Greenwich Papers in Political Economy

Was Pareto right? Is the distribution of wealth thick-tailed?

Rafael Wildauer[1] (University of Greenwich)

Ines Heck[2] (University of Greenwich)

Jakob Kapeller[3] (University Duisburg-Essen and Johannes Kepler University Linz)

Year: 2023

No: GPERC92

Abstract

We fit log-normal, exponential, Pareto type I and Pareto type II distributions to US wealth data from 1989 to 2019 and examine the goodness of fit. Unlike earlier literature this paper uses high quality data, covering the entire US population, yielding powerful and unbiased tests. Beyond the 91st percentile the type II distribution consistently provides the best fit to the data and supports the hypothesis of a thick-tailed wealth (and by extension income) distribution. In addition, our results highlight the changing shape of the tail with decreasing concentration up to the 98th percentile and increasing concentration beyond. Our results suggest that practitioners modelling the distribution of wealth in situations where only limited data is available, a type I Pareto distribution might still serve as a valuable bias correction tool but should only be fitted to the top 1% of the population.

JEL Keywords: **D31** Personal Income, Wealth, and Their Distributions; **C46** Specific Distributions; **C81** Data Estimation Methodology

Acknowledgements: We thank participants of the PEGFA research seminar series for valuable comments, all remaining errors are ours.

---

[1] Rafael Wildauer thankfully acknowledges financial support from the Austrian Federal Chamber of Labour, Vienna (AK) and the Hans Böckler Foundation under grant number 2021-544-2. Contact: r.wildauer@gre.ac.uk.

[2] Contact: Ines.Heck@gre.ac.uk

[3] Contact: Jakob.Kapeller@jku.at

Was Pareto right? Is the distribution of wealth thick-tailed?

Rafael Wildauer[1], Ines Heck[2], and Jakob Kapeller[3]

[1,2]University of Greenwich
[3]University Duisburg-Essen, Institute for Socio-Economics

This version: February 2023

Abstract

We fit log-normal, exponential, Pareto type I and Pareto type II distributions to US wealth data from 1989 to 2019 and examine the goodness of fit. Unlike earlier literature this paper uses high quality data, covering the entire US population, yielding powerful and unbiased tests. Beyond the 91^{st} percentile the type II distribution consistently provides the best fit to the data and supports the hypothesis of a thick-tailed wealth (and by extension income) distribution. In addition, our results highlight the changing shape of the tail with decreasing concentration up to the 98^{th} percentile and increasing concentration beyond. Our results suggest that practitioners modelling the distribution of wealth in situations where only limited data is available, a type I Pareto distribution might still serve as a valuable bias correction tool but should only be fitted to the top 1% of the population.

JEL Keywords: D31 Personal Income, Wealth, and Their Distributions; C46 Specific Distributions; C81 Data Estimation Methodology

1 Introduction

This paper revisits the old question about the shape of the distribution of wealth among households. Since Pareto's (1964) influential work, thick[1] tails have been documented and used in the modelling of waves, city sizes, finance (Bouchaud et al. 2004) and technology adoption (Meade & Islam 2006, Kishi 2019). See Gabaix (2016) for an extensive overview. Whether the wealth distribution exhibits a thick tail – what we will call the Pareto hypothesis (PH) – and if so how thick this tail is and how its shape changes along the distribution, is important for three reasons. First, there is a growing empirical literature relying on the assumption of a Pareto tail in its analysis. Examples include adaptations of wealth survey data for missing top wealth observations (Advani, Bangham & Leslie 2020, Bach et al. 2018, Eckerstorfer et al. 2016, Vermeulen 2016, 2018, Wildauer & Kapeller 2021, 2022), wealth tax revenue estimations (Advani, Hughson & Tarrant 2020, Kapeller et al. 2021, Tippet et al. 2021, Krenek & Schratzenstaller 2022, Apostel & O'Neill 2022) and the Distributional National Accounts (DINA) literature which aims at producing micro data sets which are consistent with national aggregates (Piketty et al. 2018, Blanchet et al. 2021, Garbinti et al. 2018, Waltl 2022, Jenkins 2016, Chakraborty et al. 2019). For all of these papers the validity of the Pareto hypothesis is a key assumption. Second, modelling tax policy for both wealth and top income taxation[2] is crucially influenced by the presence of thick tails (Saez 2001, Saez & Stantcheva 2016, 2018). Thirdly, the Pareto hypothesis can inform our theoretical understanding and modeling of wealth accumulation. If the Pareto hypothesis holds, we should focus on modelling and understanding mechanisms that allow for the result of thick tails (Benhabib & Bisin 2018).

The existing empirical evidence on the Pareto hypothesis is mixed. Some authors accept or support it (Klass et al. 2006, Levy & Solomon 1997, Sinha 2006, Ning & You-Gui 2007, Jagielski et al. 2017, Dragulescu & Yakovenko 2001), others reject it (Chan et al. 2017, Ogwang 2013) or report inconclusive results (Brzezinski 2014, Campolieti 2018, Ogwang 2011). However, most of these contributions exhibit three shortcomings. Firstly, the data used for testing the Pareto hypothesis does not adequately represent the wealth distribution. These studies heavily rely on rich list data, i.e. lists of billionaires and millionaires compiled for national business magazines or newspapers by journalists for the entertainment of their readers. The details of how these lists are constructed are often poorly documented and hardly traceable. Moreover, different magazines can report wildly different results for the same households or individuals (Capehart 2014), indicating serious measurement errors. Secondly, rich list data only covers a tiny proportion of the wealth distribution. For example, the *Forbes 400* list represent only 0.0003% of the US population and the range of wealth holdings on the list only covers a single

[1]We are following Benhabib & Bisin (2018) with their notion of thick (fat) tails which means tails following a power law decay, meaning that they decrease in magnitude more slowly than if they were to decrease, e.g., exponentially or following a log-normal distribution, which Benhabib & Bisin (2018) both classify as thin-tailed.

[2]The nature of the wealth distribution is relevant for capital income taxation, especially at the top, as a thick-tailed wealth and hence capital distribution implies a heavily skewed or thick-tailed capital income distribution. This is true even if uniform or equal returns to investment regardless of wealth levels are assumed, but even more relevant considering that the returns to more extreme wealth are often disproportionately higher (Fagereng et al. 2020, Bach et al. 2020).

order of magnitude. Thirdly, these authors test the Pareto hypothesis using the type I Pareto distribution which is characterised by its scale invariance property. This means a type I Pareto population exhibits self-similarity: The share of the richest 10% of individuals is the same as the share of the richest 10% within the top 10% and so on. This is a fairly rigid pattern which need not hold in the data. All three shortcomings can contribute to wrongly rejecting the PH and thus a more powerful test is needed.

This paper presents a test of the Pareto hypothesis which does not suffer from either of these problems. By using data from the Survey of Consumer Finances (SCF) for the United States we use the best and most granular data available on the US distribution of wealth. The SCF uses tax data to implement effective oversampling and thus addresses differential non-response problems (Avery et al. 1988, Bricker et al. 2016, Kennickell 2017, Osier 2016) while producing a representative sample of the entire population of US households. Furthermore, we directly address the exclusion of the richest 400 families on the *Forbes* list of billionaires from the SCF by using a rank correction approach (Wildauer & Kapeller 2022). We make three contributions to the existing literature. Firstly, using the SCF allows us to test the Pareto hypothesis on a much higher quality dataset compared to rich lists. This means we are less likely to wrongly reject the PH due to measurement errors. Secondly, by using data from the Survey of Consumer Finances (SCF) instead of rich-lists we can test the Pareto hypothesis for the top 10% instead the top 0.0003% of the US population. This yields a more powerful test, since wealth in the SCF between the 91^{st} percentile and the maximum stretches over three orders of magnitude rather than one in rich list data. In addition, it also allows us to assess the length of the Pareto tail. Thirdly, while the existing literature mostly relies on type I Pareto distributions, we use an estimator for type II Pareto distributions based on Castillo & Hadi (1997) which can be applied to weighted survey data and has only recently been introduced to the wealth survey literature (Heck et al. 2020, Kennickell 2021). We can thus relax the assumption of scale invariance, which the previous literature relies on, and which we demonstrate to be rather implausible. This in turn allows us to provide a detailed analysis of the changing shape of the tail along the wealth distribution.

By addressing the limitations of the previous studies, we demonstrate that the type II Pareto distribution provides a statistically superior fit compared to the type I Pareto, log-normal, or exponential distributions for the portion of the US wealth distribution above the 91^{st} percentile. This result emerges especially strongly beyond the 97^{st} percentile. Therefore, we conclude that the Pareto hypothesis is alive and well. This result does not only have important implications for the literature discussed above, it also provides support for the Pareto hypothesis with respect to the distribution of income, since the distribution of capital income closely follows the distribution of wealth and is an important factor for the top of the income distribution. Furthermore we show that the shape of the wealth distribution is not stable (scale invariant) as applied by the type I Pareto distribution. Using inverted Pareto coefficients (Blanchet et al. 2022) to measure tail thickness, We find that the tail becomes thinner between the 91^{st} and the 98^{th} percentile and starts to thicken afterwards. This means that the extent of inequality of concentration of wealth increases at the very top of the distribution, in line with recently documented heavily skewed rates of return along the wealth distribution Fagereng et al. (2020).

The rest of the paper is organised as follows: section 2 provides an overview over the related literature. Section 3 outlines our methodology and presents the data we are using. Section 4 presents our results and section 5 concludes.

2 Pareto tails in wealth data: A brief review

Most research testing wealth inequality for the goodness of the Pareto distribution's fit is based within the interdisciplinary econophysics literature, most notably publications like *Physica A*. Due to a lack of more appropriate data, many of these contributions rely on rich lists compiled by business journalists whose data sources and underlying methodology are often obscure and hard to verify. Interestingly, researchers have come to varying conclusions using the same *Forbes 400* list as a database but using different methods. Levy & Solomon (1997) were the first to use *Forbes 400* data and conclude that the Pareto distribution appears to be a good fit except for deviations at rounded levels of wealth. While Klass et al. (2006, 2007) also conclude that the Pareto distribution fits 1988-2003 data well, Chan et al. (2017) find other distributions, especially the beta Pareto distribution, to give adequate fits for 1988-2006 data. The same exercise has been performed for the 100 wealthiest Canadians as compiled by *Canadian Business* where Ogwang (2011) finds mixed evidence for 1999-2008 data and Campolieti (2018) cannot find distinguishable evidence in favour of the Pareto distribution compared to others for 1999-2017 data. A paper using 2002-2004 rich list data for India has indicated consistency with power law behaviour for the top of the wealth distribution (Sinha 2006). Chinese rich list data (2003-2005) seem to comply with this behaviour, too (Ning & You-Gui 2007). Similar research exists for the *Forbes World's Billionaires* for which Pareto behaviour, by contrast, has been rejected for the years 2000-2009 (Ogwang 2013). Brzezinski (2014) only finds that 35 % of the data sets under investigation are consistent with the Pareto distribution using 1996-2012 global, 1988-2012 American, 2006-2012 Chinese and 2004-2011 Russian rich list data.

These results could be a consequence of the data quality as different magazines can report wildly different results for the same individuals (Capehart 2014), suggesting that rich lists might not be the best data source overall. In addition, rich list authors frequently round monetary estimates of wealth, leading to what is called a heaping effect where several households worth 500 million can be observed, but none worth 476.23 million, e.g. Besides the heaping effect that was already described by Levy & Solomon (1997), other major issues concern the lists' completeness, consistency and selectivity especially at lower ranks, i.e. magazines tend to include families which they consider of interest to their readers (Bach et al. 2018). This is barely surprising since the purpose of compiling these rich lists for media outlets is hardly to create the most accurate listings of wealthy individuals and families, but rather to entertain their readers. Furthermore, a rich list extension of survey data can be problematic due to different definitions of wealth and choice of data unit. Regarding the latter, efforts have been made to rectify this issue (Bach et al. 2018, Ferschli et al. 2018), but the diverging wealth definitions seem irreconcilable to some extent. Magazines' wealth definitions can be arbitrary or constraint to publicly available information. While rich list creators hardly ever have access to liability records, surveys often refer to net wealth. Although some contributions rely on survey data and supplement rich list data for the top observations (Eckerstorfer et al. 2016, Ferschli et al. 2018, Dalitz 2018, Vermeulen 2018, Westermeier 2016), this does not resolve the measurement problems of the used rich list data.

This brings us to contributions using other types of data. As mentioned, survey data without stringent oversampling of the wealthy do not adequately represent the top of the distribution, making them unusable for goodness-of-fit testing of thick tails. Jayadev (2008) is one of the few studies which uses survey data, in this case for India. He fits a type I Pareto distribution and finds shape parameter estimates of around 2 and argues in favour of a Pareto tail based on R^2 rather than goodness-of-fit tests. There are a few studies using tax data to test the Pareto hypothesis. Jagielski et al. (2017) find a slope coefficient of $\alpha = 1.5 \pm 0.3$ for the richest 100 Norwegian individuals for 2010-2013 tax data. Again, the data are not specifically tested for their goodness of fit despite graphically the log-log plots for both income and wealth suggest a Pareto tail. The same is true for Dragulescu & Yakovenko (2001) who use the Inland Revenue's reconstructed wealth data and find a power law behaviour for wealth over £ 100,000 for the United Kingdom. This database is rather specific to the UK and similar data are unfortunately not available for most other countries.

Even though less common, some authors have also fitted Pareto type II models. Jenkins (2016) uses both types of Pareto distributions on UK income tax data and finds that ultimately the type II distribution provides a better goodness of fit (using a likelihood ratio test and probability plots). In a similar vein, Charpentier & Flachaire (2022) argue that the type I distribution is more sensitive to the choice of the cutoff compared to the type II distribution and Beirlant et al.'s (2009) Extended Pareto Distribution. In an application to 2013 US wealth data, they report a better fit of the latter two distributions relative to type I Pareto, based on graphical inspection.

Overall, the existing literature is heavily dominated by contributions using rich list data and a type I Pareto distribution, especially when goodness of fit is tested. Arguably, however, some of the methods used to test the goodness of fit are questionable, especially the rather popular Zipf plots (Cirillo 2013). The fact that studies which either use high quality (income) tax data (Jenkins 2016) or fit a type II Pareto distribution (Jenkins 2016, Charpentier & Flachaire 2022), tend to assume or support the Pareto hypothesis is already an indication that rich list-based rejections of the PH might be driven by the shortcomings of the underlying data.

3 Methodology and data

Our aim is to test the Pareto hypothesis by comparing the fit of the type I and type II Pareto distribution to thin-tailed distributions such as the exponential and log normal. To that end, we fit these four distributions to US wealth data at increasing percentile cut-offs. This section discusses the details of our approach.

3.1 Parametric wealth distributions

We examine four distributions: The log-normal distribution, the exponential distribution, the Pareto type I distribution, as traditionally used in the Pareto literature, and the Pareto type II distribution. We use the three-parameter version of the log-normal distribution, including a location parameter, which makes it more directly comparable to the Pareto type II distribution which also features three parameters. The probability density function of the former is defined as:

$$LogN : f(x|s, \beta, \mu) = \frac{1}{s(x-\mu)\sqrt{2\pi}} \exp\left[-\frac{1}{2s^2} \ln\left(\frac{x-\mu}{\beta}\right)^2\right] \tag{1}$$

where s is the shape, μ the location and β the scale parameter. The support of the density function is $x > \mu$. For the exponential distribution we use a two-parameter version also including a location parameter such that the density function is defined as:

$$Exp : f(x|\mu, \lambda) = \lambda e^{-\lambda x} e^{\lambda \mu} \tag{2}$$

where μ is the location and λ is the scale parameter and the support of the density is $x > \mu$. We fit both the log normal and the exponential distribution by quasi Maximum Likelihood where we use the complex survey weights (w_i) to expand the likelihood function for each observation.

$$\max_{\theta} \mathcal{L}(\theta|x_i, w_i) = \prod_{i=1}^{n} f(\theta|x_i)^{w_i} \tag{3}$$

where θ is the parameter vector and $f()$ the density function of either the log-normal or the exponential distribution. Since we are fitting all distributions to several different percentile cutoffs, the location parameter is set equal to the percentile threshold. For the type I Pareto distribution we use the following parametrisation:

$$PI : f(x|\alpha, \sigma) = \frac{\alpha \sigma^{\alpha}}{x^{\alpha+1}} \tag{4}$$

where α is the shape and σ the scale parameter with support $x > \sigma$. The scale parameter is equal to the percentile cutoff and the shape parameter is obtained by means of a log-rank-log-wealth regression while taking into account the SCF's exclusion of the richest 400 households on the *Forbes* list (Wildauer & Kapeller 2022) and we also incorporate Gabaix & Ibragimov's (2011) bias correction as described in Wildauer & Kapeller (2021). Using ordinary least squares is justified since it is less biased and more robust against deviations from a Pareto tail due to poor data quality than its Maximum Likelihood equivalent (Feuerverger & Hall 1999). It is important to note that the type I distribution can be approximated by an exponential distribution, especially for very large shape parameter values. The two distributions are identical as α approaches infinity (Weinberg 2016).

For the Pareto type II distribution, we use the following parametrisation:

$$PII : f(x|\alpha, \mu, \sigma) = \frac{\alpha}{\sigma}\left[1 + \frac{x-\mu}{\sigma}\right]^{-\alpha-1} \tag{5}$$

where α is the shape, σ the scale and μ the location parameter with support $x > \mu$. Using the type II Pareto distribution in this parametrisation is equivalent to using a generalised Pareto distribution with a strictly positive shape parameter. Negative shape parameters would imply a wealth cap or maximum possible wealth which is not plausible for the wealth distribution. Our approach to fitting the shape and scale parameter of the type II distribution is discussed in detail in section 3.3.

The crucial difference between the type I and type II Pareto distributions is that the former inherently assumes scale invariance. This means that the wealth concentration within in the tail remains constant throughout the tail. Blanchet et al. (2022) introduce the concept of the inverted Pareto coefficient ($b(p)$) which provides a direct measure of tail thickness which can be used to compare type I and type II distributions. They define the inverted Pareto coefficient as:

$$b(p) = E[X|X > Q(p)]/Q(p) \tag{6}$$

where $0 \leq p \leq 1$ represents the percentile rank, e.g. $p = 0.95$ would be the 95^{th} percentile. In other words, the Pareto coefficient is the average wealth above a particular threshold (wealth at percentile p), expressed as a multiple of said threshold. For the type I distribution, $b(p)$ is constant and equal to $b(p) = \alpha/(\alpha - 1)$. Therefore, a constant inverted Pareto coefficient is another way of expressing the scale invariance of a type I distribution. If wealth follows a type I Pareto distribution, average wealth above a chosen threshold is equal to a fixed multiple of that threshold, independent of the threshold itself (i.e. scale invariant). Put differently, the shape of the tail of a type I Pareto distribution does not change. In contrast, Blanchet et al. (2022) show that if wealth can be described by a type II Pareto distribution, the inverted Pareto coefficient will vary along the distribution according to:

$$b(p) = 1 + \frac{\sigma}{(\alpha - 1)[\sigma + (1 - p)^{1/\alpha}(\mu - \sigma)]} \tag{7}$$

Two possibilities arise, if $\mu > \sigma$, $b(p)$ increases as $p \to 1$ which means the tail thickness and wealth concentration increases towards the top. In contrast, if $\mu < \sigma$, $b(p)$ decreases as $p \to 1$ and the tail becomes thinner towards the top of the distribution. We will use inverted Pareto coefficients as a direct and easy-to-interpret measure of the tail's thickness. We can use it to compare tail thickness between type I and type II distributions and to assess whether the estimated type II distributions exhibit patterns of increasing or decreasing concentration along the distribution. Finally, it is worth emphasizing that Blanchet et al.'s (2022) objective is different from what we set out to do in this paper. They develop what they call Generalised Pareto curves as a tool to reconstruct and fully describe the entire (income) distribution from a small number of tabulated (income) tax data observations. Our goal is to test the Pareto hypothesis whereas Blanchet et al. (2022) rely on it as the basis for their extrapolation beyond the last bracket.

Chu et al. (2019) reviews and compares dozens of goodness-of-fit tests for Pareto distributions and concludes that the Kolmogorov-Smirnov (KS) family of tests (Clauset et al. 2009, Capehart 2014, e.g.,) exhibit the highest power. We therefore use one of them, the Cramér–von Mises (CvM) test, to compare the data against the four distributions of interest (Pareto type I and type II, exponential and log normal). The crucial difference between CvM and KS is that the test statistic of the former is based on the difference between the empirical and theoretical CDF along the entire sample rather than just at a single data point which exhibits the most extreme difference. Since the SCF does not publish the details of their sampling procedure, we cannot follow Clauset et al. (2009) in order to obtain valid p- or critical values under the null hypotheses. Therefore, we simply calculate the CvM test statistic for each estimated distribution and compare them in a pair-wise manner. This means we are assessing the relative goodness of fit of one distribution compared to another. We take the SCF's complex survey weights into account and obtain the CvM test statistic as discussed in Wildauer & Kapeller (2021) as:

$$T = n\left[\frac{1}{3}U_1^3 + \frac{1}{3}\sum_{i=1}^{n-1}\left[\left(CDF(x_{(i)}) - U_i\right)^3 - \left(CDF(x_{(i)}) - U_{i+1}\right)^3\right] + \frac{1}{3}\left[1 - U_n\right]^3\right] \quad (8)$$

where $x_{(i)}$ is the ith largest value of the sample which consists of n observations. The corresponding complex survey weights are $w_{(i)}$ for each observation. Furthermore, U_i is the CDF under the null evaluated at $x_{(i)}$, $N = \sum_{i=1}^{n} w_i$ is the sum total of the complex survey weights and the empirical CDF is defined as

$$CDF(x_{(i)}) = 1 - \frac{\sum\limits_{1 \leq j \leq i} w_{(j-1)}}{N} \quad (9)$$

3.2 Data

We use the public version of the triennial Survey of Consumer Finances (SCF), starting with the 1989 wave. The SCF, due to its unique characteristics, allows us to carry out a more powerful test of the Pareto hypothesis compared to earlier studies. Firstly, by heavily oversampling wealthy households using income tax information (Bhutta et al. 2020, Kennickell 2008), the SCF provides a highly accurate picture of the US wealth distribution up to the very top. Since the US does not administer a general net wealth tax, wealth estimates based on income tax capitalisation or estate tax data are the only tax-based sources on the US wealth distribution. The SCF uses the former in its sample design and thus provides a data set of similar quality compared to tax-based estimates. Secondly, in comparison to rich list or lower quality survey data, the SCF's coverage of the wealth distribution is unmatched. In this paper, we test for Pareto tails beyond the 91^{st} percentile which starts between \$0.7 (1992) and \$1.4 million (2019). The most affluent observations in the data set range from \$310 million (1989) to \$1.9 billion (2019), all values in 2019 Dollars. This means the SCF's wealth data spans more than three orders of magnitude and covers the entire US population compared to one order of magnitude and merely 0.003% of the population covered by the *Forbes 400* list.

A key limitation of this data set is that billionaires featured in the *Forbes 400* are excluded from the public sample due to privacy concerns. This exclusion of the top tail of the wealth distribution from publicly available data more generally is one of the reasons why researchers interested in the Pareto hypothesis explore and consider the *Forbes 400* list despite its much narrower scope. However, this problem can be dealt with simply and effectively by correcting the ranks (cumulative weights) of the survey (Wildauer & Kapeller 2022). Using this rank correction approach allows us to exploit the much richer information embedded in the SCF while being able to conduct unbiased tests of the Pareto hypothesis.

Hanna et al. (2018) provides an excellent overview over the details of the SCF in addition to the official documentation. We use all five implicates of the data; final results are averages taken at the last step of the analysis. As the publicly available version of the SCF does not include the details of the sampling structure, we use the set of 1000 replicate weights to assess the sampling based variation in our results, as reported in section 4.3.

3.3 Castillo-Hadi estimator

In the literature, the Pareto type II distribution has been avoided in wealth inequality (as well as other) research because it can be tricky to estimate: while Maximum Likelihood estimators for $\alpha < 1$ do not exist, they do for $1 < \alpha < 2$, but estimates may be problematic (Castillo & Hadi 1997). To address this, we are employing an elemental percentile method (EPM) estimator following Castillo & Hadi (1997). The survival function for the type II Pareto distribution is given by:

$$SF(x_i|\alpha, \sigma, \mu) = \left(1 + \frac{x_i - \mu}{\sigma}\right)^{-\alpha} \tag{10}$$

and can be compared to an empirical estimate of the survival function which we define based on Gabaix & Ibragimov's (2011) bias corrected version for complex survey weights (Wildauer & Kapeller 2021) as:

$$SF(x_i)_E = \frac{\left(\sum_{1 \le j \le i} w_j\right) - 0.5w_i}{N} \tag{11}$$

where $N = \sum_{i=1}^{n} w_i$ is the sum total of the complex survey weights, based on a data vector in descending order (reversed order statistics). Using the shorthand notation $\ln\left(SF(x_i)_E\right) = C_i$, we can pick a pair of two data points (x_i, x_j) and by equating the empirical and theoretical survival function obtain a system of two equations in two variables $(\alpha,\ \sigma)$:

$$-\frac{1}{\alpha}C_i = \ln\left(1 - \frac{x_i - \mu}{\sigma}\right) \tag{12}$$

$$-\frac{1}{\alpha}C_j = \ln\left(1 - \frac{x_j - \mu}{\sigma}\right) \tag{13}$$

where $\mu = x_n$ (the smallest observation in the sample of the tail) and we can readily eliminate α to obtain:

$$\frac{\ln\left(1 - \frac{x_j - \mu}{\sigma}\right)}{C_j} = \frac{\ln\left(1 - \frac{x_i - \mu}{\sigma}\right)}{C_i} \tag{14}$$

Castillo & Hadi (1997) show that for this equation a solution exists over the interval $(\sigma_0, 0)$ if $x_i < \frac{C_i X_j}{C_j}$ and over the interval (x_j, σ_0) if $x_i > \frac{C_i X_j}{C_j}$ where $\sigma_0 = \frac{X_j X_i (C_j - C_i)}{C_j X_i - C_i X_j}$. The solution can be found numerically by means of bisection. The resulting estimate $\hat{\sigma}$ can be used to obtain the second parameter estimate $\hat{\alpha} = C_i / \ln\left(1 - \frac{x_i - \mu}{\hat{\sigma}}\right)$. This algorithm is then applied to all pairs (x_i, x_j) in the dataset which yields $\frac{n(n-1)}{2}$ estimates of $(\hat{\alpha},\ \hat{\sigma})$. The final estimates are the median values across all $\frac{n(n-1)}{2}$ pairs $(\hat{\alpha},\ \hat{\sigma})$.

The CH estimator for fitting the type II distribution is computationally demanding. For example our largest sample consists of $n = 1532$ observations (91^{st} percentile in wave 2019) and thus the CH estimator requires solving $\frac{n(n-1)}{2} = 1.17 \cdot 10^6$ nonlinear systems of equations for this single percentile-year cell. Repeating this exercise for 1000 replicate weights and for 176 year-percentile cells yields 350 billion equations. Therefore, we relied on parallel computing (python's joblib package) and even more importantly on just-in-time (JIT) compilation (python's numba package).

4 Results

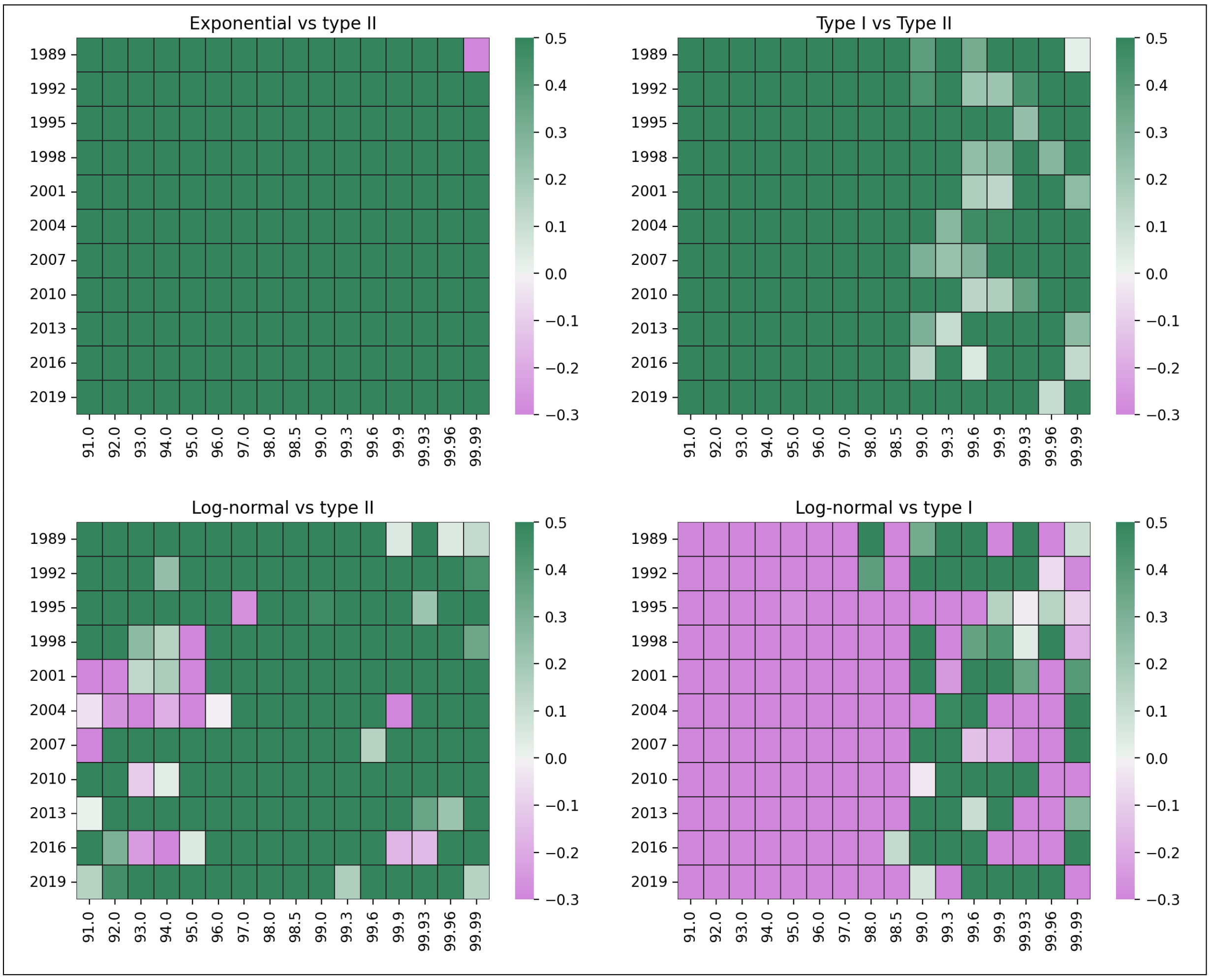
We start with the analysis of the SCF dataset using the full set of standard weights. We then analyse the shape of the estimated distributions via inverted Pareto coefficients. The last subsection provides a measure of statistical significance based on 999 subsets of replicate weights. See Kennickell & Woodburn (1999), Hanna et al. (2018) for details on the use of the SCF's replicate weights.

4.1 Modelling the US wealth distribution

As we are comparing distributions in a pair-wise manner, our results are organised accordingly. Table 1 presents the differences of the CvM test statistics of four distribution pairs. The figure in the top left compares the exponential distribution with the type II Pareto distribution. Positive values in shades of green indicate that the CvM test statistic for the fitted type II Pareto distribution is smaller and thus exhibits a better fit than the CvM test statistic based on the exponential distribution. The heat map shows that the type II Pareto, provides a superior fit for all except one cutoff-year cell. This cell belongs to the 99.99$^{\text{th}}$ percentile. We will show in section 4.3 that this outlier is not statistically significant. Overall, these results put the rich list based findings of Brzezinski (2014), who reports good fits for the (stretched) exponential distribution for some rich list data, into perspective. It is worth emphasizing that the 400 families on the *Forbes* list only represent the top 0.0003% of US households and thus less than a tenth of our highest cutoff. Our results demonstrate that any rejection of the Pareto hypothesis based on rich list data does not apply to the tail of the wealth distribution in general.

The heat map in the top right of Table 1 compares the type I against the type II Pareto distribution. Positive values in shades of green indicate a smaller CvM test statistic and thus better fit for the latter. In this pairing, the type II distribution exhibits a consistently superior fit to the type I distribution. This finding is expected as the type II distribution includes the type I distribution as a special case but provides more flexibility due to its additional parameter. Apart from differences in the estimation procedure, the type II distribution is always able to provide a fit at least as good as type I. Therefore, if the additional parameter of the type II distribution added no additional value in terms of a better fit, we would expect both distributions to yield similar fits. The fact that type II consistently outperforms its type I cousin implies that there is value in the additional parameter or, put differently, relaxing the scale invariance assumption of the type I distribution, provides a better fit. This result indicates that rejecting the Pareto hypothesis based on a type I distribution might in fact be a rejection of scale invariance rather than the Pareto hypothesis in general.

Table 1: Point estimates



The heat maps present the difference in CvM test statistics for a distribution pair. In anti-clockwise order starting in the top right they show the CvM test statistics of the type I Pareto, exponential and log-normal distribution minus the Pareto type II test statistic. Positive numbers in green shades indicate a better fit of the type II Pareto. The heat map in the bottom right shows the CvM test statistic of the log-normal distribution minus the test statistic of the type I Pareto distribution. Positive numbers in green shades indicate a better fit of the type I Pareto distribution.

The bottom two heat maps in Table 1 compare the log-normal distribution to the type I and type II Pareto distributions. Positive numbers in shades of green indicate a smaller CvM test statistic for the respective Pareto distribution and thus a better fit compared to the log-normal distribution. The heat map in the bottom left reports the fit of the type II Pareto relative to the log-normal distribution. The former provides a consistently better fit, especially for the 96th percentile cutoff and above. As will be discussed in section 4.3, there are some statistically insignificant cells such as the 99.9th percentile in 2004, the 97th percentile in 1995 or the 99.93rd percentile in 2016, but overall the type II distribution outperforms the log-normal. Even below the 96th percentile, the type II distribution fits well in most years with the notable exceptions of 2001, 2004 and 2016. We interpret these results as strong support in favour of a Pareto tail in the US wealth distribution. Our results strongly support the Pareto hypothesis for the 96th percentile and above and for many waves of the SCF, even for the 91st percentile and above. This means that the Pareto tail stretches significantly beyond the richest 400 families on the

Forbes list.

The bottom right of Table 1 shows that the type I distribution is consistently outperformed by the log-normal below the 99$^{\text{th}}$ percentile. This means that the tail implied by type I distributions fitted at low thresholds is too extreme or thick for our data. In contrast, the log-normal distribution struggles to adequately describe the tail beyond the top 1% wealthiest households in the United States. Here, the type I Pareto distribution performs better but is still rejected in favour of the log-normal in several cases. While the type I distribution only provides a shaky fit, it cannot be interpreted as a rejection of the Pareto hypothesis in general. The consistently superior fit of the more flexible type II distribution simply suggests that while the US wealth distribution exhibits a Pareto tail, it is not scale invariant as implied by the type I distribution.

4.2 Scale invariance and the shape of the US wealth distribution

The previous section established the superior fit of the type II distribution to US wealth data. While measuring the overall model fit provides important lessons for situations where available survey data does not oversample wealthy households (as is the case for many countries in the ECB's Household Finance and Consumption Survey), in the current context we can go beyond broad measures of goodness of fit. The inverted Pareto coefficient allows us to better understand why type II provides a better fit to the data than the type I distribution. Table 2 contains the difference between the inverted Pareto coefficients implied by type II and type I. The pattern which emerges is that up to the 99^{th} percentile, the fitted type II distribution yields a lower inverted Pareto coefficient and thus a thinner tail compared to the type I Pareto. This demonstrates that fitting type I distributions to wealth data starting at low thresholds, yields tails which are too extreme. This finding is consistent with the results reported by Charpentier & Flachaire (2022). The picture changes within the top 1% where the type II distribution yields thicker tails in many occasions. Overall, the additional flexibility of the type II distribution is needed for modelling wealth from low cutoffs (i.e. below the 99^{th} percentile).

Furthermore, the crucial feature of the type II distribution is that it allows for a changing pattern of concentration along the distribution. As discussed in the methodology section if $\mu > \sigma$, the thickness of the tail as measured by the inverted Pareto coefficient, increases along the distribution as $p \to 1$ and it decreases if $\mu < \sigma$. Figure 1 shows the inverted Pareto coefficients implied by the estimated parameters of the type II Pareto distribution. They decline from the 91^{st} percentile up to the 98^{th} percentile after which they start to increase. This means that around the 98^{th} percentile, the wealth distribution exhibits a structural change and the tail thickness starts to increase. This u-shaped pattern is in line with the pattern of inverted Pareto coefficients found for the distribution of income in France and the United States (Blanchet et al. 2022); however, the turning point occurs much later and the value of the inverted Pareto coefficients are much higher in the case of wealth which indicates a thicker tail as is commonly assumed in the literature.

Table 2: Difference between type II and type I inverted P coefficients

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.00	-0.04	0.52	0.30	0.19	0.30	-0.14	-0.16	-0.07	-0.05	-0.27
92.000	-0.07	-0.15	1.42	0.07	0.11	0.20	-0.24	-0.15	-0.15	-0.33	-0.30
93.000	0.03	-0.12	1.43	-0.01	0.10	0.11	-0.39	-0.10	-0.18	-0.35	-0.36
94.000	-0.12	-0.17	1.53	-0.11	-0.05	-0.05	-0.31	-0.04	-0.25	-0.14	-0.42
95.000	-0.20	-0.08	2.08	-0.15	-0.12	-0.11	-0.21	-0.11	-0.25	-0.16	-0.41
96.000	-0.04	-0.25	0.98	-0.17	-0.25	-0.14	-0.24	-0.18	-0.25	-0.36	-0.43
97.000	-0.06	-0.12	1.11	-0.41	-0.35	-0.26	-0.21	-0.31	-0.25	-0.47	-0.22
98.000	-0.27	-0.01	0.32	-0.42	-0.18	-0.29	-0.22	0.05	-0.26	-0.30	-0.07
98.500	-0.32	-0.17	-0.07	-0.20	-0.22	-0.12	0.03	0.00	-0.04	-0.07	-0.19
99.000	-0.05	0.11	-0.12	0.00	0.04	0.03	0.51	-0.05	0.20	0.23	0.03
99.100	-0.24	0.16	-0.23	0.01	-0.04	0.20	0.68	-0.07	0.12	-0.01	0.10
99.300	-0.50	0.09	-0.20	0.02	-0.05	0.25	0.47	0.08	0.37	-0.05	-0.14
99.600	-0.27	0.46	-0.14	0.25	0.19	0.29	0.86	0.31	0.92	0.34	0.05
99.900	NA	0.56	0.34	0.54	0.73	0.79	0.35	1.04	0.57	8.91	0.32
99.930	2.11	0.58	0.89	-0.22	3.70	-0.13	0.17	0.95	0.35	0.41	-0.10
99.960	-8.47	9.21	0.88	0.42	0.03	-0.33	0.15	-0.28	0.49	-0.54	1.15
99.990	-7.67	-0.02	-0.49	-3.98	1.12	1.54	2.57	0.45	-55.07	8.62	0.89

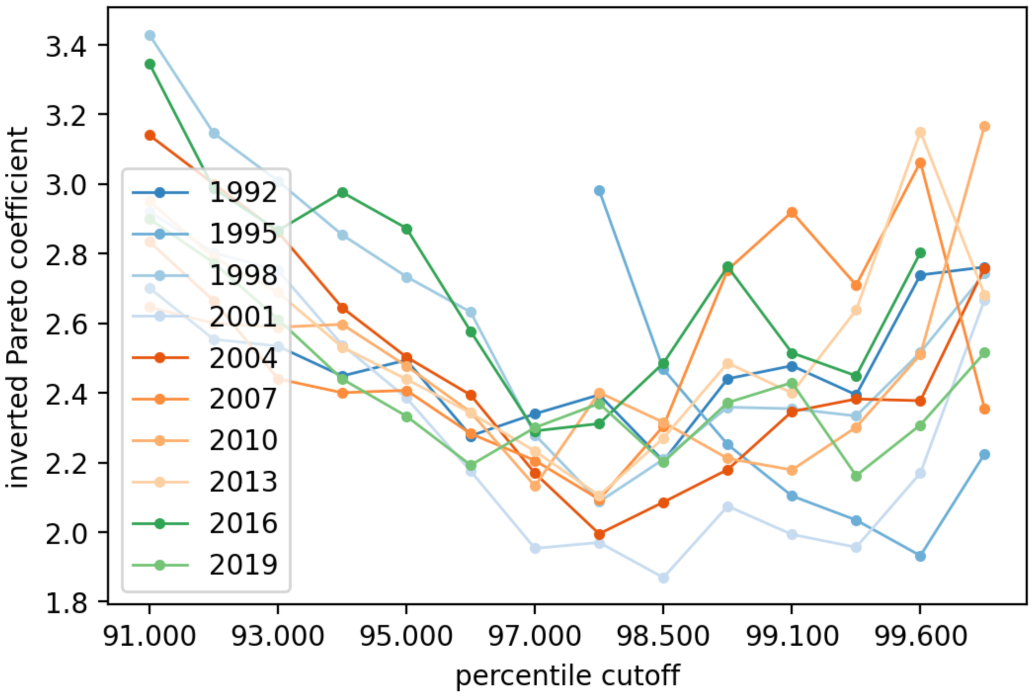
For each cell the inverted Pareto coefficient was calculated based on the estimated distribution parameters. For the type I distribution based on $b_1 = \frac{\hat{\alpha}}{\hat{\alpha}-1}$ and b_2 for the type II distribution based on equation 7. The cells in the table contain the difference: $b_2 - b_1$.

4.3 Statistical significance

The confidentiality of the SCF's sampling process requires us to use the provided sets of replicate weights to incorporate the random variation of our estimates in our analysis. We therefore repeated the goodness-of-fit tests discussed in the previous subsection for all 999 sets of replicate weights, resulting in 999 fitted parameters and CvM test statistics for each distribution and each percentile-year cell. This allows us to asses whether the results reported in the previous section are statistically significant in the sense that they hold up once the random variation in the data collection process is taken into account.

Table 3 summarises the results for all four distribution pairs. The heat map in the upper left shows that for most percentile-year cells the type II distribution provides a better fit than the exponential distribution for more than 90% of the 999 replicate weight sets. There are two noteworthy outliers at the 99.99$^{\text{th}}$ percentile: one for the 2001 wave and one for the 2010 wave, with rates of 50% and 68% respectively. Below the 99.9$^{\text{th}}$ percentile, the type II Pareto distribution provided a better fit for at least 995 out of 999 replicate weight sets in all percentile-year cells. The upper left heat map in Table 3 shows that the type II distribution consistently outperforms the type I distribution. In the 99.6$^{\text{th}}$ percentile for the 2016 wave, the type II

Figure 1: Inverted Pareto coefficient plot - type II distribution



distribution outperforms its less flexible cousin in 69% of the used replicate weight sets which is by far the lowest percentage for all cutoff-year cells comparing type I and type II distributions. This strongly supports the conclusion that adding another parameter and relaxing the scale invariance assumption of the type I distribution allows for a better fit to the data.

The bottom two heat maps of Table 3 compare the log-normal distribution to the Pareto distributions. The type II distribution outperforms the log-normal in all but two of the 120 cells beyond the 97$^{\text{th}}$ percentile. The two outliers are the 99.9$^{\text{th}}$ in the 2016 and 2004 waves. In the context of the overall results, we do not interpret these two cells as evidence against the type II distribution but as a false positive. The comparison between the type I and log-normal distribution in the bottom right of Table 3 provides a more refined picture than the point estimates from the previous section. In 77 out of the 90 percentile-year cells beyond the 99$^{\text{th}}$ percentile the type I distribution still provides a better fit than the log-normal distribution, more often than not. While clearly much worse than the fit of the type II distribution, the type I distribution nevertheless provides a decent fit for the top 1% of the US wealth distribution.

Overall, the 999 sets of replicate weights, support our main findings: First, we find strong support for the Pareto hypothesis in the context of the US wealth distribution. In particular, we find a superior fit of the type II Pareto distribution compared to the shifted log-normal for the 97$^{\text{th}}$ percentile and beyond and a better fit in many but not all years beyond the 91$^{\text{st}}$ percentile. Second, the type I Pareto provides an inferior fit compared to the type II distribution for all percentile-year cells. This demonstrates that the scale invariance property of the type I distribution might be too strong a simplifying assumption which might be a driving force behind an alleged better fit of the log-normal distribution especially below the 99$^{\text{th}}$ percentile.

Table 3: Replicate weight results

Exponential vs type II

	91.0	92.0	93.0	94.0	95.0	96.0	97.0	98.0	98.5	99.0	99.3	99.6	99.9	99.93	99.96	99.99
1992	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.99	1	0.92
1995	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.98	0.99	0.94
1998	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.96
2001	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.98	0.5
2004	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.87	0.82
2007	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99
2010	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.98	0.68
2013	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.94
2016	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.91	0.97
2019	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.96

Type I vs type II

	91.0	92.0	93.0	94.0	95.0	96.0	97.0	98.0	98.5	99.0	99.3	99.6	99.9	99.93	99.96	99.99
1992	1	1	1	1	1	1	1	0.91	0.96	0.84	0.8	0.89	0.9	0.92	0.96	0.87
1995	1	1	1	1	1	1	1	1	1	1	1	0.95	0.85	0.92	0.93	0.94
1998	1	1	1	1	0.98	0.99	1	1	0.99	0.9	0.84	0.94	0.9	0.93	0.85	0.94
2001	1	1	1	1	1	1	1	1	1	0.85	0.91	0.84	0.92	0.95	0.92	0.93
2004	0.99	0.99	0.99	1	1	1	1	1	1	0.92	0.92	0.88	0.96	0.96	0.99	0.85
2007	1	1	1	1	1	1	1	1	0.87	0.87	0.85	0.97	0.95	0.95	0.92	0.91
2010	1	1	1	1	1	1	1	0.96	0.92	0.93	0.87	0.94	0.99	0.94	0.96	0.81
2013	1	1	1	1	1	1	1	1	0.94	0.82	0.76	0.9	0.93	0.93	0.87	0.88
2016	1	1	1	1	1	1	1	1	0.97	0.92	0.93	0.69	1	0.89	0.97	0.83
2019	1	1	1	1	1	1	1	0.96	0.97	0.89	0.87	0.79	0.87	0.95	0.98	0.93

Log-normal vs type II

	91.0	92.0	93.0	94.0	95.0	96.0	97.0	98.0	98.5	99.0	99.3	99.6	99.9	99.93	99.96	99.99
1992	0.88	0.96	0.99	0.83	0.9	1	0.93	0.96	0.96	0.91	0.86	0.72	0.91	0.87	0.84	1
1995	0.98	0.92	0.98	0.9	0.88	0.92	0.66	0.97	0.84	0.95	0.98	0.91	0.83	0.82	0.78	1
1998	0.72	0.85	0.8	0.74	0.44	0.46	0.94	0.99	0.86	0.94	0.89	0.81	0.86	0.8	0.84	0.98
2001	0.56	0.51	0.41	0.41	0.36	0.94	0.99	0.99	0.98	0.94	0.94	0.86	0.81	0.58	0.96	0.99
2004	0.54	0.59	0.44	0.48	0.6	0.46	0.98	1	0.98	0.95	0.89	0.82	0.51	0.86	0.99	0.98
2007	0.1	0.6	0.99	0.98	0.87	0.98	1	0.96	0.94	0.81	0.98	0.88	0.84	0.94	0.97	0.99
2010	0.98	0.98	0.88	0.57	0.71	0.99	1	0.89	0.93	0.98	0.92	0.87	0.72	0.66	0.99	0.98
2013	0.71	0.91	0.94	0.99	0.98	0.99	1	0.98	0.95	0.95	0.92	0.77	0.89	0.93	0.91	0.92
2016	0.86	0.98	0.8	0.42	0.28	0.85	1	0.99	0.9	0.82	0.98	0.83	0.47	0.69	0.99	1
2019	0.64	0.72	0.9	0.98	1	1	0.97	0.97	0.99	0.87	0.97	0.95	0.88	1	0.98	0.96

Log-normal vs type I

	91.0	92.0	93.0	94.0	95.0	96.0	97.0	98.0	98.5	99.0	99.3	99.6	99.9	99.93	99.96	99.99
1992	0.002	0.002	0.005	0.008	0.06	0.09	0.15	0.6	0.58	0.71	0.68	0.43	0.64	0.62	0.4	0.88
1995	0.27	0.46	0.6	0.34	0.48	0.25	0.12	0.12	0.051	0.11	0.19	0.49	0.63	0.48	0.47	0.71
1998	0.13	0.11	0.069	0.052	0.029	0.062	0.012	0.021	0.28	0.75	0.69	0.52	0.69	0.55	0.59	0.56
2001	0.02	0.023	0.017	0.004	0.001	0.001	0	0.024	0.11	0.78	0.71	0.73	0.57	0.4	0.62	0.87
2004	0.12	0.13	0.051	0.057	0.054	0.023	0.002	0.004	0.17	0.79	0.76	0.63	0.34	0.64	0.48	0.86
2007	0	0	0	0	0	0	0.004	0.096	0.67	0.61	0.83	0.56	0.56	0.61	0.82	0.86
2010	0	0.001	0.004	0.03	0.054	0.036	0.024	0.49	0.63	0.67	0.74	0.59	0.38	0.52	0.69	0.89
2013	0	0	0	0	0.001	0.007	0.04	0.16	0.62	0.82	0.77	0.52	0.72	0.68	0.64	0.69
2016	0	0.002	0	0.002	0.014	0.009	0	0.13	0.47	0.7	0.77	0.69	0.15	0.53	0.39	0.87
2019	0	0	0	0	0	0	0.17	0.61	0.67	0.64	0.79	0.84	0.73	0.8	0.66	0.72

Using the SCF's 999 sets of replicate weights, each distribution is fitted 999 times to the data in each percentile-year cell. Each cell reports the proportion of cases in which the type II Pareto distribution yields a smaller CvM test statistic and thus a better fit than the distribution it was compared to. For the heat map in the bottom right, the proportion of cases in which the type I Pareto distribution yields a better fit. Shades of blue indicate shares in excess of 50%.

5 Conclusion

This paper illustrates that, indeed, Pareto was right, and the wealth distribution in the United States exhibits a thick tail. By using Cramér-von-Mises goodness-of-fit tests, we compare the type II Pareto distribution to the more traditionally used type I distribution, the log-normal and exponential distributions in a pairwise manner. The type II distribution provides by far the best fit to the data beyond the 91^{st} percentile of the US wealth distribution and especially so above the 97^{th} percentile. Compared to previous rich list based studies we are using more data of better quality to test the Pareto hypothesis: The SCF covers the entire population rather than the most affluent 0.0003% and wealth in our largest sample spans three orders of magnitude rather than one like the *Forbes 400* list. In addition, by fitting a type II Pareto distribution, we relax the scale invariance assumption of many previous studies and we effectively correct for the SCF's exclusion of the richest 400 US households from its sample design by using Wildauer

& Kapeller's (2022) rank correction approach. Altogether, this allows us to conduct a more powerful test of the Pareto hypothesis compared to the existing literature.

Beyond our primary result, the superior fit of the type II Pareto distribution to US wealth data, we also find that the log-normal provides a better fit than the type I Pareto distribution except for the top 1% of the population. We show that the shape of the wealth distribution changes along percentile cutoffs and the scale invariance assumption of the type I distribution fails to capture this. This suggests that earlier rejections of the Pareto hypothesis based on the fit of type I distributions should be interpreted as a rejection of scale invariance rather than the Pareto hypothesis itself. Many of these rejections in the literatue are also based on rich lists, so their rejections of the PH should be contextualised considering the data quality, breadth of coverage and rigidity of assumptions. In addition, our results also suggest that the tail thickness of the US wealth distribution declines between the 91^{st} and the 98^{th} percentile beyond which it increases. This increasing concentration towards the top is consistent with findings of highly skewed rates of return on wealth Fagereng et al. (2020) and theoretical models of wealth accumulation in which higher returns at the top are a result of sophistication, riskier investment, power or all of these Benhabib & Bisin (2018). Our findings can also motivate a simple test for theoretical models of wealth accumulation: Is the model able to produce a thick tail? Finally, while we did not test it directly, our findings support the Pareto hypothesis in the context of the distribution of income. This follows from the fact that the distribution of capital income closely follows the distribution of wealth and since capital income is concentrated at the top of the distribution of income. Further research will be needed in that regard.

We think these results are important in two ways. First, they provide support for those applications where researchers assume a Pareto tail, right from the outset[3]. Secondly, they provide important lessons for modelling wealth distributions in situations where only limited data or data with potential quality issues such as differential non-response are available. This means relying on Generalised Pareto interpolation is not feasible, as is the case for most countries in the ECB's Household Finance and Consumption Survey for example. Fitting a Pareto tail in such situations allows researchers to reduce the bias of raw survey data Eckerstorfer et al. (2016), Vermeulen (2018), Wildauer & Kapeller (2022). The results reported here can inform such endeavours in two ways. First, the importance of scale variance means researchers should be cautious to fit type I Pareto distributions below the 98^{th} or 99^{th} percentile. Second, relaxing the scale invariance assumption by introducing an additional parameter comes at a cost in the form of more data hungry estimators. The asymptotic equivalence of type I and type II Pareto distributions might justify using the former, especially beyond high thresholds and in situations where sample sizes are small. Both suggestions are in line with Hlasny's (2021) discussion of modelling the tail of the income distribution but more research is needed about the relative performance of different estimators of the type II distribution such as Maximum Likelihood, elemental percentile Castillo & Hadi (1997) or OLS-based approaches as in (Langousis et al. (2016), Disslbacher et al. (2020).

[3]See extensive references in the introduction

References

Advani, A., Bangham, G. & Leslie, J. (2020), 'The UK's wealth distribution and characteristics of high-wealth households', Wealth Tax Commission Evidence Paper **101**.

Advani, A., Hughson, H. & Tarrant, H. (2020), 'Revenue and distributional modelling for a wealth tax', Wealth Tax Commission Evidence Paper **113**.

Apostel, A. & O'Neill, D. W. (2022), 'A one-off wealth tax for Belgium: Revenue potential, distributional impact, and environmental effects', Ecological Economics **196**(C).
URL: *https://ideas.repec.org/a/eee/ecolec/v196y2022ics0921800922000477.html*

Avery, R. B., Elliehausen, G. E. & Kennickell, A. B. (1988), 'Measuring wealth with survey data: An evaluation of the 1983 survey of consumer finances', Review of Income and Wealth **34**(4), 339–369.

Bach, L., Calvet, L. E. & Sodini, P. (2020), 'Rich pickings? risk, return, and skill in household wealth', American Economic Review **110**(9), 2703–47.
URL: *https://www.aeaweb.org/articles?id=10.1257/aer.20170666*

Bach, S., Thiemann, A. & Zucco, A. (2018), 'Looking for the missing rich: Tracing the top tail of the wealth distribution'.

Beirlant, J., Joossens, E. & Segers, J. (2009), 'Second-order refined peaks-over-threshold modelling for heavy-tailed distributions', Journal of Statistical Planning and Inference **139**(8), 2800–2815.
URL: *https://www.sciencedirect.com/science/article/pii/S0378375809000068*

Benhabib, J. & Bisin, A. (2018), 'Skewed wealth distributions: Theory and empirics', Journal of Economic Literature **56**(4), 1261–91.

Bhutta, N., Bricker, J., Chang, A. C., Dettling, L. J., Goodman, S., Hsu, J. W., Moore, K. B., Reber, S., Volz, A. H. & Windle, R. A. (2020), 'Changes in u.s. family finances from 2016 to 2019: Evidence from the survey of consumer finances', Federal Reserve Bulletin **106**.

Blanchet, T., Chancel, L., Flores, M. & Morgan, M. (2021), 'Distributional national accounts (dina) guidelines: Methods and concepts used in the world inequality database'.
URL: *https://wid.world/document/distributional-national-accounts-guidelines-2020-concepts-and-methods-used-in-the-world-inequality-database/*

Blanchet, T., Piketty, T. & Fournier, J. (2022), 'Generalized Pareto Curves : Theory and Applications', Review of Income and Wealth **68**(1).

Bouchaud, J.-P., Gefen, Y., Potters, M. & Wyart, M. (2004), 'Fluctuations and response in financial markets: The subtle nature of randomness', Quantitative Finance **4**(2), 176–190.

Bricker, J., Krimmel, J., Henriques, A. & Sabelhaus, J. (2016), 'Measuring income and wealth at the top using administrative and survey data', Brookings Papers on Economic Activity pp. 261–312.

Brzezinski, M. (2014), 'Do wealth distributions follow power laws? Evidence from 'rich lists'', Physica A: Statistical Mechanics and its Applications **406**, 155–162.

Campolieti, M. (2018), 'Heavy-tailed distributions and the distribution of wealth: Evidence from rich lists in Canada, 1999-2017', Physica A Statistical Mechanics and its Applications **503**, 263–272.
URL: *https://ui.adsabs.harvard.edu/abs/2018PhyA..503..263C*

Capehart, K. W. (2014), 'Is the wealth of the world's billionaires not Paretian?', Physica A: Statistical Mechanics and its Applications **395**, 255–260.

Castillo, E. & Hadi, A. S. (1997), 'Fitting the generalized Pareto distribution to data', Journal of the American Statistical Association **92**(440), 1609–1620.

Chakraborty, R., Kavonius, I. K., Pérez-Duarte, S. & Vermeulen, P. (2019), 'Is the top tail of the wealth distribution the missing link between the household finance and consumption survey and national accounts?', Journal of Official Statistics **35**(1), 31–65.
URL: *https://doi.org/10.2478/jos-2019-0003*

Chan, S., Chu, J. & Nadarajah, S. (2017), 'Is the wealth of the Forbes 400 lists really Pareto distributed?', Economics Letters **152**, 9–14.

Charpentier, A. & Flachaire, E. (2022), 'Pareto models for top incomes and wealth', The Journal of Economic Inequality **20**, 1–25.
URL: *https://doi.org/10.1007/s10888-021-09514-6*

Chu, J., Dickin, O. & Nadarajah, S. (2019), 'A review of goodness of fit tests for pareto distributions', Journal of Computational and Applied Mathematics **361**, 13–41.

Cirillo, P. (2013), 'Are your data really pareto distributed?', Physica A: Statistical Mechanics and its Applications **392**(23), 5947–5962.
URL: *https://www.sciencedirect.com/science/article/pii/S0378437113006924*

Clauset, A., Shalizi, C. R. & Newman, M. E. J. (2009), 'Power-law distributions in empirical data', SIAM Review **51**(4), 661–703.

Dalitz, C. (2018), 'Estimating wealth distribution: Top tail and inequality', arXiv preprint arXiv:1807.03592 .

Disslbacher, F., Ertl, M., List, E., Mokre, P. & Schnetzer, M. (2020), On top of the top - adjusting wealth distributions using national rich lists, WorkingPaper 20, WU Vienna University of Economics and Business.

Dragulescu, A. & Yakovenko, V. M. (2001), 'Exponential and power-law probability distributions of wealth and income in the United Kingdom and the United States', Physica A: Statistical Mechanics and its Applications **299**, 213–221.

Eckerstorfer, P., Halak, J., Kapeller, J., Schütz, B., Springholz, F. & Wildauer, R. (2016), 'Correcting for the missing rich: An application to wealth survey data', Review of Income and Wealth **62**(4), 605–627.

Fagereng, A., Guiso, L., Malacrino, D. & Pistaferri, L. (2020), 'Heterogeneity and persistence in returns to wealth', Econometrica pp. 115–170.

Ferschli, B., Kapeller, J., Schütz, B. & Wildauer, R. (2018), 'ie viel bringt eine vermögenssteuer? neue aufkommensschätzungen für österreich', Wirtschafts- und Sozialwissenschaftliche Zeitschrift **41**.

Feuerverger, A. & Hall, P. (1999), 'Estimating a tail exponent by modelling departure from a pareto distribution', Annals of Statistics **27**, 760–781.

Gabaix, X. (2016), 'Power laws in economics: An introduction', Journal of Economic Perspectives **30**(1), 185–206.

Gabaix, X. & Ibragimov, R. (2011), 'Rank - 1/2: A simple way to improve the ols estimation of tail exponents', Journal of Business & Economic Statistics **29**(1), 24–39.

Garbinti, B., Goupille-Lebret, J. & Piketty, T. (2018), 'Income inequality in france, 1900–2014: Evidence from distributional national accounts (dina)', Journal of Public Economics **162**, 63–77. In Honor of Sir Tony Atkinson (1944-2017).
URL: *https://www.sciencedirect.com/science/article/pii/S0047272718300124*

Hanna, S. D., Kim, K. T. & Lindamood, S. (2018), 'Behind the numbers: Understanding the survey of consumer finances', Journal of Financial Counseling and Planning **29**(2), 410–418.
URL: *https://connect.springerpub.com/content/sgrjfcp/29/2/410*

Heck, I., Kapeller, J. & Wildauer, R. (2020), Vermögenskonzentration in Österreich. Ein Update auf Basis des HFCS 2017, Technical Report Materialien zu Wirtschaft und Gesellschaft 202, Arbeiterkammer Wien.

Hlasny, V. (2021), 'Parametric representation of the top of income distributions: Options, historical evidence, and model selection', Journal of Economic Surveys **35**(4), 1217–1256.

Jagielski, M., Czyżewski, K., Kutner, R. & Stanley, H. E. (2017), 'Income and wealth distribution of the richest norwegian individuals: An inequality analysis', Physica A: Statistical Mechanics and its Applications **474**, 330–333.

Jayadev, A. (2008), 'A power law tail in India's wealth distribution: Evidence from survey data', Physica A: Statistical Mechanics and its Applications **387**(1), 270–276.

Jenkins, S. P. (2016), 'Pareto models, top incomes and recent trends in UK income inequality', Economica **84**(334), 261–289.

Kapeller, J., Leitch, S. & Wildauer, R. (2021), 'A european wealth tax for a fair and green recovery', Greenwich Papers in Political Economy **81**.

Kennickell, A. B. (2008), 'The Role of Over-sampling of the Wealthy in the Survey of Consumer Finances', Irving Fisher Committee Bulletin **28**, 403–408.

Kennickell, A. B. (2017), 'Try, try again: Response and nonresponse in the 2009 SCF panel', Statistical Journal of the IAOS **33**(1), 203–209.

Kennickell, A. B. (2021), 'Chasing the tail: A generalized pareto distribution approach to estimating wealth inequality', Stone Center on Socio-Economic Inequality Working Paper Series .

Kennickell, A. B. & Woodburn, R. L. (1999), 'Consistent Weight Design for the 1989, 1992 and 1995 SCFs, and the Distribution of Wealth', Review of Income and Wealth **45**(2), 193–215.

Kishi, K. (2019), 'Technology diffusion, innovation size, and patent policy', European Economic Review **118**, 382–410.
URL: *https://www.sciencedirect.com/science/article/pii/S0014292119300996*

Klass, O. S., Biham, O., Levy, M., Malcai, O. & Solomon, S. (2006), 'The Forbes 400 and the Pareto wealth distribution', Economics Letters **90**(2), 290–295.

Klass, O. S., Biham, O., Levy, M., Malcai, O. & Solomon, S. (2007), 'The Forbes 400, the Pareto power-law and efficient markets', European Physical Journal B **55**, 143–147.
URL: *https://ui.adsabs.harvard.edu/abs/2007EPJB...55..143K*

Krenek, A. & Schratzenstaller, M. (2022), 'A harmonized net wealth tax in the european union', Journal of Economics and Statistics .
URL: *https://www.degruyter.com/document/doi/10.1515/jbnst-2021-0045/html*

Langousis, A., Mamalakis, A., Puliga, M. & Deidda, R. (2016), 'Threshold detection for the generalized pareto distribution: Review of representative methods and application to the NOAA NCDC daily rainfall database', Water Resources Research **52**(4), 2659–2681.

Levy, M. & Solomon, S. (1997), 'New evidence for the power-law distribution of wealth', Physica A: Statistical Mechanics and its Applications **242**(1-2), 90–94.

Meade, N. & Islam, T. (2006), 'Modelling and forecasting the diffusion of innovation – a 25-year review', International Journal of Forecasting **22**(3), 519–545. Twenty five years of forecasting.
URL: *https://www.sciencedirect.com/science/article/pii/S0169207006000197*

Ning, D. & You-Gui, W. (2007), 'Power-law tail in the Chinese wealth distribution', Chinese Physics Letters **24**(8), 2434.

Ogwang, T. (2011), 'Power laws in top wealth distributions: evidence from Canada', Empirical Economics **41**(2), 473–486.

Ogwang, T. (2013), 'Is the wealth of the world's billionaires Paretian?', Physica A: Statistical Mechanics and its Applications **392**(4), 757–762.

Osier, G. (2016), 'Unit non-response in household wealth surveys. experience from the eurosystem's household finance and consumption survey', European Central Bank Statistics Paper Series **15**.

Pareto, V. (1964), Cours d'économie politique, Vol. 1, Librairie Droz, Genf.

Piketty, T., Saez, E. & Zucman, G. (2018), 'Distributional National Accounts: Methods and Estimates for the United States*', The Quarterly Journal of Economics **133**(2), 553–609.
URL: *https://doi.org/10.1093/qje/qjx043*

Saez, E. (2001), 'Using elasticities to derive optimal income tax rates', Review of Economic Studies **68**, 205–229.

Saez, E. & Stantcheva, S. (2016), 'Generalized social marginal welfare weights for optimal tax theory', American Economic Review pp. 24–45.
URL: *http://dx.doi.org/10.1257/aer.20141362*

Saez, E. & Stantcheva, S. (2018), 'A simpler theory of optimal capital taxation', Journal of Public Economics **162**, 120–142.

Sinha, S. (2006), 'Evidence for power-law tail of the wealth distribution in India', Physica A: Statistical Mechanics and its Applications **359**, 555–562.

Tippet, B., Wildauer, R. & Onaran, Ö. (2021), The case for a progressive annual wealth tax in the uk.
URL: *http://gala.gre.ac.uk/id/eprint/33819/*

Vermeulen, P. (2016), 'Estimating the top tail of the wealth distribution', American Economic Review **106**(5), 646–50.

Vermeulen, P. (2018), 'How fat is the top tail of the wealth distribution?', Review of Income and Wealth **64**(2), 357–387.

Waltl, S. R. (2022), 'Wealth inequality: A hybrid approach toward multidimensional distributional national accounts in europe', Review of Income and Wealth **68**(1), 74–108.

Weinberg, G. V. (2016), 'Kullback–leibler divergence and the pareto–exponential approximation', SpringerPlus **5**.

Westermeier, C. (2016), Estimating top wealth shares using survey data. An empiricist's guide, Technical report, Diskussionsbeiträge.

Wildauer, R. & Kapeller, J. (2021), ‘A comment on fitting pareto tails to complex survey data’, Journal of Income Distribution .

Wildauer, R. & Kapeller, J. (2022), ‘Tracing the invisible rich: A new approach to modelling pareto tails in survey data’, Labour Economics **75**, 102145.
URL: *https://www.sciencedirect.com/science/article/pii/S0927537122000380*

APPENDIX

A Scale (in)variance

Table A1: Inverted Pareto coefficients raw data

cut	2019	2016	2013	2010	2007	2004	2001	1998	1995	1992	1989
91.000	4.57	4.37	4.17	3.73	4.31	3.63	3.63	3.88	3.74	3.48	3.36
92.000	4.40	4.36	4.09	3.59	4.29	3.63	3.63	3.90	3.59	3.47	3.31
93.000	4.25	4.16	3.93	3.40	4.36	3.63	3.55	3.83	3.60	3.34	3.17
94.000	4.06	3.85	3.78	3.25	3.92	3.66	3.56	3.76	3.56	3.24	3.14
95.000	3.74	3.77	3.54	3.23	3.54	3.58	3.44	3.60	3.49	3.05	3.09
96.000	3.45	3.77	3.33	3.15	3.31	3.40	3.42	3.53	3.52	3.06	2.89
97.000	2.98	3.51	3.06	3.01	3.02	3.36	3.18	3.52	3.40	2.79	2.78
98.000	2.74	2.95	2.82	2.59	2.67	2.95	2.65	3.07	3.45	2.64	2.73
98.500	2.69	2.66	2.55	2.55	2.42	2.60	2.48	2.69	3.30	2.63	2.61
99.000	2.49	2.55	2.38	2.46	2.23	2.34	2.17	2.50	3.04	2.41	2.44
99.100	2.47	2.62	2.41	2.45	2.20	2.26	2.19	2.48	3.01	2.38	2.48
99.300	2.50	2.58	2.32	2.34	2.28	2.23	2.16	2.44	2.79	2.39	2.47
99.600	2.38	2.41	2.21	2.26	2.26	2.21	2.03	2.28	2.40	2.27	2.20
99.900	2.38	2.34	2.26	2.21	2.22	2.11	2.03	2.25	2.00	2.24	1.95
99.930	2.39	2.61	2.25	2.24	2.19	2.33	1.97	2.30	1.91	2.22	2.10
99.960	2.20	2.55	2.11	2.31	2.06	2.08	2.11	2.11	1.95	2.05	2.08
99.990	2.22	1.90	1.85	2.04	1.70	1.68	1.64	1.69	1.77	2.14	2.42
99.995	2.22	1.75	1.91	1.49	1.64	1.58	1.45	1.93	1.82	1.95	1.48

Table A2: Forward difference in inverted Pareto coefficients raw data

cut	2019	2016	2013	2010	2007	2004	2001	1998	1995	1992	1989
91.000	-0.17	-0.01	-0.08	-0.14	-0.01	0.00	-0.00	0.02	-0.14	-0.01	-0.05
92.000	-0.15	-0.20	-0.17	-0.19	0.07	-0.01	-0.08	-0.06	0.01	-0.13	-0.14
93.000	-0.20	-0.31	-0.14	-0.15	-0.44	0.04	0.01	-0.08	-0.04	-0.09	-0.03
94.000	-0.32	-0.08	-0.24	-0.02	-0.38	-0.08	-0.12	-0.15	-0.07	-0.20	-0.05
95.000	-0.29	-0.00	-0.21	-0.08	-0.22	-0.18	-0.02	-0.08	0.04	0.01	-0.20
96.000	-0.46	-0.25	-0.27	-0.14	-0.30	-0.04	-0.25	-0.00	-0.12	-0.28	-0.11
97.000	-0.24	-0.56	-0.24	-0.43	-0.34	-0.41	-0.53	-0.45	0.05	-0.15	-0.05
98.000	-0.05	-0.29	-0.28	-0.04	-0.25	-0.35	-0.17	-0.38	-0.15	-0.01	-0.12
98.500	-0.20	-0.11	-0.17	-0.09	-0.19	-0.26	-0.31	-0.19	-0.26	-0.22	-0.17
99.000	-0.02	0.07	0.03	-0.01	-0.03	-0.08	0.01	-0.02	-0.03	-0.03	0.04
99.100	0.03	-0.04	-0.09	-0.12	0.08	-0.03	-0.03	-0.04	-0.22	0.01	-0.01
99.300	-0.13	-0.17	-0.11	-0.07	-0.03	-0.02	-0.12	-0.16	-0.40	-0.12	-0.27
99.600	0.01	-0.06	0.05	-0.06	-0.04	-0.09	-0.01	-0.02	-0.40	-0.03	-0.24
99.900	0.01	0.26	-0.01	0.03	-0.03	0.22	-0.06	0.04	-0.09	-0.02	0.14
99.930	-0.19	-0.06	-0.14	0.07	-0.13	-0.25	0.14	-0.18	0.04	-0.17	-0.02
99.960	0.02	-0.64	-0.26	-0.27	-0.35	-0.40	-0.47	-0.43	-0.18	0.09	0.34
99.990	-0.00	-0.15	0.06	-0.54	-0.06	-0.10	-0.19	0.25	0.05	-0.19	-0.93

Row 91 shows invP[92]-invP[91] and thus positive values represent increasing inverted Pareto coefficient

Table A3: Consistency raw data vs estimates

	2019	2016	2013	2010	2007	2004	2001	1998	1995	1992	1989
91.000	1	1	1	1	1	0	1	0	1	1	1
92.000	1	1	1	1	0	1	1	1	1	1	1
93.000	1	1	1	1	1	0	0	1	1	1	1
94.000	1	1	1	1	1	1	1	1	1	1	1
95.000	1	1	1	1	1	1	1	1	1	0	1
96.000	1	1	1	1	1	1	1	1	1	1	1
97.000	1	1	1	1	1	1	1	1	0	1	1
98.000	1	1	1	1	1	1	1	1	1	1	1
98.500	1	1	1	1	1	1	1	1	1	1	1
99.000	1	1	1	1	0	1	0	1	1	1	0
99.100	0	1	1	1	1	1	1	1	1	0	1
99.300	1	1	0	1	0	0	1	1	1	1	1
99.600	0	0	1	0	0	1	0	0	1	0	1
99.900	0	1	1	1	1	1	0	1	0	0	1
99.930	1	1	1	1	1	1	1	1	1	0	0
99.960	1	1	1	1	1	1	1	1	1	1	1
99.990	1	0	1	1	0	0	0	1	0	1	0

Figure A1: Scale dependence test: $\mu - \sigma$

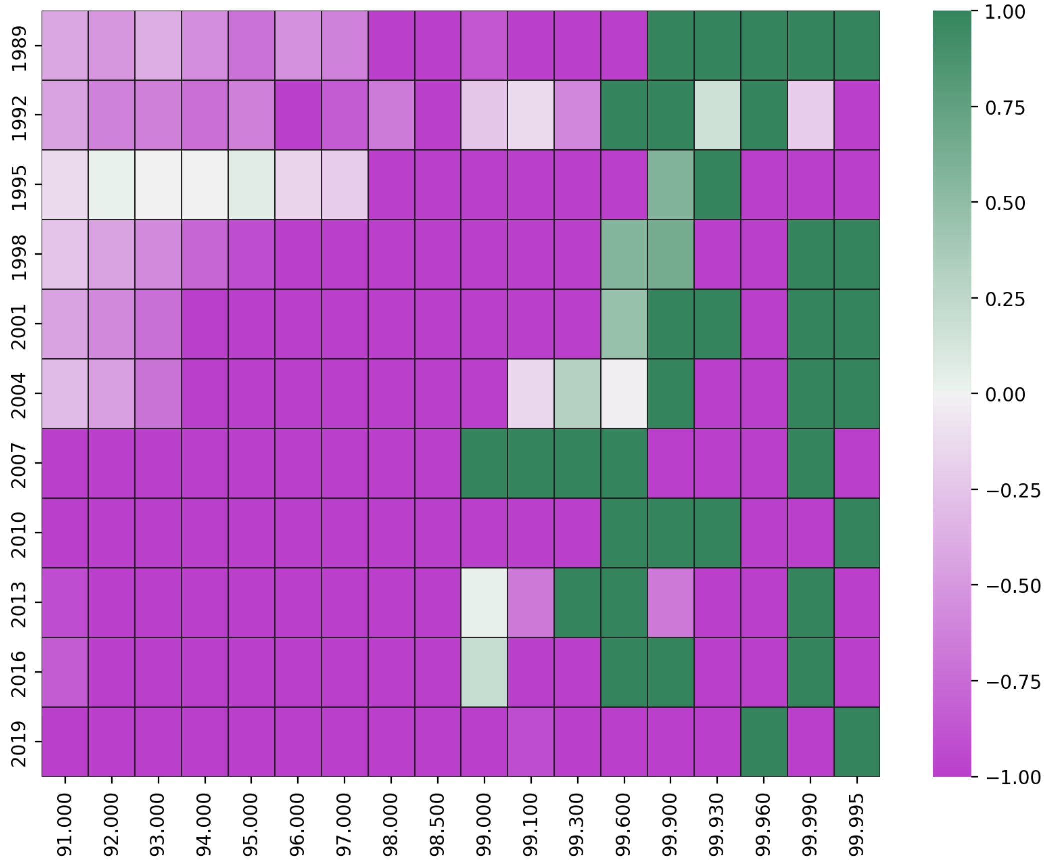


Table A4: Inverted Pareto coefficient based on type I estimates

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	2.87	2.74	3.15	3.13	2.73	2.84	2.97	2.81	3.02	3.40	3.17
92.000	2.81	2.70	3.11	3.08	2.69	2.80	2.90	2.75	2.94	3.32	3.08
93.000	2.77	2.66	3.07	3.02	2.65	2.75	2.83	2.69	2.86	3.22	2.98
94.000	2.72	2.61	3.01	2.96	2.59	2.69	2.71	2.64	2.78	3.12	2.86
95.000	2.67	2.57	2.96	2.88	2.51	2.62	2.61	2.58	2.69	3.03	2.75
96.000	2.62	2.53	2.88	2.80	2.43	2.54	2.52	2.52	2.59	2.94	2.63
97.000	2.57	2.46	2.78	2.69	2.31	2.43	2.42	2.44	2.48	2.76	2.52
98.000	2.50	2.41	2.66	2.51	2.15	2.28	2.31	2.35	2.37	2.61	2.44
98.500	2.45	2.37	2.54	2.41	2.09	2.21	2.27	2.31	2.31	2.55	2.39
99.000	2.39	2.33	2.37	2.35	2.03	2.15	2.24	2.26	2.29	2.53	2.34
99.100	2.39	2.32	2.33	2.34	2.03	2.14	2.24	2.25	2.28	2.53	2.33
99.300	2.35	2.31	2.23	2.32	2.01	2.13	2.24	2.23	2.27	2.50	2.30
99.600	2.32	2.28	2.07	2.26	1.98	2.09	2.21	2.20	2.23	2.47	2.26
99.900	2.77	2.21	1.89	2.20	1.93	1.97	2.01	2.13	2.11	2.55	2.20
99.930	3.04	2.18	1.87	2.16	1.89	1.93	1.95	2.09	2.06	2.52	2.13
99.960	3.70	2.17	1.84	2.07	1.83	1.75	1.83	2.00	1.96	2.29	2.05
99.990	6.60	1.93	1.65	1.94	1.54	1.57	1.60	1.75	1.78	2.00	1.77
99.995	3.42	1.68	1.56	1.94	1.49	1.48	1.47	1.47	1.66	1.90	1.46

Table A5: Inverted Pareto coefficient based on type II estimates

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	2.86	2.70	3.67	3.43	2.92	3.14	2.83	2.65	2.95	3.35	2.90
92.000	2.75	2.55	4.53	3.15	2.80	3.00	2.66	2.60	2.79	2.99	2.77
93.000	2.80	2.53	4.50	3.01	2.75	2.86	2.44	2.59	2.69	2.87	2.61
94.000	2.60	2.45	4.54	2.85	2.54	2.64	2.40	2.60	2.53	2.98	2.44
95.000	2.47	2.49	5.04	2.73	2.39	2.50	2.41	2.48	2.44	2.87	2.33
96.000	2.58	2.28	3.87	2.63	2.18	2.39	2.28	2.34	2.34	2.58	2.19
97.000	2.51	2.34	3.89	2.28	1.95	2.17	2.21	2.13	2.23	2.29	2.30
98.000	2.23	2.39	2.98	2.09	1.97	2.00	2.09	2.40	2.11	2.31	2.37
98.500	2.13	2.20	2.47	2.21	1.87	2.09	2.30	2.32	2.27	2.49	2.20
99.000	2.34	2.44	2.25	2.36	2.07	2.18	2.75	2.21	2.49	2.76	2.37
99.100	2.14	2.48	2.10	2.35	1.99	2.35	2.92	2.18	2.40	2.52	2.43
99.300	1.85	2.39	2.03	2.33	1.96	2.38	2.71	2.30	2.64	2.45	2.16
99.600	2.06	2.74	1.93	2.52	2.17	2.38	3.06	2.51	3.15	2.80	2.31
99.900	-5446.28	2.76	2.22	2.74	2.67	2.76	2.35	3.17	2.68	11.46	2.52
99.930	5.15	2.76	2.76	1.94	5.59	1.80	2.13	3.04	2.41	2.93	2.03
99.960	-4.77	11.39	2.72	2.49	1.86	1.42	1.98	1.72	2.45	1.75	3.20
99.990	-1.07	1.91	1.16	-2.04	2.66	3.12	4.17	2.20	-53.30	10.62	2.66
99.995	-0.07	2.93	2.12	-26.24	3.81	-4.70	1.28	3.24	4.07	-6.37	1.01

B Averaged across all implicates

Table A6: Observations beyond cutoff

year	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	923	1284	1412	1335	1363	1372	1457	1389	1442	1481	1532
92.000	878	1243	1356	1292	1323	1326	1406	1318	1371	1421	1470
93.000	835	1191	1301	1241	1276	1281	1358	1247	1301	1343	1399
94.000	793	1139	1233	1189	1217	1224	1277	1181	1226	1259	1319
95.000	749	1081	1168	1119	1144	1158	1192	1115	1141	1180	1229
96.000	698	1014	1093	1043	1073	1090	1105	1043	1044	1103	1122
97.000	636	919	1001	961	970	1009	998	951	934	982	1006
98.000	556	810	901	815	825	880	869	816	803	847	868
98.500	499	742	815	717	749	802	781	738	727	767	794
99.000	413	649	689	626	657	712	683	649	649	679	698
99.100	400	629	662	606	641	688	658	629	635	662	672
99.300	365	589	589	555	595	646	629	584	592	617	620
99.600	289	502	459	457	503	547	543	496	504	522	526
99.900	168	315	252	284	324	353	348	310	318	325	329
99.930	146	273	224	251	274	322	305	273	270	297	281
99.960	113	211	189	185	231	244	225	228	208	224	206
99.990	51	100	91	79	85	110	100	98	94	91	86
99.995	18	59	62	58	56	71	64	55	63	54	53

Table A7: Percentile cutoff values in million USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	0.8	0.7	0.7	0.9	1.2	1.3	1.2	1.3	1.1	1.4	1.4
92.000	0.9	0.8	0.8	0.9	1.3	1.4	1.4	1.4	1.3	1.5	1.5
93.000	1.0	0.9	0.9	1.0	1.5	1.5	1.5	1.7	1.5	1.8	1.8
94.000	1.2	1.0	1.0	1.2	1.6	1.7	1.9	1.9	1.7	2.2	2.1
95.000	1.3	1.2	1.1	1.4	1.9	2.0	2.3	2.2	2.1	2.5	2.6
96.000	1.6	1.4	1.3	1.7	2.3	2.4	2.9	2.6	2.6	3.0	3.3
97.000	2.0	1.8	1.7	2.1	3.0	3.0	3.9	3.3	3.4	4.0	4.7
98.000	2.7	2.5	2.2	3.2	4.7	4.5	5.8	5.0	4.8	6.2	6.6
98.500	3.4	3.0	2.8	4.4	5.9	6.1	7.6	6.1	6.4	8.3	8.0
99.000	4.6	4.2	4.1	6.1	8.5	8.6	10.3	8.1	8.7	11.1	11.1
99.100	4.8	4.5	4.4	6.5	8.9	9.5	11.1	8.7	9.1	11.5	12.0
99.300	5.7	5.2	5.6	7.6	10.4	11.1	12.3	10.5	10.9	13.6	13.8
99.600	8.7	7.5	9.2	11.4	14.8	15.2	17.1	14.8	15.9	20.4	20.0
99.900	19.4	16.7	23.4	24.7	29.5	33.5	38.4	32.4	33.7	46.4	43.5
99.930	22.3	20.6	29.0	29.6	36.4	37.0	47.5	38.6	41.7	51.5	54.3
99.960	29.9	29.9	37.1	43.6	45.5	57.4	68.5	52.1	60.0	74.0	80.9
99.990	61.8	62.1	80.3	109.0	115.4	131.8	160.1	120.1	135.4	205.6	172.5
99.995	146.0	100.8	108.9	136.5	164.7	187.1	226.6	236.9	190.7	314.0	268.4

Table A8: Estimated σ parameters for type II Pareto distribution in 1000 USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.2	1.1	0.8	1.1	1.6	1.6	2.5	2.4	2.1	2.2	2.9
92.000	1.4	1.4	0.8	1.4	1.9	1.8	3.1	2.7	2.4	3.0	3.4
93.000	1.4	1.5	0.9	1.6	2.2	2.2	4.1	2.8	2.8	3.5	4.2
94.000	1.7	1.7	1.0	2.0	2.9	2.9	4.5	3.1	3.4	3.4	5.1
95.000	2.1	1.8	1.1	2.3	3.8	3.7	4.8	3.8	4.1	4.2	6.1
96.000	2.2	2.5	1.5	2.9	5.3	4.6	6.0	4.7	4.9	5.9	7.6
97.000	2.7	2.7	1.9	4.6	7.9	6.8	7.3	6.6	6.3	8.8	7.6
98.000	4.3	3.2	3.3	6.7	8.7	10.2	9.8	6.1	8.6	10.0	8.9
98.500	5.2	4.3	4.8	6.6	11.1	9.8	8.8	7.8	8.2	10.0	12.0
99.000	5.5	4.4	7.2	7.2	10.0	10.6	8.1	10.6	8.6	10.9	12.7
99.100	6.9	4.6	9.0	7.6	11.6	9.7	8.1	11.5	9.8	13.7	12.9
99.300	10.3	5.7	10.8	9.0	13.6	10.8	10.8	11.7	9.7	16.5	18.8
99.600	10.8	6.4	16.7	10.8	14.3	15.2	13.1	13.7	11.2	18.6	22.5
99.900	9.6	15.5	22.8	24.1	24.0	27.0	42.5	24.7	34.4	20.0	47.8
99.930	11.7	20.4	21.5	55.9	22.6	74.8	61.5	32.8	48.4	57.8	79.7
99.960	15.3	13.0	46.1	46.1	87.2	230.8	102.9	126.3	63.3	195.5	63.3
99.990	57.1	62.4	95.4	35.2	81.3	121.3	101.7	201.1	105.8	131.6	250.5
99.995	24.6	1029.1	163.0	89.8	100.6	167.0	1260.7	62.1	210.2	352.5	-1434.2

Table A9: Estimated alpha parameters for type II Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.60	1.65	1.39	1.43	1.55	1.49	1.62	1.69	1.57	1.46	1.60
92.000	1.64	1.72	1.28	1.50	1.59	1.53	1.69	1.70	1.62	1.56	1.64
93.000	1.59	1.72	1.29	1.54	1.61	1.57	1.82	1.69	1.66	1.59	1.71
94.000	1.67	1.76	1.29	1.59	1.71	1.66	1.82	1.67	1.72	1.54	1.79
95.000	1.73	1.72	1.26	1.62	1.80	1.73	1.79	1.73	1.76	1.57	1.85
96.000	1.67	1.86	1.36	1.66	1.96	1.78	1.86	1.81	1.81	1.68	1.94
97.000	1.69	1.79	1.38	1.86	2.21	1.94	1.90	1.97	1.87	1.85	1.82
98.000	1.88	1.74	1.54	2.00	2.12	2.10	1.97	1.73	1.96	1.80	1.75
98.500	1.95	1.86	1.71	1.86	2.24	1.96	1.78	1.79	1.81	1.69	1.86
99.000	1.76	1.71	1.83	1.75	1.95	1.87	1.57	1.85	1.67	1.57	1.74
99.100	1.93	1.69	1.97	1.75	2.03	1.75	1.52	1.87	1.72	1.67	1.70
99.300	2.27	1.74	2.02	1.76	2.07	1.73	1.60	1.78	1.61	1.70	1.88
99.600	2.00	1.58	2.21	1.68	1.86	1.73	1.49	1.66	1.48	1.56	1.77
99.900	1.25	1.59	1.82	1.60	1.61	1.58	1.74	1.47	1.66	1.12	1.66
99.930	1.15	1.63	1.61	2.26	1.38	2.33	1.90	1.49	1.76	1.54	1.98
99.960	1.06	1.14	1.97	1.68	2.44	4.54	2.20	2.54	1.72	2.45	1.49
99.990	0.82	1.39	1.88	0.95	1.66	1.84	1.42	2.07	1.46	1.29	1.70
99.995	1.25	8.34	1.99	1.10	1.43	1.74	7.46	1.01	1.57	1.54	-2.65

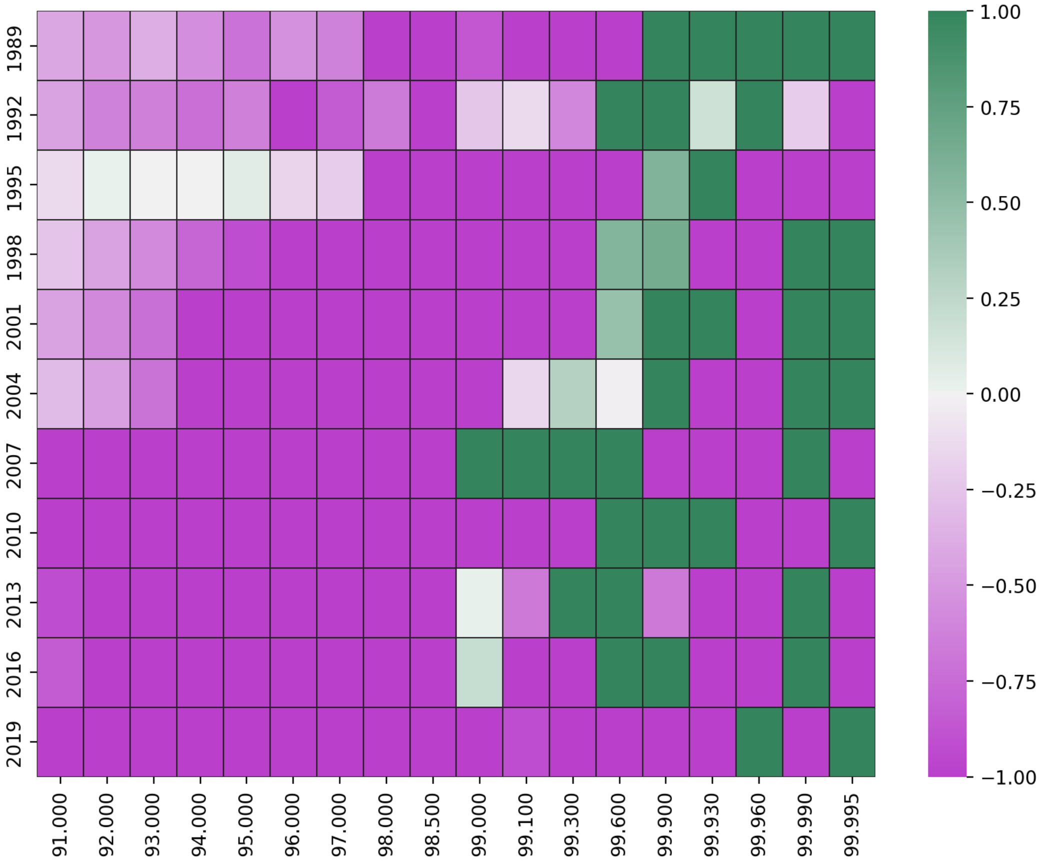
Table A10: Estimated alpha parameters for type I Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.54	1.58	1.46	1.47	1.58	1.54	1.51	1.55	1.49	1.42	1.46
92.000	1.55	1.59	1.47	1.48	1.59	1.56	1.53	1.57	1.51	1.43	1.48
93.000	1.57	1.60	1.48	1.50	1.61	1.57	1.55	1.59	1.54	1.45	1.51
94.000	1.58	1.62	1.50	1.51	1.63	1.59	1.58	1.61	1.56	1.47	1.54
95.000	1.60	1.64	1.51	1.53	1.66	1.62	1.62	1.63	1.59	1.49	1.57
96.000	1.62	1.66	1.53	1.56	1.70	1.65	1.66	1.66	1.63	1.52	1.62
97.000	1.64	1.68	1.56	1.59	1.77	1.70	1.71	1.70	1.67	1.57	1.66
98.000	1.67	1.71	1.60	1.66	1.87	1.78	1.76	1.74	1.73	1.62	1.70
98.500	1.69	1.73	1.65	1.71	1.92	1.83	1.79	1.76	1.76	1.64	1.72
99.000	1.72	1.75	1.73	1.74	1.97	1.87	1.80	1.79	1.78	1.65	1.75
99.100	1.72	1.76	1.75	1.75	1.97	1.88	1.81	1.80	1.78	1.66	1.75
99.300	1.74	1.77	1.81	1.76	1.99	1.89	1.81	1.82	1.79	1.67	1.77
99.600	1.76	1.78	1.93	1.79	2.02	1.92	1.83	1.83	1.81	1.68	1.79
99.900	1.58	1.83	2.13	1.83	2.07	2.03	1.99	1.88	1.90	1.65	1.84
99.930	1.52	1.85	2.16	1.87	2.12	2.08	2.05	1.91	1.95	1.66	1.89
99.960	1.43	1.86	2.20	1.94	2.21	2.34	2.21	2.00	2.04	1.78	1.95
99.990	1.39	2.08	2.55	2.08	2.85	2.77	2.70	2.34	2.29	2.00	2.30
99.995	3.79	2.49	2.85	2.10	3.04	3.14	3.16	3.12	2.52	2.12	3.21

Table A11: Scale invariance check: $\mu > \sigma$

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.41	-0.45	-0.13	-0.25	-0.44	-0.31	-1.29	-1.16	-0.92	-0.84	-1.57
92.000	-0.50	-0.62	0.02	-0.44	-0.58	-0.47	-1.71	-1.25	-1.18	-1.41	-1.86
93.000	-0.38	-0.63	-0.00	-0.57	-0.72	-0.71	-2.64	-1.17	-1.34	-1.67	-2.39
94.000	-0.55	-0.73	-0.00	-0.79	-1.24	-1.19	-2.60	-1.13	-1.73	-1.19	-3.04
95.000	-0.72	-0.63	0.07	-0.92	-1.87	-1.69	-2.40	-1.57	-1.99	-1.65	-3.48
96.000	-0.53	-1.12	-0.17	-1.22	-3.09	-2.21	-3.01	-2.10	-2.34	-2.89	-4.28
97.000	-0.62	-0.84	-0.21	-2.53	-4.94	-3.82	-3.34	-3.26	-2.86	-4.88	-2.97
98.000	-1.57	-0.66	-1.13	-3.53	-4.09	-5.71	-4.07	-1.04	-3.75	-3.82	-2.26
98.500	-1.85	-1.31	-1.98	-2.23	-5.18	-3.70	-1.21	-1.73	-1.83	-1.67	-3.95
99.000	-0.86	-0.25	-3.16	-1.11	-1.48	-2.00	2.21	-2.56	0.03	0.21	-1.60
99.100	-2.09	-0.13	-4.56	-1.10	-2.68	-0.15	3.06	-2.84	-0.67	-2.19	-0.92
99.300	-4.68	-0.59	-5.15	-1.39	-3.21	0.31	1.54	-1.19	1.20	-2.85	-5.01
99.600	-2.01	1.15	-7.43	0.57	0.46	-0.02	3.99	1.06	4.65	1.84	-2.43
99.900	9.81	1.19	0.58	0.64	5.53	6.52	-4.09	7.73	-0.68	26.40	-4.35
99.930	10.64	0.17	7.48	-26.39	13.81	-37.83	-14.07	5.79	-6.69	-6.38	-25.43
99.960	14.57	16.90	-9.00	-2.54	-41.64	-173.37	-34.40	-74.14	-3.33	-121.50	17.67
99.990	4.75	-0.21	-15.13	73.86	34.18	10.57	58.44	-80.91	29.69	74.07	-78.03
99.995	121.33	-928.38	-54.00	46.70	64.06	20.12	-1034.10	174.77	-19.44	-38.47	1702.66

Figure A2: Scale dependence test: $\mu - \sigma$



C Implicate 1

Table A12: Imp1: Percentile cutoff values in million USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	0.8	0.7	0.7	0.8	1.2	1.2	1.2	1.2	1.1	1.4	1.5
92.0	0.8	0.8	0.8	0.9	1.3	1.4	1.4	1.4	1.3	1.5	1.6
93.0	0.9	0.8	0.9	1.0	1.4	1.5	1.5	1.6	1.5	1.8	1.9
94.0	1.1	0.9	1.0	1.2	1.6	1.6	1.9	1.9	1.7	2.1	2.2
95.0	1.2	1.2	1.1	1.4	1.9	1.9	2.3	2.2	2.1	2.5	2.7
96.0	1.5	1.4	1.3	1.7	2.2	2.3	2.9	2.6	2.6	2.9	3.3
97.0	2.0	1.9	1.7	2.0	3.0	3.0	3.9	3.4	3.4	3.9	4.7
98.0	2.7	2.5	2.3	3.0	4.7	4.4	5.7	5.0	4.9	6.0	6.6
98.5	3.3	3.0	3.1	4.2	5.9	6.1	7.7	6.0	6.3	8.1	8.1
99.0	4.6	4.3	4.3	5.8	8.5	8.3	10.4	8.0	8.7	10.9	11.1
99.1	4.8	4.5	4.6	6.3	9.0	9.3	11.2	8.6	9.2	11.4	12.1
99.3	5.5	5.1	5.8	7.4	10.3	11.0	12.3	10.6	11.0	13.1	13.4
99.6	8.5	7.2	9.8	10.8	14.7	15.3	17.6	14.8	15.6	20.5	19.7
99.9	20.0	15.7	22.1	24.9	29.4	33.2	38.2	31.8	34.1	46.0	42.3
99.9	22.0	19.7	27.7	29.6	35.6	36.5	45.5	38.2	42.8	52.3	53.7
100.0	30.1	27.7	35.6	43.0	47.1	53.0	69.0	50.3	63.4	73.7	78.1
100.0	60.5	65.8	82.4	111.5	114.0	130.8	163.1	122.5	134.3	198.5	175.2
100.0	128.6	96.2	101.2	140.3	165.2	186.5	232.4	253.3	189.4	302.8	282.3

Table A13: Imp1: Estimated σ parameters for type II Pareto distribution in 1000 USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.0	1.1	0.9	1.0	1.5	1.5	2.4	2.3	2.1	2.2	2.7
92.000	1.2	1.3	0.8	1.3	1.8	1.7	3.0	2.7	2.5	2.8	3.3
93.000	1.4	1.5	0.9	1.4	2.1	2.1	4.0	3.0	2.8	3.1	4.0
94.000	1.6	2.0	1.0	1.9	2.8	2.7	4.2	3.1	3.5	3.1	4.8
95.000	2.2	2.0	1.4	2.2	3.7	3.7	4.7	3.8	4.1	4.0	5.9
96.000	2.2	2.7	1.8	2.7	5.5	4.8	5.8	4.9	5.0	5.7	7.8
97.000	2.8	2.8	2.4	4.8	7.9	6.5	8.1	6.0	6.6	7.9	7.8
98.000	3.8	3.0	4.2	6.7	8.2	10.5	10.2	5.8	8.6	9.9	9.9
98.500	5.6	4.1	5.3	6.4	10.7	10.3	8.9	7.8	8.9	9.8	12.0
99.000	4.4	3.6	7.5	7.1	10.4	12.1	8.5	11.2	8.7	10.6	12.1
99.100	5.3	4.2	10.7	7.3	11.2	10.5	8.8	13.3	9.6	12.6	12.3
99.300	8.0	4.9	11.7	8.3	14.0	11.0	12.7	11.2	9.5	18.2	19.9
99.600	11.8	6.2	10.5	11.6	15.6	13.2	12.6	13.6	10.8	17.3	22.9
99.900	5.6	16.7	22.4	19.1	26.4	20.8	40.2	25.0	36.4	22.2	51.0
99.930	9.4	18.3	23.0	36.1	32.7	51.7	70.5	29.4	54.9	50.0	72.5
99.960	9.4	18.0	43.9	39.0	81.0	332.5	96.7	188.7	68.2	205.1	80.9
99.990	38.2	29.6	21.0	50.5	66.9	75.7	121.8	131.4	130.8	135.5	251.7
99.995	-226.1	466.4	224.4	147.7	97.9	34.2	1313.3	27.6	159.1	987.2	-1429.8

Table A14: Imp1: Estimated alpha parameters for type II Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.48	1.67	1.39	1.43	1.53	1.49	1.59	1.67	1.57	1.48	1.57
92.000	1.54	1.71	1.26	1.48	1.56	1.50	1.67	1.71	1.64	1.54	1.62
93.000	1.58	1.75	1.30	1.50	1.59	1.54	1.78	1.73	1.64	1.54	1.68
94.000	1.59	1.88	1.31	1.57	1.68	1.64	1.78	1.68	1.73	1.51	1.75
95.000	1.77	1.82	1.39	1.60	1.78	1.73	1.78	1.74	1.77	1.55	1.83
96.000	1.69	1.96	1.46	1.64	1.98	1.81	1.83	1.83	1.81	1.67	1.97
97.000	1.72	1.85	1.54	1.89	2.20	1.92	1.97	1.88	1.90	1.78	1.83
98.000	1.80	1.73	1.74	2.03	2.04	2.14	2.02	1.70	1.95	1.80	1.86
98.500	2.02	1.86	1.80	1.85	2.16	2.00	1.79	1.78	1.84	1.67	1.88
99.000	1.64	1.60	1.91	1.75	1.98	1.97	1.59	1.90	1.67	1.56	1.72
99.100	1.72	1.67	2.20	1.73	2.00	1.81	1.56	2.03	1.70	1.63	1.67
99.300	1.91	1.67	2.17	1.71	2.08	1.75	1.71	1.76	1.58	1.77	1.94
99.600	2.08	1.59	1.79	1.73	1.93	1.65	1.47	1.65	1.44	1.54	1.81
99.900	1.00	1.67	1.79	1.43	1.65	1.43	1.67	1.47	1.67	1.19	1.69
99.930	1.08	1.57	1.66	1.73	1.66	1.93	2.03	1.43	1.85	1.45	1.91
99.960	0.88	1.30	1.94	1.55	2.33	6.02	2.10	3.23	1.81	2.47	1.70
99.990	0.78	0.93	0.92	1.08	1.51	1.43	1.66	1.70	1.65	1.31	1.69
99.995	-1.18	4.35	2.27	1.54	1.42	0.78	7.60	0.72	1.34	3.28	-2.80

Table A15: Imp1: Estimated α parameters for type I Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	1.5	1.6	1.5	1.5	1.6	1.5	1.5	1.5	1.5	1.4	1.5
92.0	1.5	1.6	1.5	1.5	1.6	1.5	1.5	1.6	1.5	1.4	1.5
93.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.5
94.0	1.6	1.6	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.5	1.6
95.0	1.6	1.6	1.5	1.5	1.7	1.6	1.6	1.6	1.6	1.5	1.6
96.0	1.6	1.7	1.6	1.5	1.7	1.6	1.7	1.7	1.6	1.5	1.6
97.0	1.6	1.7	1.6	1.6	1.8	1.7	1.7	1.7	1.7	1.6	1.7
98.0	1.7	1.7	1.6	1.6	1.9	1.8	1.8	1.7	1.7	1.6	1.7
98.5	1.7	1.7	1.7	1.7	1.9	1.8	1.8	1.8	1.8	1.6	1.7
99.0	1.7	1.8	1.8	1.7	2.0	1.8	1.8	1.8	1.8	1.6	1.8
99.1	1.7	1.8	1.8	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.3	1.7	1.8	1.8	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.6	1.8	1.8	1.9	1.8	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.9	1.6	1.8	2.1	1.8	2.1	2.0	2.0	1.9	1.9	1.6	1.8
99.9	1.5	1.8	2.1	1.8	2.2	2.0	2.0	1.9	2.0	1.7	1.9
100.0	1.4	1.8	2.1	1.9	2.3	2.2	2.2	2.0	2.1	1.8	2.0
100.0	1.3	2.1	2.4	2.1	2.9	2.5	2.8	2.4	2.3	1.9	2.4
100.0	2.1	2.4	2.6	2.2	3.1	2.5	3.2	3.3	2.5	2.0	3.3

Table A16: Imp1: Scale invariance check: $\mu > \sigma$

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.23	-0.43	-0.15	-0.20	-0.35	-0.29	-1.18	-1.10	-0.92	-0.88	-1.26
92.000	-0.36	-0.55	0.04	-0.35	-0.50	-0.36	-1.62	-1.31	-1.24	-1.30	-1.62
93.000	-0.45	-0.69	-0.04	-0.41	-0.70	-0.56	-2.46	-1.43	-1.29	-1.28	-2.11
94.000	-0.48	-1.04	-0.05	-0.72	-1.20	-1.05	-2.32	-1.19	-1.76	-0.97	-2.66
95.000	-0.93	-0.87	-0.24	-0.79	-1.82	-1.78	-2.40	-1.61	-2.07	-1.59	-3.26
96.000	-0.69	-1.37	-0.44	-1.08	-3.26	-2.44	-2.80	-2.27	-2.42	-2.78	-4.48
97.000	-0.81	-0.91	-0.63	-2.82	-4.91	-3.56	-4.24	-2.59	-3.13	-4.05	-3.10
98.000	-1.08	-0.43	-1.83	-3.71	-3.53	-6.05	-4.47	-0.76	-3.68	-3.92	-3.36
98.500	-2.24	-1.07	-2.18	-2.13	-4.72	-4.26	-1.18	-1.76	-2.58	-1.64	-3.96
99.000	0.13	0.70	-3.20	-1.27	-1.93	-3.76	1.83	-3.25	-0.08	0.33	-1.00
99.100	-0.52	0.26	-6.05	-0.98	-2.26	-1.16	2.40	-4.69	-0.42	-1.27	-0.18
99.300	-2.47	0.18	-5.95	-0.91	-3.68	-0.01	-0.47	-0.56	1.56	-5.11	-6.50
99.600	-3.29	0.96	-0.69	-0.88	-0.92	2.14	5.09	1.28	4.77	3.23	-3.19
99.900	14.32	-0.96	-0.31	5.79	2.96	12.42	-2.02	6.86	-2.26	23.86	-8.65
99.930	12.62	1.40	4.65	-6.45	2.89	-15.12	-25.04	8.86	-12.09	2.25	-18.81
99.960	20.74	9.76	-8.23	3.92	-33.92	-279.51	-27.65	-138.43	-4.77	-131.35	-2.84
99.990	22.32	36.12	61.33	60.98	47.10	55.04	41.25	-8.88	3.53	62.93	-76.58
99.995	354.64	-370.25	-123.22	-7.44	67.28	152.28	-1080.86	225.67	30.31	-684.43	1712.09

D Implicate 2

Table A17: Imp2: Percentile cutoff values in million USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	1.0	0.7	0.7	0.9	1.2	1.3	1.2	1.3	1.1	1.4	1.3
92.0	1.1	0.8	0.8	0.9	1.3	1.4	1.4	1.4	1.3	1.5	1.5
93.0	1.4	0.9	0.9	1.1	1.4	1.5	1.5	1.6	1.5	1.8	1.7
94.0	1.5	1.0	1.0	1.2	1.6	1.7	1.9	1.9	1.7	2.2	2.1
95.0	1.7	1.2	1.1	1.4	1.9	1.9	2.4	2.2	2.1	2.6	2.6
96.0	2.0	1.4	1.3	1.7	2.3	2.4	2.9	2.5	2.6	3.1	3.3
97.0	2.4	1.8	1.7	2.1	3.0	2.9	4.0	3.2	3.4	4.1	4.7
98.0	2.9	2.4	2.3	3.2	4.6	4.5	5.6	4.9	4.7	6.3	6.6
98.5	3.7	2.9	2.8	4.5	5.9	6.1	7.5	6.0	6.3	8.2	7.9
99.0	4.8	4.0	4.1	6.2	8.6	8.6	10.3	7.8	8.7	11.2	11.7
99.1	5.0	4.3	4.3	6.6	9.1	9.6	10.9	8.2	9.1	11.6	12.1
99.3	6.0	5.0	5.5	7.7	10.3	10.8	12.2	9.9	11.1	14.0	14.1
99.6	9.4	7.0	9.2	11.7	14.7	14.9	16.4	14.8	16.0	20.3	20.2
99.9	19.3	17.0	26.0	24.2	29.9	33.2	36.9	32.4	32.4	46.9	42.9
99.9	22.7	21.2	30.5	28.4	37.7	37.4	46.9	39.7	40.3	51.6	54.7
100.0	27.9	30.2	42.0	45.2	41.7	62.9	72.0	53.9	61.5	74.5	79.9
100.0	69.8	61.7	77.3	107.8	113.7	137.5	158.2	121.8	132.8	191.0	169.5
100.0	141.0	102.0	107.9	140.4	165.2	192.4	217.7	235.7	185.4	305.2	262.8

Table A18: Imp2: Estimated σ parameters for type II Pareto distribution in 1000 USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.7	1.1	0.8	1.0	1.6	1.5	2.6	2.2	2.0	2.4	2.9
92.000	2.0	1.3	0.8	1.3	1.9	1.8	3.1	2.4	2.4	3.4	3.5
93.000	1.6	1.4	0.8	1.4	2.2	2.3	4.5	2.7	2.8	3.7	4.4
94.000	1.8	1.6	0.9	1.8	3.0	2.9	4.4	2.8	3.3	3.4	5.3
95.000	1.8	1.6	1.0	2.4	4.0	3.7	4.8	3.5	4.0	4.4	6.0
96.000	2.0	2.2	1.5	2.8	5.3	4.5	6.3	4.3	4.5	6.1	7.7
97.000	2.5	2.2	1.6	4.5	8.0	7.0	6.7	6.3	5.9	8.7	7.5
98.000	5.7	3.1	2.8	7.0	9.4	9.7	9.6	5.8	8.5	10.3	8.7
98.500	4.4	4.0	4.5	6.3	10.9	10.1	7.9	6.8	8.1	11.3	12.4
99.000	6.2	4.1	6.4	7.5	9.4	10.2	6.6	10.0	8.2	12.1	11.1
99.100	8.5	3.9	7.9	7.6	10.5	8.6	6.8	11.9	9.2	15.6	13.3
99.300	12.0	4.9	10.0	10.6	13.3	11.7	8.2	13.0	8.6	16.5	16.9
99.600	8.6	7.3	15.1	9.9	15.9	18.5	11.1	13.9	9.4	17.7	21.8
99.900	8.2	14.5	19.6	25.5	22.4	33.6	44.2	28.4	41.5	17.2	48.4
99.930	7.1	15.2	20.6	79.0	13.8	92.7	66.7	32.8	66.2	51.3	74.0
99.960	11.0	12.4	14.5	55.5	161.9	110.6	54.1	78.1	57.7	185.3	73.9
99.990	33.5	70.7	49.0	46.2	88.4	297.6	144.6	161.0	138.7	195.8	320.9
99.995	38.8	222.7	112.1	99.2	124.0	114.3	1191.2	90.1	342.2	44.0	-1533.6

Table A19: Imp2: Estimated α parameters for type II Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.85	1.65	1.33	1.42	1.53	1.46	1.63	1.64	1.57	1.49	1.60
92.000	1.94	1.72	1.27	1.47	1.61	1.50	1.70	1.64	1.63	1.63	1.66
93.000	1.70	1.68	1.25	1.47	1.63	1.58	1.89	1.66	1.66	1.64	1.75
94.000	1.72	1.71	1.22	1.53	1.74	1.65	1.81	1.61	1.72	1.56	1.82
95.000	1.63	1.66	1.20	1.62	1.84	1.72	1.80	1.68	1.77	1.60	1.84
96.000	1.62	1.77	1.32	1.62	1.96	1.75	1.90	1.73	1.76	1.71	1.95
97.000	1.64	1.67	1.27	1.81	2.25	1.95	1.84	1.91	1.83	1.84	1.80
98.000	2.17	1.73	1.38	2.01	2.22	2.04	1.96	1.68	1.96	1.83	1.74
98.500	1.75	1.80	1.65	1.80	2.25	1.98	1.72	1.68	1.80	1.79	1.89
99.000	1.83	1.65	1.70	1.75	1.90	1.82	1.46	1.78	1.64	1.65	1.65
99.100	2.12	1.57	1.83	1.72	1.94	1.65	1.43	1.88	1.68	1.78	1.73
99.300	2.45	1.60	1.92	1.88	2.03	1.77	1.44	1.84	1.54	1.72	1.80
99.600	1.78	1.64	2.06	1.60	1.94	1.86	1.40	1.66	1.39	1.54	1.76
99.900	1.17	1.55	1.76	1.60	1.61	1.74	1.76	1.59	1.82	1.06	1.69
99.930	0.94	1.43	1.62	2.81	1.08	2.61	1.97	1.50	2.09	1.49	1.90
99.960	0.91	1.12	1.22	1.85	3.80	2.61	1.54	1.93	1.64	2.40	1.63
99.990	0.76	1.48	1.33	1.10	1.75	3.40	1.56	1.74	1.68	1.64	2.10
99.995	0.77	2.28	1.60	1.17	1.66	1.44	6.30	1.23	2.23	0.64	-3.27

Table A20: Imp2: Estimated α parameters for type I Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	1.6	1.6	1.5	1.5	1.6	1.5	1.5	1.5	1.5	1.4	1.5
92.0	1.7	1.6	1.5	1.5	1.6	1.5	1.5	1.6	1.5	1.4	1.5
93.0	1.7	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.5	1.5
94.0	1.7	1.6	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.5	1.5
95.0	1.7	1.6	1.5	1.5	1.7	1.6	1.6	1.6	1.6	1.5	1.6
96.0	1.7	1.6	1.5	1.6	1.7	1.6	1.7	1.6	1.6	1.5	1.6
97.0	1.7	1.7	1.6	1.6	1.8	1.7	1.7	1.7	1.7	1.6	1.7
98.0	1.7	1.7	1.6	1.7	1.9	1.8	1.7	1.7	1.7	1.6	1.7
98.5	1.7	1.7	1.6	1.7	1.9	1.8	1.8	1.7	1.8	1.7	1.7
99.0	1.7	1.7	1.7	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.1	1.7	1.7	1.7	1.8	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.3	1.8	1.7	1.8	1.8	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.6	1.8	1.8	2.0	1.8	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.9	1.6	1.8	2.2	1.8	2.1	2.1	1.9	1.9	1.9	1.7	1.9
99.9	1.5	1.9	2.2	1.9	2.1	2.1	2.0	1.9	1.9	1.7	2.0
100.0	1.4	1.9	2.2	2.0	2.1	2.4	2.1	2.0	2.1	1.8	2.1
100.0	1.3	2.1	2.5	2.2	2.9	3.1	2.4	2.3	2.3	2.0	2.4
100.0	1.9	2.5	2.8	2.3	3.1	3.4	2.8	3.2	2.6	2.1	3.4

Table A21: Imp2: Scale invariance check: $\mu > \sigma$

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.70	-0.44	-0.07	-0.17	-0.42	-0.22	-1.32	-0.95	-0.91	-0.99	-1.63
92.000	-0.89	-0.58	0.01	-0.31	-0.67	-0.40	-1.76	-0.99	-1.18	-1.91	-1.99
93.000	-0.26	-0.49	0.04	-0.34	-0.81	-0.79	-3.00	-1.11	-1.33	-1.92	-2.65
94.000	-0.29	-0.56	0.10	-0.59	-1.43	-1.25	-2.53	-0.87	-1.65	-1.19	-3.19
95.000	-0.10	-0.42	0.15	-1.02	-2.12	-1.78	-2.49	-1.30	-1.98	-1.78	-3.43
96.000	-0.03	-0.79	-0.15	-1.08	-3.04	-2.05	-3.33	-1.74	-1.96	-3.02	-4.42
97.000	-0.09	-0.43	0.07	-2.35	-5.04	-4.07	-2.70	-3.12	-2.52	-4.62	-2.77
98.000	-2.84	-0.73	-0.48	-3.79	-4.85	-5.20	-3.93	-0.89	-3.85	-4.03	-2.10
98.500	-0.65	-1.07	-1.72	-1.83	-4.99	-4.00	-0.43	-0.75	-1.84	-3.08	-4.48
99.000	-1.46	-0.16	-2.33	-1.30	-0.83	-1.55	3.68	-2.23	0.49	-0.98	0.52
99.100	-3.53	0.41	-3.59	-0.92	-1.41	1.00	4.16	-3.72	-0.03	-4.04	-1.21
99.300	-6.02	0.02	-4.44	-2.85	-3.02	-0.83	3.97	-3.08	2.51	-2.56	-2.73
99.600	0.82	-0.26	-5.86	1.83	-1.19	-3.61	5.30	0.91	6.54	2.58	-1.54
99.900	11.06	2.49	6.33	-1.32	7.51	-0.34	-7.36	3.99	-9.08	29.77	-5.48
99.930	15.66	6.00	9.93	-50.63	23.94	-55.28	-19.73	6.97	-25.89	0.24	-19.28
99.960	16.88	17.84	27.42	-10.31	-120.21	-47.76	17.85	-24.22	3.85	-110.83	5.98
99.990	36.27	-9.04	28.33	61.56	25.25	-160.10	13.59	-39.27	-5.92	-4.72	-151.45
99.995	102.20	-120.78	-4.25	41.16	41.24	78.13	-973.45	145.60	-156.79	261.27	1796.41

E Implicate 3

Table A22: Imp3: Percentile cutoff values in million USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	0.8	0.7	0.7	0.8	1.2	1.3	1.2	1.3	1.1	1.4	1.3
92.0	0.8	0.8	0.8	0.9	1.4	1.4	1.3	1.5	1.3	1.5	1.5
93.0	1.0	0.9	0.9	1.0	1.5	1.5	1.5	1.7	1.5	1.8	1.7
94.0	1.0	0.9	1.0	1.2	1.7	1.7	1.8	2.0	1.7	2.2	2.0
95.0	1.2	1.1	1.2	1.4	2.0	2.0	2.3	2.3	2.0	2.6	2.5
96.0	1.5	1.4	1.3	1.7	2.3	2.5	2.9	2.7	2.6	3.0	3.2
97.0	2.0	1.8	1.6	2.0	3.0	3.2	3.9	3.5	3.4	4.0	4.4
98.0	2.6	2.4	2.2	3.2	4.7	4.7	5.6	5.2	4.9	6.3	6.4
98.5	3.3	2.9	2.8	4.4	6.0	6.5	7.6	6.1	6.5	8.5	7.9
99.0	4.6	4.0	4.0	5.9	8.6	9.0	10.2	8.2	8.8	11.3	10.9
99.1	4.8	4.3	4.4	6.2	8.9	9.8	11.1	9.0	9.3	11.7	11.8
99.3	5.7	5.1	5.6	7.5	10.4	11.4	12.5	10.6	11.0	14.0	13.4
99.6	8.9	7.3	9.1	11.7	14.9	15.3	17.4	14.9	15.9	20.6	20.5
99.9	19.1	15.7	23.5	25.0	29.4	33.2	38.2	32.4	35.2	46.7	43.9
99.9	21.6	19.1	29.9	29.9	36.5	37.0	47.1	38.5	43.6	52.3	53.9
100.0	27.8	28.4	36.8	45.4	45.1	56.1	67.5	51.0	59.2	73.7	81.6
100.0	58.5	58.5	82.0	107.9	111.8	133.6	159.1	118.5	128.0	219.0	169.3
100.0	133.8	97.5	103.5	135.8	165.2	202.1	212.7	240.1	186.6	318.1	263.0

Table A23: Imp3: Estimated σ parameters for type II Pareto distribution in 1000 USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.2	1.0	0.9	1.1	1.9	1.7	2.5	2.7	2.1	2.2	2.6
92.000	1.3	1.2	0.8	1.3	1.9	2.1	3.0	2.9	2.4	3.0	3.2
93.000	1.4	1.3	0.9	1.5	2.2	2.5	4.0	2.9	2.8	3.7	3.9
94.000	1.7	1.7	0.9	1.9	2.9	3.1	4.6	3.1	3.4	3.4	4.9
95.000	2.3	1.7	1.0	2.2	3.8	3.9	4.8	3.8	4.1	4.2	6.0
96.000	2.2	2.1	1.4	2.8	5.3	4.8	5.9	4.9	5.0	5.9	7.0
97.000	2.4	2.4	1.8	4.5	7.9	6.7	7.2	6.8	6.7	9.7	8.2
98.000	4.1	2.9	3.2	5.8	9.2	10.4	10.6	6.2	9.2	10.1	9.2
98.500	6.3	4.1	4.7	5.9	11.6	9.4	9.1	8.3	8.7	9.9	12.2
99.000	5.9	5.1	7.5	6.7	10.0	9.9	8.7	11.6	9.5	10.8	13.4
99.100	7.9	4.9	8.5	8.2	12.6	9.6	8.2	10.5	10.8	14.1	12.7
99.300	10.6	5.7	10.8	8.3	14.5	9.4	10.5	12.3	10.8	16.5	20.4
99.600	8.0	6.3	20.9	9.0	12.6	14.4	11.9	14.7	13.9	21.7	20.1
99.900	6.2	11.4	21.4	23.4	21.4	27.3	45.2	26.0	33.3	18.3	49.9
99.930	9.8	20.6	14.0	62.8	19.8	73.0	57.1	32.4	34.6	50.7	77.5
99.960	9.1	12.6	35.5	48.6	66.4	163.7	111.2	124.9	43.2	279.3	55.0
99.990	396.9	41.8	36.3	23.5	103.5	86.4	58.3	301.9	103.8	89.0	278.6
99.995	55.9	113.0	152.3	68.0	67.5	51.5	1163.8	50.7	120.3	131.3	-1442.7

Table A24: Imp3: Estimated α parameters for type II Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.57	1.60	1.42	1.42	1.64	1.51	1.60	1.76	1.57	1.45	1.54
92.000	1.57	1.68	1.31	1.47	1.61	1.58	1.68	1.75	1.61	1.56	1.60
93.000	1.55	1.68	1.34	1.52	1.63	1.62	1.80	1.70	1.65	1.61	1.65
94.000	1.67	1.80	1.28	1.57	1.73	1.70	1.84	1.67	1.71	1.53	1.76
95.000	1.84	1.72	1.20	1.58	1.81	1.76	1.79	1.74	1.78	1.56	1.84
96.000	1.68	1.77	1.35	1.61	1.96	1.82	1.85	1.83	1.83	1.67	1.87
97.000	1.60	1.75	1.36	1.82	2.20	1.93	1.89	1.99	1.92	1.91	1.88
98.000	1.86	1.72	1.51	1.87	2.17	2.12	2.03	1.74	2.02	1.79	1.78
98.500	2.18	1.85	1.70	1.76	2.34	1.93	1.80	1.83	1.86	1.67	1.87
99.000	1.86	1.87	1.85	1.67	1.96	1.81	1.62	1.92	1.75	1.56	1.76
99.100	2.12	1.78	1.89	1.80	2.13	1.76	1.54	1.79	1.79	1.68	1.68
99.300	2.36	1.78	2.00	1.67	2.17	1.65	1.59	1.83	1.69	1.68	1.95
99.600	1.77	1.62	2.53	1.52	1.78	1.71	1.44	1.71	1.63	1.64	1.68
99.900	1.04	1.39	1.77	1.53	1.53	1.57	1.82	1.50	1.68	1.07	1.68
99.930	1.09	1.66	1.34	2.40	1.34	2.29	1.85	1.48	1.55	1.46	1.95
99.960	0.81	1.14	1.69	1.69	2.09	3.48	2.31	2.54	1.49	3.16	1.36
99.990	3.75	1.10	1.11	0.79	2.00	1.54	1.14	2.77	1.46	1.12	1.79
99.995	0.51	1.36	1.78	0.85	1.17	0.96	6.46	0.95	1.13	0.97	-2.44

Table A25: Imp3: Estimated α parameters for type I Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.4
92.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.5
93.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.5
94.0	1.5	1.6	1.5	1.5	1.7	1.6	1.6	1.6	1.6	1.5	1.5
95.0	1.6	1.6	1.5	1.5	1.7	1.6	1.6	1.6	1.6	1.5	1.5
96.0	1.6	1.7	1.5	1.5	1.7	1.7	1.7	1.7	1.6	1.5	1.6
97.0	1.6	1.7	1.5	1.6	1.8	1.7	1.7	1.7	1.7	1.6	1.6
98.0	1.6	1.7	1.6	1.6	1.9	1.8	1.8	1.8	1.8	1.6	1.7
98.5	1.7	1.7	1.6	1.7	1.9	1.8	1.8	1.8	1.8	1.6	1.7
99.0	1.7	1.7	1.7	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.7
99.1	1.7	1.8	1.7	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.7
99.3	1.7	1.8	1.8	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.7
99.6	1.7	1.8	1.9	1.7	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.9	1.5	1.8	2.1	1.8	2.1	2.0	2.0	1.9	1.9	1.6	1.8
99.9	1.4	1.8	2.1	1.8	2.2	2.1	2.1	1.9	2.0	1.7	1.8
100.0	1.2	1.8	2.1	1.8	2.3	2.3	2.2	2.0	2.0	1.8	1.9
100.0	1.1	1.9	2.4	1.8	2.9	2.7	2.6	2.3	2.2	2.1	2.1
100.0	1.1	2.3	2.5	1.8	3.1	3.1	3.0	3.1	2.4	2.1	2.9

Table A26: Imp3: Scale invariance check: $\mu > \sigma$

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.41	-0.31	-0.17	-0.23	-0.63	-0.40	-1.24	-1.46	-0.94	-0.86	-1.31
92.000	-0.43	-0.47	-0.02	-0.36	-0.50	-0.69	-1.69	-1.43	-1.13	-1.43	-1.69
93.000	-0.41	-0.49	-0.05	-0.52	-0.64	-0.93	-2.58	-1.16	-1.32	-1.90	-2.16
94.000	-0.70	-0.80	0.06	-0.72	-1.17	-1.43	-2.79	-1.05	-1.69	-1.22	-2.95
95.000	-1.10	-0.59	0.21	-0.86	-1.80	-1.91	-2.43	-1.54	-2.11	-1.62	-3.56
96.000	-0.67	-0.76	-0.11	-1.08	-2.98	-2.38	-2.99	-2.14	-2.46	-2.88	-3.83
97.000	-0.45	-0.66	-0.18	-2.43	-4.90	-3.44	-3.24	-3.31	-3.31	-5.77	-3.83
98.000	-1.50	-0.54	-1.07	-2.60	-4.46	-5.62	-4.98	-1.01	-4.32	-3.81	-2.77
98.500	-3.07	-1.21	-1.93	-1.51	-5.61	-2.88	-1.49	-2.12	-2.19	-1.40	-4.30
99.000	-1.34	-1.15	-3.45	-0.78	-1.45	-0.96	1.59	-3.45	-0.73	0.44	-2.55
99.100	-3.15	-0.59	-4.14	-2.09	-3.76	0.12	2.93	-1.47	-1.44	-2.43	-0.88
99.300	-4.90	-0.64	-5.25	-0.74	-4.08	2.06	2.01	-1.76	0.17	-2.53	-6.97
99.600	0.90	1.02	-11.82	2.75	2.32	0.91	5.51	0.18	2.05	-1.13	0.35
99.900	12.90	4.28	2.07	1.61	7.97	5.88	-7.10	6.35	1.88	28.47	-5.96
99.930	11.80	-1.50	15.88	-32.82	16.67	-36.03	-10.03	6.04	9.05	1.57	-23.63
99.960	18.64	15.81	1.28	-3.18	-21.24	-107.60	-43.71	-73.86	16.00	-205.59	26.57
99.990	-338.39	16.77	45.71	84.43	8.29	47.24	100.86	-183.38	24.28	129.94	-109.30
99.995	77.88	-15.51	-48.89	67.74	97.74	150.64	-951.17	189.39	66.36	186.88	1705.69

F Implicate 4

Table A27: Imp4: Percentile cutoff values in million USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	0.8	0.7	0.7	0.9	1.2	1.3	1.2	1.3	1.1	1.4	1.4
92.0	0.8	0.8	0.8	0.9	1.3	1.4	1.4	1.5	1.3	1.5	1.6
93.0	1.0	0.9	0.9	1.1	1.4	1.5	1.5	1.7	1.4	1.8	1.8
94.0	1.1	1.0	0.9	1.2	1.6	1.7	1.8	2.0	1.7	2.1	2.2
95.0	1.2	1.2	1.1	1.5	1.9	2.0	2.4	2.3	2.1	2.5	2.7
96.0	1.6	1.4	1.3	1.8	2.2	2.4	2.9	2.7	2.5	2.9	3.3
97.0	1.9	1.9	1.6	2.4	2.9	2.8	3.9	3.5	3.4	3.9	4.8
98.0	2.6	2.6	2.1	3.5	4.6	4.3	5.8	5.2	4.8	6.1	6.6
98.5	3.2	3.1	2.7	4.7	5.8	6.1	7.5	6.2	6.3	8.2	8.1
99.0	4.5	4.4	3.9	6.5	8.5	8.6	10.4	8.2	8.6	11.0	11.0
99.1	4.7	4.6	4.3	7.0	8.9	9.5	11.3	9.0	8.8	11.4	12.0
99.3	5.4	5.6	5.4	8.0	10.4	11.2	12.5	10.7	10.6	13.3	14.2
99.6	8.3	7.9	8.9	11.7	14.7	15.4	16.9	14.8	15.7	20.2	19.7
99.9	19.5	16.0	21.8	24.2	29.0	33.6	38.6	32.7	32.8	45.5	44.2
99.9	23.4	19.3	28.0	29.9	34.8	36.3	48.1	37.4	41.0	50.2	53.9
100.0	31.9	30.5	36.8	40.9	47.0	55.9	68.6	51.6	56.4	73.7	82.1
100.0	58.3	61.3	77.4	106.5	119.5	129.0	160.3	118.7	132.3	213.9	175.3
100.0	197.9	101.8	124.4	133.2	162.6	167.8	237.8	226.4	200.5	325.0	274.0

Table A28: Imp4: Estimated σ parameters for type II Pareto distribution in 1000 USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.1	1.3	0.8	1.4	1.5	1.6	2.6	2.6	2.0	2.1	3.2
92.000	1.3	1.6	0.8	1.8	1.9	1.8	3.0	2.8	2.4	2.7	3.5
93.000	1.3	1.7	0.8	2.1	2.1	2.1	4.0	3.0	2.8	3.2	4.2
94.000	1.7	1.7	1.1	2.6	2.6	2.7	4.5	3.3	3.3	3.4	5.0
95.000	2.0	2.0	1.0	2.7	3.3	3.3	4.5	4.2	3.9	4.1	5.9
96.000	1.8	3.1	1.3	3.6	4.9	4.5	5.5	5.1	5.0	6.0	7.6
97.000	2.8	3.3	2.0	5.0	7.3	7.1	7.1	7.0	5.8	8.8	6.7
98.000	3.2	3.8	3.3	7.7	8.2	11.1	9.7	6.5	7.8	9.8	8.6
98.500	4.8	4.9	4.7	7.0	11.0	10.0	8.9	9.3	7.9	9.5	11.6
99.000	4.9	4.5	7.2	6.9	9.4	11.0	8.5	11.0	8.3	9.8	13.0
99.100	5.6	5.1	8.1	7.0	11.6	10.3	8.1	10.7	9.8	12.1	12.6
99.300	11.1	5.2	9.6	8.4	13.4	10.1	10.7	11.3	10.5	15.6	16.4
99.600	11.3	5.4	12.8	10.6	13.0	15.3	16.4	12.7	12.1	18.8	21.7
99.900	19.9	16.8	28.4	23.7	22.3	23.7	42.1	19.6	38.6	23.6	45.1
99.930	10.7	33.2	25.8	31.8	30.9	63.4	58.3	32.0	40.8	81.1	92.7
99.960	8.2	11.2	53.3	41.9	75.5	422.1	107.0	116.5	73.8	167.1	59.5
99.990	45.8	121.8	284.3	21.2	83.8	52.1	80.7	113.6	114.4	90.7	215.9
99.995	70.6	68.0	120.7	74.2	105.6	362.0	339.3	70.5	259.6	321.3	-1370.7

Table A29: Imp4: Estimated alpha parameters for type II Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.55	1.69	1.37	1.52	1.53	1.50	1.62	1.72	1.58	1.43	1.66
92.000	1.60	1.81	1.29	1.64	1.59	1.55	1.68	1.73	1.63	1.52	1.66
93.000	1.56	1.78	1.29	1.69	1.59	1.57	1.79	1.72	1.67	1.55	1.71
94.000	1.66	1.73	1.41	1.76	1.67	1.66	1.83	1.72	1.71	1.53	1.78
95.000	1.75	1.77	1.28	1.72	1.71	1.69	1.75	1.81	1.74	1.55	1.83
96.000	1.55	2.04	1.30	1.82	1.90	1.77	1.81	1.88	1.83	1.69	1.96
97.000	1.74	1.96	1.45	1.95	2.10	1.98	1.88	2.05	1.84	1.84	1.72
98.000	1.65	1.88	1.57	2.17	2.06	2.18	1.95	1.78	1.90	1.78	1.73
98.500	1.87	1.97	1.71	1.94	2.19	2.00	1.79	1.94	1.79	1.66	1.84
99.000	1.69	1.73	1.84	1.75	1.90	1.92	1.58	1.90	1.65	1.52	1.76
99.100	1.73	1.78	1.88	1.72	2.02	1.82	1.50	1.83	1.73	1.60	1.69
99.300	2.39	1.70	1.92	1.74	2.04	1.69	1.58	1.76	1.67	1.66	1.77
99.600	2.02	1.49	1.92	1.70	1.78	1.75	1.65	1.62	1.55	1.55	1.73
99.900	1.88	1.66	1.97	1.64	1.53	1.56	1.75	1.35	1.78	1.17	1.61
99.930	1.07	2.06	1.74	1.71	1.61	2.20	1.84	1.47	1.63	1.76	2.13
99.960	0.81	1.08	2.12	1.63	2.23	7.68	2.28	2.44	1.85	2.23	1.44
99.990	0.52	2.23	4.17	0.84	1.59	1.23	1.32	1.50	1.54	1.03	1.54
99.995	3.23	1.18	1.88	0.99	1.40	3.09	2.57	1.08	1.81	1.40	-2.36

Table A30: Imp4: Estimated α parameters for type I Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.5
92.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.5
93.0	1.5	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.5	1.4	1.5
94.0	1.5	1.6	1.5	1.6	1.6	1.6	1.6	1.6	1.6	1.5	1.5
95.0	1.6	1.7	1.5	1.6	1.6	1.6	1.6	1.7	1.6	1.5	1.6
96.0	1.6	1.7	1.5	1.6	1.7	1.7	1.6	1.7	1.6	1.5	1.6
97.0	1.6	1.7	1.6	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.7
98.0	1.6	1.8	1.6	1.7	1.8	1.8	1.8	1.8	1.7	1.6	1.7
98.5	1.6	1.8	1.6	1.8	1.9	1.9	1.8	1.8	1.8	1.6	1.7
99.0	1.7	1.8	1.7	1.8	1.9	1.9	1.8	1.8	1.8	1.6	1.7
99.1	1.7	1.8	1.7	1.8	1.9	1.9	1.8	1.8	1.8	1.6	1.7
99.3	1.7	1.8	1.8	1.8	2.0	1.9	1.8	1.8	1.8	1.6	1.8
99.6	1.7	1.8	1.9	1.8	2.0	2.0	1.8	1.8	1.8	1.7	1.8
99.9	1.6	1.8	2.1	1.8	2.0	2.1	2.0	1.9	1.9	1.6	1.8
99.9	1.5	1.9	2.2	1.9	2.1	2.1	2.1	1.9	1.9	1.6	1.9
100.0	1.3	1.9	2.3	1.9	2.2	2.4	2.2	2.0	2.0	1.7	1.9
100.0	1.1	2.2	2.8	2.1	2.7	2.8	2.7	2.3	2.3	2.0	2.3
100.0	9.7	2.6	3.4	2.1	2.9	3.2	3.2	3.1	2.6	2.2	3.3

Table A31: Imp4: Scale invariance check: $\mu > \sigma$

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.36	-0.56	-0.10	-0.50	-0.35	-0.32	-1.32	-1.29	-0.91	-0.64	-1.89
92.000	-0.46	-0.84	0.02	-0.85	-0.57	-0.46	-1.65	-1.35	-1.13	-1.19	-1.87
93.000	-0.39	-0.80	0.01	-1.07	-0.64	-0.58	-2.47	-1.28	-1.33	-1.44	-2.34
94.000	-0.60	-0.66	-0.18	-1.37	-1.00	-1.05	-2.71	-1.30	-1.63	-1.24	-2.86
95.000	-0.80	-0.81	0.11	-1.21	-1.41	-1.24	-2.12	-1.94	-1.80	-1.58	-3.25
96.000	-0.26	-1.71	0.03	-1.83	-2.72	-2.08	-2.61	-2.39	-2.46	-3.06	-4.29
97.000	-0.88	-1.41	-0.43	-2.62	-4.42	-4.23	-3.20	-3.53	-2.45	-4.88	-1.81
98.000	-0.58	-1.14	-1.22	-4.24	-3.65	-6.79	-3.89	-1.33	-3.07	-3.71	-1.95
98.500	-1.58	-1.76	-2.00	-2.24	-5.16	-3.93	-1.32	-3.12	-1.58	-1.32	-3.49
99.000	-0.46	-0.06	-3.30	-0.36	-0.96	-2.49	1.92	-2.80	0.27	1.16	-2.05
99.100	-0.86	-0.48	-3.86	0.04	-2.67	-0.90	3.19	-1.76	-1.02	-0.72	-0.66
99.300	-5.72	0.36	-4.15	-0.34	-2.96	1.17	1.78	-0.54	0.12	-2.31	-2.22
99.600	-2.97	2.53	-3.91	1.11	1.78	0.07	0.54	2.14	3.58	1.38	-1.97
99.900	-0.40	-0.88	-6.63	0.53	6.79	9.91	-3.40	13.06	-5.75	21.89	-0.93
99.930	12.71	-13.86	2.15	-1.86	3.89	-27.17	-10.23	5.35	0.28	-30.90	-38.76
99.960	23.71	19.30	-16.43	-0.95	-28.57	-366.26	-38.42	-64.90	-17.44	-93.41	22.65
99.990	12.45	-60.50	-206.85	85.36	35.70	76.91	79.55	5.05	17.83	123.21	-40.64
99.995	127.38	33.86	3.74	59.01	56.96	-194.12	-101.44	155.87	-59.08	3.78	1644.66

G Implicate 5

Table A32: Imp5: Percentile cutoff values in million USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	0.8	0.7	0.7	0.8	1.2	1.2	1.2	1.3	1.1	1.4	1.3
92.0	0.9	0.8	0.8	0.9	1.3	1.4	1.4	1.4	1.2	1.6	1.5
93.0	1.0	0.9	0.9	1.0	1.4	1.5	1.5	1.6	1.4	1.8	1.7
94.0	1.1	1.0	1.0	1.2	1.6	1.7	1.9	1.8	1.7	2.2	2.1
95.0	1.3	1.3	1.1	1.4	1.9	1.9	2.4	2.1	2.1	2.6	2.6
96.0	1.5	1.4	1.3	1.7	2.2	2.4	3.0	2.6	2.6	3.1	3.4
97.0	2.0	1.9	1.7	2.0	3.0	3.0	4.0	3.1	3.4	4.0	4.7
98.0	2.6	2.6	2.2	3.1	4.7	4.5	6.0	5.0	4.8	6.3	6.8
98.5	3.4	3.1	2.8	4.1	6.0	6.0	7.5	6.1	6.5	8.5	8.1
99.0	4.7	4.2	4.1	5.9	8.5	8.7	10.4	8.1	8.6	11.1	11.0
99.1	4.8	4.6	4.5	6.4	8.9	9.5	11.1	8.5	9.0	11.4	12.0
99.3	5.7	5.1	5.7	7.6	10.6	10.9	12.2	10.5	11.0	13.9	13.6
99.6	8.6	8.2	9.1	11.0	14.9	15.2	17.3	14.6	16.1	20.5	20.1
99.9	19.4	19.2	23.8	25.2	29.7	34.2	40.1	32.6	34.0	46.7	44.2
99.9	21.7	23.7	28.8	29.9	37.6	37.6	49.8	39.0	40.5	51.0	55.2
100.0	31.7	32.7	34.1	43.6	46.9	59.2	65.3	53.9	59.6	74.5	83.0
100.0	62.1	63.5	82.3	111.4	118.3	128.3	160.0	119.3	149.8	205.8	173.2
100.0	128.6	106.4	107.8	132.7	165.2	186.7	232.6	229.1	191.8	318.7	260.2

Table A33: Imp5: Estimated σ parameters for type II Pareto distribution in 1000 USD

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.1	1.2	0.9	1.0	1.6	1.5	2.6	2.3	2.0	2.3	3.1
92.000	1.2	1.4	0.8	1.2	1.9	1.8	3.2	2.6	2.4	2.8	3.6
93.000	1.4	1.5	0.8	1.5	2.3	2.2	4.2	2.5	2.9	3.6	4.4
94.000	1.8	1.7	0.9	1.7	3.0	2.8	4.5	3.1	3.6	3.5	5.6
95.000	1.9	1.7	1.0	2.1	4.1	3.7	4.9	3.6	4.1	4.3	6.5
96.000	2.6	2.4	1.5	2.7	5.7	4.5	6.3	4.5	5.0	5.8	7.7
97.000	2.9	2.6	1.6	4.4	8.4	6.8	7.3	6.9	6.3	9.1	8.0
98.000	4.5	3.0	3.2	6.4	8.6	9.4	9.1	6.2	8.7	9.9	7.9
98.500	5.1	4.5	4.9	7.5	11.4	9.4	9.2	7.0	7.5	9.4	11.6
99.000	5.8	4.8	7.6	7.7	10.7	9.9	8.3	9.2	8.5	11.0	13.9
99.100	7.1	4.9	9.7	7.9	12.2	9.3	8.5	11.1	9.4	13.9	13.7
99.300	10.0	7.9	11.6	9.7	12.9	11.8	11.8	10.5	9.3	15.6	20.2
99.600	14.2	6.7	24.0	13.0	14.6	14.8	13.7	13.8	9.8	17.4	25.9
99.900	8.2	18.2	22.3	28.6	27.3	29.5	40.6	24.2	22.1	18.7	44.9
99.930	21.3	14.9	24.1	70.0	15.9	93.1	55.1	37.2	45.3	56.0	81.8
99.960	38.8	10.9	83.1	45.8	51.1	124.9	145.4	123.2	73.9	140.9	47.0
99.990	-228.9	47.9	86.5	34.5	63.7	94.5	103.0	297.4	41.1	146.8	185.4
99.995	184.0	4275.6	205.2	59.6	108.2	273.0	2296.2	71.8	169.8	278.6	-1394.3

Table A34: Imp5: Estimated α parameters for type II Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	1.55	1.63	1.43	1.38	1.55	1.48	1.64	1.66	1.55	1.45	1.62
92.000	1.56	1.69	1.27	1.45	1.59	1.51	1.71	1.68	1.61	1.54	1.66
93.000	1.56	1.70	1.27	1.52	1.61	1.55	1.82	1.63	1.66	1.61	1.74
94.000	1.68	1.68	1.26	1.51	1.73	1.64	1.82	1.68	1.75	1.56	1.86
95.000	1.67	1.62	1.24	1.57	1.85	1.72	1.81	1.70	1.77	1.57	1.90
96.000	1.79	1.78	1.36	1.60	1.99	1.75	1.90	1.77	1.82	1.67	1.95
97.000	1.74	1.73	1.26	1.82	2.29	1.93	1.90	2.00	1.88	1.86	1.85
98.000	1.92	1.64	1.50	1.94	2.09	2.03	1.90	1.74	1.99	1.79	1.68
98.500	1.92	1.82	1.68	1.97	2.27	1.92	1.80	1.71	1.75	1.64	1.84
99.000	1.80	1.68	1.88	1.80	1.99	1.80	1.58	1.74	1.66	1.56	1.81
99.100	1.96	1.64	2.02	1.77	2.08	1.71	1.55	1.83	1.70	1.67	1.75
99.300	2.21	1.93	2.08	1.82	2.02	1.77	1.66	1.70	1.58	1.66	1.96
99.600	2.33	1.55	2.74	1.82	1.86	1.69	1.52	1.66	1.39	1.51	1.89
99.900	1.16	1.70	1.81	1.78	1.71	1.62	1.71	1.46	1.34	1.09	1.64
99.930	1.58	1.44	1.69	2.66	1.21	2.63	1.81	1.58	1.66	1.52	2.03
99.960	1.88	1.05	2.86	1.71	1.72	2.89	2.77	2.57	1.81	2.02	1.34
99.990	-1.73	1.20	1.87	0.96	1.43	1.59	1.44	2.65	1.00	1.33	1.36
99.995	2.94	32.51	2.42	0.95	1.50	2.40	14.35	1.06	1.35	1.41	-2.40

Table A35: Imp5: Estimated α parameters for type I Pareto distribution

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.0	1.5	1.6	1.5	1.5	1.6	1.5	1.5	1.5	1.5	1.4	1.5
92.0	1.6	1.6	1.5	1.5	1.6	1.5	1.5	1.6	1.5	1.4	1.5
93.0	1.6	1.6	1.5	1.5	1.6	1.6	1.6	1.6	1.5	1.5	1.5
94.0	1.6	1.6	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.5	1.5
95.0	1.6	1.6	1.5	1.5	1.7	1.6	1.6	1.6	1.6	1.5	1.6
96.0	1.6	1.6	1.5	1.5	1.7	1.6	1.7	1.6	1.6	1.5	1.6
97.0	1.7	1.7	1.6	1.6	1.8	1.7	1.7	1.7	1.7	1.6	1.7
98.0	1.7	1.7	1.6	1.7	1.9	1.8	1.8	1.7	1.7	1.6	1.7
98.5	1.7	1.7	1.6	1.7	1.9	1.8	1.8	1.7	1.7	1.6	1.7
99.0	1.8	1.7	1.7	1.8	2.0	1.9	1.8	1.8	1.8	1.6	1.8
99.1	1.8	1.8	1.8	1.8	2.0	1.9	1.8	1.8	1.8	1.6	1.8
99.3	1.8	1.8	1.8	1.8	2.0	1.9	1.8	1.8	1.8	1.7	1.8
99.6	1.8	1.8	2.0	1.8	2.0	1.9	1.9	1.8	1.8	1.7	1.8
99.9	1.8	1.9	2.2	1.9	2.1	2.1	2.1	1.9	1.9	1.6	1.9
99.9	1.8	1.9	2.2	1.9	2.1	2.1	2.2	1.9	1.9	1.6	1.9
100.0	1.8	1.9	2.2	2.0	2.2	2.3	2.3	2.0	2.1	1.8	2.0
100.0	2.1	2.1	2.7	2.2	2.8	2.7	3.0	2.3	2.4	2.0	2.3
100.0	4.1	2.6	2.9	2.2	3.0	3.4	3.6	2.9	2.5	2.2	3.2

Table A36: Imp5: Scale invariance check: $\mu > \sigma$

year cut	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
91.000	-0.37	-0.50	-0.18	-0.16	-0.45	-0.31	-1.39	-1.02	-0.92	-0.82	-1.78
92.000	-0.38	-0.65	0.03	-0.33	-0.64	-0.43	-1.81	-1.16	-1.20	-1.21	-2.12
93.000	-0.39	-0.68	0.03	-0.53	-0.83	-0.67	-2.68	-0.87	-1.43	-1.81	-2.67
94.000	-0.67	-0.61	0.06	-0.54	-1.42	-1.18	-2.65	-1.24	-1.93	-1.34	-3.54
95.000	-0.65	-0.44	0.11	-0.73	-2.21	-1.76	-2.58	-1.43	-2.01	-1.67	-3.91
96.000	-1.03	-0.97	-0.19	-1.03	-3.43	-2.10	-3.33	-1.98	-2.43	-2.70	-4.38
97.000	-0.89	-0.77	0.13	-2.43	-5.43	-3.81	-3.31	-3.77	-2.91	-5.08	-3.32
98.000	-1.82	-0.46	-1.03	-3.29	-3.94	-4.92	-3.09	-1.21	-3.84	-3.60	-1.12
98.500	-1.69	-1.47	-2.05	-3.44	-5.42	-3.45	-1.64	-0.89	-0.94	-0.90	-3.52
99.000	-1.18	-0.56	-3.50	-1.85	-2.25	-1.24	2.05	-1.09	0.18	0.07	-2.93
99.100	-2.37	-0.22	-5.18	-1.54	-3.27	0.19	2.62	-2.56	-0.45	-2.48	-1.66
99.300	-4.30	-2.86	-5.95	-2.12	-2.32	-0.88	0.40	-0.00	1.63	-1.72	-6.65
99.600	-5.52	1.51	-14.86	-1.98	0.29	0.39	3.52	0.79	6.30	3.17	-5.80
99.900	11.17	1.00	1.46	-3.41	2.40	4.72	-0.55	8.39	11.81	27.99	-0.74
99.930	0.41	8.80	4.77	-40.17	21.67	-55.54	-5.30	1.72	-4.80	-5.09	-26.67
99.960	-7.11	21.78	-49.05	-2.17	-4.23	-65.73	-80.06	-69.31	-14.30	-66.34	35.98
99.990	291.09	15.59	-4.19	76.95	54.54	33.74	56.94	-178.09	108.72	58.98	-12.21
99.995	-55.45	-4169.22	-97.40	73.04	57.08	-86.32	-2063.60	157.32	21.99	40.15	1654.46