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Chiara Andrà[1], Guido Magnano[2], Domenico Brunetto[3] and Michela Tassone[2]

[1]University of Eastern Piedmont, Italy; chiara.andra@uniupo.it

[2]University of Torino, Italy; guido.magnano@unito.it, michela.tassone@studenti.unito.it

[3]Polytechnic of Milan, Italy; domenico.brunetto@polimi.it

We investigate the mathematical views of a sample of undergraduate mathematics students in their third year of studies, namely during the semester that precedes the Bachelor degree. We collected data about the students' mathematical views, their personal information and academic achievement, analysing the relationships among them. We also study mathematical views' influence on the appreciation of different aspects of teaching. Finally, we investigate the extent to which undergraduate studies can change a student's view of mathematics. Data are collected by means of multiple-choice demographic questions and Likert-scale affective questions. Results reveal that different views have an impact on third-year undergraduate mathematics students' achievement, as well as on the preference for more transmissive formats of the lesson, or more participative ones, based on different mathematical views.

Keywords: Beliefs, cluster analysis, gender, mathematical views, postsecondary education.

Introduction

There is an increasing number of studies on the crucial role of demographic and affective variables in tertiary mathematics learning. Gender is an example for demographic variables, especially for the *clichè* for female students of being more diligent, as this has an impact on their learning strategies and examination success (Griese, 2018). The students' views of mathematics is an example for affective variables. Roesken, Hannula and Pehkonen (2011) maintain that mathematical views comprise beliefs, wants and feelings. The authors also argue about the key role of different school backgrounds, different math curricula and different views and expectations towards mathematics in undergraduate students. To this regard, Daskalogianni and Simpson (2001) note that some beliefs, developed during school days, are carried forward in university, and this may cause difficulties. Liebendoerfer and Schukajlow (2017) consider undergraduate students' mathematics-related beliefs with respect to the nature of mathematics during the first year of undergraduate studies, and find out that different views of mathematics, hold by the students at the beginning of their studies, predict different interest towards mathematics at the end of the first and the second terms.

Many researches, like the aforementioned ones, focus on the first year of undergraduate studies and highlight difficulties in the secondary-tertiary transition. We, instead, focus on the effects of mathematical view at the end of undergraduate studies, to examine possible changes with respect to the first days at university. We firstly recall the theoretical background of our research.

Theoretical framework

Beliefs are propositions about a certain topic that are regarded as true (Philipp, 2007), and tend to form clusters as they "always come in sets or groups, never in complete independence of one another" (Green, 1971, p. 41). For this reason, according to Grigutsch, Raatz & Törner (1998), beliefs can be seen as "world views", outlining four different views: a *process-oriented* one that represents mathematics as a creative activity consisting of solving problems using different and individual ways; an *application-oriented* view that represents the utility of mathematics for real world problems as the main aspect of the nature of mathematics; a *formalist* view that represents mathematics as characterized by a strongly logical and formal structure; a *schema-oriented* view that represents mathematics as a set of calculation rules and procedures to apply for routine tasks.

Di Martino and Gregorio (2019) underline that mathematical views have an impact on an undergraduate student's choices and can prevent one to enroll in an undergraduate course only because there is some mathematics in it. Di Martino and Gregorio (2019) observe that also undergraduate *mathematics* students may face difficulties and they may develop different mathematical views that, likewise any student, may prompt them to make certain choices in place of other ones. On these premises, our research aims at answering the following four research questions. RQ1: Is there a relationship between undergraduate mathematics students' demographic variables, such as gender or school type, and their mathematical views? RQ2: Is there an effect of university studies on mathematical views? RQ3: Are there differences in students' academic achievement, such as the exam grade, with respect to their mathematical views? RQ4: Does the students' appreciation of how they are taught mathematics at university differ based on their mathematical views?

Answering to RQ1 would allow us to shed some light on the origin of different mathematical views, while answering to RQ2 would allow us to understand if and how three years of undergraduate studies in mathematics can change one's view. Answering to RQ3 and RQ4, indeed, would allow us to examine whether low- and high-achieving students tend to hold specific, yet different, world views, and the relations between such views and the aspects of teaching they mostly appreciate.

Methodology

The aim of the study is to examine the effects of undergraduate studies on students' mathematical views, in particular the achievement at the end of their studies. Hence, the participants are 93 students enrolled *in the third (and last) year* of undergraduate studies in mathematics at the University of Torino. This sample represents 37.8% of the entire population of 246 students enrolled in the third year. They participated on a voluntary basis and have been contacted via email, proposing to participate in an anonymous online survey, in May 2021. The analyzed sample is composed of 86 responses, because 7 are incomplete data.

The investigation of the mathematical views played a central role. The main idea is to figure out whether there is any correlation between affective and cognitive variables, expecially between beliefs and the students' ways of relating, in practice, to mathematics (i.e., passing exams, achieving good results, being satisfied with one's own choice of the curriculum, etcetera). To collect data on students' mathematical views, we translate into Italian a 5-points Likert-scale questionnaire developed by Erens and Eichler (2019), who design a set of 24 statements concerning the nature of mathematics based on the model proposed by Grigutsch et al. (1998). Each statement is assigned to one mathematical view. Examples of statements are: “the ideas of mathematics are of general and fundamental use to society” (A); “mathematics is a logically coherent edifice free of contradiction consisting of precisely defined terms and statements which can unequivocally be proven” (F) “there is usually more than one way to solve a task or problem in mathematics” (P), and “Mathematics consists of memorising, recalling and applying procedures” (S).

Students have been clustered with respect to mathematical views through a network analysis technique that is called *community detection* (Brunetto, Tassone and Cravero, submitted). Briefly, a network with students as nodes (i.e., with 86 nodes) has been created, and a link has been established among two nodes (i.e., two students) if both agree to the same statement concerning a mathematical view. We, thus, identified the clusters of students who are, as nodes, the most connected to each other if compared to the entire network. Four clusters have been identified and each cluster has been further characterized based on the student information collected in the survey (such as the gender, so that a cluster can have a majority of males and another one a majority of females, for example).

In order to answer RQ1, the participants were asked to declare their gender (with the possibility of choosing to not declare it), and to indicate their high school type. To this respect, in the Italian high school system there is a school type that has a strong scientific and mathematical curriculum, and usually the huge majority of undergraduate students in mathematics attended it. However, there are significant differences in the extent and depth of mathematics learning that depend on the individual teacher in each school (Lombardo, 2015), hence it makes sense to investigate the views developed by individual students even if they attended the same school type. We computed the percentages of males and females in each group defined by a specific mathematical view. Similarly, we computed the percentages of students coming from the school type with a strong mathematical background.

In order to answer RQ2, we created four 5-points Likert-scale questions and asked the participants to rate their agreement about: (a) during the undergraduate studies, one has discovered -in a positive sense- aspects of mathematics that she did not imagined before; (b) during the undergraduate studies, one has been disappointed by the way mathematics has been taught; (c) one has significantly changed her beliefs about mathematics; (d) the undergraduate studies have confirmed the view of mathematics one had when she was a freshman. We compared the averages of answers (from 1 to 5) to each question in each cluster of views of mathematics, considering an average of 3 as being neutral, an average lower (higher) than 3 as discordance (accordance) for the cluster.

In order to answer RQ3, the participants were asked to self-declare their average score at exams, because the questionnaire was anonymous, and it was not possible to recollect student data from the university official database. We computed for each group of mathematical views the percentages of students with average scores falling into one of the intervals considered “low”, “medium”, “medium high” and “excellent” in the Italian university system.

In order to answer RQ4, we borrowed the questions to be asked from a National survey on undergraduate students’ appreciation of various aspects of teaching and assessment. The questionnaire is made of four 5-points Likert-scale. We also created a series of Likert-scale questions on the distance teaching experienced during the pandemic (March 2020 - June 2021). For each mathematical view, we compared the average of answers at each question, considering an average of 3 as the group being neutral with respect to a particular aspect, an average lower than 3 as disaccordance (and we distinguished different degrees of discordance), and an average higher than 3 as accordance (at various degrees).

Results

We refer to Brunetto et al. (submitted) for more details about how the students have been associated with a mathematical view. In short, the community detection algorithm makes out 4 clusters:

- 28 students have a predominately schema-oriented view (labeled S in the following).
- 21 ones have a primarily process-oriented and a secondarily application-oriented (P-a) view.
- 20 ones hold a primarily formalist and a secondarily application-oriented (F-a) view.
- 17 students show an application-oriented (A) view.

From these data, we can see that students sorted themselves almost equally in four groups with respect to mathematical views. Having identified the clusters, the presentation of results is organized in four parts, each of one dedicated to answer one of the aforementioned research questions.

RQ1: *Is there a relationship between undergraduate mathematics students' demographic variables and their mathematical views?* Within the sample of 86 respondents, 3 students selected "other/I prefer not to reveal my gender". Table 1 shows the distribution of gender in the overall sample and within each group of students identified with the same mathematical view.

Table 1: Distribution of gender in the sample of respondents

	Overall sample	**S view**	**F-a view**	**P-a view**	**A view**
Male	35%	26%	32%	38%	53%
Female	65%	74%	68%	62%	47%

Looking at Figure 1, we infer that 29 (35%) respondents are male and 54 (65%) are female. We also notice that in cluster S there is a higher percentage (74%) of female students compared to the percentage of females (65%) in the entire sample of interviewees. Conversely, there is a significantly higher percentage (53%) of males in cluster A, compared to the distribution of gender in the entire sample of valid responses. Percentages of males and females in clusters P-a and F-a mirror the entire sample's ones with respect to gender.

As regards the secondary school type, we notice that over 75% of students come from the type of school that has a strong mathematical curriculum. Furthermore, this subsample shows to divide itself almost equally in the four groups identified for the mathematical views, but a slightly higher percentage belongs to the group of those who hold a P-a one. In the group S,, 50% of students come from secondary school types where the level of mathematics is lower, while in the other 3 groups the percentage of this kind of students resembles the general one (i.e., 25%).

RQ2: *Is there an effect of university studies on mathematical views?* Looking at Table 2, we can see that students in cluster F-a discovered positive aspects of mathematics even more than the entire sample, for which the average is high, anyway (4.5 points over 5). Only S students are less in agreement with this statement. P-a students are the most in disagreement with the statement that it was disappointing to be taught in the way it has been during the undergraduate courses. All clusters agree that beliefs about mathematics change during undergraduate studies, and all students except P-a cluster disagree that "the undergraduate studies have confirmed the view of mathematics they had when they were freshmen".

Table 2: Averages of agreement to each item in the entire sample and in each cluster. Only in case of significant difference with respect to the entire sample, the cluster average is reported

Statement	Entire sample	Cluster A	Cluster F-a	Cluster P-a	Cluster S
During the undergraduate studies, aspects of mathematics, which have not been imagined before, have been discovered in a positive sense	4.5	same of entire sample	**4.75**	same of entire sample	**3.9**
During the undergraduate studies, the way mathematics has been taught was disappointing	2.5	same of entire sample	same of entire sample	**2**	same of entire sample
My beliefs about mathematics has greatly changed during the undergraduate studies	4	same of entire sample	same of entire sample	same of entire sample	same of entire sample
The undergraduate studies have confirmed the view of mathematics they had when they were freshmen	2.7	**2.5**	**2.5**	**3**	**2**

RQ3: *Are there differences in the undergraduate mathematics students' achievement with respect to their mathematical views?* In the Italian university system, exam pass scores range from 18 (sufficient) to 30 (excellent). Scores between 18 and 20 are considered low achieving, the range 21-24 is considered medium, 25-28 medium-high and 29-30 excellent. The sample average score is summarised in Table 3. From it, we can see that the percentage of excellent students in the A group (30%) is higher than the one computed in the general sample (18%), and that the majority of excellent students tend to hold a process-oriented view. In this group, also the percentage of medium-average students (23%) is lower if compared to the general sample (40%). No excellent student belongs to the S group and the percentage of those who have a medium score average (60%) is higher if compared with the general sample (40%). F group's percentages mirror the general sample's one. These observations lead us to conclude that there are differences in the undergraduate mathematics students' scores at exams with respect to mathematical views.

Table 3: Average scores at exams and mathematical views. The percentages in brackets refer to each total reported in the last row of the table

	General sample	Application-oriented (A)	Formalist+application-oriented (F-a)	Process+application-oriented (P-a)	Schema-oriented (S)
21-24	35 (40%)	6 (35%)	7 (37%)	5 (23%)	17 (60%)
25-28	36 (42%)	6 (35%)	10 (53%)	9 (41%)	11 (40%)
29-30	15 (18%)	5 (30%)	2 (10%)	8 (36%)	0
	86	17	19	22	28

RQ4: *Does the students' appreciation of how they are taught mathematics at university differ based on their mathematical views?* Table 4 summarises the students' average appreciation of various aspects. P-a students tend to like the least step-by-step explanations and many exercises during the lessons, but to like the most being prompted to reflect. S students like the least to be provided justifications for definitions and methods, but to like the most many exercises.

Table 4: Averages of appreciation to each item in the entire sample and in each cluster. Significant differences with respect to the entire sample are in bold

Statement	Entire sample	Cluster A	Cluster F-a	Cluster P-a	Cluster S
Exhaustive, step by step explanations in frontal lessons	3.1	3.2	3	2.8	3.2
Justification of the definitions and methods	2	2.3	3	2.2	1.5
A good number of exercises	2.9	3	2.75	2	3
Stimulating reflections on a student's side	2	2	1.5	3	2

We also asked questions about the at-distance teaching that affected these students because of the pandemic, but no significant difference can be seen with respect to the different views. The sole exception is a question about the lack of opportunities to talk and interact directly with one's mates during the lockdown, to which A, F-a and P-a students show to be in accordance (rank above 3), while S ones are in discordance (rank below 3).

Discussion and preliminary conclusions

The research presented in this paper concerns a sample of undergraduate mathematics students and aims at examining the relations among their mathematical views and their academic achievement. Having considered only the third year can be seen as a limitation, but in our study this was quite unavoidable, since we needed that the students lived at least some semesters at university and we wanted that they had experience of this in pre-pandemic times.

Our results show that a schema-oriented view attracts more females, while an application-oriented view attracts more males. Among the students with a strong mathematical curriculum at secondary school, a process-oriented view is slightly predominant. This leads us to conclude that, with respect to gender and school type, mathematical views are different and, thus, it is possible that the origin of one's view depends also on their gender and/or school type. Our findings connect both to Griese's (2018) ones about females being more diligent, as the S view that seems to be predominant for females might be seen as mirroring this schema-oriented approach to learning, and to Roesken et al.'s (2011) one that students from different school types hold different views.

Furthermore, all the students in the sample strongly agree that, during the undergraduate studies, they were positively surprised by discovering aspects of mathematics they never imagined before, but for the students with a schema-oriented view such an agreement is more modest. Furthermore, for these students, it does not hold true that the undergraduate studies have confirmed the view of mathematics they had when they were freshmen, hence it seems that, for the students in this group, the mathematics that has been faced at university was different, but not always it was pleasant to discover such novelties. We know that professors who teach in the undergraduate mathematics course tend to promote a formalist, or an application-oriented, or a process-oriented view, hence we interpret S students' more modest appreciation of such a change with respect to school mathematics as an indicator of impermeability, of these students, to the novelties embodied in new views of mathematics. Liebendörfer and Schukajlow (2017) also remark that application-oriented views are positively correlated to interest in the first term, and process-oriented, schema-oriented and formalist views do not predict interest in the first or in the second terms. For the schema-oriented cluster, Libendoerfer and Schukajlow's findings seem to be confirmed also for students at the end of the third year. If we widen our focus to consider not only students' mathematical views, but also their professors' ones, we have to admit that, to our knowledge, no research study investigated the effect of different views, held by mathematics professors, on students' achievement and views in undergraduate studies. This could be a prompt for future research in this area.

With respect to the exam scores, we firstly notice that no low achieving student answered our questionnaire. This represents a limitation and, in a follow-up study, we will investigate why these students did not show up. Secondly, it emerged that no excellent student has a schema-oriented view. Conversely, most excellent students have a process- or an application-oriented view. Most students with a formalist view have a medium-high average of exam scores, whilst the majority of students with a schema-oriented view have a medium one. This leads to the conclusion that to develop a formalist, or an application- or a process-oriented view, can be considered a proxy for higher achievement at undergraduate studies. We can also notice that the students with a schema-oriented view are those who more strongly disagree with respect to exhaustive justifications of definitions and procedures during a lecture, and those who missed less the interaction with their peers during the pandemic. Somehow, these students show a sort of passive acceptance of mathematical algorithms and models and an individualistic approach to learning. We know that this kind of attitude towards mathematical learning are proxies for general weakness in mathematical achievement, as it is also confirmed in our data, being these students the ones who achieve the lowest grades.

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