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Research Article

Revisiting the Interval and Fuzzy TOPSIS Methods: Is Euclidean Distance a Suitable Tool to Measure the Differences between Fuzzy Numbers?

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Euclidean distance (ED) calculates the distance between n-coordinate points that n equals the dimension of the space these points are located. Some studies extended its application to measure the difference between fuzzy numbers (FNs).This study shows that this extension is not logical because although an n-coordinate point and an FN are denoted the same, they are conceptually different. An FN is defined by n components; however, n is not equal to the dimension of the space where the FN is located. This study illustrates this misapplication and shows that the ED between FNs does not necessarily reflect their difference. We also revisit triangular and trapezoidal fuzzy TOPSIS methods to avoid this misapplication. For this purpose, we first defuzzify the FNs using the center of gravity (COG) method and then apply the ED to measure the difference between crisp values. We use an example to illustrate that the existing fuzzy TOPSIS methods assign inaccurate weights to alternatives and may even rank them incorrectly.

1. Introduction

The Euclidean distance (ED) measures the distance between two points in an n-dimensional space. The ED is calculated based on the Pythagorean theorem expresses that the square of the straight line distance between two points in an n-dimensional space is equal to the sum of the squares of the differences between their components. Some studies extend ED application to measure the difference between fuzzy numbers (FNs). For example, Chen [1] extends the ED to measure the difference between triangular FNs (TFNs). Also, Chen et al. [2]; Wan and Li [3]; and Seiti and Hafezalkotob [4] use the ED to calculate the difference between trapezoidal FNs (TrFNs). Some researchers like Yue [5] use the ED formula to measure the difference between interval values. An interval value, also called a gray number (GN), can be considered an FN in which all values in the given interval have a membership degree of 1.

One of the most well-known techniques extended for fuzzy environments based on ED is the technique for order preferences by similarity to the ideal solution (TOPSIS). TOPSIS is a multi-attribute decision-making (MADM) technique proposed by Hwang and Yoon [6]. This method ranks the alternatives based on their EDs from positive- and negative-ideal solutions (PIS and NIS). TOPSIS has been extended for fuzzy environments. Different fuzzy TOPSIS (FTOPSIS) methods have been developed for type-1 fuzzy sets. By type-1 fuzzy sets, we mean the fuzzy sets proposed by Zadeh [7] for the first time.

Type-1 FTOPSIS methods usually define the fuzzy PIS (FPIS) and fuzzy NIS (FNIS), and rank the alternatives based on their distances from the FPIS and FNIS. The most common distance used in the literature for this purpose is the ED (See the FTOPSIS methods proposed by Chen [1]; Chen et al. [2]; Mokhtarian and Hadi-Vencheh [8]; Huang and Peng [9]; Buyukozkan and Cifci [10]; Gok [11]; Wang et al. [12]; Baykasoglu and Golcuk [13]; and Seiti and Hafezalkotob [4]. Most of these methods have been developed for TFNs, although some researchers like Chen et al. [2] and Seiti and Hafezalkotob [4] developed FTOPSIS methods for TrFNs.

FTOPSIS methods have also been developed for other types of fuzzy sets; some methods extended the ED for measuring the differences between FNs. For example, Li et al. [14] and Chen and Hong [15] developed TOPSIS for intuitionistic and interval type-2 FNs, respectively. Ye and Li [16] proposed an extended FTOPSIS method by utilizing the possibility theory. Yu et al. [17] extended TOPSIS under the interval-valued Pythagorean fuzzy environment. Mathew et al. [18] calculated the ED between alternatives and the spherical fuzzy positive and negative ideal solutions.

This study shows that extending the ED to measure the differences between FNs, including TFNs and TrFNs, suffer from ED misapplication. This misapplication causes the difference between two FNs to be incorrectly measured. In fact, the values obtained from the ED do not necessarily represent the real differences between FNs. In other words, extending the ED to measure the difference between FNs is not logical because although an n-coordinate point and an FN are denoted the same, they are conceptually different. As a result, the techniques extending the ED to measure the differences between FNs also suffer from the same misapplication. This misapplication, in turn, causes a computational error; therefore, the results of these techniques are unreliable.

The existing FTOPSIS methods usually misapply the ED to measure the differences between FNs; therefore, they may assign the wrong weights to alternatives. To avoid this misapplication, we suggest that instead of using the ED formula, the difference between two FNs is considered equal to the difference between their centroids. To show the application of this suggestion, we present a gray TOPSIS (GTOPSIS) method that considers the difference between the centroids of GNs as their difference. We also revisit the triangular and trapezoidal FTOPSIS methods. In the revisited methods, the differences between FNs are considered equal to the differences between their centroids.

The rest of this paper is organized as follows: Section 2 illustrates the misapplication of using the ED to measure the difference between FNs. Section 3 presents a GTOPSIS method and revisits the FTOPSIS methods for TFNs and TrFNs. Section 4 provides a numerical example to compare the results of a classical FTOPSIS with its revised version proposed in this study. Section 5 gives the conclusion.

2. The ED Misapplication

This section is divided into four subsections. Subsection 2.1 reviews the formulas obtained by extending the ED to measure the differences between FNs. Subsection 2.2 analyzes the ED. Subsection 2.3 discusses and illustrates the misapplication of ED to measure the difference between FNs. Subsection 2.4 obtains the differences between FNs using two different approaches: using the ED and calculating the difference between FNs based on their centroids. This subsection compares the results of these methods and uses some numerical examples to illustrate how the ED leads to a wrong difference between FNs.

2.1. Measuring the Differences between FNs Using the ED. The ED is defined below.

Definition 1. Let $X_1 \times X_2 \times, \ldots \times X_n$ be a universal set of an n-coordinate system, and $P_1 = (x_1^1, \ldots, x_1^n)$ and $P_2 = (x_2^1, \ldots, x_2^n)$ be two points of $\mathfrak{R}^n$. The ED between these points, $d(P_1, P_2)$, is obtained as follows:

$$d(P_1, P_2) = \left(\sum_{i=1}^{n} \left(x_1^i - x_2^i \right)^2 \right)^{1/2}. \quad (1)$$

The ED has been used to measure the differences between GNs. To review this application, let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two GNs. Yue [5] computes the difference between A and B as follows:

$$d(A, B) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}. \quad (2)$$

Some studies use the ED to measure the differences between FNs. For example, Chen [1] extends the ED to measure the difference between two TFNs, denoted as $K = (k_1, k_2, k_3)$ and $L = (l_1, l_2, l_3)$, as follows:

$$d(K, L) = \sqrt{\frac{1}{3} \left[(k_1 - l_1)^2 + (k_2 - l_2)^2 + (k_3 - l_3)^2 \right]}. \quad (3)$$

Chen et al. [2] use the ED to calculate the difference between two TrFNs, $M = (m_1, m_2, m_3, m_4)$ and $N = (n_1, n_2, n_3, n_4)$, as follows:

$$d(M, N) = \sqrt{\frac{1}{4} \left[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 + (m_4 - n_4)^2 \right]}. \quad (4)$$

Note that other formulas in the literature have been developed based on the ED formula to measure the difference between FNs. For example, Wan and Li [3] and Seiti and Hafezalkotob [4] measure the difference between TrFNs M and N as follows:

$$d(M,N) = \sqrt{\frac{1}{6}\left[\left(m_1-n_1\right)^2 + 2\left(m_2-n_2\right)^2 + 2\left(m_3-n_3\right)^2 + \left(m_4-n_4\right)^2\right]}. \quad (5)$$

2.2. The ED Properties. The ED calculates the distance between two n-coordinate points located in an n-dimensional space. For this purpose, first, it calculates n distinct distances between the same dimensions of given points and then combines these distances using equation (1). For example, let $X \times Y$ be a universal set of a two-coordinate system, and $A_1 = (x_1, y_1)$ and $A_2 = (x_2, y_2)$ be two ordered points. The differences between x-values and y-values of these points, called the horizontal and vertical distances, are obtained as $|x_1 - x_2|$ and $|y_1 - y_2|$, respectively. The ED between points P_1 and P_2 is calculated by combining their horizontal and vertical distances as follows:

$$d\left(A_1, A_2\right) = \left(\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2\right)^{1/2}. \quad (6)$$

The dashed lines in Figure 1(a) show the horizontal and vertical distances between points A_1 and A_2, and the solid line indicates the ED between them. Equation (6) uses the Pythagorean theorem to calculate the ED between two points in a two-coordinate system based on their horizontal and vertical distances. According to this theorem, the square of the hypotenuse in a right triangle is equal to the sum of the squares of the other two sides.

ED has been used for different purposes, including calculating the distance between two points in an n-dimensional space and measuring the difference between two FNs. However, using the ED for some purposes may come with some flaws. We provide a definition below to determine whether the ED is an appropriate tool for the given purpose.

Definition 2. (The mutual interchange property). Let $X_1 \times X_2 \times, \ldots, \times X_n$ be a universal set of an n-coordinate system and $P_1 = (x_1^1, \ldots, x_1^n)$ and $P_2 = (x_2^1, \ldots, x_2^n)$ be two points in this system. By mutually interchanging the values of the same dimension i for these points, i.e., the values of x_1^i and x_2^i, for one or more i indexes, the new points P_3 and P_4 are created. If the real distance between the new points P_3 and P_4 is equal to the real distance between P_1 and P_2, we can use the ED for the given purpose.

When the values of the same dimensions are mutually interchanged for one or more dimensions, equation (1) always calculates the same ED. However, the real distance between the new points may change. If the real distance between the new points is changed (not changed), the ED is inconsistent (consistent) with the real situation and cannot (can) be used for the given purpose. For example, by mutually interchanging the x-values of points $A_1 = (x_1, y_1)$ and $A_2 = (x_2, y_2)$ in Figure 1(a), two new order points are created as $A_3 = (x_2, y_1)$ and $A_4 = (x_1, y_2)$ shown in Figure 1(b). The ED between A_3 and A_4 is calculated as follows:

$$d\left(A_3, A_4\right) = \left(\left(x_2 - x_1\right)^2 + \left(y_1 - y_2\right)^2\right)^{1/2}. \quad (7)$$

Equations (6) and (7) indicate that although A_3 and A_4 are two different points from A_1 and A_2, the ED between A_3 and A_4 equals the ED between A_1 and A_2. This conclusion is consistent with the real distances between the points. It implies that calculating the distance between points in a two-coordinate system comes with the mutual interchange property. Therefore, the ED is an appropriate tool for this purpose.

2.3. Misapplying the ED to Measure the Difference between FNs. This section argues that the ED is not an appropriate tool to measure the difference between FNs. In particular, we show that the misapplications of equations (2)–(5) extended based on the ED to measure the differences between GNs, TFNs, and TrFNs. For this purpose, we show that these equations do not come with the mutual interchange property; therefore, they should not be used to measure the differences between FNs. For example, by mutually interchanging the lower bounds of two GNs $A = [a_1, a_2]$ and $B = [b_1, b_2]$, two new GNs are created as $C = [b_1, a_2]$ and $D = [a_1, b_2]$, provided that $a_1, b_1 \leq a_2, b_2$. The real difference between new GNs C and D is not necessarily equal to the difference between GNs A and B(See Example 1). However, equation (2) calculates equal differences between the GNs for both cases. This implies the ED is inconsistent with measuring the difference between GNs. This conclusion can be generated for FNs. We present Examples 2 and 3 to illustrate this inconsistency for TFNs and TrFNs, respectively.

A question arises: why is ED consistent with measuring the distance between two n-coordinate points located in an n-dimensional space but inconsistent with measuring the difference between two FNs? To answer this question, consider that n naturally different components characterize an n-coordinate point; each component is measured based on a different dimension. For example, let $A = (10, 20, 30)$ be a point in a three-dimensional space. Three naturally different components characterize this point: its length, width, and height; the values of these components are 10, 20, and 30, respectively. The number of components characterizing a point equals the number of dimensions of the space in which the point is located. As a result, the ED between two points in an n-dimensional space is calculated based on their differences obtained for the same dimensions based on the Pythagorean theorem.

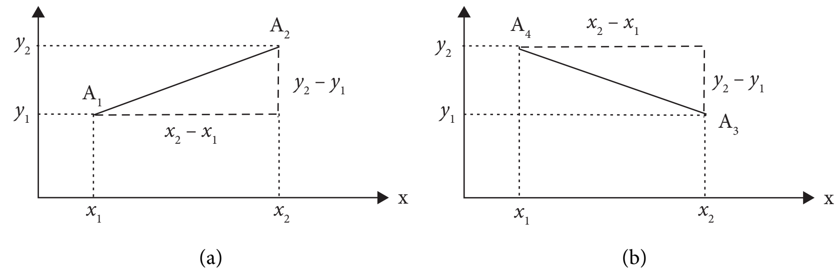

Figure 1: Mutual interchange property of ED.

However, the number of components characterizing an FN is not equal to the number of dimensions of the space in which it is located. For example, the GN, TFN, and TrFN characterized with 2, 3, and 4 components, respectively, are located in a one-dimensional space, not in two-, three-, and four-dimensional spaces. In other words, an FN is characterized by n components of the same nature; all of them are measured based on only the same dimension. For example, let $B = (10, 20, \text{and } 30)$ be a TFN representing the fuzzy set of young people. Although B is characterized by three components, i.e., 10, 20, and 30, they do not correspond to three different dimensions; these components have the same nature, i.e., age, and can be measured using only one dimension.

Both the three-coordinate point A and the TFN B are identified as (10, 20, 30). However, they are entirely conceptually different and should not be treated the same. Point A is a three-coordinate point located in a three-dimensional space, while B is a TFN located in a one-dimensional space. This clearly shows that although A and B look similar, they are conceptually quite different. Therefore, we cannot simply generalize operations that are inherently appropriate for points located in three-dimensional space to TFNs.

We can also criticize using the ED to measure the difference between FNs from the extension principle perspective. According to this principle, each fuzzy relation has been generally developed based on a crisp relation. However, the ED between FNs has been extended improperly based on the crisp ED. In other words, the ED in a crisp environment measures the distance between two n-coordinate points; each coordinate corresponds to a unique dimension. In contrast, the ED in a fuzzy environment measures the difference between two n-component FNs; all these components together correspond to only one dimension. It is clear that a point in an n-dimensional space is conceptually completely different from an FN denoted with n components.

A point in an n-dimensional space contains n heterogeneous components. For example, the three components of a point in a three-dimensional space are length, width, and height, representing three different characteristics. In contrast, an FN contains n homogenous components. For example, the three components of a TFN are the lower, middle, and upper values of the same variable; these components together represent the same characteristic. A question arises: on what logic has the ED been extended to measure the distance between one-coordinate FNs? The only answer to this question is the similarity between denoting a point in an n-dimensional space and an FN. For example, both a point in a three-dimensional space and a TFN are denoted as (a ,b, c). Despite this similarity, they are entirely different; therefore, the operations proposed for one of them cannot be extended to the other simply.

2.4. Illustrating the ED Misapplication to Measure the Difference between FNs. The previous section concluded that using the ED to measure the difference between two FNs in a one-dimensional space is meaningless. Therefore, other approaches should be used to measure the difference between FNs. One of the approaches used for this purpose is to calculate the difference between the centroids of FNs. In the following, we first review the center of gravity (COG) method used to obtain the centroid of an FN. Then, we present a theorem to show that calculating the difference between two intervals using the ED and COG methods leads to different results. It is to be noted that the proposed theorem can be extended to FNs, including TFNs and TrFNs. Next, some numerical examples are given to illustrate the ED misapplication. To show this misapplication, we compare the results of two methods between FNs: the ED and the COG methods.

Remark 1. Yager [19] proposed the COG method to obtain the centroid of the FN $\widetilde{A}$ with the membership function of $\mu_{\widetilde{A}}(x)$ as follows:

$$C(\widetilde{A}) = \frac{\int_{-\infty}^{+\infty} x.\mu_{\widetilde{A}}(x).dx}{\int_{-\infty}^{+\infty} \mu_{\widetilde{A}}(x).dx}, \tag{8}$$

where $C(\widetilde{A})$ is the centroid of $\widetilde{A}$. Equation (8) can be used to find the centroids of different types of FNs. For example, consider Figures 2(a)–2(c) representing the normal TFN $\widetilde{A} = (l, m, u)$, GN $\widetilde{B} = (l, u)$, and TrFN $\widetilde{C} = (l, m, m', u)$, respectively.

Arman et al. [20] obtained the centroids of $\widetilde{A}$, $\widetilde{B}$, and $\widetilde{C}$ using equation (8) as follows:

$$C(\widetilde{A}) = \frac{l + m + u}{3}, \tag{9}$$

$$C(\widetilde{B}) = \frac{l + u}{2}, \tag{10}$$

$$C(\widetilde{C}) = \frac{1}{3}\left[(l + m + m' + u) - \frac{(m' \times u) - (l \times m)}{(m' + u) - (l + m)}\right]. \quad (11)$$

Theorem 1. *Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two distinct GNs. Then, the difference between A and B using ED and COG leads to different results.*

Proof. The centroids of GNs A and B using equation (10) are $C(A) = a_1 + a_2/2$ and $C(B) = b_1 + b_2/2$, respectively. By contrast, assume that the difference between A and B using the ED and COG methods is the same. Therefore $\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2} = |a_1 + a_2/2 - b_1 + b_2/2| \%\Rightarrow a_1 - a_2 = b_1 - b_2$.

We know that although two new GNs are created by mutually interchanging the values of the same dimension of GNs A and B, the ED between new points is not changed. By mutually interchanging the lower bounds of A and B, two new GNs are created as $C = [b_1, a_2]$ and $D = [a_1, b_2]$, provided that $a_1, b_1 \leq a_2, b_2$. The centroids of GNs C and D using equation (10) are $C(C) = b_1 + a_2/2$ and $C(D) = a_1 + b_2/2$, respectively. Assume that the difference between C and D using the ED and COG methods leads to the same. Thus,

$$\sqrt{(b_1 - a_1)^2 + (a_2 - b_2)^2} = \left|\frac{b_1 + a_2}{2} - \frac{a_1 + b_2}{2}\right| \%\Rightarrow a_1 + a_2 = b_1 + b_2. \quad (12)$$

Since $(a_1 - a_2 = b_1 - b_2)$ and $(a_1 + a_2 = b_1 + b_2)$. So

$$\begin{aligned} &(a_1 - a_2) + (a_1 + a_2) = (b_1 - b_2) + (b_1 + b_2) \%\Rightarrow a_1 = b_1 \\ &(a_1 - a_2) - (a_1 + a_2) = (b_1 - b_2) - (b_1 + b_2) \%\Rightarrow a_2 = b_2 \end{aligned}. \quad (13)$$

This means $A = B$, and this is a contradiction. Now, the proof is completed. □

Example 1. Let $A_1 = [10, 90]$ and $B_1 = [80, 120]$ be two GNs. By interchanging the first components of these GNs, the new GNs $A_2 = [80, 90]$ and $B_2 = [10, 120]$ are created. The ED between GNs A_2 and B_2 is 76.16 using equation (2), exactly equal to that of GNs A_1 and B_1. On the other hand, the centroids of intervals A_1, B_1, A_2, and B_2 are obtained at 50, 100, 85, and 65, respectively, using equation (10). Therefore, the absolute difference between the centroids of A_1 and B_1 is $|C(A_1) - C(B_1)| = 50$, while the absolute difference between the centroids of A_2 and B_2 is $|C(A_2) - C(B_2)| = 20$. It implies that equation (2) is inconsistent with measuring the difference between GNs.

Example 2. Assume that $K_1 = (1, 3, 5)$ and $L_1 = (2, 4, 6)$ are two TFNs (Figure 3(a)). The ED between TFNs K_1 and L_1 is 1 using equation (3). L_1 is greater than K_1 because all components of L_1 are greater than their corresponding components of K_1. This is confirmed by calculating the difference between the centroids of these TFNs. The centroids of K_1 and L_1 are 3 and 4, respectively, using equation (9); therefore, the difference between them is $C(K_1) - C(L_1) = -1$.

By mutually interchanging the third components of K_1 and L_1, two new TFNs, $K_2 = (1, 3, 6)$ and $L_2 = (2, 4, 5)$, are created (Figure 3(b)). Compared to K_1 and L_1, the values of the first and second components of K_2 and L_2 have not changed; but the values of their third components have increased and decreased, respectively. Therefore, the FNs K_2 and L_2 are expected to be closer to each other compared to the FNs between K_1 and L_1. However, the ED between K_2 and L_2 is 1 using equation (3), exactly equal to the ED between K_1 and L_1. On the other hand, the centroids of K_2 and L_2 are 3.33 and 3.66, respectively, using equation (9); thus, the absolute difference between their centroids is $|C(K_2) - C(L_2)| = 0.33$ that is smaller than $|C(K_1) - C(L_1)| = 1$. It indicates that measuring the difference between TFNs based on their centroids is consistent with our expectation. This example shows that when the same elements of two TFNs are mutually interchanged, the real difference between new TFNs may change. However, equation (3) cannot discover this change and computes the same ED. As a result, the ED is not an appropriate tool for measuring the difference between two TFNs.

Example 3. Using the numbers 1 to 8, eight different two-member sets can be made, provided that the members in each set are TrFNs without using a number twice in each set. In other words, there are only eight separate sets as $A_i = \{(a, b, c, d), (e, f, g, h)\}$, $i = 1, \ldots, 8$, provided that the numbers 1 to 8 appear only once in each set, and $a < b < c < d$ and $e < f < g < h$ (See Table 1).

The ED between two TrFNs is calculated equal to 1 using equations (4) and/or (5) for all sets. It means that the ED measures the difference between two TrFNs in each set equal to 1. However, it does not reflect reality. To prove that, we obtain the centroids of TrFNs using equation (11) and then calculate the difference between the centroids of TrFNs in each set. The results are given in Table 1. This table clearly shows that the differences between the centroids better reflect the real differences between TrFNs.

3. Revisiting the Gray and Fuzzy TOPSIS Methods

This section is divided into three subsections. Subsection 3.1 presents a new gray TOPSIS method to avoid misapplying the ED distance. Subsections 3.2 and 3.2 revise two triangular and trapezoidal FTOPSIS methods, respectively, proposed by Chen [1] and Chen et al. [2]. These methods misapply the ED to measure the differences between FNs. We revise these methods to avoid this misapplication.

3.1. *The Interval value (Gray) TOPSIS.*

Consider the following gray comparison matrix.

In this matrix, $A_i\,(i = 1, \ldots, m)$ and $C_j\,(j = 1, \ldots, n)$ represent the alternative i and criterion j, respectively, and $\overline{a}_{ij} = [a_{ij}^L, a_{ij}^U]$ is the interval (gray) value of alternative i for criterion j. Here, we propose a new gray TOPSIS method consisting of six steps as follows.

Step 1. Normalizing the decision matrix

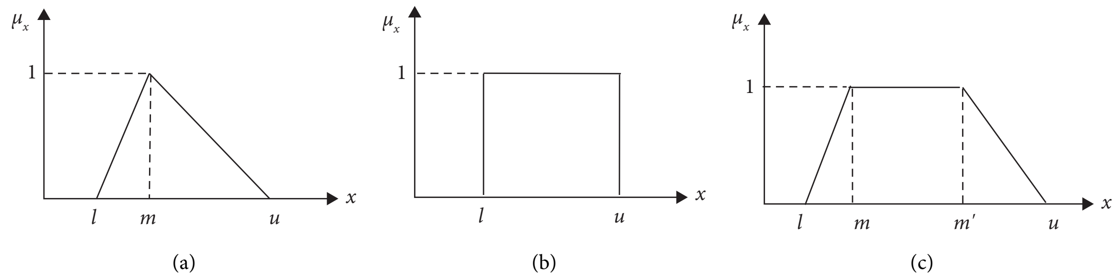

FIGURE 2: Membership function of three common FNs.

TABLE 1: Comparing TrFNs using the ED and the COG methods.

Set	TrFNs		Euclidean distance	Centroid		$C(M)-C(N)$
	TrFN M	TrFN N		$C(M)$	$C(N)$	
1	(1, 3, 5, 7)	(2, 4, 6, 8)	1	4	5	−1
2	(1, 3, 5, 8)	(2, 4, 6, 7)	1	4.296	4.714	−0.418
3	(1, 3, 6, 7)	(2, 4, 5, 8)	1	4.222	4.810	−0.588
4	(1, 4, 5, 7)	(2, 3, 6, 8)	1	4.19	4.778	−0.588
5	(2, 3, 5, 7)	(1, 4, 6, 8)	1	4.286	4.704	−0.418
6	(2, 3, 5, 8)	(1, 4, 6, 7)	1	4.583	4.417	0.166
7	(2, 3, 6, 7)	(1, 4, 5, 8)	1	4.5	4.5	0
8	(2, 4, 5, 7)	(1, 3, 6, 8)	1	4.5	4.5	0

At this step, the decision matrix $\overline{D}=[\overline{a}_{ij}]_{m\times n}$ is converted into the normalized matrix $\overline{N}=[\overline{n}_{ij}]_{m\times n}$ using the linear scale transformation as follows:

$$\begin{aligned}\overline{n}_{ij}&=\left[\frac{a_{ij}^L}{U_j^*},\frac{a_{ij}^U}{U_j^*}\right],\quad U_j^*=\max_i a_{ij}^U,\quad if\ j\in B;\\ \overline{n}_{ij}&=\left[\frac{L_j^-}{a_{ij}^L},\frac{L_j^-}{a_{ij}^U}\right],\quad L_j^-=\min_i a_{ij}^L,\quad if\ j\in C;\end{aligned}\tag{14}$$

where B and C are the sets of benefit and cost criteria, respectively, and $\overline{n}_{ij}=[n_{ij}^L,n_{ij}^U]$ is the normalized value of $\overline{a}_{ij}=[a_{ij}^L,a_{ij}^U]$.

Step 2. Weighting the normalized matrix

Let $\overline{W}=\{\overline{w}_j|j=1,\ldots,n\}$ denote the vector of interval weights of criteria as $\overline{w}_j=[\alpha_j,\beta_j]$ is the triangular fuzzy weight of criterion j. Therefore, the weighted normalized matrix $\overline{V}=[\overline{v}_{ij}]_{m\times n}$ is obtained as $\overline{V}=\overline{W}\times\overline{N}$, in which each element $\overline{v}_{ij}=\overline{w}_j.\overline{n}_{ij}$ is calculated as follows:

$$\overline{v}_{ij}=\left[v_{ij}^L,v_{ij}^U\right]=\left[\alpha_j.n_{ij}^L,\beta_j.n_{ij}^U\right],\quad \forall i,j.\tag{15}$$

Step 3. Defining the ideal solutions

The interval values of PIS and NIS for criterion j, shown as $\overline{v}_j^*=[(v_j^*)^L,(v_j^*)^U]$ and $\overline{v}_j^-=[(v_j^-)^L,(v_j^-)^U]$, respectively, can be defined as follows:

$$\begin{aligned}\overline{v}_j^*&=\{([1,1]|j\in B),[0,0]|j\in C))\},\quad j=1,\ldots,n;\\ \overline{v}_j^-&=\{([0,0]|j\in B),([1,1]|j\in C)\},\quad j=1,\ldots,n.\end{aligned}\tag{16}$$

Step 4. Transforming into crisp values based on the COG method

In this step, the gray matrix $\overline{V}$ is converted into the crisp matrix V. For this purpose, we use equation (10) to transform the gray value $\overline{v}_{ij}$ ($\forall i,j$) into the crisp value as follows:

$$v_{ij}=\frac{v_{ij}^L+v_{ij}^U}{2}.\tag{17}$$

This step also uses equation (10) to transform the interval values of PIS and NIS for criterion j into the crisp values as follows:

$$\begin{aligned}v_j^*&=\frac{\left(v_j^*\right)^L+\left(v_j^*\right)^U}{2},\\ v_j^-&=\frac{\left(v_j^-\right)^L+\left(v_j^-\right)^U}{2}.\end{aligned}\tag{18}$$

Note that if a gray PIS (or a gray NIS) is defined as (1, 1) or (0, 0), its corresponding crisp value is 1 or 0, respectively.

Step 5. Computing the EDs

The outcomes of Step 4 are a crisp weighted normalized matrix $V=[v_{ij}]_{m\times n}$ and the crisp PIS and NIS vectors, shown as $S^*=[v_j^*]_{1\times n}$ and $S^-=[v_j^*]_{1\times n}$. In this step, we compute d_i^* and d_i^-, representing the EDs between the alternative i and the crisp PIS and NIS vectors. These distances are calculated as follows:

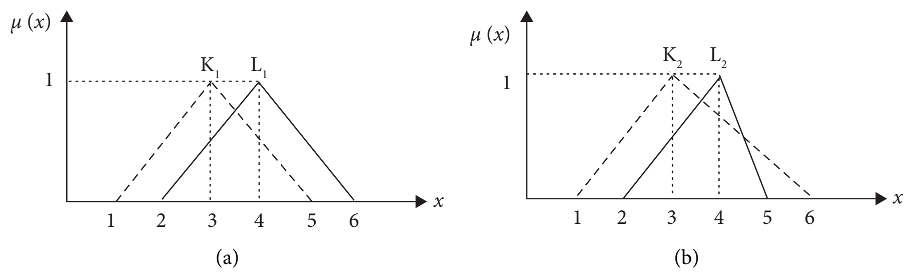

FIGURE 3: Comparing TFNs when interchanging their third components.

$$d_i^* = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^*\right)^2}, \quad i = 1,..,m,$$
$$d_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2}, \quad i = 1,..,m. \tag{19}$$

Step 6. Ranking the alternatives

This step calculates the relative closeness measures for alternatives as follows:

$$C_i^+ = \frac{d_i^-}{d_i^- + d_i^*}, \quad i = 1,\ldots,m, \tag{20}$$

C_i^+ is a utility measure. Therefore, the alternatives are ranked based on C_i^+ ascendingly.

3.2. Revisiting the Triangular FTOPSIS. Consider the following decision matrix filled with TFNs.

In this matrix, $A_i\,(i = 1,\ldots,m)$ and $C_j\,(j = 1,\ldots,n)$ represent the alternative i and criterion j, respectively, and $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$ is the triangular fuzzy value of alternative i for criterion j. The revised triangular FTOPSIS method consists of six steps as follows.

Step 1. Normalizing the decision matrix

At this step, the decision matrix $\tilde{D} = [\tilde{a}_{ij}]_{m\times n}$ is converted into the normalized matrix $\tilde{N} = [\tilde{n}_{ij}]_{m\times n}$ using the linear scale transformation as follows:

$$\tilde{n}_{ij} = \left(\frac{a_{ij}^L}{U_j^*}, \frac{a_{ij}^M}{U_j^*}, \frac{a_{ij}^U}{U_j^*}\right), \quad U_j^* = \max_i a_{ij}^U, \quad \text{if } j \in B;$$
$$\tilde{n}_{ij} = \left(\frac{L_j^-}{a_{ij}^L}, \frac{L_j^-}{a_{ij}^M}, \frac{L_j^-}{a_{ij}^U}\right), \quad L_j^- = \min_i a_{ij}^L, \quad \text{if } j \in C; \tag{21}$$

where B and C are the sets of benefit and cost criteria, respectively, and $\tilde{n}_{ij} = (n_{ij}^L, n_{ij}^M, n_{ij}^U)$ is the fuzzy normalized value of $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$.

Step 2. Weighting the normalized matrix

Let $\tilde{W} = \left\{\tilde{\omega}_j \mid j = 1,\ldots,n\right\}$ denote the vector of fuzzy weights of criteria as $\tilde{\omega}_j = (\alpha_j, \beta_j, \chi_j)$ is the triangular fuzzy weight of criterion j. Therefore, the weighted normalized matrix $\tilde{V} = [\tilde{v}_{ij}]_{m\times n}$ is obtained as $\tilde{V} = \tilde{W} \otimes \tilde{N}$, in which each element $\tilde{v}_{ij} = \tilde{\omega}_j . \tilde{n}_{ij}$ is calculated as follows:

$$\tilde{v}_{ij} = \left(v_{ij}^L, v_{ij}^M, v_{ij}^U\right) = \left(\alpha_j . n_{ij}^L, \beta_j . n_{ij}^M, \chi_j . n_{ij}^U\right), \quad \forall i, j. \tag{22}$$

Step 3. Defining the ideal solutions

The FPIS and FNIS for criterion j, shown as $\tilde{v}_j^* = ((v_j^*)^L, (v_j^*)^M, (v_j^*)^U)$ and $\tilde{v}_j^- = ((v_j^-)^L, (v_j^-)^M, (v_j^-)^U)$, respectively, can be defined as follows:

$$\tilde{v}_j^* = \{((1,1,1) \mid j \in B), (0,0,0) \mid j \in C))\}, \quad j = 1,\ldots,n;$$
$$\tilde{v}_j^- = \{((0,0,0) \mid j \in B), ((1,1,1) \mid j \in C)\}, \quad j = 1,\ldots,n. \tag{23}$$

Note that different approaches in the literature define the ideal solutions. We used the approach proposed by Chen [1]. However, the researchers can apply other approaches for future research.

Step 4. Defuzzifying based on the COG method

In this step, the weighted normalized matrix $\tilde{V}$ is converted into the crisp matrix V. For this purpose, we use equation (9) to defuzzify the triangular fuzzy value of $\tilde{v}_{ij}\,(\forall i, j)$ as follows:

$$v_{ij} = \frac{v_{ij}^L + v_{ij}^M + v_{ij}^U}{3}. \tag{24}$$

This step also uses equation (9) to defuzzify the FPIS and FNIS for criterion j as follows:

$$v_j^* = \frac{\left(v_j^*\right)^L + \left(v_j^*\right)^M + \left(v_j^*\right)^U}{3},$$
$$v_j^- = \frac{\left(v_j^-\right)^L + \left(v_j^-\right)^M + \left(v_j^-\right)^U}{3}. \tag{25}$$

Note that if a FPIS (or a FNIS) is defined as (1, 1, 1) or (0, 0, 0), its corresponding crisp value is 1 or 0, respectively.

Step 5. Computing the EDs

The outcomes of Step 4 are a crisp weighted normalized matrix $V = [v_{ij}]_{m\times n}$ and the crisp PIS and NIS vectors, shown as $S^* = [v_j^*]_{1\times n}$ and $S^- = [v_j^*]_{1\times n}$. In this step, we compute d_i^* and d_i^-, representing the EDs between the alternative i and the crisp PIS and NIS vectors. These distances are calculated as follows:

$$d_i^* = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^*\right)^2}, \quad i = 1,..,m,$$
$$d_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2}, \quad i = 1,..,m. \tag{26}$$

Step 6. Ranking the alternatives

This step calculates the relative closeness measures for alternatives as follows:

$$C_i^+ = \frac{d_i^-}{d_i^- + d_i^*}, \quad i = 1,\ldots,m, \tag{27}$$

C_i^+ is a utility measure. Therefore, the alternatives are ranked based on C_i^+ ascendingly.

3.3. Revisiting the Trapezoidal FTOPSIS. Consider the following decision matrix filled with TrFNs.

In this matrix, $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^{M'}, a_{ij}^U)$ is the trapezoidal fuzzy value of alternative i for criterion j. The revised trapezoidal FTOPSIS method consists of six steps as follows.

Step 1. Normalizing the decision matrix

This step uses the linear scale transformation to convert the decision matrix $\tilde{D} = [\tilde{a}_{ij}]_{m\times n}$ into the normalized matrix $\tilde{N} = [\tilde{n}_{ij}]_{m\times n}$ as follows:

$$\tilde{n}_{ij} = \left(\frac{a_{ij}^L}{U_j^*}, \frac{a_{ij}^M}{U_j^*}, \frac{a_{ij}^{M'}}{U_j^*}, \frac{a_{ij}^U}{U_j^*}\right), \quad U_j^* = \max_i a_{ij}^U, \quad if\ j \in B;$$
$$\tilde{n}_{ij} = \left(\frac{L_j^-}{a_{ij}^L}, \frac{L_j^-}{a_{ij}^M}, \frac{L_j^-}{a_{ij}^{M'}}, \frac{L_j^-}{a_{ij}^U}\right), \quad L_j^- = \min_i a_{ij}^L, \quad if\ j \in C; \tag{28}$$

where B and C are the sets of benefit and cost criteria, respectively, and $\tilde{n}_{ij} = (n_{ij}^L, n_{ij}^M, n_{ij}^{M'}, n_{ij}^U)$ is the fuzzy normalized value of $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^{M'}, a_{ij}^U)$.

Step 2. Weighting the normalized matrix

Assume that $\tilde{W} = \left\{\tilde{\omega}_j | j = 1,\ldots,n\right\}$ represents the vector of fuzzy weights of criteria in which $\tilde{\omega}_j = (\alpha_j, \beta_j, \beta_j', \chi_j)$ is the trapezoidal fuzzy weight of criterion j. Therefore, the weighted normalized matrix $\tilde{V} = [\tilde{v}_{ij}]_{m\times n}$ is obtained as $\tilde{V} = \tilde{W} \otimes \tilde{N}$ so that each element $\tilde{v}_{ij} = \tilde{\omega}_j.\tilde{n}_{ij}$ is calculated as follows:

$$\tilde{v}_{ij} = \left(v_{ij}^L, v_{ij}^M, v_{ij}^{M'}, v_{ij}^U\right) = \left(\alpha_j.n_{ij}^L, \beta_j.n_{ij}^M, \beta_j'.n_{ij}^{M'}, \chi_j.n_{ij}^U\right), \quad \forall i, j. \tag{29}$$

Step 3. Defining the ideal solutions

The FPIS and FNIS for criterion j, shown as $\tilde{v}_j^* = ((v_j^*)^L, (v_j^*)^M, (v_j^*)^{M'}, (v_j^*)^U)$ and $\tilde{v}_j^- = ((v_j^-)^L, (v_j^-)^M, (v_j^-)^{M'}, (v_j^-)^U)$, respectively, can be defined as follows:

$$\tilde{v}_j^* = \{((1,1,1,1)|j \in B), (0,0,0,0)|j \in C))\}, \quad j = 1,\ldots,n;$$
$$\tilde{v}_j^- = \{((0,0,0,0)|j \in B), ((1,1,1,1)|j \in C)\}, \quad j = 1,\ldots,n. \tag{30}$$

Note that we used the approach Chen et al. [2] proposed to define the ideal solutions. However, the other approaches in the literature can be used for this purpose in future research.

Step 4. Defuzzifying based on the COG method

This step converts the weighted normalized matrix $\tilde{V}$ into the crisp matrix V. For this purpose, we use equation (11) to defuzzify the trapezoidal fuzzy value of $\tilde{v}_{ij}\,(\forall i, j)$ as follows:

$$v_{ij} = \frac{1}{3}\left[\left(v_{ij}^L + v_{ij}^M + v_{ij}^{M'} + v_{ij}^U\right) - \frac{\left(v_{ij}^{M'} \times v_{ij}^U\right) - \left(v_{ij}^L \times v_{ij}^M\right)}{\left(v_{ij}^{M'} + v_{ij}^U\right) - \left(v_{ij}^L + v_{ij}^M\right)}\right]. \tag{31}$$

This step also uses equation (11) to convert the FPIS $\tilde{v}_j^*$ and FNIS $\tilde{v}_j^-$ into the crisp PIS v_j^* and the crisp NIS v_j^-, respectively, as follows:

$$v_j^* = \frac{1}{3}\left[\left(\left(v_j^*\right)^L + \left(v_j^*\right)^M + \left(v_j^*\right)^{M'} + \left(v_j^*\right)^U\right) - \frac{\left(\left(v_j^*\right)^{M'} \times \left(v_j^*\right)^U\right) - \left(\left(v_j^*\right)^L \times \left(v_j^*\right)^M\right)}{\left(\left(v_j^*\right)^{M'} + \left(v_j^*\right)^U\right) - \left(\left(v_j^*\right)^L + \left(v_j^*\right)^M\right)}\right],$$
$$v_j^- = \frac{1}{3}\left[\left(\left(v_j^-\right)^L + \left(v_j^-\right)^M + \left(v_j^-\right)^{M'} + \left(v_j^-\right)^U\right) - \frac{\left(\left(v_j^-\right)^{M'} \times \left(v_j^-\right)^U\right) - \left(\left(v_j^-\right)^L \times \left(v_j^-\right)^M\right)}{\left(\left(v_j^-\right)^{M'} + \left(v_j^-\right)^U\right) - \left(\left(v_j^-\right)^L + \left(v_j^-\right)^M\right)}\right]. \tag{32}$$

Table 2: Fuzzy triangular decision matrix.

Alternative	C_1	C_2
A	(1, 3, 5)	(3, 4, 8)
B	(2, 4, 6)	(1, 3, 5)
Weight	(0.5, 0.6, 0.6)	(0.3, 0.4, 0.4)

Table 3: The common steps of Chen's method and its revised version.

Alternative	The normalized matrix		The weighted normalized matrix	
	C_1	C_2	C_1	C_2
A	(0.167, 0.5, 0.833)	(0.375, 0.5, 1)	(0.083, 0.3, 0.583)	(0.113, 0.2, 0.5)
B	(0.333, 0.667, 1)	(0.125, 0.375, 0.625)	(0.167, 0.4, 0.7)	(0.038, 0.15, 0.313)

Table 4: The required calculation based on Chen's method.

Alternative	EDs	C_1	C_2	Summation	C_i^+	Ranking
A	d_{Aj}^* using equation (3)	0.708	0.748	1.456	0.325	1
	d_{Aj}^- using equation (3)	0.382	0.318	0.699		
B	d_{Bj}^* using equation (3)	0.618	0.841	1.459	0.317	2
	d_{Bj}^- using equation (3)	0.475	0.201	0.677		

Note that if a FPIS (or a FNIS) is defined as (1, 1, 1, 1) or (0, 0, 0, 0), its corresponding crisp value is 1 or 0, respectively.

Step 5. Computing the EDs

The outcomes of Step 4 are the crisp matrix $V = [v_{ij}]_{m\times n}$ and the crisp PIS and NIS vectors $S^* = [v_j^*]_{1\times n}$ and $S^- = [v_j^*]_{1\times n}$. This step computes d_i^* and d_i^-, representing the EDs between the alternative i and the crisp PIS and NIS vectors. These distances are calculated as follows:

$$\begin{aligned} d_i^* &= \sqrt{\sum_{j=1}^{n}\left(v_{ij}-v_j^*\right)^2}, \quad i=1,..,m, \\ d_i^- &= \sqrt{\sum_{j=1}^{n}\left(v_{ij}-v_j^-\right)^2}, \quad i=1,..,m. \end{aligned} \tag{33}$$

Step 6. Ranking the alternatives

This step calculates the relative closeness measures for alternatives as follows:

$$C_i^+ = \frac{d_i^-}{d_i^- + d_i^*}, \quad i=1,\ldots,m, \tag{34}$$

C_i^+ is a utility measure. Therefore, the alternatives are ranked based on C_i^+ ascendingly.

4. Illustrative Example

This section uses a numerical example to illustrate that the FTOPSIS method proposed by Chen [1], and its revised version presented in this study assign different weights to alternatives and may even rank them differently. Assume that we aim to rank two alternatives, A and B, considering two attributes, C_1 and C_2. The values of these alternatives for each attribute are given in Table 2 as TFNs. In this table, the weights of attributes are also given as TFNs. Both C_1 and C_2 are benefit-type attributes.

Both the FTOPSIS methods proposed by Chen and its revised version have some common steps, including normalizing the decision matrix using equations (20) and (21), and obtaining the weighted normalized matrix using equation (22). The results of these steps are given in Table 3. Since both attributes C_1 and C_2 are of the benefit type, the FPIS and FNIS are considered to be (1, 1, 1) and (0, 0, 0), respectively, for both attributes.

After the common steps, Chen's method and its revised version follow different steps described as follows.

4.1. Results Based on Chen's FTOPSIS. Chen's method obtains the EDs between the weighted normalized values and the FPIS and FNIS for each attribute, then calculates the relative closeness for alternatives, C_i^+ $(i=1,2)$, and accordingly ranks them. These calculations are given in Table 4. In this table, d_{ij}^* (d_{ij}^-), representing the ED between alternative i and FPIS (FNIS) for attribute j, is calculated using equation (3).

4.2. Results Based on the Revised FTOPSIS. The revised FTOPSIS method presented in this study converts the fuzzy weighted normalized values into crisp values using equation (9), shown in Table 5 as v_{ij}. Also, C_1 and C_2 are both benefit-type attributes; therefore, both consider (1, 1, 1) and (0, 0, 0) the FPIS and FNIS, respectively. Therefore, the crisp PIS and NIS are obtained as $A^* = \{1,1\}$ and $A^- = \{0,0\}$, respectively. Then, this method computes d_i^* and d_i^-, the EDs between the alternative i and the crisp PIS A^* and NIS A^-, respectively. Finally, it calculates the relative closeness of alternatives and accordingly ranks them. These calculations are given in Table 5.

In contrast to Chen's method, our revised FTOPSIS method ranks alternative B as the best one.

Table 5: The required calculation based on the revisited FTOPSIS.

Alternative	C_1	C_2	d^* using equation (17)	d^- using equation (18)	C_i^+	Ranking
A	$v_{A1} = 0.322$	$v_{A2} = 0.271$	0.996	0.421	0.297	2
B	$v_{B1} = 0.422$	$v_{B2} = 0.167$	1.014	0.454	0.309	1

5. Conclusion

The ED is the length of a straight line connecting two points in an n-dimensional space. It calculates n distinct differences between these points for n given dimensions and then combines them using the Pythagorean theorem. A misconception caused the ED to be applied to measure the difference between FNs. This misconception occurs because an n-coordinate point and an FN are denoted alike. Some studies considered only this similarity and generalized using the ED to measure the difference between FNs. However, they did not consider the fundamental differences between a point in an n-dimensional space and an FN denoted by n components. For example, although both a point in a three-dimensional space and a TFN are denoted exactly the same as (a, b, c), they have fundamental conceptual differences. Three components of a point in a three-dimensional space represent three completely different variables, while the three components of a TFN together represent the same variable. We showed that the ED used to measure the distance between two points in an n-dimensional space is inappropriately applied to measure the difference between two n-components FNs in a one-dimensional space.

In the literature, different fields misapply the ED to measure the difference between FNs; one of the most widely used is fuzzy MADM. This study reviewed some FTOPSIS methods that misapplyied the ED to measure the difference between FNs and revisited two FTOPSIS methods for TFNs and TrFNs to avoid this misapplication. This study also presented a GTOPSIS method that uses the COG method to measure the difference between GNs instead of the ED method. It is suggested that future research revise the other FTOPSIS methods misapplyied the ED to measure the difference between FNs. This suggestion can be generalized to other fuzzy MADM techniques. Future research can also revisit the other fields in which the ED is misapplied to measure the difference between FNs.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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