COMPLEMENTARY GENERALIZED TRANSMUTED POISSON LOMAX DISTRIBUTION WITH ITS DERIVED PROPERTIES AND APPLICATION

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Abstract

There has been a renewed interest in developing more flexible statistical distributions in recent decades. A major milestone in the methods for generating statistical distributions is undoubtedly the system of differential equation approach. There is a recent renewed interest in generating skewed distributions. In this research, a new four parameter lifetime distribution which extends the Lomax distribution is introduced by compounding the Lomax distribution with the complementary generalized transmuted Poisson family of distributions. The probability density function and cumulative distribution function as well as some basic statistical properties of the new distribution, such as moments, reliability function, hazard function, quantile function, residual life function, entropy and the order statistics were derived. Some plots of the distribution shows that it is a positively skewed distribution. The maximum likelihood estimation method is used to estimate the parameters of the new distribution. A simulation study to assess the performance of the parameters of the newly developed distribution was provided with an application to real life data to assess its potentiality. The result shows that the proposed distribution provides better fit than some generalizations of the Lomax .

Keywords: Lifetime distribution, maximum likelihood estimation, Lomax distribution, hazard function, order statistics.

1. Introduction

There have been different life data models that illustrates the pattern of failure data inengineering,environmental, financial, medical and biological sciences such as exponential, gamma and Weibull. The need for flexibility of these distributions arises and it has led to the development of many new distributions by modification or generalization of existing distributions. There are a number of classical distributions that have been used over the past decades for modeling real-life data but there are needs for extending distributions in applicable areas as lifetime analysis and reliability (Lemonte et al., 2013). Over the years, many researches have made modifications to existing distributions through various methods such as transforming the original distributions, increasing the number of parameters already existing or by the proper mix of distributions.

In many lifetime studies, the Pareto type-II which is a special case of a compound gamma distribution called Lomax distribution, is commonly used (Baharith, et al., 2019). Lomax distribution which was first introduced by Lomax (1954) for modeling business failure data, has a wide application in reliability analysis, life testing problems, information theory, business, economics, queuing problems, actuarial modeling and biological sciences (Liu et al., 2019). It is an alternative to common lifetime distributions such as exponential or gamma and used when the population is assumed to be heavy tailed (Pak et al., 2018).

In modeling failure rates, there have been literatures that proposed and studied distributions that captures the decreasing failure rates. Such failure rates arises from situations such as infant mortality in humans and over working or miss use of an engineering component (Adamidis et al, 1998). In lifetime testing and reliability studies, the exponential distribution was used in modeling problems related to failure rates (Flores et al, 2013). Barreto-Souza and Cribari-Neto (2009) generalized the distribution proposed by Kuz (2007) by including a power parameter, which considered the maximum lifetimes of the random variables from the Exponential-Poisson distribution. The power series distribution was considered when the maximum number of competing causes is assumed leading to a complementary risk scenario. Furthermore, Flores et. al., (2013) proposed a new family of distribution based on a complementary risk problem in the presence of latent risk assuming a power series. The concept assumed that there is no information about which factor was responsible for the component failure but only the maximum lifetime value among all risks is observed instead of the minimum lifetime value among all risks as in Chahkandi (2009) and Morais (2011). The distribution was called Complementary Exponential Power series, which is a counterpart of the Exponential Power series. The complementary risk problems arises in several areas such as Medical, industrial and finances as well.

Based on the Complementary Exponential Power series, Alizadeh et al, (2017) proposed the Complementary Generalized transmuted Poisson –G family of distributions which is a wider Class of continuous distribution. It was proposed to have a flexible two-parameter generator to fit real data from several fields. Some special models such as Complementary Generalized

Transmuted Poisson Weibull and Complementary Generalized Transmuted Poisson Lindley were presented and their density and hazard rates plotted and observed.

According to Alizadeh et al. (2017), the cumulative distribution function (cdf) of the Complementary Generalized Transmuted Poisson-G family for any continuous probability distribution, is defined as

$$F(x;\gamma,\varphi,\varepsilon) = \frac{e^{\gamma}-e^{\gamma\bar{G}(x;\varepsilon)}}{e^{\gamma}-1}\left[1+\ \varphi\frac{e^{\gamma\bar{G}(x;\varepsilon)}-1}{e^{\gamma}-1}\right] \quad (1)$$

Differentiating the cdf in (3.1), the corresponding pdf is given as

$$f(x;\gamma,\varphi,\varepsilon) = \frac{\gamma g(x;\varepsilon)e^{\gamma\bar{G}(x;\varepsilon)}}{e^{\gamma}-1}\left[1-\varphi\frac{(e^{\gamma}+1)-2e^{\gamma\bar{G}(x;\varepsilon)}}{e^{\gamma}-1}\right] \quad (2)$$

Where $\gamma > 0$and $|\varphi| \leq 1$ are shape parameters,$g(x;\varepsilon)$, $G(x;\varepsilon)$ and $\bar{G}(x;\varepsilon) = 1 - G(x;\varepsilon)$ are the probability density function, cumulative distribution function and corresponding reliability function respectively of the baseline distribution depending on a parameter vector ε, where $\varepsilon = (\varepsilon_k) = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots)$.

2. The Complementary Generalized Transmuted Poisson-Lomax Distribution.

We obtain the cdf of the Complementary Generalized Transmuted Poisson Lomax distribution by inserting equation (1) and equation (3) into equation (4) which yields:

$$F(x;\alpha,\beta,\gamma,\varphi) = \frac{e^{\gamma}-e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1+\varphi\frac{e^{\gamma(1+\beta x)^{-\alpha}}-1}{e^{\gamma}-1}\right]; x > 0 \quad (3)$$

The corresponding pdf of the cdf in equation (5) is given by;

$$f(x;\alpha,\beta,\gamma,\varphi) = \frac{\alpha\beta\gamma(1+\beta x)^{-(\alpha+1)}e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1-\varphi\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]; x > 0 \quad (5)$$

Where $\alpha, \gamma > 0$and $|\varphi| \leq 1$, are shape parameters, and$\beta > 0$ is the scale parameter. Therefore, equations (6) and (7) are the cdf and pdf of the Complementary Generalized Transmuted Poisson Lomax distribution.

Expansion of the distribution function of CGTPL distribution

According to Alizadeh et al. (2017), the pdf of Complementary Generalized Transmuted Poisson Lomax distribution in equation (7)can expressed as a linear expression of Exponentiated Lomax distribution, andthus reduces to

$$f(x) = \sum_{k=0}^{\infty} b_k h_{k+1}(x) \tag{6}$$

Where $h_{k+1}(x) = (k+1)\alpha\beta(1+\beta x)^{-(\alpha+1)}(1-(1+\beta x)^{-\alpha})^k$ with power parameter $k + 1$, and

$$b_k = \sum_{i=k}^{\infty} \frac{(-1)^k}{(k+1)} \binom{i}{k} \left[\frac{(1-\varphi)\gamma^{i+1}}{(e^{\gamma}-1)i!} + \frac{\varphi(2\gamma)^{i+1}}{(e^{\gamma}-1)^2 i!}\right] \tag{7}$$

The associated cdfcan be expressed by;

$$F(x) = \sum_{k=0}^{\infty} b_k H_{k+1}(x) \tag{8}$$

Where $H_{k+1}(x) = [(1-(1+\beta x)^{-\alpha})]^{(k+1)}$

3. Exhibited properties of the newly developed distribution

i) Shapes of the CGTPL PDF, CDF, Reliabilty and Hazard function

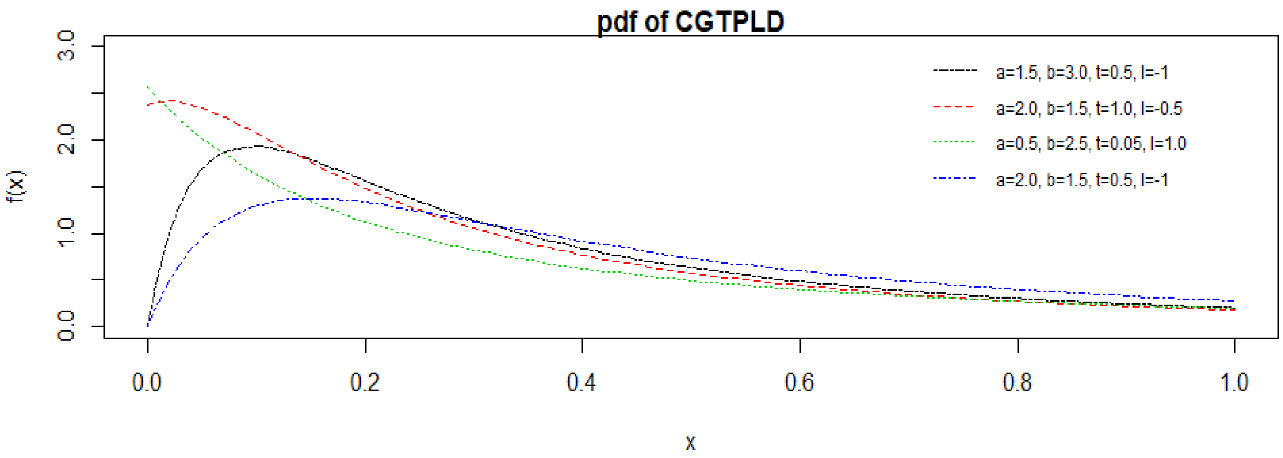

Figure 1: The pdf plot of the CGTPL distribution at different parameter value

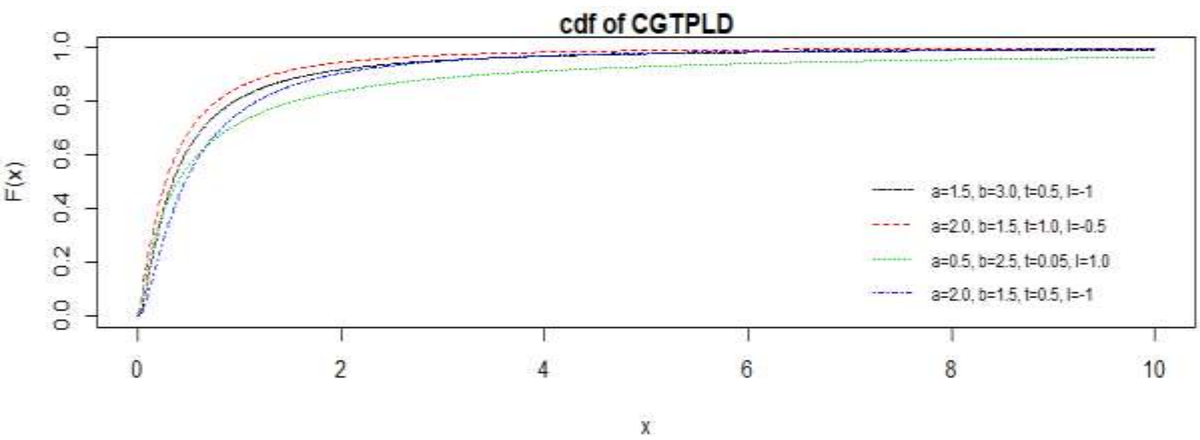

Figure 2: The cdf plot of the CGTPL distribution at different parameter value

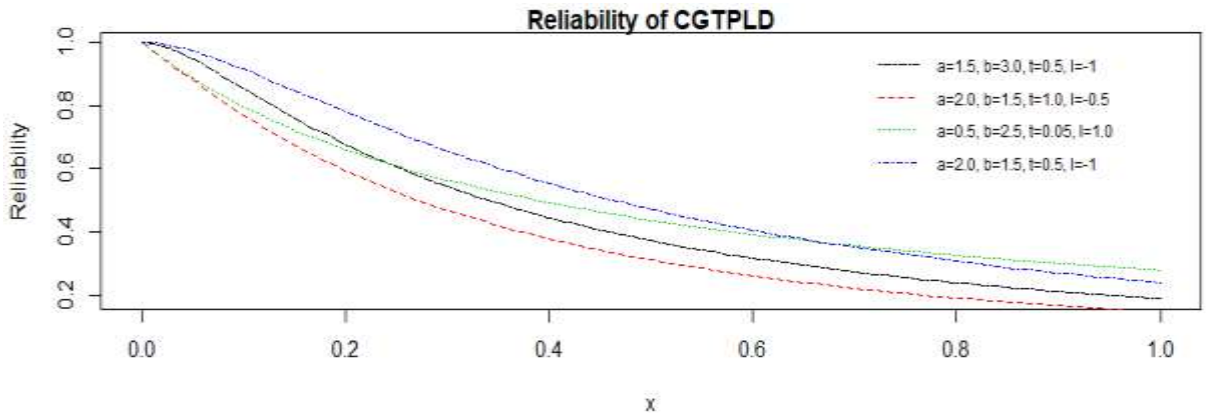

Figure 3: The reliability function plot of the CGTPL distribution at different parameter value

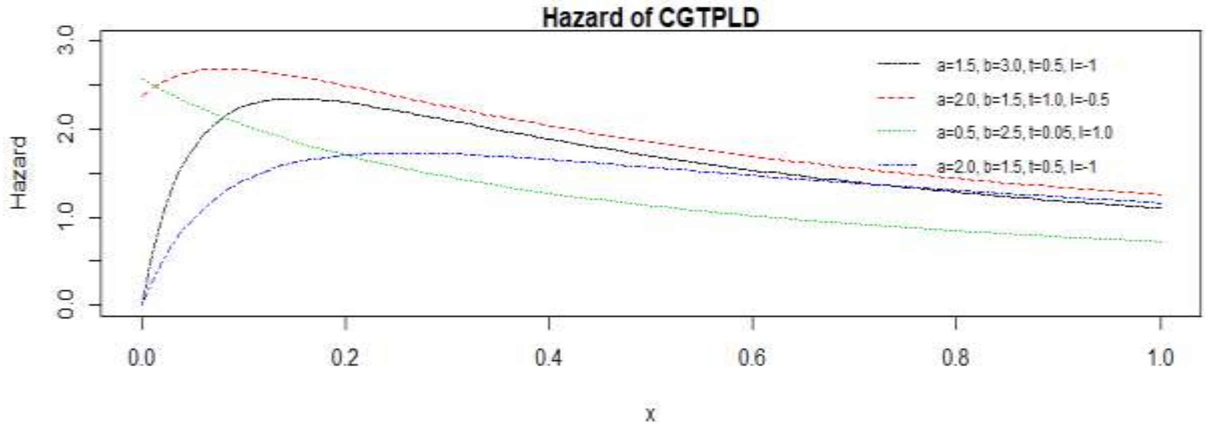

Figure 4: The hazard function plot of the CGTPL distribution at different parameter value

ii) Moments

Moments can be described as constants of a population which are used to study the various characteristics of a random variable such characteristics as the mean, the variance, skewness and kurtosis of a distribution.

The rth moment of X, where X denotes a continuous random variable, is given by;

$$\mu_r' = E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx \tag{9}$$

Where, $f(x)$ is the pdf of the Complementary Generalized Transmuted Poisson Lomax distribution given in equation (7)

The rth ordinary moment above can be derived using (8). thus, we have;

$$\mu_r' = E(x^r) = \sum_{k=0}^{\infty} b_k E(X_{k+1}^r) \tag{10}$$

Recalling from equation (8), equation (12) is further expressed by;

$$E(x^r) = \sum_{k=0}^{\infty}\sum_{i=k}^{\infty}\frac{(-1)^k}{(k+1)}\binom{i}{k}\left[\frac{(1-\varphi)\gamma^{i+1}}{(e^{\gamma}-1)i!}+\frac{\varphi(2\gamma)^{i+1}}{(e^{\gamma}-1)^2 i!}\right](k+1)\int_{-\infty}^{\infty}x^r\alpha\beta(1+\beta x)^{-(\alpha+1)}(1-(1+\beta x)^{-\alpha})^k\,dx \quad (11)$$

Where $E(X_{k+1}^r) = (k+1)\int_{-\infty}^{\infty}x^r\alpha\beta(1+\beta x)^{-(\alpha+1)}(1-(1+\beta x)^{-\alpha})^k\,dx$, which numerically can be computed by;

$$E(X_{k+1}^r) = (k+1)\int_0^1 x^r u^k du \quad (12)$$

Where u follows uniform $(0,1)$ and x is the quantile function of the baseline distribution which is the Lomax distribution given by;

$$x = (^1/_\beta)((1-u)^{-\frac{1}{\alpha}}-1) \quad (13)$$

iii) Mean

The mean of the CGTPLD can be obtained, by setting $r = 1$ in the rth moment of the distribution in equation (12), which will yield the following:

$$\mu_1' = E(x) = \sum_{k=0}^{\infty} b_k E(X_{k+1}) \quad (14)$$

The second moment is obtain from the rth moment, when r =2 in equation (12). This yields the following:

$$\mu_2' = E(x^2) = \sum_{k=0}^{\infty} b_k E(X_{k+1}^2) \quad (15)$$

iv) Variance

The rth central moment or the moment about the mean of X, can be obtained by;

$$\mu_r = E[X-\mu_1']^r = \sum_{i=0}^{r}(-1)^i\binom{r}{i}(\mu_1')^i\,\mu_{r-1}' \quad (16)$$

The Variance is obtained when n =2 in the central moment. Thus,

$$Var(X) = E(x^2) - \{E(x)\}^2 \quad (17)$$

$$= \sum_{k=0}^{\infty} b_k E(X_{k+1}^2) - \{\sum_{k=0}^{\infty} b_k E(X_{k+1})\}^2 \quad (18)$$

v) The Cumulants

The cumulantsk_c of X can be obtained directly from the ordinary moment by:

$$k_c = \mu'_c - \sum_{r=1}^{c-1}\binom{c-1}{r-1}k_r\mu'_{c-r} \tag{19}$$

The first, second, third and fourth cumulants are given as: $k_1 = \mu'_1$, $k_2 = \mu'_2 - (\mu'_1)^2$, $k_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3$ and $k_4 = \mu'_4 - 3(\mu'_2)^2 - 4\mu'_3\mu'_1 + 12\mu'_2(\mu'_1)^2 - 6(\mu'_1)^4$respectively.

The Skewess and Kurtosis of X are the third and fourth standardized cumulants given by

$$\text{Skewness} = \frac{k_3}{(k_2)^{\frac{3}{2}}} \text{and Kurtosis} = \frac{k_4}{(k_2)^2} \text{respectively.}$$

vi) Reliability Function

Reliability function can be described as the probability of the non-failure occurring before time t. The reliability function of the CGTPLD is given by;

$$R(x) = 1 - F(x) \tag{20}$$

$$= 1 - \frac{e^{\gamma} - e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1 + \varphi\frac{e^{\gamma(1+\beta x)^{-\alpha}}-1}{e^{\gamma}-1}\right] \tag{21}$$

vii)Hazard Function

The instant rate of failure at a given time t, is the hazard function. The hazard function of the CGTPLD is given by;

$$h(x) = \frac{f(x)}{R(x)} \tag{22}$$

$$= \frac{f(x) = \frac{\alpha\beta\gamma(1+\beta x)^{-(\alpha+1)}e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1 - \varphi\frac{(e^{\gamma}+1) - 2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]}{1 - \frac{e^{\gamma}-e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1 + \varphi\frac{e^{\gamma(1+\beta x)^{-\alpha}}-1}{e^{\gamma}-1}\right]} \tag{23}$$

$$= \frac{f(x) = \alpha\beta\gamma(1+\beta x)^{-(\alpha+1)}e^{\gamma(1+\beta x)^{-\alpha}}\left[1 - \varphi\frac{(e^{\gamma}+1) - 2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]}{e^{\gamma(1+\beta x)^{-\alpha}} - 1\left[1 + \varphi\frac{e^{\gamma(1+\beta x)^{-\alpha}}-1}{e^{\gamma}-1}\right]} \tag{24}$$

vii) Cumulative Hazard Function

$$H(x) = -\log[1 - F(x)] \tag{25}$$

$$= -\log\left[1 - \frac{e^{\gamma}-e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1 + \varphi\frac{e^{\gamma(1+\beta x)^{-\alpha}}-1}{e^{\gamma}-1}\right]\right] \tag{26}$$

viii) Quantile Function

The quantile function of the CGTPLD is derived by inverting $F(x) = 1 - R(x)$. Let $u = F(x)$ where u follows uniform (0,1). According to Alizadeh et al (2017), the random variable $X = Q(u)$ having equation (7) as its density has its quantile function by;

$$G(x_u) = 1 - \frac{1}{\gamma}\log\left[e^{\gamma} - \frac{(e^{\gamma}-1)[1+\varphi-\sqrt{(1+\varphi)^2-4\varphi u}}{2\varphi}\right] \tag{27}$$

Therefore, the quantile function of CGTPLD is

$$x = \frac{1}{\beta}\left(\left[\frac{1}{\gamma}\log\left[e^{\gamma} - \frac{(e^{\gamma}-1)[1+\varphi-\sqrt{(1+\varphi)^2-4\varphi u}}{2\varphi}\right]\right]^{-\frac{1}{\alpha}} - 1\right) \tag{28}$$

3. ESTIMATION

In this section, the parameters of the CGTPL distribution based on a complete sample is estimated using the maximum likelihood estimation method. Let $X_1, \ldots, X_n$ be a sample of size n independently and identically distributed random variables from CGTPLD with unknown parameters α, β, γ and φ previously defined. The pdf of the CGTPLD is given as

$$f(x;\ \alpha,\beta,\gamma,\varphi) = \frac{\alpha\beta\gamma(1+\beta x)^{-(\alpha+1)}e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\left[1 - \varphi\,\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right] \tag{29}$$

The likelihood function is given by;

$$L(x_1, x_2, \ldots, x_n\ /\ \alpha,\beta,\gamma,\varphi) = \prod_{i=1}^{n} f(x_i;\ \alpha,\beta,\gamma,\varphi) \tag{30}$$

$$L(x_1, x_2, \ldots, x_n\ /\ \alpha,\beta,\gamma,\varphi) = \prod_{i=1}^{n} \frac{\alpha\beta\gamma(1+\beta x_i)^{-(\alpha+1)}e^{\gamma(1+\beta x_i)^{-\alpha}}}{e^{\gamma}-1}\left[1 - \varphi\,\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x_i)^{-\alpha}}}{e^{\gamma}-1}\right] \tag{31}$$

Differentiating l with respect to α, β, γ and φ respectively gives;

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=0}^{n}\log(1+\beta x_i) - \gamma\sum_{i=0}^{n}(1+\beta x_i)^{-\alpha}\log(1+\beta x_i) - \sum_{i=0}^{n}\frac{2\varphi\gamma(1+\beta x_i)^{-\alpha}\log(1+\beta x_i)e^{\gamma(1+\beta x_i)^{-\alpha}}}{(e^{\gamma}-1)\left[1-\varphi\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]} = 0 \tag{32}$$

$$\frac{\partial l}{\partial \beta} = \frac{n}{\beta} - (\alpha+1)\sum_{i=1}^{n}\frac{x_i}{(1+\beta x_i)^{-\alpha}} - \gamma\alpha\sum_{i=1}^{n} x_i(1+\beta x_i)^{-(\alpha+1)} - \sum_{i=0}^{n}\frac{2\varphi\gamma\alpha x_i(1+\beta x_i)^{-(\alpha+1)}e^{\gamma(1+\beta x_i)^{-\alpha}}}{(e^{\gamma}-1)\left[1-\varphi\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]} = 0 \quad (33)$$

$$\frac{\partial l}{\partial \varphi} - \sum_{i=0}^{n}\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{(e^{\gamma}-1)\left[1-\varphi\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]} = 0 \quad (34)$$

$$\frac{\partial l}{\partial \gamma} = \frac{n}{\gamma} - \frac{n}{e^{\gamma}-1} + \sum_{i=1}^{n}(1+\beta x)^{-\alpha} - \sum_{i=0}^{n}\frac{\varphi e^{\gamma}\left[(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}\right]}{(e^{\gamma}-1)\left[1-\varphi\frac{(e^{\gamma}+1)-2e^{\gamma(1+\beta x)^{-\alpha}}}{e^{\gamma}-1}\right]} = 0 \quad (35)$$

4. Simulation Study

An evaluation of the behavior of the Maximum Likelihood estimators of the parameters of the Complementary Generalized Transmuted Poisson Lomax distribution from a Monte Carlo simulation is carried out using R software. Random samples of sizes $n = 20, 50, 75, 100\ and\ 500$ are generated over 1,000 replicates from the CGTPL distribution. The Bias and the Root Mean Square Errors, are given in **Table 1**.

Table 1: Monte carlo Simulation for the Estimates of the CGTPL Distribution

	Actual values				Bias				RMSE			
n	α	β	γ	φ	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\varphi}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\varphi}$
25	1.5	2	1	0.5	0.732	0.850	0.535	-0.172	2.805	2.785	1.061	0.281
	2	1.5	1	0.5	1.334	1.030	0.578	-0.170	5.470	2.765	1.072	0.278
	1	1	1	0.5	0.594	0.124	0.568	-0.141	7.516	0.964	1.140	0.274
	1	3	1.5	0.5	0.715	1.136	0.104	-0.151	8.445	3.662	0.992	0.279
50	1.5	2	1	0.5	0.436	0.448	0.586	-0.148	1.824	2.015	1.126	0.277
	2	1.5	1	0.5	1.009	0.447	0.687	-0.139	4.218	1.812	1.148	0.265
	1	1	1	0.5	0.150	0.104	0.620	-0.120	0.605	0.777	1.289	0.279
	1	3	1.5	0.5	0.210	0.908	0.172	-0.131	0.766	2.821	1.209	0.284
75	1.5	2	1	0.5	0.408	0.361	0.616	-0.127	2.00	1.848	1.216	0.283
	2	1.5	1	0.5	0.880	0.359	0.679	-0.139	3.948	1.697	1.189	0.275
	1	1	1	0.5	0.108	0.070	0.583	-0.113	0.530	0.667	1.238	0.284
	1	3	1.5	0.5	0.204	0.705	0.203	-0.115	1.032	2.431	1.183	0.285

100	1.5	2	1	0.5	0.230	0.298	0.604	-0.130	1.258	1.660	1.237	0.280
	2	1.5	1	0.5	0.788	0.190	0.704	-0.120	3.533	1.398	1.209	0.266
	1	1	1	0.5	0.058	0.084	0.533	-0.114	0.410	0.596	1.217	0.285
	1	3	1.5	0.5	0.084	0.770	0.182	-0.110	0.441	2.253	1.217	0.284
500	1.5	2	1	0.5	0.015	0.144	0.455	-0.112	0.415	1.032	0.959	0.262
	2	1.5	1	0.5	0.143	0.026	0.612	-0.108	0.919	0.859	1.072	0.256
	1	1	1	0.5	0.001	0.131	0.280	-0.113	0.240	0.489	0.942	0.270
	1	3	1.5	0.5	0.004	0.716	0.026	-0.082	0.256	1.755	0.944	0.260

5. Application

The data set applied to the Complementary Generalized Transmuted Poisson Lomax distribution is the data set given by Lee and Wang (2003) on the remission times (in months) of a random sample of 128 bladder cancer patients. The data set is given as follows: 0.08, 2.09, 3.48, 4.87, 6.94 , 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69.

The descriptive statistics of the data set on the remission times of a random sample of 128 bladder cancer patients is given in **Table 2**.

Table 2: Descriptive statistics of 128 Bladder Cancer Patients

Mean	**Median**	**Variance**	**SD**	**Skewness**	**Kurtosis**	**1stQu.**	**3rdQu.**	**Min.**	**Max.**
9.366	6.396	110.425	10.508	3.287	15.483	3.348	11.838	0.08	79.05

The parameter estimation for the Complementary Generalized Transmuted Poisson Lomax distribution was carried out using the maximum likelihood estimation method. The estimated parameters with their standard errors in parenthesis, are given in **Table 3**.

Table 4.3: MLEs and their Standard Errors (in parentheses) of 128 Bladder Cancer Patients

	Estimates				
Distribution					
CGTPLD*	$\hat{\alpha} = 11.0604$ (8.2796)	$\hat{\beta} = 0.0032$ (0.0004)	$\hat{\gamma} = 5.6929$ (3.4183)	$\hat{\varphi} = -0.832$ (0.148)	
Ext. PLD	$\hat{\alpha} = 0.2387$ (1.1424)	$\hat{\beta} = 124.3010$ (155.3534)	$\hat{\gamma} = 59.8378$ (242.1564)		
TE-Lomax	$\hat{\alpha} = 1.51197$ (0.2342)	$\hat{\beta} = 0.0227$ (0.017)	$\hat{\gamma} = 5.1514$ (3.7577)	$\hat{\varphi} = 0.6697$ (0.4674)	
McLomax	$\hat{\alpha} = 0.8085$ (3.364)	$\hat{\beta} = 11.2929$ (15.818)	$\hat{a} = 1.5060$ (0.243)	$\hat{b} = 4.1886$ (25.029)	$\hat{c} = 2.105$ (3.079)
KumGLomax	$\hat{\alpha} = 0.8959$ (2.4882)	$\hat{\beta} = 15.0378$ (15.0625)	$\hat{a} = 1.5224$ (0.2683)	$\hat{b} = 4.7558$ (13.9729)	

The performance of the distribution based on some model criterion selection based on the 128 bladder cancer patient data is given in **Table 4**.

Table 4: Measures of AIC, BIC HQIC and CAIC

Distribution	-logL	AIC	HQIC	CAIC	BIC
CGTPLD*	409.56	827.13	831.76	827.45	838.53
Ext. PLD	413.84	831.67	837.14	833.86	842.22
TE-Lomax	410.43	828.87	833.51	829.13	840.28
McLomax	409.91	829.82	835.62	830.14	844.09
KumGLomax	409.95	827.90	832.54	828.23	839.31

For an ordered random sample, $X_1, X_2, \ldots, X_n$, from Complementary Generalized Transmuted Poisson Lomax distribution, where the parameters α, β, γ and φ are unknown, the Kolmogorov–Smirnov D_n, Cramér von Mises W_n^2 and Anderson and Darling A_n^2, was used.

The goodness of fit statistic for the Complementary Generalized Transmute Poisson Lomax distribution in comparison with other models are given in **Table 5**.

Table 5: Goodness-of-fit tests.

Distribution	D_n	W_n^2	A_n^2
CGTPLD*	0.0344	0.0168	0.1085
Ext. PLD	0.0989	0.2268	1.4511
TE-Lomax	0.0399	0.0314	0.2275
McLomax	0.0391	0.0254	0.1685
KumGLomax	0.0389	0.0236	0.1614

The values in **Table 4** indicates that the Complementary Generalized Transmuted Poisson Lomax distribution performs better than the Extended Poisson Lomax distribution, Transmuted Exponentiated Lomax, the McDonald Lomax and the Kumaraswamy-Generalized Lomax Distribution, since it has the minimum value for AIC, CAIC, HQIC and BIC.

The values in **Table 5** indicates that the test statistics D_n, W_n^2 and A_n^2, have the smallest values for the data set under the Complementary Generalized Transmuted Poisson Lomax distribution with regards to the other models

Figure 5 shows a graphical representation of the model fit to the bladder cancer patient data. The Empirical and theoretical density plot (top left), Empirical and theoretical CDF (bottom left), Quantile-Quantile (Q-Q) plot (top right) and the Probability-Probability (P-P) plot.

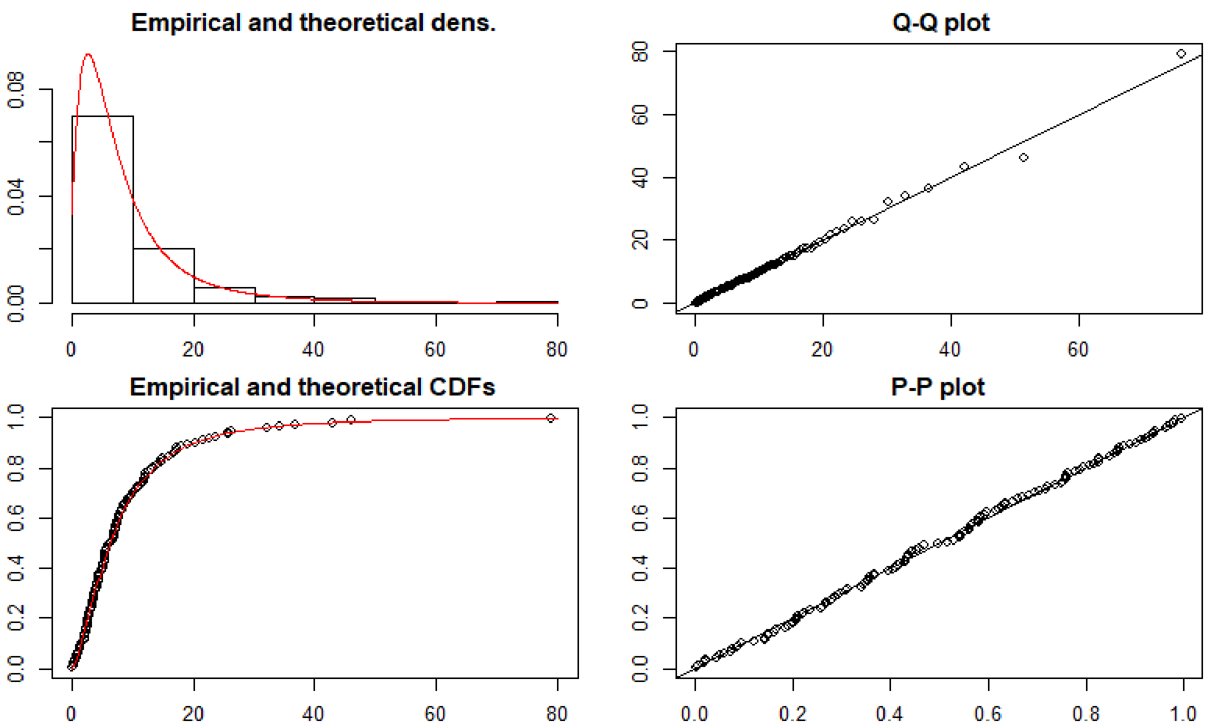

Figure 5: A Graphical representation of the model fit to the Bladder Cancer Patient data.

6. Conclusion

In this research, a mixture model of distributions was developed to model the heterogeneous survival time data. The maximum likelihood estimators of the parameters of the parametric mixture model were obtainedwe have proposed the Complementary Generalized Transmuted Poisson Lomax distribution. We derived and studied some conventional properties of the distribution such as moments, reliability function, hazard function, quantile function, residual life function, entropy and the order statistics. The parameters of the distribution were estimated using the method of Maximum Likelihood estimation and an application of the distribution to real life data set. The values of the log likelihood were in favour of the proposed model. Moreover, the AIC and BIC were also computed and both of them supported the proposed model. Indicating that the newly developed model is much morethe flexible and has a better fit than the other distributions considered.

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