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Intermediation in a Directed Search Model

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Abstract

We provide an example where establishing competitive coordination service platforms is so lucrative that they end up reducing welfare. We consider a canonical directed search model in which buyers have unit demands and sellers' capacity constraint leads to a coordination problem: in a symmetric equilibrium without intermediation some sellers receive too many and some too few buyers. We compare this equilibrium to one where sellers and buyers can choose to become intermediaries who coordinate the meetings. In this set-up, roughly one fifth of agents become intermediaries. As a result, a large part of the supply and demand in the economy vanishes. Moreover, the large amount of intermediaries actually reduces the meeting efficiency. Jointly, these effects imply that the gains from trade are lower than in the economy without intermediation.

Keywords: directed search, intermediation, middlemen

JEL Codes: D4, L1

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1 Introduction

Intermediation is a salient feature of the economy. Evident examples of the activity include real estate agents, financial intermediaries, and (on and offline) marketplaces. Stretching the interpretation a little, even grocery stores can be regarded as intermediaries. On the one hand, intermediaries facilitate trade by bringing interested parties together by overcoming coordination problems. On the other hand, intermediaries have long been disregarded as go-betweens who produce nothing, but waste resources. In this paper, we provide an example where the costs of intermediation outweigh its benefits. We focus on the intermediaries' potential to alleviate the coordination problem between buyers and sellers in a set-up in which costs of intermediation arise endogenously.[1]

We study a basic directed search model with two types of agents who must form a pair in order to produce economic surplus. For concreteness, we call two agent types buyers and sellers who have to meet to trade. The buyers have unit demands and the sellers are capacity constrained, each one possessing one unit of a good for sale. The sellers post prices competitively to attract buyers. In a symmetric equilibrium, every seller posts the same price and the buyers contact each seller with the same probability. Symmetric contact strategies combined with the capacity constraints create a coordination problem: some sellers are contacted by many buyers while some other sellers end up with no buyers.

To this setting we introduce intermediaries whose rationale is to coordinate buyers and sellers in their meetings. A key novelty of our model is that agents can choose whether to become intermediaries or remain buyers and sellers. This occupational choice determines the costs of intermediation as part of the equilibrium. Since the intermediaries are agents who could be buyers or sellers, becoming an intermediary means that some supply and demand, or potential for gains from trade, vanish. For example, if there are 100

[1] Besides overcoming coordination problems, intermediaries can facilitate trade via other means such as information production – see, e.g., Spulber (1999) for a survey of intermediaries' various tasks. We ignore such benefits of intermediation. Intermediation can also involve other costs (e.g., transaction costs) beyond those to what we consider.

buyers and 100 sellers, the theoretical gains from trade are achieved when all buyers and sellers trade, or there are 100 trades. If 10 agents of both groups become intermediaries, the theoretical gains from trade drop to 90 trades.

The occupational choice also affects the efficiency of coordination. As the intermediaries do nothing else but bring together the buyers and sellers who have contacted them, the most efficient outcome is to have just one intermediary. The agents can, however, freely enter into intermediation, reducing its efficiency. In our specific set-up, intermediation is so lucrative that too many agents become intermediaries: coordination problems become more severe than without intermediaries. Thus, competitive intermediation may reduce welfare not only because it consumes resources but also because it may reduce meeting efficiency.

To compare the economy with and without intermediation we must determine the equilibrium market structure, or which agents become intermediaries. In equilibrium the agents have to expect the same utility regardless of the role they choose; this equivalence pins down the size of the intermediation sector. The determination of the agents' expected utility requires modelling the price setting game among intermediaries. As is well known (see, e.g., Gehrig, 1993), modelling price competition among intermediaries is non-trivial since the intermediaries provide coordination services, and prices do not direct the agents' choices in the same way as in the economy with buyers and sellers only. Our solution to this problem is to assume myopic buyers and sellers in the sense that they ignore the price effects on the other side of the market. This assumption allows modelling intermediaries competing in prices without a need to take a stance on coordination issues. The equilibrium price balances competition for buyers and sellers and allows us to pin down the equilibrium where the intermediaries, buyers, and sellers fare equally well.

Next, we review the related literature. Then, we cover the benchmark directed search model in which the sellers post prices, and the buyers contact them using symmetric strategies. The frictions that arise in equilibrium are quantified, and they form the rationale for intermediation. In Section 4, we analyse the setting where the agents can choose to become intermediaries.

The main objective is to determine the number of trades, which is our efficiency measure, in each setting. The analysis of Section 4 assumes an equal number of buyers and sellers. In this case the number of trades compared to the theoretical maximum of trades is the smallest, and consequently the need for coordination services the strongest. In Section 5, we allow for unequal numbers of buyers and sellers. Depending on their ratio the equilibrium may feature only one side of the market becoming intermediaries. There is also a range of values with two (pure-strategy) equilibria. The unstable equilibrium features a small number of intermediaries, and improves the performance of the economy, while the stable equilibrium does the opposite, just like in Section 4 with an equal number of buyers and sellers.

2 Related Literature

In the seminal paper Rubinstein and Wolinsky (1987) introduce intermediaries, or middlemen, into a unit-supply, unit demand random matching framework of buyers and sellers. Middlemen can hold at most one unit of the good. Equilibrium with middlemen exists if they are at least as efficient in meeting other agents as buyers and sellers are meeting each other. If a middleman becomes the owner of the good instead of just mediating trade the buyers' and sellers' shares of the gains from trade go down.

There is a long literature on intermediation building on Rubinstein and Wolinsky (1987) (see Wright and Wong, 2014, for a survey). In this tradition, a paper close to ours is Nosal, Wong and Wright (2016) who extend the framework to study efficiency and allow the agents on the one side of the market to choose to be either producers or middlemen. Thus, intermediation entails similar opportunity costs as in our paper, since an increase in the number of intermediaries comes at the cost of having fewer producers. They find that there can be too much or too little intermediation depending on the parties' bargaining powers. In Nosal et al. (2016) the agents meet randomly, and prices are determined in pairwise bargaining, while we consider a directed search setting with price competition among intermediaries. We allow agents from both sides of the market to become intermediaries, and characterise the equilibrium where intermediares consist of the agents from one side only.

Gehrig (1993) and Spulber (1996) introduce price competion among intermediaries in search settings. In Gehrig (1993), the prices are public, whereas in Spulber (1996), the prices are observable only after time-consuming search. Rust and Hall (2003) build on Gehrig (1993) and Spulber (1996) and consider competition between market maker-type and dealer-type intermediaries. Fingleton (1997a) studies a setting where trading takes place either through a middleman who provides speed or through direct trade where markets are organised as in double auction but where putting up the market takes time. Fingleton (1997b) studies intermediation when the traders have direct trade as an alternative to using a middleman.

In traditional search models, intermediaries can only be found at random and their prices play only a limited role in the search process and allocation of goods. In contrast, in directed search models like ours, intermediaries are easy to find, and their prices crucially matter for the agents' decisions to contact intermediaries and to trade. As is common to all search models, the probability of trade also matters for the allocation of goods; thus in a competitive search equilibrium goods are allocated both by the prices and trading probabilities (see Wright et al., 2019, for a survey of directed and competitive search).

Watanabe (2010) considers intermediaries in a directed search model like we. In Watanabe (2010), an intermediary can hold larger stocks of goods than the other agents, and consequently serve more people. As the buyers are interested in both the price and the probability of acquiring a good large inventories allow higher prices than low inventories. In Gautier, Hu and Watanabe (2017), a monopoly intermediary has a choice of holding inventory and of offering a platform for buyers and sellers. The buyers and sellers have an option to trade in the decentralised market, too. In this setting an intermediary can survive in the equilibrium, and offer both the platform and inventories.

Intermediation can be regarded as a platform, or a coordination device as, e.g., in Rochet and Tirole (2003), Caillaud and Jullien (2003), and Armstrong (2006). The platform is more valuable to a particular type (e.g., buyer) the more of the opposite type (e.g., seller) uses it. In these models, demand is assumed to be sufficiently insensitive to low prices to avoid Bertrand-like or monopoly outcomes. In this strand of the literature, Ambrus and Argenziano (2009) allow firms to choose to establish one or two platforms. Even though creating platforms is costly the costs are internalised unlike in our model in which each intermediary imposes a cost to the rest of the economy in terms of lost opportunity for gains from trade. To avoid problems of multiple equilibria, Ambrus and Argenziano (2009) adopt an equilibrium concept that allows profitable coordination by consumer, whereas we impose a different restriction - myopicity - on the agents' expectations. Common to these platform competition models is that the number of platforms is restricted. In our model the intermediaries also provide coordination services but their number is determined endogenously.

In Ronayne (2019) price comparison websites act somewhat like intermediaries. In our model, intermediaries cause efficiency losses because the equilibrium price level is high enough to attract too many intermediaries. In Ronayne (2019), a higher price level in itself constitutes the adverse effects of the price comparison websites.

To summarize, typically in the literature (with some notable exceptions mentioned above) the intermediaries are assumed to exist at the outset or they may enter the economy by paying a fixed entry cost. In our model the buyers' and sellers' choice of whether or not to become an intermediary allows us to model the emergence of intermediation naturally without a need to introduce additional agent or cost types. That occupational choice also creates a meaningful general equilibrium effect: more intermediaries means fewer buyers and sellers. Thus, each new intermediary both reduces aggregate supply and demand and spreads out the remaining supply and demand into more trading locations.

3 The Economy Without Intermediaries

There are S sellers and B buyers in the economy. Ex ante all the sellers are identical, may produce a unit of an indivisible good at zero cost for sale, and value it at zero. The buyers are also identical, have unit demands and value the good at unity. The sellers post prices at which they commit to sell their unit supply. The buyers observe the prices and based on this information choose which seller to contact. If a seller is contacted by two or more buyers, the seller's good is randomly allocated to one of the buyers. If a seller is contacted by no buyers, she is unable to trade.

This situation is well-understood in a large economy where there is an infinite number of buyers and sellers; there exists a unique (symmetric) equilibrium in which all the sellers post the same price (e.g., Kultti, 1999, and Burdet, Shi and Wright, 2001). If the ratio of buyers to sellers, or the expected queue length in a symmetric equilibrium, is θ then a seller meets k buyers with probability $e^{-\theta}\theta^k/k!$.[2]

Price competition does not drive prices to zero since the sellers are capacity constrained and the buyers are interested both in the price, and the probability of attaining the good. The equilibrium price (see Appendix A for derivation) turns out to be

$$p = \frac{1 - e^{-\theta} - \theta e^{-\theta}}{1 - e^{-\theta}}. \tag{1}$$

This price is increasing in θ; the larger the ratio of buyers to sellers, the more intense the competition among buyers and the more likely that a seller can trade even if she charges a higher price.

The equilibrium price (1) balances the local monopoly power that stems from the limited capacity and the competition that stems from the large number of sellers. According to the Mortensen rule the parties in a meeting should get their marginal product, or the surplus should go to the party that initiates a match (Julien, Kennes and King, 2006). If only one buyer contacts a seller, he should get all the surplus, and if several buyers contact a seller, any one of them is dispensable and the seller should get the full

[2]This Poisson-distribution arises as a limit of a finite economy: Consider a finite number of buyers and sellers, b and s, and denote $\theta = b/s$. When the buyers contact the sellers randomly, a seller expects zero buyers with probability $(1-1/s)^b = ((1-1/s)^s)^\theta$. Allowing s and b to increase without limit while keeping θ constant, the seller's probability of meeting no buyers converges to $e^{-\theta}$, i.e., to the Poisson-θ distribution.

surplus. The Mortensen rule is satisfied when trades are consummated by auction. The economy with price posting, and the equilibrium price (1), is utilitywise equivalent to auction. Price (1) gives the conditional probability that a seller is contacted more than one buyer given an opportunity to trade. Analogously, using the equilibrium price (1), we can write the buyer's share of the surplus as $1-p = \theta e^{-\theta}/(1-e^{-\theta})$, which is the conditional probability that exactly one buyer contacts a seller given that the seller is contacted; the buyer is rewarded in proportion to his appearance given that trading takes place.

For the moment, we simplify the analysis and assume an equal number of buyers and seller.

Assumption 1. *There is a unit mass of both buyers and sellers.*

Assumption 1 (which is relaxed in Section 5) can be motivated by looking at the efficiency of the economy: Assume that the total number of agents is unity and that b of them are buyers and the rest $1-b$ sellers. Then, the ratio of buyers to sellers is given by $\theta = b/(1-b)$ and, consequently, $b = \theta/(1+\theta)$ and $1-b = 1/(1+\theta)$. Since the probability that a seller meets no buyers is $e^{-\theta}$, the number of trades in the economy is given by $\left(1-e^{-\theta}\right)/(1+\theta)$. The theoretical maximum of trades in the economy is $\min\{b, 1-b\}$. Dividing the number of trades by the theoretical maximum of trades gives an index of efficiency.

If $b \geq 1/2$, $1-b \leq b$, and the efficiency index is given by $1-e^{-\theta}$, which is increasing in θ and, hence, in b. Consequently, in the range where $b \in [1/2, 1]$, the efficiency index reaches its minimum when $b = 1/2$ or $\theta = 1$.

If $b \leq 1/2$, $1-b \geq b$, and the efficiency index becomes $(1-e^{-\theta})/\theta$. Now this index is decreasing in θ or, in b, thus reaching its minimum in the range where $b \in [0, 1/2]$ at $b = 1/2$ or $\theta = 1$.

Consequently, under Assumption 1, the economy without intermediation is the least efficient and, thus, the case for intermediation the greatest. In this case the probability that a seller meets no buyers is e^{-1}, and the number of trades consummated is given by $1-e^{-1} \approx 0.632$.

Result 1. *When buyers contact sellers directly the gains from trade, or the number of trades, is approximately* 0.632*.*

The assumption that buyers contact sellers and not vice versa is relatively innocuous. One could even allow for two markets where buyers contact sellers in one market and sellers contact buyers in the other market. Letting the agents choose between the two markets yields an outcome where the number of trades differs only in the third decimal from our simpler setting (see a working paper version (Kultti, Takalo and Vähämaa, 2018), and Halko, Kultti and Virrankoski, 2008).

4 Equilibrium Intermediation

We introduce intermediaries as buyers and sellers who decide to become meeting platforms that bring other buyers and sellers together. The intermediaries attract the buyers and sellers by posting prices for their services just like the sellers did in the benchmark model of Section 3. We restrict price competition among intermediaries in two ways. First, we assume that intermediaries charge a fee per successful trade; if j buyers and k sellers contact an intermediary, the intermediary facilitates $\min\{j, k\}$ trades, and takes a commission from each trade. This assumption means that an intermediary cannot price discriminate between the two sides of the market; as a result, the fee faced by the two market sides moves to the same direction.

Second, when an intermediary deviates, and quotes an off-equilibrium fee, we assume that agents in the one side of the market (say, buyers) take into account only the effect on the other agents on the same side of the market (the other buyers) and ignore the effect on the other side of the market (sellers). For example, consider an intermediary deviating to a lower fee. He attracts more buyers for a given number of sellers. Our two assumptions mean that sellers, too, face a lower fee and are attracted to the deviating intermediary because of the lower fee but not because of the greater number of buyers.

Modelling price competition among coordination services is known to raise difficulties, since prices do not necessarily function the same way as in the economy without intermediaries (see, e.g., Gehrig, 1993). If there are, say, two intermediaries who announce prices q_1 and q_2, $q_1 < q_2$, for their services, expectations determine the agents' decision of which intermediary to contact. For example, if everyone believes that everyone else will contact the high-price intermediary, then it is in everyone's interest to do so. These coordination problems lead to multiple equilibria.

Our short cut in the modelling of price competition among intermediaries allows for strategic behavior without coordination problems, while avoiding indeterminate and trivial outcomes. It is comparable to the techniques used in the literature to restrict the number of equilibria. For example, Gehrig (1993) restricts his attention to the Walrasian equilibrium and Spulber (1996) assumes that prices are observable only after time consuming search. Rochet and Tirole (2003), Caillaud and Jullien (2003), and Amstrong (2006), among others, restrict demand functions. In Ambrus and Argenziano (2009), agents can coordinate on coalitionally rationalizable strategies.

We may consider the setting as a three stage game: In the first stage, the buyers and sellers decide whether to become intermediaries or not. In the second stage the intermediaries post their fees. In the third stage the buyers and sellers contact the intermediaries and trade.

Definition 1. A symmetric equilibrium consists of probabilities σ_S and σ_B to become an intermediary for the sellers and buyers, the fee f posted by the intermediaries such that it is an optimal response to the other intermediaries' fees, and the buyers' and sellers' optimal contact strategies such that the buyers and sellers expect the same utility from all the intermediaries that are contacted with positive probability. Finally, the expected utilities of the intermediaries, buyers and sellers must equal.

The requirement that the buyers and sellers expect the same utility from all intermediaries that are contacted with positive probability corresponds to subgame perfectness requirement. Because there are equally many buyers and sellers, equally many of them, say z, become intermediaries, or $\sigma_S = \sigma_B = z$; otherwise the agents' expected utilities would differ. Consequently, the expected queue lengths of buyers and sellers at an intermediary are $\Omega =$

$(1-z)/(2z)$.

If intermediaries ask some fee f, we assume that a buyer and a seller who are paired share equally the remaining surplus $1-f$.[3] If intermediaries have an unequal number of buyers and sellers, they randomly ration the long side of the market, whereas the short side trades with probability one. Then, a buyer, as well as a seller, expects market utility (MU) given by

$$\sum_{k=1}^{\infty} e^{-\Omega}\frac{\Omega^k}{k!}\left[F_\Omega(k-1)+\frac{k}{\Omega}\left(1-F_\Omega(k)\right)\right]\frac{1-f}{2} \equiv MU, \tag{2}$$

in which the index k indicates the number of sellers who contact an intermediary. Here, and in what follows, we denote the cumulative distribution function of the Poisson distribution with parameter λ by $F_\lambda(k)=\sum_{i=0}^{k} e^{-\lambda}\lambda^i/i!$, i.e., $F_\Omega(k)=\sum_{i=0}^{k} e^{-\Omega}\Omega^i/i!$.

The first expression in the brackets of equation (2) is the probability that a buyer ends up with at most $k-1$ other buyers at the same intermediary, meaning that the buyer gets a good for certain. The second expression is the buyer's probability of trade when k or more other buyers contact the intermediary: if a buyer is visiting an intermediary with k sellers and $i \geq k$ other buyers, the buyer gets a good with probability $k/(i+1)$, and $\sum_{i=k}^{\infty} e^{-\Omega}(\Omega^i/i!)k/(i+1) = k/\Omega \sum_{i=k+1}^{\infty} e^{-\Omega}\Omega^i/i!$.

If an intermediary deviates and asks fee $\widetilde{f}$, the buyers' contact decisions lead to a queue length ω such that buyers expect the market utility MU, and the same applies to sellers. Because the buyers (sellers) take into account only the reactions of the other buyers (sellers), the condition that determines the relation of $\widetilde{f}$ and ω is given by

$$\sum_{k=1}^{\infty} e^{-\Omega}\frac{\Omega^k}{k!}\left[F_\omega(k-1)+\frac{k}{\omega}\left(1-F_\omega(k)\right)\right]\frac{1-\widetilde{f}}{2} = MU. \tag{3}$$

[3] We could allow for a local division of surplus that takes into account the number of buyers and sellers at an intermediary. For instance, there could be auction where the short side of the market gets all the surplus. Our calculations suggest that with auction the contact probability is less elastic, granting more monopoly power to the intermediaries. The intermediaries' fees would on average probably be higher than with the current division rule which would lead to a larger number of intermediaries and, consequently, in a more inefficient outcome.

Comparing equations (2) and (3) shows that, besides the fees, only the terms in the square brackets differ. Totally differentiating equation (3) gives

$$\frac{\partial \omega}{\partial \widetilde{f}} = -\frac{\sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left[F_\omega(k-1) + \frac{k}{\omega}\left(1 - F_\omega(k)\right)\right]}{\sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \frac{k}{\omega^2}\left(1 - F_\omega(k)\right)(1 - \widetilde{f})}, \tag{4}$$

meaning that the queue length at the deviating intermediary is decreasing in the intermediary's fee.

The deviating intermediary's objective is to choose $\widetilde{f}$ to maximize its expected profits, or utility, i.e.,

$$\max_{\widetilde{f}} \sum_{k=1}^{\infty} e^{-\omega} \frac{\omega^k}{k!} \left[\omega F_\omega(k-1) + k\left(1 - F_\omega(k)\right)\right] \widetilde{f}, \tag{5}$$

subject to equation (4). The objective function in expression (5) shows how the intermediary's expected profit is given by the expected number of trades multiplied by the fee per trade: in the intermediary's problem (5) the index in the sum keeps track of the number of sellers, the term in the square brackets displays the expected number of trades for a given number of sellers (k), and the last term is the fee charged by the intermediary. Alternatively, we may think of the expression in the brackets as the quantity that a firm with capacity k expects to sell (see, e.g., Godenhielm and Kultti, 2015); thus, expression (5) corresponds to the expected profits of a firm whose capacity is stochastic.

Using equation (4) to determine the first-order condition for the problem (5) and evaluating it at $\widetilde{f} = f$ gives the equilibrium fee

$$(6) \qquad f^* = \left(\sum_{k=0}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left(1 - F_\Omega(k+1)\right) \right) \Bigg/ \\ \left(- \sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left[\Omega F_\Omega(k-1) + k\left(1 - F_\Omega(k)\right)\right] \right. \\ + \sum_{k=0}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left[\Omega F_\Omega(k) + (k+1)\left(1 - F_\Omega(k+1)\right)\right] \\ \left. + \sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} F_\Omega(k-1) + \sum_{k=0}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left(1 - F_\Omega(k+1)\right) \right).$$

The intermediaries' equilibrium fee f^* balances the competition for the buyers and sellers, and the profitability of intermediation given the somewhat myopic expectations of the agents. Unfortunately, equation (6) does not allow for such a straightforward interpretation as equation (1). Nonetheless, equation (6) shows that the solution with one intermediary cannot be an equilibrium; when Ω, the ratio of contacting buyers (or sellers) to intermediaries, approaches infinity the fee remains positive, meaning that a monopoly intermediary would certainly get larger profits than a buyer or a seller.

As the buyers and sellers are identical in this setting and the meeting rate is given by $\Omega = (1-z)/(2z)$, the equilibrium is determined by equating the buyers' (or sellers') and the intermediaries' expected utilities, i.e., by

$$(7) \qquad \sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left[F_\Omega(k-1) + \frac{k}{\Omega}\left(1 - F_\Omega(k)\right) \right] \frac{1-f^*}{2} \\ = \sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left[\Omega F_\Omega(k-1) + k\left(1 - F_\Omega(k)\right)\right] f^*,$$

in which the left and right hand sides give the buyer's (expected) market utility (2) and the intermediary's expected profits (5), respectively, when evaluated at f^*.

Equations (6) and (7) jointly determine z^*, the equilibrium size of the intermediation sector. We find that z^* is (approximately) given by $z^* \approx 0.211$. Thus, approximately 79% of agents remain as buyers and sellers, and the meeting rate is $\Omega \approx 1.87$.

Result 2. *In equilibrium, some* 21% *of the buyers and sellers become intermediaries.*

We next determine the total number of trades, regarding it as the measure of efficiency of the economy as before. Just ignoring the fee in the expression of the expected profit of an intermediary (in the right-hand side of equation (7)) gives the expected number of trades per intermediary. Multiplying this number by the number of intermediaries yields the expected number of trades in the economy as

$$\sum_{k=1}^{\infty} e^{-\Omega} \frac{\Omega^k}{k!} \left[\Omega F_\Omega(k-1) + k \left(1 - F_\Omega(k)\right) \right] 2z^*.$$

This magnitude turns out to be approximately 0.475. Comparing this figure with the number of trades without intermediation 0.632 (Result 1) suggests that intermediation reduces welfare by roughly 25%.

Result 3. *With intermediation the gains from trade, or the number of trades, is approximately* 0.475*, which is about* 25% *less than without intermediation.*

Intermediaries reduce welfare in our model partly because they consume the resources of the economy; when a buyer or a seller decides to become an intermediary, there are fewer buyers or sellers in the economy. However, the reduction of gains from trade is not only the result of fewer buyers and sellers in the economy. Intermediation activity also attracts too many agents, failing to make the meetings more efficient.

Result 4. *If all the intermediaries are removed from the economy, and the remaining buyers contact the remaining sellers directly the number of trades is approximately* 0.499 *or approximately* 5% *higher than with intermediaries.*

To understand the result, note that the coordination problem with and without intermediaries is similar: Ideally, one should equate the number of buyers and sellers at each trading location, but buyers and sellers cannot coordinate among themselves and some trading locations have more buyers than sellers and the reverse is true for other trading locations. The difference is that with intermediaries, the number of matches per trading location is not restricted to unity. What matters is the difference between the number of buyers and sellers as the minimum determines the number of trades.

When there are only few intermediaries, the local market at each intermediary is thick. In such a case the minimum of the number of buyers and sellers is large, and the intermediaries increase the efficiency of the economy. When the number of intermediaries increases, the local market at each intermediary becomes thinner, reducing the efficiency of intermediation fast for two reasons: first, the total numbers of buyers and sellers become smaller and, second, the remaining buyers and sellers are spread out into more trading locations.

In Appendix B we show that the expected number of trades in the economy with intermediaries is bounded above by

$$(1 - z)\left(1 - e^{-\Omega}\right). \tag{8}$$

This expression can be interpreted as the number of trades in the scenario where, say, all the sellers would be distributed on the intermediaries and whenever even one buyer contacted an intermediary all the possible trades were executed. We may think the first and second terms in expression (8) as reflecting the resource cost and coordination problem with intermediaries. The second term approximates the coordination problem downwards since it fails to fully take into account the uncertainty in the numbers of sellers and buyers contacting an intermediary. Nonetheless, the second term partially captures the negative effect of an increase in the number of intermediaries on the market thickness per intermediary.

As shown in Section 3 the expected number of trades without intermediaries is given by $1 - e^{-1}$. Thus, in an economy from which the intermediaries are discarded and the remaining buyers and sellers trade directly, the expected number of trades is given by $(1 - z)(1 - e^{-1})$. This number is greater than expression (8) for $z \geq 1/3$. The resource cost in both cases is the same but the coordination problem with intermediaries worsens as z

increases whereas it is independent of z when intermediaries are discarded. As a result, once the size of the intermediation sector is sufficiently large, removing intermediaries increases trading. Since expression (8) approximates the coordination problem with intermediaries downwards, the threshold size of z is approximated upwards (cf. Results 2 and 4 and Figure 1 which imply that the true threshold is below 0.21).

Figure 1 illustrates these welfare effects of intermediation. The figure is constructed by allowing the intermediation fee to vary exogenously between zero and unity. To each fee corresponds a unique number of intermediaries such that all the agents' utilities are equal. The solid curve gives the number of trades with intermediaries, and the dashed line the number of trades when those intermediaries are removed from the economy, and the meetings between buyers and sellers take place directly. The diagonal depicts the theoretical maximum number of trades, i.e., $1 - z$.

The intersection of the vertical axis and the solid curve captures the case with one intermediary where the number of trades is at maximum as there is no coordination problem and everyone trades. The dashed line at the vertical axis gives the number of trades when that single intermediary is removed from the economy, which corresponds to the economy without intermediaries of Section 3 (Result 1). At this point, intermediation clearly improves the efficiency of trading in the economy.

Increasing the fee and concurrently the number of intermediaries has two familiar effects: First, it takes resources from the economy, and secondly it makes coordination more difficult. Both of these contribute to the reduction of welfare, as seen from the solid curve. The resource cost is also reflected in the downward sloping dashed line but, thanks to the second effect, the solid curve goes down faster than the dashed line. The point indicated by the yellow triangle shows the number of intermediaries that would generate the same number of trades as the economy without intermediaries (Result 1). At this point, welfare is still higher with intermediaries than if they were removed. At the intersection of the solid curve and dashed line the welfare is equal with intermediares and with their removal. Thus, when the number of intermediaries is sufficiently small, intermediation improves trading efficiency

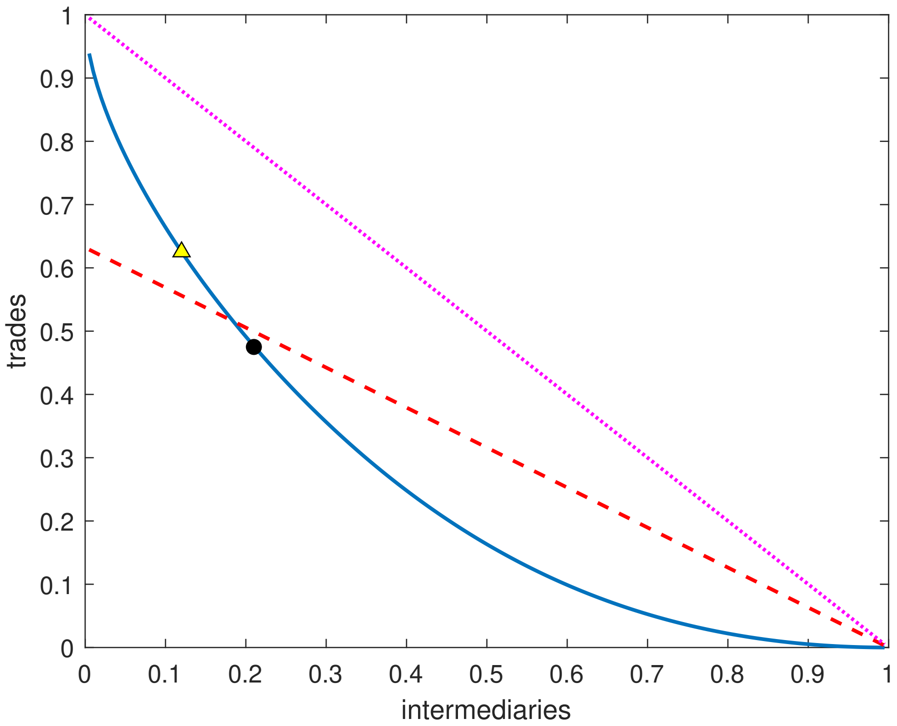

Figure 1: Number of trades and intermediaries with an exogenous fee. The solid curve and dotted line denote the actual and theoretical maximum number trades for a given fraction of intermediaries, respectively, and the dashed line gives the number of trades if that fraction of intermediaries is removed from the economy. The yellow triangle illustrates the amount of intermediaries that would deliver as many trades as the economy of Section 3 where buyers contact sellers directly and the black dot marks the equilibrium with intermediation.

but eventually there are so many intermediaries that the economy would be better off if they were removed.

The fee here works like the bargaining power in Nosal et al. (2016); when the fee is low, we are close to the vertical axis with few intermediaries and when it is high we are close to the right-vertical axis with many intermediaries. In our model, however, the fee is determined as part of equilibrium. That equilibrium is captured by the solid black circle which is to the right of the intersection. The gap in the number of trades between the solid and dashed lines at this point illustrates Result 4. This analysis also indicates how restricting entry to intermediation or intermediaries' pricing could be used to implement a more efficient outcome.

5 Unequal Number of Buyers and Sellers

We now relax Assumption 1 by letting the mass of the buyers be $B > 1$, while keeping the unit mass of sellers; the case for $B = 1 < S$ is analogous. Thus, the mass of buyers B also gives the ratio of buyers to sellers. We provide a numerical description of the main results, relegating the detailed analysis, with the expressions determining the equilibrium to Appendix C. Table 1 assembles the main results of the numerical analysis.

As can be seen from Table 1, when B is not too large, the model behaves largely as before: There is a unique equilibrium in which intermediaries come from amongst both buyers and sellers, though now more buyers than sellers choose to become intermediaries. Intermediaries reduce the efficiency of economy. This case is illustrated by Figure 2 which is constructed by varying exogenously the measure of buyers becoming intermediaries (in the horizontal axis) when $B = 1.2$ (see Appendix C for the details of the construction of Figures 2 and 3). The solid curve depicts the intermediaries' profits, and the dashed line depicts the buyers' profits. The unique equilibrium is at their intersection.

If B is sufficiently large so that all the intermediaries come from the buyers' side, there are two (pure-strategy) equilibria. Figure 3 illustrates this case when $B = 2$. In one of the equilibria, only a small number of buyers become intermediaries, making the economy more efficient than without intermediaries. In this equilibrium the intermediaries ask a low fee which makes intermediation sufficiently unattractive to keep the intermediaries' and buyers' expected utility at the same level. However, the combination of low fee with small size of the intermediation sector makes the equilibrium unstable: As can be seen from Figure 3, a small deviation in the number of intermediaries makes the incentive to become an intermediary drift away from the equilibrium value.

Table 1: Equilibrium outcomes with different amounts of buyers

		Stable				**Unstable**		
B	trades w/o i	y	z	trades	trades i r	y	z	trades
1	0.632	0.211	0.211	0.475	0.499			
1.1	0.667	0.271	0.171	0.499	0.524			
1.2	0.699	0.332	0.132	0.523	0.549			
1.3	0.728	0.392	0.092	0.547	0.574			
1.4	0.753	0.453	0.053	0.571	0.599			
1.5	0.777	0.513	0.013	0.594	0.624			
1.6	0.798	0.554	0	0.613	0.649	0.021	0	1.000
1.7	0.817	0.582	0	0.630	0.673	0.030	0	0.999
1.8	0.835	0.608	0	0.646	0.696	0.040	0	0.999
1.9	0.850	0.632	0	0.663	0.719	0.052	0	0.998
2.0	0.865	0.654	0	0.679	0.740	0.066	0	0.998
2.5	0.918	0.721	0	0.763	0.831	0.167	0	0.991

Notes: This table shows the equilibrium outcomes when we vary the initial number of buyers (the first column) from 1 to 2.5. The columns y and z show the measures of buyers and sellers who become intermediaries, respectively. The columns *trades* and *trades w/o i* show the number of trades with and without intermediation, respectively. The column *trades i r* gives the number of trades when intermediaries are removed and the remaining buyers contact the remaining sellers directly.

The other equilibrium features a large number of buyers becoming intermediaries. This case is similar to the equilibrium in the previous section. There are too many intermediaries in the sense that the economy is less efficient than without intermediaries. The intermediaries also charge a relatively high fee. This equilibrium is stable, as can be seen from Figure 3: A small decrease in the number of intermediaries increases the intermediaries profits, and gives an incentive for more agents to become intermediaries; a small increase has an opposite effect.

Columns 2 and 5 of Table 1 show how increasing B increases the number of trades in the stable equilibrium with intermediaries and without them: increasing the number of buyers makes it less likely that a seller fails to meet a buyer. Less obviously, the efficiency of intermediation (when measured by the ratio of trades with and without intermediaries) first goes down and then up when B increases. When there are so many buyers that no seller becomes an intermediary, intermediation is more efficient than with an equal number of buyers and sellers: When the ratio of buyers to sellers is large, there are

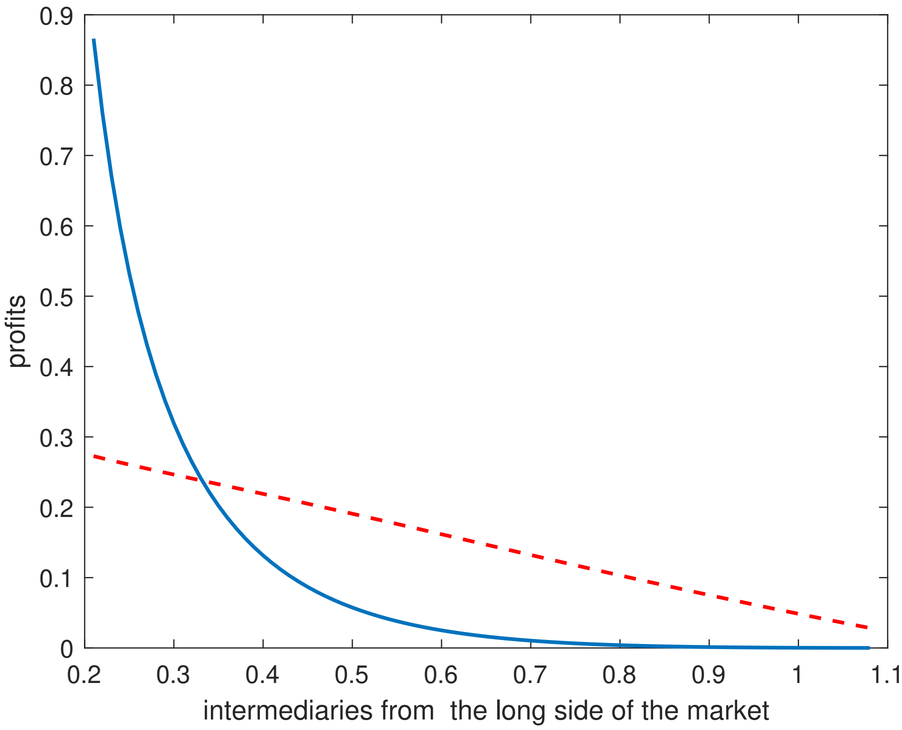

Figure 2: Equilibrium when the ratio of buyers to sellers is 1.2. The solid curve and dashed line denote the intermediaries' and buyers' profits, respectively. Their intersection marks the unique equilibrium.

many buyers who fail to trade without intermediaries. When such buyers become intermediaries, it leads to no loss of resources in the economy.

Columns 5 and 6 allow us to assess the robustness of Result 4 which highlights the coordination problem caused by too many intermediaries. As long as both buyers and sellers choose to become intermediaries the amount of trades (in the stable equilibrium) increases approximately by 5% when intermediaries are removed from the economy and the remaining buyers contact sellers directly. When only buyers choose to become intermediaries, intermediation wastes less resources and the coordination problem becomes relatively worse and these effects are amplified when B increases; e.g., removing intermediaries increases the amount of trades approximately by 6% when $B = 1.6$ and by 9% when $B = 2.5$.

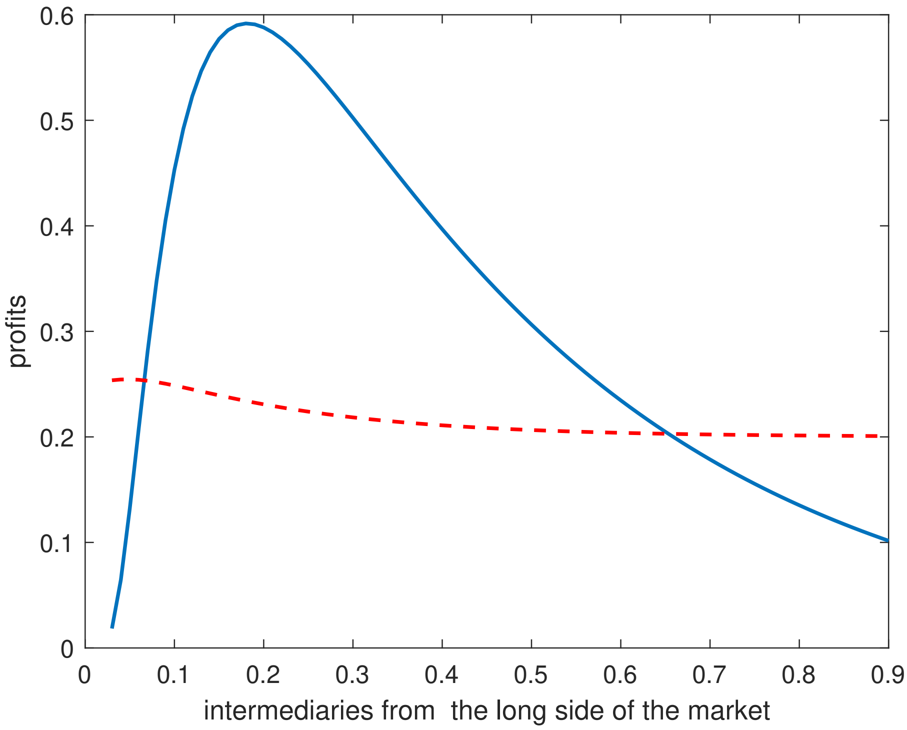

Figure 3: Equilibria when the ratio of buyers to sellers is 2. The solid curve and dashed line denote the intermediaries' and buyers' profits, respectively. Their intersections mark the two (pure-strategy) equilibria.

6 Conclusion

We investigate a canonical directed search model of buyers and sellers, and introduce intermediation there as a pure coordination or a platform service. A key feature of our model is the buyers' and sellers' choice of whether or not to become an intermediary. That choice means that more intermediaries can only come at the cost of fewer buyers and sellers. As a result, an increase in the number of intermediaries has two effects on the aggregate number of trades: First, it reduces the aggregate supply and demand. Second, it spreads out the remaining supply and demand into more trading locations. In this setting intermediation activity is so lucrative that, in equilibrium, it attracts far too many intermediaries, eliminating the gains from their coordination services. While this result may be specific to our setting, the two effects underlying the result should be part of any study of welfare effects of intermediation.

Thinking of becoming an intermediary as an occupational choice bears no exogenous entry costs in our model. One could also consider a partial equilibrium model with a continuum of intermediaries who could freely enter into the economy by paying a fixed cost. At some level of the fixed cost the equilibrium in such a model would feature the same meeting efficiency as our in model but the resource costs would differ. Conversely, at some other level of fixed costs the resource costs would be the same as in our model but the meeting efficiency would differ.

Introducing a fixed entry cost to intermediation in our model would not change the mechanism of the model, but it could be used as a policy instrument to discourage entry into intermediation as in equilibrium there are too many intermediaries. Reducing the entry cost would result in more intermediaries just like in the partial equilibrium set up. In our model, however, this reduction in entry cost would increase the price of brokerage services as the number of intermediaries and their fees move to the same direction. This result is the general equilibrium effect that arises as all occupational choices have to result in the same expected utility. In the partial equilibrium model the relation would most probably be just the opposite.

We focus on coordination problems arising from the sellers' capacity constraints and the buyers' symmetric contact strategies. Laboratory evidence directly supports the predictions of this set up (see, e.g., Helland et al., 2017). In practice, these coordination problems also appear to be familiar in many markets where sellers are capacity constrained and buyers at least partially mix. For example, Fréchette, Lizzerri and Salz (2019) provide data from the New York City showing that the fraction of time taxis spend waiting for passengers as empty varies between 30% and 65%. They also estimate that passengers spend waiting for a vacant taxi over four minutes on average on a rush hour. Many other markets with capacity constrained sellers and at least partially mixing buyers (e.g., fresh food, housing, labor, marriage, and some special professional services such as hairdressers, physicians and therapists) exhibit similar coordination problems.

In some of these markets a centralized intermediation service has emerged as a solution to the coordination problems (e.g., the National Resident Matching Program to match medical students to hospitals in the U.S. or centralized taxi dispatch in some cities) but in many markets there are no such centralized platforms. Traditionally sellers (of similar items) in these markets have mitigated these coordination problems, for example, via clustering to the same location either independently or by using intermediaries (e.g., clinics, market squares, real estate agents, taxi stands, and shopping malls). With the rise of the Internet and smart phones, physical clustering has become less relevant, and establishing intermediaries and platforms cheaper. According to our reasoning, despite the efficiency considerations suggesting a tendency towards concentration, markets where coordination services are easy to establish might be vulnerable to the creation of overly many platforms. Our paper also hints that the traditional regulatory tools such as entry regulation and price caps may be useful remedies to the problem of too many platforms.

In a desire to focus on the benefits and costs of the intermediaries' coordination services, we have made a number of short cuts that should be addressed in the future work. We simplify the price setting game between intermediaries by assuming that a trading agent only takes into account the effect of the intermediary's fee on the agent's own side of the market. Our simplification differs from those used in the literature by providing a possibility to analyse intermediary price competition without having to worry about coordination issues. However, it probably underestimates both the profitability and scale advantages of intermediation. Our simplification may thus underestimate both the incentive to become an intermediary (in which case the welfare costs of intermediation in our model are biased downwards) and a tendency towards concentration in the intermediation sector (in which case our welfare costs of intermediation are biased upwards). While we cannot deem whether our short cut is more or less realistic than those used in the literature, it is at least conceivable that in some markets agents fail to consider price effects on the other side of the market, while paying attention to the other features of the other side that are more visible and easier to understand such as the aggregate number of agents or applications.

Following Spulber (1996) we assume that buyers and sellers must use intermediaries if they exist. Allowing for the co-existence of direct and brokered trade, as e.g., in Fingleton (1997b) and Gautier et al. (2017), would – while desirable – make the characterization of equilibria more complicated and require a study of its own.

To sketch some ideas, assume that the ratio of buyers to sellers in a market with direct trading is some θ_1. Then the buyers' expected utility is $e^{-\theta_1}$ and the sellers' expected utility $1 - e^{-\theta_1} - \theta_1 e^{-\theta_1}$.[4] As long as both buyers and sellers choose to become intermediaries in the brokered market all the agents' expected utilities have to equal in equilibrium. In particular, in the market with direct trading we must have $e^{-\theta_1} = 1 - e^{-\theta_1} - \theta_1 e^{-\theta_1}$. The condition is satisfied by $\theta_1 \approx 1.146$, and it pins down the expected utility of all agents. At least for some parameter values there would probably exist three equilibria: Direct trading only, intermediation only, and their coexistence. One would also expect direct trading to be an option reducing the market power of the intermediaries and leading to a more efficient outcome.

Finally, we abstract from a number of benefits and costs of intermediation studied in the literature. Our example suggests that competitive provision of coordination services with free entry is so inefficient that it should not survive unless it performs some other functions, too. Thus, competitive coordination services with free entry might be more about overcoming informational problems than about overcoming inefficiencies of meeting technologies.

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[4]The sellers' expected utility is the equilibrium price p of equation (1) multiplied by the sellers' probability of trade $1 - e^{-\theta_1}$, and the buyers' expected utility can be obtained by multiplying $1 - p$ by their probability of trade $(1 - e^{-\theta_1})/\theta_1$.

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Appendix

A Derivation of Equation (1)

We sketch a derivation for the necessary condition for the equilibium price (1) (see Kultti, 1999, for more details). Assume that the equilibrium price is p, and that there are b buyers and s sellers in the economy. The number of trades is given by $\left(1-e^{-\theta}\right)s$, in which $\theta = b/s$ as before.

A buyer trades with probability

$$\frac{\left(1-e^{-\theta}\right)s}{b} = \frac{1-e^{-\theta}}{\theta},$$

and expects to get

$$\text{(A1)} \qquad \frac{1-e^{-\theta}}{\theta}(1-p) \equiv MU,$$

in which MU refers to market utility.

If a seller deviates to some other price $\widetilde{p}$, then she expects the queue length $\widetilde{\theta}$, which guarantees the buyers market utility of

$$\text{(A2)} \qquad MU = \frac{1-e^{-\widetilde{\theta}}}{\widetilde{\theta}}(1-\widetilde{p}).$$

The deviating seller's problem is

$$\max_{\widetilde{p}}(1-e^{-\widetilde{\theta}})\widetilde{p},$$

which may be rewritten by using equation (A2) as

$$\max_{\widetilde{\theta}} 1 - e^{-\widetilde{\theta}} - \widetilde{\theta} MU.$$

The first order condition for this problem is given by

$$e^{-\widetilde{\theta}} = MU. \tag{A3}$$

If p is an equilibrium price, then the optimal deviation is $\widetilde{p} = p$ or $\widetilde{\theta} = \theta$. As a result, equations (A1)-(A3) imply that

$$e^{-\theta} = \frac{1 - e^{-\theta}}{\theta}(1 - p).$$

Solving this equation for p gives formula (1) of the main text.

B Derivation of Expression (8)

In this appendix we provide an alternative formula for the the expected number trades in the economy with intermediaries. This formula allows us to establish the analytically tractable upper bound for the expected number trades given by expression (8) of the main text. At an intermediary the number of trades is the minimum of the number of contacting buyers and sellers. We derive the expected value of this minimum. Let random variables X and Y denote the number of buyers and sellers contacting an intermediary. For clarity of derivation, assume that X and Y have Poisson distributions with parameters λ and μ, respectively; in the end we evaluate the expectation at $\lambda = \mu = \Omega$. Then, the expected number of trades at an intermediary is given by

$$\begin{aligned} \mathbb{E}\left(min\left\{X, Y\right\}\right) &= \mathbb{E}\left(\frac{X + Y - \mid X - Y \mid}{2}\right) \\ &= \frac{1}{2}\,\mathbb{E}\left(X\right) + \frac{1}{2}\,\mathbb{E}\left(Y\right) - \frac{1}{2}\,\mathbb{E} \mid X - Y \mid . \end{aligned} \tag{B1}$$

We develop an expression for the last term of equation (B1), $\mathbb{E} \mid X - Y \mid$.

We have

$$\begin{aligned}\mathbb{E}\mid X-Y\mid) &= \sum_{k=0}^{\infty}\frac{\lambda^k}{k!}e^{-\lambda}\sum_{l=0}^{\infty}\frac{\mu^l}{l!}e^{-\mu}\mid k-l\mid \\ &= e^{-\lambda-\mu}\sum_{k=0}^{\infty}\sum_{l=0}^{k}\frac{\lambda^k\mu^l}{k!l!}(k-l)+e^{-\lambda-\mu}\sum_{k=0}^{\infty}\sum_{l=k}^{\infty}\frac{\lambda^k\mu^l}{k!l!}(l-k)\,. \qquad \text{(B2)}\end{aligned}$$

Exchanging the order of summation in the first term in the second row of equation (B2) yields

$$\mathbb{E}\mid X-Y\mid) = e^{-\lambda-\mu}\sum_{l=0}^{\infty}\sum_{k=l}^{\infty}\frac{\lambda^k\mu^l}{k!l!}(k-l)+e^{-\lambda-\mu}\sum_{k=0}^{\infty}\sum_{l=k}^{\infty}\frac{\lambda^k\mu^l}{k!l!}(l-k)\,. \qquad \text{(B3)}$$

By manipulating further the right-hand side of equation (B3) we get

$$\begin{aligned}\mathbb{E}\mid X-Y\mid) &= e^{-\lambda-\mu}\sum_{l=0}^{\infty}\sum_{k=l}^{\infty}\frac{(\lambda^k\mu^l+\lambda^l\mu^k)(k-l)}{k!l!} \\ &= e^{-\lambda-\mu}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\lambda^{k+l}\mu^l+\lambda^l\mu^{k+l})k}{(k+l)!l!} \\ &= e^{-\lambda-\mu}\sum_{k=0}^{\infty}k\left(\lambda^k+\mu^k\right)\sum_{l=0}^{\infty}\frac{\lambda^l\mu^l}{(k+l)!l!} \\ &= e^{-\lambda-\mu}\sum_{k=0}^{\infty}k\left(\left(\frac{\lambda}{\mu}\right)^{k/2}+\left(\frac{\mu}{\lambda}\right)^{k/2}\right)I_k\left(2\sqrt{\lambda\mu}\right), \qquad \text{(B4)}\end{aligned}$$

in which the first equality follows from symmetry, and the second one from starting the index of the second sum from zero, and $I_k(x)$ is the modified Bessel function of the first kind.

Inserting $\lambda=\mu=\Omega$ in the last row of equation (B4) gives

$$\mathbb{E}\mid X-Y\mid) = e^{-2\Omega}\sum_{k=0}^{\infty}k2I_k(2\Omega)\,. \qquad \text{(B5)}$$

Using $\lambda=\mu=\Omega$ and equation (B5) in equation (B1) allows us to write the expected number of trades at an intermediary as

(B6) $$\mathbb{E}\left(min\left\{X,Y\right\}\right)=\Omega-e^{-2\Omega}\sum_{k=0}^{\infty}kI_{k}\left(2\Omega\right).$$

The interpretation of equation (B6) is that with no uncertainty about the numbers of buyers and sellers contacting an intermediary the expected number of trades would be Ω or the expected queue length. The negative term comes from the difference between the numbers of buyers and sellers.

Multiplying equation (B6) by the number of intermediaries, $2z$, gives the expected number of trades in the economy with intermediaries (recall that $\Omega=(1-z)/(2z)$):

(B7) $$2z\,\mathbb{E}\left(min\left\{X,Y\right\}\right)=1-z-2ze^{-2\Omega}\sum_{k=0}^{\infty}kI_{k}\left(2\Omega\right).$$

A result by Luke (1972) gives a lower bound for the modified Bessel function of the first kind as $I_k(x)>x^k/(2^k k!)$, which implies that $I_k\left(2\Omega\right)>\Omega^k/k!$. After substituting $\Omega^k/k!$ for $I_k(2\Omega)$ in equation (B7) we find that the expected number of trades in the economy with intermediaries is bounded above by

$$\left(1-z\right)\left(1-e^{-\Omega}\right),$$

which is expression (8) of the main text. The second term in this expression approximates the coordination problem with intermediaries downwards; it gives the probability of a seller trading under the assumption that the seller always trades whenever at least one buyer contacts the seller's intermediary.

C Unequal Number of Buyers and Sellers

Let us relax Assumption 1 by assuming that the measure of buyers is $B>1$, while keeping the unit mass of sellers; the case for $1=B<S$ is analogous. Thus, B also gives the ratio of buyers to sellers.

Denote the measure of sellers who become intermediaries by z and that of buyers by y. Denote the queue length that the intermediaries expect from the buyers by $\alpha = (B - y)/(z + y)$, and the queue length they expect from the sellers by $\beta = (1 - z)/(z + y)$.

Let us adopt some more notation to keep the analysis tractable. Denote

$$\sum_{h=m}^{\infty} e^{-\lambda} \frac{\lambda^h}{h!} \equiv T^h_{\lambda,m},$$

and

$$F_\lambda(h-1) + \frac{h}{\lambda}\left(1 - F_\lambda(h)\right) \equiv [\,]_{\lambda,h},$$

in which F_λ is the distribution function of the Poisson-λ distribution.

Assume that in equilibrium the intermediaries post fee f. A buyer expects utility

$$T^k_{\beta,1}[\,]_{\alpha,k}\frac{1-f}{2} \equiv MU_b,$$

and, similarly, a seller expects utility

$$T^k_{\alpha,1}[\,]_{\beta,k}\frac{1-f}{2} \equiv MU_s.$$

Here the terms $T^k_{\beta,1}[\,]_{\alpha,k}$ and $T^k_{\alpha,1}[\,]_{\beta,k}$ give the probability that a buyer and a seller, respectively, is able to trade, and $(1-f)/2$ is their surplus conditional on trading.

We derive the equilibrium fee assuming that some intermediary deviates, and posts fee $\tilde{f}$. As in the main text the buyers take into account how the fee affects the other buyers but ignore its impact on the sellers. Then the buyers expect the other buyers to behave in such a way that the resulting queue length is $\tilde{\alpha}$ which yields them the market utility MU_b. After substituting $\tilde{\alpha}$ for α and $\tilde{f}$ for f in the formula for the buyers' utility MU_b one can totally differentiate it to get $\partial\tilde{\alpha}/\partial\tilde{f}$. Evaluating $\partial\tilde{\alpha}/\partial\tilde{f}$ at equilibrium $\tilde{f} = f$ yields the following expression

$$\text{(C1)} \qquad \frac{\partial\tilde{\alpha}}{\partial\tilde{f}} = \frac{T^k_{\beta,1}[\,]_{\alpha,k}}{T^k_{\beta,1}\frac{k}{\alpha^2}\left(1 - F_\alpha(k)\right)(1-f)}.$$

Analogous derivation using the formula for the sellers' utility MU_s yields

$$\text{(C2)} \qquad \frac{\partial \tilde{\beta}}{\partial \tilde{f}} = -\frac{T^k_{\alpha,1}[\,]_{\beta,k}}{T^k_{\alpha,1}\frac{k}{\beta^2}\left(1 - F_\beta\left(k\right)\right)\left(1 - f\right)}.$$

A deviating intermediary's problem is given compactly by

$$\text{(C3)} \qquad \max_{\tilde{f}} \tilde{\alpha} T^k_{\tilde{\beta},1}[\,]_{\tilde{\alpha},k}\tilde{f},$$

in which $\tilde{\alpha} T^k_{\tilde{\beta},1}[\,]_{\tilde{\alpha},k}$ gives the expected number of trades and $\tilde{f}$ is the intermediary's fee per trade. By using equations (C1) and (C2) the first order condition for the problem (C3) with respect to $\tilde{f}$ evaluated at $\tilde{f} = f$ can be written as

$$\begin{aligned}
&\alpha T^k_{\beta,1}[\,]_{\alpha,k}\frac{T^k_{\alpha,1}[\,]_{\beta,k}}{T^k_{\alpha,1}\frac{k}{\beta^2}\left(1 - F_\beta\left(k\right)\right)\left(1 - f\right)}f \\
&- \alpha T^k_{\beta,0}[\,]_{\alpha,k+1}\frac{T^k_{\alpha,1}[\,]_{\beta,k}}{T^k_{\alpha,1}\frac{k}{\beta^2}\left(1 - F_\beta\left(k\right)\right)\left(1 - f\right)}f \\
&- T^k_{\beta,1}F_\alpha\left(k-1\right)\frac{T^k_{\beta,1}[\,]_{\alpha,k}}{T^k_{\beta,1}\frac{k}{\alpha^2}\left(1 - F_\alpha\left(k\right)\right)\left(1 - f\right)}f \\
&+ \alpha T^k_{\beta,1}[\,]_{\alpha,k} = 0.
\end{aligned}$$

From this equation one solves for f, which gives the equilibrium fee as

$$\text{(C4)} \qquad f = \frac{N(\alpha,\beta)}{D(\alpha,\beta)},$$

in which the numerator is

$$N(\alpha,\beta) \equiv \alpha T^k_{\beta,1}[\,]_{\alpha,k} T^k_{\alpha,1}\frac{k}{\beta^2}\left(1 - F_\beta\left(k\right)\right) T^k_{\beta,1}\frac{k}{\alpha^2}\left(1 - F_\alpha\left(k\right)\right),$$

and the denominator is

$$
\begin{aligned}
D(\alpha,\beta) &\equiv -\alpha T^k_{\beta,1}[\,]_{\alpha,k} T^k_{\alpha,1}[\,]_{\beta,k} T^k_{\beta,1} \frac{k}{\alpha^2}\left(1 - F_\alpha(k)\right) \\
&+ \alpha T^k_{\beta,0}[\,]_{\alpha,k+1} T^k_{\alpha,1}[\,]_{\beta,k} T^k_{\beta,1} \frac{k}{\alpha^2}\left(1 - F_\alpha(k)\right) \\
&+ T^k_{\beta,1} F_\alpha(k-1)\, T^k_{\beta,1}[\,]_{\alpha,k} T^k_{\alpha,1} \frac{k}{\beta^2}\left(1 - F_\beta(k)\right) \\
&+ \alpha T^k_{\beta,1}[\,]_{\alpha,k} T^k_{\alpha,1} \frac{k}{\beta^2}\left(1 - F_\beta(k)\right) T^k_{\beta,1} \frac{k}{\alpha^2}\left(1 - F_\alpha(k)\right).
\end{aligned}
$$

If both z and y are positive, then all the agents have to do equally well. For the buyers this requirement means that

$$\alpha T^k_{\beta,1}[\,]_{\alpha,k} f = T^k_{\beta,1}[\,]_{\alpha,k} \frac{1-f}{2} \tag{C5}$$

and for the sellers that

$$\alpha T^k_{\beta,1}[\,]_{\alpha,k} f = T^k_{\alpha,1}[\,]_{\beta,k} \frac{1-f}{2}. \tag{C6}$$

In equations (C5) and (C6) the intermediaries' expected profits are in the left hand sides, and the right hand sides consist of the buyers' and sellers' utility, respectively. Equations (C5) and (C6) imply that $\alpha = \beta = (1-f)/(2f)$. Since $\alpha = (B-y)/(z+y)$ and $\beta = (1-z)/(z+y)$, we have $B - y = 1 - z$. Since $B > S = 1$, we see that $y > z$, i.e., more buyers than sellers become intermediaries. If B is sufficiently large, then only buyers become intermediaries.

We solve the model by first equating the intermediaries and the buyers utilities, when only buyers are allowed to become intermediaries. For this equilibrium candidate we then check whether the sellers utility is at least as large as the intermediaries utility. If this inequality does not hold, we allow both buyers and sellers to become intermediaries and find y and z such that the expected utilities are equal for all parties.

Figure 2 is constructed by inserting the equilibrium condition $z = 1 + y - B$, $B = 1.2$, and the equilibrium fee (C4) into the intermediaries' expected profit and buyers' utility (the left and right hand sides of equation (C5), respectively). Restricting $z \in \mathbb{R}_+$, this exercise gives us the intermediaries' profit and buyers' utility as functions of y, the measure of buyers becoming intermediaries. In Figure 2 the vertical axis captures the profit and utility levels, and the horizontal axis captures y. The solid curve and dashed line depict the intermediaries' expected profit and buyers' utility, respectively. Equation (C5) holds at the intersection of the solid curve and dashed line in Figure 2. By construction, also equation (C6) holds at this point.

Figure 3 is constructed analogously by inserting $z = 0$, $B = 2$, and the equilibrium fee (C4) into the intermediaries' expected profit and buyers' utility. In this case, equation (C6) never holds.