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CONTENT

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SOLVING TASKS FROM THE TOPIC PLANE EQUATION USING GEOGEBRA

ELENA KARAMAZOVA GELOVA AND MIRJANA KOCALEVA VITANOVA

Abstract. Every new generation of students have difficulties with learning mathematical subjects. The success in Mathematics is getting lower and lower, not only in Macedonia but also on global level. Mathematical materials that contain only problems with solutions without their visual representation are hardly acceptable for many students. So, visualization plays an important role in the process of teaching mathematics subjects. In this paper, we will present the solution of examples of plane equation and, by using free software, visualization of that solution is made. For solving several examples, we are going to use the Computer Algebra System (CAS) in GeoGebra. [1]

1. Introduction

When learning mathematics, students have a large selection of foreign and Macedonian literature that they use in mastering the covered topics. Many of the scripts and practicums in mathematics contain many solved tasks, but for some of them the visualization is missing. With this paper we want to visually present some tasks from the topic of equations on a plane that is studied in the subject of Mathematics 1 at technical faculties. Some students have no problem to visually present their assignment without the use of software, but for others that is a problem. Software should be used to find a solution to the problem. The visually presented solved task gives a complete picture of the solution and enables permanent storage in memory. The free software that we used to achieve our goal and create this paper is the GeoGebra software. GeoGebra is a simple mathematical program that connects geometry, algebra, and analysis. One of the main features of GeoGebra is the dynamic display. Unlike a sketch on paper, which is a static model, in GeoGebra it is possible to change certain parameters in the graphical window via changes entered in the algebra window. GeoGebra's user interface is flexible and customizable as needed.

Visualization as an approach in the teaching process is not only limited to the representation of drawings to illustrate certain objects or concepts, but it is used in every step of solving mathematical problems, especially in geometry. It is thought that mathematics is more an "abstract world" that examines objects and concepts quite different from physical phenomena that rely on visualization with all its various forms and levels. In [8] authors analyzed the perceptions and attitudes about the use of ICT tools for visualization as a "modern" approach for solving geometry problems in primary schools in Macedonia. By use of information technology and existing definitions in [7], a discrete random variable is introduced, with emphasis on variables modeling probability situations with only two outcomes. Also, examples of discrete random variable with a geometric distribution are given, which is represented visually by using GeoGebra.

Papers [1] and [2] are collections of tasks on the topic of plane equation. Paper [4] presents the procedure for determining a conditional extremity of a function with two variables and by using free software the visualization of the conditional extremum is presented. In paper [6] the answer to the question “Does the technical equipment of the classrooms bring better results in mastering the teaching program by students?” is the main goal of the research. The authors determine the quality of knowledge the students get when learning the topic “Construction of triangle and quadrangle”, with the use of free software GeoGebra and informatics/mathematics approach, by comparing the achieved results on the diagnostic and the final test of the experimental and the control group. The experimental group of students is learning the topic with the use of free software and constructions made on computer, and the control group in a classic way with a ruler and caliper and constructions made in notebooks.

Date: July 15, 2022.
Keywords. plane, equation, visualization, GeoGebra.

Most of the countries worldwide have the development of the Information Society as one of their highest priorities. Education is one of the key segments for the promotion and development of the Information Society. The quality of the educational process depends directly on the information application and its communication technologies. In [9], research has been conducted to investigate the factors that affect the motivation of teachers to use ICT in their teaching and maintain it. There is more research in which the main goal is to see the importance of ICT in the teaching process in mathematical subjects. Such is the research in paper [3] in which there are two groups of students, from two Universities: Mother Teresa Skopje and Goce Delchev Stip. In the paper mathematical content will be processed (algebra, geometry, analysis) in two different ways (some with GeoGebra and on a computer, and others without visualization and GeoGebra). Then the testing will be done, the results will be compared, and a conclusion will be made.

For increasing the motivation for learning mathematics and increasing the level of knowledge [5] there is a web application http://mathlabyrinth.azurewebsites.net. The problems that are put on the web application relate to real-life problems the students have, so students need knowledge from secondary education mathematics to solve them.

2. Main results

In this part we are going to solve several examples of plane equations visually, using GeoGebra. The examples are given below.

Example 1: Determine the equation of the plane which passes through points A (5,1,1), B (1,2, -1) and C (1,2,3).

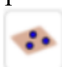
Solution:

The plane passing through the three points $A(x_1, y_1, z_1)$, $A(x_2, y_2, z_2)$, $A(x_3, y_3, z_3)$ we find with

$$\begin{vmatrix} x - x_1 & y - y_1 & z - x_1 \\ x_2 - x_1 & y_2 - x_1 & z_2 - x_1 \\ x_3 - x_1 & y_3 - x_1 & z_3 - x_1 \end{vmatrix} = 0$$

So, for the point in example 1 we got the plane

x+4y-9=0.

We can get the same plane equation much faster with the GeoGebra software. For that we need a graphic display 3D in GeoGebra. Graphic display 3D in GeoGebra we get by selecting 3D Graphics from the View menu. We enter points A, B, C through the input field and then we choose the tool  plane through three points. Next, we click on the three points, and we get the plane whose equation we can read from the algebra window.

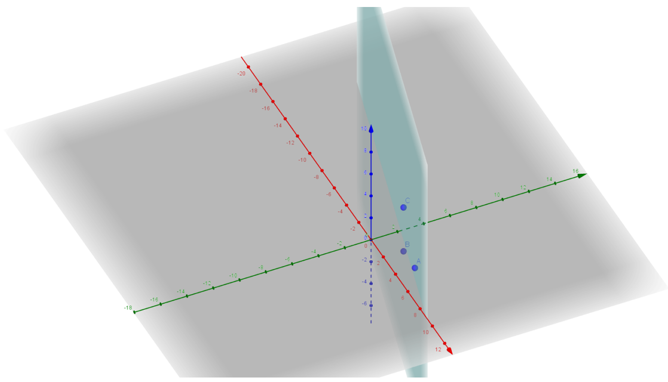
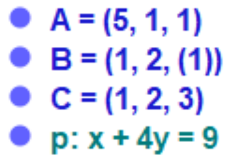


Figure 1 *Graphics view for Example 1 in GeoGebra*

Algebra windows is



Example 2: Determine the equation of the plane which passes through point M (3, -3,3) and is parallel to the plane 2x-y+z-1=0.

Solution:

The required plane passes through M (3, -3,3), so its equation as an equation of plane through a point is $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

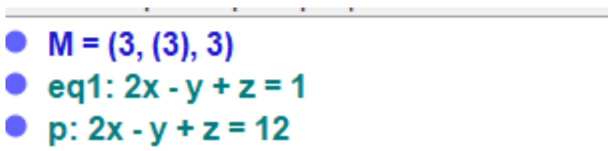
From the condition for parallelism between two planes we have $\frac{A}{2}=\frac{B}{-1}=\frac{C}{1}$

For a normal vector of the required plane, we can take the vector (2, -1,1). The required plane will have the equation

$2(x-3)-(y+3)+(z-3)=0 \Rightarrow 2x-y+z=12.$

We can also get the solution with the GeoGebra software:

We enter through the input field point M (3, -3,3) and the plane 2x-y+z-1=0. We select the tool parallel plane from the toolbar in the 3D graphic window , we click on the point M, then on the plane 2x-y+z-1=0 and we get in the algebraic window



and in graphic 3D window

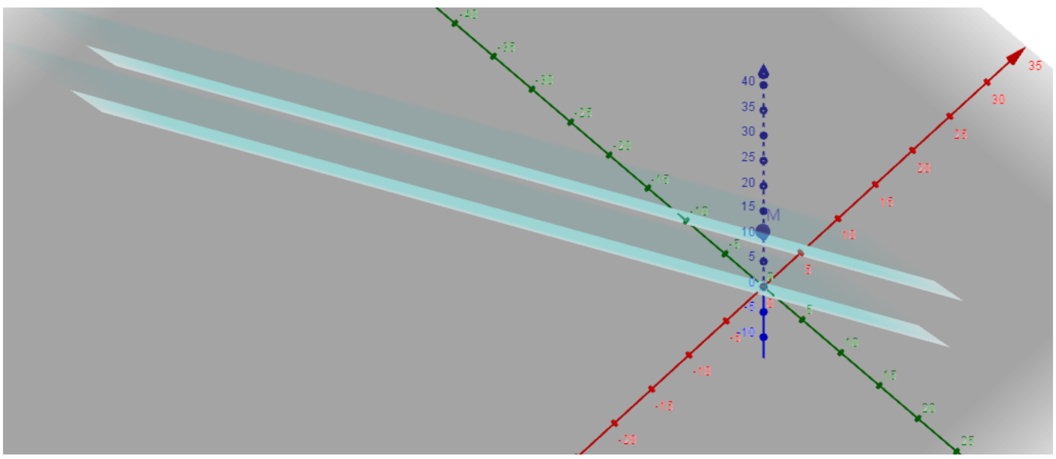
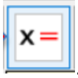


Figure 2 *Example 2 in GeoGebra*

From the algebra window we can read the equation of the newly obtained plane $2x-y+z=12$, obtained under the given conditions.

Example 3: Find the intersection point of the planes:
x-y+z-1=0
2x-z-1=0
x+3y+2z-6=0.
Solution:
We solve a system of three linear equations with three unknowns which has one solution M (1,1,1), which is the intersection point of the three plains.
The intersection point can also be obtained with the GeoGebra software using a computer algebra system CAS. We open the computer algebraic system CAS by selecting CAS from the View menu. We enter the equations one by one in the CAS window. Then we select the rows in which the equations are entered and then select the tool 

. In row 4 we get the solution:

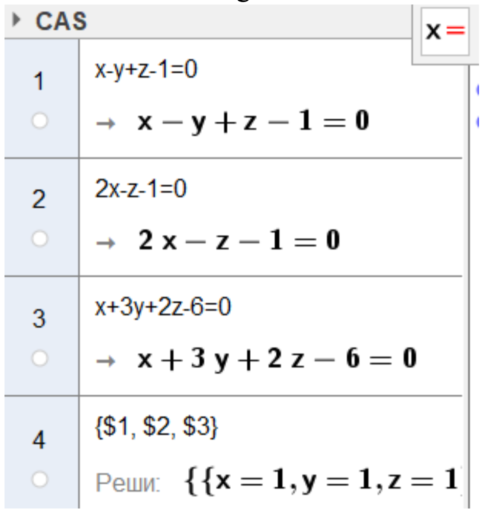


Figure 3 *Example 3*

If we enter the equations through the input field of the GeoGebra window in the graphic 3D, we can see the intersection point

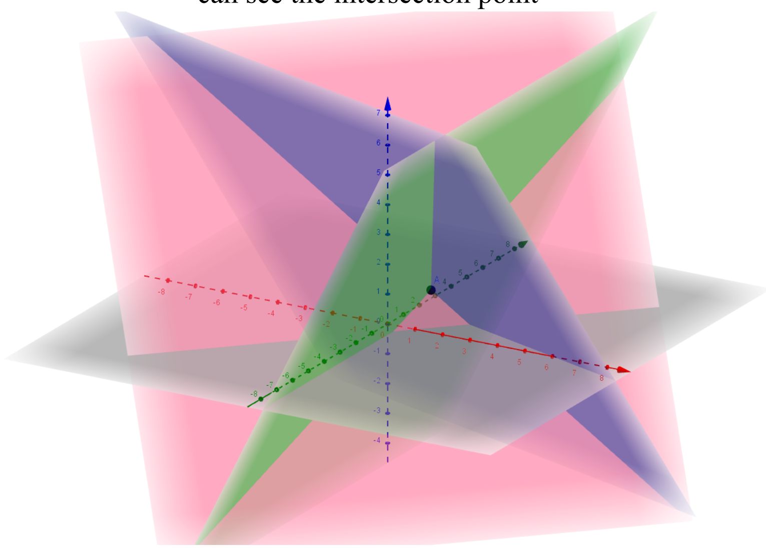
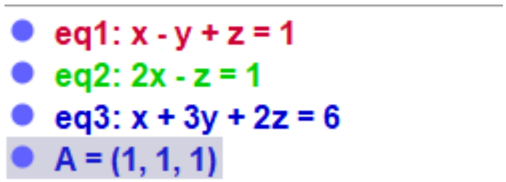


Figure 4 *Example 3 in GeoGebra*

Algebra window is



From Figure 4 we can see which equation is shown with which color.

Example 4: The points A (1, −1,1), B (4, 2,2) and C (−2,3, 4) are given:
a) Determine the equation of the plane passing through those points.
b) Determine the distance between the obtained plane and the point Q (3, −3, 3).

Solution:
a) First, we enter the points through the input field, and then we select the tool the plane through three points. Next, we click on the three points, and we get the plane whose equation we can read from the algebra window. The plane has the equation 5x-12y+21z=38

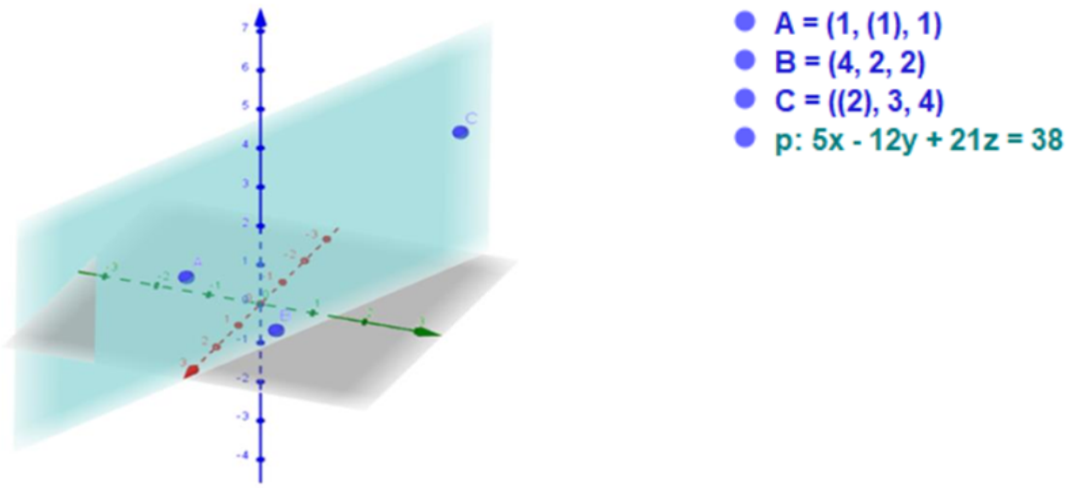
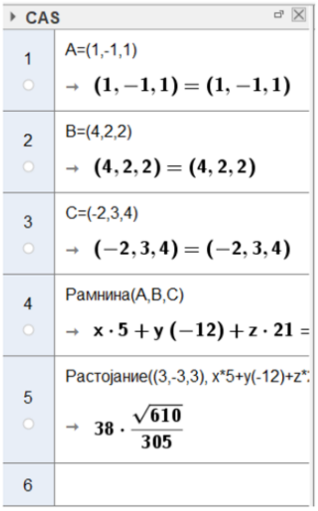


Figure 5 *Example 4 in GeoGebra*

b) To determine the distance between point Q and the plane, we open the CAS window. Here we enter the points A, B and C, then the command Plane (A, B, C) and at the end we enter the command Distance ((3, -3,3), 5x -12y + 21z = 38) as is shown in Figure below, after which we get what the distance is.



From here we can see that the required distance is $38\frac{\sqrt{610}}{305}$.

Example 5: In what mutual relation is the plane $2x+2y-7z+1=0$ with

a) $x+y-2z+2=0$ b) $4x+4y-14z+2=0$

We can solve the problem with the help of the GeoGebra software by drawing the plane in the graphic window under a) and b) and we will directly see their mutual position.

Solution: a) Because $\vec{n_1}=(2,2,-7)$ and $\vec{n_2}=(1,1,-2)$ and $\frac{2}{1}=\frac{2}{1}\neq\frac{7}{2}$ it follows that the plains intersect. There are also $\vec{n_1}\vec{n_2}=2+2+14=18\neq 0$, so the planes are not even mutually normal.

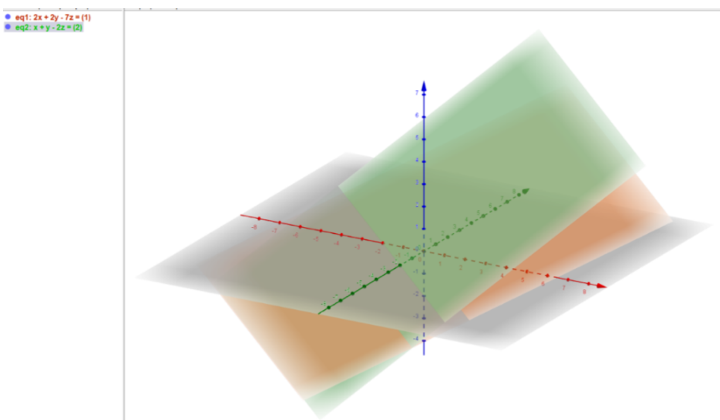


Figure 6 *Example 5 a) in GeoGebra*

One plane is in green color and the other is in brown color. We can see that the planes intersect.

b) Because $\frac{4}{2}=\frac{4}{2}=\frac{-14}{-7}=\frac{2}{1}$ it follows that the planes coincide.

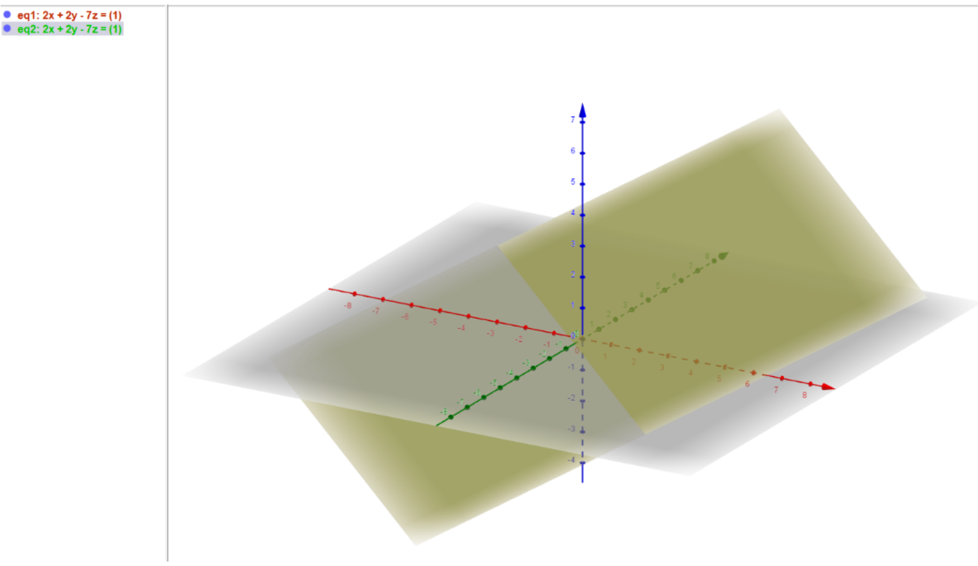


Figure 7 *Example 5 b) in GeoGebra*

Example 6: Find the angle between the planes $2x-y+2z-1=0$ and $x+y+4z-5=0$.

Solution: The first plane is normal to $\overrightarrow{n_1}=(2,-1,2)$ and the second to $\overrightarrow{n_2}=(1,1,4)$. Let φ be the angle between the planes. According to the formula for the angle between two planes, $\cos\varphi=\frac{\left|\overrightarrow{n_1}\overrightarrow{n_2}\right|}{\left|\overrightarrow{n_1}\right|\left|\overrightarrow{n_2}\right|}=\frac{\sqrt{2}}{2}$. So, $\varphi=\arccos\frac{\sqrt{2}}{2}=\frac{\pi}{4}$.

We can also get the angle directly with the GeoGebra software. For this purpose, we enter the two planes through the input field. Then, through the input field, we enter the command angle (plane, plane), i.e., we enter

Angle $(2x-y+2z-1=0,\quad x+y+4z-5=0)$ and we get

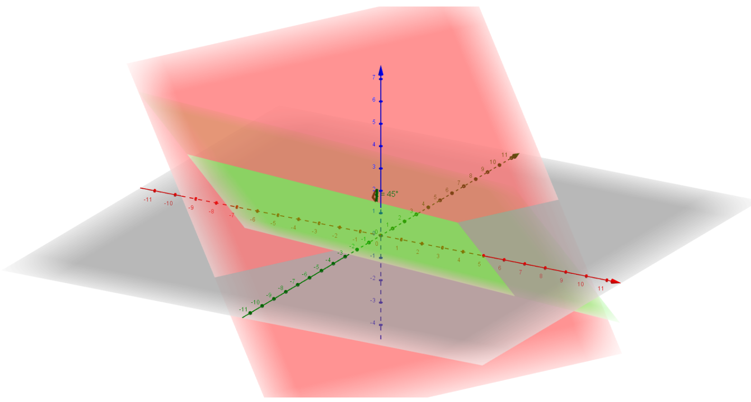


Figure 8 *Example 6 in GeoGebra*

and in the algebra window we get

eq1: 2x - y + 2z = 1

eq2: x + y + 4z = 5

α = 45°

from where we can see that the angle between the planes is 45°. The angle is also shown in the geometric window (Figure 8).

Example 7: Find the distance between the parallel planes $2x + y - 2z + 3 = 0$ and $2x + y - 2z - 12 = 0$.

Solution: Distance between parallel planes is the distance from an arbitrary point of one plane to the other plane. We choose a point from the plane $2x + y - 2z - 12 = 0$. For x=0 and z=0 we get y =12. So, point M (0,12,0) lies on the plane. Now,

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} = \frac{15}{3} = 5.$$

we can get the solution with the GeoGebra software. We enter the equations of the planes through the input field. In the 3D graphic we can see the planes. To calculate the distance through the input field, we enter the command distance (plane, plane). So, in 3D Graphics view we get:

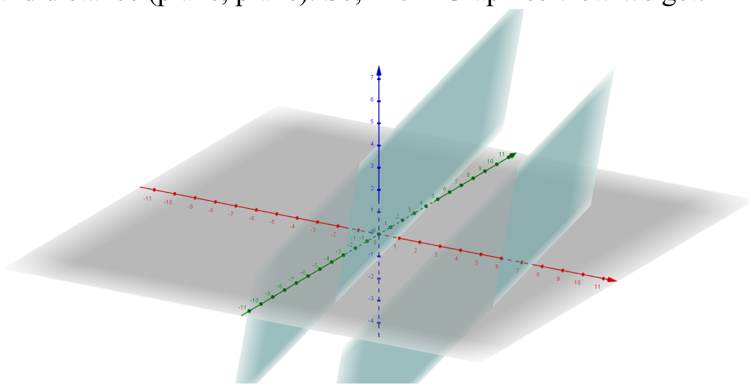
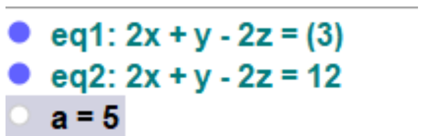


Figure 9 *Example 7 in GeoGebra*

In Algebra windows we get



from where we can see that the distance is 5.

3. Conclusion

The global problem of difficulty in mastering mathematics by students needs to be eradicated. To that end, more needs to be done. In this paper we offer a way to increase interest in mathematics with using math learning software for visual representation which will help for greater curiosity and increased motivation to work and solve problems. The software can help students to check if their solution is correct, to get a solution in advance that will guide them to solve the task etc. So, students will be motivated to study mathematics and to achieve better results. Thus, we encourage students to use free software, such as GeoGebra, while solving math problems.

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