

**GOCE DELCEV UNIVERSITY - STIP**  
**FACULTY OF COMPUTER SCIENCE**

The journal is indexed in

**EBSCO**

ISSN 2545-4803 on line

DOI: 10.46763/BJAMI

**BALKAN JOURNAL**  
**OF APPLIED MATHEMATICS**  
**AND INFORMATICS**  
**(BJAMI)**



**YEAR 2022**

**VOLUME V, Number 2**

**AIMS AND SCOPE:**

BJAMI publishes original research articles in the areas of applied mathematics and informatics.

**Topics:**

1. Computer science;
2. Computer and software engineering;
3. Information technology;
4. Computer security;
5. Electrical engineering;
6. Telecommunication;
7. Mathematics and its applications;
8. Articles of interdisciplinary of computer and information sciences with education, economics, environmental, health, and engineering.

**Managing editor**

**Mirjana Kocaleva** Ph.D.

**Zoran Zlatev** Ph.D.

**Editor in chief**

**Biljana Zlatanovska** Ph.D.

**Lectoure**

**Snezana Kirova**

**Technical editor**

**Sanja Gacov**

**Address of the editorial office**

Goce Delcev University – Stip

Faculty of philology

Krste Misirkov 10-A

PO box 201, 2000 Štip,

Republic of North Macedonia

**BALKAN JOURNAL**  
**OF APPLIED MATHEMATICS AND INFORMATICS (BJAMI), Vol 5**

**ISSN 2545-4803 on line**  
**Vol. 5, No. 2, Year 2022**

## EDITORIAL BOARD

- Adelina Plamenova Aleksieva-Petrova**, Technical University – Sofia,  
Faculty of Computer Systems and Control, Sofia, Bulgaria
- Lyudmila Stoyanova**, Technical University - Sofia , Faculty of computer systems and control,  
Department – Programming and computer technologies, Bulgaria
- Zlatko Georgiev Varbanov**, Department of Mathematics and Informatics,  
Veliko Tarnovo University, Bulgaria
- Snezana Scepanovic**, Faculty for Information Technology,  
University “Mediterranean”, Podgorica, Montenegro
- Daniela Veleva Minkovska**, Faculty of Computer Systems and Technologies,  
Technical University, Sofia, Bulgaria
- Stefka Hristova Bouyuklieva**, Department of Algebra and Geometry,  
Faculty of Mathematics and Informatics, Veliko Tarnovo University, Bulgaria
- Vesselin Velichkov**, University of Luxembourg, Faculty of Sciences,  
Technology and Communication (FSTC), Luxembourg
- Isabel Maria Baltazar Simões de Carvalho**, Instituto Superior Técnico,  
Technical University of Lisbon, Portugal
- Predrag S. Stanimirović**, University of Niš, Faculty of Sciences and Mathematics,  
Department of Mathematics and Informatics, Niš, Serbia
- Shcherbacov Victor**, Institute of Mathematics and Computer Science,  
Academy of Sciences of Moldova, Moldova
- Pedro Ricardo Morais Inácio**, Department of Computer Science,  
Universidade da Beira Interior, Portugal
- Georgi Tuparov**, Technical University of Sofia Bulgaria
- Martin Lukarevski**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Ivanka Georgieva**, South-West University, Blagoevgrad, Bulgaria
- Georgi Stojanov**, Computer Science, Mathematics, and Environmental Science Department  
The American University of Paris, France
- Iliya Guerguiev Bouyukliev**, Institute of Mathematics and Informatics,  
Bulgarian Academy of Sciences, Bulgaria
- Riste Škrekovski**, FAMNIT, University of Primorska, Koper, Slovenia
- Stela Zhelezova**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria
- Katerina Taskova**, Computational Biology and Data Mining Group,  
Faculty of Biology, Johannes Gutenberg-Universität Mainz (JGU), Mainz, Germany.
- Dragana Glušac**, Tehnical Faculty “Mihajlo Pupin”, Zrenjanin, Serbia
- Cveta Martinovska-Bande**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Blagoj Delipetrov**, European Commission Joint Research Centre, Italy
- Zoran Zdravev**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Aleksandra Mileva**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Igor Stojanovik**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Saso Koceski**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Natasa Koceska**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Aleksandar Krstev**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Biljana Zlatanovska**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Natasa Stojkovik**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Done Stojanov**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Limonka Koceva Lazarova**, Faculty of Computer Science, UGD, Republic of North Macedonia
- Tatjana Atanasova Pacemska**, Faculty of Computer Science, UGD, Republic of North Macedonia



---

## CONTENT

|                                                                                                                                                                                                       |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| <b>Sara Aneva, Marija Sterjova and Saso Gelev</b><br>SCADA SYSTEM SIMULATION FOR A PHOTOVOLTAIC ROOFTOP SYSTEM .....                                                                                  | 7   |
| <b>Elena Karamazova Gelova and Mirjana Kocaleva Vitanova</b><br>SOLVING TASKS FROM THE TOPIC PLANE EQUATION USING GEOGEBRA .....                                                                      | 17  |
| <b>Sadri Alija, Alaa Khalaf Hamoud and Fisnik Morina</b><br>PREDICTING TEXTBOOK MEDIA SELECTION USING<br>DECISION TREE ALGORITHMS .....                                                               | 27  |
| <b>Goce Stefanov And Biljana Citkuseva Dimitrovska</b><br>DESIGN OF TFT SWITCH GRID .....                                                                                                             | 35  |
| <b>Angela Tockova, Zoran Zlatev, Saso Koceski</b><br>GRAPE LEAVES DISEASE RECOGNITION USING AMAZON SAGE MAKER .....                                                                                   | 45  |
| <b>Anastasija Samardziska and Cveta Martinovska Bande</b><br>NETWORK INTRUSION DETECTION BASED ON CLASIFICATION .....                                                                                 | 57  |
| <b>Aleksandra Risteska-Kamcheski and Vlado Gicev</b><br>ANALYSIS OF THE DEFORMATION DISTRIBUTION IN THE SYSTEM<br>DEPENDING ON THE YIELD DEFORMATION.....                                             | 69  |
| <b>Aleksandra Risteska-Kamcheski and Vlado Gicev</b><br>DEPENDENCE OF ENERGY ENTERING A BUILDING FROM THE<br>INCIDENT ANGLE, THE LEVEL OF NONLINEARITY IN SOIL, AND<br>THE FOUNDATION STIFFNESS ..... | 81  |
| <b>Sijce Miovska, Aleksandar Krstev, Dejan Krstev, Sasko Dimitrov</b><br>BUSINESS PROCESS MODELING, SYSTEM ENGINEERING AND THEIR<br>APPROACH TO THEIR APPLICATION IN INDUSTRIAL CAPACITY .....        | 89  |
| <b>Sasko S. Dimitrov, Dejan Krstev, Aleksandar Krstev</b><br>MATRIX METHOD FOR LARGE SCALE SYSTEMS ANALYSIS .....                                                                                     | 99  |
| <b>Vasko Gerasimovski and Vlatko Chingoski</b><br>SMALL MODULAR NUCLEAR REACTORS – NEW PERSPECTIVES<br>IN ENERGY TRANSITION .....                                                                     | 107 |
| <b>Vesna Dimitrievska Ristovska and Petar Sekuloski</b><br>TOPOLOGICAL DATA ANALYSIS AS A TOOL FOR THE<br>CLASSIFICATION OF DIGITAL IMAGES .....                                                      | 117 |
| <b>Sasko Milev And Darko Tasevski and Blagoja Nestorovski</b><br>STRESS DISTRIBUTION ALONG THE CROSS SECTION OF THE<br>NARROWEST PART OF THE DIAPHRAGM SPRING FINGERS .....                           | 127 |



## SOLVING TASKS FROM THE TOPIC PLANE EQUATION USING GEOGEBRA

ELENA KARAMAZOVA GELOVA AND MIRJANA KOCALEVA VITANOVA

**Abstract.** Every new generation of students have difficulties with learning mathematical subjects. The success in Mathematics is getting lower and lower, not only in Macedonia but also on global level. Mathematical materials that contain only problems with solutions without their visual representation are hardly acceptable for many students. So, visualization plays an important role in the process of teaching mathematics subjects. In this paper, we will present the solution of examples of plane equation and, by using free software, visualization of that solution is made. For solving several examples, we are going to use the Computer Algebra System (CAS) in GeoGebra.<sup>1</sup>

### 1. Introduction

When learning mathematics, students have a large selection of foreign and Macedonian literature that they use in mastering the covered topics. Many of the scripts and practicums in mathematics contain many solved tasks, but for some of them the visualization is missing. With this paper we want to visually present some tasks from the topic of equations on a plane that is studied in the subject of Mathematics 1 at technical faculties. Some students have no problem to visually present their assignment without the use of software, but for others that is a problem. Software should be used to find a solution to the problem. The visually presented solved task gives a complete picture of the solution and enables permanent storage in memory. The free software that we used to achieve our goal and create this paper is the GeoGebra software. GeoGebra is a simple mathematical program that connects geometry, algebra, and analysis. One of the main features of GeoGebra is the dynamic display. Unlike a sketch on paper, which is a static model, in GeoGebra it is possible to change certain parameters in the graphical window via changes entered in the algebra window. GeoGebra's user interface is flexible and customizable as needed.

Visualization as an approach in the teaching process is not only limited to the representation of drawings to illustrate certain objects or concepts, but it is used in every step of solving mathematical problems, especially in geometry. It is thought that mathematics is more an "abstract world" that examines objects and concepts quite different from physical phenomena that rely on visualization with all its various forms and levels. In [8] authors analyzed the perceptions and attitudes about the use of ICT tools for visualization as a "modern" approach for solving geometry problems in primary schools in Macedonia. By use of information technology and existing definitions in [7], a discrete random variable is introduced, with emphasis on variables modeling probability situations with only two outcomes. Also, examples of discrete random variable with a geometric distribution are given, which is represented visually by using GeoGebra.

Papers [1] and [2] are collections of tasks on the topic of plane equation. Paper [4] presents the procedure for determining a conditional extremity of a function with two variables and by using free software the visualization of the conditional extremum is presented. In paper [6] the answer to the question "Does the technical equipment of the classrooms bring better results in mastering the teaching program by students?" is the main goal of the research. The authors determine the quality of knowledge the students get when learning the topic "Construction of triangle and quadrangle", with the use of free software GeoGebra and informatics/mathematics approach, by comparing the achieved results on the diagnostic and the final test of the experimental and the control group. The experimental group of students is learning the topic with the use of free software and constructions made on computer, and the control group in a classic way with a ruler and caliper and constructions made in notebooks.

Most of the countries worldwide have the development of the Information Society as one of their highest priorities. Education is one of the key segments for the promotion and development of the Information Society. The quality of the educational process depends directly on the information application and its communication technologies. In [9], research has been conducted to investigate the factors that affect the motivation of teachers to use ICT in their teaching and maintain it. There is more research in which the main goal is to see the importance of ICT in the teaching process in mathematical subjects. Such is the research in paper [3] in which there are two groups of students, from two Universities: Mother Teresa Skopje and Goce Delchev Stip. In the paper mathematical content will be processed (algebra, geometry, analysis) in two different ways (some with GeoGebra and on a computer, and others without visualization and GeoGebra). Then the testing will be done, the results will be compared, and a conclusion will be made.

For increasing the motivation for learning mathematics and increasing the level of knowledge [5] there is a web application <http://mathlabyrinth.azurewebsites.net>. The problems that are put on the web application relate to real-life problems the students have, so students need knowledge from secondary education mathematics to solve them.

## 2. Main results

In this part we are going to solve several examples of plane equations visually, using GeoGebra. The examples are given below.

**Example 1:** Determine the equation of the plane which passes through points A (5,1,1), B (1,2, -1) and C (1,2,3).

**Solution:**

The plane passing through the three points  $A(x_1, y_1, z_1)$ ,  $A(x_2, y_2, z_2)$ ,  $A(x_3, y_3, z_3)$  we find with

$$\begin{vmatrix} x - x_1 & y - y_1 & z - x_1 \\ x_2 - x_1 & y_2 - x_1 & z_2 - x_1 \\ x_3 - x_1 & y_3 - x_1 & z_3 - x_1 \end{vmatrix} = 0$$

So, for the point in example 1 we got the plane

$$x+4y-9=0.$$

We can get the same plane equation much faster with the GeoGebra software. For that we need a graphic display 3D in GeoGebra. Graphic display 3D in GeoGebra we get by selecting 3D Graphics



from the View menu. We enter points A, B, C through the input field and then we choose the tool plane through three points. Next, we click on the three points, and we get the plane whose equation we can read from the algebra window.

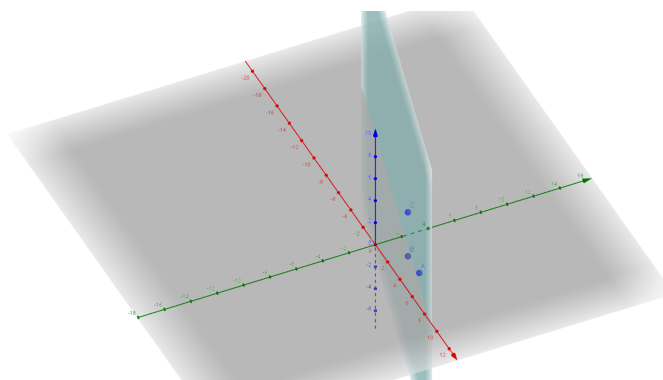


Figure 1 Graphics view for Example 1 in GeoGebra



Algebra windows is

- $A = (5, 1, 1)$
- $B = (1, 2, (1))$
- $C = (1, 2, 3)$
- $p: x + 4y = 9$

**Example 2:** Determine the equation of the plane which passes through point M (3, -3,3) and is parallel to the plane  $2x-y+z-1=0$ .

**Solution:**

The required plane passes through M (3, -3,3), so its equation as an equation of plane through a point is  $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

From the condition for parallelism between two planes we have  $\frac{A}{2} = \frac{B}{-1} = \frac{C}{1}$

For a normal vector of the required plane, we can take the vector (2, -1,1). The required plane will have the equation

$$2(x - 3) - (y + 3) + (z - 3) = 0 \Rightarrow 2x - y + z = 12.$$

We can also get the solution with the GeoGebra software:

We enter through the input field point M (3, -3,3) and the plane  $2x-y+z-1=0$ . We select the tool



parallel plane from the toolbar in the 3D graphic window, we click on the point M, then on the plane  $2x-y+z-1=0$  and we get in the algebraic window

- $M = (3, (3), 3)$
- $eq1: 2x - y + z = 1$
- $p: 2x - y + z = 12$

and in graphic 3D window

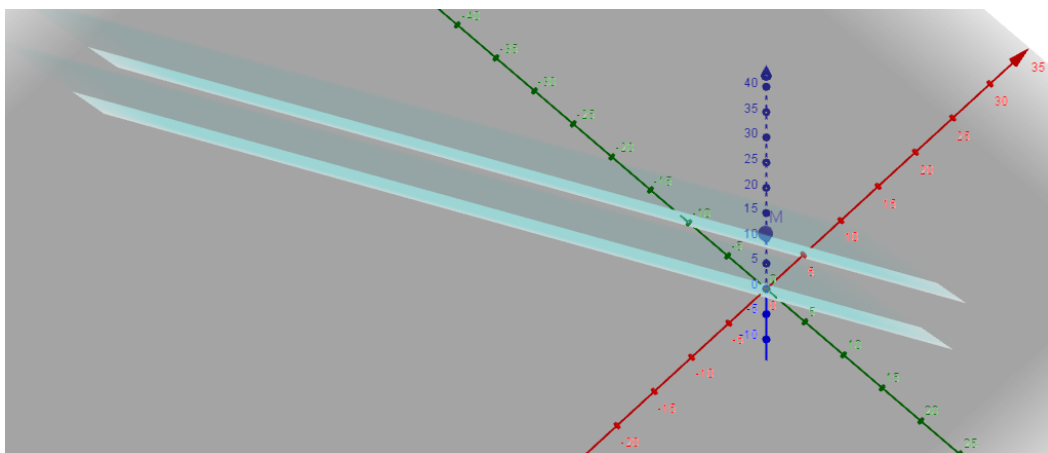


Figure 2 Example 2 in GeoGebra

From the algebra window we can read the equation of the newly obtained plane  $2x - y + z = 12$ , obtained under the given conditions.

**Example 3:** Find the intersection point of the planes:

$$x-y+z-1=0$$


$$2x-z-1=0$$

$$x+3y+2z-6=0.$$

**Solution:**

We solve a system of three linear equations with three unknowns which has one solution  $M(1,1,1)$ , which is the intersection point of the three plains.

The intersection point can also be obtained with the GeoGebra software using a computer algebra system CAS. We open the computer algebraic system CAS by selecting CAS from the View menu. We enter the equations one by one in the CAS window. Then we select the rows in which the equations are

entered and then select the tool . In row 4 we get the solution:

| CAS                   |                                   |
|-----------------------|-----------------------------------|
| 1                     | $x-y+z-1=0$                       |
| <input type="radio"/> | $\rightarrow x - y + z - 1 = 0$   |
| 2                     | $2x-z-1=0$                        |
| <input type="radio"/> | $\rightarrow 2x - z - 1 = 0$      |
| 3                     | $x+3y+2z-6=0$                     |
| <input type="radio"/> | $\rightarrow x + 3y + 2z - 6 = 0$ |
| 4                     | $\{\$1, \$2, \$3\}$               |
| <input type="radio"/> | Реш: $\{\{x = 1, y = 1, z = 1\}$  |

Figure 3 Example 3

If we enter the equations through the input field of the GeoGebra window in the graphic 3D, we can see the intersection point

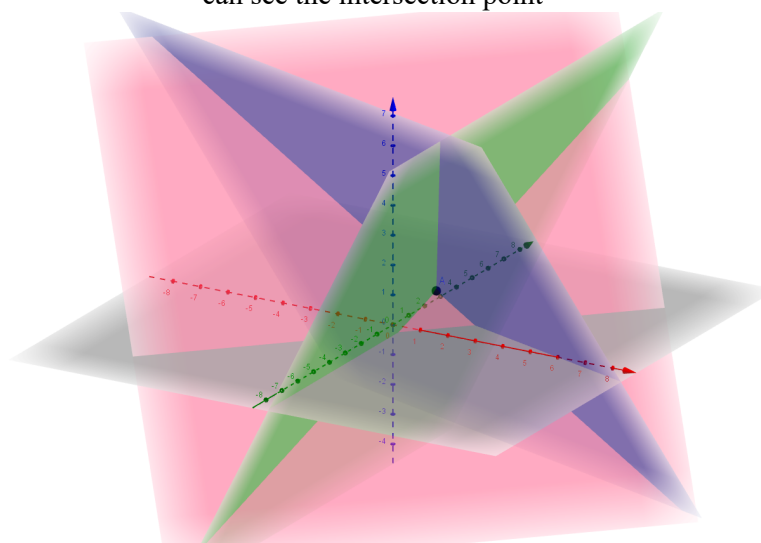
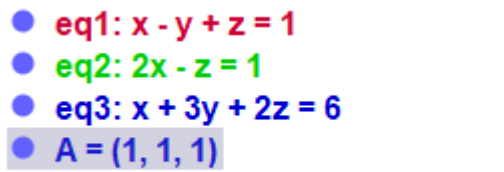


Figure 4 Example 3 in GeoGebra

Algebra window is



From Figure 4 we can see which equation is shown with which color.

**Example 4:** The points A (1, -1,1), B (4, 2,2) and C (-2,3, 4) are given:

- a) Determine the equation of the plane passing through those points.
- b) Determine the distance between the obtained plane and the point Q (3, -3, 3).

**Solution:**

a) First, we enter the points through the input field, and then we select the tool the plane through three points. Next, we click on the three points, and we get the plane whose equation we can read from the algebra window. The plane has the equation  $5x - 12y + 21z = 38$

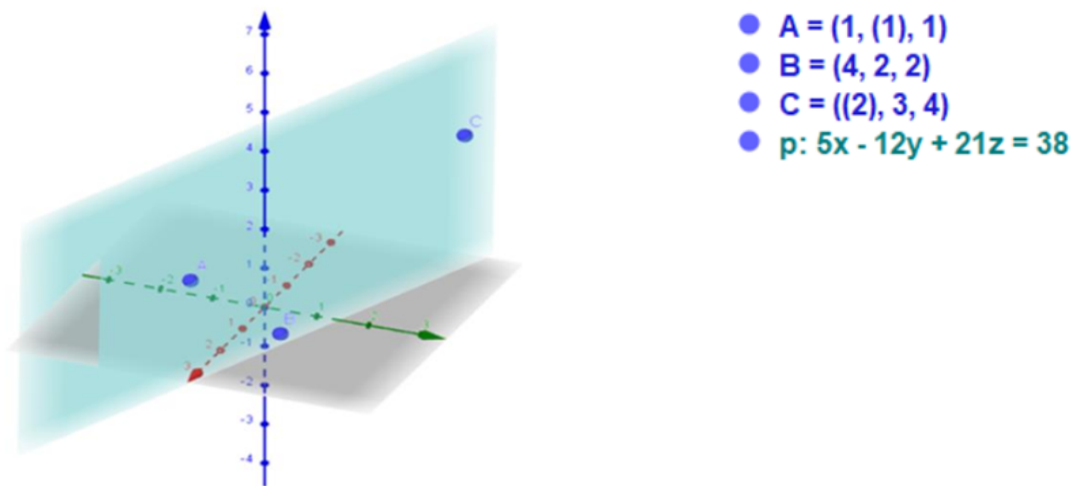


Figure 5 Example 4 in GeoGebra

b) To determine the distance between point Q and the plane, we open the CAS window. Here we enter the points A, B and C, then the command Plane (A, B, C) and at the end we enter the command Distance ((3, -3,3),  $5x - 12y + 21z = 38$ ) as is shown in Figure below, after which we get what the distance is.

| CAS |                                                                               |
|-----|-------------------------------------------------------------------------------|
| 1   | A=(1,-1,1)<br>→ (1, -1, 1) = (1, -1, 1)                                       |
| 2   | B=(4,2,2)<br>→ (4, 2, 2) = (4, 2, 2)                                          |
| 3   | C=(-2,3,4)<br>→ (-2, 3, 4) = (-2, 3, 4)                                       |
| 4   | Рамнина(A,B,C)<br>→ x · 5 + y (-12) + z · 21 =                                |
| 5   | Расстояние((3,-3,3), x*5+y(-12)+z*21):<br>→ $38 \cdot \frac{\sqrt{610}}{305}$ |
| 6   |                                                                               |

From here we can see that the required distance is  $38 \frac{\sqrt{610}}{305}$ .

**Example 5:** In what mutual relation is the plane  $2x + 2y - 7z + 1 = 0$  with

- a)  $x + y - 2z + 2 = 0$       b)  $4x + 4y - 14z + 2 = 0$

We can solve the problem with the help of the GeoGebra software by drawing the plane in the graphic window under a) and b) and we will directly see their mutual position.

**Solution:** a) Because  $\vec{n}_1 = (2, 2, -7)$  and  $\vec{n}_2 = (1, 1, -2)$  and  $\frac{2}{1} = \frac{2}{1} \neq \frac{7}{2}$  it follows that the plains intersect. There are also  $\vec{n}_1 \cdot \vec{n}_2 = 2 + 2 + 14 = 18 \neq 0$ , so the planes are not even mutually normal.

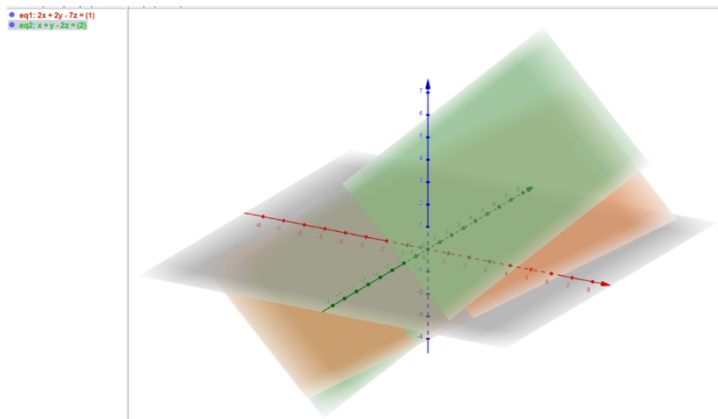


Figure 6 Example 5 a) in GeoGebra

One plane is in green color and the other is in brown color. We can see that the planes intersect.

b) Because  $\frac{4}{2} = \frac{4}{2} = \frac{-14}{-7} = \frac{2}{1}$  it follows that the planes coincide.

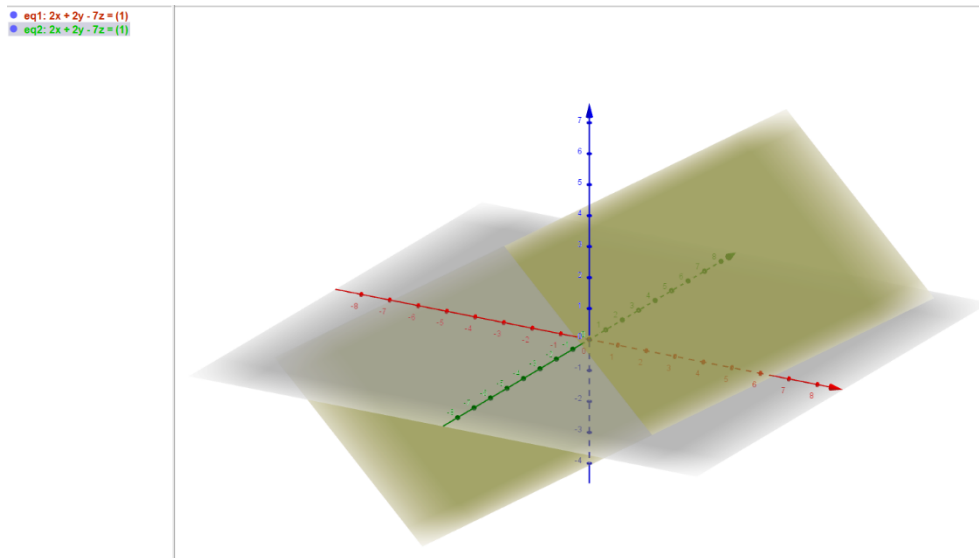


Figure 7 Example 5 b) in GeoGebra

**Example 6:** Find the angle between the planes  $2x - y + 2z - 1 = 0$  and  $x + y + 4z - 5 = 0$ .

**Solution:** The first plane is normal to  $\vec{n}_1 = (2, -1, 2)$  and the second to  $\vec{n}_2 = (1, 1, 4)$ . Let  $\varphi$  be the angle between the planes. According to the formula for the angle between two planes,  $\cos \varphi = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{\sqrt{2}}{2}$ . So,  $\varphi = \arccos \frac{\sqrt{2}}{2} = \frac{\pi}{4}$ .

We can also get the angle directly with the GeoGebra software. For this purpose, we enter the two planes through the input field. Then, through the input field, we enter the command angle (plane, plane), i.e., we enter

Angle ( $2x - y + 2z - 1 = 0$ ,  $x + y + 4z - 5 = 0$ ) and we get

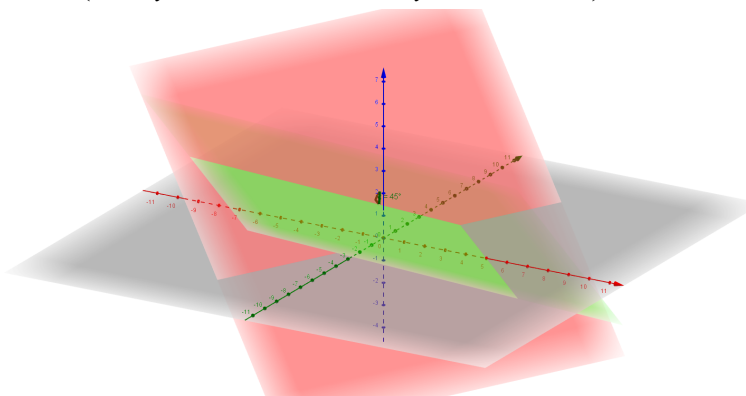


Figure 8 Example 6 in GeoGebra

and in the algebra window we get

- eq1:  $2x - y + 2z = 1$
- eq2:  $x + y + 4z = 5$
- $\alpha = 45^\circ$

from where we can see that the angle between the planes is  $45^\circ$ . The angle is also shown in the geometric window (Figure 8).

**Example 7:** Find the distance between the parallel planes  $2x + y - 2z + 3 = 0$  and  $2x + y - 2z - 12 = 0$ .

**Solution:** Distance between parallel planes is the distance from an arbitrary point of one plane to the other plane. We choose a point from the plane  $2x + y - 2z - 12 = 0$ . For  $x=0$  and  $z=0$  we get  $y=12$ . So, point  $M(0,12,0)$  lies on the plane. Now,

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} = \frac{15}{3} = 5.$$

we can get the solution with the GeoGebra software. We enter the equations of the planes through the input field. In the 3D graphic we can see the planes. To calculate the distance through the input field, we enter the command distance (plane, plane). So, in 3D Graphics view we get:

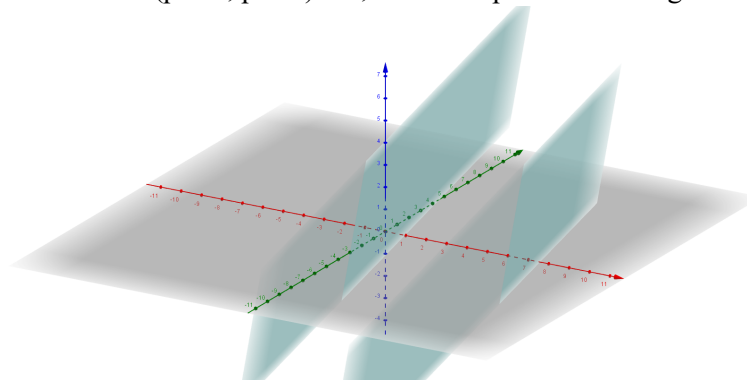
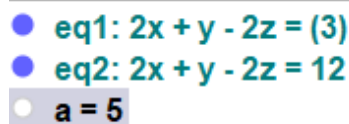


Figure 9 Example 7 in GeoGebra

In Algebra windows we get



from where we can see that the distance is 5.

### 3. Conclusion

The global problem of difficulty in mastering mathematics by students needs to be eradicated. To that end, more needs to be done. In this paper we offer a way to increase interest in mathematics with using math learning software for visual representation which will help for greater curiosity and increased motivation to work and solve problems. The software can help students to check if their solution is correct, to get a solution in advance that will guide them to solve the task etc. So, students will be motivated to study mathematics and to achieve better results. Thus, we encourage students to use free software, such as GeoGebra, while solving math problems.

## References

- [1] Goracinova Ilieva L., Zlatanovska B., Koceva Lazarova L. (2019), *Збирка задачи од математика 1 за технички факултети*, ISBN 978-608-244-682-0.
- [2] Мисајлески З. (2019), *Решени задачи по векторска и линеарна алгебра*, ISBN 978-9989-43-435-8
- [3] Trifunov Z., Jusufi Z. T., Karamazova E., Atanasova-Pacemska T. (2019), *Importance of Visualization in Math Problems at the Universities*, South East European Journal of Sustainable Development, 3 (1), pp. 17-23. ISSN 2545-4463
- [4] Trifunov Z., Jusufi Zenku T., Atanasova-Pacemska T. (2018) *Conditioned Extreme, Determination and Visualization*, Istraživanje matematičkog obrazovanja, 10 (18), pp. 33-40. ISSN 1986-518X / 2303-4890 (print)
- [5] Trifunov Z. (2017) *Increasing the motivation of the students to learn mathematics*, International Journal of Mathematics Trends and Technology, 48 (1), pp. 1-5. ISSN 2231-5373
- [6] Trifunov Z., Atanasova-Pacemska T., Pacemska S. (2015) *Use of free software in mathematics classes and presenting the results of the research*, Зборник радова са трећег међународног научног скупа одржаног 14–15. јуна 2014. године на Факултету педагошких наука Универзитета у Крагујевцу, 19. pp. 203-212.
- [7] Trifunov Z., Karamazova E. (2014) *Visualization of discrete random variables*, Proceedings of the V Congress of mathematicians of Macedonia, pp. 108-116. ISSN 978-9989-646-69-0
- [8] Atanasova-Pacemska T., Gunova V., Koceva Lazarova L., Pacemska S. (2016) Visualization of the Geometry Problems in Primary Math Education -Needs and Challenges-, IMO-Istraživanje matematičkog obrazovanja, 8 (15), pp. 33-37. ISSN 2303-4890 (print), 1986-518X (on line)
- [9] Vitanova V., Atanasova-Pacemska T. (2014) Determining the basic motivational factors of teachers to use ICT in their teaching using factor analysis, IMVI Open Mathematical Education Notes, 4, pp. 1-8. ISSN 2303-4882 (p), 1840-4383(o)

Elena Karazmova Gelova  
University of Goce Delcev, Stip,  
Faculty of computer science, Krste Misirkov 10-A,  
North Macedonia  
E-mail address: elena.gelova@ugd.edu.mk

Mirjana Kocaleva Vitanova  
Goce Delcev University of Stip,  
Faculty of computer science, Krste Misirkov 10-A,  
North Macedonia  
E-mail address: mirjana.kocaleva@ugd.edu.mk

