Free vibration investigation of submerged thin circular plate

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Free vibration of coupled system including clamped-free thin circular plate with hole submerged in three dimensional cylindrical container filled with inviscid, irrotational and compressible fluid is investigated in the present work. Numerical approach based on the finite element method (FEM) is performed using the Comsol Multiphysics software, in order to analyze qualitatively the vibration characteristics of the plate. Plate modeling is based on Kirchhoff-Love plate theory. Velocity potential is deployed to describe the fluid motion since the small oscillations induced by the plate vibration is considered. Bernoulli's equation together with potential theory are applied to get the fluid pressure on the free surface of the plate. To prove the reliability of the present numerical solution, a comparison is made with the results in the literature, which shows a very good agreement. Then, different parameters effect including fluid density, fluid height, free surface wave, hole radius and hole eccentricity on the natural frequencies of the coupled system is discussed in detail. Some three-dimensional mode shapes of the submerged plate are illustrated. Furthermore, the obtain results can serve as benchmark solutions for the vibration control, parameter identification and damage detection of plate.

Keywords: Fluid-structure interaction; Thin circular plate; Finite element procedure.

1. Introduction

Fluid-structure interaction occurs especially when an elastic structure vibrates in presence of a fluid domain. By virtue of inertia increase due to the fluid motion generated by vibrating structure, the natural frequencies are significantly lower than those in vacuum. This behavior has been justified by the concept of the gradual increase in the added mass. This study is particularly concerned with free vibration analysis of circular plate with hole submerged in fluid.

Circular plates with or without hole in contact or not with fluid are widely used in many branches of engineering, e.g. liquid storage tanks, offshore naval or marine structures, solar plates, nuclear reactor internal components, micro pumps and circular disk of butterfly valves. Also, baffles and performed plates are efficient for reducing resonant sloshing in moving tank containing liquid [Jin *et al.*, 2014; Xue *et al.*, 2017; Yu *et al.*, 2019; Ghalandari *et al.*, 2019]. On the one hand, in vacuum, the existence of hole in a circular plate can significantly affect its vibrational response.

The knowledge and understanding of the associated effects is useful to the design of structures and vibration control. Therefore, studying and analyzing the vibrational behavior of circular plate with hole, is of high importance. A number of research works have been conducted on the vibration analysis of such structures based on analytical, experimental and numerical methods, such as the FEM, energy approach and mode subtraction approach. Excellent reference sources available may be found in the literature [Khurasia and Rawtani, 1978; Leissa and Narita, 1980; Nagaya and Poltorak, 1989; Vega *et al.*, 1998; Chen *et al.*, 2006; Lee *et al.*, 2007; Jhung and Jeong, 2015].

On the other hand, several researchers have investigated the vibration of plates in contact with a fluid and there have been many excellent experimental and theoretical research papers [Kwak, 1991; Bauer, 1995; Amabili and Kwak, 1996; Ergin and Ugurlu, 2003; Jhung *et al.*, 2009; Askari *et al.*, 2013; Soltani and Reddy, 2015; Cho *et al.*, 2015; Gascon-Pérez and Garcia-Fodega, 2015]. Obviously, the presence of the fluid around the plate causes an increase in the kinetic energy, and consequently, the natural frequencies of plate coupled with fluid strongly decrease compared to those obtained in vacuum. Certainly, this will significantly affect the coupled system performance under dynamic loading. However, the vibration analysis of a circular plate with a hole submerged in fluid taking into account both the free surface wave and eccentricity of the hole is missing in the literature. A good understanding of the dynamic interactions between the plate and fluid is necessary. Due to the dynamic nature of the system, the boundaries and the fluid-structure interaction forces between the plate and the fluid do not remain constant. Therefore, the need for fluid-structure interaction modelling seems inevitable for the structure and fluid [Gascon-Pérez, 2015; Mnassri and El Baroudi, 2017; Bahaadini and Saidi, 2018; Bahaadini *et al.*, 2018a,b; Saidi *et al.*, 2019].

The aim of the present research is to investigate numerically the vibration analysis of circular plate with hole submerged in fluid. A three-dimensional finite element model is constructed using Comsol Multiphysics software and the natural frequencies and corresponding mode shapes are obtained. The results show a very good agreement with those in literature in some particular cases. Moreover, the effect of different parameters including fluid density, fluid height, free surface wave, hole size and hole eccentricity on the vibration characteristics are examined.

2. Mathematical formulation for a plate submerged in fluid

A thin circular plate with hole submerged in a fluid-filled cylindrical rigid container, where a, b and h ($\ll a$) represent the outer radius, inner radius and thickness of the plate, respectively (see Fig. 1), is considered. The cylindrical container radius is R and the fluid depth is H. The fluid domain is divided into two regions : an upper fluid region (its depth is represented by H_u) and lower fluid region (its depth is represented by H_l). The present work is based on the following assumptions : (i) the fluid is assumed to be inviscid, irrotational, and compressible, and the amplitude of

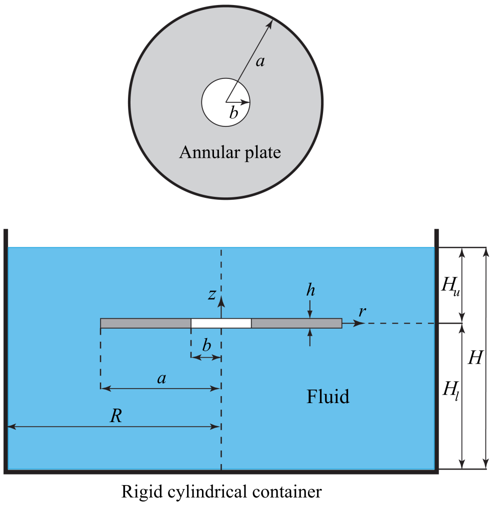


Fig. 1. Circular plate with hole submerged in fluid.

fluid motion is small in comparison with the model dimensions; (ii) the plate is made to be linearly elastic, homogeneous and isotropic; and (iii) the shear deformation and rotary inertia on the dynamics of plate are negligible. For the plate, the outer and inner boundaries are subject to the clamped and free boundary conditions, respectively. The equation of motion for the plate transverse displacement, w is

$$D\nabla^4 w + \rho_s h \partial_{tt} w = p \tag{1}$$

where D, ρ_s, h are respectively the bending rigidity, density, thickness and p is the hydrodynamic pressure on the surface of the plate. The bending rigidity of the plate is expressed as $D = Eh^3/\left(12\left(1-\nu^2\right)\right)$ where E is Young's modulus and ν is the Poisson's ratio. Since the small fluid oscillations induced by the plate vibration is considered, the fluid motion can be described using the velocity potential function Φ wich should satisfy the wave equation [Kinsler *et al.*, 1999]

$$B\nabla^2\Phi - \rho_f \partial_{tt}\Phi = 0 \tag{2}$$

where $B = \rho_f c_f^2$ is the fluid bulk modulus of elasticity, ρ_f is the fluid density and c_f is the sound wave speed. The fluid bulk modulus of elasticity is used to take into account the fluid compressibility. The fluid velocity is related to the potential function by following expression

$$\mathbf{v} = \nabla\Phi \tag{3}$$

The pressure given in Eq. (1) is related to velocity potential function by

$$p = -\rho_f \partial_t \Phi \tag{4}$$

2.1. *Plate-fluid boundary conditions*

To study the influence of fluid density on the natural frequencies of plate, or the plate effect on the free surface, boundary conditions are formulated as : (i) In the case of impermeable boundaries, which means that the fluid velocity in the normal direction to the surface is equal to zero, the boundary condition is expressed as

$$\nabla\Phi \cdot \mathbf{n} = 0 \tag{5}$$

where $\mathbf{n}$ is the normal vector at the fluid boundary. (ii) As the velocity potential is associated with the plate movement only (when the surface wave effect is neglected), the zero pressure condition is assumed at the free fluid surface, which means

$$p = -\rho_f \partial_t \Phi = 0 \tag{6}$$

(iii) When the free surface wave condition (or sloshing condition) is assumed, neglecting surface tension the sloshing condition is obtained from kinetic and kinematic conditions, yields the following relation

$$g\nabla\Phi \cdot \mathbf{n} = -\partial_{tt}\Phi \tag{7}$$

where g represents the acceleration due to gravity. (iv) During the interaction between the plate and fluid, the fluid particle and the plate move together in the normal direction of the boundary, and the interface boundary condition can be written as

$$\nabla\Phi \cdot \mathbf{n} = \partial_t w \tag{8}$$

2.2. *Numerical solution: Variational Formulation*

In this section we construct the variational formulation of the problem defined by Eqs. (1) and (2) in terms of w and Φ. The used numerical formulations include displacement formulation, potential formulation, pressure formulation and combination of some of them. Numerical solution based on FEM is used to extract frequencies and modal shapes. To compute the vibration modes of a fluid alone, the fluid is typically described either by pressure or by displacement potential variables. When the fluid is coupled with a solid, [Morand and Ohayon, 1979] introduce an alternative procedure which consists in using pressure and displacement potential simultaneously. In this section we summarize their approach and further details can be found in their book [Morand and Ohayon, 1995]. Therefore, to obtain a variational formulation for submerged plate, Eqs. (1) and (2) are multiplied by arbitrary test functions $\left(\bar{w}, \bar{\boldsymbol{\Phi}}\right)$ and integrating over the domains Ω_s and Ω_f (plate and fluid) using Green's formula and taking into account the boundary conditions yields

$$\int_{\Omega_s} D\nabla^2 w \cdot \nabla^2 \bar{w}\, d\Omega_s - \omega^2 \int_{\Omega_s} \rho_s h w \bar{w} d\Omega_s + j\omega \int_{\Omega_s} \rho_f \Phi \bar{w} d\Omega_s = 0 \tag{9}$$

$$\int_{\Omega_f} \rho_f \nabla\Phi \cdot \nabla\bar{\Phi}\, d\Omega_f - \omega^2 \int_{\Omega_f} \frac{\rho_f}{c_f^2} \Phi\bar{\Phi}\, d\Omega_f - j\omega \int_{\partial\Omega_f} \rho_f w \bar{\Phi}\, d\Gamma_f = 0 \tag{10}$$

Applying standard Galerkin discretization method wich consists in constructing an approximate solution of Eqs. (9) and (10), we have

$$\left\{\begin{pmatrix}\mathbf{K}_w & \mathbf{0}\\ \mathbf{0} & \mathbf{K}_\Phi\end{pmatrix} - \omega^2\begin{pmatrix}\mathbf{M}_w & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_\Phi\end{pmatrix} + j\omega\begin{pmatrix}\mathbf{0} & \mathbf{C}_w\\ -\mathbf{C}_\Phi & \mathbf{0}\end{pmatrix}\right\}\begin{Bmatrix}\mathbf{W}\\ \mathbf{\Phi}\end{Bmatrix} = \begin{Bmatrix}\mathbf{0}\\ \mathbf{0}\end{Bmatrix} \tag{11}$$

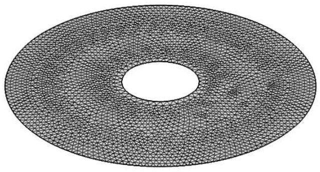
where $\mathbf{W}$ and $\mathbf{\Phi}$ are the vectors of nodal values for w and Φ, respectively, and the matrices in Eq. (11) are defined by

$$\int_{\Omega_s} D\nabla^2 w\cdot\nabla^2\bar{w}\, d\Omega_s = \bar{\mathbf{W}}^T\mathbf{K}_w\mathbf{W}\ ,\ \int_{\Omega_s}\rho_s h w\bar{w} d\Omega_s = \bar{\mathbf{W}}^T\mathbf{M}_w\mathbf{W}$$
$$\int_{\Omega_s}\rho_f\Phi\bar{w}d\Omega_s = \bar{\mathbf{W}}^T\mathbf{C}_w\mathbf{\Phi}\ ,\ \int_{\Omega_f}\rho_f\nabla\Phi\cdot\nabla\bar{\Phi}\, d\Omega_f = \bar{\mathbf{\Phi}}^T\mathbf{K}_\Phi\mathbf{\Phi}$$
$$\int_{\Omega_f}\frac{\rho_f}{c_f^2}\Phi\bar{\Phi}\, d\Omega_f = \bar{\mathbf{\Phi}}^T\mathbf{M}_\Phi\mathbf{\Phi}\ ,\ \int_{\partial\Omega_f}\rho_f w\bar{\Phi}\, d\Gamma_f = \bar{\mathbf{\Phi}}^T\mathbf{C}_\Phi\mathbf{W}$$

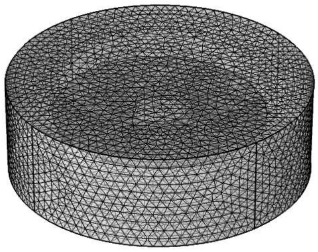
where $\bar{\mathbf{W}}$ and $\bar{\mathbf{\Phi}}$ are the vectors of nodal values for $\bar{w}$ and $\bar{\Phi}$, respectively. In order to determine natural frequencies of free vibration of submerged plate, Comsol Multiphysics code is used [Comsol, 2016] to solve Eq. (11). This modeling procedure requires two modules, one for simulating the plate and the other for the fluid. Each module provides a wide range of equations, which were needed in specifying subdomains and boundaries. For this purpose, some variables are set to make the connection between these two modules. At fluid-structure interface, kinematic and dynamic continuity has to be ensured. The complete coupled problem has to fulfill the condition that the location of the fluid-structure interface coincides for both fields. Thus, the fluid-structure interaction boundary condition concerning the fluid is of a Dirichlet type, and the fluid-structure boundary condition for the solids is given by a Neumann condition. The plate and fluid were simulated using quadratic element and quadratic Lagrange element, respectively. The plate and fluid elements at the interface shared the same nodes and had extra fine meshes to capture the details during the coupled vibrations (see Fig. 2). Thus, for the eigenvalue problem solution, which depends on finding the eigenvalues ω, is solved by the natural frequency extraction.

3. Results and discussion

This section presents results of the natural frequencies and associated mode shapes of the plate submerged in fluid-filled cylindrical container (Fig. 1). By performing modal analysis on the plate, first six frequencies are tabulated in Tables 2-4. In order to compare the present results with other established results of specific cases, we maintain the same frequency factor as the one defined by [Lee *et al.* 2007] in the case of an plate vibrating in vacuum. Thus, in other words, frequencies are normalized and introduced as the dimensionless frequency, which is defined by $\lambda = a\sqrt{\omega}\left(\rho_s h/D\right)^{1/4}$. The geometry and material properties of the plate and the compressible fluid are presented in Table 1.



(a) Plate in vacuum



(b) Submerged plate

Fig. 2. Plate mesh ($b/a = 0.25$, $a = 0.175$ m). (a) : Mesh consists of 5248 boundary elements and 200 edge elements. Number of degrees of freedom solved for : 64176 and time for solving is 21 s. (b) : Mesh consists of 99865 domain elements, 6784 boundary elements, and 308 edge elements. Number of degrees of freedom solved for : 153770 and time for solving is 151 s.

Table 1. Material and geometrical parameters.

Properties	Plate and fluid
Young's modulus, E	207 (GPa)
Density, ρ_s	7800 (kg/m^3)
Poisson's ratio, ν	0.3
Thickness, h	0.002 (m)
Density, ρ_f	1000 (kg/m^3)
Bulk modulus, B	2.199 (GPa)
Speed of sound, c_f	1483 (m/s)
Radius, R	0.25 (m)
Depth, H	0.16 (m)
Lower region depth , H_l	0.10 (m)

Tables 2 and 3 show six frequencies using indirect BIEMs method [Lee *et al.*, 2007] and the present FEM. The very good agreement is revealed by showing the reliability of the implemented algorithm in Comsol Multiphysics code. Table 2 also shows that the frequency decreases as the plate radius increases. This is related to the plate mass effect. For the plate with an eccentric hole, Table 3 shows also that the doublet frequency (multiplicity) divides into two distinct values. This multiplicity is due to the hole eccentricity. Indeed, the eccentricity generates a local modification of the mass, hence the doublet frequency [Deutsch *et al.*, 2004]. Figs. 3 and 4 show the mode shapes of the plate with an eccentric hole in vacuum and in water, respectively. Note that, Fig. 3 is identical to that given by [Lee *et al.*, 2007]. Furthermore, the convergence is very fast to obtain the desirable frequencies. In addition, the fundamental mode corresponds to the mode with $n = 0$ and $m = 0$. Thus, in tables 2-4 the parameters n and m define respectively the number of nodal diameters and nodal circles.

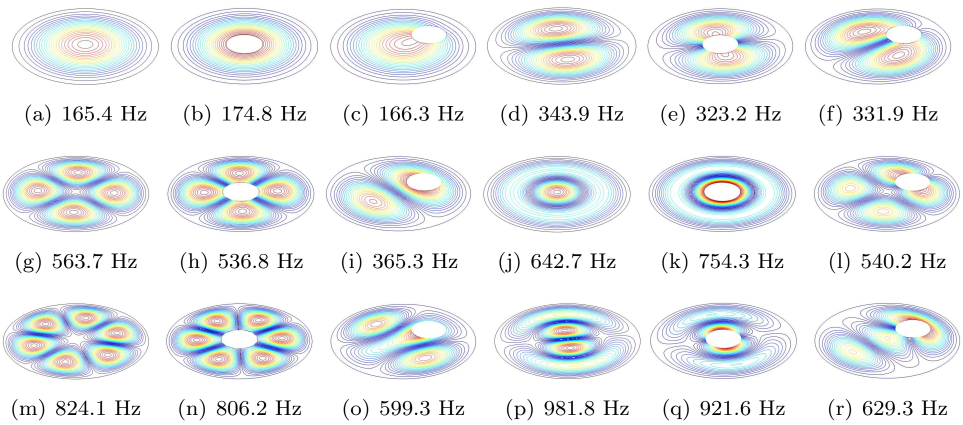
In this work, various parameters effects on plate frequencies such hole radius, hole eccentricity, fluid density and upper fluid height on the coupled frequencies are

Table 2. First six frequencies of the plate in vacuum ($b/a = 0.25$).

		$a = 0.175\,(\text{m})$	$a = 1\,(\text{m})$		
		Present	Present		Lee *et al.* 2007
Mode	(n, m)	f (Hz)	f (Hz)	λ	λ
1	$(0, 0)$	174.85	5.3829	3.2743	3.2750
2	$(1, 0)$	323.25	9.9897	4.4606	4.4610
3	$(2, 0)$	536.84	16.668	5.7618	5.7640
4	$(0, 1)$	754.34	23.3	6.8123	6.8160
5	$(3, 0)$	806.22	25.05	7.0635	7.0680
6	$(1, 1)$	921.67	28.486	7.5323	7.5360

Table 3. First six frequencies in vacuum ($b/a = 0.25$, $e = 0.45$ m). * multiplicity.

		$a = 0.175\,(\text{m})$	$a = 1\,(\text{m})$		
		Present	Present		Lee *et al.* 2007
Mode	(n, m)	f (Hz)	f (Hz)	λ	λ
1	$(0, 0)$	166.32	5.1448	3.2011	3.1870
2	$(1, 0)^*$	331.96	10.29	4.5271	4.5210
3	$(1, 0)^*$	365.31	11.292	4.7424	4.7400
4	$(2, 0)^*$	540.21	16.721	5.7709	5.7580
5	$(2, 0)^*$	599.37	18.576	6.0826	6.0600
6	$(0, 1)$	629.36	19.485	6.2296	6.2560



(a) 165.4 Hz (b) 174.8 Hz (c) 166.3 Hz (d) 343.9 Hz (e) 323.2 Hz (f) 331.9 Hz

(g) 563.7 Hz (h) 536.8 Hz (i) 365.3 Hz (j) 642.7 Hz (k) 754.3 Hz (l) 540.2 Hz

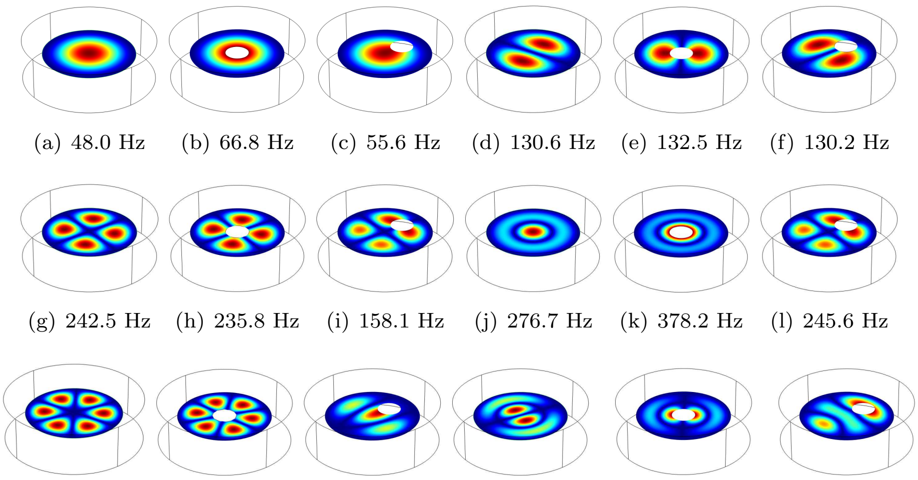
(m) 824.1 Hz (n) 806.2 Hz (o) 599.3 Hz (p) 981.8 Hz (q) 921.6 Hz (r) 629.3 Hz

Fig. 3. First six mode shapes of plate in vacuum. (a,d,g,j,m,p) Without a hole ($a = 0.175$ m). (b,e,h,k,n,q) With a concentric hole ($b/a = 0.25$). (c,f,i,l,o,r) With an eccentric hole ($e = 0.45$ m).

presented. Firstly, one investigates how hole radius affects the frequencies of plate. Variation of first ten plate modes is exhibited in Fig. 5(a) for different values of the aspect ratio b/a of the plate. As the hole radius of the plate increases, the frequencies

Table 4. Six frequencies of plate with a hole in fluid ($b/a = 0.25$, $a = 0.175$ m).

	$e = 0$		$e = 0.45$ (m)		$e = 0.6$ (m)	
Mode	(n, m)	f (Hz)	(n, m)	f (Hz)	(n, m)	f (Hz)
1	$(0, 0)$	66.824	$(0, 0)$	55.605	$(0, 0)$	51.913
2	$(1, 0)$	132.55	$(1, 0)^*$	130.25	$(1, 0)^*$	129.89
3	$(2, 0)$	235.81	$(1, 0)^*$	158.19	$(1, 0)^*$	147.38
4	$(0, 1)$	378.29	$(2, 0)^*$	245.64	$(2, 0)^*$	244.49
5	$(3, 0)$	379.38	$(2, 0)^*$	268.56	$(2, 0)^*$	260.00
6	$(1, 1)$	486.11	$(0, 1)$	298.89	$(0, 1)$	301.88



(a) 48.0 Hz (b) 66.8 Hz (c) 55.6 Hz (d) 130.6 Hz (e) 132.5 Hz (f) 130.2 Hz

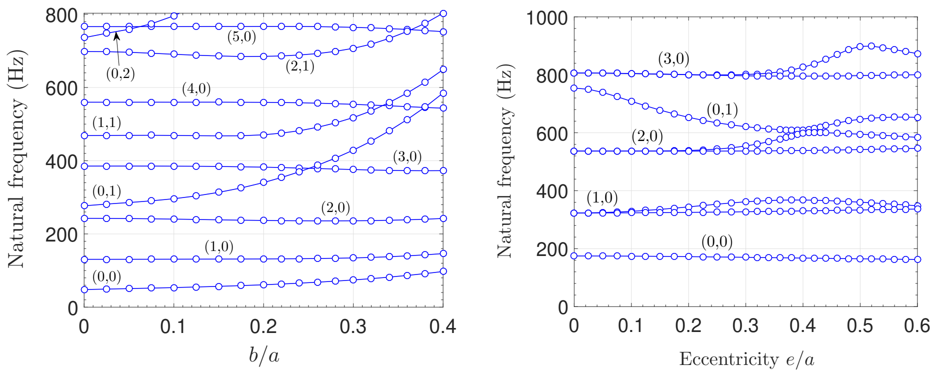
(g) 242.5 Hz (h) 235.8 Hz (i) 158.1 Hz (j) 276.7 Hz (k) 378.2 Hz (l) 245.6 Hz

(m) 385.3 Hz (n) 379.3 Hz (o) 268.5 Hz (p) 468.6 Hz (q) 486.1 Hz (r) 298.8 Hz

Fig. 4. First six mode shapes of plate in fluid. (a,d,g,j,m,p) Without a hole ($a = 0.175$ m). (b,e,h,k,n,q) With a concentric hole ($b/a = 0.25$). (c,f,i,l,o,r) With an eccentric hole ($e = 0.45$ m).

for asymmetric modes ($n > 0$) with $m = 0$ are slightly affected. However, the frequencies for asymmetric modes with $m \geqslant 1$ and for axisymmetric modes ($n = 0$) increase as the hole radius increases. In fact, when the hole radius increases, the plate effective radius decreases and this causes an increase in the plate stiffness, and as a consequence the frequencies increase. In addition, in Fig. 5, the intersection of mode curves with the frequency axis, corresponds to frequencies obtained in case of an circular plate without hole submerged in fluid. These frequencies are in good agreement with those obtained by [Askari *et al.*, 2013]. If the plate contains multi-hole, the context is much more complex. The plate stiffness decreases more than the mass and therefore the frequencies of a perforated plate are usually smaller than those of the plate without hole [Lee *et al.*, 2007; Jhung and Jeong, 2015]. This is the opposite situation from that of the plate with a single hole.

Secondly, one investigates how the hole eccentricity affects the frequencies. Figs.



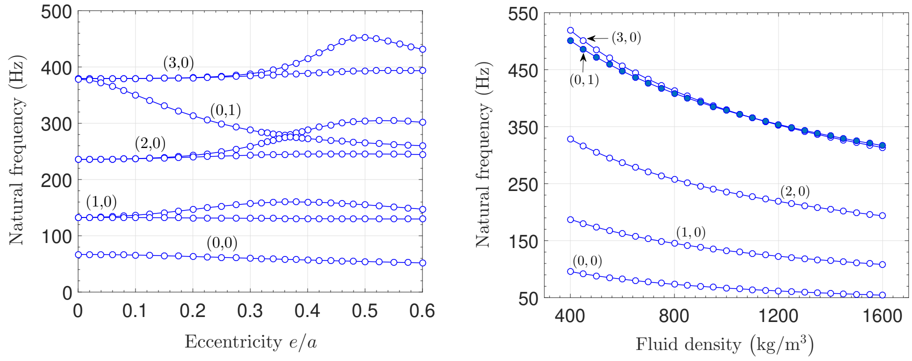
(a) (b)

Fig. 5. (a) Radius hole effect on frequencies of submerged plate. (b) Eccentricity effect on frequencies in vacuum ($b/a = 0.25$, $a = 0.175$ m).

5(b) and 6(a) show the frequencies in vacuum and in water, varying with the hole eccentricity. As can be noticed, on the one hand the presence of the fluid around the plate creates an added mass due to the fluid motion which decreases significantly the frequencies of the plate. On the other hand Figs. 5(b) and 6(a) show also that an increase in the eccentricity, leads to an axial symmetry breaking, as a consequence the doublet frequencies (multiplicity) occurring in the case of plate for asymmetric modes ($n > 0$), are progressively separated into two distinct values. We can also observe that for asymmetric modes, the first frequency varies slightly with respect to the eccentricity. However, with an increase in eccentricity, the second frequency of the asymmetric modes augments and then decreases beyond a certain eccentricity value. For example, asymmetric mode $(1, 0)$ has the peak frequency at eccentricity $e/a = 0.37$ in vacuum and in fluid, asymmetric mode $(2, 0)$ has the peak frequency at eccentricity $e/a = 0.42$ in vacuum ($e/a = 0.37$ in fluid), and asymmetric mode $(3, 0)$ has the peak frequency at $e/a = 0.52$ in vacuum ($e/a = 0.5$ in fluid). Therefore, with the asymmetric modes increasing, the eccentricity where the peak frequency is obtained increases. In addition, the fundamental mode $(0, 0)$ is slightly affected by increasing in eccentricity. However the axisymmetric mode $(0, 1)$ decreases with increasing eccentricity and increases beyond a certain eccentricity value.

Thirdly, to study the fluid density effect (added mass) on the naturel frequencies of the plate, Fig. 6(b) is plotted for first five modes. It is obvious from this figure that, when the plate vibrates in denser fluid, the frequencies take lower values.

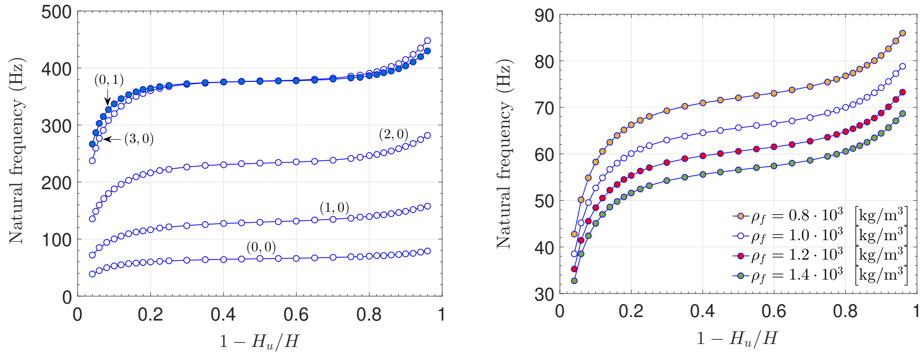
Fourthly, one investigates how the frequencies of the plate vary with the upper fluid height H_u. The frequencies are plotted in Fig. 7(a) as a function of the normalized upper fluid height. When the upper fluid height approaches zero, i.e. the plate reaches the free fluid surface, the frequencies increase regardless of (n, m), and they converge to those in the case of the plate in contact with fluid on only one side. The frequencies decrease continually, but within an interval from 0.2 to 0.8 of the



(a) (b)

Fig. 6. (a) Eccentricity effect on frequencies in fluid ($b/a = 0.25$, $a = 0.175$ m). (b) Variation of frequencies of plate versus fluid density ($b/a = 0.25$, $a = 0.175$ m).

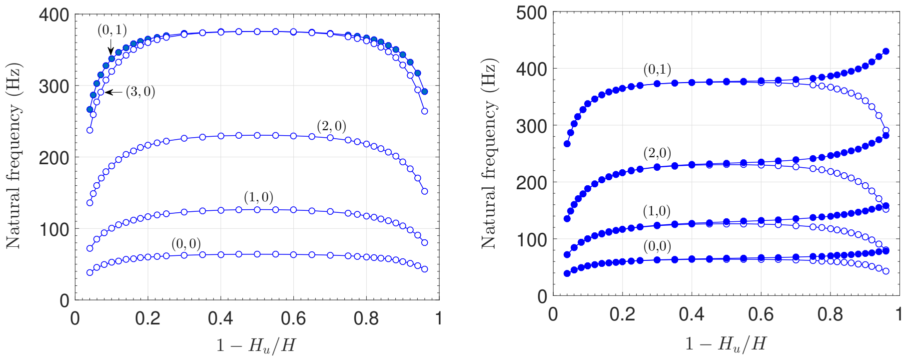
normalized $1 - H_u/H$ fluid height over the plate, they diminish slowly. However, when the plate approaches the container bottom, the frequencies decrease dramatically due to the increase of the added mass. In addition, when the plate reaches the free fluid surface, the upper fluid reacts as the in phase mode with respect to the plate [Ergin and Ugurlu, 2003], mainly the upper fluid moves vertically, and the added mass of the fluid progressively decreases with decreasing the upper fluid height. In addition, to study the effects of fluid density and upper fluid height on the naturel frequencies of the circular plate, Fig. 7(b) is plotted for some values of fluid density and normalized upper fluid height. It is evident from this figure that, when the plate is oscillating in contact with the denser fluid, the frequencies takes lower values. Note that this behavior is also found for the other modes.



(a) (b)

Fig. 7. (a) Frequencies of plate versus normalized upper fluid height in the case of free fluid surface ($b/a = 0.25$, $a = 0.175$ m). (b) Frequencies for the fundamental mode $(0, 0)$ of the plate versus normalized upper fluid height in the case of free fluid surface ($b/a = 0.25$, $a = 0.175$ m).

Fifthly, to study the case of bounded fluid, the free fluid surface is replaced by an upper rigid wall. The frequencies are plotted in Figs. 8(a) and 8(b) as a function of the normalized upper fluid height. The frequencies severely decrease when the plate approaches the upper rigid surface. In fact, the upper fluid reacts as the out of phase mode with respect to the plate [Ergin and Ugurlu, 2003], and the upper fluid moves laterally in the gap between the upper rigid wall and plate, this leads to an increase in the added mass of the fluid. Fig. 8(a) show also that with the modes increasing, the difference in the frequencies between the bounded fluid case and free fluid surface case augments. We can also remark that the frequencies in the free fluid surface case are always greater than those obtained in the bounded fluid case when the plate reaches the upper side of the container.



(a) (b)

Fig. 8. (a) Frequencies of plate versus normalized upper fluid height for the fluid-bounded case ($b/a = 0.25$, $a = 0.175$ m). (b) Fluid bounding effect on frequencies of plate ($b/a = 0.25$, $a = 0.175$ m). Free fluid surface case (●) and fluid-bounded case (○).

Finally, four first sloshing frequencies are plotted in Fig. 9 versus upper fluid height. it is well known that the submerged plate can be used as a baffle plate to reduce or suppress the sloshing waves, and simultaneously to decrease the frequencies and change the sloshing mode shapes as shown in two cases. With a decrease in the normalized upper fluid height, the sloshing frequencies increase. As the plate approaches the free fluid surface, the sloshing frequencies significantly decrease, which means that the submerged plate can be used as a baffle device to reduce or suppress the sloshing waves. In addition, first four sloshing mode shapes of the free fluid surface are illustrated in Fig. 10 for two different upper fluid heights. The fluid heights above the plate are equal to $H_u = 4.8\,\text{mm}$ and $H_u = 64\,\text{mm}$. These values correspond respectively to a positioning of the plate in the vicinity of the fluid surface and approximately at the cylindrical container half height. The sloshing modes when the plate is placed at the container bottom are identical to those without plate. Consequently, the positioning of the plate at the container bottom has no influence on the sloshing modes, whereas the positioning near the free surface greatly reduces the sloshing frequencies. In other words, sloshing frequencies converge to those without plate once the normalized upper fluid height approaches zero as shown in Fig. 10.

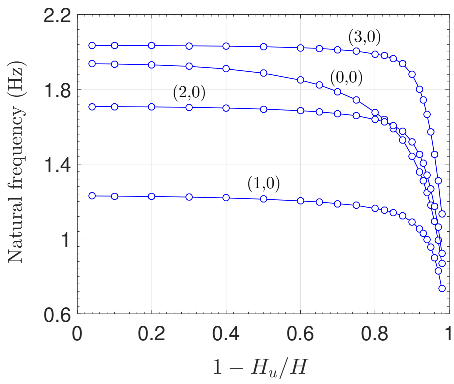
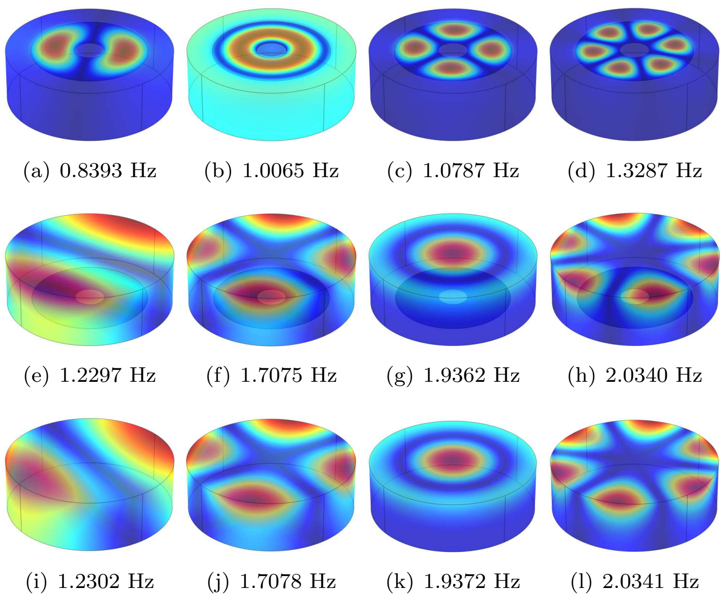


Fig. 9. Sloshing frequencies for the first four modes of plate ($b/a = 0.25$, $a = 0.175$ m).



(a) 0.8393 Hz (b) 1.0065 Hz (c) 1.0787 Hz (d) 1.3287 Hz

(e) 1.2297 Hz (f) 1.7075 Hz (g) 1.9362 Hz (h) 2.0340 Hz

(i) 1.2302 Hz (j) 1.7078 Hz (k) 1.9372 Hz (l) 2.0341 Hz

Fig. 10. First four Sloshing mode shapes ($b/a = 0.25$, $a = 0.175$ m). (a,b,c,d) for $1-H_u/H = 0.97$. (e,f,g,h) for $1 - H_u/H = 0.06$. (i,j,k,l) without plate.

4. Conclusion

A numerical analysis to investigate the frequencies of a thin circular plate with hole submerged in fluid is proposed using FEM. The effect of different parameters including fluid density, free surface wave, fluid height, hole radius and hole eccentricity on the frequencies is examined and discussed in detail. The results can serve as benchmark solutions for the vibration control, parameter identification and damage detection of thin circular plate with a hole. The following comments were made in the present study : (i) The results show a good agreement with those in literature in some particular cases by showing the reliability of the implemented algorithm in Comsol Multiphysics software and, the convergence is very fast to obtain the desirable frequencies. (ii) The hole eccentricity has a slight effect on the first frequency of asymmetric modes, but the second frequency is strongly affected by the hole eccentricity. (iii) When the plate is vibrating in fluid, the frequencies decrease and the magnitude of the decrease is more important when the fluid density increases. (iv) The hole radius of the plate affects slightly the frequencies for asymmetric modes ($n > 0$) with the radial circle $m = 0$. However, the frequencies for asymmetric modes with the radial circle $m \geqslant 1$ and for axisymmetric modes ($n = 0$) increase as the hole radius increases. (v) The frequencies of the plate initially decrease with the increasing the upper fluid height, then become about constant when the plate reaches the middle of the container. However, increasing the upper fluid height more than the container middle height leads to a decrease again in the frequencies, and this is due to the plate closeness to the container rigid bottom.

In addition, the numerical results obtained are in good agreement with existing data in some particular cases, thereby providing a satisfactory validation of the present model. In future, the following topics are of interests in order to complete this article results : (1) The cylindrical container radius effect on the frequencies may be important in order to know from which critical value of the container radius the fluid portions far from the plate have a little effect on plate vibration behavior. (2) In this study, the outer and inner boundaries of the plate are subject to the clamped and free boundary conditions, respectively. Therefore, it's recommended to investigate how the plate frequencies vary with different boundary conditions.

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