $\underline{\pi}$.

Therefore $\underline{\pi}$ cannot be destabilised by an alternative regime $\hat{\underline{\pi}}$ with the same π_L and π_H and a different structure. \square

Proof of step 4. As a regulatory price collar would only require $\pi_H \leq \kappa \pi_L$, we need to show that $\underline{\pi}$ cannot be destabilised by an equilibrium premium regime using a smaller price collar.

Consider an alternative equilibrium premium regime: $\hat{\underline{\pi}} = (\hat{\pi}_1, \hat{\pi}_2, \dots, \hat{\pi}_n)$, with minimum and maximum premiums: $\hat{\pi}_L = \min_i \hat{\pi}_i$ and $\hat{\pi}_H = \max_i \hat{\pi}_i$ respectively, such that $\hat{\pi}_H = \hat{\kappa} \hat{\pi}_L$, for some $\hat{\kappa} < \kappa$.

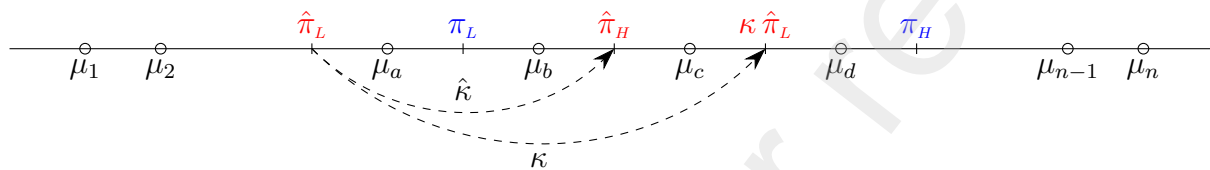


Figure 6: Theorem 1: Proof of Step 4.

- If the lower end of the new collar is set *above* the lower end of the old collar, then $\hat{\underline{\pi}}$ attracts none of the profitable low risks, and so cannot be profitable and cannot destabilise $\underline{\pi}$.
- If the lower end of the new collar is set *below* the lower end of the old collar, then $\hat{\underline{\pi}}$ generates a smaller profit contribution on all risk-groups with $\mu_i < \pi_L$.
- Equilibrium requires that this deficit be made up by charging other risks more.
- But because of the smaller $\hat{\kappa}$ and lower base of the collar, $\hat{\underline{\pi}}$ does not charge any risk-groups more.

Therefore no equilibrium solution with a smaller collar $\hat{\kappa}$ exists. \square

So, $\underline{\pi}$, as outlined in the theorem, exists, and is a unique stable equilibrium premium regime satisfying all political, regulatory and economic constraints. \blacksquare

APPENDIX B THEOREM 2

In order to prove Theorem 2, we first provide the algebraic inequalities defining the regions represented graphically in Figure 3. Recall the definitions of graph regions: \mathcal{P} : “Pooled”, \mathcal{F} : “Full risk-classification”, \mathcal{I}_L : “Intermediate, low elasticity” and \mathcal{I}_H : “Intermediate, high elasticity”, so that:

$$\begin{aligned} \mathcal{I}_L: \lambda_2 \leq \lambda_1 \leq 1 \text{ and } 1 \leq \left(1 - \frac{1}{\lambda_2}\right) / \left(1 - \frac{1}{\lambda_1}\right) \leq \mu_2 / \mu_1. & \quad \mathcal{I}_H: \lambda_2 \geq \lambda_1 \geq 1 \text{ and } 1 \leq \left(1 - \frac{1}{\lambda_2}\right) / \left(1 - \frac{1}{\lambda_1}\right) \leq \mu_2 / \mu_1. \\ \mathcal{P}: \{(\lambda_1, \lambda_2) : \lambda_2 \geq \lambda_1\} - \mathcal{I}_H. & \quad \mathcal{F}: \{(\lambda_1, \lambda_2) : \lambda_2 \leq \lambda_1\} - \mathcal{I}_L. \\ \mathcal{P}_1: \lambda_1 \leq 1 \text{ and } \lambda_2 \geq 1. & \quad \mathcal{F}_1: \lambda_1 \geq 1 \text{ and } \lambda_2 \leq 1. \\ \mathcal{P}_2: \lambda_1 \leq \lambda_2 \leq 1. & \quad \mathcal{F}_2: \{(\lambda_1, \lambda_2) : \lambda_2 \leq \lambda_1 \leq 1\} - \mathcal{I}_L. \\ \mathcal{P}_3: \{(\lambda_1, \lambda_2) : \lambda_2 \geq \lambda_1 \geq 1\} - \mathcal{I}_H. & \quad \mathcal{F}_3: \lambda_1 \geq \lambda_2 \geq 1. \end{aligned}$$

Lemma 1. For a risk-group i , with positive iso-elastic demand elasticity λ_i , m_i satisfies:

$$\lambda_i \leq 1 \Leftrightarrow m_i \leq 0.$$

Proof. This follows directly from the expression of m_i given in Equation 4.8.⁹ \square

Lemma 2. For the highest risk-group n , with positive iso-elastic demand elasticity λ_n , m_n is bounded above at 1, i.e. $m_n \leq 1$.

Proof. By the political constraint, $\pi_n \leq \mu_n$, and as $\lambda_n > 0$, $m_n \leq 1$. \square

Next note that, for the case of two risk-groups:

$$m_2 - m_1 = \frac{\pi_2}{\mu_2} \left(1 - \frac{1}{\lambda_2}\right) - \frac{\pi_1}{\mu_1} \left(1 - \frac{1}{\lambda_1}\right) = \left(1 - \frac{1}{\lambda_1}\right) \frac{\pi_2}{\mu_1} \left[\left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}}\right) \left(\frac{\mu_1}{\mu_2}\right) - \frac{\pi_1}{\pi_2} \right]. \quad (\text{B.1})$$

And, specifically, for both \mathcal{I}_L and \mathcal{I}_H ,

$$1 \leq \frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \leq \frac{\mu_2}{\mu_1} \Rightarrow \frac{\mu_1}{\mu_2} \leq \left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}}\right) \left(\frac{\mu_1}{\mu_2}\right) \leq 1. \quad (\text{B.2})$$

⁹We use the notation \geq in the following sense: $A \geq B \Rightarrow C \geq D$ is shorthand for $A > B \Rightarrow C > D$ and $A = B \Rightarrow C = D$ and $A < B \Rightarrow C < D$. A similar interpretation applies for the notation \leq .

Also, note that for any equilibrium premium regime $\underline{\pi} = (\pi_1, \pi_2)$, as π_1 increases from $\pi_1 = \mu_1$ (full risk-classification) to $\pi_1 = \pi_2$ (pooling), the ratio π_1/π_2 goes from μ_1/μ_2 to 1. So, for both \mathcal{I}_L and \mathcal{I}_H , by intermediate value theorem, there exists a premium regime $\underline{\pi}^* = (\pi_1^*, \pi_2^*)$, such that:

$$\frac{\pi_1^*}{\pi_2^*} = \left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \right) \left(\frac{\mu_1}{\mu_2} \right), \quad (\text{B.3})$$

so that Equation B.1 becomes:

$$m_2 - m_1 = \left(1 - \frac{1}{\lambda_1} \right) \frac{\pi_2}{\mu_1} \left[\frac{\pi_1^*}{\pi_2^*} - \frac{\pi_1}{\pi_2} \right]. \quad (\text{B.4})$$

Lemma 3. *Suppose there are 2 risk-groups with risks $\mu_1 < \mu_2$ and iso-elastic demand elasticities λ_1 and λ_2 respectively. For equilibrium premium regime $\underline{\pi} = (\pi_1, \pi_2)$:*

3.1. $\mathcal{P} : m_2 - m_1 \geq 0$.

3.2. $\mathcal{I}_L : m_2 - m_1 \begin{matrix} \leq \\ > \end{matrix} 0 \Leftrightarrow \frac{\pi_1}{\pi_2} \begin{matrix} \leq \\ > \end{matrix} \frac{\pi_1^*}{\pi_2^*}$.

3.3. $\mathcal{F} : m_2 - m_1 \leq 0$.

3.4. $\mathcal{I}_H : m_2 - m_1 \begin{matrix} \geq \\ < \end{matrix} 0 \Leftrightarrow \frac{\pi_1}{\pi_2} \begin{matrix} \geq \\ < \end{matrix} \frac{\pi_1^*}{\pi_2^*}$.

Proof. (of Lemma 3.1:)

For $\mathcal{P}_1 : \lambda_1 \leq 1$ and $\lambda_2 \geq 1 \Rightarrow m_1 \leq 0$ and $0 < m_2 \leq 1 \Rightarrow m_2 - m_1 \geq 0$.

For $\mathcal{P}_2 : \lambda_1 \leq \lambda_2 \leq 1 \Rightarrow 1 - \frac{1}{\lambda_1} \leq 1 - \frac{1}{\lambda_2} \leq 0$ and $\frac{\pi_1}{\mu_1} \geq \frac{\pi_2}{\mu_2} \geq 0 \Rightarrow m_2 - m_1 \geq 0$.

For $\mathcal{P}_3 : 1 \leq \lambda_1 \leq \lambda_2$ and $\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \geq \frac{\mu_2}{\mu_1} \Rightarrow \left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \right) \left(\frac{\mu_1}{\mu_2} \right) \geq 1$, $\frac{\pi_1}{\pi_2} \leq 1$ and $\lambda_1 \geq 1$

$$\Rightarrow m_2 - m_1 = \left(1 - \frac{1}{\lambda_1} \right) \frac{\pi_2}{\mu_1} \left[\left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \right) \left(\frac{\mu_1}{\mu_2} \right) - \frac{\pi_1}{\pi_2} \right] \geq 0.$$

So, for $\mathcal{P} : m_2 - m_1 \geq 0$. □

Proof. (of Lemma 3.2:) For \mathcal{I}_L , $\lambda_1 \leq 1$, so:

$$m_2 - m_1 = \left(1 - \frac{1}{\lambda_1} \right) \frac{\pi_2}{\mu_1} \left[\frac{\pi_1^*}{\pi_2^*} - \frac{\pi_1}{\pi_2} \right] \begin{matrix} \leq \\ > \end{matrix} 0 \Leftrightarrow \frac{\pi_1}{\pi_2} \begin{matrix} \leq \\ > \end{matrix} \frac{\pi_1^*}{\pi_2^*}$$

□

Proof. (of Lemma 3.3:)

For \mathcal{F}_1 : $\lambda_1 \geq 1$ and $\lambda_2 \leq 1 \Rightarrow m_1 \geq 0$ and $m_2 \leq 0 \Rightarrow m_2 - m_1 \leq 0$.

For \mathcal{F}_3 : $\lambda_1 \geq \lambda_2 \geq 1 \Rightarrow 0 \leq 1 - \frac{1}{\lambda_2} \leq 1 - \frac{1}{\lambda_1}$ and $\frac{\pi_2}{\mu_2} \leq \frac{\pi_1}{\mu_1} \Rightarrow m_2 - m_1 \leq 0$.

For \mathcal{F}_2 : $\lambda_2 \leq \lambda_1 \leq 1$ and $\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \geq \frac{\mu_2}{\mu_1} \Rightarrow \left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \right) \left(\frac{\mu_1}{\mu_2} \right) \geq 1$, $\frac{\pi_1}{\pi_2} \leq 1$ and $\lambda_1 \leq 1$
 $\Rightarrow m_2 - m_1 = \left(1 - \frac{1}{\lambda_1} \right) \frac{\pi_2}{\mu_1} \left[\left(\frac{1 - \frac{1}{\lambda_2}}{1 - \frac{1}{\lambda_1}} \right) \left(\frac{\mu_1}{\mu_2} \right) - \frac{\pi_1}{\pi_2} \right] \leq 0$.

So, for \mathcal{F} : $m_2 - m_1 \leq 0$. □

Proof. (of Lemma 3.4:) For \mathcal{I}_H , $\lambda_1 \geq 1$, so:

$$m_2 - m_1 = \left(1 - \frac{1}{\lambda_1} \right) \frac{\pi_2}{\mu_1} \left[\frac{\pi_1^*}{\pi_2^*} - \frac{\pi_1}{\pi_2} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow \frac{\pi_1}{\pi_2} \begin{matrix} \leq \\ \geq \end{matrix} \frac{\pi_1^*}{\pi_2^*}$$

□

Theorem 2. *Suppose there are two risk-groups with risks $\mu_1 < \mu_2$ and iso-elastic demand elasticities λ_1 and λ_2 respectively.*

2.1. \mathcal{P} : *Loss coverage is maximised by pooling and minimised by full risk-classification, while partial risk-classification is intermediate.*

2.2. \mathcal{I}_L : *Loss coverage is maximised by either pooling or full risk-classification depending on the population proportions and fair-premium demands.*

2.3. \mathcal{F} : *Loss coverage is maximised by full risk-classification regime and minimised by pooling, while partial risk-classification is intermediate.*

2.4. \mathcal{I}_H : *Loss coverage is maximised by a specific partial risk-classification regime.*

For the proof of Theorem 2, recall from Equation 4.10:

$$\Delta C = T (m_2 - m_1) \Delta \pi_1, \text{ where } T > 0. \quad (\text{B.5})$$

and the behaviour of $(m_2 - m_1)$ is outlined in Lemma 3.

Proof. (of Theorem 2.1:) For \mathcal{P} : $m_2 - m_1 \geq 0$. So, as π_1 increases from full risk-classification ($\pi_1 = \mu_1$) to pooling ($\pi_1 = \pi_2$), the loss coverage, C , is an increasing function. Hence, loss coverage is maximum for pooled equilibrium and minimum for full risk-classification. Partial risk-classification is intermediate. \square

Proof. (of Theorem 2.2:) For \mathcal{I}_L :

$$\frac{\Delta C}{\Delta \pi_1} = T (m_2 - m_1) \underset{>}{\leq} 0 \Leftrightarrow \frac{\pi_1}{\pi_2} \underset{>}{\leq} \frac{\pi_1^*}{\pi_2^*}. \quad (\text{B.6})$$

This implies that as π_1 increases from full risk-classification ($\pi_1 = \mu_1$) to pooling ($\pi_1 = \pi_2$), the loss coverage, C , first decreases and then increases reaching a minimum at $\pi_1 = \pi_1^*$. Hence, loss coverage is maximum at either of the two extremes, pooled or full risk-classification. \square

Proof. (of Theorem 2.3:) For \mathcal{F} : $m_2 - m_1 \leq 0$. So, as π_1 increases from full risk-classification ($\pi_1 = \mu_1$) to pooling ($\pi_1 = \pi_2$), the loss coverage, C , is a decreasing function. Hence, loss coverage is maximum for full risk-classification and minimum for pooled equilibrium. Partial risk-classification is intermediate. \square

Proof. (of Theorem 2.4:) For \mathcal{I}_H :

$$\frac{\Delta C}{\Delta \pi_1} = T (m_2 - m_1) \underset{<}{\geq} 0 \Leftrightarrow \frac{\pi_1}{\pi_2} \underset{>}{\leq} \frac{\pi_1^*}{\pi_2^*}. \quad (\text{B.7})$$

This implies that as π_1 increases from full risk-classification ($\pi_1 = \mu_1$) to pooling ($\pi_1 = \pi_2$), the loss coverage, C , first increases and then decreases reaching a maximum at $\pi_1 = \pi_1^*$. Hence, loss coverage is maximum at the partial risk-classification regime $\underline{\pi}^*$ \square

APPENDIX C THEOREM 3

Theorem 3. Suppose there are n risk-groups, with risks $\mu_1 < \mu_2 < \dots < \mu_n$ and iso-elastic demand elasticities $\lambda_1, \lambda_2, \dots, \lambda_n$ respectively.

Under a price collar κ , let $\underline{\pi}$ be the stable equilibrium premium regime, sub-dividing the risk-groups into three super-groups \mathcal{L} , \mathcal{M} and \mathcal{H} , where all risk-groups in \mathcal{L} pay the same premium π_L , all risk-groups in \mathcal{H} pay the same premium π_H , and all risk-groups in \mathcal{M} pay their fair actuarial premiums. Let μ_L and μ_H be the pooled equilibrium premiums of the risk-groups in \mathcal{L} and \mathcal{H} respectively. Further suppose:

$$\lambda_i = \begin{cases} \lambda_L & \text{if } i \in \mathcal{L}; \\ \lambda_H & \text{if } i \in \mathcal{H}. \end{cases}$$

Then the sensitivity of the loss coverage is given by:

$$\Delta C = T (m_H - m_L) \Delta \pi_L, \quad (6.1)$$

$$\text{where } T = \frac{p_L \tau_L \lambda_L \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L+1}}{1 - m_H}; \text{ and } m_i = \frac{\pi_i}{\mu_i} \left(1 - \frac{1}{\lambda_i}\right); \quad i = L, H; \quad (6.2)$$

where for the super-groups \mathcal{L} and \mathcal{H} , p_L and p_H are the aggregate proportion of population belonging to the super-groups; and τ_L and τ_H can be interpreted as the ‘fair-premium demand’ when all risk-groups in the respective super-groups are pooled and charged the same pooled premium.

Proof. As the risk-groups in \mathcal{M} do not contribute to profit or loss, the equilibrium condition can be expressed as:

$$E(\underline{\pi}) = \underbrace{\sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\pi_L}\right)^{\lambda_L} (\pi_L - \mu_i)}_{E_{\mathcal{L}}} + \underbrace{\sum_{j \in \mathcal{H}} p_j \tau_j \left(\frac{\mu_j}{\pi_H}\right)^{\lambda_H} (\pi_H - \mu_j)}_{E_{\mathcal{H}}} = 0. \quad (C.1)$$

The first term, $E_{\mathcal{L}}$, in Equation C.1, can be split as follows:

$$E_{\mathcal{L}} = \sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\pi_L}\right)^{\lambda_L} (\pi_L - \mu_i); \quad (C.2)$$

$$= \sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L} \left(\frac{\mu_i}{\mu_L}\right)^{\lambda_L} [(\pi_L - \mu_L) + (\mu_L - \mu_i)]; \quad (C.3)$$

$$= \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L} \left[\sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\mu_L}\right)^{\lambda_L} (\pi_L - \mu_L) + \sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\mu_L}\right)^{\lambda_L} (\mu_L - \mu_i) \right]; \quad (\text{C.4})$$

where μ_L is such that the second term in Equation C.4, is zero, i.e.:

$$\sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\mu_L}\right)^{\lambda_L} (\mu_L - \mu_i) = 0, \quad (\text{C.5})$$

so that μ_L can be interpreted as the pooled equilibrium premium, if the insurance market only consisted of the risk-groups in \mathcal{L} . Also, μ_L is unique and is given by:

$$\mu_L = \frac{\sum_{i \in \mathcal{L}} p_i \tau_i \mu_i^{\lambda_L + 1}}{\sum_{i \in \mathcal{L}} p_i \tau_i \mu_i^{\lambda_L}}, \quad \text{so that: } \mu_1 \leq \mu_L \leq \max_{i \in \mathcal{L}} \mu_i \leq \pi_L. \quad (\text{C.6})$$

Using such a μ_L , the expression for $E_{\mathcal{L}}$ in Equation C.4 becomes:

$$E_{\mathcal{L}} = \left[\sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\mu_L}\right)^{\lambda_L} \right] \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L} (\pi_L - \mu_L) = p_L \tau_L \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L} (\pi_L - \mu_L); \quad (\text{C.7})$$

$$\text{where } p_L = \sum_{i \in \mathcal{L}} p_i, \quad \tau_L = \sum_{i \in \mathcal{L}} \left(\frac{p_i}{p_L}\right) \tau_i \left(\frac{\mu_i}{\mu_L}\right)^{\lambda_L}. \quad (\text{C.8})$$

Note that p_L is the aggregate proportion of population belonging to the collection of risk-groups in \mathcal{L} and τ_L can be interpreted as the ‘fair-premium demand’ when all risk-groups in \mathcal{L} are pooled and charged the same pooled premium, μ_L .

A similar line of argument for the risk-groups in \mathcal{H} leads to:

$$E_{\mathcal{H}} = p_H \tau_H \left(\frac{\mu_H}{\pi_H}\right)^{\lambda_H} (\pi_H - \mu_H); \quad \text{where } p_H = \sum_{j \in \mathcal{H}} p_j, \quad \tau_H = \sum_{j \in \mathcal{H}} \left(\frac{p_j}{p_H}\right) \tau_j \left(\frac{\mu_j}{\mu_H}\right)^{\lambda_H}, \quad (\text{C.9})$$

$$\text{where } \mu_H = \frac{\sum_{j \in \mathcal{H}} p_j \tau_j \mu_j^{\lambda_H + 1}}{\sum_{j \in \mathcal{H}} p_j \tau_j \mu_j^{\lambda_H}}, \quad \text{so that: } \pi_H \leq \min_{j \in \mathcal{H}} \mu_j \leq \mu_H \leq \mu_n. \quad (\text{C.10})$$

Using the expressions for $E_{\mathcal{L}}$ and $E_{\mathcal{H}}$ in Equations C.7 and C.9 respectively, Equation C.1 becomes:

$$E(\underline{\pi}) = \underbrace{p_L \tau_L \left(\frac{\mu_L}{\pi_L}\right)^{\lambda_L} (\pi_L - \mu_L)}_{E_{\mathcal{L}}} + \underbrace{p_H \tau_H \left(\frac{\mu_H}{\pi_H}\right)^{\lambda_H} (\pi_H - \mu_H)}_{E_{\mathcal{H}}} = 0. \quad (\text{C.11})$$

Equation C.11 shows that it is possible to conceptualise \mathcal{L} and \mathcal{H} as super-groups with demand elasticities λ_L and λ_H respectively, where the true risks of the super-groups

are taken to be pooled equilibrium premiums, μ_L and μ_H , of the respective super-groups. Essentially, this reduces the problem involving more than two risk-groups to the simpler two risk-groups problem, so that the analysis of Section 4.1 can be extended directly to this situation.

The loss coverage for the stable premium regime can then be expressed as:

$$C(\underline{\pi}) = \sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\pi_L} \right)^{\lambda_L} \mu_i + \sum_{j \in \mathcal{H}} p_j \tau_j \left(\frac{\mu_j}{\pi_H} \right)^{\lambda_H} \mu_j + \sum_{m \in \mathcal{M}} p_m \tau_m \mu_m; \quad (\text{C.12})$$

$$= \sum_{i \in \mathcal{L}} p_i \tau_i \left(\frac{\mu_i}{\pi_L} \right)^{\lambda_L} \pi_L + \sum_{j \in \mathcal{H}} p_j \tau_j \left(\frac{\mu_j}{\pi_H} \right)^{\lambda_H} \pi_H + \sum_{m \in \mathcal{M}} p_m \tau_m \mu_m; \quad (\text{C.13})$$

... by the equilibrium condition in Equation C.1;

$$= p_L \tau_L \left(\frac{\mu_L}{\pi_L} \right)^{\lambda_L} \pi_L + p_H \tau_H \left(\frac{\mu_H}{\pi_H} \right)^{\lambda_H} \pi_H + \sum_{m \in \mathcal{M}} p_m \tau_m \mu_m; \quad (\text{C.14})$$

... by the definitions of p_L , τ_L , μ_L , p_H , τ_H and μ_H ;

$$= p_L \tau_L \left(\frac{\mu_L}{\pi_L} \right)^{\lambda_L} \mu_L + p_H \tau_H \left(\frac{\mu_H}{\pi_H} \right)^{\lambda_H} \mu_H + \sum_{m \in \mathcal{M}} p_m \tau_m \mu_m; \quad (\text{C.15})$$

by the equilibrium condition in Equation C.11.

Assuming the compositions of \mathcal{L} , \mathcal{M} and \mathcal{H} remain unaffected, changing π_L affects π_H without any implications for the risk-groups in \mathcal{M} . Also note that, as long as \mathcal{L} , \mathcal{M} and \mathcal{H} remain unchanged, we can follow the same steps as in Section 4.1, to get:

$$\Delta C = T (m_H - m_L) \Delta \pi_L, \quad (6.1)$$

$$\text{where } T = \frac{p_L \tau_L \lambda_L \left(\frac{\mu_L}{\pi_L} \right)^{\lambda_L + 1}}{1 - m_H}; \text{ and } m_i = \frac{\pi_i}{\mu_i} \left(1 - \frac{1}{\lambda_i} \right); \quad i = L, H. \quad (6.2)$$

□