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Tuning optical cavities by Möbius topology

Bruno Piccirillo & Verónica Vicuña-Hernández

The resonance wavelengths of optical Möbius strip microcavities can be continuously tuned via geometric phase manipulation by changing the thickness-to-width ratio of the strip.

Microring resonators have gained a prominent role in integrated optics[1], owing to their practical applications as on-chip field enhancers, spectral filters, fast modulators for optical communications, sensors, lasers, and more as well.

In general, optical microcavities trap light in small spatial volumes with characteristic linear dimensions on the order of a few wavelengths or smaller. Photon trapping is caused by resonant recirculation in a dielectric medium with a suitable refractive index contrast and resonant frequency spectra depend on the size of the resonator. Microring resonators – essentially optical waveguides with single or multiple closed loops – support whispering gallery modes (WGMs) with a high quality factor (Q) and small mode volume. Depending on the material and fabrication approach, free spectral ranges (FSRs) from below 0.01 to more than 1 THz, with a high degree of light confinement, can be achieved. The FSR is closely related with parameters such as ring radius, resonance wavelength, the microring's transverse cross section, and the polarization of light[2].

In conventional WGM microring cavities, such as a cylindrical ring resonator, tuning resonances requires changing the ring radius. For an array of microcavities, consequently, such a tuning entails redesigning the coupling architecture.

Now writing in *Nature Photonics*, J. Wang et al. report optical spin–orbit coupling in Möbius strip (MS) microcavities and experimentally demonstrate that their resonances can be precisely tuned by adjusting their transverse cross-section aspect ratio[3]. The latter directly affects the Pancharatnam–Berry (PB) phase shift originated by the parallel transport of polarization over the one-sided surface of the strip.

In conventional WGM microcavities, such as a cylindrical ring resonator, the electric field vector does not change with respect to the wave vector **k**: polarization experiences a trivial evolution when propagating along the closed loop of the cavity. As such, the optical spin–orbit interaction is irrelevant. In practice, by bending a strip waveguide having a thickness $T \gg W$ (W being the width of the strip) and attaching its ends so to form a cylinder, only resonant-wavelength light is enabled to build up in intensity over multiple roundtrips. At every turn, in fact, the light propagating through the loop retraces itself without reversing direction and self-reproduces within a multiple

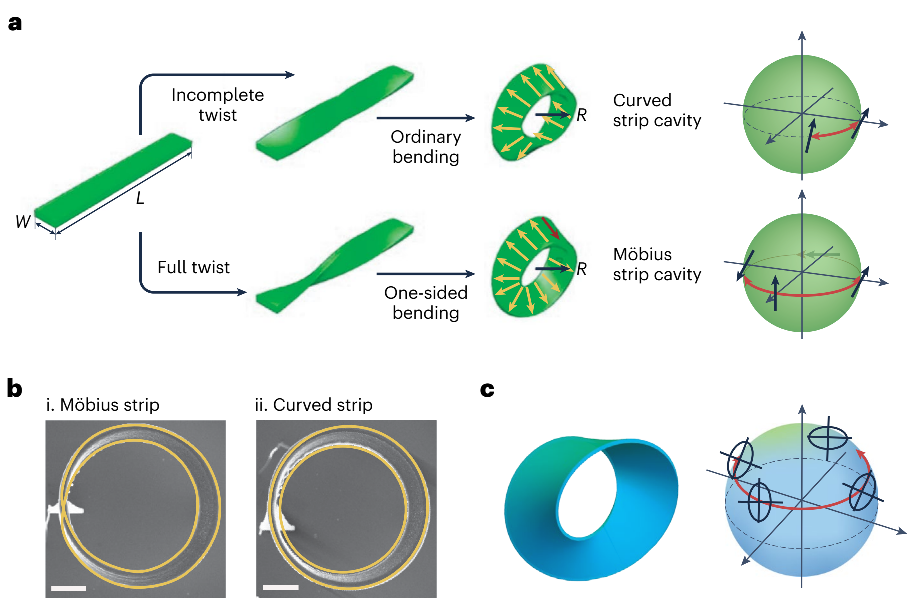

Fig. 1 | Parallel transport of 'in-plane' polarization modes in curved and Möbius strip cavities. a, A strip waveguide is bended so to attach its ends to form a curved cylinder, with ordinary double-sided surface. Though polarization changes due to parallel transport over the surface, no PB phase shift is accumulated: the closed path on the Poincaré sphere representing the polarization evolution of two superposed equatorial arcs covered back and forth in opposite directions. The strip waveguide is then folded to attach its ends after a half-twist to form a standard Möbius strip. Parallel transport of polarization produces a π rotation of the optical field, shifting the resonance spectrum by half a FSR. **b**, SEM images of fabricated Möbius- (i) and curved-strip (ii) cavities of the same size. Scale bars, 5 µm. The radius is $R \approx 10$ µm. **c**, Thick Möbius strip cavity and representation over the Poincaré sphere of the parallel transport of the major axis of the polarization ellipse of light looping within the cavity. Figure adapted from ref. [3], Springer Nature Ltd.

integer of 2π, by constructive interference. The phase shift imparted by a single loop of radius R through a strip of refractive index n is $\varphi_d = (2\pi/\lambda_0)\, n(2\pi R)$, $n(2\pi R)$ being the optical path. The self-retracing condition is:

$$2\varphi_d = \left(\frac{2\pi}{\lambda_q}\right) n\,(2\pi R) = 2\pi q, \qquad (1)$$

which implies that the roundtrip pathlength accommodates an integer number q of wavelengths.

Importantly, TE-like and TM-like polarization modes can be supported. The electric field (magnetic field) in TE(TM)-like modes, propagating along the strip, is almost zero along the propagation direction and remains parallel to the strip surface at every loop, as usual in WGMs.

In 2018, Jakob Kreismann and Martina Hentschel from the Institute of Physics at Technische Universität Illmenau in Germany theoretically studied what happens if the strip is bended so to glue up its ends after adding a half-twist, i.e. when the cavity takes the shape of a Möbius strip[4] (Fig. 1).

While the cylindrical cavity has definite sides – and it is said, therefore, to be orientable – the MS is a non-orientable surface. Without loss of generality, the implications of such topology can be analysed while focusing on TE-like or in-plane (IP) modes. In an ideal MS cavity, in the same way as in an ideal cylindrical strip cavity, the electric field, while looping, is compelled to preserve its orientation parallel to the strip surface. The optical path of the loop remains $n(2\pi R)$ as well. What is the difference, then, between the cylindrical and the Möbius strips?

The answer is that optical spin–orbit coupling is induced by MS cavity topology. In 3D space, in fact, the parallel transport of the polarization on the MS surface results in an adiabatic – slow and smooth in space – reorientation of the electric field by $\pi/2$ over half the loop. As initially pointed out by Pancharatnam[5], in 1956, and independently by Berry[6] in 1984, during light propagation, the possible evolution of polarization – running simultaneously with the dynamical phase shift associated to the optical path development – produces a phase shift $\Delta\phi$ of its own that is totally independent of the optical path, while it is exquisitely geometric in nature. The evolution of polarization, in fact, can be represented as a 1D path on the Poincaré sphere and the phase shift corresponding to a closed path is half the subtended solid angle Ω (ref. [7]), so that $\Delta\phi = \Omega/2$. Parallel transport of linear polarization on the MS surface, in a round-trip, results in a π rotation of the electric field and is represented, on the Poincaré sphere, by a closed path coincident with the equator – where all the possible linearly polarized states are located. The corresponding $\Omega/2 = \pi$ phase shift contributes, together with the dynamical phase $2\varphi_d$ in Eq. (1) to determine the resonance wavelengths of the cavity:

$$\left(\frac{2\pi}{\lambda_q}\right) n(2\pi R) = 2\pi q + \frac{\Omega}{2}. \qquad (2)$$

When $\Omega = 2\pi$, the roundtrip pathlength $n(2\pi R)$ accommodates a half-integer number $q + 1/2$ of wavelengths.

In order to measure the PB phase shift determined by Möbius topology, Wang et al. compare the resonance spectrum of an MS cavity, Eq. (2), and the resonance spectrum of a curved strip (CS) cylindrical cavity, Eq. (1), sharing the same transverse cross section dimensions, T/W, and the same radius R – and therefore the same optical path (Fig. 1a).

The microcavities were fabricated by microscale 3D printing based on two-photon polymerization of a negative photoresist IP-Dip (Fig. 1b). Indeed, several versions of the same sizes MS/CS cavities were fabricated for different values of the ratio $T/W = 0.33, 0.50, 0.57, 0.67, 0.80, 1.00$; all having the same radius $R \approx 10$ µm. As the ratio T/W increases from the minimum value 0.33 – corresponding to an almost ideally thin-strip cavity – parallel transport of the field vector becomes less and less effective due to electromagnetic inertia. Similarly to mechanical inertia – the attitude of an object to either remain at rest or continue to move at a constant velocity, unless acted upon by a force – electromagnetic inertia refers to the attitude of the electromagnetic field to preserve the essential properties it possesses without any constraint condition[8].

No polarization transport takes place when the transverse strip profile is symmetric, that is $T = W$. Consequently, when TE-like modes are envisaged, both 'in-plane' (IP) and 'out-of-plane' (OP) modes can be excited and made to travel throughout the strip with different propagation constants and, ultimately, with different effective refractive indices. Specifically, for $T/W < 0.5$, resonant light is almost elliptically polarized; while, for $T/W > 0.5$, light polarization is elliptical. In MS cavities, a PB phase shift $\Omega/2$ is accumulated also when an elliptically polarized beam loops throughout it, as the major axis of the polarization ellipse suffers parallel transport over the strip surface (Fig. 1c). However, in this case, $\Omega < 2\pi$, because the polarization evolution in a roundtrip is represented by a closed path above/below the equator on the Poincaré sphere. The spectrum of resonances, in this case, will be correspondingly shifted by a smaller FSR percentage and can still be measured by comparison with an equally sized CS cavity.

The resonant modes were characterized by measuring transmission spectra using an evanescently coupled tapered nanofibre, a popular approach for near-field delivery of light waves.

In summary, J. Wang et al. have experimentally observed that the PB phase can be generated in optical Möbius-strip microcavities. They demonstrate the possibility of 'programming' the PB phase between 0 and π, in contrast with previous theoretical predictions on optical, electronic, and magnetic Möbius-topology systems, where only a PB phase shift of π usually occurs.

MS cavities are thus an attractive future technology as they combine the opportunity for chip-scale integration with fine tuning of their resonances thanks to spin–orbit coupling providing a controllable PB phase. Importantly, the PB phase is topologically robust due to the gauge invariance of polarization parallel transport. This work is relevant to both fundamental studies and applications involving all-optical manipulation of both classical and quantum bits, and applications for photonic on-chip quantum devices.

Bruno Piccirillo [1,2] ✉ **& Verónica Vicuña-Hernández** [1,2]
[1]Department of Physics "Ettore Pancini", Università di Napoli "Federico II", Napoli, Italy. [2]INFN, National Institute for Nuclear Physics, Napoli Unit, Napoli, Italy.
✉e-mail: bruno.piccirillo@unina.it

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Competing interests

The authors declare no competing interests.