A New Alternating Predictive Observer Approach for Higher Bandwidth Control of Dual-Rate Dynamic Systems

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***Abstract*—Dual-rate dynamic systems consisting of a sensor with a relatively slow sampling rate and a controller/actuator with a fast updating rate widely exist in control systems. The control bandwidth of these dual-rate dynamic systems is severely restricted by the slow sampling rate of the sensors, resulting in various issues like sluggish dynamics of the closed-loop systems, poor robustness performance. A novel alternating predictive observer (APO) is proposed to significantly enhance the control bandwidth of a generic dual-rate dynamic systems. Specifically, at each fast controller/actuator updating period, we will first implement *the prediction step* by using the system model to predict the system output, generating a so-called *virtual measurement*, when there is no output measurement during the slow sampling period. Subsequently, *the observation step* is carried out by exploiting this virtual measurement as informative update. An APO-based output feedback controller with a fast updating rate is developed and rigorous stability of the closed-loop system is established. The superiority of the proposed method is demonstrated by applying it to control a permanent magnet synchronous motor system.**

***Index Terms*—alternating predictive observer; higher control bandwidth; dual-rate dynamic system; virtual measurement.**

I. Introduction

With the significant demand on application of advanced instruments and smart sensors in new generation of sophisticated control systems, the research of dual-rate control methods which can effectively improve the control performance, particularly control bandwidth, has attracted increased research attentions from both academia [1]–[3] and industrial sectors [4]–[6]. In these dual-rate systems, the updating rate of the controller/actuator is generally faster than the sampling rate of the sensor. The reason is that the actual measurement is usually generated at a relatively slow rate due to limitations of sensor hardware, so a fast updating controller/actuator is required to improve control performance and reduce possible vibration excitation [7]. The slow sampling rate significantly reduces the control bandwidth, resulting in significant performance degradation such as poor tracking control accuracy and dynamic disturbance rejection performance [8].

In view of the above background, some significant efforts have devoted to improve the control performance of dual-rate dynamic systems. The most promising mechanism to address this problem is to exploit state interpolation over the sampling interval [9], [10]. For discrete-time dual-rate dynamic systems, the state interpolations can be obtained by using model iteration based on the measured states at the sampling instant [11]–[13]. While for continuous-time dual-rate dynamic systems, the pre-integration approach is usually exploited to generate continuous state information over the digital sampling interval such that the missing states over the sampling interval are recovered and can be utilized for controller design [14]–[16]. These state interpolation approaches work well in the case of full state measurements [17], [18]. However, when it comes to output-feedback control problems where a state observer is required, the updating rate of the state observer is consistence with the sensor measurement rate, which significantly restricts the control bandwidth [19]–[21]. We attempt to develop a new alternating prediction and observation mechanism that renders the updating rate of the observer as fast as the controller/actuator updating rate.

A novel alternating predictive observer (APO) will be proposed as a dynamic prediction method for state estimate and prediction over the sampling interval. At each fast controller/actuator updating instant, when there is no output measurement throughout the slow sampling period, we will first implement *the prediction step* by using the system dynamic model to predict the virtual system output, generating a so-called *virtual measurement*. Subsequently, *the observation step* is carried out by exploiting this virtual measurement as the informative update of the observer. These prediction and observation steps alternate at each fast actuation period. At each slow sampling instant, the real sensor measurement is used to correct the predicted and observed states. An APO-based output feedback controller with a fast updating rate is further developed. The closed-loop system with this APO works like one that has a sensor with a sampling rate as fast as the actuation rate. Consequently, the bandwidth of the system is substantially enhanced by APO, which has the ability to predict the dynamics between the sampling interval. Rigorous stability of the closed-loop system is established and it has been shown that under some mild conditions, the state of the dual-rate system can converge asymptotically to a bounded region. Finally, the feasibility of the proposed method is validated by conducting simulation studies on a position control example of a permanent magnet synchronous motor.

II. Problem formulation

Let N^+ stands for the set of positive integers and $\mathbb{R}$ denotes the set of real numbers. Given a real matrix $A \in \mathbb{R}^{n\times m}$, A^T

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denotes the transpose of A. Given a real symmetric matrix $P = P^T \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ represent the maximum and minimum eigenvalues of matrix P, respectively.

We consider a class of continuous linear time-invariant dynamic systems as follows

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Cx(t) \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$ and $y(t) \in \mathbb{R}^{n_y}$ stand for the system state, the control input and the controlled/measurement output, respectively. A, B, and C are system matrices with compatible dimensions. Define the fast updating period of the controller as T. *The sampling rate of the sensor is slower than the updating rate of the controller, and the sampling period is supposed to be MT with $M > 1$ and $M \in N^+$.* We define the control period and sampling period as $T_c := T$ and $T_s := MT_c$, respectively. The output measurement $y(t)$ is collected by the sensor with a slow sampling period T_s, which is depicted by $y(t) = y(t_{k_s}), \forall t \in [t_{k_s}, t_{k_s+1})$ and $t_{k_s} = k_s T_s$ $(k_s = 0, 1, 2, \cdots)$ is the slow-rate sampling instant. The connection between the sampled-date measurement $y(t)$ and control input $u(t)$ is established. Suppose that the control input remains unchanged by using a zero-order holder (ZOH). The sampled-date control input in the slow sampling period $t \in [t_{k_s}, t_{k_s+1})$ is depicted by $u(t) = u(t_k), \forall t \in [t_k, t_{k+1})$ and $t_k = kT_c$ $(k = k_sM, k_sM + 1, \cdots, k_sM + M - 1)$ is the fast-rate updating instant of the controller. The above description shows that the controller updates at intervals $T_c := T$ and the sensor samples at intervals $T_s := MT_c$.

The research objective of this paper is to develop a new dual-rate dynamic controller which renders the observer works with the same updating rate as the fast updating controller even in the presence of a slow output measurement rate. Due to the effectiveness in recovering the states, the proposed control method will substantially improve the control bandwidth.

III. Controller Design

We first construct a sampled-date state observer for state reconstruction of system (1), which is given by

$$\dot{\zeta}'(t) = A\zeta'(t) + Bu(t_k) + L(y(t_k) - C\zeta'(t)) \tag{2}$$

where $\zeta'(t)$ is the estimate of $x(t)$ and $L \in \mathbb{R}^{n_x\times n_y}$ is the observer gain.

It should be highlighted that the inputs of the observer (2), namely $u(t_k)$ and $y(t_k)$, provide informative updates. However, these two inputs exhibit a considerable rate gap in the dual-rate scenario. To be specific, $u(t_k)$ is updated in each instant $t_k = kT$, e.g., a time sequence of $\{0, T, 2T, \cdots\}$, while $y(t_k)$ only provides update at the slow sampling instant in the sense that we can only access the measurement at time instant $t_k = k_sT_s = k_sMT$, e.g., a time sequence of $\{0, MT, 2MT, \cdots\}$. To solve this problem, we propose a new manner to add multiple virtual sampling points with period T during the slow sampling period $T_s = MT$. Based on the above principles, a new sampled-date APO is designed as

$$\dot{\zeta}(t) = \hat{A}\zeta(t) + Bu(t_k) + L\bar{y}(t_k),\ \forall t \in [t_k, t_{k+1}), \tag{3}$$

where $\zeta(t)$ is the estimate of $x(t)$ and $L \in \mathbb{R}^{n_x\times n_y}$ is the observer gain, $\bar{y}(t_k)$ is a mixed actual/virtual measurement and $\hat{A} = A - LC$. The mixed actual/virtual measurement $\bar{y}(t_k)$ in (3) is designed as $\forall t \in [t_k, t_{k+1})$

$$\bar{y}(t_k) = \begin{cases} y(t_k), k = k_sM, \\ y_p(t_k), k = k_sM + 1, \cdots, k_sM + M - 1. \end{cases} \tag{4}$$

where $y(t_k)$ and $y_p(t_k)$ stand for actual measurement and virtual measurement, respectively. The actual output $y(t_k)$ in the sampling instant $k = k_sM$ is obtained by the real sensor measurement. *An important concept of virtual output measurement is newly defined in this paper to facilitate the APO design.* To be specific, the virtual output measurement $y_p(t_k)$ in non-sampling instant, e.g., $k \neq k_sM$ of the sensor is inferred and obtained by using system dynamic model, the control input and the estimated state in the last fast-rate updating instant t_{k-1}. We use the integrations of (1) to obtain

$$x(t_k) = \bar{A}x(t_{k-1}) + \bar{B}u(t_{k-1}),\ y(t_k) = Cx(t_k) \tag{5}$$

where $\bar{A} = \mathrm{e}^{AT}$, $\bar{B} = \int_0^T \mathrm{e}^{A\tau}\,\mathrm{d}\tau B$. From (5), the output measurement prediction $y_p(t_k)$ is obtained and given by

$$\zeta_p(t_k) = \bar{A}\zeta(t_{k-1}) + \bar{B}u(t_{k-1}),\ y_p(t_k) = C\zeta_p(t_k) \tag{6}$$

It can be seen from (3) and (4) that at the actual sampling instant, APO uses the actual measurement $y(t_k)$ of sensor, which can effectively improve the observation effect and make the observation effect close to the actual measurement to achieve the correction effect. Then the continuous-time APO (3) with a sampler provides the same state estimate in sampling instant t_k as the following discrete-time observer

$$\zeta(t_{k+1}) = F\zeta(t_k) + Gu(t_k) + H\bar{y}(t_k) \tag{7}$$

where $F = e^{\hat{A}T}$, $G = \int_0^T e^{\hat{A}\tau} d\tau B$ and $H = \int_0^T e^{\hat{A}\tau} d\tau L$.

Since the discrete-time APO (7) and the sampled-data APO (3) will produce exactly the same estimate $\zeta(t_k)$, we will use the discrete-time APO (7) for the design of the output feedback controller and practical implementation, while adopting the sampled-data APO (3) for stability analysis to restore a more realistic sample-date system. Consequently, the fast updating output feedback control law is constructed as

$$u(t_k) = K\zeta(t_k),\ \forall t \in [t_k, t_{k+1}) \tag{8}$$

where $K \in \mathbb{R}^{n_u\times n_x}$ is the feedback control gain.

To show the essential differences between the proposed APOBC approach and the existing paradigm of interpolation-based dual-rate control approaches, a schematic diagram showing the signal flow and updating rate of observer, predictor and sensor measurement is given in Fig. 1. As clearly shown in Fig. 1, the updating period of the state observer is MT for the traditional approach, while it is merely T for the proposed APOBC approach. This indicates that the APOBC approach can significantly enhance the sampling efficiency in a soft manner, and consequently exhibits great potential to enhance the control bandwidth of the dual-rate systems.

IV. Stability analysis

The dynamics of the dual-rate closed-loop dynamic systems consisting of (1) are formulated in this subsection. It should be pointed out that, according to (4), we divide the analysis into two cases.

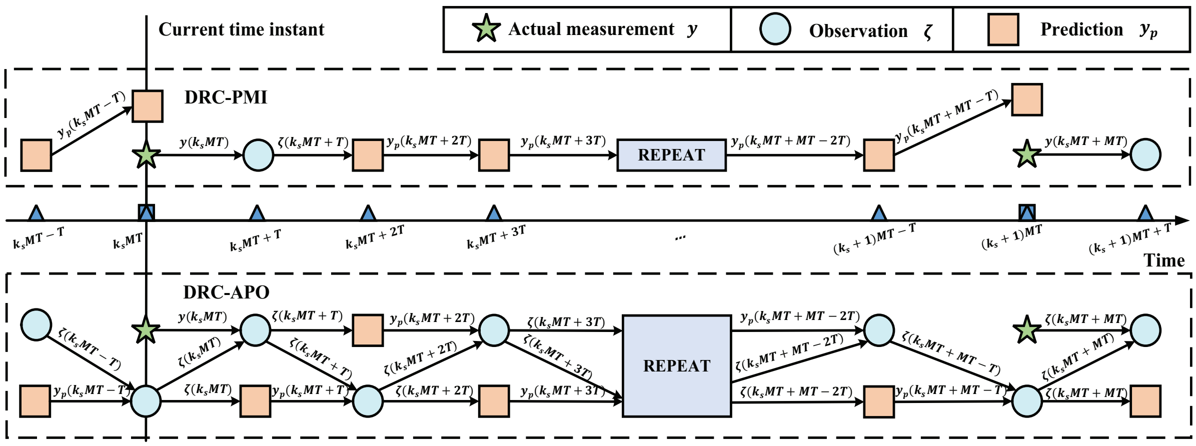

Fig. 1. Schematic diagram of signal flows, updating rates and logic of the observation, the prediction and the sensor measurement of the DRC-PMI (up) and DRC-APO (down) approaches.

Case 1: $t \in [t_k, t_{k+1})(k = k_sM+1, k_sM+2, \ldots, k_sM+M-1)$. Defining the state estimation error of APO as $\xi(t) = x(t) - \zeta(t)$ and substituting the control law (8) into (1) and (3) leads to the following closed-loop system

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} K\zeta(t_k) - \begin{bmatrix} 0 \\ L \end{bmatrix} (\bar{y}(t_k) - C\zeta(t)) \tag{9}$$

which can be further rewritten as

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = & \begin{bmatrix} A+BK & -BK \\ 0 & \hat{A} \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} \\ & + \begin{bmatrix} B \\ 0 \end{bmatrix} K(\zeta(t_k) - \zeta(t)) - \begin{bmatrix} 0 \\ L \end{bmatrix} (y(t_k) \\ & - y(t)) - \begin{bmatrix} 0 \\ L \end{bmatrix} (y_p(t_k) - y(t_k)) \end{aligned} \tag{10}$$

Case 2: $t \in [t_k, t_{k+1})(k = k_sM)$. Similar to the deviation process in *Case 1*, the dual-rate closed-loop dynamic system in *Case 2* is the same as (10) without $y_p(t_k) - y(t_k)$. Consequently, *Case 2* can be considered as a special scenario of *Case 1* when the prediction error $y_p(t_k) - y(t_k)$ is zero and the system is less restricted under *Case 2*.

To facilitate the establishment of stability, denote the system matrix as $\tilde{A} = \begin{bmatrix} A+BK & -BK \\ 0 & \hat{A} \end{bmatrix}$.

Since there exists matrices L and K such that $\hat{A} = A - LC$ and $A + BK$ are both Hurwitz stable matrices respectively, the system matrix $\tilde{A}$ is also a Hurwitz matrix. Consequently, there exists a symmetric, positive definite matrix $P = P^T > 0$ such that $\tilde{A}^T P + P\tilde{A} = -I$. We define a candidate Lyapunov function as $V(\chi) = \chi^T P \chi$ with $\chi(t) = [x(t), \xi(t)]^T$. The derivative of $V(\chi(t))$ along system (10) is

$$\begin{aligned} \dot{V}(\chi(t)) = & -\|\chi(t)\|^2 + 2\chi^T(t)P \begin{bmatrix} B \\ 0 \end{bmatrix} K(\zeta(t_k) - \zeta(t)) \\ & - 2\chi^T(t)P \begin{bmatrix} 0 \\ L \end{bmatrix} (y(t_k) - y(t)) \\ & - 2\chi^T(t)P \begin{bmatrix} 0 \\ L \end{bmatrix} (y_p(t_k) - y(t_k)) \end{aligned} \tag{11}$$

The last three terms in (11) are estimated subsequently for further establishment. To begin with, keeping $\zeta(t_k) = x(t_k) - \xi(t_k)$ in mind, one can obtained from the dynamic system (1) that $\forall t \in [t_k, t_{k+1})$ and $\forall \tau \in [t_k, t]$

$$|\dot{y}(\tau)| \leq \frac{\|CA\|\sqrt{V(\chi(\tau))} + 2\|CBK\|\sqrt{V(\chi(t_k))}}{\sqrt{\lambda_{\min}(P)}} \tag{12}$$

It can be further derived from (12) that

$$|y(t_k) - y(t)| \leq \int_{t_k}^{t} |C\dot{x}(\tau)| \mathrm{d}\tau \leq \alpha_1 V_m(t, t_k) \tag{13}$$

where $V_m(t, t_k) = \sqrt{V_{\max}(t)}(t - t_k)$, $\alpha_1 = (\|CA\| + 2\|CBK\|)/\sqrt{\lambda_{\min}(P)}$ and $V_{\max}(t) = \max_{\tau \in [t_k, t]} V(\chi(\tau))$.

Next we will proceed to estimate the term $K(\zeta(t_k) - \zeta(t))$. Note that $|K(\zeta(t_k) - \zeta(t))| \leq \int_{t_k}^{t} |K\dot{\zeta}(\tau)| \mathrm{d}\tau$. Similar with (12) and (13) and one has that from (3) that $\forall \tau \in [t_k, t]$

$$|K(\zeta(t_k) - \zeta(t))| \leq \alpha_2 V_m(t, t_k) + \|KL\| |e_p(t_k)| \tag{14}$$

where $\alpha_2 = (2\|K\hat{A}\| + 2\|KBK\| + \|KLC\|)/\sqrt{\lambda_{\min}(P)}$ and $e_p(t_k) = y_p(t_k) - y(t_k)$.

From (3), (7) and (6), we can also write a continuous form of virtual output $y_p(t_k)$ which is $\dot{y}_p(t) = CA\zeta(t) + CBu(t_k)$. Noticing that $\dot{y}(t) - \dot{y}_p(t) = CA\xi(t)$, we can get

$$|y_p(t_k) - y(t_k)| \leq \alpha_3 V_m(t, t_k) \tag{15}$$

where $\alpha_3 = \|CA\|/\sqrt{\lambda_{\min}(P)}$.

Substituting (13), (14) and (15) into (11), one has that

$$\begin{aligned} \dot{V}(\chi(t)) \leq & -\|\chi(t)\|^2 + 2\alpha_2\|\chi(t)\|\|PB\|V_m(t, t_k) \\ & + 2\alpha_1\|\chi(t)\|\|PL\|V_m(t, t_k) \\ & + 2\alpha_3\|\chi(t)\|\|PB\|\|KL\|V_m(t, t_k) \\ & + 2\alpha_3\|\chi(t)\|\|PL\|V_m(t, t_k) \end{aligned} \tag{16}$$

Noting that $\|B\| = c_1$, $\|P\| = \lambda_{\max}(P)$ and $\|\chi(t)\| \leq \sqrt{V(\chi(t))/\lambda_{\min}(P)}$, (16) can be rewritten as

$$\dot{V}(\chi(t)) \leq -\|\chi(t)\|^2 + \alpha_4\sqrt{V(\chi(t))}V_m(t, t_k) \tag{17}$$

where $\alpha_4 = 2(\alpha_2 c_1 + \|L\|\alpha_1 + c_1\alpha_3\|KL\| + \alpha_3\|L\|)\lambda_{\max}(P)/\sqrt{\lambda_{\min}(P)}$. Suppose that the parameters satisfy the following condition

$$\alpha_4 \lambda_{\max}(P) T < 1 \tag{18}$$

In what follows, we will use (17) together with the parameter conditions (18) to prove the following

$$\max_{\forall \tau \in [t_k, t_{k+1}]} V(\chi(\tau)) = V(\chi(t_k)) \tag{19}$$

Suppose that (19) does not hold, which indicates that there exists a time instant $t_1 \in [t_k, t_{k+1}]$ such that $V(\chi(t_1)) > V(\chi(t_k))$. With (18) in mind, we can obtain from (17) that for $\chi(t_k) \neq 0$, $\dot{V}(\chi(t_k)) < 0$, which implies $V(\chi(t_k))$ will decrease in a short time period starting from t_k. Hence, there exists a time instant $t_2 \in [t_k, t_1]$ such that: (i) $V(\chi(t_2)) = V(\chi(t_k))$, (ii) $\dot{V}(\chi(t_2)) > 0$, and (iii) $V(\chi(t)) \leq V(\chi(t_k))$, $\forall t \in [t_k, t_2]$. Based on the above three points, we can get from (17)

$$\dot{V}(\chi(t_2)) \leq -(1 - \alpha_4 \lambda_{\max}(P) T) \|\chi(t_2)\|^2 < 0 \tag{20}$$

which contradicts to the assumption $\dot{V}(\chi(t_2)) > 0$. Thus, we can conclude that $V(\chi(t))$ is decreasing monotonically during $t \in [t_k, t_{k+1})$ and (19) is true. Then it follows from (17) that

$$\dot{V}(\chi(t)) \leq -\frac{V(\chi(t))}{\lambda_{\max}(P)} + \alpha_4 \sqrt{V(\chi(t))}\sqrt{V(\chi(t_k))} T \tag{21}$$

Let $\kappa(t) = \sqrt{V(\chi(t))/V(\chi(t_k))}$ and we can get $\dot{\kappa}(t) \leq -\kappa(t)/(2\lambda_{\max}(P)) + (T\alpha_4)/2$. Noticing that $\kappa(t_k) = 1$, one has that $\kappa(t_{k+1}) \leq \varphi$ where

$$\varphi = e^{-\frac{T}{2\lambda_{\max}(P)}} + (1 - e^{-\frac{T}{2\lambda_{\max}(P)}}) T \alpha_4 \lambda_{\max}(P) \tag{22}$$

which implies $V(\chi(t_{k+1})) \leq \varphi^2 V(\chi(t_k))$. From (18), one has that $\varphi < 1$, which implies that $V(\chi(t_k))$ converges to zero as k tends to infinity and system (1) is globally stabilized by the proposed APOBC law (8).

V. Application of a AC servo system

The position control-oriented mathematical model of a permanent magnet synchronous motor (PMSM) system is described as follows

$$\begin{aligned} \frac{\mathrm{d}i_q}{\mathrm{d}t} &= -\frac{R}{L_q} i_q - \frac{p\Psi_f}{L_q}\omega + \frac{1}{L_q} u_q \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{K_T}{J} i_q - \frac{B_\omega}{J}\omega, \ \ \frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega \end{aligned} \tag{23}$$

where i_q is q axis currents, ω is electrical angular velocity, θ is electrical angular position. The electrical and mechanical parameters $R = 0.54\ \Omega$ is stator resistance, $\Psi_f = 0.61$ Wb is permanent magnet flux, $L_q = 0.0096$ H represent q axis inductors, $p = 4$ is number of pole pairs, $B_\omega = 0.0001$ Nm/rad s^{-1} is the viscous friction coefficient, $J = 0.016$ kgm^2 is rotational inertia, and $K_T = 3.66$ Nm/A is the torque constant, respectively.

Define the state vector as $x(t) = [\theta(t), \omega(t), \dot{\omega}(t)]^T$, and the control input as $u(t) = u_q(t)$. The PMSM model (23) is rewritten as the same as (1) and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{RB_\omega + K_T p \Psi_f}{JL_q} & -\frac{R}{L_q} - \frac{B_\omega}{J} \end{bmatrix},$$

$B = \begin{bmatrix} 0 & 0 & \frac{K_T}{JL_q} \end{bmatrix}^T$, and $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

Suppose that the reference position of the motor system is given by $y_d(t) = \sin(\sigma t)$ and σ is a real number. A slightly modified tracking controller based on the proposed APOBC approach is designed as $u(t) = K\hat{\tilde{x}}(t) + \bar{y}_d(t)$, $\hat{\tilde{x}}(t) = \left[\hat{\theta}(t) - y_d(t)\ \hat{\omega}(t) - \dot{y}_d(t)\ \dot{\hat{\omega}}(t) - \ddot{y}_d(t)\right]^T$ and $\bar{y}_d(t) = (\dddot{y}_d(t) + \frac{RB_\omega + K_T p\Psi_f}{JL_q}\dot{y}_d(t) + \frac{RJ + B_\omega L_q}{L_q J}\ddot{y}_d(t))/(\frac{K_T}{JL_q})$.

To highlight the benefit of the proposed dual-rate control with APO (**DRC-APO**), quantitative comparisons with two existing popular control approaches (i.e., single-rate control (**SRC**) and pure model interpolation (**DRC-PMI**)) for dual-rate dynamic systems, where **SRC** represents the case when both the observer and controller update in a period of MT, while **DRC-PMI** indicates that the observer is updated with a period of MT while the controller is updating with a faster period of T by exploiting model-based pure state interpolation. Different from these two existing control approaches, both the

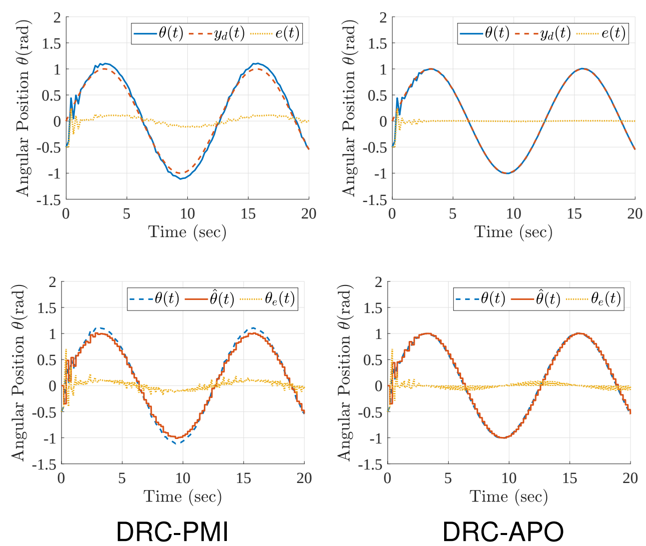

Fig. 2. The comparison of the two methods when the reference position output $y_d = \sin(0.5t)$ and $e(t) = \theta(t) - y_d(t)$, $\theta_e(t) = \theta(t) - \hat{\theta}(t)$.

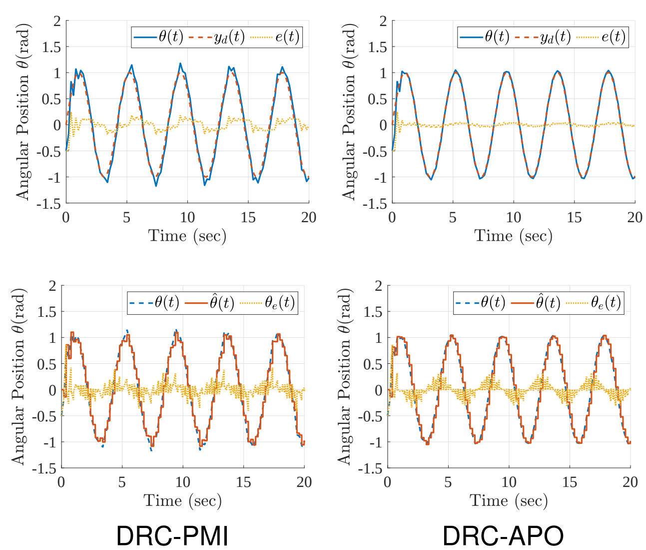

Fig. 3. The comparison of the two methods when the reference position output $y_d = \sin(1.5t)$.

TABLE I
COMPARISON OF MAXIMUM TRACKING ERROR RATIO

Reference	SRC	DRC-PMI	DRC-APO
sin(t/2)	-	11.38%	1.02%
sin(t)	-	13.89%	2.93%
sin(3t/2)	-	18.21%	5.42%
sin(2t)	-	24.73%	9.91%
sin(5t/2)	-	33.85%	16.73%
sin(3t)	-	42.03%	25.92%

controller and observer update with a fast updating period of T for the proposed **DRC-APO**. The output measurement is only available during actual slow sampling instant $t = k_s MT$. The initial system value is $x(t) = [-0.5, 1, -1]^T$ and the parameters of above three control methods are selected as $T = T_c = 0.2s$, $M = 5$, $T_s = 1s$, $K = [-20\ \ -0.1\ \ -0.01]$ and $L = [0.45\ 0.9\ 0.9]^T$, respectively.

To demonstrate the improvement of dynamic control performance of the proposed APOBC approach, two simulation scenarios with position references $y_d = \sin(0.5t)$ and $y_d = \sin(1.5t)$ are selected, where the state observation curve, output tracking curve and error curve are shown in Figs. 2 and 3. Because the system under **SRC** is unstable, it is not plotted in figures. As clearly shown by the simulation results, compared with **SRC** and **DRC-PMI**, **DRC-APO** delivers a much higher state estimation accuracy as well as much better transient and static position reference tracking performances.

To further demonstrate the claimed benefits of the proposed APOBC approach, a group of simulations with different harmonic frequencies of the position references ranging from 0.5 rad/s to 3 rad/s, have been carried out for position tracking performance comparisons among the three controllers. For the sake of illustration, we set the maximum tracking error ratio which is defined as the ratio of the maximum tracking error to the amplitude of the reference position output. Then the maximum tracking errors ratio are shown in Table. 1 for comparisons among **SRC**, **DRC-PMI** and **DRC-APO**. To conclude, the proposed APOBC approach has the advantages of simple implementation, better dynamic and static tracking performance, and higher control bandwidth.

VI. CONCLUSION

In this paper, a new alternating predictive observer (APO) based control (APOBC) approach has been proposed to significantly improve the control performance of a class of dual-rate dynamic systems. A promising feature within the proposed APOBC method is that updating rate of the observer can be set as fast as the controller/actuator updating rate even the sampling rate of the sensor is relatively slow. To be specific, the APOBC updates the output information of the state observer by exploiting several virtual sampling points during the slow sampling period when there is no actual sensor measurement. The asymptotic stability of the resultant control system under APOBC has been established under some standard conditions by using Lyapunov stability theory. Finally, simulation studies on a position tracking control system has been conducted, which show that the proposed APOBC approach can achieve much better dynamic and static tracking control performance as well as much higher control bandwidth. This paper is a basic version. APO is only a preliminary version at present, and the proposed APO will be validated against the background of more complex and practical systems in the future.

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