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From thin objects to thin concepts?

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Abstract

In this short paper we consider Linnebo's thin/thick dichotomy: first, we show that it does not overlap with the very common one between abstract/concrete objects; second, on the basis of some difficulties with the distinction, we propose, as a possible way out, to move from thin/thick objects to thin/thick concepts.

1 | INTRODUCTION

In his wide-ranging and deep *Thin Objects*, (Linnebo, 2018), advances an abstractionist account in the philosophy of mathematics. One of the conceptual cores of his work is the distinction between *thin* and *thick* objects. The characterisation he provides is as follows:

(Thin)　An object is *thin* if it does not require very much to the world in order to exist (*Preface, xi*)

Analogously, and unsurprisingly, he adds the following characterisation of thick objects:

(Thick)　An object is *thick* if it makes substantial requests to the world in order to exist.

As it is easy to observe, this is a preliminary intuitive characterisation, grounded on the concept of 'request to the world', which deserves a deeper analysis (for a comparison, see Rayo, 2013). Of course, Linnebo's proposal is not limited to introducing the thin/thick distinction, but it provides a logical engine that allows him to account for thin objects. This device is constituted by abstraction principles (AP).

In this short paper we argue for two points. First, we show that the dichotomy thin/thick—contrary to some *prima facie* evidence—does not overlap with the very common in philosophy of mathematics dichotomy abstract/concrete. Second, we sketch a possible extension of Linnebo's original intuition of thin/thick towards the notion of conceptual grounding. We proceed as

follows: first, we introduce the *abstraction principles* (AP); then, we isolate some difficulties of Linnebo's use of them for the thin/thick distinction; finally, we sketch the proposal to move from thin/thick objects to thin/thick concepts.

2 | ABSTRACTION PRINCIPLES AND THE DISTINCTION THIN/ THICK OBJECTS

According to Linnebo, the logical machinery that allows us to introduce the distinction thin/ thick objects are so-called *abstraction principles*, for short (AP). Therefore, given the importance of (AP) for the issue we are dealing with, we use this section to briefly remind the reader what abstraction principles are.

In a nutshell, an easy way to introduce (AP) (on this topic, see the seminal work of Hale & Wright (2001)) is by the Fregean notion of identity criteria. Frege suggests that an identity criterion has the function of providing a general way of answering the following question:

(Q)   how can we know whether *a* is identical to *b*?

Frege observes that:

> If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*, even if it is not always in our power to apply this criterion. (see Frege, 1884, §62)

The example of identity criterion, proposed by Frege, takes parallelism as providing the identity of directions of lines. He remarked that:

> The judgement 'line *a* is parallel to line *b*' … can be taken as an identity. If we do this, we obtain the concept of *a* direction, and say: 'the direction of line *a* is identical with the direction of line *b*'. (see Frege, 1884, §64)

The proposed example suggests that the question to be answered by an identity criterion can be stated in the following general way:

(OQ)   If *a* and *b* are *Ks*, what is it for the object *a* to be identical to *b*?

But if we review the first passage quoted from Frege and (Q), one could observe that the ontological reading formulated above is not the only one. There is a second plausible reading of an identity criterion, which can be expressed in the following terms:

(EQ)   If *a* and *b* are *Ks*, how can we know that *a* is the same as *b*?

The identity criterion, in this perspective, concerns the knowledge of identity between entities *a* and *b* of a sort *K*. Following Frege's second quotation and (OQ), the Fregean criterion for directions can be expressed thus:

(1)   The direction of line *a* is identical to the direction of line *b* if, and only if, line *a* is parallel to line *b*

and can be usefully formalized in the following way:

(D) $\forall x \forall y (o(x) = o(y) \leftrightarrow P(x,y))$

where 'x' and 'y' range over lines, 'o' is a letter for 'the direction of' and 'P' for 'is parallel to'. In (D), the identity sign is flanked by terms constructed with a functional letter, while the right-hand side of the biconditional introduces a relation among entities with no functional letter.

Frege introduces the idea of identity criteria in a context where he wonders how we can grasp or formulate the concept of *number* and he thinks of the identity criterion for numbers as an explication of the sense of:

(2) The number which belongs to the concept F is the same as that which belongs to the concept G.[1]

An identity criterion for (2) is developed in the same lines of (D): the identity sign is flanked by terms constructed with a functional letter, and the right-hand side of the biconditional introduces a relation among entities different from the entities for which the criterion is formulated (in (2) in terms of equinumerosity).

According to Williamson, (D) is an example of what he calls *a two-level identity criterion* (Williamson, 1990, p. 145–146). In the case of *two-level identity criteria*, the conditions of identity concern objects that are *not* of the same kind of objects for which the identity criterion is provided. On the contrary, in the case of *one-level* identity criteria—e.g. the *axiom of extensionality* for sets—the conditions of identity concern objects that *are of* the same kind of objects for which the identity criterion is provided. As Williamson points out:

> The idea of a two-level criterion of identity has an obvious advantage. No formula could be more basic (in any relevant sense) than '$x = y$', but some might be more basic than '$ox = oy$', by removing the symbol 'o' and inserting something more basic than it. (Williasmon, 1990, p.147)

Linnebo follows Frege who introduces (1) as an identity criterion. Moreover, on Williamson's distinction he observed that:

> From a logical point of view, a two-level criterion of identity is just the same as an abstraction principle. (Linnebo, 2018, p. 35)

So, following Linnebo's above quotation, abstraction principles (AP) *are*, from a logical point of view, two-level identity criteria.

Notice, by the way, that the (AP) discussion has also touched upon the issue of the functions that can be attributed to these principles. One can indeed think they have an epistemic function, as principles governing certain concepts and their relations; or a semantic function, as tools for

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[1] Frege was ultimately dissatisfied with this route to identity criteria. He observes that in the sentence 'the direction of line a is identical to the direction of line b' the direction of a plays the part of an object, and our definition affords us a means of recognising this object as the same again, in case it should happen to crop up in some other guise, say as the direction of b. But this means does not provide for all cases … That says nothing as to whether the proposition: 'the direction of line a is identical to q' should be affirmed or denied, except for the one case where q is given in the form of 'the direction of b' (see Frege, 1884,§66). The reason is that, in Frege's opinion, the nature of certain objects is entirely clarified only if one can find a way to refer to them such that it would determine the truth-value of any identity sentence concerning the given objects, without any restriction. For these reasons, one could say, for Frege, (OQ) should be better reformulated in this way:
(OQ1) If a is K, what is it for the object a to be identical with b?
Now, the probelm is: What do we need to obtain the universal definiteness of identity questions concerning a K? Frege is absolutely clear about this: we need the concept of K ('What we lack is the concept of direction'; Frege (1884, §66)). But this amounts to acknowledging that a criterion of identity for objects of the K-kind does not provide the concept of K which it was supposed to provide. Hence, Frege gives up the plan of obtaining the concepts of direction and of number from the corresponding identity criteria.

specifying reference to certain classes of entities; and, finally, a metaphysical function as principles making explicit grounding and dependence relations between certain entities.

How does Linnebo exploit (AP) in order to carry on his project? Linnebo's strategy is summarised as follows:

> My strategy for making sense of thin objects has a simple structure. I begin with the Fregean idea that an object, in the most general sense of the word, is a possible referent of a singular term. The question of what objects there are is thus transformed into the question of what forms of singular reference are possible. This means that any account that makes singular reference easy to achieve makes it correspondingly easy for objects to exist. A second Fregean idea is now invoked to argue that singular reference can indeed be easy to achieve. According to this second idea, there is a close link between reference and criteria of identity. Roughly speaking, it suffices for a singular term to refer that the term has been associated with a specification of the would-be referent, which figures in an appropriate criterion of identity. For instance, it suffices for a direction term to refer that it has been associated with a line and is subject to a criterion of identity that takes two lines to specify the same direction just in case they are parallel. In this way, the second Fregean idea makes easy reference available. And by means of the first Fregean idea, easy reference ensures easy being. (Linnebo, 2018, xii)

In a nutshell, Linnebo explains (1) the notion of object as reference of a singular term; and (2) the reference of singular terms by means of identity criteria. And (AP) are special types of identity criteria:

> My preferred way of understanding abstraction principles is simply as a special type of criterion of identity. (Linnebo, 2018, xiii)

Linnebo defends a claim that he calls *Reference by Abstraction*, according to which if we have a predicative abstraction principle (where 'on this form of abstraction, any question about the "new" abstracta can be reduced to a question about the "old" on which we abstract. A paradigm example is the case of directions where we abstract on lines to obtain their directions' Linnebo (2018, xiii)) and the relation of equivalence on its right-hand side holds, then the singular terms on the left part (e.g.: 'the direction of *a*') refer to abstract objects.

Linnebo's second thesis (2) is connected to the idea of *reconceptualisation*:

> The material made available by the explanans [for instance, the parallelism between lines] is reconceptualized in a way that brings out the existence of [directions]. (Linnebo, 2018, p. 30)

In this way 'a truth about parallelism is thus reconceptualized in a way that reveals a new object, namely a direction, which was not involved in the original truth' (Linnebo, 2018, p. 23). This new object is a *thin* object. Putting together the two theses (1) and (2), we have that

(i) the singular terms on the left side refer to objects;
(ii) if the right side is true, then this fact 'suffices' (Linnebo, 2018, 11) for the existence of the objects which we refer to on the left side.

This completes Linnebo's abstractionist path to thin objects. It is worth noticing that (AP) provides both a device of reference and an existential condition for thin objects.

3 | THIN/THICK OBJECTS AND METAPHYSICAL GROUNDING

The relational character of the thin/thick distinction naturally connect them to another flourishing and widespread notion in the contemporary literature, namely the notion of metaphysical grounding. Grounding is one of the most discussed notions in contemporary philosophy. Roughly, grounding is a type of non-causal, primitive relation (or operation) such that the grounded entities, usually facts, are somehow explained, determined or constituted by the grounding entities. The *grounding revolution* (Schaffer, 2016, p. 91) contributed to clarifying the meaning of locutions such as 'in virtue of' and 'because'. (The literature on grounding is massive. Some basic and introductory papers on it are: (Fine, 2002); (Fine, 2012); (Clark & Liggins, 2012); (Trogdon, 2013); and (Bliss & Trogdon, 2021).)

In particular, the relational feature of the thin/thick distinction might lead one to think that thin and thick objects are linked to each other by a metaphysical grounding relation: the (equivalence relations holding between) thick objects ground (the identity and distinctness of) thin objects. Let us take, as an example, the thick object corresponding to the thick fact that a and b are parallel, and, as a thin object, the fact that a and b have the same direction; then one might be willing to claim that the thick object of two lines being parallel grounds the thin object of two lines having the same direction (see Linnebo, 2018, p. 18).[2] Although this might sound as a natural and interesting connection, as has been discussed at length by (Carrara & De Florio, 2020), several problems arise when the (AP) are interpreted in terms of metaphysical grounding and thus, as it stands, it is not a viable road. Let us briefly resume why.

There is at least one *prima facie* reason for reading (AP) as grounding metaphysical principles: to say that x and y are distinct objects, or the same object, seems to imply that there is something *in virtue of which* x and y are distinct objects/are the same object, i.e. a fact that *grounds* the distinctness or identity of the object(s) at play.

A first kind of problem of this reading of (AP) as *grounding principles* has to do with the *discrimination* of the facts involved in (AP).

First, let us establish some terminology. We use square brackets to denote facts: if A is a sentence, $[A]$ is the fact that A. By angle brackets $\langle \dots \rangle$ we denote propositions; so, $\langle A \rangle$ is the proposition that A. To indicate the grounding relation, we will use the symbol $\rhd$.

Then, the problem is that the facts occurring in (AP) must be distinct. That is, generally speaking, if

$$[A] \rhd [B]$$

then the fact that A must be different from the fact that B; if not, we lose the anti-reflexivity of the grounding relation. An analogous point is considered in (Rosen, 2010), pp. 123–124. According to Rosen, a good guide for finding grounding relations among facts is given by the reduction relations among propositions. He sums up this strategy in the so-called *Grounding-Reduction Link*:

> If $\langle p \rangle$ is true and $\langle p \rangle \Leftarrow \langle q \rangle$, then $[q] \rhd [p]$

In words, if the proposition that p is true and *p's being the case consists in q's being the case*, then the fact that q grounds the fact that p.

Now, construing (AP) as reductions à la Rosen is perfectly plausible; in this case, we would have the following:

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[2]As a reviewer observed, the moving from thick/thin objects to thick/thin facts might not be accepted by Linnebo: he might not be happy with this extension of the notion of thick and thin objects to facts. Indeed, if the paradigm thin object was 'the direction of a', now we have a new thin object 'the fact that a and b have the same direction'. But why should he not accept a commitment to facts? A commitment to these entities does not seem to be more demanding than a commitment to objects alone.

If $\quad \langle d(a) = d(b) \rangle \quad$ is $\quad$ true $\quad$ and $\quad \langle d(a) = d(b) \rangle \Leftarrow \langle Par(a,b) \rangle$, then $[Par(a,b)] \rhd [d(a) = d(b)]$.

In words, if it is true that the direction of a is identical to the direction of b, and if their identity consists in being parallel a and b, then the fact that the direction of a is identical to the direction of b is grounded on the fact that a and b are parallel. However, things are not so easy:

> […] The [Grounding-Reduction] Link presents us with a real puzzle. After all, if our definition of square is correct, then surely the fact that $ABCD$ is a square and the fact that $ABCD$ is an equilateral rectangle are not different facts; they are one and the same. But then the grounding-Reduction Link must be mistaken, since every instance of it will amount to a violation of irreflexivity. (Rosen, 2010, p. 124)

Mutatis mutandis the same type of criticism is at work for the direction case. Is there a way to solve the problem?

Let us briefly consider Rosen's solution to the problem. He observes that:

> We can resist this [critique] by insisting that the operation of replacing a worldly item in a fact with its real definition never yields the same fact again. It yields a new fact that 'unpacks' or 'analyzes' the original. (Rosen, 2010, p. 124)

What Rosen means here by the notion of 'unpacking' a fact is not perfectly clear; the example he provides is the following:

> Suppose for the sake of argument that to be the number two just is to be the successor of 1. [In our notation: $\forall x \ (\langle x = 2 \rangle \Leftarrow \langle x = s(1) \rangle)$.] One might accept this while rejecting the exotic view that the number 2 somehow contains the number 1 as a part or constituent. Simply from the fact that 1 figures in the *definition* of 2, it does not follow that 1 is a part of 2. But now propositions (and facts) are individuated by their constituents. […] The former contains 2 as a constituent, but need not contain the successor function or the number 1; the latter contains *successor* and the number 1, but need not contain the number 2. (Rosen, 2010, p. 125)

Briefly put, Rosen's idea amounts to identifying facts and propositions through their constituents, as individuals, functions, attributes and so on. Therefore, according to him, the fact that $3 = 2 + 1$ is different from the fact that $3 = s(1) + 1$. Once this has been assumed, the nexuses of conceptual reduction are reliable guides to the genuine grounding relations.

To import Rosen's intuition to our case, we should maintain that in

$$\forall x \forall y ((o(x) = o(y) \leftrightarrow P(x,y))) \tag{1}$$

the fact that certain a and b are parallel, that is, $[P(x,y)]$, is different from the fact that the direction of a is identical to the direction of b, that is, $[o(x) = o(y)]$. The constituents of the facts at issue are different: in the former, we have lines, whereas in the latter, we have directions of lines.

Does this strategy work? Well, following Rosen's suggestion, it seems that we have to accept that $[3 = 2 + 1] \neq [3 = s(1) + 1]$, even if, arithmetically, $2 = s(1)$. So it should be a(n arithmetical) fact, that $2 = s(1)$. Now, understanding how this sub-fact must not enter into the constitution of both the facts $[3 = 2 + 1]$ and $[3 = s(1) + 1]$ is not easy; otherwise, if $2 = s(1)$ was a relevant fact, how could these facts be different? Put in other terms, how can we explain that these facts

are different in virtue of the distinction of their constituents even though these constituents are identical?

Admittedly, one could argue for Rosen's account by saying that even if $2 = s(1)$ is a fact, this fact does not enter as a constituent in those other two facts. The general idea is that, according to many grounding theorists, grounding is a hyper-intensional relation: necessarily equivalent facts can be discriminated with respect to their grounding relation. Let us concede that grounding is a hyper-intensional relation and let us follow this train of thought. Accordingly, the two facts $[3 = 2 + 1]$ and $[3 = s(1) + 1]$ are different in virtue of their constituents. But if we plug, so to speak, the fact that $2 = s(1)$ into the facts under examination, we would obtain two composed facts as follows: $[[3 = 2 + 1], [2 = s(1)]]$ and $[[3 = s(1) + 1], [2 = s(1)]]$. Now, are these two facts identical or different? If they are identical, they are so in virtue of their constituents. However, not *all* the constituents are identical and connected by the same relationship. Therefore, the reason of their identity has to be identified with what they have in common, that is, that $2 = s(1)$. But this is exactly the same reason we advanced to say that the original facts ($[3 = 2 + 1]$ and $[3 = s(1) + 1]$) are identical.

So far we have discussed some metaphysical trouble arising from an arithmetical example. But what about the relation of metaphysical grounded induced on (AP)? In order to avoid irreflexivity, we have to claim that $[d(x) = d(y)] \neq [P(x,y)]$. Is this plausible? Well, the two facts have, at least *prima facie*, different constituents and hence, it seems plausible to claim that they are not identical. However, there is another metaphysical problem lurking in this reading. It concerns the nature of the so-called *identity facts*. As is well known, the right part of any (AP) is an identity statement. But what is an identity fact? It seems, indeed, that within $[d(x) = d(y)]$ there is just one individual, the direction of x (viz, of y). But then, $[d(x) = d(y)]$ has the same constituents of $[d(x) = d(x)]$. However, the latter seems to be a kind of 'logical fact', a universal feature of reality: everything is self-identical. Thus, it is far from clear what is the nature of the identity facts presupposed by a metaphysical construal of the relation of grounding in (AP).

To conclude: although metaphysical grounding might sound like a natural and interesting connection, it is not a viable road. However, not everything is lost. Metaphysical grounding is indeed not the only type of grounding on the market. Quite recently, the so-called *conceptual* grounding has started attracting philosophers' attention. Conceptual grounding is a relation among truths, which is objective, non-causal and explanatory in nature (as it is the case with metaphysical grounding), but which holds in virtue of the concepts these truths contains (see, e.g., Carrara & De Florio, 2020). Typical examples of conceptual grounding are the following:

1. that animal is a vixen because it is a fox and it is a female,
2. Tobias is European because he is German,
3. the area of the square $ABCD$ is bigger than the area of the square $EFGH$, because the side AB is bigger than the side EF.

Sentences 1–3 above convey a grounding relation, namely a relation among truths that is explanatory in nature and that holds in virtue of the links among concepts these sentences contain, i.e., the link between a square and the side of a square, or the link between vixen-animal-fox. Hence, while the metaphysical grounding relation connects objects and facts in the world and arranges them in a hierarchy of fundamentality where less fundamental facts or objects ground more fundamental ones, the conceptual grounding relation relies on connections among concepts in our theories and arranges them in a hierarchy of complexity where less complex concepts ground more complex ones.

There are two features of the relation of conceptual grounding which have been highlighted recently and which are important for our purposes. On the one hand, several scholars (see, e.g., Poggiolesi & Genco, 2021) have been underlining the key role that mathematics plays for

conceptual grounding. In other words, paradigmatic and highly interesting cases of conceptual grounding are examples coming from the mathematical world, namely a world where truths are connected in virtue of the mathematical concepts they contain. In the work of (Bolzano, 2015), plenty of examples of this type can be found. On the other hand, arguments have been provided in defense of a neat distinction between metaphysical and conceptual grounding (see, e.g., Betti, 2010; Smithson, 2020). In other words, according to these works, each example of grounding is either an example of metaphysical grounding or an example of conceptual grounding, and the two notions, together with the two different levels they involve, should not be conflated. Putting these two features together, we have that we can have examples of grounding among mathematical truths: these are cases of conceptual grounding and not of metaphysical grounding.

In virtue of these last remarks, let us go back to (AP) sentences. Not only does the literature on conceptual grounding confirm the problems we have encountered with a metaphysical reading, it also provides a new and fruitful way of looking at them: they convey relations of conceptual grounding. In other words, we can think of the concept of direction as grounded in the concept of parallelism; as we can think of the concept of number as grounded in the concept of equinumerosity. But what does it imply for the dichotomy thin and thick objects introduced by Linnebo that (AP) can be read in terms of conceptual grounding, and not in terms of metaphysical grounding? We dedicate the last section of the paper to the brief analysis of this possibility.

4 | FROM THIN/THICK OBJECTS TO THIN/THICK CONCEPTS

Let us sum up what we have been discussing so far. First of all, we have introduced the (AP) and we have underlined some of their relevant features. Among them, we have shown that, in Linnebo's view, (AP) lead to the distinction between thin and thick objects. But the distinction between thin and thick objects invites[3] in its turn a reading in terms of metaphysical grounding: given that thick objects seem to be determined by thin objects, this is precisely when the relation of metaphysical grounding comes into play. However, we have shown that the metaphysical grounding reading is not a viable option, rather one should think of the (AP) as conveying a conceptual grounding relation, where conceptual and metaphysical grounding are two separate and distinct notions. Hence, reflection on the (AP) leads to the identification of a tension between two different and separate levels: one is the ontological level proper of the distinction between thin and thick objects, the other is the conceptual level proper of the notion of conceptual grounding. A way out from this tension is to think the distinction between thin and thick rather than as applied to objects, as applied to concepts.[4]

In other terms, given the distinction between ontological and conceptual levels, and the difficulty shown by Carrara and De Florio of interpreting (AP) at the former level, one natural solution is to move the whole discussion to the latter level. This of course implies and involves a clear definition of what thin and thick concepts are. This paper is not the place to fully develop such an idea; nevertheless, we would like to outline two possible and promising lines of research.

The first consists in looking at the distinction between thin and thick concepts in epistemic terms; in other words, the dichotomy between thin and thick concepts could be articulated on the basis of the epistemic resources required in order to understand some given concepts. Henceforth, given a class of thick concepts, a class of related thin concepts is such when the

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[3]If the term *invites* sounds too strong, than at least it should be compatible with.

[4]We note that, in relation to concepts, the thick/thin terminology has been used to mark a rather different distinction. For example, thick ethical concepts are those that combine evaluation and description; thin ethical concepts are those that are purely evaluative. Our use of thick/thin to distinguish different kinds of concepts is not intended to track this distinction. We thank the reviewer for this clarification.

epistemic resources requested to size them are not demanding. This perspective stands as quite natural and applies very well to our example (D), where we can assume to work with the concept of *parallel lines*, as well as with the concept of *lines having a same direction*. While the former is a thick concept, the latter is a thin concept: indeed given the thick concept of *parallel lines*, it is not very demanding to grasp the concept of *two lines having the same direction*. Finally, note that the epistemic perspective might be thought of as linked with the more formal notion of computational complexity (see, e.g., Wagner & Wechsung, 1986) and hence could give rise to a development in clear and rigorous terms.

The second way one could describe the distinction between thin and think concepts is compositional: in other worlds, we characterise the class of thin and the class of thick concepts on the base of the composition of concepts themselves. A concept belongs to the class of thin concepts if it contains a related thick concept, where the containment relation can assume several and different forms. Although this perspective most likely stands as less intuitive and does not straightforwardly apply to our example (D), it is, however, connected to an illustrious philosophical tradition that ranges from Porphyry to Leibniz and Bolzano and Kant (see, e.g., Margolis & Laurence, 2021).

Consider for example the concept of *European*, which can be thought of as composed by the concepts *French*, *Italian*, *German* and so on. Then, while *French*, *Italian*, *German* are thick concepts, *European* is a thin one as it contains the other two as its parts. Moreover, consider the concept of *vixen*, which can be thought of as composed by the concepts of *fox* and the concept of *female*. Then, while *fox* and *female* are thick concepts, *vixen* is a thin one as it contains the other two as its parts.

Both the epistemic and compositional perspectives have solid background and promising outcomes, although of course they both need to be further developed. Moreover, an analysis of their relationships and possible interconnections also deserves a detailed study. As they stand, they represent sound and promising lines of research that are worth being pursued.

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