

1 **Sensitivity Analysis and Quantification of the Role of**
2 **Governing Transport Mechanisms and Parameters in**
3 **a Gas Flow Model for Low Permeability Porous Media**

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8 **Abstract** Recent models represent gas (methane) migration in low permeability
9 media as a weighted sum of various contributions, each associated with a given
10 flow regime. These models typically embed numerous chemical/physical parame-
11 ters that cannot be easily and unambiguously evaluated via experimental investiga-
12 tions. In this context, modern sensitivity analysis techniques enable us to diagnose
13 the behavior of a given model through the quantification of the importance and
14 role of model input uncertainties with respect to a target model output. Here,
15 we rely on two global sensitivity analysis approaches and metrics (i.e., variance-
16 based Sobol' indices and moment-based AMA indices) to assess the behavior of
17 a recent interpretive model that conceptualizes gas migration as the sum of a
18 surface diffusion mechanism and two weighted bulk flow components. We quan-
19 titatively investigate the impact of (i) each uncertain model parameter and (ii)
20 the type of their associated probability distribution on the evaluation of methane
21 flow. We then derive the structure of an effective diffusion coefficient embedding
22 all complex mechanisms of the model considered and allowing quantification of
23 the relative contribution of each flow mechanism to the overall gas flow.

24 **Article Highlights**

- 25 – Relative importance of parameters driving gas flow in low permeability media
26 is assessed.
27 – The Influence of parameter probability distribution on gas flow statistics is
28 appraised.
29 – A simple effective diffusion model embedding major methane flow mechanisms
30 is derived.

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31 **Keywords** Gas flow · Nanopores · Bulk flow · Methane · Sensitivity Analysis

32 1 Introduction

33 Methane is recognized as a potential energy source to assist transition to a carbon free energy landscape (Hughes, 2013), considerable reserves of methane being associated with subsurface reservoirs worldwide (U.S. Energy Information Administration, 2015). After its generation, this gas typically accumulates in reservoir regions subdued to low permeability layers (i.e., caprocks) that prevent its upward migration (Dembicki-Jr., 2017). Due to the partial sealing efficiency of caprocks, some amount of gaseous phase hydrocarbons might cross such barrier and reservoir gas can then be released into the overburden to (eventually) reach the surface (Schlömer and Krooss, 1997; Schloemer and Krooss, 2004). In this context, appropriate modeling approaches to quantify gas migration in low-permeability geomaterials can assist the appraisal of the feasibility of a methane recovery project.

44 A variety of models depicting gas movement in low permeability geomaterials have been proposed (Wu et al., 2016; Sun et al., 2017; Rani et al., 2018; Wang et al., 2019). These models typically estimate the mass flow rate of gas as the result of a combination of various gas transport mechanisms taking place across the porous system. Parameters associated with these models, describing chemical, mechanical, flow, and transport features governing feedbacks between gas and the host rock matrix are always affected by uncertainty. The conceptual model of Wu et al. (2016) depicts the mass flow rate of a gas across a low permeability medium as the sum of three key processes: (i) a surface diffusion, and two weighted bulk diffusion components corresponding to (ii) slip flow and (iii) Knudsen diffusion. This model takes into account changes in the porous system caused by mechanical deformation and adsorption/desorption dynamics. The model embeds numerous parameters which are typically estimated through (direct or indirect) laboratory-scale experiments. Considering the set of complex mechanisms involved, these types of experiments are costly, time demanding, and their results are prone to uncertainty. The latter is also related to the intrinsic difficulties linked to replicating operational field conditions at the laboratory scale as well as to the challenges stemming from transferability of results to heterogeneous field scale settings (Pan et al., 2010; Yuan et al., 2014; Tan et al., 2018).

63 Due to our still incomplete knowledge of the critical mechanisms driving gas movement in low permeability media (Singh and Myong, 2018; Javadpour et al., 2021) and the complexities associated with the estimation of model parameters, model outputs should be carefully analyzed considering all possible (aleatoric and epistemic) sources of uncertainty. In this sense, sensitivity analysis approaches are important tools enabling us to (i) quantify uncertainty, (ii) enhance our understanding of the relationships between model inputs and outputs, and (iii) tackle the challenges of model- and data-driven design of experiments (Dell’Oca et al., 2017). Hence, sensitivity analysis techniques may be effectively used in the context of methane flow modeling efforts to (i) quantify and rank the contribution of our lack of knowledge on each model parameter to the uncertainty associated with model outputs; (ii) identify model input-output relationships; and (iii) enhance the quality of parameter estimation workflows, upon focusing efforts on parameters with the highest influence to target model outputs (Saltelli et al., 2010;

Dell’Oca et al., 2020). In cases where parameters associated with a model have already been estimated (e.g., through model calibration), the main purpose of a Global Sensitivity Analysis, GSA, is to quantify the uncertainty still remaining after model calibration, thus guiding additional efforts for its characterization (e.g., Dell’Oca et al. (2020) and references therein). The probability density function (pdf) associated with each model parameter at this stage will possibly be different from the one employed before model calibration and some model parameters might be associated with a reduced uncertainty. In cases where processes are described through black-box models, GSA can be employed to quantify the influence that the variability of hyperparameters embedded in these models can have on their outcomes. We note that if uncertainty of some model parameters is further constrained, for example through stochastic inverse modeling (e.g., Ceresa et al. (2021)), results of the uncertainty quantification might also change. In this work we illustrate the methodological framework and the workflow required for GSA of a methane flow model and provide the elements to perform such an analysis for diverse scenarios. In order to assist this process, we provide a repository with scripts developed during this work (see declaration section).

In this work we rely on GSA approaches to study the behavior of the aforementioned gas migration model targeting low permeability media. While previous works focus on only a few selected model parameters (Song et al., 2016; Wu et al., 2017; Sun et al., 2017), a comprehensive diagnosis of the system behavior based on rigorous and modern GSA approaches taking into account the way all model parameters influence model output uncertainty is still missing. Here, we do so by implementing two GSA techniques, respectively based on the evaluation of (i) the classical (variance-based) Sobol’ indices (Saltelli and Sobol’, 1995) and (ii) the recent moment-based GSA metrics proposed by Dell’Oca et al. (2017). We recall that GSA approaches relying on Sobol’ indices are widely used to quantify the relative expected reduction of variance of the target model output due to the knowledge of (or conditioning on) a given parameter. These have been employed in several applications, including diagnosis of models related to, e.g., flood risk assessment (Koks et al., 2015), overpressure risk assessment in sedimentary basins (Colombo et al., 2017), and energy storage (Xiao et al., 2021). A critical limitation of variance-based GSA methodologies is that the uncertainty of the output is considered to be completely characterized by its variance. Such an assumption can lead to an incomplete characterization of the system behavior. The moment-based GSA approach introduced by Dell’Oca et al. (2017) is designed to enhance our capability to evidence model behavior upon including the quantification of model parameter uncertainty on the (statistical) moments of the pdf of a model output of interest. As such, this comprehensive approach yields information enabling us to characterize various aspects of uncertainty, without being limited solely to the concept of variance. The ensuing indices (termed AMA indices, after the initials of the authors (Dell’Oca et al., 2017)) have been effectively employed in a variety of contexts, including geophysical analyses related to gravimetric responses due to pumping tests (Maina et al., 2021), biochemical degradation of compounds such as glyphosate in soils (la Cecilia et al., 2020), and groundwater flow, including its feedbacks with evapotranspiration (Bianchi Janetti et al., 2019; Maina and Siirila-Woodburn, 2020).

This work is organized as follows: Section 2.1 briefly illustrates the complete model we consider to describe methane flow in low permeability media. The main

126 theoretical elements of the GSA approaches employed are described in Section
 127 2.2. Key results of the GSA are presented in Section 3. **Here, we also derive and**
 128 **discuss novel effective diffusive formulations, which have** the ability to encapsulate
 129 all physical-chemical mechanism included in the full methane flow model described
 130 in Section 2.1. Finally, conclusions are drawn in Section 4.

131 2 Materials and Methods

132 2.1 Gas flow in low permeability media

133 Models adopted to quantify gas migration in low permeability media can be clas-
 134 sified according to their complexity, in terms of, e.g., conceptualization and math-
 135 ematical rendering of the embedded processes, as well as number of their char-
 136 acteristic parameters. Among existing models associated with a high degree of
 137 complexity and including multiple transport processes jointly contributing to the
 138 total gas migration across the system (Mehmani et al., 2013; Wu et al., 2015a,
 139 2016, 2017; Sun et al., 2017; Zhang et al., 2018; Javadpour et al., 2021), here we
 140 consider the model of Wu et al. (2016). The selected model allows considering me-
 141 chanical deformation as well as relevant features associated with real gases such
 142 as variations in the gas viscosity (η), and the effects of the compressibility (C_g)
 143 and gas deviation (Z) factors caused by pressure and temperature changes.

144 The model introduced by Wu et al. (2016) rests on a conceptual picture ac-
 145 cording to which the total mass flow rate of gas per unit of area (J) is rendered
 146 through the sum of (i) a surface diffusion (J_s) and two weighted bulk diffusion
 147 components, corresponding to (ii) slip flow (J_v), and (iii) Knudsen diffusion (J_k),
 148 i.e.,

$$J = J_s + w_v J_v + w_k J_k. \quad (1)$$

149 The surface diffusion component is given by (Wu et al., 2015b)

$$J_s = -\zeta_{ms} \frac{D_s C_{sc}}{p} \frac{\partial p}{\partial l}, \quad (2)$$

150 where p is (gas) pore pressure and $\frac{\partial p}{\partial l}$ represents the strength of the driving force
 151 through the system, corresponding to the spatial gradient of gas pore pressure.
 152 The (dimensionless) coefficient ζ_{ms} is intended to take into account the possibility
 153 of applying the model (originally developed for capillary tubes) to a complex pore
 154 space and is defined in Equation (17) of the Appendix where it is shown that
 155 ζ_{ms} depends on porosity (ϕ), tortuosity (τ), pore size (r) (i.e. pore radius), and
 156 gas coverage on the geomaterial (θ). The term D_s in Equation (2) is the surface
 157 diffusion coefficient, which is expressed (as shown in Equation (25)) in terms of gas
 158 temperature (T), isosteric adsorption heat of the geomaterial (ΔH), a parameter
 159 (κ) related to the blockage/migration ratio of the adsorbed molecules, and θ .
 160 Finally, C_{sc} , defined in Equation (28), is the adsorbed concentration, which in
 161 turn depends on θ and on the gas molecule diameter (d_m).

162 The model proposed by Wu et al. (2016) allows representing the mechanical
 163 deformation of the pore space (in terms of variation of permeability and porosity
 164 with pressure) through power-law relationships and making use of the classical
 165 Kozeny-Carman equation. Here, we rest on their original model formulation, which

166 naturally leads to Equations (19) and (20), clearly evidencing that both r and ϕ
 167 evolve with p as a function of a reference pore radius (r_o) and reference porosity
 168 (ϕ_o), respectively.

169 The weight coefficients of the slip flow (w_v) and Knudsen diffusion (w_k) com-
 170 ponents in Equation (1) are given by (Wu et al., 2016)

$$w_v = \frac{1}{1 + K_n}, \quad (3)$$

171

$$w_k = \frac{1}{1 + 1/K_n}. \quad (4)$$

172 Here, K_n is the (dimensionless) Knudsen number defined as

$$K_n = \frac{\lambda}{2r}, \quad (5)$$

173 with

$$\lambda = \frac{\eta}{p} \sqrt{\frac{\pi ZRT}{2M}}, \quad (6)$$

174 where M and R are the gas molar mass and universal constant, respectively. Note
 175 that K_n relates the mean free path of the gas molecules (λ) to a representative
 176 length of the system (Civan, 2010), here taken as the pore diameter.

177 The slip flow component is dominant in systems where $K_n < 0.1$ (Ziarani and
 178 Aguilera, 2012) and can be evaluated as (Karniadakis et al., 2005; Wu et al., 2016)

179

$$J_v = -\zeta_{mb} \frac{r^2 p M}{8\eta ZRT} (1 + \alpha K_n) \left(1 + \frac{4K_n}{1 + K_n}\right) \frac{\partial p}{\partial l}. \quad (7)$$

180 Here, ζ_{mb} is intended to take into account the possibility of applying the slip
 181 flow formulation (7) to a complex pore space (see Eq. (23)) and α is the rarified
 182 effect coefficient for a real gas which, according to Karniadakis et al. (2005), is
 183 evaluated through Equation (24).

184 The Knudsen flow component is dominant in systems where $K_n > 10$ (Ziarani
 185 and Aguilera, 2012) and is evaluated as (Darabi et al., 2012; Liu et al., 2016)

$$J_k = -\frac{2}{3} \zeta_{mb} r \delta^{D_f - 2} \left(\frac{8ZM}{\pi RT}\right)^{1/2} \frac{p}{Z} C_g \frac{\partial p}{\partial l}. \quad (8)$$

186 Here, D_f represents the fractal dimension of the pore surface and δ denotes the
 187 ratio between d_m and r .

188 We conclude by noting that the model here described includes a total of 15 pa-
 189 rameters, which are related to the richness of physical processes embedded therein
 190 (See also Section 3.3). **All quantities here introduced are listed in Table 1 and in**
 191 **the list of symbols and nomenclature Section.**

192 2.2 Global Sensitivity Analysis

193 We perform a rigorous sensitivity analysis of the model illustrated in Section 2.1 to
194 diagnose its behavior with reference to the estimate of methane flow as driven by
195 imperfect knowledge of the associated parameters. Here, the uncertainty associated
196 with the selection of the interpretative model is not analyzed. Nevertheless, one
197 could also consider quantification of uncertainty of model outputs in the presence
198 of uncertain interpretive models. In this context, uncertainty of a target variable
199 which might result from the use of a collection of interpretive (conceptual and
200 mathematical) models, could be assessed upon relying, for example, on the ap-
201 proach illustrated by Dell’Oca et al. (2020). Our analysis is intended to yield a
202 robust quantification of the relative importance of uncertain model parameters to
203 a model output of interest. As mentioned in the Introduction, we rely on two GSA
204 approaches, corresponding to (i) the classical variance-based technique grounded
205 on the evaluation of the well-known Sobol’ indices (Saltelli and Sobol’, 1995) and
206 (ii) the moment-based GSA framework introduced by Dell’Oca et al. (2017).

207 Model parameters are treated as statistically independent, as the amount
208 of available information does not enable us to clearly identify cross-correlations
209 amongst parameters and to quantify joint distributions. We consider three differ-
210 ing characterizations of pdf describing uncertainty of model parameters: (a) all pa-
211 rameters are represented through uniform pdfs, (b) all parameters are represented
212 by truncated Gaussian pdfs, and (c) the reference pore radius is characterized by
213 a (truncated) log-normal pdf, while all remaining parameters are associated with
214 uniform pdfs. Case *a* is representative of an approach where information on the
215 considered parameters is limited so that all parameter values within the identi-
216 fied range of variability are equally weighted in the analysis (other studies relying
217 on the same assumption include, e.g., Ciriello et al. (2013); Laloy et al. (2013);
218 Sochala and Le Maître (2013); Bianchi Janetti et al. (2019); Dell’Oca et al. (2020)).
219 Case *b* is implemented as an alternative uninformed case, making use of the widely
220 adopted hypothesis that model parameters are normally distributed. Case *c* takes
221 advantage of the findings of Naraghi et al. (2018) who provide some experimen-
222 tal evidence suggesting that the pdf of pore radii in shales can be interpreted
223 through a log-normal model. Our choice of performing sensitivity analyses accord-
224 ing to configurations associated with diverse pdfs characterizing uncertain model
225 parameters enables us to analyze the influence of model parameter pdf (which is
226 generally unknown a priori) on the results of the GSA and, ultimately, on gas flow
227 forecasting.

228 Considering the computational cost associated with multiple model evaluations
229 (corresponding to 10^{-4} seconds per simulation on an Intel Xeon Gold 6148 CPU
230 @ 2.4 GHz) required for these analyses and the corresponding cost for random
231 sampling across the considered high dimensionality model parameter space, our
232 analyses rest on 10^8 model evaluations. The latter has been deemed to constitute
233 an acceptable trade-off between the need to obtain stable results and computa-
234 tional efforts (details not shown). The pressure gradient acting on the system is
235 set as a given boundary condition (and equal to 0.1MPa/m) in all test cases.

2.2.1 Variance-based Sobol' Indices

Sobol' indices (Saltelli and Sobol', 1995) can assist the appraisal and quantification of the relative expected reduction of the variance of a target model output due to knowledge of (or conditioning on) a given model parameter, which would otherwise be subject to uncertainty. In this context, considering a model output y , which depends on N random parameters collected in vector $\mathbf{x} = (x_1, x_2, \dots, x_N)$ and defined within the space $\Gamma = \Gamma_1 \times \Gamma_2 \times \dots \times \Gamma_N$ ($\Gamma_i = [x_{i,min}, x_{i,max}]$ corresponding to the support of the i -th parameter, x_i), the principal Sobol' index S_{x_i} associated with a given model parameter x_i is evaluated as

$$S_{x_i} = \frac{V[E[y|x_i]]}{V[y]}. \quad (9)$$

Here, $E[\cdot]$ and $V[\cdot]$ represent expectation and variance operators, respectively; the notation $y|x_i$ denotes conditioning of y on x_i . Note that S_{x_i} describes the relative contribution to $V[y]$ due to variability of only x_i . Joint contributions of x_i with other model parameters included in \mathbf{x} to the variance of y are embedded in the total Sobol' indices (details not shown). We recall that relying on Sobol' indices to diagnose the relative importance of uncertain model parameters to model outputs is tantamount to identifying uncertainty with the concept of variance of a pdf. As such, while Sobol' indices are characterized by a conceptual simplicity and straightforward implementation and use, they provide only limited information about the way variations of model parameters can influence the complete pdf of model outputs.

2.2.2 Moment-Based AMA Indices

The recent moment-based GSA approach proposed by Dell'Oca et al. (2017, 2020) rests on the idea that the quantification of the effects of model parameter uncertainty on various statistical moments of the ensuing pdf of model outputs can provide enhanced understanding of model functioning. Dell'Oca et al. (2017) introduce Moment-Based sensitivity metrics (termed AMA indices) according to which one can evaluate the influence of uncertain model parameter on key elements of the model output pdf, as embedded in its associated statistical moments. The AMA indices are defined as follows (Dell'Oca et al. (2017)):

$$AMAM_{x_i} = \frac{1}{|M[y]|} E[|M[y] - M[y|x_i]|]. \quad (10)$$

Here, $AMAM_{x_i}$ represents the indices associated with a model parameter x_i and a given statistical moment M of the pdf of model output y (considering the first four statistical moments of y , $M = E$ for the mean, $M = V$ for the variance, $M = \gamma$ for the skewness, and $M = k$ for the kurtosis). The AMA indices are intended to quantify the expected change of each statistical moment of y due to our knowledge of x_i . Large values of these indices indicate that variations of the associated parameter strongly affect the statistical moments of y .

Table 1 Ranges of variability for the methane migration model uncertain parameters considered in the GSA. Coefficient of variation, criteria for the selection of the range of variability, and reference considered for the definition of each range of variability is also listed.

Parameter - (Units) - Symbol	Range (CV%)	Criteria support	Reference
Reference pore radius - (nm) - r_o	2-100 (55)	Literature	Wu et al. (2016)
Reference porosity - (-) - ϕ_o	0.005-0.1 (52)	Literature	Li et al. (2006)
Pore pressure - (MPa) - p	0.5-50 (57)	Literature	Wu et al. (2016)
Tortuosity - (-) - τ	2.8-5.8 (20)	Literature	Mohd Amin et al. (2014)
Temperature - (K) - T	337-473 (10)	Literature	Chiquet et al. (2007)
Overburden pressure - (MPa) - p_c	51-90 (16)	Literature	Chiquet et al. (2007)
Porosity exponent - (-) - q	0.014-0.056 (35)	Literature	Dong et al. (2010)
Pore radius exponent - (-) - t	0.02-0.04 (19)	Literature	Dong et al. (2010)
Block/migration ratio - (-) - κ	0.1-2 (52)	Literature	Wu et al. (2015b)
Fractal dimension - (-) - D_f	2.1-2.9 (9)	Theoretical Limits	Coppens (1999)
Isosteric adsorption heat - (J/mol) - ΔH	12000-16000 (8)	Literature	Wu et al. (2015b)
Reference Langmuir pressure - (Pa) - p_{L_o}	41-128 (30)	CV	Wu et al. (2015b)
Parameter - (-) - α_0	1.02-1.36 (8)	Literature	Karniadakis et al. (2005)
Parameter - (-) - α_1	2-6 (30)	CV	Karniadakis et al. (2005)
Parameter - (-) - β	0.2-0.6 (30)	CV	Karniadakis et al. (2005)

272 3 Results and Discussion

273 3.1 GSA of methane flow model

274 The 15 uncertain model parameters of model (1) are considered to vary across the
 275 support defined through the ranges of variability listed in Table 1. These ranges
 276 have been designed upon considering available literature references (values typi-
 277 cally employed for the model parameters in low permeability geomaterials). **With**
 278 **reference to three of the model parameters, i.e., L_{p_o} , α_1 and β , only very limited**
 279 **information is available from the literature, to the best of our knowledge (Karni-**
 280 **adakis et al., 2005). Thus, we take the values considered by Wu et al. (2017) and**
 281 **Karniadakis et al. (2005) as the centers of corresponding intervals of variability**
 282 **associated with a given coefficient of variation, that we set equal to 30%, which**
 283 **enables us to imprint these parameters with a sufficiently broad range of variabil-**
 284 **ity, similar to what found for the remaining uncertain parameters (see Table 1).**
 285 Finally, we allow the fractal dimension D_f to vary within its theoretical bounds
 286 (i.e., $2 < D_f < 3$) (Coppens, 1999; Coppens and Dammers, 2006). Methane prop-
 287 erties (such as viscosity, compressibility, and deviation factor) are estimated using
 288 miniREFPROP (Lemmon et al., 2018), a tool that incorporates equations of state
 289 for a variety of gas species. With reference to methane miniREFPROP relies on
 290 the equation of state proposed by Setzmann and Wagner (1991).

291 Table 2 lists the moment-based GSA indices related to mean ($AMAE_{x_i}$), vari-
 292 ance ($AMAV_{x_i}$), skewness ($AMA\gamma_{x_i}$), and kurtosis ($AMAK_{x_i}$) of J as well as the
 293 principal Sobol' indices (S_{x_i}) evaluated for methane flow rate values rendered by
 294 Eq. (1) for the case in which all model parameters are modeled as independent
 295 and identically distributed random variables, each characterized by a uniform pdf
 296 (Case a).

297 While the strength of the influence of the reference pore radius (r_o) on the
 298 model output is not the same for the (first four) statistical moments, the AMA
 299 indices clearly suggest that conditioning on r_o has (overall) the strongest impact
 300 on the first four statistical moments of methane flow. This is then followed by ref-
 301 erence porosity, pore pressure, tortuosity, and temperature. While the remaining

Table 2 Moment-based GSA indices $AMAM_{x_i}$ and Sobol' principal indices S_{x_i} for all x_i parameters included in Equation (1). All model parameters are described by uniform pdfs (Case *a*). Values of each metric identifying the most influential parameters are reported in bold.

x_i	$AMAE_{x_i}$	$AMAV_{x_i}$	S_{x_i}	$AMA\gamma_{x_i}$	$AMAk_{x_i}$
r_o	0.728	0.798	0.417	0.562	0.757
ϕ_o	0.453	0.643	0.160	0.345	0.464
p	0.335	0.484	0.091	0.208	0.476
τ	0.181	0.356	0.026	0.114	0.213
T	0.094	0.163	0.007	0.027	0.046
q	0.061	0.119	0.003	0.011	0.022
t	0.057	0.114	0.003	0.01	0.021
p_c	0.028	0.063	0.001	0.008	0.014
κ	0.010	0.005	0	0.004	0.007
ΔH	0.001	0.002	0	0.002	0.005
D_f	0.002	0.003	0	0.002	0.004
pL_o	0.002	0.003	0	0.002	0.004
α_0	0.001	0.002	0	0.002	0.004
α_1	0.001	0.002	0	0.002	0.004
β	0.001	0.002	0	0.002	0.004

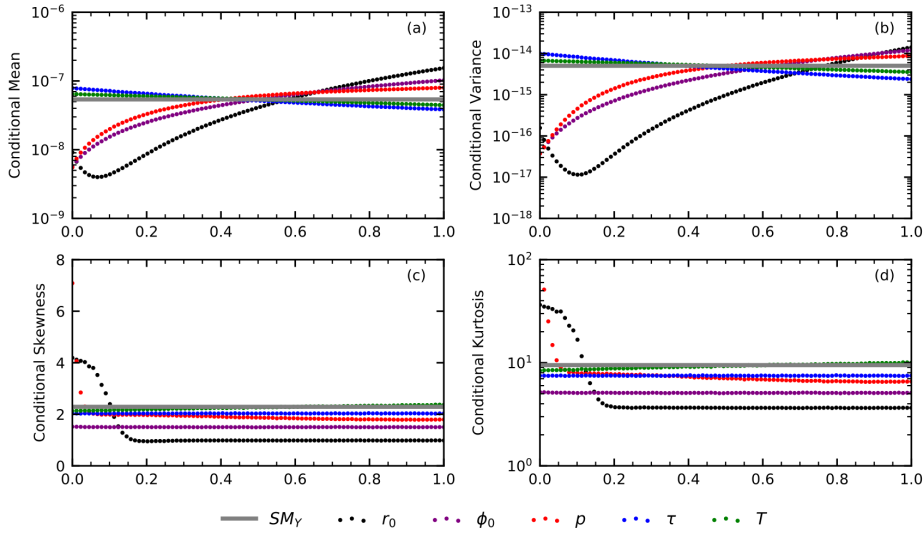


Fig. 1 First four statistical moments of methane flow J (Ton/m² year) conditional on values of the most influential model parameters (see Table 2): (a) expected value, (b) variance, (c) skewness, and (d) kurtosis. The corresponding unconditional moments (i.e. SM_Y) are also depicted (gray bold horizontal lines). Intervals of variation of the uncertain model parameters are rescaled within the unit interval for graphical representation purposes. All model parameters are described by uniform pdfs (Case *a*).

302 model uncertain parameters still exert some influence on the (first four) statistical
 303 moments of J (as evidenced by the non-zero values of AMA indices), the strength
 304 of their influence can be considered as marginal when compared to the above men-
 305 tioned quantities, which are seen to be key in driving the main features of the pdf
 306 of methane flow. In the following we denote as *most influential parameters* for
 307 metrics $AMAM_{x_i}$ or S_{x_i} all parameters x_i where $AMAM_{x_i} / \sum_{x_i} AMAM_{x_i} \geq 5\%$
 308 or $S_{x_i} / \sum_{x_i} S_{x_i} \geq 5\%$, respectively. Most influential parameters identified by each

metric are reported in bold in Table 2. Values of the Sobol' principal indices are generally consistent with the results stemming from the moment-based GSA, even as τ and T are not identified as influential to the model output according to the Sobol' principal index. This result is consistent with the observation that conditional variance can be larger or smaller than its unconditional counterpart (see also Fig. 1b) in a way that its integral over Γ_T vanishes. A similar effect associated with the principal Sobol' indices was identified by Dell'Oca et al. (2017) with reference to the Ishigami function, which is a widely used analytical benchmark in sensitivity studies.

Figure 1 depicts the first four statistical moments of J conditioned on values of the five most influential uncertain parameters selected on the basis of Table 2. Uncertain parameters are normalized to span the unit interval, for ease of interpretation. Unconditional moments are also depicted as a reference. **We note that when considering conditioning on the model parameters which have been identified as non-influential according to the metrics employed, the difference between conditional and unconditional moments is negligible (details not reported).**

As expected, conditioning on values of the reference pore radius (r_o) yields the most marked effects to all of the statistical moments considered (see black dotted curves in Figure 1). Mean and variance of methane flow generally increase with r_o . A minimum mean methane flow value is attained for $2 < r_o < 15$ nm (corresponding to the range of normalized values comprised between 0 and 0.15 in Figure 1). The dominant transport mechanism for $r_o < 15$ nm is surface diffusion, the strength of its contribution decreasing with increasing r_o . As r_o increases, the strength of the contribution related to surface diffusion decreases faster than the corresponding increase of the slip flow contribution, thus resulting in a minimum value for the expected methane flow for values of the reference pore radius comprised in the aforementioned range. Otherwise, skewness and kurtosis (i) are affected by variations of the reference pore radius when the latter is smaller than 20 nm (corresponding to a normalized value of 0.18); and (ii) are generally constant for $r_o > 20$ nm. Nevertheless, we note that these (statistical) moments are still remarkably different from their unconditional counterparts even for large r_o values, thus evidencing the impact of acquired knowledge on r_o on reducing the asymmetry (as rendered by the skewness) and the peakedness and tailedness (i.e., the probability associated with extreme values, as rendered by the kurtosis) of the methane flow pdf.

Conditioning on pore pressure imprints variations to the statistics of the model output which are qualitatively similar to those associated with r_o . Larger values of mean and variance of J are linked to larger values of p . This result descends from the linear relationship between pore pressure and slip flow (Equation (7)), the latter being the dominant mechanism in systems formed by larger pores. Conditional skewness and kurtosis are constant (albeit different from their unconditional counterpart) across most of the variability range of p , sharp variations of these quantities being associated with conditioning on low values of p (i.e., corresponding to pore pressure values smaller than 10 MPa). Our findings about the influence of p on J are consistent with the results of Sun et al. (2017). These authors find that increasing values of pore pressure lead to an increase of apparent permeability (which is in turn linearly proportional to gas flow) for $r_o > 10$ nm. Wu et al. (2016) document a similar behavior due to the dominance of the slip flow

component (which is proportional to p ; see Equation (7)) in systems characterized by large pores.

While the impact of reference porosity and tortuosity is not analyzed in any of the available previous studies (Wu et al., 2015b, 2016, 2017; Sun et al., 2017; Zhang et al., 2018), our results rank ϕ_o and τ as the second and fourth most influential parameters in the evaluation of the pdf of J , respectively (see Table 2). The correction factors for bulk (eq. (23)) and surface (eq. (17)) diffusion flow increase linearly with reference porosity. Thus, increased values of ϕ_o yield corresponding increases of the methane flow (and hence of its first two statistical moments) independent of the dominant transport mechanism. Conditional mean and variance of J decrease with increasing values of tortuosity. This is in line with the observation that all gas transport mechanisms are characterized by an inverse proportionality between J and τ through the correction factor which is related to these processes taking place within a porous domain. These elements are consistent with a physical picture according to which fluid flow rates across a porous geomaterial are expected to increase and decrease with increasing porosity and tortuosity, respectively. Unlike pore pressure and reference pore radius, conditioning on reference porosity and tortuosity yields a reduction of skewness and kurtosis of the pdf of J , whose conditional values remain constant independent of the value of ϕ_o and/or τ .

Conditioning on temperature (T) affects the mean and variance of the methane flow pdf in a way which is qualitatively similar to the effect of tortuosity (albeit quantitatively to a lesser extent) due to the inverse proportionality between J and T . Otherwise, the overall shape of the pdf of J is not significantly influenced by the knowledge of T , values of conditional skewness and kurtosis practically coinciding with their unconditional counterparts.

The results listed in Table 2 suggest that statistical moments of methane flow are virtually insensitive to the remaining parameters (i.e., 10 of the 15 model parameters). Therefore, setting any of these parameters at given values within the variability space considered in our analysis yields only minor changes in the prediction of J . In this context, our results suggest that methane flow can be assessed with an acceptable degree of reliability even in the presence of scarce information about several parameters embedded in Equation (1) such as, e.g., the overburden pressure (i.e., p_c), the power-law exponents related to porosity (i.e., q) and pore radius (i.e., t), the fractal dimension of the pore surface (i.e., D_f), or the isosteric adsorption heat of the geomaterial (i.e., ΔH). Further to this, our results suggest the opportunity to prioritize allocation of resources to robust characterization of (in descending order) reference pore radius, reference porosity, pore pressure, tortuosity, and temperature.

3.2 Impact of the model parameter pdfs on GSA results

In this section we analyze the impact of the choice of model parameter distribution on the pdf of J . As described in Section 2.2, we compare the GSA outcomes obtained with a uniform pdf for all model parameters (Case *a*) and illustrated in Section 3.1 against those computed when (i) all model parameters are characterized through (truncated) Gaussian pdfs (Case *b*) and (ii) r_o is described by a (truncated) log-normal pdf while the remaining parameters are described as in

Table 3 Moment-based GSA indices $AMAM_{x_i}$ and Sobol' principal indices S_{x_i} for all x_i parameters included in Equation (1). All model parameters are described by truncated Gaussian distributions (Case *b*). Values of each metric identifying the most influential parameters are reported in bold.

x_i	$AMAE_{x_i}$	$AMAV_{x_i}$	S_{x_i}	$AMA\gamma_{x_i}$	$AMAK_{x_i}$
r_o	0.787	0.828	0.761	0.608	0.692
ϕ_o	0.452	0.674	0.242	0.306	0.402
p	0.321	0.481	0.131	0.152	0.302
τ	0.182	0.363	0.041	0.088	0.157
T	0.100	0.178	0.012	0.027	0.042
q	0.063	0.122	0.005	0.010	0.018
t	0.059	0.117	0.004	0.009	0.016
p_c	0.025	0.056	0.001	0.006	0.011
κ	0.007	0.005	0	0.005	0.008
ΔH	0.001	0.002	0	0.003	0.007
D_f	0.001	0.003	0	0.002	0.005
pL_o	0.001	0.002	0	0.003	0.006
α_0	0.001	0.002	0	0.002	0.005
α_1	0.001	0.002	0	0.003	0.006
β	0.001	0.002	0	0.003	0.006

403 Case *a* (Case *c*). To provide a consistent comparison, Gaussian and log-normal
 404 pdfs are defined to honor the same mean and variance of the scenario associated
 405 with Case *a*.

406 Table 3 lists values of AMA and principal Sobol' indices for each of the pa-
 407 rameters embedded in Equation (1) for Case *b*. Results of Table 3 and Table 2 are
 408 qualitatively similar, i.e., the GSA yields similar results considering a uniform or
 409 a (truncated) Gaussian pdf for all model parameters. Our results imbue us with
 410 confidence about the documented ranking of parameter importance, with reference
 411 pore radius, reference porosity, pore pressure, tortuosity, and temperature identi-
 412 fied as the model parameters being the key drivers to the evaluation of the major
 413 features of the pdf of methane flow. Values of statistical moments of J conditioned
 414 on model parameters for Case *b* are very similar to those depicted in Figure 1 for
 415 Case *a* (details not shown).

416

417 Table 4 lists the AMA and the principal Sobol' indices associated with J for
 418 Case *c*. In this case, it is even more evident that the uncertainty of r_o is strongly
 419 dominant on the evaluation of the pdf of methane flow. Additionally, the block-
 420 age/migration ratio of the adsorbed molecules (κ) gains importance with respect
 421 to previous cases, quantitatively impacting the pdf of J to an extent which is simi-
 422 lar to what exhibited by temperature. This feature is attributed to the abundance
 423 of small pores in this scenario, which favors the dominance of the surface diffusion
 424 flow mechanism (linked to parameter κ).

425 Figure 2 depicts the first four statistical moments of methane flow conditioned
 426 on values of influential uncertain parameters for Case *c* (see Table 4). Uncondi-
 427 tional moments are also shown as a reference. Overall, the results are qualitatively
 428 similar to those embedded in Figure 1 for Case *a*. The unconditional mean and
 429 variance of J in Case *c* are reduced (to approximately one-fourth and one-sixth,
 430 respectively) with respect to the corresponding values for Case *a*. Otherwise, un-
 431 conditional skewness and kurtosis increase by about 2.6 and 6 times, respectively.

Table 4 Moment-based GSA indices $AMAM_{x_i}$ and Sobol' principal indices S_{x_i} for all x_i parameters included in Equation (1). Reference pore radius (r_o) is described by a (truncated) lognormal distribution and the remaining model parameters are described by uniform distributions (Case *c*). Values of each metric identifying the most influential parameters are reported in bold.

x_i	$AMAE_{x_i}$	$AMAV_{x_i}$	S_{x_i}	$AMA\gamma_{x_i}$	$AMAK_{x_i}$
r_o	3.332	3.649	2.803	0.788	0.883
ϕ_o	0.452	0.690	0.064	0.212	0.404
p	0.192	0.507	0.012	0.152	0.263
τ	0.181	0.358	0.011	0.070	0.167
T	0.090	0.173	0.003	0.024	0.050
q	0.063	0.121	0.001	0.008	0.020
t	0.041	0.112	0.001	0.008	0.020
p_c	0.023	0.061	0	0.007	0.016
κ	0.112	0.010	0.005	0.021	0.027
ΔH	0.002	0.006	0	0.004	0.010
D_f	0.02	0.008	0	0.011	0.018
pL_o	0.025	0.007	0	0.011	0.017
α_0	0.002	0.006	0	0.005	0.013
α_1	0.002	0.006	0	0.005	0.012
β	0.002	0.006	0	0.004	0.010

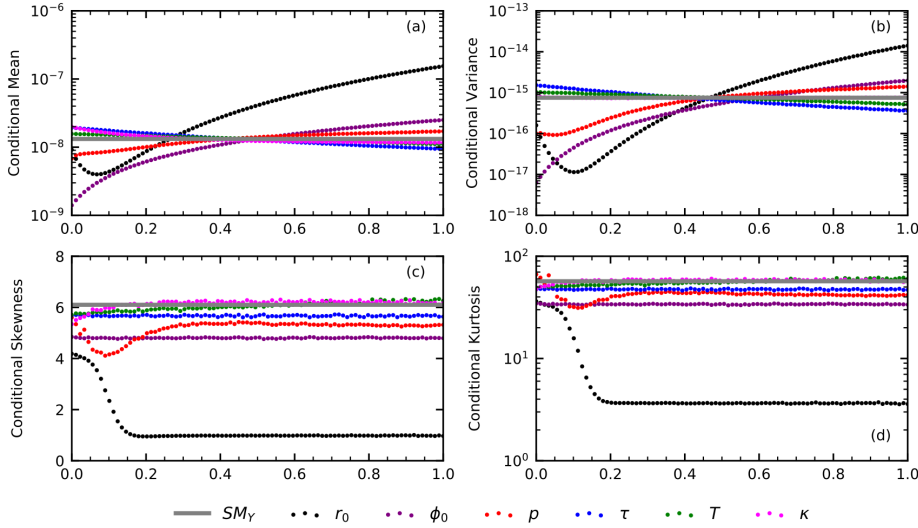


Fig. 2 First four statistical moments of methane flow J (Ton/m² year) conditional on values of the most influential model parameters (see Table 4): (a) expected value, (b) variance, (c) skewness, and (d) kurtosis. The corresponding unconditional moments (i.e. SM_Y) are also depicted (gray bold horizontal lines). Intervals of variation of the uncertain model parameters are rescaled within the unit interval for graphical representation purposes. r_o is described by a truncated log-normal pdf and the remaining model parameters are described by uniform pdfs (Case *c*).

432 These behaviors are attributed to the larger frequency of small reference pore ra-
 433 dius values considered in Case *c* with respect to Case *a* (and *b*). Low values of
 434 reference pore radius are associated with large values of surface diffusion and to
 435 small values of mean and variance of methane flow. Conditioning on r_o and ϕ_o
 436 imprints variations to the model output mean and variance across the entire range
 437 of variability of these parameters (Figure 2). We further note that conditioning

438 on r_o strongly reduces skewness and kurtosis of the pdf of J , thus reducing the
 439 probability associated with extreme (large) values of J .

440 Conditioning on p induces variations in the (first four) statistical moments
 441 of the model output. Conditioning on larger values of this quantity yields the
 442 highest values of mean and variance of the model output. A minimum in the
 443 values of conditional variance, skewness, and kurtosis is observed in the interval
 444 $1\text{MPa} < p < 15\text{MPa}$. Finally, the blockage/migration ratio of adsorbed molecules
 445 displays (a small but noticeable) influence on the model output pdf. Mean and
 446 variance of J decrease with increasing values of κ . This behavior is expected,
 447 given the nature of κ , high values of this parameter being related to significant
 448 blockage of gas molecules on the geomaterial surface.

449 3.3 Scaling of gas flow model and identification of dominant flow mechanisms

450 A pure diffusion modeling approach has been shown to represent with an accept-
 451 able degree of accuracy the movement of methane in low permeability media (Lu
 452 et al., 2015). Such a model embeds all physics governing the system dynamics in
 453 a unique parameter (i.e., a diffusion coefficient D) and, under steady-state condi-
 454 tions, the mass flow-rate of methane can be expressed as:

$$J_d = -D \frac{\partial C}{\partial l}, \quad (11)$$

455 where $\partial C/\partial l$ represents the spatial gradient of methane concentration (C), i.e., the
 456 driving force of the system. Considering an isothermal system under single-phase
 457 flow and introducing the density of methane, $\rho = pM/RTZ$, Equation (11) can be
 458 written as:

$$J_d = -\frac{DM}{RTZ} \left(1 - \frac{p}{Z} \frac{dZ}{dp}\right) \frac{\partial p}{\partial l}. \quad (12)$$

459 We complete our set of results and discussion by noting that the model illus-
 460 trated in Section 2.1 coincides with a pure diffusion model (Equation (12)) under
 461 single-phase conditions, as we illustrate in the following.

462 Equation (1) can be written as:

$$J = -B \frac{\partial p}{\partial l}, \quad (13)$$

463 with $B = B_v + B_k + B_{ss}$, where

$$\begin{aligned} B_v &= w_v \zeta_{mb} \frac{r^2 p M}{8 \eta Z R T} (1 + \alpha K_n) \left(1 + \frac{4 K_n}{1 + K_n}\right), \\ B_k &= w_k \frac{2}{3} \zeta_{mb} r \delta^{D_f - 2} \left(\frac{8 Z M}{\pi R T}\right)^{1/2} \frac{p}{Z} C_g, \\ B_{ss} &= \zeta_{ms} \frac{D_s C_{sc}}{p}. \end{aligned} \quad (14)$$

464 Comparing Equations (12) and (13), it can be seen that the diffusion coefficient
 465 D can be decomposed according to each flow mechanism as:

$$D = D_v + D_k + D_{ss}, \quad (15)$$

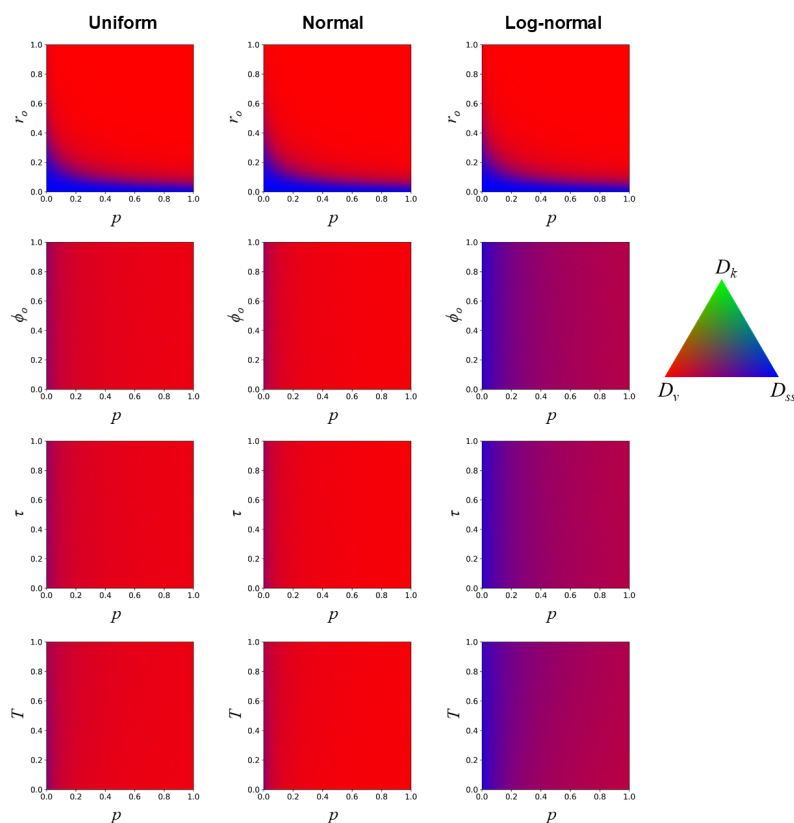


Fig. 3 Relative contribution of the effective diffusion coefficients (D_v , D_k , and D_{ss}) to the overall diffusion coefficient D rendered by Equations (15) and (16). Intervals of variation of the uncertain model parameters are rescaled within the unit interval for graphical representation purposes.

466 with

$$D_i = \frac{B_i RTZ}{M \left(1 - \frac{p}{Z} \frac{dZ}{dp}\right)}, \quad (16)$$

467 where $i = v, k, ss$. Note that we introduce three effective diffusion coefficients
 468 in Equation (15). These are respectively associated with the slip flow (D_v), the
 469 Knudsen diffusion (D_k), and the surface diffusion (D_{ss}) components of model (1)
 470 and are to the best of our knowledge, new for the flow model considered in this
 471 work. The variety of mechanisms included in model (1) are fully encapsulated in
 472 an overall diffusion coefficient D as illustrated in Equations (12), (15), and (16),
 473 where the contribution of each of the processes described in Section 2.1 is clearly
 474 recognizable.

475 Figure 3 depicts color maps quantifying the relative strength of the contribution
 476 of the three flow mechanism (slip flow in red, Knudsen diffusion in green, and
 477 surface diffusion in blue) to the overall diffusion coefficient defined by Equation
 478 (15) considering various combinations of all uncertain parameters embedded in

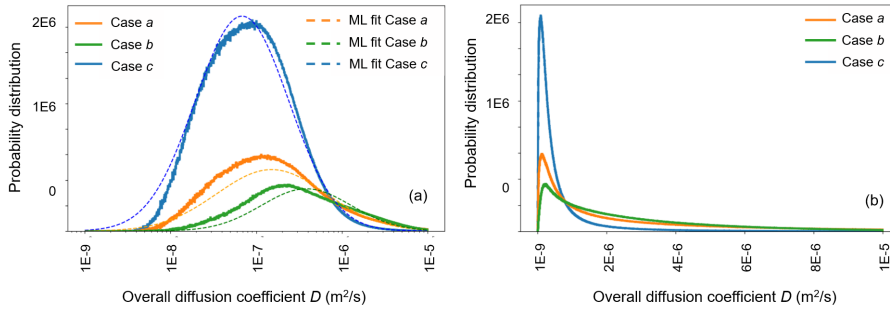


Fig. 4 Probability density functions (in logarithmic (a) and natural (b) scale) of the overall diffusion coefficient rendered by Equation (15) for model parameters characterized by: (i) uniform distributions (Case *a*), (ii) truncated normal distributions (Case *b*), and (iii) uniform distributions with the exception of r_o which is represented by a log-normal distribution (Case *c*). Dashed curves represent a ML- based fit with a log-normal model for each case

Table 5 Sample mean, variance, coefficient of variation, skewness, and kurtosis of the overall diffusion coefficient D (m^2/s) (Equation (15)) together with parameters of log-normal models (μ and σ) evaluated through ML fits against sample pdfs.

Feature	Case <i>a</i>	Case <i>b</i>	Case <i>c</i>
Mean ($\times 10^{-6}$)	3.14	2.96	0.78
Variance ($\times 10^{-12}$)	16.3	9.53	2.46
CV	1.29	1.04	2.01
Skewness	2.21	1.95	5.90
Kurtosis	9	8.1	53.54
μ	-13.53	-13.31	-14.92
σ	1.47	1.21	1.26

479 Equation (1) for all scenarios investigated. Each sub-plot depicts the average value
 480 of the ratio D_i/D as a function of two parameters (i.e., averaging is performed with
 481 respect to uncertain parameters with the exception of the two varying along the
 482 (normalized) axes of the subplots), selected amongst those which were classified
 483 as most influential to the system (see Sections 3.1 and 3.2).

484 Our results indicate that the dominant flow mechanism in defining the overall
 485 diffusion coefficient (and consequently the methane flow) is slip flow (in red in
 486 Figure 3) in all of the analyzed cases. An exception is observed for small values
 487 of the reference pore radius and/or small pore pressure, where surface diffusion is
 488 dominant. The contribution of Knudsen diffusion mechanism is always negligible.
 489 This suggests that it is possible to simplify Equation (1) by neglecting the Knudsen
 490 diffusion mechanism in the evaluation of methane flow.

491 Further simplifications of the methane flux model illustrated in Section 2.1
 492 can be considered when the dominance of a given flow mechanism can be clearly
 493 established. For example, Figure 3 suggests that the identification of the dominant
 494 flow mechanism is affected by the pdf of the uncertain model parameters. If r_o
 495 is represented by a Gaussian (or uniform) pdf, J is mainly dominated by slip flow
 496 or surface diffusion with a sharp transition zone between these two mechanisms.
 497 Otherwise, when r_o is represented by a log-normal pdf both mechanisms (i.e.,
 498 slip flow and surface diffusion) may play an important role in the estimation of
 499 methane migration independent of the value of the model parameters.

500 Finally, we evaluate the pdf of the overall diffusion coefficient (D) by making
501 use of Equations (15) and (16) for all scenarios analyzed. Sample pdfs as well
502 as corresponding Maximum Likelihood (ML) fits of log-normal distributions are
503 depicted in Figure 4 in logarithmic and natural scales. Positive skewness and large
504 kurtosis are evident for all cases, these being larger for Case c , as illustrated in
505 Section 3.2. These results reinforce the observation of higher frequencies of low J
506 values in Case c with respect to the other settings investigated. Sample statistical
507 moments (mean, variance, coefficient of variation, skewness, and kurtosis) of the
508 pdf of D are listed in Table 5 together with the parameters of the ML-based
509 log-normal models. The overall diffusion coefficient can vary across about four
510 orders of magnitude (i.e., between 10^{-9} and 10^{-5} m²/s). As expected, the largest
511 variance of D is associated with Case a , where all parameters of model (1) are
512 characterized by uniform pdfs. Otherwise, the largest coefficient of variation of D
513 is associated with Case c . Finally, we remark that the results embedded in Figure
514 4 can be of practical assistance, as they allow for fast evaluations of the probability
515 that methane flow in low permeability media exceeds a given threshold value.

516 4 Conclusions

517 We perform a rigorous Global Sensitivity Analysis (GSA) to assess the impact of
518 uncertain model parameters on the evaluation of methane flow (J) in low per-
519 meability media, such as caprocks. **We study three cases that consider differing**
520 **characterizations of the probability density function (pdf) describing model uncer-**
521 **tain parameters to assess the impact of this choice on the results of the analysis.**
522 **Such cases are: (i) all model parameters represented through uniform pdfs, (ii) all**
523 **model parameters represented through (truncated) Gaussian pdfs, and (iii) refer-**
524 **ence pore radius characterized by a (truncated) log-normal pdf while all remaining**
525 **parameters are associated with uniform pdfs.**

526 Our work leads to the following main conclusions:

- 527 1. The uncertainty of methane flow is governed by uncertainty in the reference
528 pore radius, followed (in decreasing order of importance) by reference poros-
529 ity, pore pressure, tortuosity, temperature, and (to a lesser extent) blockage
530 migration ratio of adsorbed molecules. The remaining parameters of the in-
531 vestigated model (Section 2.1) being practically uninfluential. This result can
532 assist future efforts to allocate resources during experimental activities aimed
533 at characterizing methane flow in caprocks.
- 534 2. The gas flow model introduced by Wu et al. (2016) (Section 2.1) can be related
535 to a simple pure diffusion model by introducing an overall diffusion coefficient
536 (D). The latter represented by the contribution of three effective diffusion
537 coefficients, each associated with a well-defined flow mechanism. The ensuing
538 mathematical structure of D allows distinguishing the relative contribution of
539 all flow mechanisms to the overall methane flow. The relationship we derive also
540 enables one to estimate the pdf of D when the model parameters are uncertain.
541 The latter is a useful tool which can assist the probabilistic evaluation of J
542 even in the absence of the detailed amount of information which is typically
543 required to characterize the full methane flow model.
- 544 3. The shape of the pdf employed to characterize uncertain model parameters
545 affects the results of our GSA. Additionally, it has a marked effect in the defi-

546 nition of the dominating transport mechanisms of the model. With reference to
 547 the model parameter variability considered in this study, as evaluated on the
 548 basis of available information, our results suggest that the dominant transport
 549 mechanism is slip flow. Surface diffusion plays also an important role, espe-
 550 cially for low values of reference pore radius and pore pressure, while Knudsen
 551 diffusion is negligible in all of the test cases analyzed.

552 **Appendix: Additional mathematical details related to the description**
 553 **of the gas flow model introduced in Section 2.1**

554 The correction factor ζ_{ms} is given by

$$\zeta_{ms} = \frac{\phi}{\tau} \left(1 - \frac{r_{ad}}{r}\right)^2 \left[\left(1 - \frac{r_{ad}}{r}\right)^{-2} - 1 \right], \quad (17)$$

555 with

$$r_{ad} = r - d_m \theta, \quad (18)$$

556

$$r = r_o \left(\frac{p_c - p}{p_o} \right)^{-t}, \quad (19)$$

557

$$\phi = \phi_o \left(\frac{p_c - p}{p_o} \right)^{-q}, \quad (20)$$

558 where r_{ad} is thickness of the adsorbed gas layer, d_m is gas molecule diameter, p_c
 559 is overburden pressure, p_o is atmospheric pressure, and θ is evaluated through a
 560 Langmuir equilibrium isotherm as:

$$\theta = \frac{p/Z}{p_L + p/Z}, \quad (21)$$

561 where p_L is a Langmuir pressure evaluated with

$$p_L = p_{L_o} \exp\left(-\frac{\Delta H}{RT}\right). \quad (22)$$

562 The correction factor ζ_{mb} is expressed as

$$\zeta_{mb} = \frac{\phi}{\tau} \left(1 - \frac{r_{ad}}{r}\right)^2. \quad (23)$$

563 The value of α (in Eq. (7)) is evaluated through

$$\alpha = \alpha_0 \frac{2}{\pi} \tan^{-1} \left(\alpha_1 K_n^\beta \right), \quad (24)$$

564 where uncertain parameters α_0 , α_1 , and β allow to represent the variation of α as
 565 a function of Knudsen number (K_n). Here, α_0 represents the maximum value of
 566 α for large values of K_n , α_1 governs the values of α for small values of K_n , and β
 567 defines the slope of the relationship between K_n and α for low values of K_n .

568 The surface diffusion coefficient (D_s) is given by

$$D_s = D_s^0 \frac{(1 - \theta) + \frac{\kappa}{2}\theta(2 - \theta) + H(1 - \kappa)(1 - \kappa)\frac{\kappa}{2}\theta^2}{(1 - \theta + \frac{\kappa}{2}\theta)^2}, \quad (25)$$

569 with

$$D_s^0 = 8.29 \times 10^{-7} \exp\left(-\frac{\Delta H^{0.8}}{RT}\right), \quad (26)$$

570

$$H(1 - \kappa) = \begin{cases} 0, & \text{if } \kappa \geq 1 \\ 1, & 0 \leq \kappa \leq 1 \end{cases}. \quad (27)$$

571 The adsorbed concentration (C_{sc}) is given by

$$C_{sc} = \frac{4\theta M}{\pi d_m^3 N_A}, \quad (28)$$

572 where N_A is the Avogadro Constant (6.02×10^{-23} /mol).

573 List of Symbols and Nomenclature

Symbol	Refers to	Units	Evaluated
C_g	Gas compressibility	1/MPa	MiniREFPROP
C_{sc}	Adsorbed concentration	kg/m ³	Equation 28
d_m	Gas molecule diameter	nm	0.38
$\partial p / \partial l$	Gradient of gas pore pressure	MPa/m	0.1
D	Overall diffusion coefficient	m ² /s	Equation 15
D_s	Surface diffusion coefficient	m ² /s	Equation 25
D_k	Knudsen effective diffusion coefficient	m ² /s	Equation 16
D_{ss}	Surface effective diffusion coefficient	m ² /s	Equation 16
D_v	Slip flow effective diffusion coefficient	m ² /s	Equation 16
J	Mass flux of gas per unit of area	kg/(m ² s)	Equation 1
J_k	Knudsen diffusion	kg/(m ² s)	Equation 8
J_s	Surface diffusion	kg/(m ² s)	Equation 2
J_v	Slip flow	kg/(m ² s)	Equation 7
Kn	Knudsen number	-	Equation 5
M	Gas molar mass	kg/mol	1.6×10^{-2}
p_L	Langmuir pressure	MPa	Equation 22
p_o	Atmospheric pressure	MPa	0.1
r	Pore size	nm	Equation 19
R	Universal gas constant	J/(mol K)	8,3144
r_{ad}	Thickness of adsorbed gas layer	nm	Equation 18
w_k	Knudsen diffusion flux weight factor	-	Equation 4
w_v	Slip mass flux weight factor	-	Equation 3
Z	Gas deviation factor	-	MiniREFPROP
α	Rarified effect coefficient for gas	-	Equation 24
ζ_{ms}	Correction factor of surface diffusion	-	Equation 17
ζ_{mb}	Correction factor bulk flow	-	Equation 23

η	Gas viscosity	Pa s	MiniREFPROP
θ	Gas coverage of the geomaterial	-	Equation 21
λ	Mean free path of gas molecules	m	Equation 6
ϕ	Porosity	-	Equation 20

574 Declarations

575 *Funding.* The authors acknowledge financial support from Geolog srl.

576 *Competing interests.* Not applicable

577 *Availability of data and material* All data used in the paper will be retained by the
578 authors for at least 5 years after publication and will be available to the readers
579 upon request.

580 *Code availability.* Codes used in this paper are available in the following github
581 repository https://github.com/rlsandovalp/Sensitivity_Analysis

582 References

- 583 Bianchi Janetti E, Guadagnini L, Riva M, Guadagnini A (2019) Global sensitiv-
584 ity analyses of multiple conceptual models with uncertain parameters driving
585 groundwater flow in a regional-scale sedimentary aquifer. *Journal of Hydrology*
586 574(September 2018):544–556, DOI 10.1016/j.jhydrol.2019.04.035
- 587 la Cecilia D, Porta GM, Tang FH, Riva M, Maggi F (2020) Probabilistic indicators
588 for soil and groundwater contamination risk assessment. *Ecological Indicators*
589 115:106,424, DOI 10.1016/j.ecolind.2020.106424
- 590 Ceresa L, Guadagnini A, Porta GM, Riva M (2021) Formulation and probabilistic
591 assessment of reversible biodegradation pathway of Diclofenac in groundwater.
592 *Water Research* 204:117,466, DOI 10.1016/J.WATRES.2021.117466
- 593 Chiquet P, Daridon JL, Broseta D, Thibeau S (2007) CO₂/water interfacial ten-
594 sions under pressure and temperature conditions of CO₂ geological storage. *En-
595 ergy Conversion and Management* 48(3):736–744, DOI 10.1016/J.ENCONMAN.
596 2006.09.011
- 597 Ciriello V, Guadagnini A, Di Federico V, Edery Y, Berkowitz B (2013) Compara-
598 tive analysis of formulations for conservative transport in porous media through
599 sensitivity-based parameter calibration. *Water Resources Research* 49(9):5206–
600 5220, DOI 10.1002/wrcr.20395
- 601 Civan F (2010) Effective correlation of apparent gas permeability in tight
602 porous media. *Transport in Porous Media* 82(2):375–384, DOI 10.1007/
603 s11242-009-9432-z
- 604 Colombo I, Porta GM, Ruffo P, Guadagnini A (2017) Uncertainty quantifica-
605 tion of overpressure buildup through inverse modeling of compaction processes
606 in sedimentary basins. *Hydrogeology Journal* 25(2):385–403, DOI 10.1007/
607 s10040-016-1493-9
- 608 Coppens MO (1999) The effect of fractal surface roughness on diffusion and reac-
609 tion in porous catalysts – from fundamentals to practical applications. *Catalysis*
610 *Today* 53(2):225–243, DOI 10.1016/S0920-5861(99)00118-2

- 611 Coppens MO, Dammers AJ (2006) Effects of heterogeneity on diffusion in
612 nanopores—From inorganic materials to protein crystals and ion channels. *Fluid*
613 *Phase Equilibria* 241(1-2):308–316, DOI 10.1016/J.FLUID.2005.12.039
- 614 Darabi H, Ettehad A, Javadpour F, Sepehrnoori K (2012) Gas flow in ultra-tight
615 shale strata. *Journal of Fluid Mechanics* 710:641–658, DOI 10.1017/jfm.2012.424
- 616 Dell’Oca A, Riva M, Guadagnini A (2017) Moment-based metrics for global sen-
617 sitivity analysis of hydrological systems. *Hydrology and Earth System Sciences*
618 21(12):6219–6234, DOI 10.5194/hess-21-6219-2017
- 619 Dell’Oca A, Riva M, Guadagnini A (2020) Global Sensitivity Analysis for Multi-
620 ple Interpretive Models With Uncertain Parameters. *Water Resources Research*
621 56(2):1–20, DOI 10.1029/2019WR025754
- 622 Dembicki-Jr H (2017) *Petroleum Geochemistry for Exploration and Production*.
623 Candice Janco, DOI 10.1016/j.jhsb.2006.10.005
- 624 Dong JJ, Hsu JY, Wu WJ, Shimamoto T, Hung JH, Yeh EC, Wu YH, Sone H
625 (2010) Stress-dependence of the permeability and porosity of sandstone and
626 shale from TCDP Hole-A. *International Journal of Rock Mechanics and Mining*
627 *Sciences* 47(7):1141–1157, DOI 10.1016/J.IJRMMS.2010.06.019
- 628 Hughes JD (2013) Energy: A reality check on the shale revolution. *Nature*
629 494(7437):307–308, DOI 10.1038/494307a
- 630 Javadpour F, Singh H, Rabbani A, Babaei M, Enayati S (2021) Gas Flow Mod-
631 els of Shale: A Review. *Energy and Fuels* 35(4):2999–3010, DOI 10.1021/acs.
632 energyfuels.0c04381
- 633 Karniadakis G, Beskok A, Aluru N (2005) *Microflows and Nanoflows*. (Springer
634 Science+Business Media, Inc, DOI 10.1103/PhysRevE.67.041106
- 635 Koks EE, Bočkarjova M, de Moel H, Aerts JC (2015) Integrated direct and in-
636 direct flood risk modeling: Development and sensitivity analysis. *Risk Analysis*
637 35(5):882–900, DOI 10.1111/risa.12300
- 638 Laloy E, Rogiers B, Vrugt JA, Mallants D, Jacques D (2013) Efficient posterior
639 exploration of a high-dimensional groundwater model from two-stage Markov
640 chain Monte Carlo simulation and polynomial chaos expansion. *Water Resources*
641 *Research* 49(5):2664–2682, DOI 10.1002/wrcr.20226
- 642 Lemmon EW, Bell I, Huber ML, McLinden MO (2018) NIST Standard Refer-
643 ence Database 23: Reference Fluid Thermodynamic and Transport Properties-
644 REFPROP, Version 10.0, National Institute of Standards and Technology. DOI
645 <https://doi.org/10.18434/T4/1502528>
- 646 Li Z, Dong M, Li S, Huang S (2006) CO₂ sequestration in depleted oil and gas
647 reservoirs—caprock characterization and storage capacity. *Energy Conversion*
648 *and Management* 47(11-12):1372–1382, DOI 10.1016/J.ENCONMAN.2005.08.
649 023
- 650 Liu J, Wang JG, Gao F, Ju Y, Zhang X, Zhang LC (2016) Flow Consistency Be-
651 tween Non-Darcy Flow in Fracture Network and Nonlinear Diffusion in Matrix
652 to Gas Production Rate in Fractured Shale Gas Reservoirs. *Transport in Porous*
653 *Media* 111(1):97–121, DOI 10.1007/s11242-015-0583-9
- 654 Lu J, Larson TE, Smyth RC (2015) Carbon isotope effects of methane transport
655 through Anahuac Shale - A core gas study. *Journal of Geochemical Exploration*
656 148:138–149, DOI 10.1016/j.gexplo.2014.09.005
- 657 Maina FZ, Siirila-Woodburn ER (2020) The Role of Subsurface Flow on Evapo-
658 transpiration: A Global Sensitivity Analysis. *Water Resources Research* 56(7):1–
659 20, DOI 10.1029/2019WR026612

- 660 Maina FZ, Guadagnini A, Riva M (2021) Impact of multiple uncertainties on
661 gravimetric variations across randomly heterogeneous aquifers during pumping.
662 *Advances in Water Resources* 154:103,978, DOI 10.1016/J.ADVWATRES.2021.
663 103978
- 664 Mehmani A, Prodanović M, Javadpour F (2013) Multiscale, Multiphysics Network
665 Modeling of Shale Matrix Gas Flows. *Transport in Porous Media* 99(2):377–390,
666 DOI 10.1007/s11242-013-0191-5
- 667 Mohd Amin S, Weiss DJ, Blunt MJ (2014) Reactive transport modelling of geologic
668 CO₂ sequestration in saline aquifers: The influence of pure CO₂ and of mixtures
669 of CO₂ with CH₄ on the sealing capacity of cap rock at 37°C and 100bar.
670 *Chemical Geology* 367:39–50, DOI 10.1016/J.CHEMGEO.2014.01.002
- 671 Naraghi ME, Javadpour F, Ko LT (2018) An Object-Based Shale Permeability
672 Model: Non-Darcy Gas Flow, Sorption, and Surface Diffusion Effects. *Transport*
673 *in Porous Media* 125(1):23–39, DOI 10.1007/s11242-017-0992-z
- 674 Pan Z, Connell LD, Camilleri M, Connelly L (2010) Effects of matrix moisture on
675 gas diffusion and flow in coal. *Fuel* 89(11):3207–3217, DOI 10.1016/j.fuel.2010.
676 05.038
- 677 Rani S, Prusty BK, Pal SK (2018) Adsorption kinetics and diffusion modeling of
678 CH₄ and CO₂ in Indian shales. *Fuel* 216(November 2017):61–70, DOI 10.1016/
679 j.fuel.2017.11.124
- 680 Saltelli A, Sobol' IM (1995) Sensitivity analysis for nonlinear mathematical models:
681 numerical experience (in Russian). *Mathematical models and computer experi-*
682 *ment* 7(11):16–28
- 683 Saltelli A, Annoni P, Azzini I, Campolongo F, Ratto M, Tarantola S (2010) Vari-
684 ance based sensitivity analysis of model output. Design and estimator for the to-
685 tal sensitivity index. *Computer Physics Communications* 181(2):259–270, DOI
686 10.1016/j.cpc.2009.09.018
- 687 Schloemer S, Krooss BM (2004) Molecular transport of methane, ethane and ni-
688 trogen and the influence of diffusion on the chemical and isotopic composition
689 of natural gas accumulations. *Geofluids* 4(1):81–108, DOI 10.1111/j.1468-8123.
690 2004.00076.x
- 691 Schlömer S, Krooss BM (1997) Experimental characterisation of the hydrocarbon
692 sealing efficiency of cap rocks. *Marine and Petroleum Geology* 14(5):565–580,
693 DOI 10.1016/S0264-8172(97)00022-6
- 694 Setzmann U, Wagner W (1991) A New Equation of State and Tables of Thermo-
695 dynamic Properties for Methane Covering the Range from the Melting Line to
696 625 K at Pressures up to 100 MPa. *Journal of Physical and Chemical Reference*
697 *Data* 20(6):1061–1155, DOI 10.1063/1.555898
- 698 Singh H, Myong RS (2018) Critical Review of Fluid Flow Physics at Micro- to
699 Nano-scale Porous Media Applications in the Energy Sector. *Advances in Ma-*
700 *terials Science and Engineering* 2018, DOI 10.1155/2018/9565240
- 701 Sochala P, Le Maitre OP (2013) Polynomial Chaos expansion for subsurface flows
702 with uncertain soil parameters. *Advances in Water Resources* 62:139–154, DOI
703 10.1016/j.advwatres.2013.10.003
- 704 Song W, Yao J, Li Y, Sun H, Zhang L, Yang Y, Zhao J, Sui H (2016) Apparent
705 gas permeability in an organic-rich shale reservoir. *Fuel* 181:973–984, DOI 10.
706 1016/j.fuel.2016.05.011
- 707 Sun Z, Li X, Shi J, Zhang T, Sun F (2017) Apparent permeability model for
708 real gas transport through shale gas reservoirs considering water distribution

- 709 characteristic. *International Journal of Heat and Mass Transfer* 115:1008–1019,
710 DOI 10.1016/j.ijheatmasstransfer.2017.07.123
- 711 Tan Y, Pan Z, Liu J, Kang J, Zhou F, Connell LD, Yang Y (2018) Experimental
712 study of impact of anisotropy and heterogeneity on gas flow in coal. Part I:
713 Diffusion and adsorption. *Fuel* 232(15):444–453, DOI 10.1016/j.fuel.2018.05.173
- 714 US Energy Information Administration (2015) World Shale Resource Assessments.
715 URL <https://www.eia.gov/analysis/studies/worldshalegas/>
- 716 Wang T, Tian S, Li G, Zhang P (2019) Analytical Model for Real Gas Trans-
717 port in Shale Reservoirs with Surface Diffusion of Adsorbed Gas. *Industrial and*
718 *Engineering Chemistry Research* 58(51):23,481–23,489, DOI 10.1021/acs.iecr.
719 9b05630
- 720 Wu K, Chen Z, Li X (2015a) Real gas transport through nanopores of varying
721 cross-section type and shape in shale gas reservoirs. *Chemical Engineering Jour-*
722 *nal* 281:813–825, DOI 10.1016/j.ccej.2015.07.012
- 723 Wu K, Li X, Wang C, Yu W, Chen Z (2015b) Model for surface diffusion of
724 adsorbed gas in nanopores of shale gas reservoirs. *Industrial and Engineering*
725 *Chemistry Research* 54(12):3225–3236, DOI 10.1021/ie504030v
- 726 Wu K, Chen Z, Li X, Guo C, Wei M (2016) A model for multiple transport mecha-
727 nisms through nanopores of shale gas reservoirs with real gas effect-adsorption-
728 mechanic coupling. *International Journal of Heat and Mass Transfer* 93:408–426,
729 DOI 10.1016/j.ijheatmasstransfer.2015.10.003
- 730 Wu K, Chen Z, Li X, Xu J, Li J, Wang K, Wang H, Wang S, Dong X (2017) Flow
731 behavior of gas confined in nanoporous shale at high pressure: Real gas effect.
732 *Fuel* 205:173–183, DOI 10.1016/j.fuel.2017.05.055
- 733 Xiao S, Praditia T, Oladyshkin S, Nowak W (2021) Global sensitivity analysis
734 of a CaO/Ca(OH)₂ thermochemical energy storage model for parametric effect
735 analysis. *Applied Energy* 285(December 2020):116,456, DOI 10.1016/j.apenergy.
736 2021.116456
- 737 Yuan W, Pan Z, Li X, Yang Y, Zhao C, Connell LD, Li S, He J (2014) Experi-
738 mental study and modelling of methane adsorption and diffusion in shale. *Fuel*
739 117(PART A):509–519, DOI 10.1016/j.fuel.2013.09.046
- 740 Zhang Q, Su Y, Wang W, Lu M, Sheng G (2018) Gas transport behaviors in
741 shale nanopores based on multiple mechanisms and macroscale modeling. *Inter-*
742 *national Journal of Heat and Mass Transfer* 125:845–857, DOI 10.1016/j.
743 ijheatmasstransfer.2018.04.129
- 744 Ziarani AS, Aguilera R (2012) Knudsen’s Permeability Correction for Tight
745 Porous Media. *Transport in Porous Media* 91(1):239–260, DOI 10.1007/
746 s11242-011-9842-6