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Address of the editorial office
Goce Delcev University – Stip
Faculty of philology
Krste Misirkov 10-A
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CONTENT

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MATRIX METHOD FOR LARGE SCALE SYSTEMS ANALYSIS

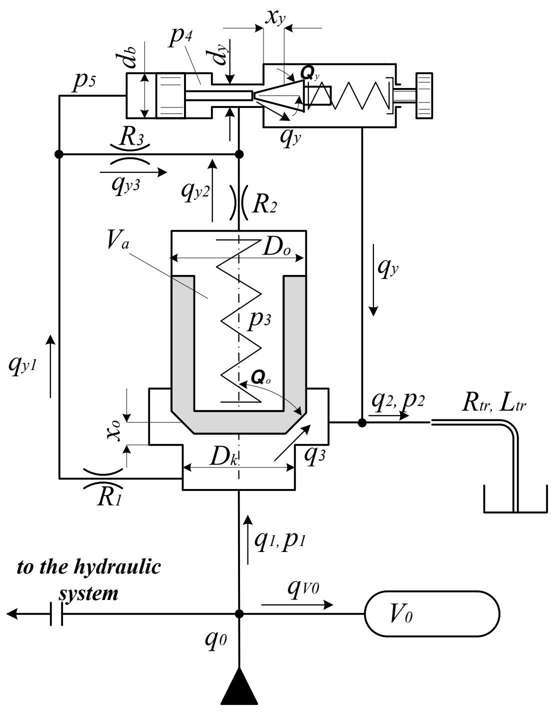
SASKO S. DIMITROV, DEJAN KRSTEV, ALEKSANDAR KRSTEV

Abstract: The mathematical model of a pilot operated pressure relief valve is described by a large system of nonlinear differential and algebraic equations. To analyze the stability condition of the valve, linearization of the mathematical model and getting the transfer function, i.e., the characteristic equation of the system is necessary. Obtaining the transfer function with elimination of the intermediate parameters is very complex and sometimes impossible. In this paper, a state space matrix method is used to reduce the large system of equation and to get the transfer function of the system.

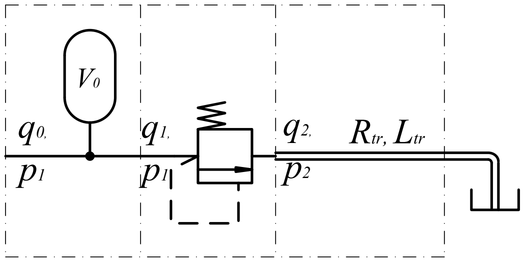
1. Introduction

The dynamic behavior of the pilot operated pressure relief valve is described by a large system of nonlinear differential and algebraic equations. To analyze the stability of the dynamic system contained of a valve, a volume of oil in front of it, and an outlet pipeline a linearization of the mathematical model of the system around the steady state it is necessary. Very often it is difficult to obtain the transfer function of large-scale systems with mathematical elimination because of the complexity of the mathematical model. To obtain the transfer function of the dynamic system, the matrix method in the state space presented in [3] is used. The transfer function and the characteristic equation of the system is found and analysis of the location of the characteristic equation roots and the stability criteria of the system has been done.

In Fig. 1 a functional diagram and its symbolic representation of the pilot operated pressure relief valve with compressible volume of oil at its inlet and return pipeline at its outlet has been shown. The system contains three successively connected subsystems. The outlet parameters of the previous subsystem are inlet parameters to the next subsystem, Fig. 1-b.



a)



b)

Figure.1 Functional diagram of the valve

2. Linear mathematical model of the valve

The mathematical model of the plane pilot operated pressure relief valve, without the inlet volume of oil and the outlet pipeline, is described by the following equation:

- *Equation of motion of the closing element of the pilot valve*

$$m_{by} \cdot \frac{d^2 x_y}{dt^2} + c_y \cdot (h_y + x_y) + r_y \cdot x_y \cdot p_{4,2} = p_{5,4} \cdot A_b + p_{4,2} \cdot A_y \tag{2.1}$$

Where $m_{by} = m_b + m_y$ – is the sum of masses of the compensating control piston and the cone of the pilot valve; c_y- the spring constant of the pilot valve; h_y- the previous spring deformation of the pilot valve; x_y- the displacement of the pilot valve; r_y – the coefficient of the hydrodynamic force of the pilot valve; $p_{4,2}$ – the pressure drop at the pilot valve; $p_{5,4}$ – the pressure drop at the orifice R_3; A_b - the area of the compensating control piston; A_y - the area of the seat of the pilot valve.

With the linearization of the equation (2.3) around the steady state, and if we introduce a dimensionless time with the substitution $t = \tau \cdot T_y$, $T_y = \sqrt{\frac{m_{by}}{c_y + r_y \cdot (p_{4,2})_0}}$ - time constant of the pilot valve, the following is obtained:

$$\ddot{X}_y = -X_y + k_{py_1} \cdot P_5 - k_{py_3} \cdot P_4 - k_{py_2} \cdot P_2 \tag{2.2}$$

Where the coefficients are: $k_{py_1} = \frac{A_b \cdot (p_{1,2})_0}{\left(c_y + r_y \cdot (p_{4,2})_0\right) \cdot x_{y,o}}$; $k_{py_2} = \frac{(A_y - r_y \cdot x_{y,o}) \cdot (p_{1,2})_0}{\left(c_y + r_y \cdot (p_{4,2})_0\right) \cdot x_{y,o}}$; $k_{py_3} = k_{py_1} - k_{py_2}$, X_y, P_5 and P_4 – the linear values of the appropriate parameters.

With introducing the substitution $\dot{X}_y = X_{vy}$, (2) is transforming into:

$$\dot{X}_y = X_{vy} \tag{2.3}$$

$$\dot{X}_{vy} = -X_y + k_{py_1} \cdot P_5 - k_{py_3} \cdot P_4 - k_{py_2} \cdot P_2 \tag{2.4}$$

X_{vy} - the linear velocity of the closing element of the pilot valve.

- *Equation of motion of the closing element of the main valve*

$$m_0 \cdot \frac{d^2 x_0}{dt^2} + c_0 \cdot (h_0 + x_0) + r_0 \cdot x_0 \cdot p_{1,2} = p_1 \cdot A_k + p_2 \cdot \Delta A - p_3 \cdot A_0 \tag{2.5}$$

Where m_o – is the mass of the closing element of the main valve; c_o- the spring constant of the main valve; h_o- the previous spring deformation of the main valve; x_o- the displacement of the main valve; r_o – the coefficient of the hydrodynamic force of the main valve; $p_{1,2}$ – the pressure drop at the main valve; A_k - the area of the main valve seat; A_0 - the area of the closing element of the main valve.

With the linearization of the equation (3) around the steady state, we obtain:

$$\ddot{X}_o = \frac{1}{T_{oy}} \left(-X_o + k_{p_1} \cdot P_1 - k_{p_2} \cdot P_2 - k_{p_3} \cdot P_3\right) \tag{2.6}$$

Where the coefficients are: $k_{p_1} = \frac{(A_k - r_o \cdot x_{o,o}) \cdot (p_{1,2})_0}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_0\right)}$; $k_{p_2} = \frac{(A_k - A_0 - r_o \cdot x_{o,o}) \cdot (p_{1,2})_0}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_0\right)}$ $k_{p_3} = \frac{A_0 \cdot (p_{1,2})_0}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_0\right)}$, X_o, P_1 and P_3 – the linear values of the appropriate parameters. The time constant of the main valve is: $T_{oy} = \frac{T_o}{T_y}$, $T_o = \sqrt{\frac{m_0}{c_0 + r_0 \cdot (p_{1,2})_0}}$.

Introducing the substitution $\dot{X}_o = X_{vo}$, (2.6) is transforming into:

$$\dot{X}_o = X_{vo} \tag{2.7}$$

$$\dot{X}_{vo} = \frac{1}{T_{oy}}\left(-X_o + k_{p_1} \cdot P_1 - k_{p_2} \cdot P_2 - k_{p_3} \cdot P_3\right) \tag{2.8}$$

X_{vo} - the linear velocity of the closing element of the main valve.

- *Equation for pressure drop in the resistance* R_1

$$p_1 = p_5 + R_{1l} \cdot q_{y_1} + R_{1m} \cdot q_{y_1}^2 + L_1 \cdot \frac{dq_{y_1}}{dt} \tag{2.9}$$

Where R_{1l} – is the linear resistance in the orifice R_1; R_{1m}- the local resistance in the orifice R_1; L_1 – the inertial resistance in the orifice R_1; q_{y_1} – the pilot oil flow.

With the linearization of the equation (2.9) around the steady state, we obtain:

$$\dot{Q}_{y_1} = \frac{1}{T_{r_1}} \cdot \left[P_1 - P_5 - R_1 \cdot Q_{y_1}\right] \tag{2.10}$$

Where $R_1 = \left(R_{1l} + 2 \cdot R_{1m} \cdot q_{y_1,0}\right) \cdot \frac{q_{y_1,0}}{(p_{1,2})_0}$ – is the dimensionless coefficient of the linear and local resistance of the orifice R_1; $T_{r_1} = \frac{L_1 \cdot q_{y_1,0}}{(p_{1,2})_0 \cdot T_y}$ - the dimensionless coefficient of the inertial resistance of the orifice R_1, Q_{y_1} – the linear value of the pilot flow q_{y_1}.

- *Equation for pressure drop in the resistance* R_3

$$p_5 = p_4 + R_{3l} \cdot q_{y_3} + R_{3m} \cdot q_{y_3}^2 + L_3 \cdot \frac{dq_{y_3}}{dt} \tag{2.11}$$

Where R_{3l} – is the linear resistance in the orifice R_3; R_{3m}- the local resistance in the orifice R_3; L_3 – the inertial resistance in the orifice R_3; q_{y_3} – the pilot oil flow through the orifice R_3.

With the linearization of the equation (2.11) around the steady state, the following is obtained:

$$\dot{Q}_{y_3} = \frac{1}{T_{r_3}} \cdot \left[P_5 - P_4 - R_3 \cdot Q_{y_3}\right] \tag{2.12}$$

Where $R_3 = \left(R_{3l} + 2 \cdot R_{3m} \cdot q_{y_1,0}\right) \cdot \frac{q_{y_1,0}}{(p_{1,2})_0}$ – is the dimensionless coefficient of the linear and local resistance of the orifice R_3; $T_{r_3} = \frac{L_3 \cdot q_{y_1,0}}{(p_{1,2})_0 \cdot T_y}$ - the dimensionless coefficient of the inertial resistance of the orifice R_3, Q_{y_3} - the linear value of the pilot flow q_{y_3}.

- *Equation of compressibility in the spring chamber in the main valve*

$$q_{y_2} = A_0 \cdot \frac{dx_0}{dt} - \frac{V_a}{K} \cdot \frac{dp_3}{dt} \tag{2.13}$$

Where V_a – is the volume of oil in the spring chamber of the main valve; K - bulk modulus of the oil.

With the linearization of the equation (2.13) around the steady state, we obtain:

$$\dot{P}_3 = \frac{1}{T_{V_a}} \cdot \left(T_{A_o} \cdot \dot{X}_o + Q_{y_2}\right) \tag{2.14}$$

$T_{V_a} = \frac{V_a}{K} \cdot \frac{(p_{1,2})_0}{q_{y_1,0} \cdot T_y}$ – is the dimensionless coefficient of the oil volume in the spring chamber in the main valve; $T_{A_o} = \frac{A_0 \cdot x_{o,o}}{q_{y_1,0} \cdot T_y}$ - the dimensionless coefficient of the closing element of the main valve.

- *Equation for pressure drop in the resistance* R_2

$$p_3 = p_4 + R_{2l} \cdot q_{y_2} + R_{2m} \cdot q_{y_2}^2 + L_2 \cdot \frac{dq_{y_2}}{dt} \tag{2.15}$$

Where R_{2l} – is the linear resistance in the orifice R_2; R_{2m}- the local resistance in the orifice R_2; L_2 – the inertial resistance in the orifice R_2; q_{y_2} – the pilot oil flow through the orifice R_2.

With the linearization of the equation (2.15) around the steady state, this is obtained:

$$\dot{Q}_{y_2} = \frac{1}{T_{r_2}} \cdot \left[P_3 - R_2 \cdot Q_{y_2} - P_4\right] \tag{2.16}$$

Where $R_2 = R_{2l} \cdot \frac{q_{y_{1,0}}}{(p_{1,2})_0}$ – is the dimensionless coefficient of the linear resistance of the orifice R_2; $T_{r_2} = \frac{L_2 \cdot q_{y_{1,0}}}{(p_{1,2})_0 \cdot T_y}$ - the dimensionless coefficient of the inertial resistance of the orifice R_2, Q_{y_2} - the linear value of the pilot flow q_{y_2}.

- *Equation of continuity in front of the compensating control piston*

$$q_{y_1} = q_{y_3} + \frac{V_b}{K} \cdot \frac{dp_5}{dt} + A_y \cdot \frac{dx_y}{dt} \tag{2.17}$$

Where V_b – is the oil volume in front of the compensating control piston.
With the linearization of the equation (2.18) around the steady state, this is obtained:

$$\dot{P}_5 = \frac{1}{T_{V_b}} \cdot \left(-T_{A_y} \cdot X_{vy} + Q_{y_1} - Q_{y_3}\right) \tag{2.18}$$

$T_{V_b} = \frac{V_b}{K} \cdot \frac{(p_{1,2})_0}{q_{y_{1,0}} \cdot T_y}$ – is the dimensionless coefficient of the oil volume in front of the compensating control piston; $T_{A_y} = \frac{A_y \cdot x_{y,0}}{q_{y_{1,0}} \cdot T_y}$ - the dimensionless coefficient of the pilot valve seat.

- *Flow equation through the pilot valve*

$$K_y \cdot x_y \cdot \sqrt{p_{4,2}} = q_{y_3} + q_{y_2} - A_y \cdot \frac{dx_y}{dt} + A_b \cdot \frac{dx_y}{dt} \tag{2.19}$$

$K_y = \mu_y \cdot \pi \cdot d_y \cdot \sqrt{\frac{2}{\rho}}$ – is the coefficient of the pilot valve.
With the linearization of the equation (2.19) around the steady state, we obtain:

$$0.5 \cdot P_{4,2} = -X_y + \left(T_{A_b} - T_{A_y}\right) \cdot X_{vy} + Q_{y_3} + Q_{y_2} \tag{2.20}$$

- *Flow equation through the main valve*

$$q_3 = \mu_0 \cdot \pi \cdot D_k \cdot x_0 \cdot \sqrt{\frac{2}{\rho} \cdot p_{1,2}} = K_0 \cdot x_0 \cdot \sqrt{p_{1,2}} \tag{2.21}$$

Where μ_0 – is the flow coefficient of the main valve; D_k - the seat diameter of the main valve;
$K_0 = \mu_0 \cdot \pi \cdot D_k \cdot \sqrt{\frac{2}{\rho}}$ – the coefficient of the main valve.
With the linearization of the equation (2.21) around the steady state, the following is obtained:

$$Q_3 = X_0 + \frac{1}{2} \cdot P_{1,2} \tag{2.22}$$

- *Equation of continuity in front of the main valve*

$$q_1 = q_3 + q_{y_1} + A_k \cdot \frac{dx_0}{dt} \tag{2.23}$$

With the linearization of the equation (2.23) around the steady state, we obtain:

$$-(1 - a_1) \cdot Q_3 = a_1 \cdot Q_{y_1} + T_{A_k} \cdot X_{v0} - Q_1 \tag{2.24}$$

Where $a_1 = \frac{q_{y_1}}{q_1}$ – is the dimensionless coefficient; $T_{A_k} = \frac{A_k \cdot x_{0,0}}{q_{1,0} \cdot T_y}$ - the dimensionless coefficient of the main valve seat.

As can be seen from the above consideration, the linear mathematical model of the pilot operated pressure relief valve with a compensating control piston is described by a large system of differential and algebraic equations. To obtain the transfer function of the valve as a system a complex mathematical transformation and elimination [2] is needed. Very often it is difficult to obtain the transfer function of large-scale systems with mathematical elimination because of the complexity of the mathematical model. To solve this problem, it is possible to use the matrix method in the state space [3].

3. State space method for large scale systems analyses

The dynamic characteristics of the valve as a subsystem of the whole system with oil volume in front of the valve and the outlet pipeline in the state space can be represented as follows:

$$\dot{\vec{x_i}} = A_i \cdot \vec{x_i} + B_i \cdot \vec{u_i}$$
$$\vec{y_i} = C_i \cdot \vec{x_i} + D_i \cdot \vec{u_i} \qquad (3.1)$$

Where x_i, u_i and y_i are phase coordinates, inlet coordinates and outlet coordinates; A_i - system matrix, B_i - control matrix, C_i - outlet matrix and D_i - linkage matrix.

To express the dynamics of the valve in the form (3.1), first the transformation is necessary of the output equations to eliminate the intermediate coordinates, as P_4, Q_3 etc. If it is expressed: the phase coordinates: $x_1 = X_y$, $x_2 = X_{vy}$, $x_3 = X_o$, $x_4 = X_{vo}$, $x_5 = Q_{y_1}$, $x_6 = Q_{y_3}$, $x_7 = P_3$, $x_8 = Q_{y_2}$ and $x_9 = P_5$; the intermediate coordinates: $v_1 = P_4$, $v_2 = P_1$ and $v_3 = Q_3$; the inlet coordinate: $u = Q_1$; and the outlet coordinate: $y = P_1 = v_2$, the mathematical model of the valve can be expressed with the system of matrix equations:

$$\dot{\vec{x}} = K \cdot \vec{x} + L \cdot \vec{v} + R \cdot \vec{u}$$
$$P \cdot \vec{v} = Q \cdot \vec{x} + W \cdot \vec{u} \qquad (3.2)$$
$$\vec{y} = N \cdot \vec{x} + M \cdot \vec{v} + S \cdot \vec{u}$$

Eliminating the vector of the intermediate coordinates in the system (3.2) matrices *A, B, C* and *D* are obtained.

$$A = K + L \cdot P^{-1} \cdot Q$$
$$B = L \cdot P^{-1} \cdot W + R$$
$$C = M \cdot P^{-1} \cdot Q + N \qquad (3.3)$$
$$D = M \cdot P^{-1} \cdot W + S$$

- **Transfer function of the valve**

According to the mathematical model (2.1) – (2.24), the matrix *K, L, R, P, Q, W, M* and *S* are:

$$Q = \begin{bmatrix} -1 & T_{A_b} - T_{A_y} & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & T_{A_k} & a_1 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad P = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & -0.5 & 1 \\ 0 & 0 & -1 \end{bmatrix}; \quad W = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix};$$

$$N = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]; \quad M = [0 \quad 1 \quad 0]; \quad S = [0];$$

$$K = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{py_1} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{oy}^2} & 0 & 0 & 0 & -\frac{k_{p_3}}{T_{oy}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R_1}{T_{r_1}} & 0 & 0 & 0 & -\frac{1}{T_{r_1}} \\ 0 & 0 & 0 & 0 & 0 & -\frac{R_3}{T_{r_3}} & 0 & 0 & -\frac{1}{T_{r_3}} \\ 0 & 0 & 0 & \frac{T_{A_o}}{T_{V_a}} & 0 & 0 & 0 & -\frac{1}{T_{V_a}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{r_2}} & -\frac{R_2}{T_{r_2}} & 0 \\ 0 & -\frac{T_{A_y}}{T_{V_b}} & 0 & 0 & \frac{1}{T_{V_b}} & -\frac{1}{T_{V_b}} & 0 & 0 & 0 \end{bmatrix}; \quad L = \begin{bmatrix} 0 & 0 & 0 \\ -k_{py_3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{k_{p_1}}{T_{oy}^2} & 0 \\ 0 & \frac{1}{T_{r_1}} & 0 \\ -\frac{1}{T_{r_3}} & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{T_{r_2}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad R = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Inserting matrices *K, L, R, P, Q, W, M* and *S* into (3.3) and then inserting matrices *A, B, C* and *D* in the state equations (3.1), it is possible to obtain the transfer function of the valve. For this purpose, a MATLAB m-script file was created for calculating the transfer function of the valve $W_{kl} = \frac{P_1}{Q_1}$.

The subject of investigation was the *Denison* pressure relief valve type *R4V 06*, shown in Fig. 2 [4]. Working pressure is *150 bar* and the flow is *30 l/min*. In front of the valve a volume of oil *0.5 l* was assumed. The valve outlet was connected with the tank by the pipeline with a diameter of *20 mm* and length *1.5 m*.

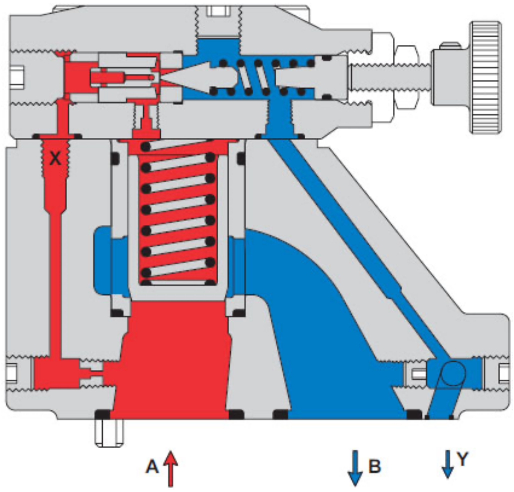


Figure 2. The analyzed valve type *R4V 06-Denison* [4]

The other parameters of the valve are: $d_y = 5\,mm$, $c_y = 250\,\frac{N}{mm}$, $d_b = 5.5\,mm$, $\mu_y = 0.7$, $d_{dr1} = d_{dr3} = 0.8\,mm$, $d_{dr2} = 0.6\,mm$, $l_{dr1} = l_{dr3} = 1\,mm$, $D_k = 28.5\,mm$, $D_o = 28\,mm$, $c_o = 7\,\frac{N}{mm}$, $h_o = 16.5\,mm$, the parameters of the oil are: $\nu = 34\,cSt$, $\rho = 890\,\frac{kg}{m^3}$ and $K = 1.45 \cdot 10^9\,N/m^2$

- **Transfer function of the rest of the system**
  - *Flow equation through the output pipeline*

$$p_2 = R_{l,tr} \cdot q_2 + L_{tr} \cdot \frac{dq_2}{dt} \tag{3.4}$$

Where $R_{l,tr}$ – is the linear resistance of the output pipeline; L_{tr} – the inertial resistance of the output pipeline; $q_2 = q_1$ - the flow in the output pipeline.
With the linearization of the equation (3.4) around the steady state, thee following is obtained:

$$P_2 = (R_T + T_T \cdot s) \cdot Q_2 \tag{3.5}$$

Where $R_T = R_{l,tr} \cdot \frac{q_{1,0}}{(p_{1,2})_0}$ – is the dimensionless coefficient of linear resistance of the output pipeline; $T_T = \frac{L_{tr} \cdot q_{1,0}}{(p_{1,2})_0 \cdot T_y}$ - the dimensionless coefficient of inertial resistance of the output pipeline, P_2, Q_2 - the linear values of the appropriate parameters.

The transfer function of the output pipeline as a subsystem is:

$$W_{tr} = \frac{P_2}{Q_2} = R_T + T_T \cdot s \tag{3.6}$$

- *Equation of continuity at inlet port of the system*

$$q_0 = q_1 + \frac{V_0}{K} \cdot \frac{dp_1}{dt} = q_1 + q_{V_0} \tag{3.7}$$

Where V_0 – is the volume of oil in front of the valve; $q_{V_0} = \frac{V_0}{K} \cdot \frac{dp_1}{dt}$ - the flow into the volume V_0.
With the linearization of the equation (3.7) around the steady state, this is obtained:

$$Q_0 = Q_1 + T_{V_0} \cdot s \cdot P_1 = Q_1 + Q_{V_0} \tag{3.8}$$

T_{V_0} - the dimensionless coefficient of the inlet volume of oil; $Q_{V_0} = T_{V_0} \cdot s \cdot P_1$ - the linear flow into the volume V_0.

The transfer function of the inlet volume of oil as a subsystem is:

$$W_{V_0} = \frac{Q_{V_0}}{P_1} = T_{V_0} \cdot s \tag{3.9}$$

The block diagram of the whole system containing the pressure relief valve, the outline pipeline, and the inlet volume of oil is presented in Fig. 3.

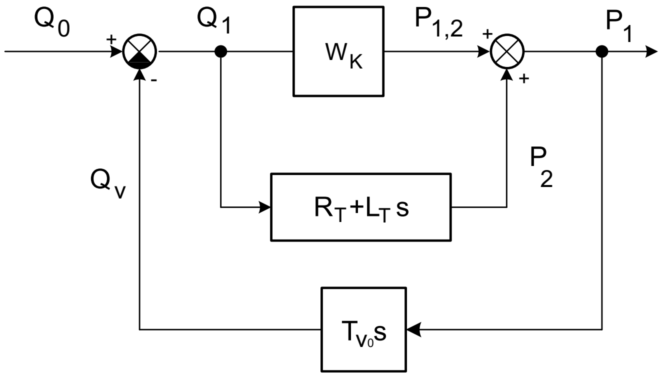


Figure 3. The block diagram of the system

- **Total transfer function of the system**

Knowing the transfer functions of the separate subsystems: valve, volume of oil in front of it and outlet pipeline; the total transfer function $W_0 = \frac{P_1}{Q_0}$, using the mentioned MATLAB m-script file, would be of 11-th order. In general form it is:

$$W_0 = \frac{a_{10}s^{10} + a_9s^9 + a_8s^8 + a_7s^7 + a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}{b_{11}s^{11} + b_{10}s^{10} + b_9s^9 + b_8s^8 + b_7s^7 + b_6s^6 + b_5s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s + b_0} \tag{3.10}$$

The characteristic equation of the system is the denominator of the equation (30) equal to zero. The frequency characteristic of the valve is shown in Fig. 4 which is analog to the frequency characteristic presented in [3].

To work the valve properly it is necessary that all the roots of the characteristic equation, i.e., all the poles of the total transfer function be negative.

The amplitude characteristic of W_k shows the presence of two resonance frequencies: the first one has a peak of $-4\ dB$ at the resonance frequencies $f_r = 70 - 90\ Hz$ and is determined by the natural frequency of the control valve. The second is above 10000 Hz, during which the main valve poppet practically does not move and the valve reacts as a local resistance with an amplitude of $6\ dB$, corresponding to $A_\omega = \frac{P_1}{Q_1} = 2$.

The phase characteristic of W_k exceeds $+90°$ in the range of frequencies between $420\ Hz$ and $600\ Hz$ and the valve in this range will work in an unstable manner at a certain volume of oil V_0 at the inlet. The presence of an inlet volume and an outlet pipeline changes the transfer function in the form $\frac{P_1(s)}{Q_0(s)} = W_0(s)$, where the phase characteristic (Phase W_0) does not exceed $+90°$ and the valve is stable at any volume of oil at the inlet. In order to obtain a transfer function $W_0(s)$ as close as possible to the plain valve $W_k(s)$, it is necessary that the volume V_0 be as small as possible and the return pipeline has very little inertial and active resistance.

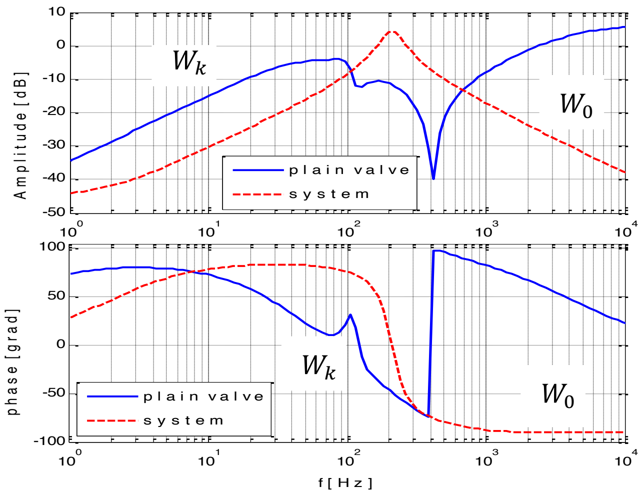


Figure 4. Frequency characteristic of the valve

4. Conclusion

The mathematical model of a pilot operated pressure relief valve is described by a large system of nonlinear differential and algebraic equations. To analyze the stability condition of the valve the linearization of the mathematical model and getting the transfer function, i.e., the characteristic equation of the system is necessary. Obtaining the transfer function with the elimination of the intermediate parameters is very complex and sometimes impossible. In this paper a state space matrix method is used to reduce the large system of equations and to get the transfer function of the system.

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Sasko Dimitrov
University of Goce Delcev,
Faculty of Mechanical Engineering,
Krste Misirkov 10-A, North Macedonia
sasko.dimitrov@ugd.edu.mk

Dejan Krstev
University of Goce Delcev,
Faculty of Mechanical Engineering,
Krste Misirkov 10-A, North Macedonia
dejan.krstev@ugd.edu.mk

Aleksandar Krstev
University of Goce Delcev,
Faculty of Computer Science,
Krste Misirkov 10-A, North Macedonia
aleksandar.krstev@ugd.edu.mk