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Abstract

The ultimate goal of asset pricing theory is to provide a general equilibrium explanation of how asset returns and consumption are jointly determined. Corporate financing, investment, hedging, liquidity, and payout decisions are endogenously determined, and have an effect on this equilibrium. In turn, firms' decisions appear to be influenced by stock market activity. Chapter 2, *"Collateral-Based Asset Pricing"*, constructs an asset pricing model, the Corporate CAPM, from firms' hedging behavior. Chapter 3, *"The Relative Leverage Premium"*, revisits the complex relationship between financial leverage and expected equity returns through the lens of the tradeoff theory of capital structure. Chapter 4, *"Dynamic Corporate Liquidity"*, studies the corporate finance implications of hedging, an economic mechanism with fundamental asset pricing implications.

Chapter 1

Introduction

The ultimate goal of asset pricing theory is to provide a general equilibrium explanation of how asset returns and consumption are jointly determined. Corporate financing, investment, hedging, liquidity, and payout decisions are endogenously determined, and have an effect on this equilibrium. In turn, firms' decisions appear to be influenced by stock market activity. Related research in dynamic corporate finance contributes to this goal. The drivers of corporate investment, financing, and liquidity policies can be better undrestood through the lens of dynamic quantitative models of investment and financing, and of dynamic contracting models.

In Chapter 2, *"Collateral-Based Asset Pricing"*, I build on recent corporate finance studies that show that hedging is a first-order driver of corporate decisions. I use firms' hedging behavior to build a novel asset pricing model, the Corporate CAPM. I propose a dynamic contracting framework in which firms hedge by transferring resources to future states that are most important for firm's value. Firms have limited funds because of collateral constraints that endogenously arise from agency conflicts between firms and lenders. The amount of resources firms can devote to hedging is therefore limited. In the model, firms' hedging behavior is informative on the shareholders' stochastic discount factor, which measures the value of each state. As a consequence, discount rates can be inferred from firm's observed investment, financing, and hedging policies. On the corporate finance side, a calibrated version of the model is broadly consistent with observed corporate policies of US listed firms. On the asset pricing side, the Corporate CAPM is successful in pricing different test assets, also in comparison to leading asset pricing models.

In Chapter 3, *"The Relative Leverage Premium"*, co-authored with Filippo Ippolito and Claudio Tebaldi, we revisit the relationship between financial leverage and expected equity returns. The existing empirical evidence on the relationship between leverage and expected equity returns is inconclusive, both in terms of sign and significance. We re-examine this evidence in a setting in which firms pursue an optimal leverage policy dynamically in the presence of frictions, and can temporarily deviate from their optimal capital structure. These frictions produce heterogeneity in the cross-section of observed leverage, and make the relationship between returns and leverage complex to examine empirically. We remove this heterogeneity by looking at relative leverage, computed as the difference between observed and target leverage. We estimate target leverage using three different proxies: the partial adjustment model of Flannery and Rangan [2006], four-digit SIC code industry median leverage, and firm median leverage. We show that, for all proxies, relative leverage is positively and significantly related to expected equity returns. This suggests a novel interpretation of the relation between leverage and equity returns, through a capital structure channel that operates via capital structure imbalances.

Chapter 4, *"Dynamic Corporate Liquidity"*, co-authored with Boris Nikolov and Lukas Schmid, focuses on firms' liquidity policies. When external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks. Making optimal use of liquid funds means transferring them to times and states where they are most valuable. We examine the determinants of corporate liquidity management in a dynamic model where stochastic investment opportunities and cash shortfalls provide liquidity needs. Firms can transfer liquidity across time using cash, and across states drawing on credit lines subject to debt capacity constraints. We generate empirical and quantitative predictions by means of calibration. Small and constrained firms use cash to provide liquidity to fund investment opportunities, while large and unconstrained firms manage their liquidity needs by means of credit lines. In the time series, equity issuances are used to replenish cash balances, and credit lines to fund unanticipated investment opportunities. We find strong support for our predictions in the data. Overall, the model provides a quantitatively and empirically successful framework explaining corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

Chapter 2

Collateral-Based Asset Pricing

2.1 Introduction

Stochastic discount factors are the cornerstone of modern asset pricing. They allow to compute asset prices as the expected discounted value of future cashflows. Asset pricing theory ordinarily derives a stochastic discount factor from the optimizing behavior of an investor who decides over consumption and portfolio allocations. In this paper, I instead build upon corporate finance theory to identify a stochastic discount factor from firms' policies.[1] This leads to a novel asset pricing model, the Corporate CAPM.

Specifically, I recover a stochastic discount factor from firms' hedging behavior. Hedging is not only a pivotal economic mechanism in corporate finance, but also a fundamental channel through which firms transfer resources across states of the world.[2] A firm that hedges a state reveals information on the importance of that state for its own value. The value of each state is also measured by the owners' stochastic discount factor. Therefore, the stochastic discount factor can be identified through observed firms' decisions, and used to price the assets in the economy. The concept of hedging I entertain here draws on the close connection between collateralized financing and risk management recognized by Rampini and Viswanathan [2010], and Rampini and Viswanathan [2013]. Hedging and financing both involve promises to pay from the firm to external lenders in some states of the world. Collateral constraints arising from limited enforcement restrict such promises, and hence the amount of resources firms can effectively transfer across states.

The Corporate CAPM expresses the stochastic discount factor in terms of firms' characteristics, and can be approximated as a linear two-factor model. The factors are a

[1]Related works by Cochrane [1993], Belo [2010], and Jermann [2010] also attempt to recover a stochastic discount factor from the production side of the economy.

[2]Rampini and Viswanathan [2010], Bolton et al. [2011], and Rampini and Viswanathan [2013], highlight the importance of hedging to understand firm's growth, investment, and financing policies. Nikolov et al. [2013], and Li and Whited [2013] document the quantitative importance of hedging for firms' policies.

"hedging" factor, which equals the change in firms' net worth[3], and a "profitability" factor, which is associated to the change in firms' productivity. I implement asset pricing tests with the Generalized Method of Moments (GMM) to assess the empirical performance of the model. As the recent empirical literature recommends (Lewellen et al. [2010], Daniel and Titman [2012]), I consider different test assets in empirical tests, namely the Fama-French 25 portfolios sorted by size and book-to-market equity, the 30 Fama-French industry portfolios, and 25 portfolios sorted by market and HML beta as in Yogo [2006]. Overall, the Corporate CAPM finds support in the data. The model prices the test assets well, and delivers low pricing errors even in comparison to leading asset pricing models, as the CAPM, the Consumption CAPM, and the Fama and French three-factor model. Historically, asset pricing models obtained from consumption-based stochastic discount factors have not succeeded in accounting for the variation of expected returns across stocks. One important reason for their empirical failure is the smoothness of consumption data. This prevents expected returns to line up with covariances with consumption aggregates, as these models predict. On the contrary, the Corporate CAPM gets traction because it links the stochastic discount factor to firms' characteristics, which exhibit larger fluctuations.

My theoretical framework is a dynamic contracting model. Hedging is in fact an inherently dynamic process. Firms engage in hedging to transfer resources from today to future times and states when they are more valuable. For instance, a firm might hedge specific future states to finance profitable investment opportunities, or to pay out more dividends in bad times. In the model, firms have valuable investment opportunities that arise stochastically over time. However, they have limited funds, and they sign contracts with external lenders to aid external financing of profitable investments. Contracts have limited enforcement. The entrepreneur has the option to renege the contract and divert capital for their own private benefit. In equilibrium, this limited commitment problem endogenously imposes a collateral constraint, and firms implicitly borrow constrained against their equity value. In this context, value maximization provides a rationale to hedge more valuable states, in a tradeoff with their funding needs for current investment and distributions. Firms' debt capacity is limited, and firms can preserve it for specific future states by optimally contracting state-contingent repayments with the lender. A firm can therefore hedge any future state by arranging a low repayment in the case that state occurs. Hence, firms can in effect transfer resources (net worth) across states.[4]

[3] As standard in the dynamic contracting literature, net worth is the firm's counterpart of household wealth, and captures how constrained a company is with respect to funds to allocate to investment, and distributions.

[4] As previous studies discuss, hedging is practically implemented with combinations of traditional debt instruments and other financial instruments like lines of credit and financial derivatives. In particular, credit lines appear to be a prominent implementation of hedging. Sufi [2009] reports that credit lines constitute more than 80 percent of bank debt for public firms in the US. Colla et al. [2013] report that

In this setting, the stochastic discount factor reflects which state must have led a firm to optimally make its observed decisions, and can be backed out from the firms' state-by-state first-order conditions with respect to debt repayments. Conditional on how financially constrained they are, firms implement investment and financing policies to transfer resources to most important states, where the stochastic discount factor is high.

On the corporate finance side, I solve the model numerically and I find that a calibrated version is quantitatively consistent with basic stylized facts about corporate investment and financing, and with key aggregated asset pricing moments. To solve the model, and to determine the properties of the optimal contract, I formulate the contracting problem recursively as an infinite-horizon dynamic programming problem. The problem has a nonstandard topological structure because of the presence of the objective function, the firm's equity value, in the borrowing constraint. I use Knaster-Tarski (Tarski [1955]) fixed point theorem[5] to prove the problem has a well-defined equilibrium. In addition, the number of decision variables is high because of state-contingent hedging decisions. To deal with this issue, I introduce an equivalent mixed-integer programming representation of the dynamic programming problem. The equivalent problem is a natural extension of the extant linear programming methods for dynamic programming to the specific topological structure of the model. These methods have been introduced in finance by Trick and Zin [1993], and then extended to large state spaces by Nikolov et al. [2013]. As in Nikolov et al. [2013], I exploit a separation oracle, an auxiliary linear programming problem, to achieve computational efficiency.

This paper lies at the intersection of three lines of research. First, it relates to the large literature that develops quantitative production models to investigate the cross-section of equity returns. Recent contributions include Zhang [2005], Livdan et al. [2009], Gomes and Schmid [2010], Garlappi and Yan [2011], Obreja [2013], and Bazdrech et al. [2013]. With respect to these papers, my focus is to obtain a stochastic discount factor, instead of rationalize observed spreads in returns with respect to specific firms' characteristics. Second, the paper builds upon the literature on hedging and dynamic contracting in corporate finance, that refers to Rampini and Viswanathan [2010], Rampini and Viswanathan [2013], Rampini et al. [2013], and whose quantitative implications have been examined in Li and Whited [2013], and Nikolov et al. [2013]. In this context, this paper analyzes the asset pricing implications of contracting models of hedging. Finally, this work is closely related to the literature that attempts to identify a stochastic discount factor in production models from firms' policies and data, as in Cochrane [1991], Cochrane [1993], Cochrane [1996], Jermann [2010], and Belo [2010]. The key difference with these works is the economic mechanism that allows to identify the stochastic discount factor from firms'

the drawn part of credit lines accounts for 22 percent of their total debt.

[5]See Aliprantis and Border [2006], and Kamihigashi [2012].

decisions.[6]

This work has potential implications for future research. As Cochrane [2011] discusses, research in asset pricing ultimately aims at understanding how asset returns and consumption are jointly determined in general equilibrium. In this perspective, the identification of a stochastic discount factor from the production side of the economy imposes additional restrictions that may provide further guidance for modeling the consumption side of the economy, rather than representing a competing approach. On the empirical side, new testable hypotheses for cross-sectional differences in returns can be developed from the present framework, especially from the observation that variables that describe firms' policies enter the stochastic discount factor directly.

The paper is organized as follows. Section 2.2 develops the key intuition of the paper in a two-period example. Section 2.3 presents the dynamic contracting model, describes its properties, and the numerical solution method. Section 2.4 introduces the key asset pricing result of the paper, the Corporate CAPM. Section 2.5 assesses the quantitative performance of the calibrated model for providing a reasonable description of corporate investment and financing decisions. Section 2.6 presents the empirical tests of the Corporate CAPM. Section 4.7 concludes.

2.2 A Two-Period Example

The goal of this section is to convey the main idea of this work with a simple example. Typical production models do not lead to an explicit expression for the stochastic discount factor, but only to pricing equations for asset returns. Here I show that when firms transfer resources across states of nature through risk management, the stochastic discount factor can be instead backed out from firms' optimization conditions. The argument proceeds as follows. I first illustrate in a two-period model why when firms cannot implement risk management the stochastic discount factor cannot be obtained from the firm's problem. I then show why introducing hedging decisions allows to do so.

Consider a model with two periods: today, and tomorrow. Three states of nature, rainy, foggy, and sunny, can possibly occur tomorrow, with probabilities π_R, π_F, and π_S respectively. Consider a firm with an initial wealth endowment w that has access to a production technology. The production technology delivers a stochastic output $A(S)f(k) > 0$ in the sunny state tomorrow, $A(F)f(k) > 0$ in the foggy state tomorrow, and $A(R)f(k) > 0$ in the rainy state tomorrow, with $A(S) > A(F) > A(R)$. $f(\cdot)$

[6] Cochrane [1991] and Cochrane [1996] assume that the stochastic discount factor depends on returns on real investment, Cochrane [1993] and Belo [2010] rely on a representation of production sets in which firms can affect idiosyncratic productivity shocks, and Jermann [2010] investigates the equity premium by taking advantage of state-contingent technologies. Here, the relevant state-contingent action that allows to identify the stochastic discount factor is based on the corporate finance theory of hedging, in the context of dynamic contracting.

is a production function, and k denotes investment in real capital. The economy ends tomorrow: capital fully depreciates, and a liquidating dividends $d(S)$, $d(F)$, and $d(R)$ are distributed in the sunny, foggy, and rainy states respectively. The firm can borrow from a competitive, risk neutral, and deep-pocket lender at a constant rate R.[7] The firm's problem is to decide over capital k and debt a repayment b to maximize the expected discounted value of its profits, that is

$$\begin{aligned} U(w) &= \max_{k,b} d + \pi_S M(S)d(S) + \pi_F M(F)d(F) + \pi_R M(R)d(R) && (2.1) \\ & \quad s.t. \\ w+b &= d+k && (2.2) \\ d(s) &= A(s)f(k) - Rb \qquad s \in \{S,F,R\} && (2.3) \\ d &\geq 0 && (2.4) \\ d(s) &\geq 0 \qquad s \in \{S,F,R\} && (2.5) \end{aligned}$$

$M(S)$, $M(F)$, and $M(R)$ are the realizations of the owners' stochastic discount factor in the sunny, foggy, and rainy states[8], Equation (2.1) is the budget constraint today, and simply equates sources and uses of funds, where d is today's dividend. Equations (2.4), and (2.5) rule out negative dividends. Equation (2.4) states that the firm has access to no other external funds, while Equations (2.5) guarantee debt is actually riskfree and is repaid tomorrow in all states.[9] Equations (2.4) and (2.5) determine limits on the amount the firm can borrow. b must therefore lie in the closed interval $[k-w, A(R)f(k)]$. Equations (2.5) can be interpreted as collateral constraints, which states that the firm can borrow up to the cash flow it obtains whatever tomorrow's weather is.[10]

Denote by λ the Lagrange multiplier on constraint (2.4), and by $\pi_s\lambda_s$ the Lagrange multipliers on constraints (2.5). The first-order conditions of this problem lead to the usual pricing equations for the return on real capital and for the loan interest rate:

$$E[(M(s)+\lambda_s)R^k(s)] = 1+\lambda \tag{2.6}$$

$$E[(M(s)+\lambda_s)R] = 1+\lambda \tag{2.7}$$

where $s \in \{S,F,R\}$ is an index for the state, and $R^k(s) \equiv A(s)f_k(k)$. Two points are worth noting. First, the pricing equations contain additional terms related to the Lagrange multipliers on the constraints. This reflects the fact that the typical assumption

[7] Section 2.3 discusses this assumption. Appendix A reports the lender's problem.

[8] This objective only requires that a stochastic discount factor exists. This is the case in the absence of arbitrage opportunities. The objective therefore captures the idea that physical assets and riskfree debt are priced consistently with other securities that investors can trade.

[9] Because $d(S) \geq d(F) \geq d(R)$, the constraints in Equation (2.5) for $s \in \{S,F\}$ are never active.

[10] In the full model, the collateral constraint arises endogenously as an outcome of dynamic contracting.

of free portfolio formation is violated (see Cochrane [2001], Chapter 4).[11] Intuitively, the firm trades real capital and loans.[12] If, for example, the collateral constraint in the rainy state in (2.5) is binding, the firm cannot freely tilt its portfolio of assets by increasing its debt stock and leaving its capital stock unchanged. The presence of Lagrange multipliers accounts exactly for this restriction. In fact, when the constraints are not binding, Equations (2.6) and (2.7) reduce to $E[M(s)R^k(s)] = 1$ and $E[M(s)R] = 1$. However, if free portfolio formation holds for the representative household, the stochastic discount factor $M(s)$ prices all the assets the household trades (such as equities). Second, and most important, the firm's optimality conditions do not allow to get an expression for the stochastic discount factor in each state. This happens because the firm cannot transfer resources across the rainy, foggy, and sunny states, or equivalently from today to one future state only. As Equations (2.3) show, by changing its capital and debt decisions in the feasible set, the firm *jointly* increases its payoff in all three states. A unit more of capital generates more output in both states in proportions determined by $A(S)$, $A(F)$, and $A(R)$, and a unit more of debt reduces the payoff by R in both states. A simple algebraic manipulation of Equation (2.3) indeed shows that the payout in the sunny state can be rewritten as a fixed function of the payoffs in the foggy and rainy states as:

$$d(S) = d(R) + \frac{A(S) - A(R)}{A(F) - A(R)}(d(F) - d(R)) \tag{2.8}$$

Panel A of Figure 2.1 makes this idea clear. The solid lines represent the possibility set for the firm's equity payoffs in the sunny and in the rainy state tomorrow for different choices of capital and debt. For simplicity I keep the payoff in the foggy state fixed, although this result hold for every other pair of states. It is immediate to notice that the feasible sets for the payoffs have a kink. In the consumption side of the economy, the condition that these Leontief-type payoffs must be tangent to an indifference curve form the familiar relation $p = E\left[\beta \frac{u'(c(s))}{u'(c)} d(s)\right] = E[M(s)d(s)]$, where p is the price of the firm's equity, c is today's consumption, $u(\cdot)$ is the investor's utility function, and β is his time discount factor. The indifference curves are related to the marginal rate of substitution between today's and tomorrow's consumption, and their slope allows to identify $M(s)$. However, the dashed lines show that any point the firm is willing to choose is consistent with many indifference curves.

[Insert Figure 2.1 Here]

Consider now the same problem in which the firm is allowed to hedge by setting different debt repayments $b(S)$, $b(F)$ and $b(R)$ for the sunny, foggy, and rainy states. The

[11] This is very common in models with financial constraints, and the "corrected" discount factor is sometimes denoted as "the firm's discount factor". See for example Mendoza [2000], and Rampini and Viswanathan [2013].

[12] For simplicity assume these assets cannot be traded by the household directly.

firm's problem becomes:

$$\begin{aligned} U(w) &= \max_{k,b} d + \pi_S M(S)d(S) + \pi_F M(F)d(F) + \pi_R M(R)d(R) & (2.9) \\ & s.t. & \\ w + b &= d + k & (2.10) \\ d(s) &= A(s)f(k) - Rb(s) \qquad s \in \{S, F, R\} & (2.11) \\ d &\geq 0 & (2.12) \\ d(s) &\geq 0 \qquad s \in \{S, F, R\} & (2.13) \end{aligned}$$

where the amount of debt financing raised today from the risk-neutral lender is $b = E[b(s)]$.[13] The first-order conditions with respect to k, $b(s)$, $s \in \{S, F, R\}$, are:

$$E[(M(s) + \lambda_s)R^k(s)] = 1 + \lambda \tag{2.14}$$

$$M(s) = \frac{1 + \lambda - \lambda_s R}{R} \tag{2.15}$$

Equation (2.14) is the familiar pricing equation for capital, while Equation (2.15) provides an expression for the stochastic discount factor that must have let to the observed firm's policy. Notice that the difference in the discount rates of lenders and borrowers does not imply the presence of arbitrage opportunities. The stochastic discount factor in fact adapts such that equity claims are priced consistently with the presence of a risk-neutral lender that allows the firm to implement limited risk sharing. This is apparent comparing the stochastic discount factor in Equation (2.15) with the one for the case without collateral constraints, that is $M(s) = \frac{1}{R}$. In this case the firm guarantees full insurance to the owners, their marginal utility across states is equalized, and equity claims are prices as if the firm were risk neutral. Appendix A discusses this case. Because the stochastic discount factor is higher in most valuable states, the firm trades off dividend distributions today (with a higher λ) in order to pay out in most important states tomorrow (with a lower λ_s), even though the latter reduces the payout in other states or makes it overall more volatile. With risk-averse investors, most important states are those where aggregate consumption is low and firms are less productive, such as the rainy state in this example. Contingent claims that pay out more in those states are therefore more valuable for investors. In addition, because the solution of the firm's problem depends on its wealth w, two firms with different initial wealths in general implement different policies. This does not mean that there is a stochastic discount factor for each firm. Instead, the firm changes its investment and financing policy in a state-contingent way, depending on whether the state is either sunny, foggy, or rainy. As a consequence, in principle, both firms' policies

[13] As I discuss in Section 4.3.2, in the complete model lenders offer an elastic supply of credit at all future times and dates at the riskfree rate.

(and data) could be used as a reference point to back out the stochastic discount factor and to price other assets. In Section 2.4, I refer to this result as the *relativity property*.

Panel B of Figure 2.1 illustrates why the stochastic discount factor can be recovered in the presence of hedging. Firms are able to set $b(S)$ and $b(R)$ and determine their payout profile in both the rainy and the sunny states. Contingent claim hyperplanes are therefore differentiable (linear), and indifference curves must be tangent to them at the decision point.

2.3 The Dynamic Limited Enforcement Model

This section develops a discrete-time dynamic agency model in a neoclassical environment. Entrepreneurs make investment and financing decisions with an infinite time horizon. This ensures they take into account the expected consequences of current actions for the feasibility of future decisions. Dynamic financing is subject to limited enforcement constraints.[14] Firms borrow constrained against their equity value from competitive lenders, and implement state contingent debt repayments up to their debt capacity. The state contingent nature of the contract allows firms to transfer resources to states and times where they are more valuable. In Subsections 4.3.1 and 4.3.2, I detail the technology and the industry environment, and the financial contracting problem. In Subsection 2.3.3, I rewrite the contracting model as a recursive dynamic programming problem. Despite its conceptual simplicity, this problem has two nonstandard features. First, conventional dynamic programming results do not apply because the equity value enters the enforcement constraint. Second, the presence of state contingent debt repayments as decision variables makes the problem virtually intractable with conventional iterative numerical methods. In Subsection 4.3.4 I address these two issues. Using a fixed point argument, I first show the existence and uniqueness of the value function as the solution of a dynamic programming problem with an appropriate initial condition. Then, I extend the linear programming techniques in Trick and Zin [1993], Trick and Zin [1997], Nikolov et al. [2013], and Schmid and Steri [2013], and propose a computationally efficient solution method based on mixed-integer programming. Finally, in Subsection 2.3.5, I characterize the solution illustrate the qualitative firm optimal investment and financing policies.

2.3.1 Technology and Competitive Environment

A continuum of perfectly competitive firms operates in an industry. Each firm produces a homogeneous product, whose price is normalized to one. In period t, a fraction ϕ of new

[14] Related contracting problems are proposed, for example, by Albuquerque and Hopenhayn [2004], Rampini and Viswanathan [2010], Rampini and Viswanathan [2013], Li and Whited [2013], and Schmid and Steri [2013].

firms randomly enters the industry. Existing firms become unproductive and exit with probability ϕ, so that the total mass of operating firms is unchanged over time.

An entrant i arrives with some initial capital stock $k_{i,0}$. Entrants engage in a long-term contract with lenders to obtain external financing. Firms have access to a production technology that generates a stochastic stream of profits

$$\Pi(k_{i,t}, s_{i,t}) \equiv A(s_{i,t}) k_{i,t}^{\alpha}$$

where $s_{i,t}$ is a shorthand for the state $\{x_t, z_{i,t}\}$, $k_{i,t}$ is the capital input of firm i at time t, $\alpha \in (0,1)$ is the curvature parameter of the production function, which exhibits decreasing returns to scale, and $A(s_{i,t})$ is a stochastic process describing productivity. Here $A(s_{i,t}) = x_t z_{i,t}$, where x_t and $z_{i,t}$ are respectively aggregate and firm-specific technology shocks. The idiosyncratic shock $z_{i,t}$ is the driving force of firm-level heterogeneity, and generates a nontrivial cross-section of firms, while the aggregate shock x_t describes the overall technological level of the economy. $z_{i,t}$ and x_t follow Markov processes with finite support Z and X, and stationary transition functions $Q_z(z_{i,t+1}|z_{i,t})$ and $Q_x(x_{t+1}|x_t)$ as follows:

$$\log(z_{i,t+1}) = \rho_z \log(z_{i,t}) + \sigma_z \epsilon^z_{i,t+1} \tag{2.16a}$$

$$\log(x_{t+1}) = (1-\rho_x)\mu_x + \rho_x \log(x_t) + \sigma_x \epsilon^x_{t+1} \tag{2.16b}$$

where $\epsilon^z_{i,t}$ and $\epsilon^z_{j,t}$ are uncorrelated for every $i \neq j$, and ϵ^x_t is uncorrelated with $\epsilon^z_{i,t}$ for every i. $\epsilon^z_{i,t}$ and ϵ^x_t are truncated iid standard normal variables. The capital stock $k_{i,t}$ obeys the law of motion

$$k_{i,t+1} = (1-\delta)k_{i,t} + i_{i,t+1}$$

where δ is the depreciation rate and $i_{i,t+1}$ denotes corporate investment.

2.3.2 The Contracting Framework

Upon arriving in the industry, the firm enters a long-term contractual relationship with an outside lender. The contract not only provides initial funding, but also financing over the firm's lifecycle. Following several previous studies, lenders are risk neutral and have "deep pockets", that is they offer an elastic supply of credit in all times and states. This assumption can be interpreted as a reduced form for lenders having a very large amount of funds to achieve a sufficient diversification of risks arising from granting individual loans. The risk neutrality assumption is convenient because it allows not to put addictional structure on the lenders' possible stochastic discount factor. This allows to avoid to explicitly model lenders' decisions and ownership structure, such as bankers' decisions over portfolios of loans and deposits.[15]

Entrepreneurs are risk averse and discount future dividend payouts with a stochastic discount factor process $\{M(x_{t+\tau})\}_{\tau=0}^{\infty}$.[16] Risk-neutral lenders' discount factor is instead $R_t \equiv E_t[M(x_{t+1})]$.

The timing of events over a firm's lifecycle is as follows. As soon as a firm enters the industry, it signs a long-term contract with the lender to obtain initial funding. Then, at the beginning of each period, the firm first faces the exogenous exit shock, and the state $s_{i,t}$ realizes. There are no information asymmetries because $s_{i,t}$ is publicly known. The entrepreneur has limited liability, and the firm defaults if its value after observing the shock goes to zero. Second, firm's decisions and operations occur: inputs are purchased, production takes place, revenues are collected, transfers to and from lenders are made, and dividends are distributed. Third, the firm chooses either to renege the contract or to continue operations. This limited enforcement problem is discussed in more detail below. Panel A of Figure 2.2 summarizes the intra-period timing. In this setup, the contract has one side commitment. While there is a limited commitment problem on the firm's side, the lender honors the long-term contract. This feature becomes apparent in the recursive formulation in Subsection 2.3.3, where a lender's promise-keeping constraint is part of the problem.

In the remainder of this subsection, I define the specifics of long-term contracts, and detail the limited enforcement problem. Following Albuquerque and Hopenhayn [2004], I define feasible and enforceable contracts. Then, I specify the firm's optimization problem by introducing equilibrium contracts. Equilibrium contracts define the Pareto frontier between the value for the firm (which I interpret as equity) and the value for the lender (which I interpret as debt), and impose restrictions the realizations of corporate policies that can be observed in the data.

A long-term contract for a firm i that enters the industry specifies a sequence of capital advancements $\{k_{i,t}\}_{t=0}^{\infty}$, a sequence of transfers $\{\tau_{i,t}\}_{t=0}^{\infty}$ from the firm to the lender, and a sequence of dividend payments $\{d_{i,t}\}_{t=0}^{\infty}$ to the firm's shareholders.[17] The aforementioned investment, financing, and dividend policies are fully state-contingent, and depend on the

[15]In expected utility theory, the risk neutrality assumption captures the evidence for which wealthy individuals behave as if they were risk neutral (Rabin [2000]). Indeed, in models with large investors, the latter are typically modeled as risk neutral or as agents with CARA utility. See also Jaffee and Russell [1976], and Gale and Hellwig [1985]. For general equilibrium models of households, and intermediary capital see, for example, Gertler and Kiyotaki [2010], Gertler and Karadi [2011], and He and Krishnamurthy [2013].

[16]The effective discount factor accounts for the probability that the firm exits the industry, that is:

$$M(x_{t+\tau}) = \widehat{M}(x_{t+\tau})(1-\phi)$$

[17]By convention, positive transfers represent repayments to the lender, while negative transfer are inflows for the firm.

entire history $h_{i,t} \equiv \{k_{i,j-1}, \tau_{i,j-1}, d_{i,j-1}, s_{i,j}\}_{j=1}^{t}$ of previous policies and both aggregate and idiosyncratic shocks. The current shock is part of the history, consistent with the timing described above. Importantly, the contract jointly specifies financing, dividend, and investment policies, in close analogy with the covenants that are routinely found in loan agreements. On the firm's perspective, a contract must be budget feasible, that is the firm's internally generated profits must suffice to cover investment expenses, debt repayments, and dividend distributions. In addition, the entrepreneur cannot raise additional funds by issuing equity, that is $d_{i,t} \geq 0$ for all t. The latter condition prevents the firm from raising costless external equity (i.e. to have negative dividends). Without this constraint, the contracting problem would be trivial. Finally, the contract must be consistent with the entrepreneur's limited liability, that is the value of the firm must be non-negative to prevent non-strategic default.

Definition 1 (Feasible Contract) *Let $\mathcal{H}_i$ be the set of all possible histories for firm i. A feasible contract is a mapping $\mathcal{C}_i : \mathcal{H}_i \to \mathbb{R}^3$ such that for all $h_{i,t} \in \mathcal{H}_i$, $(k_{i,t}, \tau_{i,t}, d_{i,t}) = \mathcal{C}_i(h_{i,t})$, and, for all t:*

$$d_{i,t} \geq 0 \tag{2.17a}$$

$$d_{i,t} + \tau_{i,t} + [k_{i,t+1} - (1-\delta)k_{i,t}] \leq \Pi(k_{i,t}, s_{i,t}) \tag{2.17b}$$

$$E_t \left[\sum_{\tau=0}^{\infty} M(x_{t+\tau}) d_{i,t+\tau} \right] \geq 0 \tag{2.17c}$$

The contract has limited enforcement. The entrepreneur's incentive problem is illustrated in the extensive form game in Panel B of Figure 2.2. Each period t, after observing the shocks and choosing investment, financing, and payout policies, the entrepreneur faces an outside opportunity of total value $O(k_{i,t+1}, s_{i,t})$. The value of the outside opportunity is common knowledge to both parties, and depends on the newly purchased capital stock, and on the current state of the economy. Different interpretations of the outside opportunity can be entertained. For instance, the entrepreneur may liquidate the capital and disappear. The entrepreneur can choose either to renege the contract, divert the capital stock, and use it to pursue an outside opportunity, or to continue operations. In the former scenario, lenders liquidate the firm, and the liquidation value is split between the two parties. In particular, the entrepreneur is left with $\theta k_{i,t+1}$, while the lender expropriates $(1-\theta)k_{i,t+1}$. In equilibrium, the entrepreneur therefore compares the value of continuing running the firm with its share of the liquidation value.[18] Incentive compatibility requires the diversion value not to exceed the value of staying in the contractual relationship. In Panel B of Figure 2.2, this corresponds to the subgame perfect equilibrium $\{\overline{R}, L\}$ in

[18]Notice that the value of the outside opportunity is irrelevant in this setup, because the lender always chooses to liquidate the firm. The strategy $\overline{L}$ in fact delivers a null payoff to the lender, and is therefore dominated by L. This is equivalent to assume $O(k_{i,t+1}, s_{i,t}) = \theta k_{i,t+1}$ in the case of liquidation.

which the firm never reneges the contract because of the threat by the lender to liquidate the firm. This leads to the following definition of enforceable contract (or self-enforcing contract).

[Insert Figure 2.2 Here]

Definition 2 (Enforceable Contract) *A feasible contract $\mathcal{C}_i(\cdot)$ is enforceable if after any history $h_{i,t}$ and for all t, the following enforcement constraint is satisfied:*

$$\theta k_{i,t+1} \leq E_t \left[\sum_{\tau=1}^{\infty} M(x_{t+\tau}) d_{i,t+\tau} \right] \tag{2.18}$$

In equilibrium, contracts must be consistent with both the firm and the lender maximizing their lifetime utility. Since lenders are competitive, equilibrium long-term contracts attain the maximum initial value for the borrower with the lender breaking even. The lender's participation constraint therefore states that the expected discounted value of repayments is non-negative.

Definition 3 (Equilibrium Contract) *An equilibrium contract $\mathcal{C}_i(\cdot)$ is an enforceable contract such that the borrower maximizes*

$$E_0 \left[\sum_{t=0}^{\infty} M(x_t) d_{i,t} \right] \tag{2.19}$$

subject to the lender's participation constraint

$$E_0 \left[\sum_{t=0}^{\infty} R_t^{-t} \tau_{i,t} \right] \geq 0 \tag{2.20}$$

2.3.3 Recursive Formulation

Dealing with equilibrium contracts specified as sequence problems would require to keep track of an infinite sequence of occasionally binding constraints. This is due to the enforcement constraints in Equation (2.18), which must be satisfied in all future periods t. In this section, I formulate the problem recursively, so that dynamic programming techniques can be applied. I propose two recursive formulations. First, following Spear and Srivastava [1987] and Abreu et al. [1990], I formulate the dynamic limited enforcement model in recursive form with firm's capital and promised utility to the lender as endogenous state variables. This formulation allows to interpret optimal contracts as equity/debt pairs on a Pareto frontier.

I define promised utility $b_{i,t}$ at time t as the value of future debt transfers to the lender, that is:

$$b_{i,t} \equiv \sum_{j=0}^{\infty} \tau_{i,t} \tag{2.21}$$

With this definition, Spear and Srivastava [1987] show that the equilibrium contracting problem defined in (2.19), subject to (2.17a), (2.17b), (2.17c), (2.18), and (2.20), has a stationary representation as a dynamic programming problem. This leads to the following formulation:

$$\begin{aligned}
V(k_{i,t}, b_{i,t}, s_{i,t}) &= \max_{\{d_{i,t}, k_{i,t+1}, b(s_{i,t+1})\}} d_{i,t} + E_t\left[M(x_{t+1})V(k_{i,t+1}, b(s_{i,t+1}), s_{i,t+1})\right] && (2.22)\\
& s.t. && \\
d_{i,t} &\geq 0 && (2.23)\\
d_{i,t} &\leq \Pi(k_{i,t}, s_{i,t}) - I_{i,t} - \tau_{i,t} && (2.24)\\
I_{i,t} &= k_{i,t+1} - (1-\delta)k_{i,t} && (2.25)\\
\tau_{i,t} &= R_t b_{i,t} - E_t[b(s_{i,t+1})] && (2.26)\\
V(k_{i,t}, b_{i,t}, s_{i,t}) &\geq 0 && (2.27)\\
\theta k_{i,t+1} &\leq E_t\left[M(x_{t+1})V(k_{i,t+1}, b(s_{i,t+1}), s_{i,t+1})\right] && (2.28)\\
b_{i,0} &\geq 0 && (2.29)
\end{aligned}$$

In this formulation, equilibrium contracts maximize the firm's equity value, using promised utility and the capital stock as endogenous state variables. In analogy with the sequential formulation of the contract, Constraint (2.23) is the dividend non-negativity constraint, Constraint (2.24) is the budget constraint, where the auxiliary variables $I_{i,t}$ and $\tau_{i,t}$ define the current investment expense and transfer to the lender. The law of motions of $I_{i,t}$ and $\tau_{i,t}$ are specified in Constraints (2.25) and (2.26). Constraint (2.26) can be interpreted as a promise-keeping constraint for the lender. Constraint (2.27) is the limited-liability constraint for the borrower. Constraint (2.28) is the enforcement constraint, which states that the diversion value cannot exceed the continuation value. Thus, reneging the contract is never optimal. Finally, contracts are initialized such that the participation constraint (2.29) for the lender is satisfied.

The problem can be further simplified by reducing the dimension of the state space. This can be achieved using net worth as a state variable, in line with Abreu et al. [1990], Rampini and Viswanathan [2010], and Rampini and Viswanathan [2013]. Realized net worth in state $s_{i,t+1}$ is defined as $w(s_{i,t+1}) \equiv \Pi(k_{i,t}, s_{i,t}) + (1-\delta)k_{i,t+1} - R_t b(s_{i,t+1})$, and determines the amount of resources that are available to the firm in a certain state, net of liabilities. Intuitively, net worth is the corporate counterpart of household wealth, and captures how constrained a company is in terms of resources to allocate to investment, and distributions. This leads to the following lemma.

Lemma 1 (Recursive Problem) *The constrained optimization problem in (2.22)-(2.29) is equivalent to:*

$$\begin{aligned} V(w_{i,t}, s_{i,t}) &= \max_{\{d_{i,t}, k_{i,t+1}, b(s_{i,t+1})\}} d_{i,t} + E_t\left[M(x_{t+1})V(w(s_{i,t+1}), s_{i,t+1})\right] && (2.30) \\ & \quad s.t. \\ d_{i,t} &\geq 0 && (2.31) \\ w_{i,t} &\geq d_{i,t} + k_{i,t+1} - E_t[b(s_{i,t+1})] && (2.32) \\ w(s_{i,t+1}) &\leq \Pi(k_{i,t+1}, s_{i,t+1}) + (1-\delta)k_{i,t+1} - R_{t+1}b(s_{i,t+1}) \quad \forall s_{it+1} && (2.33) \\ \theta k_{i,t+1} &\leq E_t\left[M(x_{t+1})V(w(s_{i,t+1}), s_{i,t+1})\right] && (2.34) \\ b_{i,0} &\geq 0 && (2.35) \end{aligned}$$

The recursive formulation in terms of net worth not only improves the computational efficiency of the numerical solution because of the smaller state-space, but is also convenient to introduce the notion of hedging. As I discuss in more detail in Subsection 2.3.5, the firm has a limited borrowing capacity because of the enforcement constraint. In this formulation, the firm has the possibility to choose state-contingent promised utility (debt repayments) $b(s_{i,t+1})$ for each state.[19] The firm can therefore choose to hedge a specific state s at time $t+1$ by choosing a lower debt repayment $b(s)$. Other conditions equal, hedging a state has three effects. First, the firm saves debt capacity by relaxing the enforcement constraint. Second, as Equation (2.33) shows, the firm increases its net worth in state s at time $t+1$, by lowering its required repayment. As a result, more resources are available for investments and distributions in state s. Third, as Equation (2.32) illustrates, a lower repayment in some future state implies a lower amount of external debt raised at time t, and less net worth available for today's investment and distributions. In sum, the firm implements hedging by transferring net worth from today to specific future states tomorrow. Because the firm's debt capacity is limited by the borrowing constraint, the company faces a tradeoff between raising funds today, and preserving them for specific states that may occur tomorrow.

2.3.4 Model Solution

Because the objective function itself appears on the right-hand side of the enforcement constraint, the dynamic programming problem in (2.30)-(2.35) is not a standard convex optimization problem. In particular, verifying the discounting property of Blackwell's sufficient conditions would require the knowledge of the solution to be determined. The

[19] As Rampini and Viswanathan [2010], Rampini and Viswanathan [2013], and Nikolov et al. [2013] discuss, state-contingent debt can be implemented using credit lines, forward, and futures.

solution of the functional equation may therefore not be unique. However, a different approach based on Knaster-Tarski fixed-point theorem allows to establish two results. First, the value function is the unique fixed point of the Bellman operator in a restricted functional space. The lower boundary of this functional space is the zero function, while the upper boundary is the solution to a planner's problem in which the enforcement constraint is removed. Second, the sequence of functions obtained by iterating the Bellman operator from the lower bound converges pointwise to such a fixed point. This leads to the following lemma:

Lemma 2 (Fixed Point) *Assume $M(x_{t+1}) = \beta M_0(x_{t+1})$, with $\beta < 1$, and*

$$\lim_{n\uparrow\infty} \beta^n E_t \left[M_0(x_{t+1}) V(w(s_{i,t+1}), s_{i,t+1}) \right] = 0 \tag{2.36}$$

Let T be the Bellman operator associated with the problem (2.30)- (2.35), $V^{UB}(w_{i,t}, s_{i,t})$ the solution of the same problem without constraint (2.34), and $V^{LB}(w_{i,t}, s_{i,t})$ a function over the same domain of $V(w_{i,t}, s_{i,t})$ such that $V^{LB}(w_{i,t}, s_{i,t}) \leq V(w_{i,t}, s_{i,t})$. Then:
i) The value function is the unique fixed point of T in the order interval between $V^{LB}(w_{i,t}, s_{i,t})$ and $V^{UB}(w_{i,t}, s_{i,t})$.
ii) The sequence of functions $\{T^n V^{LB}(w_{i,t}, s_{i,t})\}_{n=1}^{\infty}$ converges to $V(w_{i,t}, s_{i,t})$ pointwise.

The previous lemma provides an operating procedure to solve for the equilibrium contract. The solution can be obtained by value function iteration from the any initial condition $V^{LB}(w_{i,t}, s_{i,t}) \leq V(w_{i,t}, s_{i,t})$, such as the null function. Assumption (2.36) is guaranteed if the first-best solution $V^{UB}(w_{i,t}, s_{i,t})$ is bounded, and as long as the time-discount factor β in $M(x_{t+1})$ is less than one, and $M_0(x_{t+1})$ is finite. The last two conditions are generally guaranteed in common specifications of the stochastic discount factor.

Unfortunately, because of the large number of control variables (capital, and one debt variable for each future state), the previous iterative solution strategy is plagued by a severe curse of dimensionality, and cannot be practically implemented.[20] In particular, the maximization step is critical. For each iteration, determining the combination of control variables that maximizes the sum of distributions and the continuation value for each state would imply to search over a grid of $nk \cdot nb^{nx \cdot nz}$ points, where nk, nb, nx, and nz are respectively the number of grid points for capital, promised utility, the aggregate, and idiosyncratic shocks. To deal with this computational issue, I start from the linear programming representation of dynamic programming problems with infinite horizon (Ross [1983]). I then propose an equivalent mixed-linear programming representation of

[20]As Rust [1996] discusses, a possible alternative to new computational methods for the solution of large-scale dynamic programming problems is massively parallel policy iteration. However, hardware requirements for massive parallel computation are enormous.

the dynamic programming problem. On this representation, I find a numerical solution by extending the constraint generation algorithm in Trick and Zin [1993].[21] Specifically, I take advantage of a separation oracle, an auxiliary linear programming problem, to deal with large state spaces and achieve computational efficiency. In Appendix C, I derive the key results on which the solution method is based, and I provide details on the implementation of the computational procedure adopted.

2.3.5 Optimal Policies

In this section, I characterize the optimal policy of the firms in the model through their first-order conditions.[22] The optimality conditions show how investment, financing, hedging, and payout policies are intimately related, and illustrate the qualitative mechanisms that drive firm's decisions. Because the problem has no closed-form solution, the following analysis is based on the economic interpretation of the Lagrange multipliers as shadow values.

Before introducing the optimality conditions, the numerical illustration in Figure 2.3 summarizes a few key properties of the firm's value and policy functions.[23] In Figure 2.3 the model is solved numerically under the baseline parametrization in Table 4.2. All policies, unless otherwise specified, refer to the middle state for both the aggregate and the idiosyncratic shocks. Panel A depicts firm's value as a function of current net worth. The value function is increasing and weakly concave in net worth. In particular, it is strictly concave up to a cutoff value w^C, then it becomes linear. As typical in the contracting literature, w^C defines two regions. For $w_{i,t} \geq w^C$, an additional unit of net worth translates into a one-for-one increase in equity because the total real value of the contract is not affected. If $w_{i,t} < w^C$, instead, additional net worth alters the entrepreneur incentives, and the equity value increases with a slope greater than one. Panel B shows the payout policy of the firm. The firm pays no dividends up to w^C, then the payout function is linear in net worth. Notice that the value function is strictly concave precisely in the region where no dividends are paid. Panels C and D present the investment policy $k_{i,t+1}$ and the amount of debt raised $E_t[b(s_{i,t+1})]$. From the threshold w^C onwards, the firm is reaching a "first-best" optimal level of capital. Instead, for $w_{i,t} < w^C$, the firm is constrained in its investment, because the sum of its net worth and the raised debt finance does not suffice to achieve the "first-best" capital stock. Finally, Panels E and F depict the hedging policy of the firm with respect to aggregate and idiosyncratic states. The solid lines represent the

[21] As Denardo [1970] discusses, when discounting is present, Howard [1960] policy iteration corresponds exactly to block pivoting in the *full* equivalent linear program. Constraint generation considers sequences of smaller problems to obtain the solution.

[22] In a similar framework, Thomas and Worrall [1994] prove that the value function is differentiable. Their result extends to this model.

[23] Some properties can be established also analytically, and are rather standard in the hedging literature. I omit them, and refer the reader to Rampini and Viswanathan [2013] for the details.

repayments the equilibrium contract specifies for the middle state, the dashed red lines refer to one state down, and the dash-dotted green lines to one state up. In general, which states the firm hedges depend on the parameter values in the model, and especially on the persistence of the autoregressive processes in Equations (2.16b) and (2.16a). Under the baseline parametrization, Panel E shows that the firm is implementing a lower repayment in the lower state, where the stochastic discount factor is high. On the contrary, Panel F shows that firms have an incentive to hedge more profitable idiosyncratic states, because of the persistence of investment opportunities over time. When aggregate states are concerned, this effect is instead dominated by the one on discount rates.

[Insert Figure 2.3 Here]

I now define $\mu_{i,t}$, $\nu_{i,t}$, and $\lambda_{i,t}$ as the Lagrange multipliers on the dividend non-negativity constraint (2.31), on the budget constraint at time t (2.32), and on the borrowing constraint (2.34). Denote by $\nu(s_{i,t+1})$ the marginal value of net worth in the state $s_{i,t+1}$ at time $t+1$, that is $\nu(s_{i,t+1}) \equiv V_w(w(s_{i,t+1}), s_{i,t+1})$. Notice that by the envelope condition, the marginal value of net worth at time t equals $V_w(w_{i,t}, s_{i,t})$.

The first-order condition with respect to dividends $d_{i,t}$ is:

$$\overbrace{\nu_{i,t}}^{\text{Marginal Benefit of Payouts}} = \overbrace{1 + \mu_{i,t}}^{\text{Marginal Cost of Payouts}} \tag{2.37}$$

The payout policy of the firm balances the cost and the benefits of allocating an additional unit of current net worth to dividend distributions. The investment policy $k_{i,t+1}$ can be illustrated with the corresponding first-order condition:

$$\overbrace{E_t[M(x_{t+1})\nu(s_{i,t+1})(\Pi_k(k_{i,t+1}, s_{i,t+1}) + (1-\delta))]}^{\text{Marginal Benefit of Investment}} = \overbrace{\frac{\nu_{i,t} + \theta\lambda_{i,t}}{1 + \lambda_{i,t}}}^{\text{Marginal Cost of Investment}} \tag{2.38}$$

The left-hand side of Equation (2.38) represents the marginal benefit of an additional unit of capital. Investing one unit more increases realized net worth in every future state by the return on physical capital $\Pi_k(k_{i,t+1}, s_{i,t+1}) + (1-\delta)$. The marginal benefit of investment is the expectation of these returns, accounting for the different importance of future states. Here, the effective discount factor for cash flows from invested capital is $M(x_{t+1})\nu(s_{i,t+1})$. The first component $M(x_{t+1})$ is the stochastic discount factor of the owners, while the second component $\nu(s_{i,t+1})$ relates to the concavity of the value function. The latter term is familiar in models of financial constraints. Specifically, it accounts for the different marginal value of firm's net worth across future states, and effectively renders the firm more risk averse. The right-hand side is instead the effective marginal cost of increasing the capital stock by one unit. In addition to the shadow cost $\nu(s_{i,t+1})$ of reducing net worth at time t, there are two correction terms, $\theta\lambda_{i,t}$ and $1+\lambda_{i,t}$, that reflect the presence of the borrowing constraint. Increasing investment has an effect on both sides of the enforcement constraint (2.34). First, it makes it more tight by increasing the diversion value of capital on the left-hand side, with a shadow value of $\theta\lambda_{i,t}$ for the firm. Second, it increases future net worth and, because the value function is increasing in it, also the continuation value on the right-hand side of (2.34) raises. This lowers the shadow value of investing for the firm, as the term $1+\lambda_{i,t}$ at the denominator of (2.38) captures.

Finally, the first-order conditions with respect to state-contingent debt $b(s_{i,t+1})$ in the contract describes the firm financing and hedging policies:

$$\overbrace{R_t \nu(s_{i,t+1}) M(x_{t+1})}^{\text{Marginal Benefit of Hedging}} = \overbrace{\frac{\nu_{i,t}}{1+\lambda_{i,t}}}^{\text{Marginal Cost of Hedging}} \tag{2.39}$$

Equation (2.39) illustrates the key tradeoff between raising less external resources today and hedging a specific future state $s_{i,t+1}$ by contracting, and implementing, a lower state-contingent repayment $b(s_{i,t+1})$. For this reason, Equation (2.39) highlights how financing and hedging policies are profoundly related. Specifically, the left-hand side represents the marginal benefit of hedging a specific state $s_{i,t+1}$ by reducing the corresponding repayment $b(s_{i,t+1})$, where R_t is the interest rate charged by the risk-neutral lender. As in Equation (2.38), the effective value of the state for the firm is $M(x_{t+1})\nu(s_{i,t+1})$. The right-hand side instead measures the cost of reduced current net worth. The shadow value of the lower amount of resources available for investment and financing is measured by $\nu_{i,t}$. The term $1+\lambda_{i,t}$ reflects a less tight borrowing constraint because of the increased continuation value, as a consequence of hedging the state $s_{i,t+1}$. In fact, a lower repayment $b(s_{i,t+1})$ increases net worth $w(s_{i,t+1})$, and in turn relaxes the borrowing constraint.

2.4 The Corporate CAPM

This section introduces the key asset pricing results of this paper. I first derive the stochastic discount factor in terms of firm's policies and characteristics. This leads to an asset pricing model, which I refer to as the Corporate CAPM. Finally, I discuss the aggregation properties of the asset pricing model and, in particular, a property I dub as the relativity property. The latter is an irrelevance results which states that any subset of firms in the economy can be used to back out the stochastic discount factor. Operatively, this property allows to choose different benchmark sets with respect to which stock prices and returns can be computed.

Proposition 1 (The Corporate CAPM) *i) The stochastic discount factor can be backed* *out from the firm's optimality conditions as follows:*

$$M(x_{t+1}) = \frac{1}{R_t}\frac{1}{1+\lambda_{i,t}}\frac{V_w(w_{i,t},x_t,z_{i,t})}{V_w(w(s_{i,t+1}),x_{t+1},z_{i,t+1})} \tag{2.40}$$

ii) The stochastic discount factor can be approximated as a linear function of observable firm-level variables, and quantities that are predetermined at time t, that is:

$$\log M(x_{t+1}) \approx \mu^M_{i,t} - \overline{a}_{i,t}(w(s_{i,t+1}) - w_{i,t}) - \overline{b}_{i,t}\left(\frac{\rho_{i,t+1}}{\rho^A_{t+1}} - \frac{\rho_{i,t}}{\rho^A_t}\right) - \overline{c}_{i,t}\left(\rho^A_{t+1} - \rho^A_t\right) \tag{2.41}$$

where $\rho_{i,t}$ and $\rho^A_{i,t}$ relate to idiosyncratic and aggregate productivity respectively:

$$\rho_{i,t} \equiv z_{i,t} = \frac{\Pi(k_{i,t},s_{i,t})}{k^\alpha_{i,t}}$$
$$\rho^A_{i,t} \equiv x_{i,t} = \frac{\Pi^A(k_{i,t},s_{i,t})}{\left(k^A_{i,t}\right)^\alpha}$$

and $\mu^M_{i,t} \equiv \log\frac{1}{R_t} + \log\frac{1}{1+\lambda_{i,t}}$, $\overline{a}_{i,t}$, $\overline{b}_{i,t}$, and $\overline{c}_{i,t}$ are predetermined variables at time t, with

$$\mu^M_{i,t} \equiv \log\frac{1}{R_t} + \log\frac{1}{1+\lambda_{i,t}} \tag{2.42}$$
$$\overline{a}_{i,t} \equiv \frac{V_{ww}(w_{i,t},s_{i,t})}{V_w(w_{i,t},s_{i,t})} \tag{2.43}$$
$$\overline{b}_{i,t} \equiv \frac{V_{wz}(w_{i,t},s_{i,t})}{V_w(w_{i,t},s_{i,t})} \tag{2.44}$$
$$\overline{c}_{i,t} \equiv \frac{V_{wx}(w_{i,t},s_{i,t})}{V_w(w_{i,t},s_{i,t})} \tag{2.45}$$

The first part of the proposition obtains a stochastic discount factor from firms' decisions. Equation (2.40) is the counterpart of Equation (2.15) in the two-period example of Section 2.2. This result reflects the key intuition of the paper, that I develop in Section 2.2, and that Panel B of Figure 2.1 illustrates. The possibility to negotiate state-contingent debt repayments with the lenders allows firms to transfer resources across states. Firms have a rationale for hedging because of the endogenous collateral constraint, and have a motive to transfer net worth to most important states, where the stochastic discount factor is high. It is important to notice that in the absence of state-contingent debt, the stochastic discount factor cannot be recovered. This is the case in Panel A of Figure 2.1, in which firms cannot implement state-contingent decisions. The resulting first-order condition would not deliver a stochastic discount factor for each state, but only one equation containing an expectation over all future states, along the lines of (2.7).

Specifically, the stochastic discount factor relates to the firm's policy through the Lagrange multiplier $\lambda_{i,t}$ on the borrowing constraint, and the growth rate of the marginal value of net worth. The left-hand side is the stochastic discount factor, which essentially measures the value of an aggregate state for equity pricing. The right-hand side instead illustrates how the optimal decisions of heterogenous firms adapt to the aggregate state to maximize the value for their shareholders. Backing out the stochastic discount factor therefore amounts to investigate what state must have led a firm to optimally make its observed investment and financing decisions. In the absence of state-contingent financing, realized net worth in individual future states could not instead be influenced by firm's decisions, but would vary across states only because of exogenous shocks. Firms' decisions would not therefore be informative of the stochastic discount factor.

The economic mechanism driving the result in Equation (2.40) relates to firms' hedging behavior. Firms have a motive to transfer resources (net worth) to states that are most important for their shareholder value. This policy would lower the marginal value of net worth in those states. However, investors' risk aversion implies that most important states are "bad times", in which marginal utility of consumption in high, and consumption is low. The term $\frac{1}{1+\lambda_{i,t}}$ accounts for firms being financially constrained. The more financially constrained they are, the higher the shadow value $\lambda_{i,t}$ of extra borrowing, the less their effective ability to transfer resources to most important states, in spite of their hedging motives. This is consistent with the models of Rampini and Viswanathan [2010], and Rampini and Viswanathan [2013], and the evidence in Rampini et al. [2013] and Nikolov et al. [2013], according to which more constrained firms hedge less.

It is important to notice that *all* the state variables of the problem determine the policies of firms, and in turn affect their hedging abilities and the needs. From an empirical viewpoint, this result implies that firms' characteristics enter the stochastic discount factor directly. This mechanism is similar to the way, on the consumption side of the economy, the state variables of the representative household's problem enter the stochastic discount factor in the intertemporal CAPM of Merton [1973]. The second part of the proposition provides an approximated linear representation of the Corporate CAPM, in terms of observable variables and quantities that are pre-determined at time t. Such an approximation delivers the following result:

Proposition 2 (Expected Return-Beta Representation) *The expected excess return on a security $E_t[R_{i,t+1} - R_t^f]$ is given by the following expression:*

$$E_t[R_{i,t+1} - R_t^f] \approx \widetilde{\lambda}_{j,t}^T \beta_{i,t} \tag{2.46}$$

where R_t^f is the riskfree return (or a riskfree equivalent), and the parameters $\widetilde{\lambda}_{j,t}^T$ and $\beta_{i,t}$ are given by

$$\widetilde{\lambda}_{j,t} = \begin{bmatrix} \bar{a}_{j,t} & \bar{b}_{j,t} & \bar{c}_{j,t} \end{bmatrix} \sigma_{j,t}$$

$$\beta_{i,t} = \sigma_{j,t}^{-1} \begin{bmatrix} Cov_t(w(s_{j,t+1}) - w_{j,t}, R_{i,t+1} - R_t^f) \\ Cov_t\left(\frac{\rho_{j,t+1}}{\rho_{t+1}^A} - \frac{\rho_{j,t}}{\rho_t^A}, R_{j,t+1} - R_t^f\right) \\ Cov_t(\rho_{t+1}^A - \rho_t^A, R_{i,t+1} - R_t^f) \end{bmatrix}$$

and $\sigma_{j,t}$ is the covariance matrix of $\left[\begin{array}{ccc} w(s_{j,t+1}) - w_{j,t} & \frac{\rho_{j,t+1}}{\rho_{t+1}^A} - \frac{\rho_{j,t}}{\rho_t^A} & \rho_{t+1}^A - \rho_t^A \end{array} \right]^T$.

Proposition 2 is an equivalent expected return/beta representation of the Corporate CAPM. This formulation emphasizes how expected excess equity returns are determined by the covariance with three factors: the "hedging" factor $w(s_{j,t+1}) - w_{j,t}$, and "idiosyncratic profitability" factor $\frac{\rho_{j,t+1}}{\rho_{t+1}^A} - \frac{\rho_{j,t}}{\rho_t^A}$, and the "aggregate profitability" factor $\rho_{t+1}^A - \rho_t^A$. As usual, $\beta_{i,t}$ can be interpreted as price of risk, and $\widetilde{\lambda}_{j,t}$ as quantity of risk. In the proposition, the index j refers to a benchmark firm with respect to which the factors are computed. The presence of two profitability factors denotes that in some states the j-th firm may be able to generate more resources either because all firms are more profitable (high aggregate productivity), or because it is more profitable with respect to the average (high idiosyncratic productivity). In both cases, firm's realized net worth increases in the state, and this affects the firm's hedging policy. Despite this result, in empirical tests it is convenient to aggregate firms to avoid the measurement problems that arise from separating idiosyncratic and aggregate productivity. The next proposition shows how firms can be conveniently aggregated to implement empirical tests of the model.

Proposition 3 (Aggregation) *Consider an arbitrary subset Ω of N firms in the cross-section.*
i) The expression of the stochastic discount factor in Equation (2.41) and its covariance representation can be restated in terms of averages across firms in Ω as follows:

$$\log M(x_{t+1}) \approx \frac{1}{N} \sum_{j\in\Omega} \left[\log \mu_{i,t}^M - \overline{a}_{j,t}(w(s_{j,t+1}) - w_{j,t}) - \overline{b}_{j,t}\left(\frac{\rho_{j,t+1}}{\rho_{t+1}^A} - \frac{\rho_{j,t}}{\rho_t^A}\right) - \overline{c}_{j,t}(\rho_{t+1}^A - \rho_t^A) \right] \tag{2.47}$$

and

$$E_t[R_{i,t+1} - R_t^f] \approx \widetilde{\lambda}_{j,t}^T \beta_{j,t} \tag{2.48}$$

with

$$\widetilde{\lambda}_{j,t} = -\sigma_{\Omega t}$$

$$\beta_{i,t} = \sigma_{\Omega t}^{-1} \begin{bmatrix} Cov_t\left(\frac{1}{N}\sum_{j\in\Omega} -\overline{a}_{j,t}(w(s_{j,t+1}) - w_{j,t}), R_{i,t+1} - R_t^f\right) \\ Cov_t\left(\frac{1}{N}\sum_{j\in\Omega} -\overline{b}_{j,t}\left(\frac{\rho_{j,t+1}}{\rho_{t+1}^A} - \frac{\rho_{j,t}}{\rho_t^A}\right), R_{i,t+1} - R_t^f\right) \\ Cov_t\left(\frac{1}{N}\sum_{j\in\Omega} -\overline{c}_{j,t}\left(\rho_{t+1}^A - \rho_t^A\right), R_{i,t+1} - R_t^f\right) \end{bmatrix}$$

and $\sigma_{\Omega t}$ *is the covariance matrix of* $\left[\begin{array}{ccc} \frac{1}{N}\sum_{j\in\Omega}\overline{a}_{j,t}(w(s_{j,t+1})-w_{j,t}) & \frac{1}{N}\sum_{j\in\Omega}\overline{b}_{j,t}\left(\frac{\rho_{j,t+1}}{\rho^A_{t+1}}-\frac{\rho_{j,t}}{\rho^A_t}\right) & \frac{1}{N}\sum_{j\in\Omega}\overline{c}_{j,t}\left(\rho^A_{t+1}-\rho^A_t\right) \end{array} \right]^T$ *ii) If* $N \to \infty$*, then:*

$$\log M(x_{t+1}) \approx \frac{1}{N}\sum_{j\in\Omega}\left[\log \mu^M_{i,t} - \overline{a}_{j,t}(w(s_{j,t+1}) - w_{j,t}) - \overline{c}_{j,t}(\rho^A_{t+1} - \rho^A_t)\right] \tag{2.49}$$

with the following expected return/beta representation

$$\beta_{i,t} = \sigma^{-1}_{\Omega t}\left[\begin{array}{c} Cov_t\left(\frac{1}{N}\sum_{j\in\Omega} -\overline{a}_{j,t}(w(s_{j,t+1}) - w_{j,t}), R_{i,t+1} - R^f_t\right) \\ Cov_t\left(\frac{1}{N}\sum_{j\in\Omega} -\overline{c}_{j,t}\left(\rho^A_{t+1} - \rho^A_t\right), R_{i,t+1} - R^f_t\right) \end{array} \right]$$

and $\sigma_{\Omega t}$ *is the covariance matrix of* $\left[\begin{array}{cc} \frac{1}{N}\sum_{j\in\Omega}\overline{a}_{j,t}(w(s_{j,t+1})-w_{j,t}) & \frac{1}{N}\sum_{j\in\Omega}\overline{c}_{j,t}\left(\rho^A_{t+1}-\rho^A_t\right) \end{array} \right]^T$

The first part of the proposition provides a theoretical irrelevance result, to which I refer as the relativity property. This can be illustrated as follows. In the model, all firms maximize the value for the owner, in that they use the same the stochastic discount factor to discount expected future profits. In the model, the left-hand side of Equation (2.47) is therefore constant, and the stochastic discount factor can be backed out by averaging out the right-hand side for any subset of firms Ω in the economy (e.g. an industry).[24] The property essentially states that the reference set of firms with respect to which expected returns are evaluated can be arbitrarily chosen. This differs from macro models with a representative agent, that dictate that factors must necessarily be aggregated quantities. The second part of the proposition provides an aggregation result when the number of firms used to construct the stochastic discount factor is large enough. In this case, the idiosyncratic productivity factor zeros out because of averaging out a large number of firms in the cross-section. This result is useful in implementing empirical tests of the model. Using individual firms to back out the stochastic discount factor can be problematic for two main reasons. First, as in any economic model, there are omitted forces that can affect individual firms much more than sample averages, such as product market competition, labor market frictions, or investment adjustment costs. Second, testing the model in a small sample of firms would pose the challenge of measuring and disentangling aggregate and idiosyncratic productivities. Such a task would lead to technical difficulties, and is subject to misspecification errors, as discussed for example in Burnside et al. [1996] and Ábrahám and White [2006].

[24]In case Ω is a weighted portfolio of firms, all the sample averages are replaced by weighted averages.

2.5 Quantitative Analysis

I resort to calibration to evaluate the quantitative ability of the model to rationalize firm's observed policies. Calibration restricts some structural parameter values to replicate some key quantities in the data. Ideally, a one-to-one mapping between parameters and moments provides a sufficient condition for identification. Such a close mapping is hard to accomplish in any economic model, because firm's investment and financing decisions are intertwined, and the model parameters affect all the data moments.

To identify the key parameters in the model, I break them down into three groups. The first group includes parameters whose value can be restricted from existing quantitative works or mapped directly into data moments. The second group refers to parameters that can be identified using some aggregate asset pricing moments. The third group includes parameters that I set to obtain a match between the simulated data moments from the model, and the actual data moments. Panel C of Table 4.2 reports parameter values, while Panel A and B respectively show simulated and actual moments that pertain to corporate policies, and to aggregate asset pricing quantities. All data are described in Appendix D.

[Insert Table 4.2 Here]

In the numerical solution of the model, I follow the recent literature on cross-sectional asset pricing and specify an exogenous process for the stochastic discount factor (Berk et al. [1999], Carlson et al. [2004], Zhang [2005], Gomes and Schmid [2010]). Since the goal of this section is to provide evidence that the model is quantitatively successful on the corporate side for a sensible choice of a pricing kernel, this strategy seems reasonable. All calibrations are based on annual data, consistent with the quantitative corporate finance literature. I follow Zhang [2005] and I specify the pricing kernel as follows:

$$\log M(x_{t+1}) = \log \beta + [(\gamma_0 + \gamma_1(x_{t+1} - \mu_x)](x_{t+1} - x_t) \tag{2.50}$$

where β, $\gamma_0 > 0$, and $\gamma_1 < 0$ are constant parameters.[25]

The parameters that pertain to the first group are the depreciation rate δ, the persistence ρ_x and the volatility σ_x of the aggregate shock process, and the exit rate ϕ. The depreciation rate is set to 0.15, to approximately match the depreciation rate for US listed firms in my sample. This value is the same used in Hennessy and Whited [2007], and DeAngelo et al. [2011]. ρ_x and σ_x are set to 0.95^4 and $0.007 \cdot 4$ to correspond, on an annual frequency and with the autoregressive specification in (2.16b), to the quarterly values of 0.95 and 0.007 in Cooley and Prescott [1995]. As in Gomes and Schmid [2010], I set the fraction of incumbents ϕ to 0.02, in line with the study of Covas and Den Haan [2012].

[25] For a in-depth discussion of this assumption and of the properties of the pricing kernel see Berk et al. [1999] and Zhang [2005].

The second set of parameters consists of those in the stochastic discount factor, β, γ_0, and γ_1. I pin down their value, using the strategy in Zhang [2005], to match three aggregate moments: the mean and volatility of the real interest rate, and the average Sharpe ratio. The parametrization in Equation (2.50) for the pricing kernel is convenient in that the real interest rate R_t^f and the maximum Sharpe ratio S_t are:

$$R_t^f = \beta^{-1} e^{-(\mu_m + \frac{1}{2}\sigma_m^2)} \tag{2.51}$$

$$S_t = \frac{\sqrt{e^{\sigma_m^2}(e^{\sigma_m^2} - 1)}}{e^{\frac{\sigma_m^2}{2}}} \tag{2.52}$$

with

$$\mu_m = [(\gamma_0 + \gamma_1(x_t - \mu_x)](x_t - \mu_x)(1 - \rho_x) \tag{2.53}$$

$$\sigma_m = [(\gamma_0 + \gamma_1(x_t - \mu_x)]\sigma_x \tag{2.54}$$

This strategy yields $\beta = 0.94$, $\gamma_0 = 12.5$, and $\gamma_1 = -120$, and gives a real interest rate of 2.99% per year, an annual interest rate volatility of 3.75%, and a Sharpe ratio of 0.35. These values are close to the corresponding data moments of 2.2%, 4.35%, and 0.41.

Finally, I pick 13 moments to match the remaining 5 parameters in the third group. I roughly categorize these moments as representing firm's investment, financing, and equity returns. On the investment side, I choose moments that relate to operating income, investment, and Tobin's Q. On the financing side, I consider the mean, variance, and serial correlation of leverage. On the asset pricing side, I pick the mean and the average of market excess return, and the average volatility of individual stock returns. The resulting parameter values appear to be reasonable. The curvature α is 0.76, in the range of values reported by Hennessy and Whited [2005], Hennessy and Whited [2007], and DeAngelo et al. [2011] on annual data. The persistence and volatility ρ_z and σ_z of idiosyncratic productivity shocks are within one standard error of the estimates in Hennessy and Whited [2007], in which there are no capital adjustment costs as in the present framework. The parameter μ_x is a scale parameter, that determines the scale of the simulated economy and the steady-state capital stock. Finally, there is little guidance for the value of θ, which represents the fraction of capital that the entrepreneur effectively diverts in the case of liquidation. I set $\theta = 0.3$, which is in line with values of related quantities in existing models, such as DeAngelo et al. [2011] and Nikolov et al. [2013].

Panels A and B of Table 4.2 show that the model is broadly successful in matching both aggregate asset pricing moments, and moments that relate to corporate investment and financing. The model performance may further improve by adding other frictions and considering additional moments. However, the absence of these frictions like capital adjustment costs and fixed operating costs considerably simplifies the analysis. Because the focus of this work is to derive a stochastic discount factor from an optimal contracting framework, I privilege model parsimony over an improvement of the quantitative fit of the model.

[Insert Table 4.2 Here]

In Tables 2.2 to 2.5 I perform comparative statics exercises with respect to the parameters in the third group, and to the volatility and persistence of aggregate shocks. This allows to assess the quantitative effect of the parameters in the model on the data moments. Specifically, for each parameter except the scale parameter μ_x, I consider three possible values (low, medium, and high) and I evaluate how the data moments vary in the each scenario.

Table 2.2 considers the curvature α of the production function. This parameter has a strong effect on all moments. A higher curvature leads to lower, less volatile, and less autocorrelated operating income (profitability), higher and more volatile investment, and lower Tobin's Q. Higher returns to scale also imply higher, more volatile, and less autocorrelated leverage. Distributions decrease with the curvature of the production function. On the asset pricing side, higher values of α imply lower average excess returns, and more volatile returns, both at the aggregate level and at the individual stock level.

[Insert Table 2.2 Here]

Another key parameter of the model is θ, the fraction of capital that the entrepreneur can divert in the case of liquidation. *Ceteris paribus*, higher values of theta render the collateral constraint tighter. This parameter has primarily an effect on leverage: higher values of θ reduce firm's leverage, and render it less volatile. A higher fraction of capital that the firm can potentially divert also leads to an increase in operating income, in the investment-to-capital ratio, in Tobin's Q, and in average excess returns.

[Insert Table 2.3 Here]

Tables 2.4 and 2.5 refer to the volatility and persistence of aggregate shocks (Panel A), and idiosyncratic shocks (Panel B). While σ_z only affects the uncertainty of firm-specific investment opportunities, σ_x also drives the probability that important states for firm value, where the stochastic discount factor is high, occur. High values of σ_x lead to lower and more volatile operating income, to higher and more volatile investment-to-capital ratios, to lower average dividends, to higher mean excess returns, and to more volatile returns. Data moments are quantitatively less sensitive to σ_z than to σ_x, in that the former does not affect the stochastic discount factor directly. Qualitatively, the effect of σ_z is roughly similar to that of σ_x, with the exception that higher values of σ_z imply higher average distributions, in that firm-specific investment opportunities are very risky.

Analogously, the persistence parameter ρ_x captures not only how good times are likely to follow other good times in term of investment opportunities, but also the persistence of the process driving discount rates. Higher values for it imply higher and more autocorrelated operating income, lower and less volatile investment, more correlated leverage, and less volatile returns. As in the case of volatility, the parameter that drives the aggregate shocks appears to have a stronger quantitative effect on moments. Qualitatively, however, higher values for ρ_z lead to higher and more volatile operating income and investment, and to higher and more volatile returns. As I discuss in Subsection 2.3.5, firms have strong motives to hedge aggregate states, and higher values of ρ_x imply a lower probability of a change in the state of the economy.

[Insert Table 2.4 Here]

[Insert Table 2.5 Here]

2.6 The Corporate CAPM: Empirical Evaluation

In this section, I test the implications of the Corporate CAPM in the data. Because the focus of this work is on differences in risk premia across assets, I examine the implications of the model for cross-sectional expected excess returns. To do so, I test the following restrictions on the pricing errors of a vector of excess returns R^e_{t+1}:

$$E_t[M(x_{t+1}R^e_{t+1})] = 0 \tag{2.55}$$

where $M(x_{t+1})$ is defined in Equation (2.49). The model with excess returns does not identify the intercept $\mu^M_{i,t}$ of the stochastic discount factor in Proposition 3. The intercept is in fact predetermined at time t, and can be normalized in empirical tests (Cochrane [2001], Yogo [2006], Belo [2010]). I implement empirical tests by GMM using yearly data from 1965 to 2010. Estimation is by two-step GMM, with the initial weighting matrix attaching equal weights to all assets. Appendix E provides details on the estimation procedure, and replicates the empirical tests with an alternative measure of the productivity factor based on Fernald [2009]. The latter analysis controls for possible misspecifications in measuring aggregate productivity ρ^A_t as a Solow residual, as discussed by Burnside et al. [1996]. All data are described in Appendix D.

The test assets are: (i) the 25 Fama-French portfolios sorted by size and book-to-market equity, (ii) the 30 Fama-French industry portfolios, (iii) 25 portfolios sorted by market and HML beta, and (iv) all the previous portfolios together. The 25 Fama-French portfolios are chosen because they capture the value and the size premia, which have received considerable attention in the literature. As in Lewellen et al. [2010], I include the 30 Fama-French industry portfolios to relax the tight factor structure of the 25 Fama-French portfolio. At Lewellen et al. [2010] document, the 30 industry portfolios represent a challenging test for all leading asset pricing models. Following Yogo [2006], I also include the beta-sorted portfolios, in order to address the critique in Daniel and Titman [2012].

As Equation (2.49) shows, if the number of firms with respect to which the factors are computed is large enough, the Corporate CAPM reduces to a two-factor conditional model. In other words, the coefficients $\overline{a}_{i,t}$ and $\overline{c}_{i,t}$ are time varying and depend on firms' characteristics. In the next two subsections, I therefore implement both unconditional and conditional tests. Unconditional tests treat $\overline{a}_{i,t}$ and $\overline{c}_{i,t}$ as constant parameters. Unconditional tests are reported for comparability with previous studies. In conditional tests, I instead use a model-based identification strategy. More precisely, I use the quantitative policy function of the model from Section 2.5 to find a parsimonious functional form for the time-varying coefficients in terms of constant parameters and observable variables. As aggregation properties in Proposition 3 illustrate, in order to implement empirical tests a level of aggregation must be specified. For comparability with previous studies that use aggregate data, in both Subsections 2.6.1 and 2.6.2 I aggregate data at the market level. In Subsection 2.6.4 I instead carry out empirical tests using the five Fama-French industries (consumer goods, manufacturing, hi-tech, healthcare, other) as references.

2.6.1 Unconditional Tests

If $\overline{a}_{i,t}$ and $\overline{c}_{i,t}$ are constant terms, Proposition 3 leads to a two factor model where the net worth and profitability factors are averaged across all firms. Table 2.6 presents the estimation results. Coefficient estimates for the two factors and the corresponding HAC standard errors are reported. The table also reports the following goodness-of-fit measures based on first-stage inference: the mean absolute pricing error (MAE), and the cross-sectional R^2 of a regression of realized average excess returns on predicted average excess returns, computed as in Campbell and Vuolteenaho [2004]. As a measure of model misspecification I report the Hansen-Jagannathan (HJ) distance (Hansen and Jagannathan [1997]). The HJ-distance can be interpretd as the minimum distance between the proposed stochastic discount factor and and the set of correct stochastic discount factors for a given set of test assets. Finally, the table includes two formal tests of the model: the J-test of overidentifying restrictions (Hansen and Singleton [1982]), and a test of the null hypothesis of zero HJ-distance (Jagannathan and Wang [1996]). Although several studies[26] document the statistical power of both tests is low in the context of asset pricing tests, and their small-sample properties vary to a great extent with the sample size and the test assets, I report them for comparability with previous studies.

Unconditional tests suggest that the Corporate CAPM finds support in the data. The

[26]See, for example, Ferson and Foerster [1994], Ahn and Gadarowski [2004], and Lewellen et al. [2010].

first two rows of Table 2.6 report GMM estimates of the coefficients on the net worth and profitability factor for all the test assets. Although conditional tests are a more appropriate setting to discuss the sign restrictions on the coefficients, the unconditional estimates are overall in line with the predicted signs for $\overline{a}_{i,t}$ and $\overline{c}_{i,t}$ from the model. Column 4 shows that when all test assets are considered, the coefficients on net worth and profitability factors have a negative and a positive sign respectively, as the model predicts. As Columns 1-3 show, the coefficient on the profitability factor is positive even for all test assets individually. In addition, while the 25 portfolios sorted by size and book-to-market and the risk-sorted portfolios do not individually lead to statistically significant estimates of the coefficients for the net worth factor, the estimates for the 30 industry portfolios clearly identify a negative coefficient. Such a negative coefficient remains significant when all portfolios are considered together, with a point estimate of -6.334, more than four standard errors from zero. This result supports the recommendation in Lewellen et al. [2010] to include the Fama-French industry portfolios in tests of asset pricing models.

The Corporate CAPM appears to capture most of the variation in expected returns across the test assets. Mean absolute pricing errors range from 0.676% to 0.838% per annum. Cross-sectional R^2 are also high, ranging from 0.771 for the industry portfolios, to 0.923 for the 25 size/book-to-market porfolios. Remarkably, the model is successful in pricing the Fama-French 30 Industry portfolios. In fact, as Lewellen et al. [2010] document, these test assets represent a challenge for all leading asset pricing models. Finally, although the results of formal tests should be interpreted with extreme caution for the reasons above, both the tests based on the HJ distance and the J statistic cannot statistically reject the model.[27]

[Insert Table 2.6 Here]

Figure 2.4 provides a visual summary of the performance of the model. Panels A through D report predicted versus realized average returns for the four sets of test assets. If priced correctly, the portfolio should lie along the 45-degree line. The figure clearly shows that the pricing performance of the Corporate CAPM is more than satisfactory.

[Insert Figure 2.4 Here]

2.6.2 Conditional Tests

In this section I implement conditional tests of the Corporate CAPM. Because the change in aggregate profitability does not vary across firms, Equation (2.49) leads to the following

[27]The values of the HJ distance for the case of all portfolios together is not reported because, as Cochrane [1996] discusses, the cross-moment matrix of returns is nearly singular when the number of test assets is large.

specification for the stochastic discount factor:

$$\log M(x_{t+1}) \approx \mu^M - \frac{1}{N}\sum_{j\in\Omega}\left[\overline{a}_{j,t}(w(s_{j,t+1}) - w_{j,t})\right] - \overline{c}_t(\rho^A_{t+1} - \rho^A_t) \qquad (2.56)$$

where $\overline{c}_t \equiv \frac{1}{N}\sum_{j\in\Omega}\overline{c}_{j,t}$.

As the theoretical argument in Hansen and Richard [1987] remarks, testing conditional models is conceptually difficult because they inherently depend on the information structure of the agents in the economy. In empirical work, the most common testing strategy is to specify the conditional parts of the model as linear functions of some set of observable variables, such as the default and term spreads, the consumption-to-wealth ratio of Lettau and Ludvigson [2001], and the aggregate dividend yield. Other approaches make use of higher frequency data, such as the MIDAS techniques in Ghysels et al. [2004] and Ghysels et al. [2005].

In the implementation of conditional tests, I use the policy function of the model to specify a parsimonious functional form for $\overline{a}_{j,t}$ and $\overline{c}_t$. I adopt a model-based identification strategy for three reasons. First, the annual data frequency of my sample is not well-suited to implement methods that take advantage of high frequency data. Second, the coefficients $\overline{a}_{j,t}$ and $\overline{c}_t$ depend on the state variables of the model, rather than on the observable variables usually considered in conditional tests based on macroecomic factors. Third, as Brandt and Chapman [2006] discuss, a linear approximation for the functional forms of the coefficients in the model may result in large misspecifications. Admittedly, the information set investors access in the real world is larger than the state variables of the contracting model. However, as Hansen and Richard [1987] show, by the law of iterated expectations a conditional model can be tested by "conditioning down" finer information sets to coarser ones.[28]

Panels C and F of Figure 2.5 plot the building blocks for the conditional tests in this section, namely the coefficients $\overline{a}_{i,t}$ and $\overline{c}_{i,t}$ for the firm i. The coefficient $\overline{a}_{i,t}$ is negative and increasing in current net worth, and its graph is highly nonlinear, especially for firms with low net worth. The negative sign of $\overline{a}_{i,t}$ follows directly from the shape of the value function. Panels A and B depict respectively the denominator and the numerator of $\overline{a}_{i,t}$, as defined in Proposition 1. The graph in Panel A is the marginal value of net worth, which is positive because the value function is increasing in net worth. The graph in Panel B is its derivative with respect to net worth, which is negative because of the concavity of the value function. Analogously, the coefficient $\overline{c}_{i,t}$ is approximately linear and decreasing in the current aggregate shock x_t which, as Proposition 1 shows, can be measured in the data as the Solow residual ρ^A_t. Panels D and E depict the denominator and the numerator

[28]As Cochrane [2001] points out, all the moments computed with respect to the coarser information set must exist.

of $\overline{c}_{i,t}$ under the baseline calibration in Table 4.2.

[Insert Figure 2.5 Here]

To carry on conditional tests, I look for an approximation of $\overline{a}_{i,t}$ and $\overline{c}_t$ in terms of observable variables. To do so, I run regressions on the model solution to identify a functional form for $\overline{a}_{i,t}$ and $\overline{c}_t$ in terms of net worth $w_{i,t}$ and ρ_t^A. While both coefficients in principle depend on all the state variables of the model, my goal is to find a parsimonious functional form for them, which possibly involves only a subset of the state variables. Table 2.7 reports the estimates for a nonlinear regression of $\overline{a}_{i,t}$ on the function $a_0 \frac{1}{1+a_1 w_{i,t}}$, where a_0 and a_1 are constant parameters, and the estimates for a linear regression of $\overline{c}_t$ on ρ_t^A. The nonlinear regression is implemented with the algorithm in Levenberg [1944] and Marquardt [1963], as described in the caption of the table. While the model as no closed-form solution, the approximations for both coefficient delivers a good fit, with R^2 statistics of 0.969 and 0.999 respectively. The regressions produce estimates of -35.424, 7.489, 4.142, and -17.623 for a_0, a_1, the intercept c_0, and the slope c_1. Given the limited number of observations on an annual frequency, to avoid overfitting and noisy estimates in the GMM tests of the model, I only estimate a_0 and c_0, while I set a_1 and c_1 to the values reported above.

[Insert Table 2.7 Here]

Table 2.8 reports the results for the estimation. The results are consistent with those of the unconditional tests in Table 2.6. The estimates of a_0 and c_0 have the expected sign when all test assets are considered in Column 4. The estimates in Columns 1-3 confirm that, as in unconditional tests, the Fama-French 30 industry portfolios play an important role in the inference. Finally, the Corporate CAPM appears to have a good pricing performance, with mean absolute pricing errors below 0.8% per year, and R^2 statistics well above 0.8.

[Insert Table 2.8 Here]

2.6.3 Comparison Among Models

Table 2.9 compares the pricing performance of the Corporate CAPM and that of the most popular existing asset pricing models. I consider three other models: the CAPM (Column 1), the Fama and French three-factor model (Column 2), and the Consumption CAPM (Column 3). Columns 4 and 5 report the results for both unconditional and conditional tests of the Corporate CAPM. In terms of test assets, Panel A refers to the Fama-French 25 portfolios, Panel B to the 25 portfolios sorted by HML and market beta, Panel C to the 30 Fama-French industry portfolios, and Panel D to all portfolios together.

[Insert Table 2.9 Here]

As in previous studies, the CAPM and the Consumption CAPM are not successful in pricing the tests assets. The MAE is high, ranging from 1.362% per annum to 1.911% per annum, and the R^2 is consistently low across all test assets. The Fama-French model instead performs rather well, with mean absolute pricing errors ranging from 0.673% to 1.095% per year, and R^2 between 0.630 for the 30 Fama-French portfolios and 0.915 for the portfolios sorted by size and book-to-market. With respect to these two indicators, the Corporate CAPM outperforms all models on all test assets, both in its unconditional and conditional specification. Not surprisingly, and consistent with the findings in Ahn and Gadarowski [2004], Burnside [2010], Lewellen et al. [2010], and Daniel and Titman [2012], the formal tests based on HJ and J statistics are uninformative, and are unable to reject any model. Although these findings should be interpreted with caution due to the well-known issues with the testing framework, the Corporate CAPM seems to have a satisfactory pricing performance.

[Insert Figure 2.6 Here]

Figure 2.6 summarizes the previous comparison among models, in line with Figure 2.4. Panels A through D depict predicted versus realized average excess returns for the CAPM, the Fama-French model, the Consumption CAPM, and the Corporate CAPM. The figure refers to all the test assets together. Panels A and C show that the points are far from the 45-degree line for the CAPM and the Consumption CAPM, while they line up fairly well for the Fama and French's model (Panel B), and especially for the Corporate CAPM (Panel D).

2.6.4 Industry Breakdowns

As I discuss in Section 2.4, Proposition 3 provides an irrelevance result that I dub as the relativity property. In the model, as long as the number of firms used in the aggregation process is large, any choice of the set of benchmark firms for the computation of the factors allows to back out the same approximate stochastic discount factor.

Table 2.10 reports unconditional (Panel A) and conditional (Panel B) tests of the Corporate CAPM with respect to five large reference industry, namely the Fama-French industries (consumer goods, manufacturing, hi-tech, healthcare, other). The test assets are all the previous portfolios together. The results appear to be consistent with the relativity property. Regardless of the reference industry, mean absolute pricing errors are rather low, with R^2 statistics between 0.720 to 0.854. In addition, the estimates for the coefficients on the net worth and profitability factors are respectively negative and positive as predicted by the model.

These results represent a starting point to understand common procedures that focus on "comparable" firms, and that practitioners ordinarily use for company valuation, such as relative valuation based on multiples or bottom-up betas (Damodaran [2008]). In fact, unlike classical macro-based asset pricing models, the present framework allows to formally introduce the concept of benchmark set of firms. Future research may extend the present model to analyze the conditions under which the irrelevance result breaks, and attempt to rationalize such commonly used practices.

[Insert Table 2.10 Here]

2.7 Conclusions

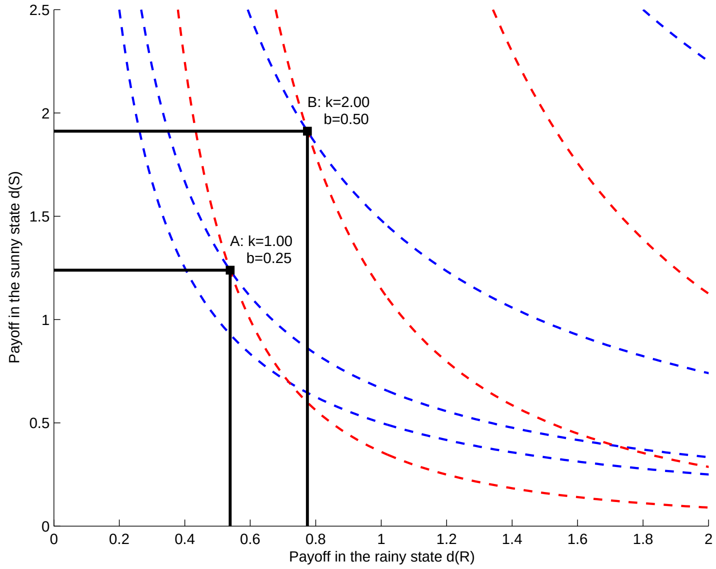
Recent corporate finance studies document that hedging motives represent a key determinant of corporate decisions. In a dynamic contracting model, I recover a stochastic discount factor from firm's investment and financing policies. This leads to a novel asset pricing model, the Corporate CAPM. In the model, firms hedge by transferring resources to future states where they are more valuable. Firms have limited funds because of collateral constraints that endogenously arise from agency conflicts between firms and lenders. The amount of resources firms can devote to hedging is therefore limited. In this context, the shareholders' stochastic discount factor measures the importance of each state for firm's value. Value maximization provides a motive for firms to hedge most important states, in a tradeoff with their funding needs for current investment and distributions. On the corporate finance side, a calibrated version of the model is quantitatively consistent with investment, financing, and payout policies of US listed firms. On the asset pricing side, the Corporate CAPM finds support in the data. The model performs well in pricing different test assets, also in comparison to popular asset pricing models, namely the CAPM, the Consumption CAPM, and the Fama and French three-factor model.

This work has implications for future research not only for production-based asset pricing, but also for consumption-based models, and for empirical work on the cross-section of expected returns. The present framework may represent a complementary tool to advance the understanding of the consumption side of the economy. As Cochrane [2011] points out, the ultimate goal of asset pricing theory should be to provide a general equilibrium explanation of how asset returns and consumption are jointly determined. In general equilibrium, the stochastic discount factor obtained from both the production and consumption side of the economy must have consistent properties. These additional restrictions may provide guidance in modeling the household side on the economy. Another implication of this paper is that the state variables of the firm's optimization problem, in other words the determinants of firms' decisions, enter the stochastic discount factor directly. For empirical work, this observation may provide insights for the development of new testable hypotheses for cross-sectional differences in returns. Finally, an asset pricing result in this paper is what I refer to as the relativity property: any subset of firms in the economy can be used as a benchmark to recover the stochastic discount factor, and compute prices and returns. The relativity property is an irrelevance result because, literally, the model predicts that the choice of the set of benchmark firms does not matter. However, practioners often adopt procedures that focus on comparable firms to evaluate riskiness and compute equity returns.[29] The irrelevance result in this framework may represent a benchmark for future research, both theoretical and empirical. I leave these as possible topics for future research.

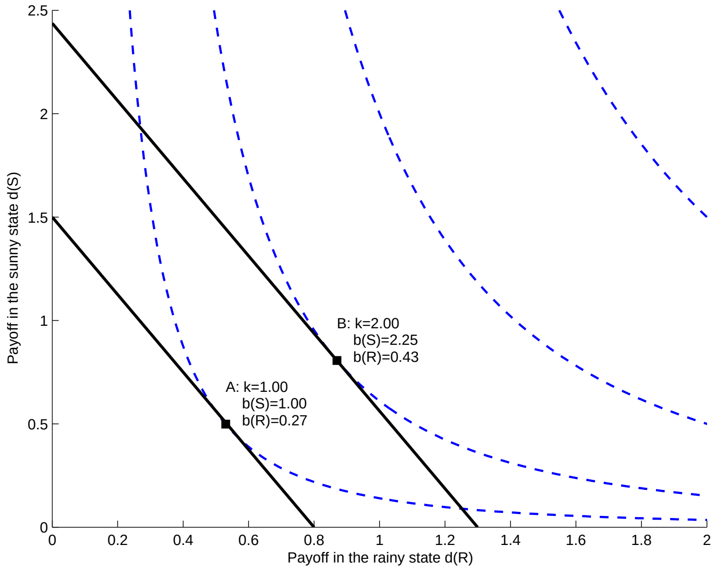
[29]Examples are the use of bottom-up betas, and relative valuation based on multiples. See, for example, Damodaran [2008].

Figure 2.1. Collateral-Based Asset Pricing: Illustration

Panel A: No Hedging



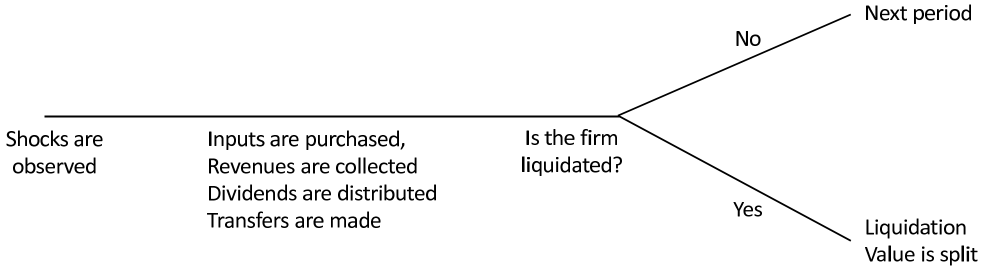
Panel B: Hedging



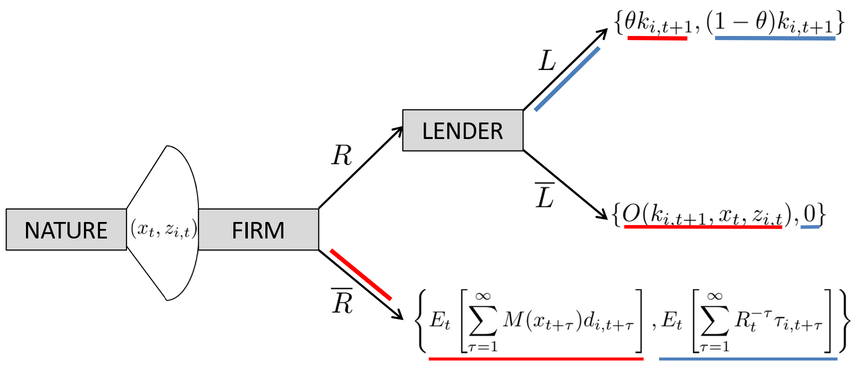
The figure illustrates the set of possible payoffs of a firm with and without hedging in the context of the example in Section 2.2. Panel A depicts the case of no hedging, while Panel B introduces hedging. In Panel A, the thick solid lines represent the firm's payoff in the sunny $(d(S))$ and rainy $(d(R))$ states for a given payout $d(F)$ in the foggy state. k in capital investment, and b is the debt stock. Blue and red dashed lines represent two possible sets of indifference curves for the representative investor. The equilibrium marginal rate of substitution, and hence the stochastic discount factor, cannot be backed out because the kinks at any decision point are consistent with more than one indifference curve. In Panel B, the firm can transfer resources across states by arranging state-contingent debt repayments $b(S)$, $b(F)$, and $b(R)$ in the sunny, foggy, and rainy states, in the presence of collateral constraints. The payout set is linear, and in equilibrium its slope must be equal to the slope of indifference curves.

Figure 2.2. The Dynamic Limited Enforcement Problem

Panel A: Intraperiod Timing

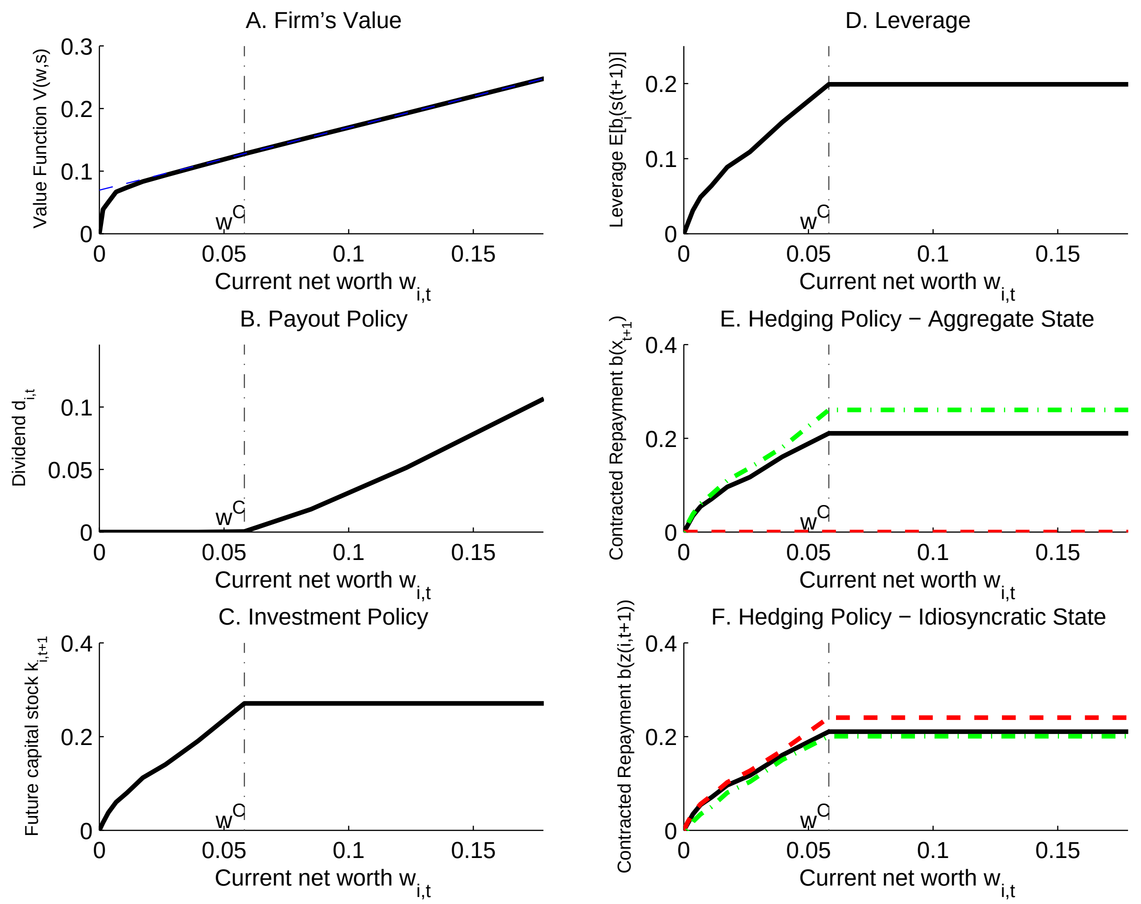


Panel B: Extensive Form Game



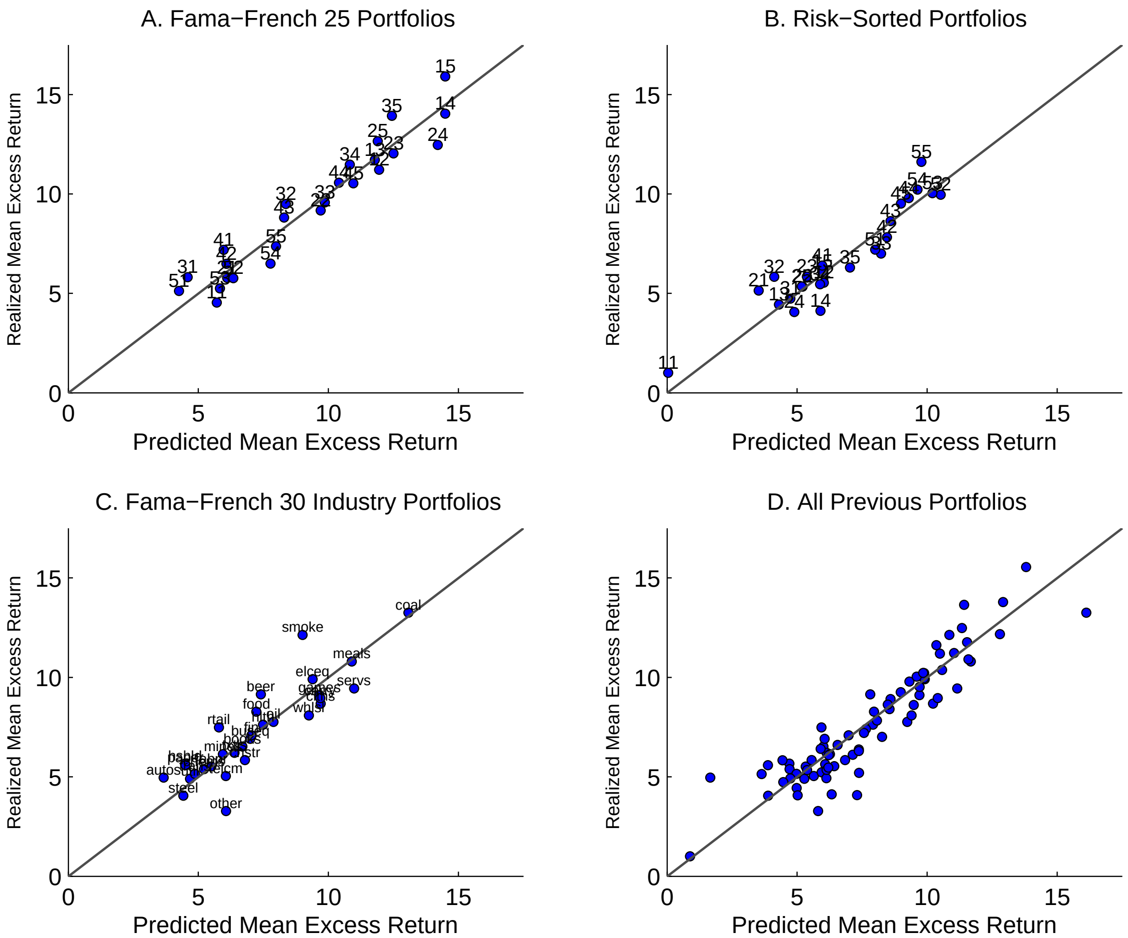
The figure depicts the timing of events in the dynamic limited enforcement problem, as described in the text. Panel A represents the sequence of events that occur each period after the long-term contract between the lender and the borrower is signed. Panel B shows the extensive form of the game from which enforcement constraints arise as an equilibrium outcome. In Panel B, red lines and blue lines represent optimal strategies and payoffs for the firm and the lender respectively. The possible strategies for the borrower are either to renege the contract ($\overline{R}$), or to continue running the firm (R). If the borrower decides to renege the contract, The possible strategies for the lender are either to liquidate the firm (L), or to not liquidate the firm ($\overline{L}$). At time t and for firm i, $M(x_t)$ denotes the stochastic discount factor, R_t is the risk-neutral lender's discount rate, $d_{i,t}$ the dividend payment, $\tau_{i,t}$ the repayment to the lender, $k_{i,t}$ the firm's capital stock, $O(k_{i,t+1}, s_{i,t})$ the value of the outside opportunity for the entrepreneur, and $1-\theta$ the fraction of capital the lender can expropriate upon liquidation. $s_{i,t}$ in the state of the economy, and consists of an aggregate shock x_t, and of a firm-specific shock $z_{i,t}$.

Figure 2.3. Firm's policy: Illustration



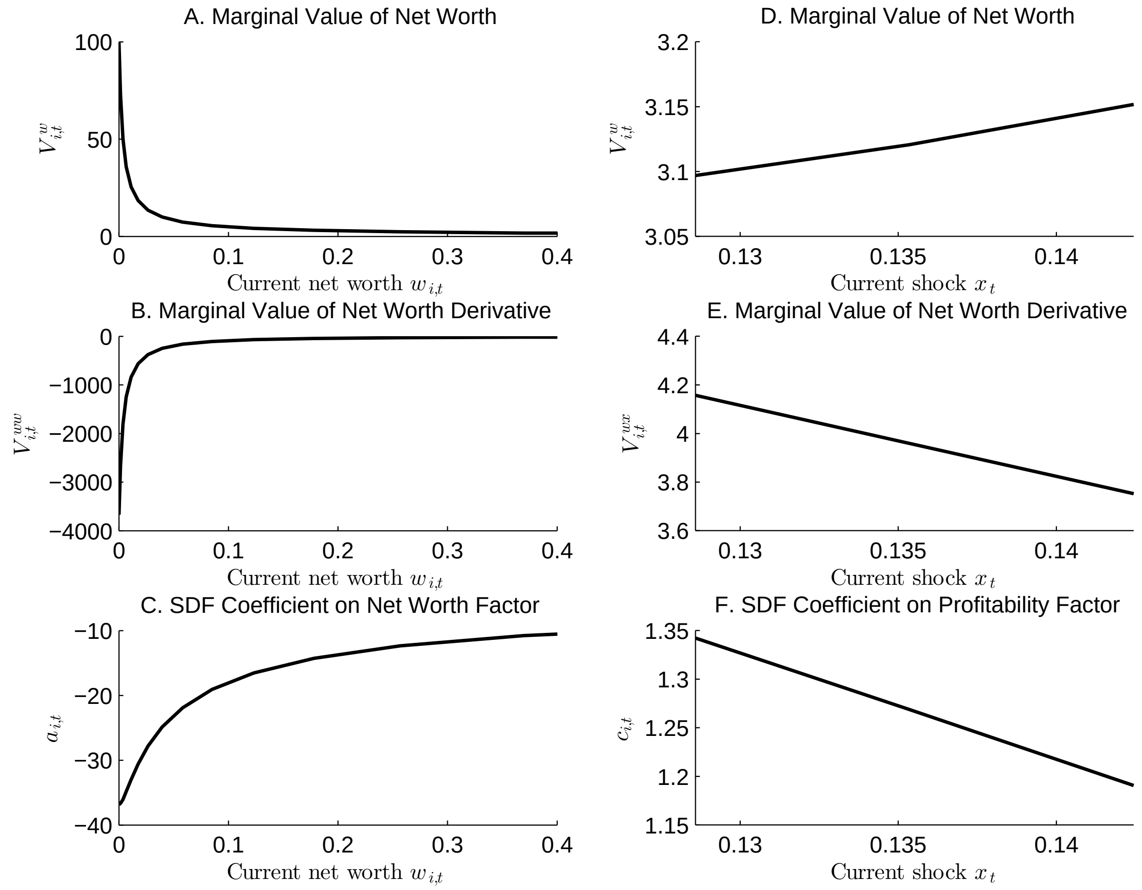
The figure illustrates the investment, payout, financing, and hedging policy of the firm as a function of current net worth $w_{i,t}$. The model is solved under the baseline calibration in Table 4.2. Panels A through F show: firm's equity value $V(w(s_{i,t}), s_{i,t})$, dividend payouts $d_{i,t}$ the new capital stock $k_{i,t+1}$, the observed debt stock $E[b(s_{i,t+1})]$, the debt repayment in three different aggregate states $b(x_{t+1})$, and the debt repayment in three different idiosyncratic states $b(z_{i,t+1})$. In all Panels, w^C denotes the net worth cutoff that delimits the region in which the firm is paying dividends. In Panel A, the dashed blue line represents the 45-degree slope of the value function in the region where dividends are paid. In Panels E and F, the solid line refer to the repayment in the middle state, the dashed red line to the one-state-down repayment, and the dash-dotted green line to the one-state-up repayment.

Figure 2.4. Predicted vs Realized Excess Returns: Corporate CAPM.



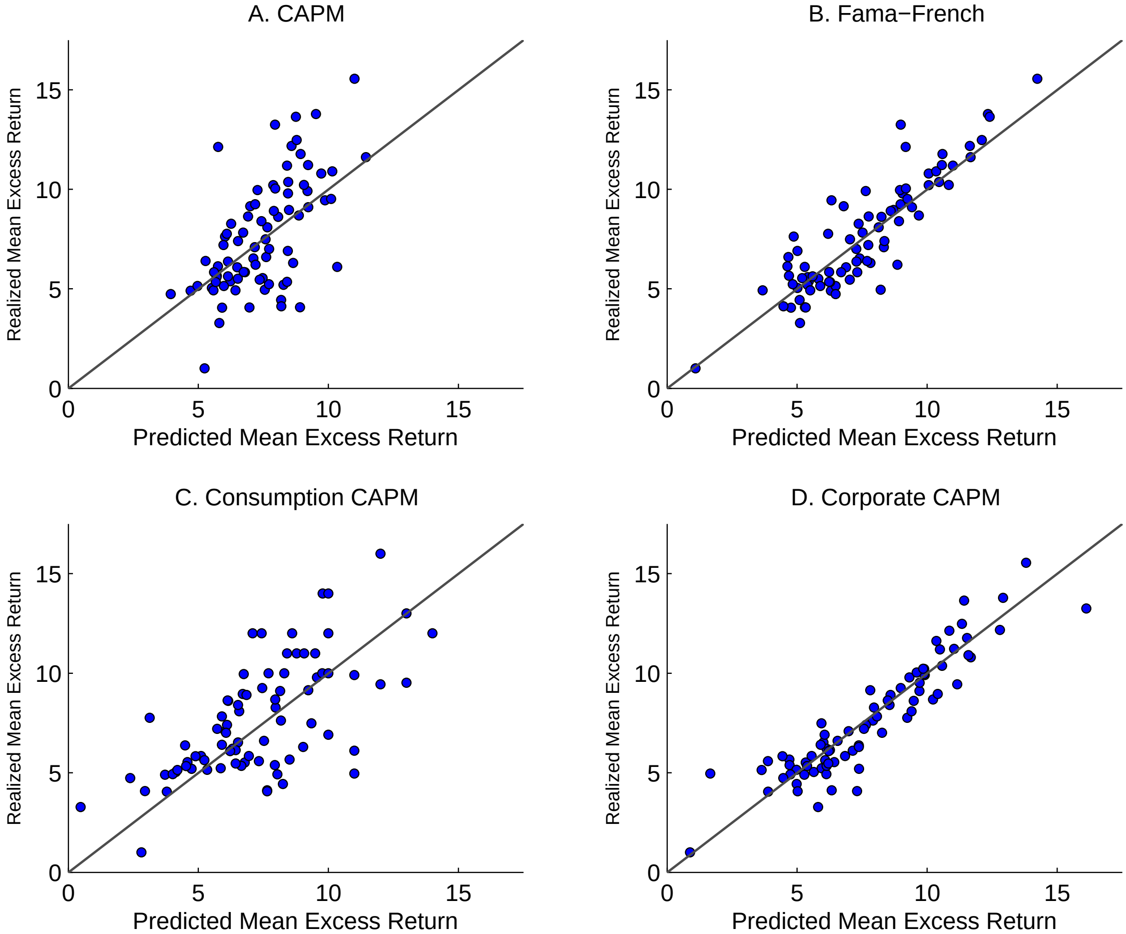
The figure illustrates annual predicted and realized excess returns for the first-stage GMM estimation of the Corporate CAPM as in Table 2.6. Panels A through D refer to the following test assets: the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios on pre-ranking market and HML betas as in Yogo [2006], the 30 Fama-French industry portfolios, and all the previous portfolios. In Panel A, the first digit of the label corresponds to the size quintile, and the second digit to the book-to-market equity quintile. In Panel B, the first digit of the label corresponds to the pre-ranking HML beta quintile, and the second digit to the market beta within each HML beta group. In Panel C, the labels are mnemonics for Fama and French 30-Industry classification as on Kenneth French's website. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

Figure 2.5. Coefficients on the Hedging and Profitability Factors



Panels A through C depict the marginal value of net worth $V_w(w_{i,t}, s_{i,t})$, its derivative with respect to current net worth $V_{ww}(w_{i,t}, s_{i,t})$, and the coefficient $\overline{a}_{i,t} \equiv \frac{V_{ww}(w_{i,t},s_{i,t})}{V_w(w_{i,t},s_{i,t})}$, all as a function of current net worth $w_{i,t}$. The pictures refer to the steady state for both aggregate and idiosyncratic shocks. Panels D through F depict the marginal value of net worth $V_w(w_{i,t}, s_{i,t})$, its derivative with respect to current net worth $V_{ww}(w_{i,t}, s_{i,t})$, and the coefficient $\overline{c}_{i,t} \equiv \frac{V_{wx}(w_{i,t},s_{i,t})}{V_w(w_{i,t},s_{i,t})}$, all as a function of the current aggregate shock x_t. The pictures refer to the steady state for both net worth and aggregate shocks. The coefficients $\overline{a}_{i,t}$ and $\overline{c}_{i,t}$ on the net worth and aggregate profitability factors are aggregated for conditional tests and lead to the specification of the Corporate CAPM in Equation (2.56).

Figure 2.6. Predicted vs Realized Excess Returns: Comparison Among Models.



The figure illustrates predicted and realized excess returns for the first-stage GMM estimation of different asset pricing models. All returns are annual and in excess of the riskfree rate. The test assets are the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios on pre-ranking market and HML betas as in Yogo [2006], and the 30 Fama-French industry portfolios, all together. Panels A through D refer to the asset pricing models estimated in Table 2.9: the CAPM, the three factor model of Fama and French, the Consumption CAPM, and the Corporate CAPM (unconditional estimation). Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

Table 2.1. Model Calibration.

The table reports actual and simulated moments, together with the corresponding choice of structural parameters. Panel A reports a set of moments that refers to corporate policies, and the corresponding data values. Calculations of data moments in Panel A are based on a sample of nonfinancial, unregulated firms from the annual 2012 Compustat Industrial database. The sample period is from 1988 to 2001. Operating income is defined as $(x_{t+1}z_{t+1}k_t^{\alpha})/k_t$, investment as $i_t = k_{t+1} - (1-\delta)k_t$, leverage as $E[b(s_{t+1})]/(E[b(s_{t+1})] + V(w_t, s_t))$, distributions as d_t/k_t and Tobin's Q as $(V(w_t, s_t) + E[b(s_{t+1})])/k_t$. Panel B reports a set of simulated aggregate asset pricing moments, whose data counterparts are from previous studies. Panel C reports the chosen values for structural parameters. Parameters in Group I are those whose value can be restricted from previous works or maps directly into data moments. Parameters in Group II pertain to the pricing kernel and are set to match the average real riskfree rate, the real riskfree rate volatility, and the average Sharpe ratio. Parameters in Group III are set to match simulated moments to data moments. α is the curvature of the production function, θ is the fraction of diverted capital in case of liquidation, δ is the depreciation rate, β, γ_0, and γ_1 are the parameters in the stochastic discount factor, μ_x, ρ_x, σ_x are the parameters driving the dynamics of the aggregate shock, ρ_z, and σ_z are the parameters driving the dynamics of the idiosyncratic shock, and ϕ is the fraction of incumbents per period.

Panel A: Corporate Policy Moments		
	Simulated Moments	Data Moments
Mean of operating income	0.2115	0.1387
Variance of operating income	0.0077	0.0068
Serial correlation of operating income	0.6706	0.7920
Mean of investment	0.1609	0.2018
Variance of investment	0.0568	0.0516
Mean of leverage	0.3931	0.2820
Variance of leverage	0.0427	0.0546
Serial correlation of leverage	0.6493	0.7723
Average distributions	0.0486	0.0310
Mean Tobin's Q	1.6522	1.5594

Panel B: Aggregate Moments		
	Simulated Moments	Data Moments
Mean of riskfree rate	0.0219	0.0290
Volatility of riskfree rate	0.0375	0.0300
Mean of Sharpe Ratio	0.3499	0.4100
Average excess returns	0.0627	0.0790
Variance of aggregate returns	0.0228	0.0317
Mean of firm-level return variances	0.0804	0.1149

Panel C: Calibrated Parameters												
Group I				Group II			Group III					
δ	ρ_x	σ_x	ϕ	β	γ_0	γ_1	α	θ	μ_x	ρ_z	σ_z	
0.1500	0.8145	0.0280	0.0200	0.9400	12.5	-120	0.7600	0.3000	-2.0	0.8700	0.0750	

Table 2.2. Comparative Statics: Curvature of the Production Function.

The table reports simulated data moments for three different values of the curvature of the production function α. As in Table 4.2, the table reports a set of moments that refers to corporate policies, and the aggregate asset pricing moments that are not calibrated to the parameters in the pricing kernel (Group II in Table 4.2). Operating income is defined as $(x_{t+1}z_{t+1}k_t^{\alpha})/k_t$, investment as $i_t = k_{t+1} - (1-\delta)k_t$, book leverage as $E[b(s_{t+1})]/k_t$, market leverage as $E[b(s_{t+1})]/(E[b(s_{t+1})] + V(w_t, s_t))$, distributions as d_t/k_t and Tobin's Q as $(V(w_t, s_t) + E[b(s_{t+1})])/k_t$.

Curvature of the Production Function	$\alpha = 0.4000$	$\alpha = 0.6500$	$\alpha = 0.9000$
A. Corporate Policy Moments			
Mean of operating income	0.4467	0.2862	0.1672
Variance of operating income	0.0401	0.0075	0.0018
Serial correlation of operating income	0.6918	0.6777	0.5325
Mean of investment	0.1556	0.1691	0.1774
Variance of investment	0.0109	0.0277	0.1983
Mean of leverage	0.2433	0.3302	0.5613
Variance of market leverage	0.0144	0.0307	0.0306
Serial correlation of market leverage	0.7459	0.7807	0.3423
Average distributions	0.1804	0.1023	0.0291
Mean Tobin's Q	2.8122	2.0801	1.3531
B. Aggregate Asset Pricing Moments			
Average excess returns	0.0778	0.0581	-0.0242
Variance of aggregate returns	0.0141	0.0191	0.0707
Mean of firm-level return variances	0.0211	0.0289	0.1618

Table 2.3. Comparative Statics: Pledgeability Parameter.

The table reports simulated data moments for three different values of the fraction of diverted capital in case of liquidation θ. As in Table 4.2, the table reports a set of moments that refers to corporate policies, and the aggregate asset pricing moments that are not calibrated to the parameters in the pricing kernel (Group II in Table 4.2). Operating income is defined as $(x_{t+1}z_{t+1}k_t^{\alpha})/k_t$, investment as $i_t = k_{t+1}-(1-\delta)k_t$, book leverage as $E[b(s_{t+1})]/k_t$, market leverage as $E[b(s_{t+1})]/(E[b(s_{t+1})]+V(w_t, s_t))$, distributions as d_t/k_t and Tobin's Q as $(V(w_t, s_t) + E[b(s_{t+1})])/k_t$.

Pledgeability Parameter	$\theta = 0.1000$	$\theta = 0.4000$	$\theta = 0.7000$
A. Corporate Policy Moments			
Mean of operating income	0.2123	0.2402	0.2606
Variance of operating income	0.0080	0.0056	0.0067
Serial correlation of operating income	0.5296	0.6714	0.6843
Mean of investment	0.1783	0.2442	0.2736
Variance of investment	0.4371	0.4442	0.3685
Mean of leverage	0.4276	0.3621	0.3227
Variance of market leverage	0.0534	0.0490	0.0436
Serial correlation of market leverage	0.7300	0.7747	0.7529
Average distributions	0.1092	0.0697	0.0820
Mean Tobin's Q	1.5040	1.8070	1.8362
B. Aggregate Asset Pricing Moments			
Average excess returns	0.0461	0.0809	0.0945
Variance of aggregate returns	0.0547	0.0722	0.0373
Mean of firm-level return variances	0.0915	0.1072	0.1125

Table 2.4. Comparative Statics: Volatility of Productivity Shocks.

The table reports simulated data moments for three different values of the volatility of aggregate productivity σ_x (Panel A), and idiosyncratic productivity σ_z (Panel B). As in Table 4.2, the table reports a set of moments that refers to corporate policies, and the aggregate asset pricing moments that are not calibrated to the parameters in the pricing kernel (Group II in Table 4.2). Operating income is defined as $(x_{t+1}z_{t+1}k_t^{\alpha})/k_t$, investment as $i_t = k_{t+1} - (1-\delta)k_t$, book leverage as $E[b(s_{t+1})]/k_t$, market leverage as $E[b(s_{t+1})]/(E[b(s_{t+1})] + V(w_t, s_t))$, distributions as d_t/k_t and Tobin's Q as $(V(w_t, s_t) + E[b(s_{t+1})])/k_t$.

Panel A. Volatility of Aggregate Shocks	$\sigma_x = 0.0050$	$\sigma_x = 0.0250$	$\sigma_x = 0.0450$
A. Corporate Policy Moments			
Mean of operating income	0.2992	0.2543	0.2153
Variance of operating income	0.0027	0.0080	0.0108
Serial correlation of operating income	0.2773	0.6913	0.6673
Mean of investment	0.1731	0.3613	1.1805
Variance of investment	0.1197	1.1622	16.8060
Mean of leverage	0.3782	0.4170	0.2872
Variance of market leverage	0.0325	0.0448	0.0515
Serial correlation of market leverage	0.7181	0.7254	0.4187
Average distributions	0.0965	0.0854	0.0021
Mean Tobin's Q	1.7250	1.7035	2.1871
B. Aggregate Asset Pricing Moments			
Average excess returns	0.0539	0.1002	0.3471
Variance of aggregate returns	0.0265	0.0787	2.7614
Mean of firm-level return variances	0.0945	0.1661	1.8517

Panel B. Volatility of Idiosyncratic Shocks	$\sigma_z = 0.0250$	$\sigma_z = 0.1250$	$\sigma_z = 0.2250$
A. Corporate Policy Moments			
Mean of operating income	0.2355	0.2301	0.2440
Variance of operating income	0.0059	0.0054	0.0103
Serial correlation of operating income	0.6448	0.6114	0.5750
Mean of investment	0.1746	0.1871	0.1998
Variance of investment	0.0638	0.0825	0.1672
Mean of leverage	0.4419	0.4112	0.3019
Variance of market leverage	0.0383	0.0396	0.0414
Serial correlation of market leverage	0.6742	0.7506	0.7535
Average distributions	0.0501	0.0339	0.0958
Mean Tobin's Q	1.6616	1.8544	2.1161
B. Aggregate Asset Pricing Moments			
Average excess returns	0.0332	0.0471	0.0977
Variance of aggregate returns	0.0322	0.0487	0.0618
Mean of firm-level return variances	0.0608	0.0753	0.1560

Table 2.5. Comparative Statics: Persistence of Productivity Shocks.

The table reports simulated data moments for three different values of the persistence of aggregate productivity ρ_x (Panel A), and idiosyncratic productivity ρ_z (Panel B). As in Table 4.2, the table reports a set of moments that refers to corporate policies, and the aggregate asset pricing moments that are not calibrated to the parameters in the pricing kernel (Group II in Table 4.2). Operating income is defined as $(x_{t+1}z_{t+1}k_t^{\alpha})/k_t$, investment as $i_t = k_{t+1} - (1-\delta)k_t$, book leverage as $E[b(s_{t+1})]/k_t$, market leverage as $E[b(s_{t+1})]/(E[b(s_{t+1})] + V(w_t, s_t))$, distributions as d_t/k_t and Tobin's Q as $(V(w_t, s_t) + E[b(s_{t+1})])/k_t$.

Panel A. Persistence of Aggregate Shocks	$\rho_x = 0.6500$	$\rho_x = 0.8000$	$\rho_x = 0.9500$
A. Corporate Policy Moments			
Mean of operating income	0.2480	0.2412	0.2600
Variance of operating income	0.0088	0.0070	0.0054
Serial correlation of operating income	0.6396	0.6923	0.7037
Mean of investment	0.8602	0.3097	0.2213
Variance of investment	5.3475	0.8147	0.3225
Mean of leverage	0.3594	0.3955	0.3405
Variance of market leverage	0.0666	0.0502	0.0664
Serial correlation of market leverage	0.7092	0.8047	0.9082
Average distributions	0.0893	0.0828	0.0910
Mean Tobin's Q	2.1677	1.6384	1.7656
B. Aggregate Asset Pricing Moments			
Average excess returns	0.1088	0.0719	0.0898
Variance of aggregate returns	0.1877	0.0950	0.0496
Mean of firm-level return variances	0.2284	0.1256	0.0587

Panel B. Persistence of Idiosyncratic Shocks	$\rho_z = 0.1000$	$\rho_z = 0.5000$	$\rho_z = 0.9000$
A. Corporate Policy Moments			
Mean of operating income	0.2251	0.2351	0.2377
Variance of operating income	0.0058	0.0090	0.0090
Serial correlation of operating income	0.6523	0.6619	0.6689
Mean of investment	0.1616	0.3816	0.4212
Variance of investment	0.1244	0.9144	1.7224
Mean of leverage	0.4181	0.4693	0.4111
Variance of market leverage	0.0448	0.0536	0.0439
Serial correlation of market leverage	0.7924	0.5810	0.6837
Average distributions	0.0912	0.0579	0.0785
Mean Tobin's Q	1.5041	1.4552	1.6718
B. Aggregate Asset Pricing Moments			
Average excess returns	0.0365	0.0737	0.0960
Variance of aggregate returns	0.0298	0.0968	0.1024
Mean of firm-level return variances	0.0389	0.1211	0.1864

Table 2.6. Unconditional Tests of the Corporate CAPM.

The table reports the estimated factor loading on the net worth, and profitability factors for the Corporate CAPM. The test assets are the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios sorted on pre-ranking market and HML betas as in Yogo [2006], the 30 Fama-French industry portfolios, and all the previous portfolios. All returns are annual and in excess of the riskfree rate. In this specification, the model is estimated unconditionally, and the curvature parameter α is set to the calibrated value of 0.76. Estimation is by two-step GMM. HAC standard errors are in parentheses. The kernel is Newey-West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho [2004]. The latter two statistics are based on first-stage estimates. HJ denotes the Hansen-Jagannathan distance, computed as in Jagannathan and Wang [1996]. $p(HJ)$ is the p-value for the HJ test corrected for degrees of freedom as in Ferson and Foerster [1994]. J and $p(J)$ denote the test statistic and the p-value for a test of overidentifying restrictions. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

	Test Assets			
Estimate	25 S/BM	FF 30 Ind	Risk-Sorted	All
Net Worth	2.853	-8.392	0.879	-6.344
	(1.623)	(0.978)	(1.574)	(1.367)
Profitability	23.145	26.927	50.785	27.621
	(1.285)	(1.572)	(5.826)	(5.831)
MAE (%)	0.764	0.790	0.676	0.838
R^2	0.923	0.771	0.872	0.846
HJ Distance	0.773	0.828	0.669	-
$p(HJ)$	(0.768)	(0.982)	(0.913)	-
J	22.333	22.405	17.730	22.487
$p(J)$	(0.500)	(0.762)	(0.772)	(1.000)

Table 2.7. Conditional Tests: Nonlinear Regression for $\overline{a}_{i,t}$ and $\overline{c}_t$.

The table reports estimated coefficients and the R^2 for a nonlinear regression of the time-varying coefficient $\overline{a}_{i,t}$, and of a linear regression of $\overline{c}_t$, for the conditional specification of empirical tests of the Corporate CAPM. The values of $\overline{a}_{i,t}$ are regressed from the numerical solution of the model on the endogenous state variable $w_{i,t}$, with the functional form:

$$a_0 \frac{1}{1 + a_1 w_{i,t}}$$

Estimation is based the algorithm in Levenberg [1944] and Marquardt [1963]. The values of $\overline{c}_t \equiv \frac{1}{N} \sum_{j=1}^{N} \overline{c}_{j,t}$ are regressed from the numerical solution of the model on the state variable x_t, with the functional form:

$$c_0 + c_1 \rho_t^A$$

Standard errors are in parentheses. The R^2 is from a cross-sectional regression of fitted on actual values.

Dependent Variable: $\overline{a}_{i,t}$			
Functional Form	a_0	a_1	R^2
$a_0 \cdot \frac{1}{1+a_1 w_{i,t}}$	-35.424	7.489	0.969
	(0.295)	(0.271)	

Dependent Variable: $\overline{c}_t$			
Functional Form	c_0	c_1	R^2
$c_0 + c_1 \rho_t^A$	4.142	-17.623	0.999
	(0.072)	(0.529)	

Table 2.8. Conditional Tests of the Corporate CAPM

The table reports the estimated factor loading on the net worth, and profitability factors for the Corporate CAPM. The test assets are the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios sorted on pre-ranking market and HML betas as in Yogo [2006], the 30 Fama-French industry portfolios, and all the previous portfolios. All returns are annual and in excess of the riskfree rate. In this specification, the model is estimated conditionally with the stochastic discount factor in Equation (2.56), in which the coefficient $\overline{a}_{i,t}$ for "net worth" factor is time varying, and as in Table 2.7, is parametrized as:

$$a_0 \frac{1}{1 + a_1 w_{i,t}}$$

and the estimated coefficient for the "profitability factor" is parametrized as:

$$c_0 + c_1 \rho_t^A$$

The table reports the estimates for a_0 and c_0, while a_1 is set to 7.489, and c_1 is set to -17.623 as estimated in Table 2.7. The curvature parameter α is set to the calibrated value of 0.76. Estimation is by two-step GMM. Standard errors are in parentheses, and are computed with HAC standard error. The kernel is Newey West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho [2004]. The latter two statistics are based on first-stage estimates. HJ denotes the Hansen-Jagannathan distance, computed as in Jagannathan and Wang [1996]. $p(HJ)$ is the p-value for the HJ test corrected for degrees of freedom as in Ferson and Foerster [1994]. J and $p(J)$ denote the test statistic and the p-value for a test of overidentifying restrictions. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

	Test Assets			
Estimate	25 S/BM	FF 30 Ind	Risk-Sorted	All
Net Worth	19.536	-97.200	7.272	-21.700
	(38.975)	(14.240)	(40.198)	(5.009)
Profitability	34.361	27.007	40.231	28.097
	(2.788)	(0.934)	(3.613)	(4.160)
MAE (%)	0.634	0.784	0.557	0.722
R^2	0.944	0.820	0.911	0.888
HJ Distance	0.876	0.810	0.783	-
$p(HJ)$	(0.711)	(0.981)	(0.901)	-
J	22.714	22.863	18.997	22.254
$p(J)$	(0.478)	(0.740)	(0.701)	(1.000)

Table 2.9. Comparison Among Models.

Columns 1 through 5 report performance measures for the CAPM, the three factor model of Fama and French, the consumption CAPM, and the Corporate CAPM. For the Corporate CAPM, the results for unconditional estimates are in Column 5, and those for conditional estimates are in Column 6. Panels A through D refer to the following test assets: the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios on pre-ranking market and HML betas as in Yogo [2006], the 30 Fama-French industry portfolios, and all the previous portfolios. All returns are annual and in excess of the riskfree rate. Estimation is by two-step GMM. HAC standard error are in parentheses. The kernel is Newey-West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho [2004]. The latter two statistics are based on first-stage estimates. HJ denotes the Hansen-Jagannathan distance, computed as in Jagannathan and Wang [1996]. p(HJ) is the p-value for the HJ test corrected for degrees of freedom as in Ferson and Foerster [1994]. J and $p(J)$ denote the test statistic and the p-value for a test of overidentifying restrictions. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

	CAPM	Fama-French	CCAPM	Corporate CAPM (Unconditional)	Corporate CAPM (Conditional)
			Panel A. 25 Fama-French Portfolios		
MAE (%)	1.764	0.673	1.414	0.752	0.634
R^2	0.510	0.915	0.586	0.923	0.944
HJ	0.871	0.863	0.869	0.804	0.876
$p(HJ)$	(0.736)	(0.293)	(0.885)	(0.735)	(0.711)
J	19.508	21.459	20.913	21.968	22.714
$p(J)$	(0.724)	(0.493)	(0.644)	(0.522)	(0.478)
			Panel B. 25 Risk-Sorted Portfolios		
MAE (%)	1.857	0.815	1.911	0.758	0.557
R^2	0.217	0.837	0.196	0.852	0.911
HJ	0.761	0.761	0.773	0.693	0.713
$p(HJ)$	(0.912)	(0.683)	(0.942)	(0.894)	(0.901)
J	19.897	19.886	21.809	20.354	18.997
$p(J)$	(0.703)	(0.590)	(0.591)	(0.620)	(0.701)
			Panel C. 30 Fama-French Industry Portfolios		
MAE (%)	1.362	1.095	1.629	0.935	1.015
R^2	0.264	0.630	0.159	0.743	0.784
HJ	0.846	0.848	0.877	0.822	0.820
$p(HJ)$	(0.988)	(0.906)	(0.993)	(0.982)	(0.981)
J	20.232	20.838	22.306	22.112	22.863
$p(J)$	(0.886)	(0.794)	(0.807)	(0.776)	(0.740)
			Panel D. All 80 Portfolios		
MAE (%)	1.703	0.990	1.829	0.838	0.722
R^2	0.378	0.791	0.349	0.846	0.888
J	22.349	22.358	22.475	22.487	22.254
$p(J)$	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)

Table 2.10. The Corporate CAPM: Industry Breakdowns.

The table reports the estimated factor loading on the net worth, and profitability factors for the Corporate CAPM. The test assets are the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios on pre-ranking market and HML betas as in Yogo [2006], and the 30 Fama-French industry portfolios, all together. All returns are annual and in excess of the riskfree rate. The first row reports the reference set of firms with respect to the Corporate CAPM factors are computed, and corresponds to Fama and French's five-industry classification. Panel A refers to unconditional tests, implemented as in Table 2.6. Panel B refers to conditional tests, implemented as in Table 2.8. Estimation is by two-step GMM. HAC standard errors are in parentheses. The kernel is Newey-West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho [2004]. The latter two statistics are based on first-stage estimates. J and $p(J)$ denote the test statistic and the p-value for a test of overidentifying restrictions. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

Panel A: Unconditional Tests					
	Reference Industry				
Estimate	Cnsmr	Manuf	HiTec	Hlth	Other
Net Worth	-8.567	-2.148	-2.359	-3.795	-5.224
	(1.976)	(0.480)	(0.508)	(0.821)	(1.105)
Profitability	34.680	15.858	11.321	19.065	20.690
	(7.337)	(3.346)	(2.392)	(4.025)	(4.379)
MAE (%)	0.930	0.959	0.898	1.214	0.895
R^2	0.822	0.854	0.841	0.830	0.823
J	22.487	22.481	22.465	22.470	22.432
$p(J)$	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)

Panel B: Conditional Tests					
	Reference Industry				
Estimate	Cnsmr	Manuf	HiTec	Hlth	Other
Net Worth	-17.700	-27.000	-20.300	-18.900	-47.500
	(7.239)	(5.861)	(4.772)	(4.462)	(10.343)
Profitability	12.975	4.902	6.526	10.107	7.189
	(2.751)	(1.034)	(1.384)	(2.132)	(1.529)
MAE (%)	1.212	1.297	0.856	0.951	0.864
R^2	0.720	0.728	0.847	0.835	0.822
J	22.485	22.480	22.487	22.475	22.457
$p(J)$	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)

2.A Two-Period Example: Additional Results

2.A.1 The Lenders' Problem

In the model, lenders have deep pockets and agree to provide an amount b to the firm in change of state-contingent repayments $Rb(s)$ tomorrow. Lenders are risk neutral, and they make zero profits because of competition among them. The lender's problem is:

$$
\begin{aligned}
U^L &= \max_b -b + E\left[\frac{Rb(s)}{R}\right] && (2.A.1)\\
& s.t. \\
& -b + E\left[\frac{Rb(s)}{R}\right] \geq 0 && (2.A.2)
\end{aligned}
$$

The second equation is the incentive rationality constraint of lendes. Because of competition, Equation (2.A.2) is satisfied with equality, and $b = E[b(s)]$. Therefore, the supply curve is perfectly elastic, and the price b is constant regardless of demand. Notice that if a lender would try to ask more than b, another lender would undercut it. Incentive rationality constraints are therefore always binding.

2.A.2 Perfect Risk Sharing

This subsection presents the two-period problem without constraints on the implementable state-contingent transfers $b(s)$. The problem in (2.9)-(2.12) becomes:

$$
\begin{aligned}
U(w) &= \max_{k,b} d + \pi_S M(S)d(S) + \pi_F M(F)d(F) + \pi_R M(R)d(R) && (2.A.3)\\
& s.t. \\
w + b &= d + k && (2.A.4)\\
d(s) &= A(s)f(k) - Rb(s) \qquad s \in \{S, F, R\} && (2.A.5)
\end{aligned}
$$

The first-order conditions with respect to capital and state-contingent debt are:

$$E[M(s)R^k(s)] = 1 \qquad (2.A.6)$$

$$M(s) = \frac{1}{R} \qquad (2.A.7)$$

Equation (2.A.7) shows that in this case the stochastic discount factor is constant. In other words, the firm is able to hedge and fully insure the owners by equalizing their marginal utility across states. With perfect risk sharing, equity claims would therefore be priced as if the firm is risk neutral. This case emphasizes that different discount rates between lenders and borrowers do not imply the presence of arbitrage opportunities in the market.

2.B Proofs of Propositions

Proof of Lemma 1. By the definition of net worth, Equation (2.33) must also hold for the current state, which is measurable respect to the information set at time t. Hence

$$w_{i,t} \leq \Pi(k_{i,t}, s_{i,t}) + (1-\delta)k_{i,t} - R_t b_{i,t} \tag{2.B.1}$$

Because free disposal is never optimal, Equations (2.24) , (2.32) and (2.33) are always binding. This yields:

$$\Pi(k_{i,t}, s_{i,t}) + (1-\delta)k_{i,t} - R_t b_{i,t} = d_{i,t} + k_{i,t+1} - E_t[b(s_{i,t+1})] \tag{2.B.2}$$

Equations (2.32) and (2.33), and Equations (2.24), (2.25) and (2.26) are therefore equivalent. The enforcement constraint in conjunction with the dividend non-negativity constraint imply that the limited liability constraint is always satisfied. This constraint is therefore redundant, and can be omitted from the problem. In fact, because at the optimum $V(k_{i,t}, b_{i,t}, s_{i,t}) = d_{i,t} + E_t\left[M(x_{t+1})V(k_{i,t+1}, b(s_{i,t+1}), s_{i,t+1})\right]$, Equation (2.28) can be rewritten as

$$V(k_{i,t}, b_{i,t}, s_{i,t}) \geq \theta k_{i,t+1} + d_{i,t} \tag{2.B.3}$$

By (2.23), $d_{i,t} \geq 0$. Thus:

$$V(k_{i,t}, b_{i,t}, s_{i,t}) \geq \theta k_{i,t+1} + d_{i,t} \geq \theta k_{i,t+1} \tag{2.B.4}$$

which implies (2.27) because the fact that $\lim_{k_{i,t}\downarrow 0} \Pi(k_{i,t}, s_{i,t}) = \infty$ makes optimal capital always strictly positive. Because only $w_{i,t}$, and not its individual components predetermined at time t, affect the return function $d_{i,t}$, the two formulations are equivalent. □

Proof of Lemma 2. Denote by Y the set of the possible values for the state variables $w_{i,t}$ and $s_{i,t}$, by $\Gamma(y)$ the set of possible actions $k_{i,t+1}$ and $b(s_{i,t+1})$ for each $y \in Y$. Let V be the set of functions from Y to $(-\infty, \infty)$. In the remainder of the proof, I use the shorthands V^{LB} for $V^{LB}(w_{i,t}, s_{i,t})$, V^{UB} for $V^{UB}(w_{i,t}, s_{i,t})$, and V^* for $V(w_{i,t}, s_{i,t})$. Denote by $\leq$ be partial order operator for the functions on V, and by T the Bellman operator defined by

$$(Tv)(y) = \sup_{a\in\Gamma(y)} \left(d(y,a) + E_t\left[\beta M_0(x_{t+1})v(y')\right]\right), \qquad y, y' \in Y, v \in V \tag{2.B.5}$$

In this setting, the number of states is assumed to be finite, and by no arbitrage we have $M_0(\cdot) > 0$. Therefore, from the definition of T, it follows that T is monotone. Furthermore, $T(V^{UB}) \leq V^{UB}$, and $T(V^{LB}) \geq V^{LB}$. Under these conditions, the Knaster-Tarski fixed-point theorem (Aliprantis and Border [2006], Theorem 1.10) guarantees that the Bellman operator has at least one fixed point V^{FP} in $[V^{LB}, V^{UB}]$. Define the sequence V_n^{LB}, with $n = 0, 1, 2, ...$ such that $V_0^{LB} = V^{LB}$, and $V_{n+1}^{LB} = TV_n^{LB}$. Since any fixed point of T in $[V^{LB}, V^{UB}]$ is bounded above by V^{UB}, the increasing sequence V_n^{LB} must converge to a fixed point $\hat{V}^{LB}$ in $[V^{LB}, V^{UB}]$. By definition of fixed point, $V^{FP} = TV^{FP}$, and, by construction, $V_n^{LB} \leq V^{FP}$, for all n. Thus, $\hat{V}^{LB} \leq V^{FP}$. By (2.36), and since the number of states is finite, the conclusion of Theorem 4.3 in Stokey and Lucas [1989] go through. Therefore $V^* = V^{FP}$. Finally, the assumptions for Lemma 4.3 in Kamihigashi [2012] are satisfied, and this guarantees that $V^* \leq \hat{V}^{LB}$. As a consequence, the following chain of inequalities holds:

$$V^* \leq \hat{V}^{LB} \leq V^{FP} = V^* \tag{2.B.6}$$

This establishes that the uniqueness result in part (i), and the convergence results in part (ii). □

Proof of Proposition 1. Part (i). As Equation (2.39) states, the first-order conditions of problem (2.30)-(2.35) with respect to $b(s_{i,t+1})$ are:

$$R_t \nu(s_{i,t+1}) M(x_{t+1}) = \frac{\nu_{i,t}}{1+\lambda_{i,t}} \tag{2.B.7}$$

Solving the previous equation for $M(x_{t+1})$, the stochastic discount factor can be obtained as:

$$M(x_{t+1}) = \frac{\nu_{i,t}}{R_t V_w(w(s_{i,t+1}), x_{t+1}, z_{i,t+1})(1+\lambda_{i,t})} \tag{2.B.8}$$

The envelope condition (2.30)-(2.35) with respect to the state variable $w_{i,t}$ is:

$$\nu_{i,t} = V_w(w_{i,t}, x_t, z_{i,t}) \tag{2.B.9}$$

Plugging the expression of the multiplier $\nu_{i,t}$ from Equation (2.B.9) into (2.B.8) yields:

$$\begin{aligned} M(x_{t+1}) &= \frac{V_w(w_{i,t}, x_t, z_{i,t})}{R_t V_w(w(s_{i,t+1}), x_{t+1}, z_{i,t+1})(1+\lambda_{i,t})} = \\ &= \mu^M_{i,t} \frac{V_w(w_{i,t}, x_t, z_{i,t})}{V_w(w(s_{i,t+1}), x_{t+1}, z_{i,t+1})} \end{aligned} \tag{2.B.10}$$

Part (ii). Taking the log of both sides of (2.40) yields

$$\log M(x_{t+1}) = \mu^M_{i,t} + \log \frac{V_w(w_{i,t}, x_t, z_{i,t})}{V_w(w(s_{i,t+1}), x_{t+1}, z_{i,t+1})} \tag{2.B.11}$$

Define $f(w(s_{i,t+1}, s_{i,t+1}) \equiv \log \frac{V_w(w(s_{i,t+1}), x_{t+1}, z_{i,t+1})}{V_w(w_{i,t}, x_t, z_{i,t})}$. A first-order Taylor expansion of $f(w(s_{i,t+1}, s_{i,t+1})$ around the previous period realization $(w_{i,t}, s_{i,t})$ leads to:

$$\begin{aligned} f(w(s_{i,t+1}, s_{i,t+1}) &\simeq f(w_{i,t}, s_{i,t}) + f_w(w_{i,t}, s_{i,t})(w(s_{i,t+1}) - w_{i,t}) + f_z(w_{i,t}, s_{i,t})(z_{i,t+1} - z_{i,t}) \\ &+ f_x(w_{i,t}, s_{i,t})(x_{i,t+1} - x_{i,t}) \end{aligned} \tag{2.B.12}$$

Since

$$\begin{aligned} f(w_{i,t}, s_{i,t}) &= 1 && (2.B.13) \\ f_w(w_{i,t}, s_{i,t}) &= \frac{V_{ww}(w_{i,t}, s_{i,t})}{V_w(w_{i,t}, s_{i,t})} && (2.B.14) \\ f_z(w_{i,t}, s_{i,t}) &= \frac{V_{wz}(w_{i,t}, s_{i,t})}{V_w(w_{i,t}, s_{i,t})} && (2.B.15) \\ f_x(w_{i,t}, s_{i,t}) &= \frac{V_{wx}(w_{i,t}, s_{i,t})}{V_w(w_{i,t}, s_{i,t})} && (2.B.16) \end{aligned}$$

and because, expressing x_t as a Solow residual and recovering $z_{i,t}$ as a function of it, I obtain:

$$\begin{aligned} x_t &= \rho^A_t && (2.B.17) \\ z_{i,t} &= \frac{\rho_{i,t}}{\rho^A_t} && (2.B.18) \end{aligned}$$

Then Equation (2.B.12) simplifies as

$$\log \frac{V_w(w(s_{i,t+1}), x_{t+1}, z_{i,t+1})}{V_w(w_{i,t}, x_t, z_{i,t})} = \bar{a}_{i,t}(w(s_{i,t+1}) - w_{i,t}) + \bar{b}_{i,t}\left(\frac{\rho_{i,t+1}}{\rho^A_{t+1}} - \frac{\rho_{i,t}}{\rho^A_t}\right) + \bar{c}_{i,t}\left(\rho^A_{t+1} - \rho^A_t\right) \quad (2.B.19)$$

Plugging (2.B.19) into (2.B.11) yields the result. □

Proof of Proposition 2. The stochastic discount factor can be log-linearized at the first-order as

$$\frac{M(x_{t+1})}{E_t[M(x_{t+1})]} \approx 1 + \log M(x_{t+1}) - E_t[\log M(x_{t+1})] \quad (2.B.20)$$

that, using equation (2.41), can be written as:

$$\frac{M(x_{t+1})}{E_t[M(x_{t+1})]} \approx \mu^M_{j,t} - \bar{a}_{j,t}(w(s_{j,t+1}) - w_{j,t}) - \bar{b}_{j,t}\left(\frac{\rho_{j,t+1}}{\rho^A_{t+1}} - \frac{\rho_{j,t}}{\rho^A_t}\right) - \bar{c}_{j,t}\left(\rho^A_{t+1} - \rho^A_t\right) \quad (2.B.21)$$

The SDF can therefore be approximated with a two-factor linear representation, that is

$$\frac{M(x_{t+1})}{E_t[M(x_{t+1})]} \approx \mu^M_{j,t} - \bar{a}_{j,t} f^1_{j,t+1} - \bar{b}_{j,t} f^2_{j,t+1} - \bar{c}_{j,t} f^3_{j,t+1} \quad (2.B.22)$$

with

$$f^1_{j,t+1} \equiv w(s_{i,t+1}) - w_{i,t} \quad (2.B.23)$$

$$f^2_{j,t+1} \equiv \frac{\rho_{j,t+1}}{\rho^A_{t+1}} - \frac{\rho_{j,t}}{\rho^A_t} \quad (2.B.24)$$

$$f^3_{j,t+1} \equiv \rho^A_{t+1} - \rho^A_t \quad (2.B.25)$$

$M(x_{t+1})$ is a valid stochastic discount factor for equity returns $R_{i,t+1}$, and for the riskfree return R^f_t. Therefore:

$$E_t[M(x_{t+1})R_{i,t+1}] = E_t[M(x_{t+1})R^f_t] = 1 \quad (2.B.26)$$

The previous equation can be rewritten as

$$E_t\left[M(x_{t+1})(R_{i,t+1} - R^f_t)\right] = 0 \quad (2.B.27)$$

The constant in the SDF is measurable with respect to the time-t information set. Thus, I obtain

$$E_t\left[\frac{M(x_{t+1})}{E_t[M(x_{t+1})]}(R_{i,t+1} - R^f_t)\right] = 0 \quad (2.B.28)$$

that is

$$Cov_t\left[\frac{M(x_{t+1})}{E_t[M(x_{t+1})]}, R_{i,t+1} - R^f_t\right] + E_t\left[\frac{M(x_{t+1})}{E_t[M(x_{t+1})]}\right] E_t[R_{i,t+1} - R^f_t] = 0 \quad (2.B.29)$$

Substituting the approximated expression for the SDF in equation (2.B.22):

$$\begin{aligned} E_t[R_{i,t+1} - R^f_t] &\approx -Cov_t\left[\mu^M_{j,t} - \bar{a}_{j,t} f^1_{j,t+1} - \bar{b}_{j,t} f^2_{j,t+1} - \bar{c}_{j,t} f^3_{j,t+1}, R_{i,t+1} - R^f_t\right] = \quad (2.B.30) \\ &= -Cov_t\left[-\bar{a}_{j,t} f^1_{j,t+1} - \bar{b}_{j,t} f^2_{j,t+1} - \bar{c}_{j,t} f^3_{j,t+1}, R_{i,t+1} - R^f_t\right] \end{aligned}$$

Consider the column vector $f_{j,t+1}$ obtained by stacking $f^1_{j,t+1}$, $f^2_{j,t+1}$, and $f^3_{j,t+1}$. The variance-covariance matrix of the factors is

$$Var_t\left[f_{j,t+1}\right] \equiv Var_t \begin{bmatrix} f^1_{j,t+1} \\ f^2_{j,t+1} \\ f^3_{j,t+1} \end{bmatrix} = \begin{bmatrix} Var_t(f^1_{j,t+1}) & Cov_t(f^1_{j,t+1}, f^2_{j,t+1}) & Cov_t(f^1_{j,t+1}, f^3_{j,t+1}) \\ Cov_t(f^2_{j,t+1}, f^1_{j,t+1}) & Var_t(f^2_{j.t+1}) & Cov_t(f^2_{j,t+1}, f^3_{j,t+1}) \\ Cov_t(f^3_{j,t+1}, f^1_{j,t+1}) & Cov_t(f^3_{j,t+1}, f^2_{j,t+1}) & Var_t(f^3_{j.t+1}) \end{bmatrix} \tag{2.B.31}$$

and the vector $\widetilde{b}_{j,t}$ as

$$\widetilde{b}_{j,t} \equiv \begin{bmatrix} -\overline{a}_{j,t} \\ -\overline{b}_{j,t} \\ -\overline{c}_{j,t} \end{bmatrix}$$

Then, it follows that:

$$\begin{aligned} E_t[R_{i,t+1} - R^f_t] &\approx -\widetilde{b}^T_{j,t} Cov_t\left[f_{j,t+1}, R_{i,t+1} - R^f_t\right] = && (2.B.32) \\ &= -\widetilde{b}^T_{j,t} Var_t\left[f_{j,t+1}\right] Var_t\left[f_{j,t+1}\right]^{-1} Cov_t\left[f_{j,t+1}, R_{i,t+1} - R^f_t\right] = \\ &= \widetilde{\lambda}^T_{j,t}\beta_{i,t} \end{aligned}$$

where

$$\widetilde{\lambda}^T_{j,t} \equiv -\widetilde{b}^T_{j,t} Var_t\left[f_{j,t+1}\right] \tag{2.B.33}$$

$$\beta_{j,t} \equiv Var_t\left[f_{j,t+1}\right]^{-1} Cov_t\left[f_{j,t+1}, R_{i,t+1} - R^f_t\right] \tag{2.B.34}$$

Substituting back the explicit expressions for $f^1_{j,t+1}$, $f^2_{j,t+1}$, and $f^3_{j,t+1}$ completes the proof. □

Proof of Proposition 3. Part (i). The current aggregate state imposes a restriction of firms' investment, and financing policy such that the left-hand side of equation (2.41) is equalized across firms. Therefore:

$$\begin{aligned} \frac{1}{N}\sum_{j\in\Omega} \log M(x_{t+1}) &= \log M(x_{t+1}) \approx && (2.B.35) \\ &\approx \frac{1}{N}\sum_{j\in\Omega}\left[\mu^M_{j,t} - \overline{a}_{j,t}(w(s_{j,t+1}) - w_{j,t}) - \overline{b}_{j,t}\left(\frac{\rho_{j,t+1}}{\rho^A_{t+1}} - \frac{\rho_{j,t}}{\rho^A_t}\right) - \overline{c}_{j,t}\left(\rho^A_{t+1} - \rho^A_t\right)\right] \end{aligned}$$

The proof of the covariance representation in Equation (2.48) follows as in the previous proof by replacing $\begin{bmatrix} f^1_{j,t+1} \\ f^2_{j,t+1} \\ f^3_{j,t+1} \end{bmatrix}$ with $\begin{bmatrix} \frac{1}{N}\sum_{j\in\Omega} \overline{a}_{j,t} f^1_{j,t+1} \\ \frac{1}{N}\sum_{j\in\Omega} \overline{b}_{j,t} f^2_{j,t+1} \\ \frac{1}{N}\sum_{j\in\Omega} \overline{c}_{j,t} f^3_{j,t+1} \end{bmatrix}$.

Part (ii). Because $\frac{\rho_{j,t+1}}{\rho^A_{t+1}} - \frac{\rho_{j,t}}{\rho^A_t} = z_{j,t+1} - z_{j,t}$ has zero mean, the process for $z_{j,t}$ has a finite support, and $z_{j,t}$ and $z_{i,t}$ are independent for each $i \neq j$, the assumptions in Pruitt [1966] and Rohatgi [1971] hold and, for $N \to \infty$:

$$\frac{1}{N}\sum_{j\in\Omega} \overline{b}_{j,t}(z_{j,t+1} - z_{j,t}) \to 0 \tag{2.B.36}$$

by the strong law of large numbers. □

2.C Solution by Mixed-Integer Programming

In this section, I discuss the numerical solution method of the model. I introduce the main results on which the solution algorithm is based, and I provide details on its implementation. I start considering the perfect enforcement problem without the borrowing constraint (2.34), and I show the equivalence between the dynamic program and the linear program, along the lines of Ross [1983].

Lemma 3 (Perfect Enforcement Problem as a Linear Program) *The solution of problem (2.30) subject to (2.31), (2.32), (2.33), and (2.35) on a discrete grid is equivalent to the solution of the following linear programming problem:*

$$\min_{v_{w,s}} \sum_{w=1}^{nw} \sum_{s=1}^{nx \cdot nz} v_{w,s} \tag{2.C.1}$$

$$s.t.$$

$$v_{w,s} \geq d_{w,s,a} + \sum_{s'=1}^{nx \cdot nz} \pi(s'|s) M(s') v_{a,s'} \qquad \forall w, s, a \tag{2.C.2}$$

where nw, nx, and nz are the number of grid points on the grids for $w_{i,t}$, x_t, and $z_{i,t}$ respectively, $v_{w,s}$ is the value function on the grid point indexed by w and s, a is an index for an action on the grid for both capital and state-contingent debt repayments, and $d_{w,s,a}$ denotes the payout corresponding to the action a starting from the state indexed by w and s.

Proof For a generic policy correspondence $g(w, s)$ define the functional operator T^g as

$$(T^g f)(w, s) \equiv d(w, s, g(w, s)) + E_t \left[M(x_{t+1}) f(w(g(w, s)), s')\right] \tag{2.C.3}$$

where $f(\cdot)$ is a function to which the operator is applied. $d(w, s, g(w, s))$ denotes the dividend corresponding to the action $g(w, s)$ if the current state is (w, s), and $w(g(w, s))$ denotes future net worth in state s' if the action $g(w, s)$ is undertaken. Notice that this operator is not the Bellman operator because there is no maximization involved. T^g in instead a "policy iteration" operator, which simply iterates on the function $f(\cdot)$ using the policy rule specified by $g(w, s)$. T^g is a monotone operator, that is if $f_1(w, s) \leq f_2(w, s)$ pointwise, then $T^g(f_1) \leq T^g(f_2)$. In fact, if $f_1(w, s) \leq f_2(w, s)$, and because the number of exogenous states is finite:

$$E_t \left[M(x_{t+1}) f_1(w(g(w, s)), s')\right] \leq E_t \left[M(x_{t+1}) f_2(w(g(w, s)), s')\right] \tag{2.C.4}$$

Adding $d(w, s, g(w, s))$ to both sides of (2.C.4) yields $T^g(f_1) \leq T^g(f_2)$. Now consider a function $v(w, s)$ that satisfies all the constraints in (2.C.2). The monotonicity of T^g implies, for $n = 0, 1, ...$, that $v \geq T^g(v)$, $T^g(v) \geq (T^g)^2(v)$, ..., $(T^g)^n(v) \geq (T^g)^{n+1}(v)$. Then:

$$v \geq \lim_{n \uparrow \infty} (T^g)^n(v) \tag{2.C.5}$$

By the dominated convergence theorem, the sequence $(T^g)^n(v)$ converges to a limit point $v^g(w, s)$ such that $v \geq v^g(w,s)$. Choosing $g(w,s) = g^*(w,s)$, where $g^*(w,s)$ is the optimal policy function that yields the solution $v^*(w,s)$ of the dynamic programming problem in (2.30)-(2.32) as a fixed point of the Bellman operator, I obtain:

$$v(w,s) \geq v^{g*}(w,s) = v^*(w,s) \tag{2.C.6}$$

Therefore, $v(w,s)$ is the smallest pointwise function that, for each state (w,s), satisfies the constraints in (2.C.2). Hence, this function is the solution of any minimization problem

$$\min_{v_{w,s}} \sum_{w=1}^{nw} \sum_{s=1}^{nx \cdot nz} \zeta_{w,s} v_{w,s} \tag{2.C.7}$$

subject to (2.C.2) with $\zeta_{w,s} > 0$. In particular, the objective (2.C.1) solves the problem with $\zeta_{w,s} \equiv 1$. □

The previous lemma shows that the linear programming solution method does not require the Bellman operator be a contraction mapping. I now incorporate the borrowing constraint (2.34) into the linear programming representation above. Because the dynamic programming problem with perfect enforcement has a unique solution, there is only one binding constraint (i.e. one optimal action a on the grid) for each state (w,s) in the equivalent linear programming representation. The enforcement constraint (2.34) dictates that the optimal action $a^*_{w,s}$ for each state (w,s) satisfies

$$\theta k(a^*_{w,s}) \leq \sum_{s'=1}^{nx \cdot nz} \pi(s'|s) M(s') v_{a^*_{w,s},s'} \qquad \forall w,s \tag{2.C.8}$$

where $k(a^*_{w,s})$ denotes the point on the capital grid corresponding to the action $a^*_{w,s}$. In the following lemma, I show that the linear programming representation augmented with constraints (2.C.8) can be solved as a mixed-integer programming problem.

Lemma 4 (Equivalent Mixed-Integer Programming Representation) *The problem in (2.C.1)-(2.C.2) with the borrowing constraints in (2.C.8) is equivalent to:*

$$\min_{v_{w,s}} \sum_{w=1}^{nw} \sum_{s=1}^{nx \cdot nz} v_{w,s} + \sum_{w=1}^{nw} \sum_{s=1}^{nx \cdot nz} \sum_{a \in \Gamma(w,s)} \epsilon \cdot D_{w,s,a} \tag{2.C.9}$$

$$s.t.$$

$$d_{w,s,a} + \sum_{s'=1}^{nx \cdot nz} \pi(s'|s) M(s') v_{a,s'} \leq v_{w,s} \qquad \forall w,s,a \tag{2.C.10}$$

$$-v_{w,s} + d_{w,s,a} + \sum_{s'=1}^{nx \cdot nz} \pi(s'|s) M(s') v_{a,s'} + N D_{w,s,a} \geq 0 \qquad \forall w,s,a \tag{2.C.11}$$

$$\sum_{s'=1}^{nx \cdot nz} \pi(s'|s) M(s') v_{a,s'} + N D_{w,s,a} \geq \theta k(a) \qquad \forall w,s,a \tag{2.C.12}$$

where $D_{w,s,a}$ are binary variables, $\epsilon \to 0$ is a positive small number, $N \to \infty$ is a positive *large number, and* $\Gamma(w,s)$ *is the set of feasible actions if the current state is* (w,s)*.*

Proof The constraints in (2.C.8) must be active only for the optimal action in each state. The mixed-integer representation achieves this goal by introducing the set of binary variables $D_{w,s,a}$ in the objective and in the constraints (2.C.11) and (2.C.12).[30] Specifically, the term $\sum_{w=1}^{nw}\sum_{s=1}^{nx\cdot nz}\sum_{a\in\Gamma(w,s)}\epsilon\cdot D_{w,s,a}$ in the objective initializes all the binary variables to zero without affecting the objective for ϵ small enough. If N is large enough, Equations (2.C.11) force $D_{w,s,a}$ to one if the corresponding constraint in (2.C.10) is slack. As a result, $D_{w,s,a}$ equals zero only in correspondence of the optimal action $a^*_{w,s}$ for each state (w,s). Finally, when $D_{w,s,a}$ equals zero, the corresponding enforcement constraint in (2.C.12) becomes active. This representation of the enforcement constraints is therefore equivalent to the formulation in (2.C.8). □

It is important to remark that the mixed-integer problem in the previous lemma is in general *less* constrained than the "first-best" problem with perfect enforcement. In fact, some actions that are feasible in the "first-best" problem do not satisfy the borrowing constraints, and are excluded from $\Gamma(w,s)$. Consistent with this observation, the minimized objective in the problem with limited enforcement is better and, as I show below, results in a lower optimal equity value for each state.

As Trick and Zin [1993] discuss, solving the full mixed-integer program (as well as the full linear problem) would require to store a huge matrix, because the number of constraints in the problem is very large. This would be impractical, in that hardware, memory, and computational requirements would be enormous. For this reason, I resort to constraint generation, which is a standard technique in operation research to solve problems with a large number of constraints. Specifically, constraint generation begins with the solution a relaxed problem with the same objective and only a subset of the constraints. Then, the procedure identifies the remaining constraints in the full problem that are violated. A subset of the violated constraints is then added to the relaxed problem according to a selection rule. The procedure is iterated until all constraints are satisfied. The next lemma proposes a constraint generation algorithm, and shows it converges to the unique fixed point in Lemma 2.

Lemma 5 (Constraint Generation) *The sequence of functions* $\{v^n(w,s)\}_{n=1}^{\infty}$ *generated by the following algorithm converges to the fixed point* $V(w,s)$ *specified in Lemma 2:*
1. solve the problem in Lemma 4 with only the constraints corresponding to zero capital and zero debt for each state (w,s)*;*
2. if all constraints $a\in\Gamma^n(w,s)$*, for all* (w,s)*, are satisfied, terminate the algorithm (where* $\Gamma^n(w,s)$ *is the set of feasible actions at iteration* n*);*
3. for each state (w,s) *add the constraint* $a\in\Gamma^n(w,s)$ *that generates the highest violation in (2.C.10) with respect to the current solution* $v^n(w,s)$*;*

[30] For a review of representations of disjunctive constraints with mixed-integer formulations see, for example, Vielma [2013].

4. solve the problem with the current set of constraints;

5. go back to step 2.

Proof The initial set of constraints is feasible in the full problem in Lemma 4 and yields an initial value function $v^1(w,s)$. Adding constraints to the problems as in step 3 renders it more constrained and, because the objective involves a minimization, yields to a higher objective function. As the proof of Lemma 4 and Proposition 5.1 in Ross [1983] illustrate, any choice of $\zeta_{w,s} > 0$ in the objective function results in equivalent problems. This implies that, at iteration n and for each grid point (w,s), $v^{n-1}(w,s) \leq v^n(w,s)$. The sequence $\{v^n(w,s)\}_{n=1}^{\infty}$ is therefore an increasing sequence in a compact set, because the solution of the full problem lies in the order interval $[v^1(w,s), v^{FB}(w,s)]$, where $v^{FB}(w,s)$ in the solution of the problem with perfect enforcement in (2.C.1)-(2.C.2). $v^1(w,s)$ and $v^{FB}(w,s)$ respectively define $V^{LB}(w_{i,t}, s_{i,t})$ and $V^{UB}(w_{i,t}, s_{i,t})$ in Lemma 2. An increasing sequence in a compact set converges to a limit point that, by construction, is the solution of the full mixed-integer problem in Lemma 4. By Lemma 2, the equivalent dynamic programming problem has a unique fixed point in $[V^{LB}(w_{i,t}, s_{i,t}), V^{UB}(w_{i,t}, s_{i,t})]$. Therefore the constraint generation procedure yields the equilibrium contract. □

The constraint generation algorithm above extends the procedure in Trick and Zin [1993], and Trick and Zin [1997]. The procedure starts from a solution which is feasible in that it does not violate the enforcement constraint. Then, at iteration n and for each state (w,s), constraints are added using the same rule which is used in value function iteration, namely maximizing the sum of distributions and the expected continuation value given the current maximized value $v^n(w,s)$. In the mixed-integer programming representation, this rule corresponds to selecting the most violated constraint for each state in the feasible set $\Gamma^n(w,s)$. As Trick and Zin [1993] document and the results in Pucci de Farias and Van Roy [2003] suggest, constraint generation allow to achieve significance speed gains. Most important, it avoids to solve the full problem, which would be computationally too demanding.

However, to make the method implementable, one last critical issue must be addressed. The selection of the most violated constraint in the third step of the constraint generation procedure requires searching over a huge vector of grid points for all the choice variables. The computational and memory requirement would still be excessive for a problem with many controls variables. In this setting, this issue is exacerbated by the presence of state-contingent actions. To make the constraint generation operational, I use a separation oracle, that is an auxiliary linear programming problem that identifies the most violated constraint. Separation oracles are standard tools in operation research (Nemhauser and Wolsey [1988], Schrijver [1998], Cook et al. [2011]), and have been recently used in corporate finance by Nikolov et al. [2013]. I detail and describe the separation oracle for this problem at the end of this appendix. Operatively, the problem is solved using the algorithm in Lemma 5, and the separation oracle. Codes are implemented with Matlab®, and the solver for the mixed-integer programming problems is CPLEX®. Matlab® and CPLEX® are interfaced through the CPLEX Class API®. The workstation has with a

CPU with 8 cores and 32GB of RAM. The model is solved with three grid points for the aggregate shock, seven grid points for the idiosyncratic shock, 500 grid points for capital and each state-contingent debt variable, and 27 grid points for net worth. Following McGrattan [1997], the grid for net worth is not evenly spaced, but more points are collocated in the low net worth region, where the curvature of value function is more relevant. Simulated data from the model are based on panels of 5000 firms and 2000 time periods.

Separation Oracle

$$\max_{a=\{k',b(s')\}} d_{w,s,a} + \sum_{s'=1}^{nx\cdot nz} \pi(s'|s)M(s')v_{a,s'} - v_{w,s} \tag{2.C.13}$$

$s.t.$

$$\underline{k} \le k' \le \overline{k} \tag{2.C.14}$$

$$\underline{b} \le b(s') \le \overline{b} \qquad \forall s' \tag{2.C.15}$$

$$\sum_{s'=1}^{nx\cdot nz} \pi(s'|s)M(s')v_{w(s'),s'} \ge \theta k' \tag{2.C.16}$$

$$0 \le p(i_k) \le 1 \qquad \forall i_k = 1, ..., n_k \tag{2.C.17}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{2.C.18}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k)k^G(i_k) \tag{2.C.19}$$

$$d_{w,s,a} = w - k' + \sum_{s'=1}^{nx\cdot nz} \pi(s'|s)M(s')b(s') \tag{2.C.20}$$

$$d_{w,s,a} \ge 0 \tag{2.C.21}$$

$$f(k') = \sum_{i_k=1}^{n_k} p(i_k)(k^G(i_k))^\alpha \tag{2.C.22}$$

$$w(s') = A(s')f(k') + k'(1-\delta) - R_t b(s') \qquad \forall s' = 1...nx\cdot nz \tag{2.C.23}$$

Equations (2.C.14) and (2.C.15) define the bounds for capital and debt, Equation (2.C.16) is the enforcement constraint and allows to select feasible actions from $\Gamma^n(w,s)$, Equations (2.C.17) and (2.C.18) define the variables $p(i_k)$ that have the role to select a grid point for capital on the grid $k^G(i_k)$ and linearize the term k'^α in the production function, Equation (2.C.19) picks the grid point for the chosen capital stock from $k^G(i_k)$, Equations (2.C.20) and (2.C.21) define dividends and impose their positivity, Equation (2.C.22) computes the nonlinear term in capital in the production function, and Equation (2.C.23) defines future net worth in each state s'. The solution of the separation oracle for state-contingent debt is a continuous variable and is interpolated to the nearest point on the corresponding grid.

2.D Data

2.D.1 Corporate Data

Firm-level data for the computation of data moments in Table 4.2 are from the annual 2012 Compustat Industrial database. As in Hennessy and Whited [2005] and DeAngelo et al. [2011], I consider the sample period from 1988 to 2001 because the tax code has no large structural breaks. Following standard procedures, I exclude firms with SIC codes between 4900 and 4999, 6000 and 6999, and larger than 9000. I delete firm-year observations with missing data, and those for which total assets (item [at]), the capital stock (item [ppegt]), or sales (item [sale]) are either zero or negative. The data moments in Panel A of Table 4.2 are computed as follows: operating income is the ratio between items [oibdp] and [at]; investment is the difference between items [capx] and [sppe], divided by [ppegt]; leverage is the sum of items [dltt] and [dlc], divided by the sum of [dltt], [dlc], and the total value of equity (the product of the share price [prcc_f] and the number of outstanding shares [csho]); distributions are the ratio of items [dvc] and [at]; and Tobin's Q is the sum of [dltt], [dlc], and the value of equity [prcc_f] $\cdot$ [csho], all divided by [at]. Aggregate asset pricing moments are measured as in Zhang [2005].

2.D.2 Data About Assets and Factors

The empirical analyses is Section 2.6 use data about portfolios and factors to test the Corporate CAPM, the CAPM, the Consumption CAPM, and the Fama-French three-factor model. The sample period is from 1965 to 2010.

The Corporate CAPM factors are constructed from the Compustat/CRSP merged dataset. In order to prevent look-ahead bias, fiscal years are matched to calendar years with the procedure in Fama and French [1992]. Specifically, returns on the test assets formed in June of year t are matched to accounting data from the last fiscal year ending in calendar year $t-1$. This guarantees a gap of at least six months between accounting data and the date of portfolio formation. In constructing the factors for the Corporate CAPM, net worth is measured as the book value of equity, consistent with the accounting definition in the contracting model. Following Daniel and Titman [2006], the book value of equity is computed using redemption, liquidation, and par value of preferred shares, and accounting for investment tax credits and postretirement benefits. The data items used ([seq], [ceq], [pstk], [at], [lt], [mib], [pstkl], [pstkrv], [txditc]) are obtained from merging the Compustat/CRSP merged with CRSP. The profitability factor is computed as a Solow residual, where profitability of individual firms is the ratio between the item [oibdp], and of the item [at] to the power of α. As I discuss in Appendix E, I also consider a different measure of aggregate productivity. These data are obtained from John Fernald's website. Data on the market return, HML, SMB, the riskfree rate, and the five-industry classification in Table 2.10 are from Kenneth French's website. Data on consumption growth in nondurable and services are from the US national accounts.

Regarding the test assets, the returns on the Fama-French 25 portfolios sorted by size and book-to-market equity, and the returns on the Fama-French 30 Industry portfolios are from Kenneth French's website. The returns on the portfolios sorted by market and HML betas are computed as in Yogo [2006]. Post-ranking betas are obtained using the procedure in Fama and French [1992].

2.E Empirical Tests

2.E.1 Testing Procedure

Empirical tests for all the asset pricing models in Section 2.6 are implemented in stochastic discount factor form along the lines of Cochrane [2001], to which I refer for a textbook treatment of such tests. Since all models are tested on excess returns, the mean of the stochastic discount factor is not identified. I follow Yogo [2006] and I normalize the constant in the stochastic discount factor to $\mu^M_{i,t-1} = 1 + \mu'_f b$, so that the mean of the stochastic discount factor $M(x_t) = \mu^M_{i,t-1} + (f_t - \mu_f)'b$ is equal to one. As Burnside [2010] points out, this normalization appears to be less sensitive to misspecifications when excess returns are considered. A generic element $\hat{\theta}$ of the parameter space is a pair (b', μ'_f) of vectors of size K. This approach recognizes that the mean of the factors μ'_f is estimated, and accounts for sampling variation induced by this fact, and is particularly well-suited when factors are not excess returns on traded assets.

Denote by $y_t \equiv (R^e_t, f_t)$ the vector of data obtained by stacking the excess returns on the N test assets and the K factors. Then the set of $N + K$ moments conditions is:

$$g(\hat{\theta}, y_t) \equiv \begin{bmatrix} M(x_t)R^e_t \\ f_t - \mu_f \end{bmatrix} \tag{2.E.1}$$

A sufficient condition for local identification is that the covariance matrix of factors and returns has full rank (Newey and McFadden [1994]). The objective function for the GMM estimation is:

$$\min_{\hat{\theta}\in\Theta} E^T[g'(\hat{\theta}, y_t)]WE^T[g(\hat{\theta}, y_t)] \tag{2.E.2}$$

where the operator $E^T(\cdot)$ denotes the sample mean for a time series of length T, and W is the positive definite weighting matrix. Estimation is by two-step GMM, with HAC standard errors. The kernel is Newey-West with a lag length of 1 year. The first-stage weighting matrix puts an equal weight on the moment conditions for excess returns and, following Yogo [2006], is specified as

$$W = \begin{bmatrix} (K/N) \cdot I_N & 0 \\ 0 & \widehat{\sigma}_F^{-1} \end{bmatrix} \tag{2.E.3}$$

where $\widehat{\sigma}_F$ is a consistent estimate of the covariance matrix of the factors. The R^2 and MAE reported in the text are from first-stage estimations. The R^2 measure is computed as in Campbell and Vuolteenaho [2004]. The J-test of overidentifying restrictions is performend as in Hansen and Singleton [1982], and the Hansen-Jagannathan distance, its test-statistics, and its p-value are worked out as in Appendix C of Jagannathan and Wang [1996].

2.E.2 Alternative Measure of Productivity

Tables 2.E.1 and 2.E.2 are replicas of Tables 2.6 and 2.8 with an alternative productivity measure from Fernald [2009]. This measure accounts for the presence of labor input in the production function, and for the possible misspecification in the computation of Solow residuals. Potential misspecifications are mainly related to failures of standard measures to control for capital utilization, as Burnside et al. [1996] suggest. The results in the tables are qualitatively similar to those in the main text. The pricing performance is sastisfactory and, as column 4 of both tables show, the signs of the estimates are in line with the predictions of the model.

Table 2.E.1. Alternative Productivity Measure: Unconditional Tests of the Corporate CAPM

The table reports the estimated factor loading on the net worth, and profitability factors for the Corporate CAPM. The profitability measure is from Fernald [2009]. The test assets are the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios sorted on pre-ranking market and HML betas as in Yogo [2006], the 30 Fama-French industry portfolios, and all the previous portfolios. All returns are annual and in excess of the riskfree rate. In this specification, the model is estimated unconditionally. Estimation is by two-step GMM. HAC standard errors are in parentheses. The kernel is Newey West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho [2004]. The latter two statistics are based on first-stage estimates. HJ denotes the Hansen-Jagannathan distance, computed as in Jagannathan and Wang [1996]. $p(HJ)$ is the p-value for the HJ test corrected for degrees of freedom as in Ferson and Foerster [1994]. J and $p(J)$ denote the test statistic and the p-value for a test of overidentifying restrictions. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

	Test Assets			
Estimate	25 S/BM	FF 30 Ind	Risk-Sorted	All
Net Worth	-2.013	2.868	-1.929	-6.074
	(1.834)	(1.174)	(1.983)	(1.385)
Profitability	58.740	44.300	45.056	28.196
	(5.134)	(2.717)	(4.085)	(5.954)
MAE (%)	0.597	0.963	0.814	0.782
R^2	0.943	0.754	0.829	0.873
HJ Distance	0.831	0.839	0.698	-
$p(HJ)$	(0.654)	(0.973)	(0.793)	-
J	21.780	22.079	21.103	22.474
$p(J)$	(0.534)	(0.778)	(0.575)	(1.000)

Table 2.E.2. Alternative Productivity Measure: Conditional Tests of the Corporate CAPM

The table reports the estimated factor loading on the net worth, and profitability factors for the Corporate CAPM. The profitability measure is from Fernald [2009]. The test assets are the 25 Fama and French's portfolios sorted on size and book-to-market equity, 25 risk-sorted portfolios sorted on pre-ranking market and HML betas as in Yogo [2006], the 30 Fama-French industry portfolios, and all the previous portfolios. All returns are annual and in excess of the riskfree rate. In this specification, the model is estimated conditionally with the stochastic discount factor in Equation (2.56), in which the coefficient $\overline{a}_{i,t}$ for "net worth" factor is time varying, and as in Table 2.7, is parametrized as:

$$a_0 \frac{1}{1 + a_1 w_{i,t}}$$

and the estimated coefficient for the "profitability factor" is parametrized as:

$$c_0 + c_1 \rho_t^A$$

The table reports the estimates for a_0 and c_0, while a_1 is set to 7.489, and c_1 is set to -17.623 as estimated in Table 2.7. Estimation is by two-step GMM. Standard errors are in parentheses, and are computed with HAC standard error. The kernel is Newey West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho [2004]. The latter two statistics are based on first-stage estimates. HJ denotes the Hansen-Jagannathan distance, computed as in Jagannathan and Wang [1996]. $p(HJ)$ is the p-value for the HJ test corrected for degrees of freedom as in Ferson and Foerster [1994]. J and $p(J)$ denote the test statistic and the p-value for a test of overidentifying restrictions. Accounting data for the construction of the Corporate CAPM factors are from Compustat Annual. The sample period is from 1965 to 2010.

	Test Assets			
Estimate	25 S/BM	FF 30 Ind	Risk-Sorted	All
Net Worth	19.536	-97.200	7.272	-21.700
	(38.975)	(14.240)	(40.198)	(5.009)
Profitability	34.361	27.007	40.231	28.097
	(2.788)	(0.934)	(3.613)	(4.160)
MAE (%)	0.634	0.784	0.557	0.722
R^2	0.944	0.820	0.911	0.888
HJ Distance	0.876	0.810	0.783	-
$p(HJ)$	(0.706)	(0.976)	(0.887)	-
J	22.714	22.863	18.997	22.254
$p(J)$	(0.478)	(0.740)	(0.701)	(1.000)

Chapter 3

The Relative Leverage Premium

3.1 Introduction

In their famous second proposition Modigliani and Miller [1958] suggest a positive relationship between equity returns and leverage. Their finding is widely regarded as a theoretical pillar of modern corporate finance and provides important normative indications on how to rationalize a firm's capital structure. However, whether their prediction holds in a world in which markets are not frictionless is still a controversial issue. Bhandari [1988] finds a positive relationship between stock returns and market leverage, after controlling for market beta and size. Fama and French [1992] find that the natural logarithms of market and book leverage have opposite coefficients for returns, and propose the difference between these two variables, i.e. the book-to-market ratio, as an alternative explanatory variable. After controlling for the book-to-market ratio, George and Hwang [2010] show that stocks of firms in the highest quintile of book leverage earn lower average returns, while the opposite holds for those in the lowest quintile. Penman et al. [1992] report a negative relation with returns for both market and book leverage.

While in a Modigliani and Miller's world firms achieve their desired capital structure instantaneously, in the presence of adjustment costs and other frictions firms can temporarily deviate from the optimum.[1] As discussed by Korteweg [2010], a non-frictionless dynamic environment generates heterogeneity in the cross-section of observed leverage ratios. The equity of firms with the same observed leverage but with a different target ratio may bear a different risk exposure and be priced differently. By simply looking at the relationship between observed leverage and returns, one disregards any possible role of target leverage ratios. Intuitively, a leverage ratio of 50 percent may be adequate for a large firm with plenty of tangible assets, while it may be excessive for a high growth firm.

[1]For a review of dynamic and static trade-off theories see Frank and Goyal [2008]. More specifically, see Fischer et al. [1989], Leland [1994], Goldstein et al. [2001], Leary and Roberts [2005], Strebulaev [2007].

These two firms likely have different optimal ratios and risk profiles. One should account for the role played by heterogeneous target ratios when examining the cross-section stock returns. Ultimately, it may not be leverage per se that matters for equity returns, but rather a measure of leverage which accounts for firm-level heterogeneity. In this paper we refer to such measure as *relative leverage*.

A simple way to account for the heterogeneity in capital structure is to consider the existence of a target leverage, as predicted by the trade off theory of capital structure, according to which firms optimize their capital structure by balancing costs and benefits of debt. We estimate target leverage with a partial-adjustment model originally developed by Flannery and Rangan [2006], and later examined by Lemmon et al. [2008], Huang and Ritter [2009], Faulkender et al. [2010], and Flannery and Oztekin [2012]. We carry out the estimation both in sample and on a rolling basis, to deal with possible look-ahead biases. We refer to the rolling estimation procedure as out-of-sample estimation. We decompose observed leverage into target leverage and deviation from target. A firm has positive *relative leverage* if above target, and negative otherwise. We then examine the relationship between *relative leverage* and equity returns in the cross-section. Our main objective is to test whether positive (negative) deviations from target are associated with higher (lower) expected returns.

The main finding of the paper is that *relative leverage* is positively and strongly related to expected equity returns across our entire investigation period (1965-2009), as well as across several sub-periods (1965-1979, 1980-1994, and 1995-2009). Our results are robust to a set of out-of-sample robustness checks that control for a possible look-ahead bias, and to different estimation procedures of the target. They hold both for book and market leverage ratios. *Relative leverage* is significant after controlling for the underlying leverage measure, either market or book, and for target leverage. The *relative leverage premium* appears to be larger than the premia associated with size and book-to-market.

The main contribution of this paper is empirical, as it uncovers potentially puzzling new evidence on the determinants of equity returns. While a full structural explanation is beyond the scope of this paper, our results offer two clear indications. First, to ascertain the role of capital structure in the cross-section of expected equity return, it is necessary to remove the heterogeneity caused by dynamic adjustment costs. Second, the methodology that we employ to remove firm heterogeneity leads to a new measure of leverage that is strongly related to expected returns in the cross-section.

It is possible to conceive more than one story that rationalizes the *relative leverage premium*. In the construct of Zhang [2005], Gomes and Schmid [2010], and Ozdagli [2012], the partial irreversibility of investments affects the relationship between leverage and returns. Over-leveraged firms have less residual debt capacity than under-leveraged firms, and are thus less flexible in adjusting capital investment. Inflexible firms have higher risk, because it is harder for them to smooth their dividend streams in the face of an aggregate exogenous shock. It then follows that over-leveraged firms should earn higher expected equity returns in the cross-section. For an alternative story, suppose that firms find it costly to have a different leverage ratio than their target, and that target leverage varies less than leverage over the business cycle.[2] Then, over-leveraged firms move further away from their target in a recession, because the value of their assets decreases more than their debt level. For similar reasons, under-leveraged firms converge towards target during a recession. Then, for a risk-averse investor, stocks of under-leveraged firms are counter-cyclical, because they deliver a higher payoff in bad times, when consumption is low and marginal utility of consumption is high. Symmetrically, over-leveraged firms are pro-cyclical because they allow investors to consume more when consumption is high.

Our analysis begins by sorting stocks into quintiles by observed market leverage (market debt ratio (MDR)) and *relative leverage*, and illustrate that there is no clear pattern in average returns as one moves from low to high MDR. On the contrary, average returns show a strong positive correlation with *relative leverage* within every quintile of MDR (Figure 1). This indicates that the positive relationship between *relative leverage* and average stock returns is observed for both over-leveraged and under-leveraged firms. Moreover, the premium (discount) associated with *relative leverage* is fairly symmetric. On average, a deviation of 10% between observed and target leverage corresponds to a premium (discount) of about 0.4% per month for over- (under-) leveraged firms. Average returns of firms on target are around 1.5% per month.

(Insert Figure 3.1 here)

We then follow the Fama and MacBeth [1973] (FMB) regression approach and examine the time-series averages of the estimated coefficients of monthly cross-sectional regressions of stock returns on size, book-to-market equity, momentum and *relative leverage*. We find that *relative leverage* plays a dominant role in the cross-section of expected equity returns. *Relative leverage* has an average coefficient of 3.509 in the period 1965-2009, 20.73 standard errors from zero. The explanatory power of (log) book-to-market equity is weak if both (log) size and *relative leverage* are included in the same regression. The positive relation between average returns and *relative leverage* is strong in all regressions specifications and in all sub-periods, also after controlling for momentum. For robustness, in the FMB regressions we employ *relative leverage* based on out-of-sample estimates of target leverage. Our results remain strong in the out-of-sample estimation.

Next, we compare the explanatory power of *relative leverage* with that of observed market leverage. For robustness, we also compute *relative leverage* and observed leverage at book values. Our findings provide support to Gomes and Schmid [2010] and Obreja [2010], in that neither MDR nor book leverage are important in the cross-section of

[2] Lemmon et al. [2008] find that estimated target debt ratios tend to vary slowly over time and that there are strong firm specific effects.

expected returns after controlling for size and book-to-market. On the contrary, *relative leverage* at market and book value is strongly significant after controlling for observed leverage (respectively at market and book value). Our results reconcile the conflicting evidence on whether one should use book rather than market leverage as a determinant of cross-sectional equity returns. We show that the explanatory power of *relative leverage* is qualitatively unaffected by whether one computes leverage at either market or book value.

Finally, we examine the implications of our results for factor asset pricing models. Following the approach of Chan et al. [1998], itself based on Fama and French [1993], we define a factor mimicking portfolio based on the *relative leverage premium.* We define the OMU (over- minus under-leveraged) factor and run orthogonalizing regressions to show that the explanatory power of OMU is not subsumed by the Fama and French's (FF) factors, RMRF, SMB, and HML. We compare the pricing ability of a multi-factor model including RMRF, HML, and OMU with that of the FF three-factor model and of the CAPM. We find that the model including OMU is able to correctly price more assets than the FF model and CAPM, with lower average pricing errors. These results suggest that (i) the *relative leverage premium* is not captured by the FF model, and (ii) a factor model that includes a mimicking portfolio based on *relative leverage* is consistent with the *relative leverage premium* being a compensation for risk rather than an arbitrage opportunity.

Section 3.2 discusses the estimation of target leverage based on the partial-adjustment model of Flannery and Rangan [2006]. Sections 3.3 and 3.4 present the empirical results of our asset pricing tests. Section 4.7 summarizes and discusses our findings. The Appendix describes the data and variables employed in our analysis, it provides details on the out-of-sample estimation procedure and on the GMM asset pricing tests, and it reports a number of robustness checks.

3.2 The decomposition of leverage

In this section we implement the leverage decomposition of Flannery and Rangan [2006] (FR) which allows us to identify the firm-specific components of total leverage. Following FR we measure leverage as the market debt ratio, defined as

$$MDR_{i,t} = \frac{D_{i,t}}{D_{i,t} + ME_{i,t}} \tag{3.1}$$

where $D_{i,t}$ denotes the stock of interest-bearing debt of firm i in period t and $ME_{i,t}$ is the stock market capitalization of firm i in period t. We then consider the partial-adjustment model of FR, according to which firms (partially) adjust their leverage over time towards the desired level $MDR^*_{i,t+1}$ at a speed of adjustment λ:

$$MDR_{i,t+1} - MDR_{i,t} = \lambda(MDR^*_{i,t+1} - MDR_{i,t}) + \epsilon_{i,t+1} \tag{3.2}$$

with

$$MDR^*_{i,t+1} = \beta X_{i,t} \tag{3.3}$$

$MDR^*_{i,t+1}$ is modeled as a linear function of a set of firm-specific characteristics $X_{i,t}$, and varies both over time and across firms. Equations (3.2) and (3.3) lead to the following estimable model:

$$MDR_{i,t+1} = (\lambda\beta)X_{i,t} + (1-\lambda)MDR_{i,t} + \epsilon_{i,t+1} \tag{3.4}$$

FR interpret $MDR^*_{i,t+1}$ as a proxy of a firm's target leverage within the framework of the trade-off theory of capital structure. Accordingly, the variables in $X_{i,t}$ are firm-specific characteristics that the literature on the trade-off theory has identified as relevant for capital structure. The parameter λ can be interpreted as the percentage reduction in the gap between actual and target leverage that occurred over one period.

Differently from other models previously employed in the literature (e.g. Hovakimian et al. [2001]; Korajczyk and Levy [2003]), the specification suggested by FR is consistent with the *dynamic* models of capital structure in presence of frictions. Common to other specifications is the absence of lagged MDR in the estimation of (3.4). Excluding lagged MDR amounts to assuming that a firm's target leverage coincides with its observed leverage.[3] The high and significant loading of $MDR_{i,t}$ in the empirical estimate of (3.4) is consistent with Leary and Roberts [2005] and Strebulaev [2007], according to which the existence of frictions prevents firms from instantaneously adjusting towards their desired capital structure. The empirical estimation of (3.4) leads to a decomposition of $MDR_{i,t}$ into a target-related component $(\lambda\beta)X_{i,t-1}$, an autoregressive component $(1-\lambda)MDR_{i,t-1}$, and a residual component $\epsilon_{i,t}$.

3.2.1 Estimation of the partial adjustment model

Table 3.1 reports different specifications for Equation (3.4).[4] FR and Lemmon et al. [2008] underline the importance of including unobservable firm fixed effects in $X_{i,t}$. Columns 2

[3]If $MDR_{i,t+1}$ is *expected* to equal $MDR^*_{i,t+1}$, then $\lambda = 1$ in the estimation of Equation (3.4), i.e. firms immediately adjust their capital structure to the desired level. In this case, the partial-adjustment model in (3.2) simplifies to

$$MDR_{i,t+1} = MDR^*_{i,t+1} + \epsilon_{i,t+1}$$

that is

$$E[MDR_{i,t+1}] = E[MDR^*_{i,t+1}]$$

[4]See Appendix A for definitions of variables and a discussion of the data employed in this section.

and 3 include these effects, and accordingly the regressions are estimated as a dynamic panel data model.

(Insert Table 3.1 here)

Flannery and Hankins [2010] find that the technique that generates the most accurate parameter estimates in Equation (3.4) is the system GMM of Blundell and Bond [1998]. Therefore, as in Lemmon et al. [2008], Lockhart [2010], and Faulkender et al. [2010], in our “base” specification of column 3 we estimate the partial-adjustment model (3.4) using Blundell and Bond system GMM.[5]

The results of our estimations are provided in Table 3.1 and are in line with previous work. In particular, our estimate of the adjustment speed λ in column (3) is 23.8% which is similar to that obtained by others. As expected, our estimate of the autoregressive term $1 - \lambda$ (0.762) lies in the interval between the pooled OLS estimate in column 1 (0.845), which is expected to be biased upwards, and the fixed-effect estimate in column 2 (0.647), which is expected to be biased downwards (Hsiao [2003]). While the estimated value of the speed of adjustment λ depends significantly on the methodology employed, the estimation of target leverage is much less sensitive to different estimation techniques. Simulation results provided by Flannery and Hankins [2010] suggest that the econometric techniques employed in the recent literature generally exhibit satisfactory finite-sample performance (in terms of average bias) in estimating firm-specific target debt ratios $MDR^*_{i,t+1}$. In our analysis of cross-section returns, we use the regression specification of column 3. However, if the target is estimated as in Flannery and Rangan [2006] - our column 2 - results are qualitatively unaffected.

For the purpose of Section 3.3, it is useful to define the leverage-related variables that we employ in our asset pricing tests. These variables are: 1) *relative leverage* obtained as the difference between observed and target leverage, 2) *distance*, which is the absolute value of *relative leverage*, 3) *over-leverage* which is the maximum between *relative leverage* and zero, and 4) *under-leverage* which is the negative of the minimum between *relative leverage* and zero. Noting that $M\hat{D}R^*_{i,t}$ denotes the estimated firm-specific target for firm i in period t, obtained from the regression equation in column 3 of Table 3.1, we have:

$$Rel\_Lev_{i,t} \equiv MDR_{i,t} - M\hat{D}R^*_{i,t} \tag{3.5}$$
$$Distance_{i,t} \equiv \|MDR_{i,t} - M\hat{D}R^*_{i,t}\| \tag{3.6}$$
$$Overlev_{i,t} \equiv max\{Rel\_Lev_{i,t}, 0\} \tag{3.7}$$
$$Underlev_{i,t} \equiv -min\{Rel\_Lev_{i,t}, 0\} \tag{3.8}$$

[5] In the estimation of Equation (3.4) with the Blundell and Bond system GMM, we consider all right-hand-side variables as predetermined with a lag length of one year. Only year dummies are regarded as fully exogenous. The inclusion of further lags has no significant influence on results.

3.3 Relative leverage and expected returns

Leverage and accounting variables that we use in our tests are matched to monthly return series as in Fama and French [1992]. The matching procedure is described in Appendix A. Table 3.2 displays a correlation matrix for the main variables of our analysis. In the first column, MDR and *relative leverage* present a high average cross-sectional correlation (0.425) but are far from identical, as can be seen from column two. In particular, over-leverage has a higher correlation with MDR than under-leverage, which indicates that *relative leverage* differs from MDR more for under-leveraged firms than for over-leveraged ones. Furthermore, a correlation of 0.162 between MDR and distance indicates that firms with high levels of observed leverage tend to deviate from their target debt ratios by a greater amount (in absolute value). However, distance is correlated to over-leverage and under-leverage with coefficients of 0.450 and 0.597 respectively: this suggests that under-leveraged firms are on average more distant from target than over-leveraged firms.

In addition, the table shows that all our leverage-related variables are correlated to the variables normally known to affect the cross-section of expected equity returns. Specifically, the natural logarithm of market capitalization is negatively related to the absolute deviation from target leverage with a mean correlation of -0.121, while the natural logarithm of book-to-market equity is positively related to *relative leverage* - with a correlation of 0.138. Both these interactions are stronger for over-leverage, while under-leverage is weakly correlated to log(size) and log(bm). Consistent with previous studies, observed debt ratios are negatively correlated to log(size) and positively correlated to log(bm). Our measure of momentum is correlated to *relative leverage* with a coefficient of 0.137, and it also presents cross-sectional correlations coefficients of similar magnitude with over-leverage (0.106) and under-leverage (-0.109).

(Insert Table 3.2 here)

Figure 3.2 illustrates how *relative leverage* interacts with firm size, which we compute as in Fama and French [1992]. Along the horizontal axis we report deciles of *relative leverage*, while along the vertical axis we have size. Both over-leveraged and under-leveraged firms appear to be smaller. Hennessy and Whited [2007] find that firm's size is a well-suited proxy for high costs of external financing, in that costs of adjustment prevents small firms to rebalance their capital structure as frequently as large firms. Therefore, Figure 3.2 may be interpreted as evidence that smaller firms tend to be further away from the desired level of leverage than larger firms due to the presence of adjustment costs.

(Insert Figure 3.2 here)

More generally, the sorts of Table 3.3 allow to examine separately the effects of observed leverage and *relative leverage* on expected stock returns. Portfolios are formed each June by independently ranking stocks into five groups by market debt ratios and *relative leverage*. The panels from top-left to bottom-right respectively report averages of monthly time series of 1) returns, 2) MDR, 3) BE/ME, 4) number of firms, 5) log(size), and 6) momentum.

Starting from the "Average Return" panel, we observe that no clear pattern exists in average returns as firms move from low to high MDR (vertical shift). Low MDR stocks are weakly associated with higher returns than high MDR stocks within the first three quintiles of *relative leverage* (first three columns). However, this trend is inverted in the last two columns. Moreover, these effects do not appear to be monotonic across quintiles of MDR. This evidence suggests that sorting by MDR produces little variation in average returns. On the contrary, average returns show a strong positive relation with *relative leverage* within every quintile of MDR. Average percent monthly returns of stocks in the lowest *relative leverage* quintile range from 0.56 and 0.96, while they are between 2.19 and 2.57 for stocks with the highest values of *relative leverage*. Moreover, average returns appear to increase monotonically across *relative leverage* quintiles. This suggests that *relative leverage* is positively related to stock returns for both over-leveraged and under-leveraged firms. As a consequence, the direction of deviations from target capital structure seems relevant in explaining expected returns.

The "MDR" panel indicates that MDR is roughly constant across *relative leverage* quintiles. Therefore, with reference to the "average return" panel, the positive relationship between returns and *relative leverage* is not due to higher MDR.

In the "log(size)" panel, we observe a U-shaped relationship between *relative leverage* and size. This pattern is consistent with the presence of costs of external financing that prevent small firms to rebalance their capital structure frequently. Hence, small firms are expected to deviate from optimal capital structure more than large firms.

The "BE/ME" panel shows the well-known positive relationship between BE/ME and MDR. However, there is no evident relationship between BE/ME and *relative leverage* within any MDR quintile. Thus, the positive correlation between the book-to-market ratio and *relative leverage* in Table 3.2 is likely the result of the positive correlation between MDR and *relative leverage*.

Finally, the "Momentum" panel shows that profits due to momentum are higher for firms with high *relative leverage*. This stresses the importance to account for the interaction of the momentum variable with *relative leverage*.

(Insert Table 3.3 here)

Figure 3.3 depicts average monthly returns of stocks of firms sorted according to *relative leverage*. Panel A refers to the full sample, while Panels B, C and D refer to the subperiods 1965-1979, 1980-1994, and 1995-2009 respectively. The figure emphasizes the magnitude of the *relative leverage* premium and shows a certain symmetry around the estimated target MDR ratios (vertical line). In all four panels, firms that are over-leveraged by 7.5% to 12.5% consistently earn average returns of about 2% per month, while firms that are under-leveraged by 7.5% to 12.5% earn average returns of about 1% per month. Average returns of firms on target are around 1.5% per month.

(Insert Figure 3.3 here)

3.3.1 The relative leverage premium

Table 3.4 reports time-series averages of the estimated coefficients of monthly cross-sectional regressions of stock returns on size, book-to-market equity, momentum and *relative leverage*. As in Fama and French [1992] and George and Hwang [2010], we follow the regression approach in Fama and MacBeth [1973] (FMB). We report FMB tests with a Newey-West correction with lag-length of 2 to assess which regressors have a coefficient that is significantly different from zero. The FMB regressions in Table 3.4 take the following form:

$$R_{i,t} = \beta_0 + \beta_1 log(size_{i,t-1}) + \beta_2 log(bm_{i,t-1}) + \beta_3 mom_{i,t-1} + \beta_4 Rel\_Lev_{i,t-1} + \epsilon_{i,t} \quad (3.9)$$

where $R_{i,t}$ denotes realized returns, $size_{i,t-1}$ market capitalization, $bm_{i,t-1}$ book-to-market equity, $mom_{i,t-1}$ momentum, and $Rel\_Lev_{i,t-1}$ *relative leverage*.

The results of Table 3.4 highlight the dominant role played by *relative leverage* in the cross-section of expected equity returns. In the regression of column 7 of Panel A, *relative leverage* has an average slope of 3.509%, with a t-statistic of 20.73. In the same regression, the natural logarithm of market capitalization has a slope of -0.221%, while the slope of (log) book-to-market equity is not statistically different from zero. Comparing across columns 6 and 7, we notice that the explanatory power of (log) book-to-market drops significantly when both (log) size and *relative leverage* are included in the same regression. The coefficient of book-to-market is significant when it is the only variable in the regression (column 2), and when it interacts separately either with size (column 4) or *relative leverage* (column 6). The positive relation between average returns and *relative leverage* persists across all regressions specifications, also after including momentum (column 8). The estimated slopes for *relative leverage* range from 3.509% to 4.003%, with Newey-West t-statistics between 18.68 and 23.39.

(Insert Table 3.4 here)

In Table 3.11 of Appendix D, we report sub-period evidence on the estimation of the FMB regressions. The coefficient of *relative leverage premium* is strong in each of the three sub-periods 1965-1979, 1980-1994, and 1995-2009. In comparison with *relative* *leverage*, the explanatory power of size and book-to-market appears much less stable in the sub-periods. Size has a strong effect on average returns in the 1995-2009 sub-period, both in terms of estimated slope and significance, while its effect is weak in the sub-period 1965-1979. Book-to-market has a strong effect in the years 1980-1994, while its slope is not statistically different from zero in the years 1965-1979 and 1995-2009.

To eliminate potential biases due to in-sample estimation of target leverage, we replicate the results of Panel A of Table 3.4 using an out-of-sample estimation procedure. Results are reported in Panel B of Table 3.4. We employ the years 1965-1987 as the estimation period, and for each year between 1987 and 2009 we estimate Equation 3.4 on a rolling basis. As the estimation of target leverage contains firm fixed effects, we ensure that the estimation of these effects remains stable when Equation 3.4 is estimated on a rolling basis.[6] To ensure stability of fixed effects estimation, we impose conditions on their convergence and exclude observations that do not satisfy these criteria.[7] Additional details on the rolling estimation procedure are provided in Appendix B.

The regressions in Panel B of Table 3.4 show that the significance of *relative leverage* remains strong in the FMB regressions also for the out-of-sample estimation. In column 1, *relative leverage* is highly significant after controlling for (log) size and (log) book-to-market, with a positive slope of 2.166, and a t-statistic of 8.779. This slope can be interpreted as the average monthly return of a self-financing portfolio with unit *relative leverage*, that hedges the effects of size and book-to-market in the period 1990-2009. In the Fama and MacBeth [1973] approach, the standard error is computed as the standard deviation of monthly returns on this portfolio, divided by the square root of the number of months in the sample (234 in this case). Hence, a t-statistic of 8.779 can be approximately translated into an annualized Sharpe ratio of 1.725, assuming an average monthly risk-

[6]Lemmon et al. [2008] highlight the importance of using fixed effects as proxies for in the estimation of partial adjustment models. The inclusion of firm fixed-effects finds theoretical support in models where investment and financing interact, such as Hennessy and Whited [2007], and Gomes and Schmid [2010]. Specifically, firm-specific unobservable shocks generate heterogeneity in firm-level investment opportunities that, in turn, determines optimal leverage decisions. In Table 3.14 of Appendix D, we estimate the partial adjustment model including only the firm fixed effect as a determinant of target leverage, and show that key results are qualitatively unchanged with this specification. This finding further stresses the importance of allowing for firm-specific unobservable heterogeneity in the estimation of target leverage.

[7]Firm fixed-effect estimates are likely to be noisier than for other variables because they are based only on individual time series variations. With a rolling estimation procedure, fixed effects are computed on the basis of one observation for the first year a firm appears in the panel, two observations for the second year, three for the third, and so on. Due to the unbalanced nature of the panel, and to the shorter time period required by the out-of-sample estimation, ensuring convergence in the estimation of fixed effects becomes necessary. For this reason we impose convergence conditions. See Appendix B for further details on the estimation procedure described here. Table 3.17 in Appendix D provides evidence that our results are robust to several alternative convergence conditions. Table 3.18 in Appendix D shows that our results are qualitatively unchanged if we consider a shorter estimation period, namely from 1965 to 1972.

free rate of 30 basis points.[8] The regression in column 2 shows that after controlling for momentum, our results are qualitatively in line with those reported in Panel A.[9]

To provide a term of comparison between the out and in-sample estimates, in columns 3-6 we provide in-sample estimations based on the same period employed for the out-of-sample estimates (1990-2009) of columns 1-2. In columns 3-4 we estimate *relative leverage* with Blundell and Bond [1998] system GMM, while in columns 5-6 we employ the LSDV approach. A comparison across columns shows that results are similar for both out and in-sample estimates. In addition, the coefficients of columns 3-4 are similar to those in columns 5-6. This suggests that using the LSDV estimator instead of Blundell and Bond [1998] system GMM has no serious impact on our findings, consistent with the evidence in Flannery and Hankins [2010].

In-sample estimation is convenient for two reasons: 1) it allows asset pricing tests over the whole period 1965-2009 and the collection of useful sub-period evidence; 2) due to the asymptotic properties of fixed-effect estimators, longer time-series help mitigate the finite-sample bias in the estimation of firm fixed-effects. The cost of using in-sample estimation is that Fama-MacBeth coefficients do not correspond to directly implementable trading strategies. Nonetheless, our out-of-sample test bring out that there exists a convergence condition such that investors can actually implement these strategies, and our results are not driven by look-ahead bias.

As *relative leverage* is the result of a previous estimation, our FMB regressions may suffer from an errors-in-variables bias.[10] The errors-in-variables problem may bias the estimated coefficients towards zero (Greene [2008]). Therefore, insofar as our estimate of *relative leverage* contains errors, the FMB regressions of Table 3.4 generate more conservative estimates than in the absence of errors.[11] This suggests that there is a *relative leverage premium* despite of a potential errors-in-variables bias.[12]

In sum, the FMB regressions provide support to a *relative leverage premium*, and indicate that *relative leverage* plays an important role in explaining the cross-section of expected equity returns, also after controlling for size, book-to-market equity, and momentum.

[8]We estimate the monthly risk-free rate using data from Kenneth French's website for the period from July 1990 to December 2009.

[9]To evaluate the economic relevance of the results of a FMB regression, one has to account for the length of the sample period. The procedure to compute the implied Sharpe ratio from a FMB regression is such that the magnitude of the t-statistics depends on the number of periods employed. Therefore, one should expect the t-statistics in Panel B to be lower than in Panel A even if *relative leverage* is estimated in-sample, as in this case.

[10]This kind of problem exists also for the CAPM beta in Fama and French [1992], and for the distress measures in George and Hwang [2010].

[11]However, this may not be the case if multiple variables in the regression are measured with error.

[12]For a broader discussion of the errors-in-variables bias see Kim [2010] and Carmichael and Coen [2008].

3.3.2 Symmetry of the relative leverage premium

The regressions in Table 3.5 investigate the relative importance that the following four measures of leverage have in explaining equity returns: *relative leverage*, distance, over-leverage, and under-leverage. Panel A covers the entire sample from 1965 to 2009. The slope of distance is significant when *relative leverage* is not included (column 3), with a slope of -0.709 and a t-statistic of -2.278. However, confirming our informal tests, the regression in column 6 of panel A shows that when distance and *relative leverage* are included in the same regression, the slope of distance is not statistically different from zero, with a t-statistic of 0.07. Columns 4 and 5 of panel A confirm that the *relative leverage premium* is not driven separately by either under-leveraged or over-leveraged firms. Over- and under-leverage are statistically insignificant when they are included in the same regression with *relative leverage*.[13]

Columns 1 and 2 of Panel B replicate columns 2 and 6 of Panel A, using out-of-sample estimation. Both over- and under-leverage matter when target leverage is estimated out-of-sample. Over- and under-leverage are both statistically significant in the regression in column 1, with slopes of 2.416 (t-statistic = 4.496) and -1.668 (t-statistic = -3.859) respectively. When *relative leverage* and distance are included in the same regression (column 2), distance is not statistically significant, with an estimated coefficient of 0.374 and a t-statistic of 0.914. As in Table 3.4, the results of the out-of-sample estimation are in line with those of the in-sample estimation on the same period, both using Blundell and Bond [1998] system GMM (columns 3-4) and the LSDV approach (columns 5-6).

Returning to Panel A of Table 3.5, the regression in column 2 shows that over- and under-leverage have slopes of similar magnitude (3.496 and -3.450 respectively), but opposite sign. This suggests that their difference, i.e. *relative leverage*, is what matters in explaining returns, which is consistent with the results of columns 4 and 5. We perform a Wald test of the linear restriction that the slope of over-leverage is equal, in absolute value, to the slope of under-leverage. The test does not reject the null hypothesis that the restriction holds with an F-stat of 0.02 and a p-value of 0.897.[14]

The symmetry of the *relative leverage premium* may be generated mechanically by the fact that MDR is a function of returns, because leverage decreases when the market value of equity increases.[15] The relationship between size and *relative leverage* is not straightforward as for other variables measured as ratios, like market capitalization and

[13]In Table 3.12 of Appendix D we provide sub-period evidence, and show that the marginal effect of *relative leverage* dominates both distance, over-leverage and under-leverage in each sub-period

[14]Sub-period evidence in Table 3.12 of Appendix D confirms that the over- and under-leverage have statistically equal slopes (in absolute values). In particular, Wald tests cannot reject this restriction with p-values of 0.1664 in the sub-period 1965-1979 (column 2 of panel A), of 0.1914 in the sub-period 1980-1994 (column 2 of panel B), and of 0.8018 in the sub-period 1995-2009 (column 2 of panel C).

[15]The evidence in Berk [1995] hints that the mechanical relationship may affect all size-related anomalies.

book-to-market equity. However, if target leverage estimates are not systematically affected by stock price changes, for a higher (respectively, lower) stock price the market debt ratio is lower (higher), and *relative leverage* is lower (higher). Since expected returns are by construction inversely related to stock prices, a perfectly symmetric *relative leverage premium* may naturally reflect such a mechanical effect for both over- and under-leveraged firms.

There are three reasons why we believe that our results are not driven by a mechanical relationship between MDR and prices. First, MDR is computed using data from fiscal year end $t-1$, and then matched to returns from July of year t to June on year $t+1$. Hence, the stock price component of MDR is unlikely to drive the relationship between *relative leverage* and returns, because there is a minimum gap of six months between the accounting data used to compute MDR and returns. Second, as we show in the next section, the relationship between *relative leverage* and returns holds also if leverage is measured at book values. In that case, the stock price is not included in the calculation of the value of equity and leverage. Third, if a purely mechanical relationship drives our results, a higher degree of symmetry should be observed when target leverage is estimated using only firm fixed effects. In such a case, the assumption that target leverage estimates are not also influenced by stock prices is more likely to hold. In fact, no other determinants of target leverage that are likely to be affected by movements in stock prices, such as the market-to-book value of assets, are included in the partial adjustment model. On the contrary, the absolute values of the coefficients for under- and over-leverage are closer in Panel A of Table 3.5 (3.496 and 3.450 in column 2) than in Table 3.14 (2.835 and 3.304 in column 3).

In sum, our results indicate that there is a linear relationship between expected stock returns and *relative leverage*. As Figure 3.3 suggests, the premium associated with over-leverage is comparable to the discount associated with under-leverage.

(Insert Table 3.5 here)

3.3.3 Relative vs. observed leverage

As discussed in the introduction, the empirical evidence about leverage in the cross-section of expected equity returns is mixed. It is not clear yet whether (observed) leverage is an important variable in explaining expected equity returns. In this section, we explore this issue and compare the explanatory power of observed leverage with that of *relative leverage*. The aim of this section is to show that *relative leverage*, rather than observed leverage, is the relevant variable to account for in the cross-section of average stock returns. To this end we include observed leverage and *relative leverage* in the same regression and test the significance of the two coefficients.

A potential source of disagreement among the studies that examine the role of leverage in the cross-section of expected stock returns is whether one should consider market or book leverage. As discussed in Flannery and Rangan [2006], the corporate finance literature largely focuses on market debt ratios. However, in the interest of completeness, we run the comparison between relative and observed leverage both in book and market value terms. We then have two pairs of variables: observed leverage at book and market values, and *relative leverage* at book and market values. This requires us to introduce two new variables: book leverage (BDR) which is computed as the book value of debt (DLTT+DLC) divided by the sum of itself plus the book value of equity - measured as in Fama and French [1993]. The second variable is Rel_Lev(book) which denotes *relative leverage* with respect to BDR. In the construction of Rel_Lev(book) we follow the same steps employed for the FR decomposition of *relative leverage* at market values, discussed in Appendix A.

Our results are presented in Table 3.6.[16] The comparison between relative and observed leverage is carried out in columns 3 and 6, respectively for market and book values. Notice that observed leverage can be decomposed as the sum of *relative leverage* plus target leverage. Therefore, we can equivalently interpret the regressions in columns 3 and 6 as tests on the significance of target leverage for explaining equity returns, after controlling for *relative leverage*.

Columns 1 and 2 of panel A respectively estimate the slope of market leverage in a univariate regression and with size and book-to-market equity as control variables. Consistent with Gomes and Schmid [2010] and Obreja [2010], expected returns are positively and significantly related to MDR in a univariate setting (with a slope of 0.987 and a t-statistic of 3.296), but they are insignificant after controlling for size and book-to-market. The regressions for book leverage in columns 4 and 5 are also in line with the predictions of Gomes and Schmid [2010] and Obreja [2010]. In particular, reading from column 4, the explanatory power of book leverage is lower than that of market leverage, with an estimated slope of 0.332, 1.833 standard errors from zero. Also, book leverage is insignificant at the multivariate level, after controlling for market capitalization and book-to-market equity.

Regressions in columns 3 and 6 show that when relative and observed leverage are included in the same regression, *relative leverage* is clearly more important than observed leverage for explaining expected stock returns, both in economic and statistical terms. Regardless of whether market or book leverage is employed, *relative leverage* is highly significant with estimated slopes of 3.992% (with t-statistic 21.32) for market values, and 1.542% (with t-statistic 12.17) for book values.

Observed leverage, both at market and book values, remains significant after account-

[16]In Table 3.13 of Appendix D we provide the corresponding sub-period evidence.

ing for *relative leverage*. More precisely, in column 3 MDR is still significant, with a negative slope of -1.000 and a t-statistic of -3.725. The residual explanatory power is much lower for book-valued variables (column 6): the estimated slope of BDR is -0.386, with a t-statistic of -1.775. We look at Panels A, B, and C of Table 3.13 in Appendix D to examine the evidence for the various sub-periods, and find that the significance of observed leverage is concentrated only in the 1980-1994 sub-period. On the contrary, observed leverage is not statistically significant at the 5% level neither in the years 1965-1979, nor in the years 1995-2009. Furthermore, in the years 1980-1994, the significance level of observed leverage is much lower than that of *relative leverage*. Considering the instability of the significance of observed leverage across different estimation periods, we conclude that idiosyncratic residual effects drive the results on observed leverage. Our out-of-sample findings in Panel B are consistent with this interpretation. As columns 1 and 2 show, MDR and BDR are no longer statistically significant after controlling for *relative leverage*. In column 1, the slope for MDR is -0.507, with a t-statistic of -0.507. In column 2, the slope for book leverage is -0.079, with a t-statistic of -0.221. Because we impose a convergence condition in our out-of-sample analysis, the results in columns 1-2 refer to firms for which target leverage estimates are less noisy.

(Insert Table 3.6 here)

3.4 Implications for factor pricing models

In this section we investigate the implications of the above results for the pricing of assets. We want to ascertain if the introduction of a new factor based on *relative leverage* can improve the pricing performance of existing factor models. We take the Fama and French (FF) three factor model as a benchmark. We compare its pricing performance to that of a multi-factor model that contains a factor-mimicking portfolio based on the *relative leverage premium*. If we find that the mimicking portfolio helps in pricing assets, we interpret the result as consistent with a rational *relative leverage premium* coherent with no-arbitrage in the stock market, in the spirit of the Arbitrage Pricing Theory (APT) of Ross [1976].

As the evidence in Fama and French [2008] suggests, many anomalies do not require the introduction of new factors, but can be explained by the three factor model. In addition, even if the three factor model does not succeed in explaining the *relative leverage premium*, it is not straightforward whether a new multi-factor model can explain the spread in average returns associated with *relative leverage*. If not, the *relative leverage premium* represents a pricing anomaly that may be exploited by investors.

3.4.1 Factor-mimicking portfolios

To define a factor mimicking portfolio for *relative leverage*, we base our approach on Chan et al. [1998], itself inspired by Fama and French [1993]. We rank firms with respect to *relative leverage* at the end of June of each year t, and assign them to portfolios from July of year t to June of year $t+1$. Stocks are assigned to portfolios on the basis of the distribution of *relative leverages* of NYSE firms only. We define the OMU (over-minus under-leveraged) factor as the difference between the average monthly return of stocks with *relative leverage* above the 80th percentile for NYSE firms, minus the average monthly return of stocks with *relative leverage* below the 20th percentile for NYSE firms.

In order to test the hypothesis that OMU is useful to price other assets, we choose 27 portfolios independently sorted on size, book-to-market equity, and *relative leverage* as test assets. In this way we can test whether the FF model explains average returns on a set of diversified assets that exhibit dispersion against size, book-to-market, and *relative leverage*. Individual stocks are re-assigned to equally-weighted portfolios every June on the basis of NYSE breakpoints for the three variables. They are grouped in terciles of size, book-to-market, and *relative leverage*.

To dispel the possibility that our results are driven by the specific test assets that we have chosen, we carry out additional tests. We select further sets of 25 portfolios following two-way independent sorts in quintiles. More precisely, two-way sorts are based on the following pairs of variables: size and book-to-market; size and *relative leverage*; book-to-market and *relative leverage*; momentum and size; momentum and book-to-market; momentum and *relative leverage*. The breakpoints and returns of these portfolios are determined with the same procedure described above.

3.4.2 Orthogonalizing regressions and factor model identification

In this section we test whether the FF factors, RMRF, SMB, HML, and OMU provide the same information for pricing assets. In particular, we want to assess whether OMU is redundant because it is proxied by the other factors.

The "orthogonalizing" regressions in Table 3.7 suggest that the explanatory power of OMU is not subsumed by RMRF, SMB, and HML. The regression of OMU on RMRF, SMB and HML in column 1 reports a statistically significant intercept which means that OMU cannot be replaced by a linear combination of the FF factors. This implies that, *ex-ante*, RMRF, SMB, and HML do not encompass OMU in the explanation of returns. Column 3 and 4 report similar results for the HML factor and the market factor RMRF respectively. Neither of these two factors can be regarded as redundant due to a significant intercept. On the contrary, as shown in column 2, when SMB is regressed on the other factors, the intercept is not statistically different from zero. This suggests that the pricing ability of SMB is proxied by the joint effects of RMRF, HML, and OMU[17].

(Insert Table 3.7 here)

3.4.3 Horse race

From our discussion in the previous section we can use a parsimonious model that includes only RMRF, OMU, and HML, and excludes SMB. We then compare the pricing performance of this model with the FF model. For completeness, we also report the results for CAPM. Our results are displayed in Table 3.8.

Using the 27 portfolios sorted on size, book-to-market equity, and *relative leverage* as test assets, the table shows that the model including RMRF, HML and OMU dominates the FF model and CAPM. Panel A provides descriptive statistics for the raw monthly returns of the 27 portfolios in the 1965-2009 period, showing the spreads in mean returns associated with size, book-to-market equity, and *relative leverage*. Panels B, C and D report the estimated pricing errors a_i and t-statistics $t(a_i)$ for the hypothesis that $a_i = 0$. These figures are based on the joint GMM estimation of the time series regressions of portfolio excess returns respectively on the factors of the FF model, of CAPM and of the model that contains RMRF, HML and OMU. T-tests are based on Newey-West standard errors with a lag length of 4. Panels B, C, and D also report, for each model, the average absolute pricing error, the mean-squared pricing error, the number of pricing errors out of 27 that are significantly different from zero at the 1% significance level, and the Gibbons et al. [1989] (GRS) test statistics.

To account for heteroskedasticity and error autocorrelation in the time-series regressions, we also implement Wald-type tests for the joint distribution of pricing errors in the GMM estimation. As can be expected, the results of Wald-type tests are qualitatively similar to those of GRS tests, and they are reported in Table 3.15 in Appendix D. Finally, since both GRS and Wald-type tests are known to over-reject the null hypothesis in finite samples, we estimate each model in stochastic discount factor form by efficient iterated GMM as in Cochrane [1996]. We report Hansen's J test statistics for a chi-square test

[17] Consider the FF's model augmented with the OMU factor, that is:

$$E[R_{i,t} - r_t^f] = a_i + b_i E[RMRF_t] + s_i E[SMB_t] + h_i E[HML_t] + o_i E[OMU_t]$$

If the orthogonalizing regression of SMB on RMRF, HML and OMU yields to an estimated intercept indistinguishable from zero, we have

$$E[SMB_t] = \beta E[RMRF_t] + \gamma E[HML_t] + \delta[OMU_t]$$

that is

$$E[R_{i,t} - r_t^f] = a_i + (b_i + \beta) E[RMRF_t] + (h_i + \gamma) E[HML_t] + (o_i + \delta) E[OMU_t]$$

of over-identifying restriction and its p-value. As Cochrane [1996] discusses, the test of over-identifying restrictions is based on the null hypothesis that the model is not rejected by the data. Accordingly, low p-values should be interpreted as evidence that the model fails in pricing the test assets. We provide additional details on the implementation of our GMM tests in Appendix C.

Panel B shows that the estimated intercepts for the FF model are generally high and statistically different from zero, with very high t-statistics. The FF model fails in pricing 17 out of 27 test assets at the 1% significance level, with monthly mean absolute and squared intercepts of 0.437 and 0.287 respectively. In particular, the FF model does not capture the spread in returns associated with *relative leverage*. This can be seen by the resulting trend in the pricing errors. Panel C shows that CAPM fails in pricing 18 out of 27 portfolios at the 1% significance level, with an average absolute pricing error 0.564% per month. The mean squared pricing error is even higher than that of the FF model. Panel D tests the pricing performance of the model that contains RMRF, HML, and OMU. This model provides the best description of variation in expected returns for the 27 portfolios. Only 2 intercepts out of 27 are statistically distinguishable from zero at the 1% significance level. The pricing error is also lower than that of the FF model and CAPM. For RMRF, our results offer support to the explanation of Fama and French [1993] that the market factor helps explain why average stock returns are higher than the risk-free rate. While (unreported) factor loadings for HML and OMU vary across the test assets and explain variations in expected returns, estimated factor loadings for RMRF are close to one for all portfolios. Finally, GRS test statistics and J test statistics are extremely high for both the FF model and the CAPM. Therefore, both of them fail to price the test assets. On the contrary, the test of over-identifying restrictions (J=21.46, p-value=0.55) fails to reject the model with RMRF, HML, and OMU. In Appendix D, Table 3.16, we also consider the pricing performance of a four-factor model including RMRF, SMB, HML and momentum. The results are qualitatively the same.

(Insert Table 3.8 here)

In Table 3.9 we compare the pricing performance of the three factor models on the remaining test assets. For convenience, we only report the average absolute pricing error, the mean squared pricing error, the GRS test statistic, the number of intercepts that are statistically different from zero at the 1% confidence level, the J test-statistics, and its p-value. Consistent with the results of Table 3.8, both the FF model and CAPM are unable to explain spreads in average returns when portfolios are sorted on *relative leverage*. More precisely, FF and CAPM report statistically significant intercepts for almost all portfolios sorted on *relative leverage*, market capitalization, book-to-market and momentum. The pricing errors are also high, with high values of the GRS and J statistics. On the contrary, the model with RMRF, HML and OMU reports statistically significant pricing errors only for 2 out of 25 portfolios sorted by size and *relative leverage*. None of the intercepts is significant for both the book-to-market/relative leverage and the momentum/relative leverage sorting. Average absolute and squared pricing errors, as well as GRS statistics and J statistics, are much lower than those of the FF model and CAPM. In addition, for all three sets of test assets, the test for over-identifying restrictions fails to reject the model with sizeable p-values. In summary, on these three sets of assets, the multi-factor model with RMRF, HML, and OMU in the one that provides the best description of expected returns.

The pricing performance of the model with RMRF, HML, and OMU does not appear to be limited to the test assets that include *relative leverage* as a sorting variable. It performs well also on the 25 portfolios sorted on size and book-to-market, on size and momentum, and on book-to-market and momentum. Average absolute pricing errors never exceed 0.3% per month, and the model rarely produces statistically significant intercepts at the 1% level. The pricing performance of both CAPM and FF improves only if *relative leverage* is not used to identify assets. However, even under this condition, CAPM originates significant pricing errors for individual assets (14 for the size/book-to-market sorts, 10 for the size/momentum sorts, 20 for the book-to-market/momentum sorts), and generates high mean absolute and squared pricing errors. The FF model provides a good description of average returns for portfolios formed on size and book-to-market, and on size and momentum. On these two sets of assets, its mean absolute and squared pricing errors are slightly lower than those of the model including RMRF, HML and OMU. Finally, on the 25 portfolios sorted by book-to-market and momentum, the FF model reports significant pricing errors in 18 cases, even though the average absolute and squared pricing errors are not as high as for the sorting based on *relative leverage*.

In sum, the results of this section indicate that a factor based on *relative leverage* helps price expected returns across assets. In the APT framework, our findings are consistent with a rational *relative leverage premium*, that is with the existence of a source of systematic risk that should be considered to price assets under no arbitrage in the stock market.

(Insert Table 3.9 here)

3.5 Discussion and conclusions

Leary and Roberts [2005] and Strebulaev [2007] show that in the presence of frictions firms cannot always reach the desired level of leverage. This implies that firms may be temporarily over- or under-leveraged, as their leverage is above or below the desired target. In this paper we start by estimating *relative leverage* as the difference between observed and target leverage, individually for each firm. This allows us to remove part of the heterogeneity in the cross-section of leverage in a way that accounts for firm specific characteristics. We then employ *relative leverage* as a variable to explain expected equity returns.

We find that expected equity returns are increasing in *relative leverage*. The relation is significant over all sub-periods after controlling for size, book-to-market, momentum, and observed leverage. On the contrary, observed leverage does not appear to play a relevant role in explaining equity returns. Our empirical evidence helps clarify the relationship between expected returns and leverage. The significance of *relative leverage* as an explanatory variable for equity returns is robust to out-of-sample estimates of target leverage.

We envisage three possible explanations for our results. First, our findings may be sample specific. However, considering the stability of our results across various sub-periods, it seems unlikely that the *relative leverage premium* is confined to any specific sample. Second, our findings may due to mis-pricing. However, our tests in Section 3.4 suggest that the *relative leverage premium* is consistent with a linear multi-factor model in the absence of arbitrage (Ross [1976]). Third, the *relative leverage premium* arises in a framework of rational asset pricing. In this interpretation over-leveraged (under-leveraged) firms should be riskier (safer) for investors.

While our contribution is essentially empirical, the third scenario poses an interesting challenge for future theoretical research. Our findings suggest the need for a structural model to rationalize the *relative leverage premium* and its asset pricing implications. As a starting point, we propose two possible stories for the *relative leverage premium.*

The first one hinges upon the investment inflexibility mechanism proposed, amongst others, by Zhang [2005], Livdan et al. [2009], Li et al. [2009], Gomes and Schmid [2010], Obreja [2010], and Ozdagli [2012] in an investment-based asset pricing framework. Over-leveraged firms are less flexible than under-leveraged firms in adjusting capital investment and smoothing dividends, and therefore are expected to earn higher equity returns in equilibrium. We speculate that inflexibility is generated by two frictions: collateral constraints and investment adjustment costs. Collateral constraints impose an upper bound on a firm's leverage ratio. And, when binding, they either reduce a firm's ability to exploit future investment opportunities, or require costly equity issues. Investment adjustment costs prevent firms from adjusting instantaneously to their desired leverage ratio. Adjustment costs generate path dependence in investment and financing decisions, and temporary deviations from the firm's optimal leverage ratio. *Target leverage* is the optimal leverage ratio that firms have in the absence of adjustment costs, in line with the investment literature (e.g. Caballero and Engel [1999]). As the difference between realized and target leverage, *relative leverage* captures the degree of a firm's inflexibility.

Another possible story stems from the trade-off theory of capital structure, according to which there is a cost of not being at the optimum leverage ratio. Suppose that in a recession a firm's assets A decrease in value more than its outstanding amount of debt D. For example, the drop in value of the assets can be due to lower expected future cash flows. *Ceteris paribus*, a firm's leverage D/A increases because of the systematic shock that affects A. As a result, in a recession over-leveraged firms tend to move further away from their desired target leverage, and experience higher costs and lower cash flows, due to their (sub-optimal) capital structure. The returns of over-leveraged firms are then pro-cyclical because they are lower in a recession than in a boom. The opposite holds for an under-leveraged firm. Accordingly, risk-averse investors expect higher returns from over-leveraged firms than from under-leveraged ones.[18]

Finally, we suggest another possible avenue for future research. We conjecture that long-run equity returns may be primarily related to target leverage[19], while short-term returns relate more to *relative leverage*. Intuitively, long-run returns reflect business risk more than temporary deviations from target leverage. Furthermore, in the long-run adjustment costs matter less, and firms are closer to their target. Therefore, target leverage reflects asset beta and determines long-run returns, while *relative leverage* relates to short-run returns where adjustment costs matter the most.[20]

[18]It is worth noting that the above interpretation relies on the assumption that firms have a target leverage, but it does not require firms to exhibit a targeting behavior. Therefore, it is not inconsistent with Chang and Dasgupta [2009] and Iliev and Welch [2010].

[19]See DeAngelo et al. [2011] for a definition of target leverage as a long-run objective.

[20]We thank Arthur Korteweg for suggesting this idea.

Figure 3.1. Average Return for 25 Portfolios Sorted by Relative Leverage and Observed Leverage.

Using monthly data from July 1965 to December 2009, stocks are sorted independently every June in quintiles based on their values of MDR (Obs_Lev), and *relative leverage* (Rel_Lev). Time-series averages of monthly cross-sectional returns are reported. Portfolios are formed by matching MDR and *relative leverage* to monthly returns as in Fama and French (1992).

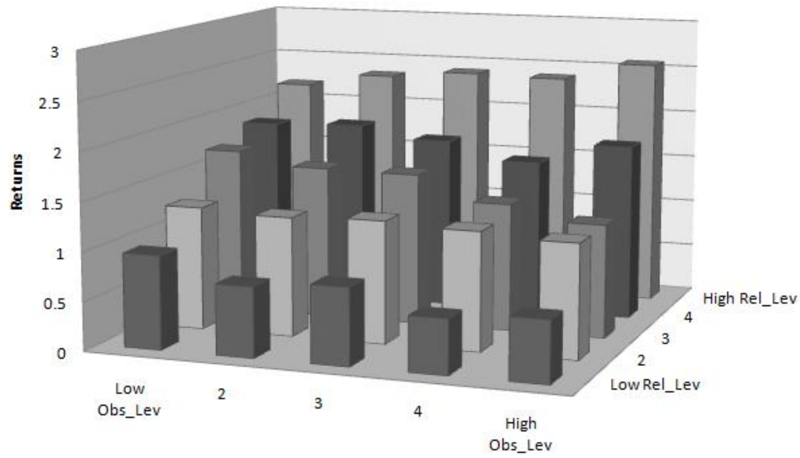


Figure 3.2. Market Capitalization and Deviations from Target Debt Ratio

Using monthly data from July 1965 to December 2009, stocks are sorted every June in deciles based on their values of *relative leverage*. OL (over-leveraged) denotes the top decile, and UL (under-leveraged) denotes the bottom decile. Time-series averages of market capitalizations are reported. Portfolios are formed by matching *relative leverage* to monthly stock prices as in Fama and French (1992).

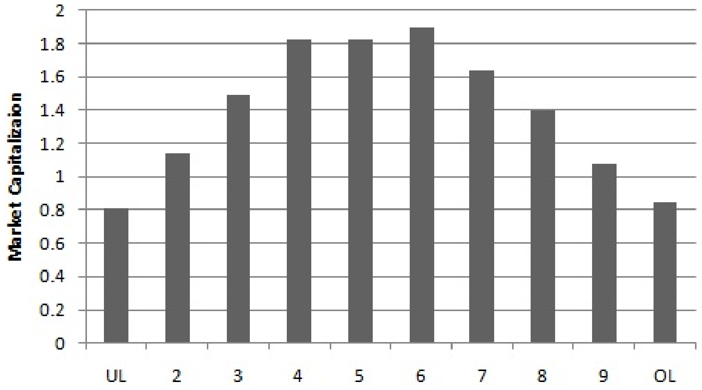


Figure 3.3. Average Returns for 9 Portfolios Sorted by Relative Leverage

Using monthly data from July 1965 to December 2009, stocks are sorted every June in nine portfolios on the basis of the equally-spaced breakpoints of *relative leverage* reported on the horizontal axis. The vertical blue line indicates target leverage. Time-series averages of monthly cross-sectional returns are reported. Panel A refers to the period between July 1965 and December 2009, Panel B to the sub-period between July 1965 and December 1979, Panel C to the sub-period between January 1980 and December 1994, and Panel D to the sub-period between January 1995 and December 2009. Portfolios are formed by matching *relative leverage* to monthly returns as in Fama and French (1992).

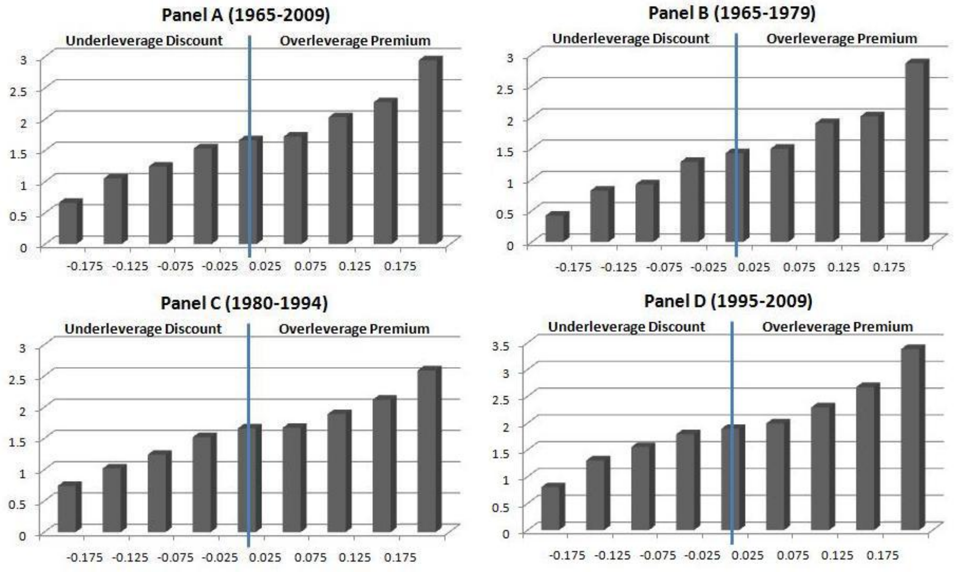


Figure 3.4. Average Returns for 9 Portfolios Sorted by Out-of-Sample Relative Leverage (1990-2009)

Using monthly data from July 1990 to December 2009, stocks are sorted every June in nine portfolios on the basis of the equally-spaced breakpoints of *relative leverage* reported on the horizontal axis. *Relative leverage* is estimated out-of-sample, as described in Appendix B. The vertical blue line indicates target leverage. Time-series averages of monthly cross-sectional returns are reported. Portfolios are formed by matching MDR and *relative leverage* to monthly returns as in Fama and French (1992).

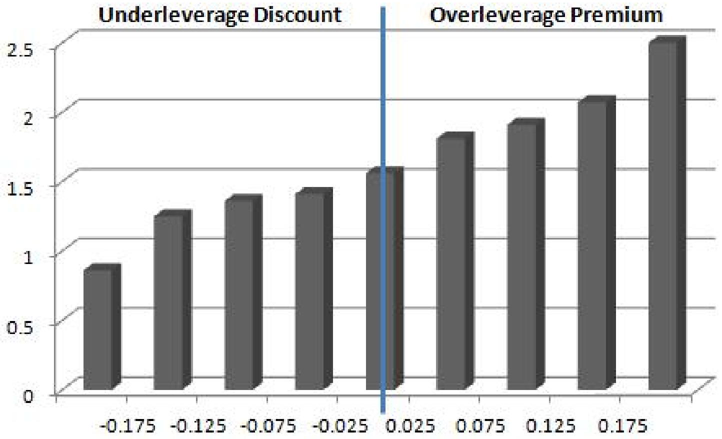


Table 3.1. Panel Regressions Estimating Target Leverage.
Columns (1),(2),(3) report estimates for the model:
$MDR_{i,t+1} = (\lambda\beta)X_{i,t} + (1-\lambda)MDR_{i,t} + \epsilon_{i,t+1}$
Column (1) reports pooled OLS estimates; Column (2) reports estimates with the Least Squares Dummy Variable (LSDV) model as in Flannery and Rangan [2006]; and column (3) reports Blundell and Bond's system GMM estimates (our "base" specification). The dependent variable in all regressions is the market debt ratio (MDR). λ represents adjustment speed. EBIT_TA is profitability, MB is the book-to-market value of assets, DEP_TA is depreciation over total assets, lnTA is the logarithm of total assets, FA_TA is the fixed-to-total assets ratio, R&D_TA is the ratio between R&D expenses and total assets, R&D_DUM is a dummy equal to one for firms with missing values for R&D expenses, and Ind_Median is the median industry MDR calculated for each year for two-digit SIC code industries. Variables that are not expressed as ratios are deflated by the consumer price index in 1983 dollars. All variables are winsorized at the 1% level, and all right-hand side variables are lagged by one year. The sample is the same described in Table 3.10 of Appendix A.

VARIABLES	(1) OLS	(2) LSDV	(3) BB ("base")
λ	0.155**	0.353**	0.238**
	(450.1)	(244.3)	(94.28)
EBIT_TA	-0.259**	-0.207**	-0.240**
	(-14.61)	(-19.14)	(-5.844)
MB	-0.0111**	-0.0026**	-0.0089**
	(-6.473)	(-2.677)	(-3.496)
DEP_TA	-1.419**	-0.799**	-2.181**
	(-15.92)	(-13.44)	(-9.545)
LnTA	0.0164**	0.0646**	0.0295**
	(12.41)	(36.28)	(5.569)
FA_TA(-1)	0.206**	0.160**	0.279**
	(15.32)	(12.94)	(5.971)
R&D_DUM	0.0566**	0.0035	0.1168**
	(10.41)	(0.737)	(6.262)
R&D_TA	-0.501**	-0.116**	-0.344**
	(-11.77)	(-3.781)	(-4.299)
Ind_Median	0.178**	0.018	-0.022
	(7.949)	(0.995)	(-0.408)
Constant	-0.247**	-1.020**	-0.403**
	(-5.452)	(-28.46)	(-3.876)
Adjusted R-squared	0.779	0.466	-
Firm FE	No	Yes	Yes
Year FE	Yes	Yes	Yes

t-statistics in parentheses
** p<0.01, * p<0.05

Table 3.2. Time-Series Average of Cross-Sectional Correlations.
We report time-series averages of monthly cross-sectional correlations using monthly data from July 1965 to December 2009. MDR is the market debt ratio. Log(size) is the natural logarithm of market capitalization. Log(bm) is the natural logarithm of book-to-market equity, Rel_Lev is *relative leverage*, Distance is the distance from target leverage, Overlev is over-leverage, Underlev is under-leverage, and Momentum is the continuously compounded return from month $t-12$ to month $t-2$.

	MDR	Rel_Lev	Distance	Overlev	Underlev	Log(size)	Log(bm)	Momentum
MDR	1.000							
Rel_Lev	0.425	1.000						
Distance	0.162	-0.130	1.000					
Overlev	0.467	0.774	0.450	1.000				
Underlev	-0.233	-0.840	0.597	-0.361	1.000			
Log(size)	-0.167	-0.090	-0.121	-0.152	-0.004	1.000		
Log(bm)	0.435	0.138	0.071	0.163	-0.063	-0.318	1.000	
Momentum	0.056	0.137	-0.007	0.106	-0.109	-0.011	0.084	1.000

Table 3.3. Cross-Sectional Patterns: *Relative Leverage* vs MDR.

Using monthly data from July 1965 to December 2009, stocks are sorted independently every June in quintiles based on their values of MDR and *relative leverage*. Time-series averages of monthly cross-sectional firm characteristics are reported. Average return is the percent average monthly return, number of firms is the average number of companies each month in each group, MDR is the market debt ratio, Log(size) is the natural logarithm of market capitalization, BE/ME is book-to-market equity, Momentum is the continuously compounded return from month $t-12$ to month $t-2$.

	LowRL	2	3	4	HighRL	LowRL	2	3	4	HighRL
	Average Return					Number of firms				
LowMDR	0.96	1.27	1.71	1.87	2.19	191.84	182.19	174.78	113.59	34.45
2	0.72	1.23	1.58	1.90	2.33	104.71	86.73	80.40	67.84	25.74
3	0.79	1.26	1.57	1.78	2.40	75.82	61.70	66.44	75.57	52.55
4	0.56	1.22	1.32	1.60	2.38	54.97	45.80	53.52	77.54	101.52
HighMDR	0.63	1.17	1.17	1.82	2.57	36.46	33.94	40.66	72.09	208.84
	MDR					Log(size)				
LowMDR	0.03	0.03	0.03	0.03	0.03	4.87	5.22	5.11	4.83	4.58
2	0.13	0.13	0.14	0.14	0.15	5.18	5.56	5.61	5.28	4.51
3	0.24	0.24	0.24	0.24	0.25	4.96	5.45	5.53	5.35	4.71
4	0.36	0.36	0.37	0.36	0.37	4.64	5.08	5.09	5.10	4.64
HighMDR	0.56	0.57	0.57	0.58	0.62	4.00	4.30	4.42	4.47	4.12
	BE/ME					Momentum				
LowMDR	0.64	0.53	0.55	0.52	0.45	-7.07	-4.49	-2.41	-1.09	2.17
2	0.68	0.63	0.64	0.67	0.67	-8.35	-3.85	-0.58	3.32	3.84
3	0.80	0.80	0.79	0.79	0.82	-8.58	-2.53	-0.44	1.75	6.94
4	0.98	0.97	0.94	0.97	0.95	-10.03	-2.87	-1.16	0.76	6.55
HighMDR	1.23	1.23	1.24	1.28	1.25	-14.35	-3.90	-2.54	0.79	7.13

Table 3.4. *Relative Leverage*, Size, Book-to-Market, Momentum.
In Panel A, each month between July 1965 to December 2009, we estimate cross-sectional regression of stock returns on size, book-to-market ratio, *relative leverage* and momentum. In Panel B, in the regressions in columns 1-2, *relative leverage* is estimated out-of-sample, as described in Appendix B. The regressions in columns 3-4 are estimated in-sample on the period July 1990-December 2009 using Blundell and Bond [1998] system GMM, as described in Section 3.2. The regressions in columns 5-6 are estimated in-sample on the period July 1990-December 2009 using the Least Squares Dummy Variable (LSDV) estimator, with the specification in column 2 of table 3.1. The independent variables are matched to monthly returns in line with Fama and French (1992). The sample includes a total of 1,113,498 and 1,084,367 firm-month observations for Panel A and Panel B respectively. We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	ret	ret	ret	ret	ret	ret	ret	ret
Panel A: 1965-2009 (534 months)								
Log(size)	-0.273			-0.237	-0.243		-0.221	-0.213
	(-5.964)			(-4.915)	(-5.384)		(-4.644)	(-4.667)
Log(bm)		0.383		0.173		0.288	0.0960	0.103
		(5.080)		(2.157)		(3.947)	(1.228)	(1.335)
Rel_Lev			4.003		3.655	3.706	3.509	3.522
			(18.68)		(18.96)	(20.71)	(20.73)	(23.39)
Momentum								0.002
								(1.010)
Constant	1.047	1.677	1.628	1.171	1.193	1.738	1.270	1.132
	(4.045)	(5.695)	(5.244)	(4.837)	(4.528)	(5.797)	(5.141)	(4.854)

t-statistics in parentheses

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ret	ret	ret	ret	ret	ret
Panel B: 1990-2009 (234 months)						
	Out-of-sample Est.		BB Est.		LSDV Est.	
log(size)	-0.238	-0.236	-0.251	-0.248	-0.237	-0.235
	(-3.732)	(-3.927)	(-3.916)	(-4.119)	(-3.701)	(-3.886)
log(bm)	0.069	0.068	0.052	0.052	0.039	0.040
	(0.616)	(0.681)	(0.459)	(0.519)	(0.351)	(0.400)
Rel_Lev	2.166	2.175	3.486	3.606	3.867	3.966
	(8.779)	(8.850)	(12.85)	(13.73)	(12.77)	(13.58)
Momentum		-0.003		-0.003		-0.003
		(-0.993)		(-1.143)		(-1.185)
Constant	1.321	1.071	1.342	1.088	1.340	1.090
	(3.860)	(3.460)	(3.901)	(3.494)	(3.900)	(3.504)

t-statistics in parentheses

Table 3.5. *Relative Leverage*, Over-Leverage, Under-Leverage and Distance.
In Panel A, each month between July 1965 to December 2009, we estimate cross-sectional regressions of stock returns on size, book-to-market ratio, *relative leverage*, distance, over-leverage and under-leverage. In Panel B, in the regressions in columns 1-3, *relative leverage*, under-leverage, and over-leverage are estimated out-of-sample, as described in Appendix B. In the regressions in columns 4-6 they are estimated in-sample on the period July 1990-December 2009 using Blundell and Bond [1998] system GMM, as described in Section 3.2. In the regressions in columns 7-9, they are estimated in-sample on the period July 1990-December 2009 using the Least Squares Dummy Variable (LSDV) estimator, with the specification in column 2 of table 3.1. The independent variables are matched to monthly returns in line with Fama and French (1992). The sample includes a total of 1,084,367 and 446,933 firm-month observations for Panel A and Panel B respectively. We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2. Distance, Overlev and Underlev are defined as:

$$Distance_{i,t} = abs\{Rel\_Lev_{i,t}, 0\}$$
$$Overlev_{i,t} = max\{Rel\_Lev_{i,t}, 0\}$$
$$Underlev_{i,t} = -min\{Rel\_Lev_{i,t}, 0\}$$

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ret	ret	ret	ret	ret	ret
Panel A: 1965-2009 (534 months)						
Log(size)	-0.221	-0.222	-0.234	-0.222	-0.222	-0.222
	(-4.644)	(-4.700)	(-4.905)	(-4.700)	(-4.700)	(-4.700)
Log(bm)	0.0960	0.0962	0.153	0.0962	0.0962	0.0962
	(1.228)	(1.230)	(1.915)	(1.230)	(1.230)	(1.230)
Overlev		3.496		0.0457		
		(7.690)		(0.0716)		
Underlev		-3.450			0.0457	
		(-11.01)			(0.0716)	
Rel_Lev	3.509			3.450	3.496	3.473
	(20.73)			(11.01)	(7.690)	(15.44)
Distance			-0.709			0.0229
			(-2.278)			(0.0716)
Constant	1.270	1.270	1.304	1.270	1.270	1.270
	(5.141)	(5.358)	(5.478)	(5.358)	(5.358)	(5.358)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret
Panel B: 1990-2009 (234 months)						
	Out-of-sample Est.		BB Est.		LSDV Est.	
log(size)	-0.236	-0.236	-0.249	-0.249	-0.236	-0.236
	(-3.699)	(-3.699)	(-3.890)	(-3.890)	(-3.674)	(-3.674)
log(bm)	0.0690	0.0690	0.0498	0.0498	0.0396	0.0396
	(0.621)	(0.621)	(0.446)	(0.446)	(0.358)	(0.358)
Overlev	2.416		3.492		3.659	
	(4.496)		(5.045)		(5.257)	
Underlev	-1.668		-3.020		-3.654	
	(-3.859)		(-5.285)		(-5.937)	
Rel_Lev		2.042		3.256		3.657
		(7.701)		(9.611)		(10.74)
Distance		0.374		0.236		0.002
		(0.914)		(0.440)		(0.004)
Constant	1.292	1.292	1.326	1.326	1.347	1.347
	(4.011)	(4.011)	(4.147)	(4.147)	(4.241)	(4.241)

t-statistics in parentheses

Table 3.6. *Relative Leverage*, Market Leverage, Book Leverage.
In Panel A, each month between July 1965 to December 2009, we estimate cross-sectional regressions of stock returns on size, book-to-market ratio, *relative leverage*, market leverage, and book leverage. In Panel B, in the regressions in columns 1-3, *relative leverage*, under-leverage, and over-leverage are estimated out-of-sample, as described in Appendix B. In the regressions in columns 4-6 they are estimated in-sample on the period July 1990-December 2009 using Blundell and Bond [1998] system GMM, as described in Section 3.2. In the regressions in columns 7-9, they are estimated in-sample on the period July 1990-December 2009 using the Least Squares Dummy Variable (LSDV) estimator, with the specification in column 2 of table 3.1. The independent variables are matched to monthly returns in line with Fama and French (1992). The sample includes a total of 1,084,367 and 446,933 firm-month observations for Panel A and Panel B respectively. We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2.

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ret	ret	ret	ret	ret	ret
Panel A: 1965-2009 (534 months)						
Log(size)		-0.233	-0.223		-0.236	-0.221
		(-4.931)	(-4.749)		(-5.025)	(-4.682)
Log(bm)		0.135	0.186		0.163	0.198
		(1.798)	(2.488)		(2.082)	(2.501)
MDR	0.987	0.282	-1.000			
	(3.296)	(1.140)	(-3.725)			
Rel_Lev			3.992			
			(21.32)			
BDR				0.332	0.146	-0.386
				(1.833)	(0.701)	(-1.775)
Rel_Lev (book)						1.542
						(12.17)
Constant	1.285	1.101	1.573	1.455	1.137	1.385
	(4.397)	(4.806)	(6.706)	(5.099)	(4.888)	(6.053)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret
Panel B: 1990-2009 (234 months)						
	Out-of-sample Est.		BB Est.		LSDV Est.	
log(size)	-0.220	-0.243	-0.229	-0.233	-0.211	-0.236
	(-3.666)	(-4.075)	(-3.788)	(-3.887)	(-3.492)	(-3.930)
log(bm)	0.139	0.142	0.166	0.160	0.178	0.159
	(1.510)	(1.353)	(1.817)	(1.517)	(1.948)	(1.518)
MDR	-0.507		-1.078		-1.394	
	(-1.226)		(-2.473)		(-3.137)	
Rel_Lev	2.241		4.057		4.805	
	(7.946)		(11.34)		(12.33)	
BDR		-0.0791		-0.361		-0.259
		(-0.221)		(-1.023)		(-0.715)
Rel_Lev (book)		0.757		1.599		1.294
		(3.197)		(6.026)		(4.400)
Constant	1.478	1.381	1.675	1.491	1.759	1.461
	(4.448)	(4.055)	(4.959)	(4.410)	(5.240)	(4.300)

t-statistics in parentheses

Table 3.7. Orthogonalizing Regressions.

We report orthogonalizing time-series regressions of factors OMU, SMB, HML, and RMRF using monthly data from July 1965 to December 2009. Monthly series of the Fama and French factors are from Kenneth French's website, while OMU (Over-leveraged Minus Under-leveraged) is defined as the difference between the average monthly return of stocks with *relative leverage* above the 80th percentile for NYSE firms, minus the average monthly return of stocks with *relative leverage* below the 20th percentile for NYSE firms. In order to define OMU, firms are assigned to portfolios at the end of June of each year. Reported t-statistics are based on standard errors corrected for heteroskedasticity as suggested by Davidson and MacKinnon [1993].

	(1)	(2)	(3)	(4)
VARIABLES	OMU	SMB	HML	RMRF
RMRF	-0.0245	0.178***	-0.136***	
	(-0.918)	(4.632)	(-3.680)	
SMB	0.125***		-0.194***	0.353***
	(3.810)		(-3.239)	(3.753)
HML	0.285***	-0.266***		-0.370***
	(6.708)	(-2.811)		(-3.582)
OMU		0.352***	0.585***	-0.136
		(3.475)	(6.657)	(-0.942)
Constant	1.532***	-0.285	-0.462**	0.693**
	(19.61)	(-1.465)	(-2.583)	(2.423)
Observations	534	534	534	534
R-squared	0.196	0.153	0.274	0.162

Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.8. Comparison of Pricing Performance of Factor Models.
At the end of June of each year between 1965 and 2009, stocks are allocated to 27 portfolios by independently ranking them into three groups on the basis of their values of size, book-to-market equity, and *relative leverage*. Individual stocks are re-assigned to equally-weighted portfolios every June on the basis of NYSE breakpoints for the three variables. UL (OL) denotes the portfolio of stocks with *relative leverage* below (above) that of the lowest (highest) tercile of NYSE firms. Low (High) denotes the portfolio of stocks with book-to-market equity below (above) that of the lowest (highest) tercile of NYSE firms. Small (Large) denotes the portfolio of stocks with market capitalization below (above) that of the lowest (highest) tercile of NYSE firms. Panel A shows average returns for the 27 portfolios and their standard deviations. Panel B reports estimated pricing errors from iterated GMM estimation of time-series regressions of the excess returns of the 27 portfolios on the Fama and French factors. Panel C reports estimated pricing errors from iterated GMM estimation of time-series regressions of the excess returns of the 27 portfolios on the market factor RMRF. Panel D reports estimated pricing errors from iterated GMM estimation of time-series regressions of the excess return of the 27 portfolios on RMRF, OMU, and HML. Reported t-statistics are based on Newey-West standard errors with a lag length of 4. Panels B, C, and D also report average absolute pricing errors, average squared pricing errors, number of failures "#Fail", GRS test statistics, Hansen's test statistics for a J test of over-identifying restriction "J", and its p-value "p(J)". #Fail is defined as the number of pricing errors out of 27 that are significantly different from zero at the 1% level. "J" refers to the model estimated in stochastic discount factor form by GMM.

Panel A: Descriptive Statistics

		Average Return			St. Dev		
RL	Size/BM	Low	2	High	Low	2	High
	Small	0.61	0.74	0.99	8.62	6.98	6.56
UL	2	0.10	0.47	0.47	7.40	6.40	6.38
	Large	0.09	0.24	0.42	5.83	5.61	5.94
	Small	1.27	1.25	1.47	8.61	6.73	6.55
2	2	0.87	0.91	1.02	6.58	5.92	5.96
	Large	0.68	0.70	0.78	5.25	5.18	5.50
	Small	2.28	1.83	2.06	8.45	7.03	6.68
OL	2	1.39	1.34	1.45	6.86	6.07	6.61
	Large	1.05	0.96	1.00	5.65	5.51	5.86

Panel B: FF Model ($R_{i,t} - r_t^f = a_i + b_i RMRF_t + s_i SMB_t + h_i HML_t + \epsilon_{i,t}$)							
		a_i			$t(a_i)$		
RL	Size/BM	Low	2	High	Low	2	High
	Small	-0.29	-0.04	0.11	-2.22	-0.37	0.91
UL	2	-0.57	-0.35	-0.39	-4.92	-3.42	-3.24
	Large	-0.28	-0.34	-0.26	-3.33	-3.29	-1.91
	Small	0.52	0.48	0.59	3.69	4.34	5.52
2	2	0.33	0.22	0.21	3.57	2.25	1.91
	Large	0.34	0.11	0.14	4.95	1.17	1.31
	Small	1.36	0.92	1.17	8.63	7.98	10.25
UL	2	0.69	0.58	0.46	6.90	5.31	3.70
	Large	0.65	0.24	0.16	6.88	2.16	1.35

Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)
0.437	0.287	17	21.79	71.8	0.00

Panel C: CAPM ($R_{i,t} - r_t^f = a_i + b_i RMRF_t + \epsilon_{i,t}$)							
		a_i			$t(a_i)$		
RL	Size/BM	Low	2	High	Low	2	High
	Small	-0.18	0.23	0.54	-0.80	1.23	2.68
UL	2	-0.59	-0.11	-0.00	-4.17	-0.79	-0.01
	Large	-0.40	-0.21	-0.03	-4.59	-1.81	-0.17
	Small	0.55	0.71	1.01	2.42	3.77	5.05
2	2	0.28	0.44	0.55	1.97	3.28	3.62
	Large	0.22	0.28	0.38	2.89	2.52	2.85
	Small	1.54	1.24	1.63	5.85	6.28	7.54
OL	2	0.76	0.86	0.89	5.34	5.77	4.85
	Large	0.55	0.51	0.55	5.48	3.99	3.39

Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)
0.564	0.486	18	20.72	75.9	0.00

Panel D: $R_{i,t} - r_t^f = a_i + b_i RMRF_t + h_i HML_t + o_i OMU_t + \epsilon_{i,t}$							
		a_i			$t(a_i)$		
RL	Size/BM	Low	2	High	Low	2	High
	Small	0.12	0.17	0.08	0.25	0.45	0.20
UL	2	0.06	0.18	0.16	0.22	0.61	0.51
	Large	0.19	0.28	0.25	1.51	1.72	1.13
	Small	0.45	0.24	0.11	1.00	0.70	0.27
2	2	0.28	0.05	0.12	1.38	0.23	0.55
	Large	0.46	0.30	0.36	4.10	2.32	2.36
	Small	0.44	0.30	0.23	0.97	0.81	0.57
OL	2	0.53	0.23	0.22	2.10	1.07	0.67
	Large	0.51	0.04	0.29	3.51	0.25	1.33

Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)
0.246	0.073	2	3.37	21.46	0.55

Table 3.9. Comparison of Pricing Performance of Factor Models.

At the end on June of each year between 1965 and 2009, stocks are allocated to 25 portfolios by independently ranking them into five groups on the basis of their values of size (S), book-to-market equity (BM), momentum (M), and *relative leverage* (RL). Individual stocks are re-assigned to equally-weighted portfolios every June on the basis of NYSE breakpoints for the four variables. The table shows average absolute pricing errors, average squared pricing errors, number of failures "#Fail", GRS test statistics, Hansen's test statistics for a J test of over-identifying restriction "J", and its p-value "p(J)" for the Fama and French model, the CAPM, and a model including the market factor RMRF, HML, and OMU. #Fail is defined as the number of pricing errors out of 25 that are significantly different from zero at the 1% level. These tests are based on Newey-West standard errors with a lag length of 4. Time-series regressions are estimated by iterated GMM. "J" refers to the model estimated in stochastic discount factor form by GMM.

	FF Model						CAPM						RMRF+HML+OMU					
Test Assets	Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)	Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)	Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)
25 port. (5 S, 5 BM)	0.200	0.086	6	3.38	27.9	0.14	0.425	0.287	14	3.77	35.0	0.05	0.279	0.103	3	2.96	26.4	0.19
25 port. (5 RL, 5 BM)	0.512	0.380	19	19.84	84.0	0.00	0.719	0.724	18	18.80	85.2	0.00	0.217	0.058	0	1.83	16.9	0.72
25 port. (5 RL, 5 S)	0.428	0.269	20	19.10	60.8	0.00	0.564	0.484	20	19.29	82.6	0.00	0.238	0.067	2	2.40	18.1	0.64
25 port. (5 S, 5 M)	0.226	0.097	8	2.96	28.5	0.13	0.391	0.250	10	2.87	38.1	0.02	0.244	0.127	2	2.27	15.5	0.80
25 port. (5 BM, 5 M)	0.347	0.162	18	4.67	39.1	0.01	0.612	0.484	20	4.48	51.7	0.00	0.239	0.113	0	2.80	39.0	0.01
25 port. (5 RL, 5 M)	0.505	0.368	20	20.29	67.8	0.00	0.672	0.672	19	20.38	87.6	0.00	0.201	0.083	0	2.75	25.9	0.21

Appendix

Appendix A describes the data and variables used for the estimation of Flannery and Rangan [2006] partial adjustment model in Section 3.2, and for the asset pricing tests in Sections 3.3, and 3.4. Appendix B provides more details on the out-of-sample estimation procedure. Appendix C discusses the GMM-based asset pricing tests that we carry out in Section 3.4. Finally, Appendix D implements several robustness tests with a particular focus on possible look-ahead biases.

Appendix A: data and variables

A.1. Data and variables for the estimation of target leverage

For the leverage decomposition of FR, we use the Compustat Industrial Annual database over the period 1965-2009 including all companies listed on AMEX, NYSE, and NASDAQ, and excluding foreign firms that are not incorporated in the United States. We exclude financials (SIC codes 6000-6999) and utilities (SIC codes 4900-4999) because of their special characteristics.

Our measure of leverage is MDR as defined in (3.1), and is computed as the book value of short-term plus long-term interest bearing debt (Compustat items DLTT+DLC) divided by the market value of assets (DLTT+DLC + PRCC_F*CSHO). As in FR, $X_{i,t}$ contains the following variables:[21] Profitability (EBIT_TA): Earnings before interest and taxes (EBIT) over total assets (AT); Market Value over Assets (MB): Book value of liabilities plus market value of equity (DLTT+DLC + PRCC_F*CSHO) over total assets [AT]; Depreciation (DEP_TA): Depreciation (DP) over total assets (AT); Size (lnTA): Logarithm of total assets (AT); Tangibility (FA_TA): Property, plant, and equipment (PPENT) over total assets (AT); R&D expenses (R&D_TA): R&D expenses (XRD) over total assets (AT); R&D Dummy (R&D_DUM): Dummy equal to one for firms with missing values for R&D expenses (XRD); Industry MDR (Ind_Median): Median industry MDR calculated for each year for two-digit SIC code industries; a firm fixed effect.

Following standard procedures, all the previous variables (including MDR) are winsorized at the 1st and 99th percentiles to mitigate the influence of extreme observations. All variables are based on fiscal years. When included, year dummies are based on calendar years. Table 3.10 provides summary statistics for the variables listed above.

A.2. Data and variables for the analysis of returns

In our asset pricing tests we use monthly stock prices and returns for firms on NYSE, AMEX, Nasdaq covered by the Center of Research in Security Prices (CRSP) from 1965 to 2009. We exclude financial companies (SIC codes 6000-6999) and utility companies (SIC codes 4900-4999), and foreign firms not incorporated in the United States. De-listing returns are included in monthly returns.

We match these monthly data to annual income statement and balance sheet data from the CRSP/COMPUSTAT merged database. We follow the matching procedure of Fama and French [1992], which ensures a minimum gap of six months between fiscal year-ends and returns. More precisely, we match monthly prices and returns from July of calendar year t to June of calendar year $t+1$ with data from each company's latest fiscal

[21] Variables that are not expressed as ratios are deflated by the consumer price index in 1983 dollars.

Table 3.10. Flannery and Rangan [2006] Decomposition: Summary Statistics.
Sample includes all Compustat firms traded on NYSE,AMEX and NASDAQ between 1965 and 2009. Financial firms and utilities are excluded. In the sample there are 9,058 firms and 115,710 firm-year observations. MDR is the market debt ratio, EBIT_TA is profitability, MB is the book-to-market value of assets, DEP_TA is depreciation over total assets, lnTA is the logarithm of total assets, FA_TA is the fixed-to-total assets ratio, R&D_TA is the ratio between R&D expenses and total assets, R&D_DUM is a dummy equal to one for firms with missing values for R&D expenses, and Ind_Median is the median industry MDR calculated for each year for two-digit SIC code industries. Variables that are not expressed as ratios are deflated by the consumer price index in 1983 dollars. All variables are winsorized at the 1% level.

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**
MDR	0.24	0.23	0	0.87
EBIT_TA	0.06	0.17	-0.76	0.36
MB	1.61	1.5	0.33	9.49
DEP_TA	0.04	0.03	0	0.18
LnTA	18.69	1.85	14.73	23.49
FA_TA	0.3	0.22	0.01	0.89
R&D_DUM	0.04	0.08	0	0.47
R&D_TA	0.52	0.5	0	1
Ind_Median	0.19	0.14	0	0.98

year ending in calendar year $t-1$.

In our tests, we consider the natural logarithm of market capitalization, the natural logarithm of book-to-market equity, and momentum as control variables. Market capitalization - defined as the product of a company's stock price times the number of outstanding shares - is measured at June of calendar year t for the returns between July of calendar year t and June of calendar year $t+1$. We measure book-to-market equity as the ratio between a firm book equity and its market capitalization at the end of December of calendar year $t-1$. Following Fama and French [1993], we compute book equity as the sum of shareholders' equity, balance sheet deferred taxes and investments, and tax credits if available, minus the book value of preferred stocks. Depending on data availability, we estimate the book value of preferred stocks using, in this order, their redemption, liquidation or par value. Since we consider the natural logarithm of book-to-market equity in our tests, we eliminate firms with negative book equity from our analysis. Finally, similar to Fama and French [2008], we measure momentum as the continuously compounded return from month $t-12$ to month $t-2$.

In Section 3.3.3 we also consider book-valued debt ratios (BDR), defined as the book value of short-term plus long-term interest bearing debt (DLTT + DLC) divided by the book value of assets (DLTT + DLC) + book value of equity (BE). For BDR, we estimate a *relative leverage* measure following the same procedure as in Section 3.2. More precisely, our *relative leverage* measure for BDR is obtained by re-estimating Equation (3.4) with

BDR as dependent variable.[22]

All annual series are matched to monthly data from CRSP as described before. Therefore, we match leverage, *relative leverage*, over-leverage, under-leverage, and distance measures from fiscal year $t-1$ to monthly returns from July of year t to June of year $t+1$. In Section 3.4 and Appendix A we also employ monthly series of Fama and French's factors RMRF, HML, SMB, of the risk-free rate RF, and of the momentum factor MOM. We obtain these data from Kenneth French's website.

[22] Accordingly, MDRInd is replaced by the industry median of BDR.

Appendix B: out-of-sample estimation

In this appendix, we provide details and robustness checks on the out-of-sample estimation of *relative leverage*. First, notice that the properties of the fixed effect estimators are asymptotic.[23] Thus, while consistent, fixed effects estimates may be biased in finite samples. This may be a problem in our unbalanced panel, because the length of the time-series for many firms is quite short.

For the out-of-sample analysis, we consider the years 1987-2009 as the period for which we want to explain equity returns. For each year t between 1987 and 2009 we estimate Equation 3.4 on a rolling basis, i.e. using data from 1965 to t. Target leverage for the i-th stock is estimated in year t only if firm i's fixed effect estimate is stable enough. Since fixed effects estimates are consistent, once stability is reached in a given year, it will also apply in the following years. More specifically, stability of the estimate for firm i in year t is achieved if and only if there exists a period t^* between 1989 and t such that the fixed effect estimate F_{i,t^*} can be computed[24] and satisfies

$$F_{i,t^*} - F_{i,t^*-1} < 0.05$$

and

$$F_{i,t^*} - F_{i,t^*-2} < 0.05.$$

Once the stability criterion is satisfied for firm i, the fixed effect estimate F_{i,t^*} at t^* is used to compute target leverage $MDR^*_{i,t}$ for every $t \geq t^*$ for which Equation 3.4 produces an estimate of target leverage for firm i.[25] For computational convenience, in the rolling estimation, we use the LSDV estimator instead of the Blundell and Bond [1998] system GMM. Once we have obtained target leverage estimates for the period 1989-2009 and computed *relative leverage* as in Equation 3.5, we match annual series to monthly returns according to the procedure described in Appendix A. This yields monthly time series from July 1990 to December 2009.

Figure 3.4 depicts average monthly returns for the period 1990-2009, sorting firms according to the out-of-sample approach described above. As in Figure 3.3, a clear *relative leverage* premium emerges. Consistent with our previous findings, the premium is fairly symmetrical around the target.

(Insert Figure 3.4 here)

[23]Independently of the estimation method that one employs (LSDV or Blundell and Bond [1998] system GMM), firm fixed effects are determined such that the sample mean of each *individual*'s time series of residuals equals zero (Baltagi [2008]).

[24]We require that there are no gaps in the firm i's time series of $F_{i,s}$, for $t^* < s < t$.

[25]Occasionally, due to missing data for one or more regressors in $X_{i,t-1}$, it is not possible to estimate $MDR^*_{i,t}$ for every $t \geq t^*$. When this occurs, we do not include the firm in the analysis, and check again the stability criterion.

In Table 3.18 of Appendix D we repeat the procedure described above with a shorter burn-in period from 1965 to 1972. This leads to monthly time series from July 1975 to December 2009. Our results remain qualitatively unchanged.

We employ the 0.05 threshold in the out-of-sample estimates of Panels B of Tables 3.4, 3.5, and 3.6. In Table 3.17 of Appendix D we consider different convergence criteria, with tighter and looser bounds, and based on estimates for more than two consecutive years. This does not qualitatively affect our results.

Appendix C: GMM-based asset pricing tests

In Section 3.4 we test the CAPM, the FF3 model, and a model with RMRF, HML, and OMU both in their expected return-beta and in their stochastic discount factor representation. Both tests are based on Generalized Method of Moments (GMM) estimation. For an exhaustive treatment of these types of tests, we refer the reader to Hansen [1982], and Cochrane [1996].

For each set of N test assets we contemplate in Tables 3.8 and 3.9, the return-beta equations for all N assets in vector form can be written as

$$r_t - r_t^f = \alpha + \beta' f_t + \epsilon_t$$

where $f_t = [RMRF_t]'$ for the CAPM, $f_t = [RMRF_t\ SMB_t\ HML_t]'$ for the FF3 model, and $f_t = [RMRF_t\ HML_t\ OMU_t]'$ for the model with RMRF, HML, and OMU. The moment conditions that we consider to map the regression equations above into the GMM framework are

$$M_T(\alpha, \beta) \equiv \begin{bmatrix} E[\epsilon_t] \\ E[\epsilon_t f_t] \end{bmatrix}$$

The GMM objective is to estimate α and β to minimize the following quadratic form of the moment conditions

$$J_T(\alpha, \beta) \equiv M_T(\alpha, \beta)' W M_T(\alpha, \beta)$$

where W is a weighting matrix. As Ferson and Foerster [1994] suggest, we use iterative efficient GMM estimation to improve finite-sample performance. Specifically, we initially set $W = W_0$, and we estimate the model by GMM to get a consistent estimate of the covariance matrix of moments S. We then impose $W = S^{-1}$ to obtain efficient next-stage estimates. We iterate the last step until convergence is achieved. In our analysis, we choose $W_0 = (\tilde{f}_t{}' \tilde{f}_t) \otimes I_N$, where $\tilde{f}_t{}' = [1\ f_t']$ and I_N is an identity matrix of size N. At each step, we estimate a heteroskedasticity and autocorrelation consistent matrix S using a Newey-West kernel with 4 lags.

In this framework, the asymptotic variance matrix of GMM estimates $[\hat{\alpha}\ \hat{\beta}]$ is

$$Var \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \frac{1}{T} D^{-1} S D^{-1\prime}$$

where T is the number of monthly observations in our sample, and D is defined as

$$D \equiv \frac{\partial M_T}{\partial [\alpha'\ \beta']} = - \begin{bmatrix} 1 & E[f_t] \\ E[f_t] & E[f_t^2] \end{bmatrix} \otimes I_N$$

Therefore, a Wald-type test for null hypothesis that the N pricing errors are jointly zero can be easily implemented. In fact:

$$\hat{\alpha}' Var(\hat{\alpha})^{-1}\hat{\alpha} \xrightarrow{d} \chi^2_N$$

Since the GMM system is just identified (the number of parameters to estimate and the number of moment conditions are both equal to $N(K+1)$, where K is the number of factors in each model), the point estimates $\hat{\alpha}$ and $\hat{\beta}$ coincide with OLS estimates of time-series regressions for the individual assets. However, standard errors differ in that we do not assume homoscedasticity and independent errors over time in the construction of the S matrix. Under these two assumptions, the Wald-type test above boils down to the asymptotic counterpart of the celebrated GRS test.

In order to estimate the three models in stochastic discount factor form, we apply the iterative GMM procedure described above to the following set of moment conditions

$$M_T(b) \equiv E[m_{t+1}(1+r_t)-1]$$

The stochastic discount factor m_{t+1} is parametrized as $m_{t+1} = b_0 + b_1 RMRF_{t+1}$ for the CAPM, $m_{t+1} = b_0 + b_1 RMRF_{t+1} + b_2 SMB_{t+1} + b_3 HML_{t+1}$ for the FF3 model, and $m_{t+1} = b_0 + b_1 RMRF_{t+1} + b_2 HML_{t+1} + b_3 OMU_{t+1}$ for the model with RMRF, HML, and OMU. Following Cochrane [1996], we now set the initial weighting matrix $W_0 = I_N$.

In this setting, we can test whether the pricing errors are jointly zero using the J test of over-identifying restrictions. In fact

$$J \equiv T M_T(\hat{b})' \hat{S}^{-1} M_T(\hat{b})$$

converges to a chi-square distribution with $N-K-1$ degrees of freedom. All inverse matrices are computed as generalized inverses.

Appendix D: robustness checks

In this section we report extensive robustness checks on the results reported in the main text. Tables 3.11, 3.12, and 3.13 assess the sub-period robustness of our results from FMB regressions. They are respectively replicas of Tables 3.4, 3.5, and 3.6 on the 1965-1979, 1980-1994, and 1995-2009 sub-samples. They show that the relationship between *relative leverage* and expected returns is strong and stable across sub-periods, while the residual significance of target leverage is mostly driven by the 1980-1994 sub-period evidence.

Table 3.14 replicates the key results in Tables 3.4, 3.5, and 3.6, where target leverage is estimated with only a firm-fixed effect as a determinant. Our key results are qualitatively unchanged. This stresses the importance of allowing for firm-specific unobservable heterogeneity in the estimation of target leverage, as Lemmon et al. [2008] point out.

Tables 3.15, and 3.16 provide additional evidence supporting the asset pricing tests in Section 3.4. Table 3.15 reports results of autocorrelation and heteroskedasticity consistent Wald-type tests for the test assets we consider in Tables 3.8, and 3.9. These tests are detailed in Appendix C. Table 3.16 extends the results in Table 3.8 to the four-factor model of Carhart [1997]. Overall, these two tables provide evidence that the *relative leverage premium* is not captured by existing factor models, even if we consider momentum as a potential risk source, and that our results are not driven by specific statistical testing procedures.

Tables 3.17, and 3.18 provide evidence that our out-of-sample results are robust to different choices of convergence criterion and burn-in period, as detailed in Appendix B.

Table 3.11. *Relative Leverage*, Size, Book-to-Market, Momentum: Subperiod Evidence.
This table is a replica of Table 3.4 for specific sub-periods. We estimate monthly cross-sectional regression of stock returns on size, book-to-market ratio, *relative leverage* and momentum. Panel A refers to the period between July 1965 and December 1979, Panel B to the period between January 1980 and December 1994, and Panel C to the period between January 1995 and December 2009. The independent variables are matched to monthly returns in line with Fama and French (1992). We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	ret	ret	ret	ret	ret	ret	ret	ret
Panel A: 1965-1979 (174 months)								
Log(size)	-0.242			-0.213	-0.189		-0.177	-0.158
	(-2.578)			(-2.245)	(-2.067)		(-1.914)	(-1.847)
Log(bm)		0.383		0.136		0.279	0.0709	0.0743
		(2.540)		(0.968)		(1.930)	(0.532)	(0.551)
Rel_Lev			4.252		3.689	3.906	3.410	3.188
			(13.02)		(15.14)	(15.33)	(15.95)	(15.83)
Momentum								0.005
								(1.688)
Constant	0.690	1.280	1.327	0.795	0.894	1.364	0.963	0.905
	(1.707)	(2.442)	(2.280)	(2.206)	(2.090)	(2.496)	(2.498)	(2.400)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret	(7) ret	(8) ret
Panel B: 1980-1994 (180 months)								
Log(size)	-0.199			-0.162	-0.172		-0.150	-0.156
	(-3.226)			(-2.645)	(-2.849)		(-2.481)	(-2.606)
Log(bm)		0.391		0.277		0.250	0.151	0.140
		(3.415)		(2.368)		(2.221)	(1.295)	(1.226)
Rel_Lev			3.403		3.101	3.180	3.043	2.916
			(10.47)		(10.96)	(12.62)	(13.04)	(12.79)
Momentum								0.005
								(2.552)
Constant	1.142	1.783	1.609	1.355	1.199	1.753	1.354	1.322
	(2.778)	(4.025)	(3.480)	(3.570)	(2.894)	(3.944)	(3.544)	(3.529)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret	(7) ret	(8) ret
				Panel C: 1995-2009 (180 months)				
Log(size)	-0.377			-0.335	-0.366		-0.336	-0.323
	(-4.787)			(-3.724)	(-4.662)		(-3.730)	(-3.687)
Log(bm)		0.376		0.105		0.335	0.0650	0.0930
		(2.966)		(0.672)		(2.745)	(0.424)	(0.624)
Rel_Lev			4.361		4.176	4.038	4.071	4.451
			(9.975)		(9.757)	(10.38)	(10.63)	(15.01)
Momentum								-0.005
								(-1.343)
Constant	1.296	1.955	1.937	1.350	1.475	2.085	1.482	1.160
	(2.513)	(3.521)	(3.440)	(2.708)	(2.854)	(3.728)	(2.956)	(2.560)

t-statistics in parentheses

Table 3.12. *Relative Leverage*, Over-Leverage, Under-Leverage and Distance: Subperiod Evidence.

This table is a replica of Table 3.5 for different sub-periods. We estimate monthly cross-sectional regression of stock returns on size, book-to-market ratio, *relative leverage*, distance, over-leverage and under-leverage. Panel A refers to the period between July 1965 and December 1979, Panel B to the period between January 1980 and December 1994, and Panel C to the period between January 1995 and December 2009.The independent variables are matched to monthly returns in line with Fama and French (1992). We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2. Distance, Overlev and Underlev are defined as:

$$Distance_{i,t} = abs\{Rel\_Lev_{i,t}, 0\}$$
$$Overlev_{i,t} = max\{Rel\_Lev_{i,t}, 0\}$$
$$Underlev_{i,t} = -min\{Rel\_Lev_{i,t}, 0\}$$

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ret	ret	ret	ret	ret	ret
Panel A: 1965-1979 (174 months)						
Log(size)	-0.177	-0.173	-0.191	-0.173	-0.173	-0.173
	(-1.914)	(-1.896)	(-2.044)	(-1.896)	(-1.896)	(-1.896)
Log(bm)	0.0709	0.0656	0.0815	0.0656	0.0656	0.0656
	(0.532)	(0.487)	(0.585)	(0.487)	(0.487)	(0.487)
Overlev		4.377		1.555		
		(5.203)		(1.384)		
Underlev		-2.822			1.555	
		(-5.636)			(1.384)	
Rel_Lev	3.410			2.822	4.377	3.599
	(15.95)			(5.636)	(5.203)	(8.905)
Distance			0.662			0.778
			(1.344)			(1.384)
Constant	0.963	0.895	0.847	0.895	0.895	0.895
	(2.498)	(2.332)	(2.202)	(2.332)	(2.332)	(2.332)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret
Panel B: 1980-1994 (180 months)						
Log(size)	-0.150	-0.156	-0.168	-0.156	-0.156	-0.156
	(-2.481)	(-2.629)	(-2.794)	(-2.629)	(-2.629)	(-2.629)
Log(bm)	0.151	0.155	0.261	0.155	0.155	0.155
	(1.295)	(1.321)	(2.228)	(1.321)	(1.321)	(1.321)
Overlev		2.426		-1.037		
		(4.253)		(-1.306)		
Underlev		-3.463			-1.037	
		(-9.161)			(-1.306)	
Rel_Lev	3.043			3.463	2.426	2.944
	(13.04)			(9.161)	(4.253)	(10.64)
Distance			-0.841			-0.518
			(-2.061)			(-1.306)
Constant	1.354	1.400	1.443	1.400	1.400	1.400
	(3.544)	(3.744)	(3.844)	(3.744)	(3.744)	(3.744)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret
Panel C: 1995-2009 (180 months)						
Log(size)	-0.336	-0.335	-0.342	-0.335	-0.335	-0.335
	(-3.730)	(-3.733)	(-3.814)	(-3.733)	(-3.733)	(-3.733)
Log(bm)	0.0650	0.0672	0.115	0.0672	0.0672	0.0672
	(0.424)	(0.442)	(0.737)	(0.442)	(0.442)	(0.442)
Overlev		3.715		-0.331		
		(4.137)		(-0.251)		
Underlev		-4.046			-0.331	
		(-5.834)			(-0.251)	
Rel_Lev	4.071			4.046	3.715	3.881
	(10.63)			(5.834)	(4.137)	(8.466)
Distance			-1.901			-0.165
			(-2.969)			(-0.251)
Constant	1.482	1.501	1.606	1.501	1.501	1.501
	(2.956)	(3.239)	(3.456)	(3.239)	(3.239)	(3.239)

t-statistics in parentheses

Table 3.13. *Relative Leverage*, Market Leverage, Book Leverage: Subperiod Evidence.

This table is a replica of Table 3.6 for different sub-periods. We estimate monthly cross-sectional regression of stock returns on size, book-to-market ratio, *relative leverage*, market leverage, and book leverage. Panel A refers to the period between July 1965 and December 1979, Panel B to the period between January 1980 and December 1994, and Panel C to the period between January 1995 and December 2009. The independent variables are matched to monthly returns in line with Fama and French (1992). We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2.

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ret	ret	ret	ret	ret	ret
Panel A: 1965-1979 (174 months)						
Log(size)		-0.201	-0.181		-0.209	-0.190
		(-2.189)	(-1.986)		(-2.264)	(-2.039)
Log(bm)		0.0587	0.150		0.115	0.161
		(0.383)	(0.987)		(0.828)	(1.148)
MDR	1.378	0.576	-0.847			
	(2.768)	(1.482)	(-1.875)			
Rel_Lev			3.791			
			(12.68)			
BDR				0.722	0.312	-0.364
				(1.808)	(0.938)	(-0.937)
Rel_Lev(book)						1.662
						(7.448)
Constant	0.861	0.660	1.246	1.059	0.723	1.041
	(1.812)	(2.059)	(3.562)	(2.322)	(2.275)	(3.302)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret
Panel B: 1980-1994 (180 months)						
Log(size)		-0.163	-0.158		-0.164	-0.145
		(-2.698)	(-2.642)		(-2.705)	(-2.388)
Log(bm)		0.260	0.273		0.267	0.302
		(2.210)	(2.342)		(2.295)	(2.608)
MDR	0.816	0.0656	-1.330			
	(2.448)	(0.265)	(-4.936)			
Rel_Lev			3.853			
			(15.02)			
BDR				0.203	-0.0287	-0.596
				(1.038)	(-0.129)	(-2.482)
Rel_Lev(book)						1.635
						(9.295)
Constant	1.385	1.330	1.740	1.548	1.358	1.629
	(3.103)	(3.751)	(4.847)	(3.629)	(3.784)	(4.509)

t-statistics in parentheses

VARIABLES	(1) ret	(2) ret	(3) ret	(4) ret	(5) ret	(6) ret
Panel C: 1995-2009 (180 months)						
Log(size)		-0.334	-0.329		-0.335	-0.329
		(-3.751)	(-3.707)		(-3.833)	(-3.751)
Log(bm)		0.0851	0.133		0.106	0.128
		(0.724)	(1.132)		(0.707)	(0.843)
MDR	0.780	0.214	-0.818			
	(1.166)	(0.369)	(-1.341)			
Rel_Lev			4.327			
			(10.90)			
BDR				0.0831	0.161	-0.197
				(0.265)	(0.334)	(-0.422)
Rel_Lev(book)						1.333
						(5.300)
Constant	1.594	1.299	1.722	1.744	1.317	1.474
	(2.742)	(2.667)	(3.514)	(2.997)	(2.631)	(3.037)

t-statistics in parentheses

Table 3.14. Estimation of Target Leverage Using Only Fixed Effects.
Each month between July 1965 to December 2009, we estimate cross-sectional regressions of stock returns on size, book-to-market ratio, *relative leverage*, market debt ratio, over-leverage and under-leverage. *Relative leverage* is estimated using Blundell and Bond [1998] system GMM, as described in Section 3.2, where $X_{i,t}$ contains only a firm fixed-effect. The independent variables are matched to monthly returns in line with Fama and French (1992). We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2. Overlev and Underlev are defined as:

$$Overlev_{i,t} = max\{Rel\_Lev_{i,t}, 0\}$$
$$Underlev_{i,t} = -min\{Rel\_Lev_{i,t}, 0\}$$

	(1)	(2)	(3)
VARIABLES	ret	ret	ret
Sample Period: 1965-2009 (534 months)			
Log(size)	-0.232	-0.234	-0.234
	(-5.229)	(-5.375)	(-5.350)
Log(bm)	0.077	0.163	0.081
	(1.113)	(2.468)	(1.177)
MDR		-0.974	
		(-4.292)	
Rel_Lev	3.200	3.638	
	(21.15)	(21.28)	
Overlev			2.835
			(8.342)
Underlev			-3.304
			(-14.09)
Constant	1.248	1.530	1.269
	(5.561)	(7.078)	(5.836)

t-statistics in parentheses

Table 3.15. Autocorrelation and Heteroskedasticity Consistent Wald Tests.
At the end of June of each year between 1965 and 2009, stocks are allocated to portfolios by independently ranking them into groups on the basis of their values of size, book-to-market equity, *relative leverage*, and momentum. Individual stocks are re-assigned to equally-weighted portfolios every June on the basis of NYSE breakpoints for the four variables. The first row refers to 27 portfolios ranked by size (three groups), book-to-market equity (three groups), and *relative leverage* (three groups). Rows 2-7 refer to 25 portfolios ranked into five groups by the reported sorting variables. The table shows autocorrelation and heteroskedasticity consistent Wald test statistics for the Fama and French model, the CAPM, and a model including the market factor RMRF, HML, and OMU. These tests are based on Newey-West standard errors with a lag length of 4.

	Wald Test Statistics ($H_0 : a_i = 0$)		
Test Assets	FF Model	CAPM	RMRF+HML+OMU
27 port. (3 S, 3 BM, 3 RL)	525.8	506.7	88.8
25 port. (5 S, 5 BM)	93.6	87.9	67.4
25 port. (5 RL, 5 BM)	456.9	414.7	50.5
25 port. (5 RL, 5 S)	433.5	431.0	44.8
25 port. (5 S, 5 M)	72.8	65.2	39.2
25 port. (5 BM, 5 M)	116.6	120.1	73.9
25 port. (5 RL, 5 M)	468.7	490.9	56.6

Table 3.16. Comparison of Pricing Performance of Factor Models: Carhart Model. At the end of June of each year between 1965 and 2009, stocks are allocated to 27 portfolios by independently ranking them into three groups on the basis of their values of size, book-to-market equity, and *relative leverage*. Individual stocks are re-assigned to equally-weighted portfolios every June on the basis of NYSE breakpoints for the three variables. UL (OL) denotes the portfolio of stocks with *relative leverage* below (above) that of the lowest (highest) tercile of NYSE firms. Low (High) denotes the portfolio of stocks with book-to-market equity below (above) that of the lowest (highest) tercile of NYSE firms. Small (Large) denotes the portfolio of stocks with market capitalization below (above) that of the lowest (highest) tercile of NYSE firms. We report estimated pricing errors from iterated GMM estimation of time-series regressions of the excess returns of the 27 portfolios on a four-factor model including the Fama and French factors and the momentum factor, as in Carhart [1997]. Reported t-statistics are based on Newey-West standard errors with a lag length of 4. We also report average absolute pricing errors, average squared pricing errors, number of failures "#Fail", GRS test statistics, Hansen's test statistics for a J test of over-identifying restriction "J", and its p-value "p(J)". #Fail is defined as the number of pricing errors out of 27 that are significantly different from zero at the 1% level. "J" refers to the model estimated in stochastic discount factor form by GMM.

$$R_{i,t} - r_t^f = a_i + b_i RMRF_t + s_i SMB_t + h_i HML_t + m_i MOM_t + \epsilon_{i,t}$$

		a_i			$t(a_i)$		
RL	Size/BM	Low	2	High	Low	2	High
	Small	0.01	0.19	0.32	0.04	1.74	2.38
UL	2	-0.29	-0.10	-0.16	-2.47	-0.99	-1.23
	Large	-0.09	-0.19	-0.08	-1.17	-1.88	-0.60
	Small	0.70	0.65	0.74	4.36	5.69	6.08
2	2	0.41	0.30	0.32	4.54	2.83	2.77
	Large	0.44	0.20	0.23	5.92	2.36	2.11
	Small	1.45	1.05	1.29	8.58	8.00	9.89
OL	2	0.76	0.68	0.63	6.79	6.08	4.68
	Large	0.68	0.33	0.32	6.81	2.97	2.86

Avg($\|a_i\|$)	Avg(a_i^2)	#Fail	GRS	J	p(J)
0.467	0.349	16	19.95	37.6	0.02

Table 3.17. Out-of-Sample Convergence Criteria.
We estimate *Relative leverage*, under-leverage, and over-leverage out-of-sample following the procedure described in Appendix B. We then estimate cross-sectional regressions of stock returns on size, book-to-market ratio, *relative leverage*, over-leverage, under-leverage, and market leverage. In the regressions in columns 1-3, the fixed effect estimate F_{i,t^*} must satisfy the convergence conditions $F_{i,t^*} - F_{i,t^*-1} < 0.01$, and $F_{i,t^*} - F_{i,t^*-2} < 0.01$ for a firm to be included in the analysis. In the regressions in columns 4-6, the fixed effect estimate F_{i,t^*} must satisfy the convergence conditions $F_{i,t^*} - F_{i,t^*-1} < 0.10$, and $F_{i,t^*} - F_{i,t^*-2} < 0.10$ for a firm to be included in the analysis. In the regressions in columns 7-9, the fixed effect estimate F_{i,t^*} must satisfy the convergence conditions $F_{i,t^*} - F_{i,t^*-1} < 0.05$, $F_{i,t^*} - F_{i,t^*-2} < 0.05$, and $F_{i,t^*} - F_{i,t^*-3} < 0.05$ for a firm to be included in the analysis. The independent variables are matched to monthly returns in line with Fama and French (1992). We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	ret	ret	ret	ret	ret	ret	ret	ret	ret
	Sample Period: 1990-2009 (234 months)								
	Tight Bounds			Loose Bounds			Three-Period Bounds		
Log(size)	-0.227	-0.217	-0.225	-0.239	-0.229	-0.237	-0.219	-0.209	-0.217
	(-3.610)	(-3.554)	(-3.570)	(-3.748)	(-3.700)	(-3.709)	(-3.423)	(-3.391)	(-3.385)
Log(bm)	0.041	0.094	0.038	0.069	0.119	0.067	0.071	0.116	0.069
	(0.367)	(1.027)	(0.349)	(0.615)	(1.291)	(0.607)	(0.625)	(1.231)	(0.616)
MDR		-0.518			-0.540			-0.478	
		(-1.247)			(-1.284)			(-1.117)	
Rel_Lev	2.093	2.406		2.063	2.359		2.042	2.310	
	(7.303)	(7.595)		(8.659)	(9.072)		(7.167)	(7.531)	
Overlev			2.481			2.382			2.337
			(4.249)			(4.341)			(3.872)
Underlev			-1.196			-1.486			-1.432
			(-2.316)			(-3.659)			(-2.844)
Constant	1.285	1.433	1.240	1.320	1.469	1.283	1.356	1.492	1.322
	(3.792)	(4.435)	(3.915)	(3.861)	(4.423)	(3.976)	(3.897)	(4.507)	(4.086)

t-statistics in parentheses

Table 3.18. Out-of-Sample Evidence on the 1975-2009 Testing Period.
We estimate *Relative leverage*, under-leverage, and over-leverage out-of-sample following the procedure described in Appendix B, with a shorter burn-in period is from 1965 to 1972. We then estimate cross-sectional regressions of stock returns on size, book-to-market ratio, *relative leverage*, over-leverage, under-leverage, and market leverage. The independent variables are matched to monthly returns in line with Fama and French (1992). This yields to monthly time series from July 1975 to December 2009. The sample includes a total of 692,745 firm-month observations. We report Fama-MacBeth coefficient estimates and t-statistics based on Newey-West standard errors with a lag length of 2.

	(1)	(2)	(3)
VARIABLES	ret	ret	ret
Sample Period: 1975-2009 (414 months)			
Log(size)	-0.189	-0.184	-0.188
	(-4.268)	(-4.215)	(-4.243)
Log(bm)	0.138	0.183	0.135
	(1.812)	(2.589)	(1.788)
MDR		-0.459	
		(-1.766)	
Rel_Lev	1.863	2.077	
	(10.55)	(10.82)	
Overlev			2.203
			(5.747)
Underlev			-1.283
			(-4.693)
Constant	1.399	1.530	1.353
	(5.437)	(6.162)	(5.508)

t-statistics in parentheses

Chapter 4

Dynamic Corporate Liquidity

4.1 Introduction

When external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks. Making optimal use of liquid funds means transferring them to times and states where they are most valuable. Liquid funds may be valuable because they aid financing of a profitable investment opportunity, or because they help covering cash shortfalls. Anticipations of such future states thus provide a rationale for corporate liquidity management and renders it inherently dynamic. One way to implement liquidity management is using uncontingent instruments, such as holding cash, which transfers liquid funds across all states symmetrically. We will refer to such policies as unconditional liquidity management. Alternative instruments, such as credit lines or derivatives, have a more state-contingent flavor in that corporations may draw on them to transfer funds to specific states only. We will refer to such policies as conditional liquidity management.

In practice, we see firms engaging in many combinations of conditional and unconditional liquidity management policies, yet there is relatively little work attempting to understand the determinants of these choices. In this paper, our objective is to take a step towards filling this gap. We do so by proposing a dynamic model of corporate policies that explicitly allows corporations to transfer liquid funds unconditionally using cash and conditionally by drawing on credit lines. The result is a quantitative theory of optimal liquidity management based on the trade-off between conditionali liquidity subject to collateral constraints and unconditional, unconstrained liquidity. In the model, liquidity needs arise from stochastic investment opportunities and cash shortfalls in the context of high leverage. By solving the model numerically, we provide novel empirical predictions on the cross-sectional and time-series determinants of corporations' liquidity policies. We test these predictions empirically using data on credit lines from CapitalIQ and find strong support for them. The model thus provides a quantitatively and empirically successful framework explaining corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

In the model, firms attempt to take advantage of profitable investment opportunities that arise stochastically. However, due to capital market imperfections, issuing equity entails costs such that firms will find it beneficial to exploit the tax benefits of leverage by issuing debt. However, we assume that debt needs to be collateralized by capital so that all debt is secured. This means that firms' debt capacity is endogenously bounded. In this context, a rationale for liquidity management arises. Firms can transfer liquidity unconditionally across all states by saving, that is, by holding cash. On the other hand, firms can preserve debt capacity in a state-contingent way by drawing on their credit lines as economic conditions dictate. This allows firms to transfer liquidity conditionally to specific states only. We show that the model predicts that firms will exploit conditional and unconditional liquidity management highly differentially both in the cross-section and in the time series. Calibrating the model, we find that such differential use of liquidity management provides a coherent explanation for many stylized facts about firms joint investment, financing and liquidity policies.

Our model rationalizes the empirical evidence that firms simultaneously hold cash and debt, hence corroborating the notion that cash is not negative debt. Within the context of our model, the intuition is simple. While debt and credit lines jointly allow for state-contingency within the limits of debt capacity, holding cash allows to transfer liquidity beyond collateral constraints in case of high financing needs. Such high financing needs most likely arise when firms have many profitable investment opportunities. In this context, the model predicts that small firms and constrained firms (as measured by net worth) hold more cash, all else equal. This is a pattern well documented in the data, indicating that such firms mostly manage liquidity by means of unconditional instruments. On the other hand, large firms and relatively unconstrained firms are predicted to hold less cash and have more undrawn credit, indicating that they rely more conditional policies for liquidity management. We confirm this prediction using data on credit lines from CapitalIQ. Our model also replicates the well documented positive relationship between leverage and size.

An important implication of the model is that empirically we carefully need to distinguish between small firms (as measured by the capital stock) and constrained firms (as measured by net worth). Indeed, these variables are the two relevant (endogenous) state variables in the model. While the two variables are indeed somewhat correlated, we document the need of distinguishing them by means of two way sorts on relevant variables on capital and undrawn credit. These sorts suggest that the main driver of cash holdings is capital, while financial constraints matter less. Since low capital implies valuable growth opportunities (in a model with decreasing returns to scale), this suggests that unconditional liquidity management mostly serves to transfer funds to states with high investment opportunities. On the other hand, the amount of undrawn credit mostly varies with net worth, controlling for capital. Indeed, unconstrained firms have more slack on their credit lines, so that the transfer more funds to valuable states conditionally. Symmetrically, constrained firms mostly exhaust their debt capacity. This is consistent with the notion, developed in Rampini and Viswanathan [2010], and Rampini and Viswanathan [2012a], that constrained firms hedge less, and that if they do, they do it unconditionally using cash. We find strong support for these predictions in the data, suggesting the need to distinguish between size and financial constraints, in contrast to most commonly used financial constraint indicators in empirical work. Moreover, these findings suggest that cross-sectionally we can distinguish firms whose liquidity management is mostly dictated by preserving liquidity for investment opportunities, which we label 'upstate hedging', as opposed to firms preserving liquidity in order to cover cash shortfalls, which we label 'downstate hedging'. In particular, our findings suggest that different instruments serve such liquidity needs better. Figure 1 illustrates our results.

Our analysis points to the importance of examining financing and liquidity policies in the context of investment opportunities, and in particular, investment frictions. While it is well known that financing policies in dynamic investment models exhibit considerable sensitivity to the specification of investment technologies, we reinforce such results in the context of measures of firms' liquidity management. Obstructions to frictionless adjustment of the capital stock in dynamic corporate models are most commonly represented by means of a convex (quadratic) adjustment cost. Our results clearly indicate that fixed costs of adjustment are important to understand liquidity management at the firm level, and cash holdings in particular.

Our paper is at the intersection of several converging lines of literature. In particular we interpret the quantitative literature on dynamic investment and financing (as started by Gomes [2001], Hennessy and Whited [2005], and Hennessy and Whited [2007]) further in light of the recently emerged literature on dynamic risk management and hedging in the context of collateralized debt (Rampini and Viswanathan [2010], Rampini and Viswanathan [2012a]). We build on Rampini and Viswanathan by modeling state-contingent debt subject to collateral constraints. While Rampini and Viswanathan operate in a dynamic optimal contracting framework, we take the form of the contracts as exogenously given and interpret them in the wider context of commonly used frictions in the dynamic financing literature, such as equity issuance costs and investment frictions. Most importantly, we allow firms to use cash as a form of liquidity management. While these leads to a distinct set of empirical predictions, we moreover view our paper as contributing more to the quantitative and empirical literature rather than the one on optimal security design.

Our paper is closely related to the emerging literature on firm policies and cash holdings. A non-exhaustive list includes Nikolov and Whited [2009], Morellec and Nikolov [2009], Hugonnier, Malamud and Morellec (2011), Bolton et al. [2011], Falato et al. [2013], Bolton et al. [2012], and Eisfeldt and Muir [2013]. Our main departure from this line of literature is that we allow for conditional liquidity management that we interpret in the context of credit lines. Our empirical results suggest that this is a relevant model feature. In this context, our paper is most closely related to Bolton, Chen and Wang (2011, 2012), who allow firms to access credit lines and hedge aggregate shocks using derivatives. On the other hand, for tractability, these authors operate within an AK-framework which allows to reduce the number of state variables and to obtain analytical solutions up to an ordinary differential equation. However, our empirical results suggest that distinguishing between the capital stock and net worth as state variables is empirically relevant.

From a computational viewpoint, we introduce linear programming methods into dynamic corporate finance. Accounting for conditional liquidity management by means of state-contingent policies introduces a large number of control variables into our setup which would render our model subject to the curse of dimensionality for standard computational methods. We exploit and extend linear programming methods to circumvent this problem and efficiently solve for the value and policy functions in this class of problems. Linear programming methods, while common in operations research, have been introduced into economics and finance by Trick and Zin (1993, 1997). We extend their methods to setups common in corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin's constraint generation algorithm.

This paper is structured as follows. After presenting some stylized empirical evidence on corporate liquidity management in section 2, we present our model in section 3. We qualitatively examine the determinants of corporations' joint investment, financing and liquidity policies in section 4. After detailing our approach to calibration and identification of our quantitative model in section 5, we present cross-sectional implications in section 6 and time-series implications by means of generalized nonlinear impulse response functions in section 7. Section 8 concludes.

[Insert Figure 4.1 Here]

4.2 Stylized Facts on Corporate Liquidity

In this section we revisit the key empirical facts about firms' joint liquidity management, cash, and capital structure decisions. This evidence can be rationalized and interpreted within our model. In Table 4.1, we present stylized evidence by sorting firms on the empirical counterpart of the two state variables of our model, namely net worth, and capital stock. The sorts in table 4.1 are based on a sample of manufacturing firms from the merged Compustat Annual and Capital IQ datasets, for the period 2001-2011. As in the models of Rampini and Viswanathan [2010], and Rampini and Viswanathan [2012a], net worth determines the amount of resources that are available to the firm in a certain state of the world. Net worth is the sum of realized cash flows from current investment, capital net of depreciation, and cash holdings, net of debt repayments. Intuitively, net worth is the firm's counterpart of household's wealth. Therefore, net worth captures how constrained a company is with respect to funds to allocate to investment, risk management, and distributions. Consistent with the definition in our model, we proxy net worth as the book value of shareholder equity as in Rampini et al. [2012]. Capital stock is measured as the book value of property, plant, and equipment. For each year, firms that are above (below) the 67th (33th) percentile of net worth are classified as relatively unconstrained (constrained). Using the same procedure, firms are classified as large or small on the basis of their capital stock.

An important caveat that limits empirical evidence on corporate risk management is that firm's hedging is unobservable. Existing studies focus on specific industries and types of hedging to draw inference. For example, Tufano [1996] considers hedging of output price in the gold mining industry, while Rampini et al. [2012] investigate hedging of input (fuel) price for airlines. In our model, firms can transfer conditional liquidity by keeping slack on their collateral constraints, that is by saving debt capacity in a state-contingent way. As Rampini and Viswanathan [2012a] discuss, an important implementation of conditional liquidity management relies on loan commitments. This implementation appears to be important in practice, because credit lines play a first-order role for firm's financing. As Sufi [2009] points out, over 80 percent of bank debt held by public firms is in the form of lines of credit. Moreover, Colla et al. [2013] report that the drawn part alone of credit lines accounts for more than 20 percent of the debt structure of US listed firms. On the contrary, the overall quantitative importance of risk management based on derivatives is debatable. For instance, Guay and Kothari [2003] find that even large firms implement little hedging through financial derivatives. In table 4.1, we report the undrawn fraction of credits from lines of credit from the Capital IQ dataset. For the aforementioned reasons, and because of data limitations, we consider this indicator as a proxy of how much firms are slack on their collateral constraints for providing stylized evidence. This choice is consistent with the definition of conditional liquidity in our model. Despite there are reasons other than hedging for which firms do not fully draw from their credit lines, such as limited investment needs, we expect to observe cross-sectional differences in the fraction of undrawn debt capacity across net worth and capital clusters.

Panel A shows one-way sorts by net worth and capital. We report mean cash-to-asset and debt-to-asset ratios, and the average fraction of undrawn credit from credit lines. More constrained and smaller firms have larger cash holdings, consistent with existing empirical studies, such as Denis and Sibilkov [2009], and Almeida et al. [2004]. Consistent with some constrained firms having low cash holdings, as in Denis and Sibilkov [2009], the pattern is more pronounced for the sort on capital. Small firms in our sample have an average cash-to-asset ratio of 23.6 percent, compared to 9.6 percent for large firms. Constrained firms instead hold 16.7% of their asset in cash and cash equivalents, while the ratio falls to 11.7% for unconstrained firms. Regarding leverage, the cross-sectional patterns for sorts on net worth and capital have opposite directions. The sort on capital highlights the well-known positive relationship between leverage and size, that several studies document. Firms with low net worth appear to have more debt than those with high net worth, namely 35.1% versus 23.1% of total assets. Finally, relatively unconstrained firms appear to have more undrawn credit, in line with the result in Rampini et al. [2012] that firms with high net worth hedge more (0.922 versus 0.716). Large firms also appear to keep more slack on their credit lines, despite the pattern is not as clear as for the sort on net worth.

As Rampini et al. [2012] discuss, patterns that relate the corporate policy to net worth are largely unexplored. In our framework, net worth measures how constrained is a firm with respect to the amount of available resources. Other proxies of financial constraints used in the empirical literature capture different dimensions. For example, bond ratings proxy for distance to default. Remarkably, size, typically measured as the book value of total assets, is one of those proxies. In our model, net worth and capital are two different state variables. Therefore, in panel B, we report two-way sorts to revisit and provide new insights about the key stylized facts on debt, cash, and risk management with respect to these two variables. Distinguishing between net worth and capital allows to uncover stylized evidence that can be useful to understand firms' conditional and unconditional liquidity and hedging policies.

Concerning cash holdings, our two-way sorts show that capital is the main variable that influences cash. Small firms hold more cash than large firms for each cluster of new worth. The "Cash holdings" panel shows that the cash-to-asset ratio of small firms is around three times higher than that of large firms. Remarkably, after controlling for capital, unconstrained firms appear to hold more cash than constrained firms. This evidence is consistent with the finding in Denis and Sibilkov [2009] that some constrained firms have low cash holdings, despite small firms hold more cash than large firms. The "Leverage" panel highlights that large constrained firms have very high debt ratios (69.6%

of total assets), much higher than large firms with high net worth (25.1% of total assets). A similar pattern, but less strong, can be observed for less constrained firms, which are likely to have more internal resources (38.1% versus 8.9% for the second cluster, and 25.1% versus 10.5% for unconstrained firms). Finally, the joint effect of net worth and capital on undrawn credit suggest that unconstrained firms are more slack on their credit lines, while capital does not appear to play a very important role. This pattern is consistent with the evidence in Rampini et al. [2012], and emphasizes the importance to distinguish between net worth and size per se.

Finally, as Strebulaev and Whited [2012] point out, an interesting piece of evidence, which existing dynamic models of investment and financing are generally unable to rationalize, is that firms simultaneously hold cash and debt.[1]

[Insert Table 4.1 Here]

In summary, the key stylized facts on corporate liquidity, financing, and hedging can be summarized as follows:

- Firms with low capital and high net worth have higher cash holdings;
- Firms with high capital and low net worth have higher leverage;
- Firms with high net worth are more slack on their lines of credit;
- Firms simultaneously hold cash and debt.

[1] An exception is Gamba and Triantis [2008].

4.3 The Model

This section provides a dynamic neoclassical model of investment, financing, and corporate liquidity. Managers decide at each period in a infinite-horizon environment. This ensures that they take into account the expected consequences of today's decisions for the feasibility of future decisions. They jointly decide over (i) investment in real capital, (ii) debt and equity issues, (iii) cash holdings, and (iv) state-contingent hedging in order to maximize shareholders' wealth. The feasible set of managers' decisions is limited by the presence of real and financial frictions. As in Rampini and Viswanathan [2010], dynamic debt financing is subject to collateral constraints that limit firms' debt capacity. Collateral constraints reflect limited enforcement problems that prevent creditors from accurately assessing the firm's ability to repay debt. State-contingent hedging can hence be interpreted as conserving debt capacity to finance future investments, in presence of uncertainty and limited debt capacity. State-contingent liquidity management can be implemented, for example, by loan commitments, or by purchasing traded securities to hedge shocks which can affect firms' cash flows and investment opportunities. On the real side, adjusting the real capital stock entails both fixed and smooth costs, as in Cooper and Haltiwanger [2006]. In addition, following the existing literature, firms face costly equity issues, and costs of maintaining cash balances.

4.3.1 Technology and Investment

We consider the problem of a value-maximizing firm in a perfectly competitive environment. Time is discrete. The operating profit for firm i in period t depends upon the capital stock $k_{i,t}$ and a shock $z_{i,t}$, as described by the expression

$$\Pi(k_{i,t}, z_{i,t}) = (1-\tau) z_{i,t} k_{i,t}^{\alpha} - f \tag{4.1}$$

The production function exhibits decreasing returns to scale with $0 < \alpha < 1$. As in Gomes [2001], we assume there is a per-period fixed production cost $f \geq 0$. $\tau \geq 0$ is the corporate tax rate. The variable $z_{i,t}$ reflects shocks to demand, input prices, or productivity. $z_{i,t}$ is assumed to be lognormal and to obey the Markovian law of motion

$$\log(z_{i,t+1}) = \mu_z(1-\rho_z) + \rho_z \log(z_{i,t}) + \sigma_z \epsilon_{i,t+1} \tag{4.2}$$

where $\epsilon_{i,t+1}$ is a truncated standard normally distributed random variable. The parametrization in equation (4.2) ensures that the transition probability has the Feller property. In addition, we require that $z_{i,t}$ lies in a close and bounded (therefore compact) set by imposing large bounds on the values of $\epsilon_{i,t+1}$. $k_{i,t}$ falls into the compact set $[0, \bar{K}]$ without loss of generality. Following Gomes [2001], $\bar{K}$ can be defined as

$$\Pi(\bar{K}, \bar{Z}) = \delta \bar{K} \tag{4.3}$$

where $\bar{Z}$ in the upper bound for $z_{i,t}$, and δ is the depreciation rate of capital. Hence, a capital stock larger than $\bar{K}$ cannot be observed, because not economically profitable. The compactness of the state space for $k_{i,t}$ and $z_{i,t}$, and the continuity of $\Pi(k_{i,t}, z_{i,t})$, ensure that $\Pi(k_{i,t}, z_{i,t})$ is bounded. This is a necessary condition for the existence of a solution for the firm's problem. At the beginning of each period the firm is allowed to scale its operations by choosing next period capital stock $k_{i,t+1}$. This is accomplished through investment $i_{i,t}$, which is defined by the standard capital accumulation rule

$$k_{i,t+1} = k_{i,t}(1 - \delta) + i_{i,t} \tag{4.4}$$

Investment is subject to capital adjustment costs. Following Cooper and Haltiwanger [2006], we include both fixed and convex adjustment cost components. We parametrize capital adjustment costs with the following functional form:

$$\Psi(k_{i,t+1}, k_{i,t}) \equiv \left(\psi_0^+ k_{i,t} + \frac{1}{2}\psi^+ \left(\frac{i_{i,t}}{k_{i,t}} \right)^2 k_{i,t} \right) \mathbf{1}_{\{k_{i,t+1} > (1-\delta) k_{i,t}\}} + \tag{4.5}$$

$$\left(\psi_0^- k_{i,t} + \frac{1}{2}\psi^- \left(\frac{i_{i,t}}{k_{i,t}} \right)^2 k_{i,t} \right) \mathbf{1}_{\{k_{i,t+1} < (1-\delta) k_{i,t}\}}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function, and the parameters ψ_0^+ and ψ_0^- govern fixed-adjustment costs of investing and disinvesting respectively. Non-convex costs of adjustment are typically intended to capture indivisibilities in capital, increasing returns to the installation of new capital, and increasing returns to retraining and restructuring of production activity. ψ^+ and ψ^- instead drive the convex component of adjustment costs. We consider asymmetric adjustment costs because, as in Zhang [2005], disinvesting is typically more costly than increasing capital. Both convex and non-convex costs are proportional to the initial capital stock $k_{i,t}$ to eliminate any size effect.

4.3.2 Financing and Liquidity Management

Investment and distributions to shareholders can be financed with three potential sources: internally generated cash flows, riskfree debt (net of repayments), and external equity. In addition, firms have the option to hoard cash for future investments. As in Rampini and Viswanathan [2010], we model one-period state-contingent debt. Formally, $(1 + r)b_{i,t+1}(z(i, t + 1))$ represents the face value to be repaid at time $t + 1$ in the state of the world $s(t + 1)$ corresponding to the realization of the shock $z(i, t + 1)$, where r is the one-period rate of return.[2] In other words, the firm is borrowing from deep-pocket lenders who are willing to lend in all states and dates at the rate of return r. To simplify notation, we introduce the shorthand $b_i(s(t+1))$ for the decision variables $b_{i,t+1}(z(i,t+1))$. The value of new debt issues at time t in state $s(t)$ is

$$E_t[b_i(s(t+1))] - (1 + r(1-\tau))b_i(s(t)) \tag{4.6}$$

where the operator $E_t[\cdot]$ denotes the expectation under the manager's probability measure conditional to her information set at time t. In equation (4.6), the term $E_t[b_i(s(t+1))]$ represents the observed debt stock on the firm balance sheet in period t, which is determined by risk-neutral security pricing in the capital market.[3] $1 + r(1-\tau)$ is the effective interest rate paid by the firm, after accounting for the tax shield of debt. Firms are subject to collateral constraints, that impose an upper bound on the amount of one-period state-contingent debt that a firm can issue. Assuming that future cash flows are not pledgeable, collateral constraints take the form:

$$(1 + r(1-\tau))b_i(s(t+1)) \leq \theta(1-\delta)k_{i,t+1} \tag{4.7}$$

Up to a fraction θ of the resale value of the firm's tangible capital can be used as collateral for state-contingent debt at time $t+1$ in state $s(t+1)$. Rampini and Viswanathan [2012a] prove that collateral constraints of this form are equivalent to limited enforcement constraints. Intuitively, in a dynamic agency problem, the enterpreneur can abscond with a part of tangible capital. The lender is mindful of this possibility, and she cannot precisely gauge the firm ability to support debt. Therefore, he imposes participation and enforcement constraints that limit the share of capital she is willing to finance. We characterize risk management and the conditional corporate liquidity policy by defining

[2]Because our focus in not on endogenous costs of distress, as in Hennessy and Whited [2005] we make the assumption of riskfree debt in the interest of tractability. Given the high number of decision variables and the presence of occasionally non-binding constraints and non-convex costs, solving the model is computationally intensive. The introduction of endogenous default costs would disproportionately increase the computational burden.

[3]To see this, one can apply the basic asset pricing formula to the state-contingent claim with payoffs $(1+r)b_i(s(t+1))$ at time $t+1$. Today's market valuation of the debt stock under the measure of deep-pocket investors is therefore

$$E_t[M(s(t+1))(1+r)b_i(s(t+1))]$$

where $M(s(t+1))$ is the stochastic discount factor. Under risk neutrality, $M(s(t+1)) = \frac{1}{1+r}$. As a consequence:

$$E_t\left[\frac{1}{1+r}(1+r)b_i(s(t+1))\right] = E_t[b_i(s(t+1))]$$

conditional hedging $h_i^C(s(t+1))$ as the slacks on the state-contingent collateral contraints:

$$h_i^C(s(t+1)) \equiv \theta(1-\delta)k_{i,t+1} - (1+r(1-\tau))b_i(s(t+1)) \tag{4.8}$$

The higher $h_i^C(s(t+1))$, the larger the amount of debt capacity the firm is preserving for possible investment opportunities that may arise conditionally on the realization of the state $s(t+1)$. This means that firms can *conditionally* manage its liquidity, that is they can preserve their ability to raise debt and support investment in states in which their cash flows are low, and they have less internally generated resources. There is a clear-cut tradeoff between *conditional hedging* against future income shortfalls, and available funds for current investment. The amount of raised debt $E_t[b_i(s(t+1))]$ in equation (4.8) is supported by the promised payments in future states. Therefore, the higher $h_i^C(s(t+1))$, the more firms are tranferring resources from today to future states, and the lower $E_t[b_i(s(t+1))]$. As Rampini and Viswanathan [2010] discuss, state-contingent debt contracts can be implemented in practice by arranging loan commitments or purchasing derivative securities. The model with state-contingent debt $b_i(s(t+1))$ is also equivalent to a model in which debt is not state-contingent, but the firm can conditionally transfer liquidity by purchasing Arrow-Debreu securities.[4]

Conditional hedging is not the only way firms can transfer liquid funds. Firms can hoard cash and implement *unconditional hedging*. Hoarding cash is equivalent to unconditionally transferring resources from today to *all* future states, including those in which investment can be financed by internally generated funds. As for *conditional hedging*, there is a tradeoff between current investment and saving resources for the future. However, as we are going to discuss in section 4.4, *conditional hedging* is preferable to *unconditional hedging* because it allows to tranfer resources to the future states where they are needed the most. Nevertheless, the presence of capital adjustment costs as in equation (4.5) makes cash hoarding optimal for smaller firms that would not otherwise be able to invest to an economically profitable scale, even if they exhaust their debt capacity. For this reason, and consistent with empirical evidence, our model predicts that firms can simultaneously hold debt and cash instead of using cash for repaying debt. This mechanism corroborates the intuition in Acharya et al. [2007] that cash is not negative debt. We denote cash holdings in period t as $c_{i,t}$. Firms earn the after-tax riskfree interest rate $r(1-\tau)$ on their cash balances, but also bear costs for holding them. Previous studies motivate the costs of holding cash by agency costs, and different lending and borrowing rates. Following DeAngelo et al. [2011], we model these costs through an "agency parameter" $0 \leq \gamma \leq 1$. We interpret γ as the one-period rate to which cash holdings deteriorate in value. Accordingly, the total hedging for firm i at time $t+1$ in state $s(t+1)$ is the

[4]Technically, recalling that collateral constraints are equivalent to limited enforcement constraints, this interpretation is possible because the market is complete in the set of enforceable payoffs.

amount of resources available from both *conditional hedging* and *unconditional hedging*, that is:

$$h_i^T(s(t+1)) \equiv h_i^C(s(t+1)) + (1 + r(1-\tau) - \gamma)c_{i,t+1} \tag{4.9}$$

Finally, the firm can raise external equity. We assume seasoned equity offers are costly, so that it is never optimal for the firm to simultaneously pay dividends and issue equity. Following Hennessy and Whited [2005], we model equity flotation costs with a fixed and a proportional component. We indicate *net equity payout* at time t as $e_{i,t}$. When $e_{i,t} < 0$ the firm is raising equity, while $e_{i,t} \geq 0$ means that the firm is making distributions to shareholders. Equity issuance costs are given by:

$$(\lambda_0 + \lambda_1 |e_{i,t}|)\mathbf{1}_{\{e_{i,t}<0\}} \tag{4.10}$$

The parameters $\lambda_0 \geq 0$ and $\lambda_1 \geq 0$ drive the fixed and the proportional component, respectively. The indicator function denotes that the firm faces these costs only in the region where the the net payout is negative. Accordingly, distributions to shareholders $d_{i,t}$ are the equity payout net of issuance costs:

$$d_{i,t} = e_{i,t} - (\lambda_0 + \lambda_1 |e_{i,t}|)\mathbf{1}_{\{e_{i,t}<0\}} \tag{4.11}$$

4.3.3 The Firm Problem

Managers determine investment, financing, and risk management to maximize the wealth of shareholders, which is the risk-neutral security price in the capital market. Hence, in period t, they decide over real capital $k_{i,t+1}$, cash $c_{i,t+1}$, and state-contingent debt $b_i(s(t+1))$, for each state $s(t+1)$. As we discuss in section 4.3.1, the choice set for capital is compact. Collateral constraints in equation (4.7) imply that state contingent debt variables are bounded between 0 and $\frac{\theta(1-\delta)k_{i,t+1}}{1+r(1-\tau)}$. To ensure compactness of the feasible set for $c_{i,t+1}$, we impose an arbitrarily high bound $\bar{C}$ on cash holdings. This bound is imposed without loss of generality because of the assumption of costly cash balances. Intuitively, even when the marginal productivity of real capital is low, it is never optimal for the firm to invest in liquid assets and have unbounded savings. Cash can be distributed as dividends right away, and shareholders discount future dividends at a rate r per period, while the rate of return for each unit of cash is $r(1-\tau) - \gamma$. The overall choice set is therefore compact.

Despite the large number of choice variables in the firm problem, the current state can be more efficiently summarized by introducing *realized net worth* as a state variable. Realized net worth at time t in the (realized) state $s(t)$ for firm i is given by:

$$w_{i,t} \equiv \Pi(k_{i,t}, z_{i,t}) + k_{i,t}(1-\delta) - (1 + r(1-\tau))b_i(s(t)) + (1 + r(1-\tau) - \gamma)c_{i,t} + \tau\delta k_{i,t} \tag{4.12}$$

As in Rampini and Viswanathan [2012a], net worth measures the amount of resources that are available to the firm in a certain state. It includes cash flows from current investment, value of capital net of depreciation, and value of cash holdings, all net of due debt payments. Intuitively, net worth is the corporate counterpart of household's wealth (Rampini and Viswanathan [2012b]). Therefore, net worth is a measure of how constrained a firm is in terms of available funds to allocate to investment, risk management, and distributions to shareholders. In our model, the presence of capital adjustment costs implies that the current stock of capital $k_{i,t}$ is also a relevant state variable. In fact, the knowledge of net worth and of the choice variables does not suffice to determine distributions to shareholders $d_{i,t}$ that appear in the objective function, because the adjustment costs $\Psi(k_{i,t+1}, k_{i,t})$ also directly depend on the current stock of capital. The current state is therefore summarized by the vector $(w_{i,t}, k_{i,t}, z_{i,t})$. The set of state variables is compact because $k_{i,t}$ and $z_{i,t}$ are bounded, and from equation (4.12) it is straightforward that net worth lies in a closed and bounded interval $[\underline{W}, \bar{W}]$.

Investment, financing, and liquidity management decisions are intimately related. They should satisfy the following budget identities between sources and uses of funds both at time t, and for each state at time $t+1$:

$$w_{i,t} + E_t[b_i(s(t+1))] = e_{i,t} + k_{i,t+1} + \Psi(k_{i,t+1}, k_{i,t}) + c_{i,t+1} \tag{4.13a}$$

$$\begin{aligned} w_i(s(t+1)) = \Pi(k_{i,t+1}, z_{i,t+1}) + k_{i,t+1}(1-\delta) - (1 + r(1-\tau))b_i(s(t+1)) + \\ + (1 + r(1-\tau) - \gamma)c_{i,t+1} + \tau\delta k_{i,t} \end{aligned} \tag{4.13b}$$

where $w_i(s(t+1))$ denotes net worth at time $t+1$ is state $s(t+1)$.

The firm objective function is to maximize the equity value $V(k_{i,t}, w_{i,t}, z_{i,t})$, that is the discounted value of distributions to shareholders. By the Bellman's principle of optimality, the equity value be computed as the solution to the dynamic programming problem

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max\left\{0, \max_{k_{i,t+1}, c_{i,t+1}, b_i(s(t+1))} \left\{d_{i,t} + \frac{1}{1+r} E_t[V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})]\right\}\right\} \tag{4.14}$$

subject to the constraints in (4.4), (4.5), (4.7), (4.11), and (4.13). In equation (4.14), $V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})$ denotes the continuation value for equity, which depends on the future state $(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})$ and on the values of the choice variables at time t. The first maximum captures instead the possibility of default in current period, in which case the shareholders get nothing. To sum up, the complete firm problem is the following:

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max\left\{0, \max_{k_{i,t+1}, c_{i,t+1}, b_i(s(t+1))} \left\{d_{i,t} + \frac{1}{1+r} E_t[V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})]\right\}\right\} \tag{4.15}$$

$s.t.$

$$w_{i,t} + E_t[b_i(s(t+1))] \geq e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1}) \tag{4.16a}$$

$$\begin{aligned} w_i(s(t+1)) \leq\ & (1-\tau)\Pi(k_{i,t+1}, z_{i,t+1}) + k_{i,t+1}(1-\delta) - (1+r(1-\tau))b_i(s(t+1)) + \\ & + (1+r(1-\tau)-\gamma)c_{i,t+1} + \tau\delta k_{i,t+1} \qquad \forall s(t+1) \end{aligned} \tag{4.16b}$$

$$(1+r(1-\tau))b_i(s(t+1)) \leq \theta(1-\delta)k_{i,t+1} \qquad \forall s(t+1) \tag{4.16c}$$

$$b_i(s(t+1)) \geq 0 \qquad \forall s(t+1) \tag{4.16d}$$

$$c_{i,t+1} \geq 0 \tag{4.16e}$$

4.3.4 Model Solution

Because of the presence of occasionally non-binding collateral constraints, and because costs of equity issues and capital adjustment depend on indicator functions, the model cannot be solved numerically by interior points methods. In principle, the model could be solved on a discrete grid by value function iteration or policy function iteration. The Bellman operator in equation (4.14) is indeed a contraction mapping, in that Blackwell's sufficient conditions hold in this framework. Therefore, the fixed point of the functional equation (4.14) is well-defined. For a standard formal proof in a similar framework, we refer to Hennessy and Whited [2005]. Unfortunately, there is a computational hurdle that prevents the solution of the model with standard techniques. Due to the large number of control variables (capital, cash, and one debt variable for each future state), value function iteration and policy iteration cannot be practically implemented. In particular, the maximization step is critical. Determining for each state the combination of control variables that maximizes the sum of distributions and the continuation value implies to store and maximize over a vector of $nk \times nc \times nb^{nz}$ elements, where nk, nc, nb, and nz are the number of grid points for capital, cash, debt, and the shock. As in Rust [1997], this problem is plagued by a curse of dimensionality, since the amount of computer memory and CPU time required increases exponentially with the number of control variables. As a consequence, even for modest values for nz, such a vector becomes too large even to be stored.

We overcome this difficulty by exploiting the linear programming representation of dynamic programming problems with infinite horizon (Ross [1983]), as in Trick and Zin [1993], and Trick and Zin [1997]. This technique has not been historically widely used. Despite it often allows to achieve significant speed gains over iterative methods, it requires in turn to store huge matrices and arrays that make it impractical for complex enough models. Specifically, we extend the constraint generation algorithm developed by Trick and Zin [1993], and we rely on a *separation oracle*, an auxiliary mixed integer programming problem, to avoid dealing with large vectors at all. As in Trick and Zin [1993], the contrained generation algorithm converges to the fixed point faster than traditional iterative methods. Moreover, the separation oracle allows to efficiently implement the maximization step because of a remarkable feature of our model, namely the relatively small number of state variables in spite of the large number of control variables. With this method, we manage to solve the model in a reasonable time (around ten minutes on our workstation).

4.4 Investment, Financing, and Liquidity Management

4.4.1 Hedging Formulation

Lemma 6 (Hedging formulation)
The constrained optimization problem (4.15) is equivalent to:

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max\left\{0, \max_{k_{i,t+1}, h^U_{i,t+1}, h^C_i(s(t+1))}\left\{e_{i,t} - \Lambda(e_{i,t}) + \frac{1}{1+r}E_t[V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})]\right\}\right\} \tag{4.17}$$

s.t.

$$w_{i,t} \geq e_{i,t} + E_t\left[\frac{h^C_i(s(t+1))}{1+r(1-\tau)}\right] + \frac{h^U_{i,t+1}}{1+r(1-\tau)-\gamma} + Pk_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1}) \tag{4.18a}$$

$$w_i(s(t+1)) \leq (1-\tau)\Pi(k_{i,t+1}, z_{i,t+1}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h^T_i(s(t+1)) \quad \forall s(t+1) \tag{4.18b}$$

$$h^C_i(s(t+1)) \geq 0 \qquad \forall s(t+1) \tag{4.18c}$$

$$h^C_i(s(t+1)) \leq \theta(1-\delta)k_{i,t+1} \qquad \forall s(t+1) \tag{4.18d}$$

$$h^U_{i,t+1} \geq 0 \tag{4.18e}$$

where $P \equiv 1 - \frac{\theta(1-\delta)}{1+r(1-\tau)}$ *is the fraction of each unit of capital paid down by the firm at time* t, $h^C_i(s(t+1)) \equiv \theta(1-\delta)k_{i,t+1} - (1+r(1-\tau))b_i(s(t+1))$ *is conditional hedging for state* $s(t+1)$, $h^U_i(s(t+1)) \equiv h^U_{i,t+1} = (1+r(1-\tau)-\gamma)c_{i,t+1}$ *is unconditional hedging for all states at time* $t+1$, *and* $h^T_i(s(t+1)) \equiv h^C_i(s(t+1)) + h^U_{i,t+1}$ *is total hedging.*

The hedging formulation is particularly instructive because it emphasizes the role of dynamic liquidity management. The problem (4.17) can be equivalently interpreted as a problem where firms pledge all their collateral, and transfer resources (net worth) from t to $t+1$ both conditionally, to specific states, and unconditionally, to all future states. Regarding conditional liquidity, firms decide to purchase $\frac{h_i^C(s(t+1))}{1+r(1-\tau)}$ Arrow-Debreu securities at time t in order to obtain a payoff of $h_i^C(s(t+1))$ is state $s(t+1)$ next period. Constraints (4.18c) and (4.18d) impose bounds on the amount of conditional hedging the firm can implement. The collateral constraint imposes a lower bound, that corresponds to exhausting all debt capacity. Constraint (4.18d) states that the maximum amount of liquid funds that a firm can transfer to state $s(t+1)$ corresponds to its debt capacity, that is to the firm having zero debt due in state $s(t+1)$. Unconditional hedging instead consists of hoarding an amount of cash $\frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma}$, in order to get to obtain a payoff $h_{i,t+1}^U$ in all future states at time $t+1$. The hedging formulation provides a preliminary intuition on the different nature of conditional and unconditional liquidity management. Equations (4.18a) and (4.18b) hint that transferring liquid funds conditionally is more efficient than doing so unconditionally if the firm needs to transfer resources only to some states (for example to bad states). Transferring funds to future states involves subtracting resources available to be distributed to shareholders $e_{i,t}$ and to be paid down to make investment possible $Pk_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$. If, for example, a firms needs to transfer an amount M only to the specific state $s(t+1)$ (for example the lowest state), the amount of resources it needs at time t is $\pi(s(t), s(t+1))\frac{M}{1+r(1-\tau)}$, where $0 \leq \pi(s(t), s(t+1)) < 1$ is the transition probability from state $s(t)$ to state $s(t+1)$. On the contrary, implementing unconditional hedging for the same purpose would require to subtract $\frac{M}{1+r(1-\tau)-\gamma}$. So, why should firms engage in unconditional liquidity management at all? Constraint (4.18d) states that the maximum amount of liquid funds that a firm can transfer conditionally is bounded by its total debt capacity $\theta(1-\delta)k_{i,t+1}$. Therefore, whenever it is optimal for the firm to have total hedging greater than this amount, hoarding cash becomes necessary. As a result, endougenously, cash is not negative debt, and consistent with empirical evidence we can observe firms simultaneously holding cash and debt.[5] As the quantitative analysis in section 4.5 emphasizes, capital adjustment costs $\Psi(k_{i,t}, k_{i,t+1})$ play an important role, both qualitatively and quantitatively. Specifically, they allow to differentiate between firms that are constrained in terms of net worth, and small firms, and rationalize patterns that are observed in the data. Equation (4.18a) points up that different current and future investment needs yield to different needs of tranferring net worth to future states. This creates sharp differences in corporate liquidity policy of large and small firms. Suppose, for example, that adjustment costs are quadratic in the investment-to-capital ratio. With

[5] As DeAngelo et al. [2011] discuss, in frameworks in which firms never optimally hold cash and debt together, it is not necessary to model them using two separate positive control variables. In our model, letting negative debt being cash by relaxing constraint (4.18d) would not only prevent firms from simultanously holding cash and debt, but also assume that state-contingent cash securities exist, which is unrealistic.

decreasing returns to scale, small firms with high investment needs would be better off in spreading investment over multiple periods to avoid incurring disproportionately high adjustment costs. Therefore, they may find optimal to hedge more, by saving debt capacity in a state contingent and possibly by hoarding cash. This creates a dependence between investment and liquidity needs, and, as a consequence, between size and risk management.

4.4.2 Optimal Policy

Proposition 4 (Optimality conditions)
Denote by λ^w, $\frac{\pi(s(t),s(t+1))\lambda^w_{s(t+1)}}{1+r}$, $\frac{\pi(s(t),s(t+1))\underline{\lambda}^C_{s(t+1)}}{1+r}$, $\frac{\pi(s(t),s(t+1))\overline{\lambda}^C_{s(t+1)}}{1+r}$, *and* $\underline{\lambda}^U$ *the multipliers on constraints (4.18a), (4.18b), (4.18c), (4.18d), and (4.18e) respectively, where* $\pi(s(t), s(t+1))$ *is the Markovian transition probability from state* $s(t)$ *to state* $s(t+1)$. *Assume that the equity cost function* $\Lambda(e_{i,t})$ *is differentiable in* $e(i,t)$.[6] *Then, the first order conditions for the hedging formulation (4.17) can be expressed as follows:*

$$\lambda^w = 1 - \frac{\partial\Lambda(e_{i,t})}{\partial e_{i,t}} \tag{4.19a}$$

$$\lambda^w\left(P + \frac{\partial\Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}\right) = \frac{1}{1+r}E_t[\lambda^w_{s(t+1)}V^k(s(t+1)) + \overline{\lambda}^C_{s(t+1)}H^k] \tag{4.19b}$$

$$\lambda^w\frac{1}{1+r(1-\tau)-\gamma} = \frac{1}{1+r}E_t[\lambda^w_{s(t+1)}] + \underline{\lambda}^U \tag{4.19c}$$

$$\frac{1}{1+r(1-\tau)}\lambda^w = [(\underline{\lambda}^C_{s(t+1)} - \overline{\lambda}^C_{s(t+1)}) + \lambda^w_{s(t+1)}]\frac{1}{1+r} \qquad \forall s(t+1) \tag{4.19d}$$

where

$$V^k(s(t+1)) = (1-\tau)\frac{\partial\Pi(k_{i,t+1}, z_{i,t+1})}{\partial k_{i,t+1}} + \tau\delta + (1-\theta)(1-\delta) \qquad \forall s(t+1) \tag{4.20a}$$

$$H^k = \theta(1-\delta) \tag{4.20b}$$

The envelope conditions imply:

$$\frac{\partial V(w_{i,t}, z_{i,t})}{\partial w_{i,t}} = \lambda^w \tag{4.21a}$$

$$\frac{\partial V(w_{i,t+1}, z_{i,t+1})}{\partial w_{i,t+1}} = \lambda^w_{s(t+1)} \qquad \forall s(t+1) \tag{4.21b}$$

[6] In our model, we choose a functional form for equity flotation costs with a fixed and a proportional component, which is non-differentiable for $e(i,t) = 0$ (its derivative at zero exists only in a distributional sense). This assumption is not critical for our qualitative analysis. Alternatively, one can approximate $\Lambda(e_{i,t})$ with $0.5(1+tanh(Ne(i,t)))$, with N large enough, in the neighborhood of zero. A similar argument applies to the adjustment cost function $\Psi(k_{i,t}, k_{i,t+1})$ in case fixed costs are included.

Moreover, the investment Euler equation is:

$$P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}} = E_t[M^w(s(t), s(t+1))V^k(s(t+1))] + E_t[M^h(s(t), s(t+1))H^k] \quad (4.22)$$

where $M^w(s(t), s(t+1)) \equiv \frac{1}{1+r}\frac{\lambda^w_{s(t+1)}}{\lambda^w}$ *and* $M^h(s(t), s(t+1)) \equiv \frac{1}{1+r}\frac{\overline{\lambda}^C_{s(t+1)}}{\lambda^w}$ *are stochastic discount factors. In addition:*

$$M^w(s(t), s(t+1)) = \frac{1}{1+r(1-\tau)} - \frac{1}{1+r}\frac{\underline{\lambda}^C_{s(t+1)} + \overline{\lambda}^C_{s(t+1)}}{\lambda^w} \quad (4.23)$$

The optimality conditions illustrate how investment, financing, liquidity and payout policies are intimately related, and shed light on the qualitative mechanism that drive firm's decisions. Moreover, they allow to understand the rationale for liquidity management, and which future states firms optimally hedge. Since the problem has no closed-form solution, the following analysis relies on the economic interpretation of the Lagrange multipliers as shadow values.

Equation (4.19b) relates the costs and benefits of investing an additional unit of real capital at time $t+1$. The left hand side represent the marginal cost of investing. An additional unit of capital requires that the firm puts P money down and pays capital adjustment costs. The cost of doing so is $(P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}})\lambda^w$. The multiplier λ^w accounts for the shadow loss in firm value of relaxing the resource constraint (4.18a) at time t (resource constraints are always binding). The right hand side is the marginal benefit of an additional unit of investment, discounted back to time t by the shareholders' discount factor $\frac{1}{1+r}$. The benefits correspond to the two terms on the right hand side. First, the expected value of the additional investment $V^k(s(t+1))$ across all future possible states, that consists of marginal changes in profits, of tax benefits, and of the liquidation value of the share of capital not pledged to lenders. Second, the expected increase in debt capacity available for conditional hedging H^k in all states. The multipliers $\lambda^w_{s(t+1)}$ and $\overline{\lambda}^C_{s(t+1)}$ instead account respectively for the additional future net worth (constraint (4.18b)), and for the additional debt capacity (constraint (4.18d)) available to transfer conditional liquidity to state $s(t+1)$ because of the additional unit capital installed (in case this constraint is binding).

$$\overbrace{\lambda^w(P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}})}^{\text{Marginal cost of investment}} = \overbrace{\frac{1}{1+r}E_t[\underbrace{\lambda^w_{s(t+1)}V^k(s(t+1))}_{\text{Net worth}} + \underbrace{\overline{\lambda}^C_{s(t+1)}H^k}_{\text{Debt capacity}}]}^{\text{Marginal benefit of investment}} \quad (4.24)$$

Equation (4.19c) describes the unconditional liquidity policy of the firm. Similar to equation (4.19b), the left-hand side $\lambda^w\frac{1}{1+r(1-\tau)-\gamma}$ is the cost of allocating a unit of current net worth to cash hoarding, in order to tranfer one unit of cash to all future states at $t+1$. The right-hand side is the value of this additional unit of net worth available in all states $\frac{1}{1+r}E_t[\lambda^w_{s(t+1)}]$. In addition, the term $\underline{\lambda}^U$ accounts for the possibility that the constraint on positive cash is binding.[7]

$$\overbrace{\lambda^w \frac{1}{1+r(1-\tau)-\gamma}}^{\text{Marginal cost of unconditional liquidity}} = \overbrace{\underbrace{\frac{1}{1+r}E_t[\lambda^w_{s(t+1)}]}_{\text{Net worth}} + \underbrace{\underline{\lambda}^U}_{\text{Positive cash holdings}}}^{\text{Marginal benefit of unconditional liquidity}} \tag{4.25}$$

Equation (4.19d) describes the conditional liquidity policy of the firm. As for unconditional liquidity management, the marginal cost of allocating one unit of net worth to risk management is $\lambda^w \frac{1}{1+r(1-\tau)}$ (the agency parameter $\gamma > 0$ makes it more costly for unconditional hedging). It is however more interesting to examine the right-hand side, and to compare it to the optimality conditions for unconditional liquidity management in equation (4.19c). As for cash hoarding, the benefits are discounted to time t through the manager's discount factor $\frac{1}{1+r}$. However, the value of additional net worth potentially available for the state $s(t+1)$ is $\lambda^w_{s(t+1)}$. In equation equation (4.19c), the value of the net worth tranferred to state $s(t+1)$ is only $\pi(s(t), s(t+1))\lambda^w_{s(t+1)}$.[8] This supports the statement in section 4.4.1 that conditional liquidity management is preferrable to unconditional liquidity management because with the same amount of net worth at time t it allows to transfer more resources to a *specific* state $s(t+1)$. The term $\underline{\lambda}^C_{s(t+1)} - \overline{\lambda}^C_{s(t+1)}$ instead illustrates why firms may be interested in managing its liquidity both conditionally and unconditionally at the same time. Specifically, since in our model conditional hedging can be implemented only saving debt capacity in a state contingent way, the amount of conditional liquidity is limited by the constraints (4.18c) and (4.18d). The term $\underline{\lambda}^C_{s(t+1)}$ accounts for the presence of occasionally binding state-contingent collateral constraints, that may become active and limit the amount of state-contingent debt that a firm can hold given the amount of pleadgeable capital $k_{i,t+1}$. Simmetrically, the multiplier $\overline{\lambda}^C_{s(t+1)}$ is different from zero in case the firm would like to transfer more resources conditionally, but its amount is limited because the firm has alredy zero debt due in state $s(t+1)$. The limited amount of implementable conditional hedging through liquidity management implies that firms can simultaneously hold cash and debt. To see this, suppose that the firm is interested in hedging a specific state, such as the lowest state $\underline{s}$, as much as pos-

[7]This term is more meaningful in case we interpret the first-order condition on unconditional hedging for a reduction of one unit. In this case, the marginal benefit is the additional amount $\lambda^w \frac{1}{1+r(1-\tau)-\gamma}$ available at time t for investment, distributions, and conditional hedging, and the marginal cost is the sum of the value of one less unit of net worth available in all states, and of the shadow value of being able to reduce further cash if constraint (4.18e) binds.

[8]To better see this, notice that the expectation in equation (4.19c) is $\sum_{s=1}^{S} \pi(s(t), s)\lambda^w_s$, where S is the total number of states.

sible. *Ceteris paribus*, the maximum amount of resources that the firm can tranfer to $\underline{s}$ corresponds to exhausting all debt capacity in all states except $\underline{s}$. This implies that no debt is due in state $\underline{s}$. Moreover, the firm can transfer the net worth raised by the state-contingent debt issues in all states excluding $\underline{s}$, to all future states, including $\underline{s}$, by hoarding cash. As a result, the firm would hold cash and debt together.

$$\overbrace{\frac{1}{1+r(1-\tau)}\lambda^w}^{\text{Marginal cost of conditional hedging}} = \overbrace{[\underbrace{(\underline{\lambda}^C_{s(t+1)} - \overline{\lambda}^C_{s(t+1)})}_{\text{Limited conditional liquidity}} + \underbrace{\lambda^w_{s(t+1)}}_{\text{Net worth}}]\frac{1}{1+r}}^{\text{Marginal benefit of conditional liquidity}} \qquad \forall s(t+1) \tag{4.26}$$

The payout policy instead balances the marginal cost of allocating a unit of net worth to dividend distributions or, viceversa, to issue equity to increment net worth by one unit. In case of equity issues, there is not a one-to-one correspondence between raised equity and increased net worth because of equity flotation costs.

$$\underbrace{\overbrace{\lambda^w}^{\text{Marginal benefit of issuing equity}}}_{\text{Marginal cost of paying dividends}} = \overbrace{\underbrace{1}_{\text{Marginal benefit of paying dividends}} - \frac{\partial \Lambda(e_{i,t})}{\partial e}}^{\text{Marginal cost of issuing equity}} \tag{4.27}$$

The Euler condition (4.22) clarifies the important matter of the firm's rationale for liquidity management, and of which states it is optimal to hedge. The Euler equation can be interpreted as a pricing relationship. The left-hand side can be seen as the valuation of the paid down share $P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}$ per unit of capital. The right-hand hand side shows that this value is supported by two terms. The term $E_t[M^w(s(t), s(t+1))V^k(s(t+1))]$ is the stochastically discounted valuation of the benefits $V^k(s(t+1))$ of investing an additional unit. $M^w(s(t), s(t+1))$ is the firm's stochastic discount factor, and is equal to $\frac{1}{1+r}\frac{\lambda^w_{s(t+1)}}{\lambda^w}$. The concavity properties of the value function imply that the marginal value of a certain level of net worth is higher in bad times, so that the stochastic discount factor puts more weight on bad states through the Lagrange multipliers. Indeed, envelope conditions (4.21a) and (4.21b) show how Langrange multipliers are related to the shape of the value function, so that $\lambda^w_{s(t+1)}$ is decreasing in $w_i(s(t+1))$. In a valuation perspective, since a larger share of P is supported by those states, the firm behaves as if it were risk-averse. This provides incentives to implement liquidity management by preserving net worth for investments and distributions for bad future states, where internally generated cash flows and *future realized* net worth are, other conditions equal, lower. Viceversa, the payoff from investments $V^k(s(t+1))$ suggests that the firm may want to hedge good states as well. If the law of motion of shocks to capital productivity $z_{i,t}$ is persistent enough, the payoff of investing in good (bad) times is higher (lower) because the firm expects a sequence of good (bad) shock realizations. The firm will therefore save resources for good states and boost investment in good times. If this is the case, the marginal value of net worth is not necessarily lower in bad states anymore. An instructive benchmark case is the case with independent productivity shocks. In such a scenario, the expected productivity of capital $\frac{\partial \Pi(k_{i,t+1}, z_{i,t+1})}{\partial k_{i,t+1}}$ is independent of the current state. As a consequence, firms only hedge bad states because of the properties of the discount factor $M^w(s(t), s(t+1))$. In practice, however, the productivity process in quite persistent. Therefore, the matter of whether firms hedge good or bad states (or both), and how much, is a purely quantitative question. Also, it is a quantitative question whether firms hedge at all. As in Rampini and Viswanathan [2012a], firms that are particularly constrained may not hedge, and prefer to allocate their scarce resources to current investment and distributions. The second term on the right-hand side instead H^k reflects that capital is valuable also because it serves as collateral, it increases debt capacity and, as a consequence, the amount of conditional liquidity management implementable in all states. The stochastic discount factor $M^h(s(t), s(t+1))$ depends on the multiplier $\overline{\lambda}^C_{s(t+1)}$. Therefore, the value of increased debt capacity is higher is states where firms hold no debt because conditional liquidity is more valuable.

$$\overbrace{P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}}^{\text{Value of paid-down capital}} = \overbrace{E_t[M^w(s(t), s(t+1))V^k(s(t+1))]}^{\text{Discounted investment profits}} + \overbrace{E_t[M^h(s(t), s(t+1))H^k]}^{\text{Debt capacity}} \tag{4.28}$$

Finally, equation (4.23) explicitly relates the stochastic discount factor $M^w(s(t), s(t+1))$, which appears in the investment Euler equation, to the hedging policy of the firm. The multipliers $\underline{\lambda}^C_{s(t+1)}$, and $\overline{\lambda}^C_{s(t+1)}$ differ from zero respectively when the firm exhausts all its debt capacity in state $s(t+1)$, and when the firm has zero debt in state $s(t+1)$. These multipliers enter the Euler equation because of market incompleteness. Given the stochastic nature of the model, firms anticipate that colleteral and debt positivity constraints may bind in the future, and this affect their investment and liquidity management policy. By transferring liquid funds conditionally, the firm can therefore influence the relative importance of different states for determining the value of paid-down capital. For example, if a company borrows constrained in the low state $\underline{s}$ and saves all its debt capacity for future investment in the high state $\overline{s}$, the stochastic discount factor puts more weight on the high state, namely $\frac{1}{1+r(1-\tau)} + \frac{1}{1+r}\frac{\overline{\lambda}^C_{\overline{s}}}{\lambda^w}$ versus $\frac{1}{1+r(1-\tau)} - \frac{1}{1+r}\frac{\underline{\lambda}^C_{\underline{s}}}{\lambda^w}$.

$$M^w(s(t), s(t+1)) = \underbrace{\frac{1}{1+r(1-\tau)}}_{\text{Unconditional component}} - \underbrace{\frac{1}{1+r}\frac{\overbrace{\underline{\lambda}^C_{s(t+1)}}^{\text{Debt capacity}} + \overbrace{\overline{\lambda}^C_{s(t+1)}}^{\text{Positive debt}}}{\lambda^w}}_{\text{State-contingent component}} \tag{4.29}$$

4.4.3 Numerical Illustration

We provide numerical examples to illustrate the analytical analysis in section 4.4.2, and to better understand the qualitative importance of different types of capital adjustment costs for corporate investment and liquidity policy. In the interest of clarity, in all the examples we solve the model with three possible states and in absence of equity issues, and report the policy for the middle state. The details of the parametrizations are reported in the captions of figures 4.2 to 4.7.

Figure 4.2 refers to the case with no adjustment costs and independent investment opportunities. Specifically, Markovian transition probabilities are uniform (equal to one third for each pair of states), so that the expected capital productivity is the same for every state at time t. Panels A and B depict investment and payout as a function of current net worth. Similar to Rampini and Viswanathan [2012a], there esist a threshold of net worth below which investment is increasing, and dividends are zero. Above the threshold investment is constant and dividends are linear. Panel C shows that the value function is weakly concave in net worth. This is an important property, because the firm's stochastic discount factor in equation (4.28) is equal to $\frac{1}{1+r}\frac{\lambda^w_{s(t+1)}}{\lambda^w}$. As a consequence, the firm behaves as if risk averse with respect to net worth. Such a behavior is clearly visible in panel F. As we pointed out in the previous section, with independent productivity, the firm implements downstate hedging. In this example, it saves all its debt capacity for the low state for almost all levels of net worth. The dashed line (conditional hedging for the low state), and the thin line (available debt capacity) are indeed very close. The amount of hedging decreases for the middle states (solid line), and is equal to zero for the high state (dashed-dotted line). Panel E shows the cash policy of the firm. When hedging needs exceed the available debt capacity, that is the amount of implementable conditional hedging, and the firm is unconstrained enough in terms of net worth, it implements unconditional hedging too. This way, additional resources are transferred to the low state. As a consequence, as panel D depicts, cash is not negative debt, and it is optimal for the firm to simultaneously hold them.

[Insert Figure 4.2 Here]

Figure 4.3 removes the assumption of independent investment opportunities, and introduces some persistence. In particular, the firm has now a probability of one half to stay in the current state, and of one quarter to move to another state. The policy is generally similar to that in figure 4.2, except for conditional liquidity management. The dashed-dotted line in Panel F is no longer equal to zero, meaning that the firm hedges upstate as well. Intuitively, with independent investment opportunities, the firm has no incentive to hedge the state where the marginal value of future net worth is lower. However, as equation (4.28) states, if there is a high probability that periods of high profits are followed by periods of high profits, expected future productivity is higher in good states. Therefore, the firm may rationally save resources for future investments in states where investment opportunities are likely to remain good.

[Insert Figure 4.3 Here]

Figures 4.4 to 4.7 emphasize the importance of capital adjustment costs to disentangle net worth from capital, and rationalize the patterns in table 4.1. We consider, one at a time, the four types of adjustment costs in the general functional form (4.5), namely convex investment costs, fixed investment costs, convex disinvestment costs, and fixed disinvestment costs. This approach allows to see how the firm implements conditional and unconditional liquidity management for investment and disinvestment motives. Moreover, we can assess how the investment, liquidity, and risk management policy differs if we consider either fixed or smooth costs.

Figure 4.4 illustrates investment and liquidity management in presence of smooth investment costs. Panels A to C show how, for some values of the current capital stock, the policy is similar to the case with no adjustment costs. *Conditional on capital*, unconstrained firms transfer more liquidity, both conditionally and unconditionally. However, Panels D to F depict how the level of current capital now influences investment and hedging decisions, *conditional on net worth.* Panel D reports the optimal investment-to-capital ratio as a function of firm's size. Because of decreasing returns to scale in the production function, capital installment is relatively more profitable for small firms, which have higher investment needs. Because adjustment costs are quandratically increasing in the investment-to-capital ratio, smaller firms cannot istantaneously adjust to the desired capital level. Partial adjustment is hence optimal, and small firms transfer net worth for (costly) investment to future states, both conditionally (panel F), and unconditionally (panel E). This behavior results in small firms having more cash. Remarkably, these patterns are qualitatively consistent with the stylized facts we revisit in the two-way sorts of table 4.1.

[Insert Figure 4.4 Here]

Figure 4.5 shows instead the case of liquidity management for investment in presence of fixed capital adjustment costs. As panel D clearly shows, the firm has a standard (S,s) policy as a function of current capital.[9] In the figure, k^* denotes the "frictionless" level of capital in absence of investment adjustment costs, defined as in Caballero et al. [1995], and Caballero and Engel [1999]. Intuitively, the more the firm deviates from the "target" level, the higher the cost it bears. As a consequence, when the disequilibrium $|k_{i,t} - k^*|$ is large, it is optimal to pay the fixed cost and to re-adjust the capital level to k^*. This

[9] For an exhaustive treatment of models with fixed costs we refer to Stokey [2008].

policy determines an inaction region bounded by the low barrier k^D, and by the high barrier k^U. In this region, optimal investment is zero. Panels E and F emphasize how firms transfers conditional and unconditional liquid funds precisely in the inaction region. Intuitively, since they are not currently investing, they transfer some net worth to future states, instead of paying it off as dividends.

[Insert Figure 4.5 Here]

Finally, figures 4.6 and 4.7 analyze the case of costly disinvestment with convex and fixed costs respectively. In these cases, firms implement conditional and unconditional liquidity management to cover future costs of disinvestment. This mechanism is similar to the one in Gamba and Triantis [2008], where firms hold cash and debt together because of the presence of transaction costs of issuing debt. Panel F of figure 4.6 shows how the firm hedges the low state, where disinvestment needs, and costs, are higher. In addition, if the firm is small, current investment needs are high, as panel D depicts. As a consequence, the firm borrows constrained against the middle and the good state, and hoards cash (Panel E) to transfer additional net worth to the bad state as well.

[Insert Figure 4.6 Here]

In the case with fixed disinvestment costs, the firm still transfers resources to the low state (Panel F). In the inaction region, collateral constraints imply that the firm's debt capacity is higher because of the capital in excess to the "frictionless" target capital stock. Therefore, in this region, firms are able to save more conditional liquidity for the bad state, and they need to hoard less cash, as Panel E shows. In this example, also large firms hold cash. Different from the investment case, disinvestment generates internal resources from capital liquidation. Firms can keep part of these resources as cash reserves, and hedge future investment and disinvestment needs.

[Insert Figure 4.7 Here]

In the full model, all these types of adjustment costs are present. Therefore, it is a quantitative question how much each type of cost is important, and whether firms hedge mainly for either investment or disinvestment reasons. In sections 4.5 and 4.6 we analyze the quantitative implications of the model.

4.5 Calibration and Identification

In order to assess the quantitative implications of the model, and to perform counterfactual comparative statics, we calibrate the model to match a set of data moments. In this process, it is important to understand how the parameters of the model can be identified. Ideally, a one-to-one mapping between the structural parameters and a set of data moments provides a sufficient condition for identification. Such a close mapping in difficult to obtain in every economic model, and all the model parameters affect all the data moments to some extent. However, although firm's investment, financing, and liquidity management decisions are intertwined, we can still classify the moments roughly as representing the firm's investment, financing, and hedging decisions. We first discuss the implementation of state-contingent debt with credit lines. This provides a mapping between the concept of conditional liquidity in the model, and the data of leverage an lines of credits from Compustat and Capital IQ. Then we describe how parameter values are set, and discuss the quantitative performance of the model.

4.5.1 Implementation with Lines of Credit

As we discuss in Section 4.2, conditional liquidity management entails transfers of net worth to specific states, which are inherently unobservable. This feature renders our structural approach particularly suitable to investigate corporate liquidity management. To identify conditional liquidity management, we take advantage of data on credit lines from the Capital IQ dataset. Capital IQ reports the drawn fraction of funds from firms' credit lines. This metric is particularly useful because, consistent with the model, reflects differences in the fraction of debt capacity that firms preserve to conditionally transfer liquidity. The following proposition shows how state-contingent debt can be implemented in the model with a combination of traditional state-uncontingent debt intruments, such as bank loans or corporate bonds, and lines of credit.

Proposition 5 (Implementation with Credit Lines) *State-contingent debt* $b_i(s(t+1))$ *can be implemented by the following combination of securities: state-uncontingent debt* $D_{i,t+1} \geq 0$*, and a secured line of credit* $C_i^L(s(t+1))$*, with interest rate* r*, and limit* $\overline{C_{i,t+1}}$*. The firm arranges a loan* $L_{i,t}$ *at time* t *of size*

$$L_{i,t} = E_t\left[\frac{D_{i,t+1}}{1+r(1-\tau)}\right] \tag{4.30}$$

where the uncontingent debt claim is

$$D_{i,t+1} = (1+r(1-\tau))E_t[b_i(s(t+1))] \tag{4.31}$$

and saves state-contingent debt capacity by drawing $(1+r(1-\tau))(E_t[b_i(s(t+1))]-b_i(s(t+1)))$ *from the credit line in each state* $s(t+1) \in S$*, that is:*

$$C_i^L(s(t+1)) = (1+r(1-\tau))(E_t[b_i(s(t+1))] - b_i(s(t+1))) \tag{4.32}$$

The limit of the credit line is defined as

$$\overline{C_{i,t+1}} = (1+r(1-\tau))E_t[b_i(s(t+1))]$$

The proposition illustrates how firms can implement conditional liquidity management combining available securities, namely standard debt and credit lines. This provides a mapping between the variables in the model and the corresponding data moments. We use this mapping to compare the mean, the variance, and the serial correlation of undrawn debt capacity in the model and in the data. More precisely, in this implementation firms borrow the expected amount of required debt financing $E_t[b_i(s(t+1))]$ using standard uncontingent debt. Liquidity is then drawn from credit lines to fulfill unanticipated funding needs in the amount $(1+r(1-\tau))(E_t[b_i(s(t+1))] - b_i(s(t+1)))$ in each future state $s(t+1)$. The limit $\overline{C_{i,t+1}}$ on the credit line is set such that the total amount borrowed never exceeds the firm's debt capacity $\theta(1-\delta)k_{i,t+1}$.

Of course, the implementation with credit lines is not the only possibility for firms to engage in conditional liquidity management. For example, as Rampini and Viswanathan [2010] discuss, other possibilities involve the use of forwards and futures. In general, the state-contingent debt variables $b_i(s(t+1))$ in the model encompass different possible implementations. However, in quantitative analyses, taking a stand on a specific implementation provides a closer mapping between the model and the data. In this respect, as we discuss in Section 4.2, credit lines appear to be very important in practice, while even larger firms appear to implement little hedging through financial derivatives. For these reasons, and because of data limitations, we rely on the implementation in Proposition 5 in the following quantitative analysis.

4.5.2 Parameter Values and Model Fit

The model parameters we set in order to obtain a close match between the simulated data moments from the model, and the real data moments, are the production function curvature α, the operating leverage parameter f, the shock serial correlation ρ_z, the shock standard deviation σ_z, the fixed and convex physical adjustment cost parameters ψ_0^+, ψ_0^-, ψ^+, and ψ^-, the debt capacity parameter θ, the agency parameter γ, and the equity issuance fixed and proportional unit costs λ_0, and λ_1.[10]

[10] Following DeAngelo et al. [2011], we instead fix the tax rate parameter τ to the the statutory tax rate in the United States (0.35), the interest rate r to be approximately equal to the real interest rate in the 20th century (0.015), and δ to be approximately equal to the depreciation rate in our sample (0.15).

We pick 19 moments to match. On the investment side, we choose moments that relate to operating income, investment, and Tobin's Q. Average operating income is primarily affected by the curvature of the production function α, and by the operating leverage parameter f. The variance of operating income and its first-order autocorrelation instead capture the parameters σ_z and ρ_z that govern the dynamics of the shock process $z_{i,t}$. The investment moments we match are the mean, the variance, the serial correlation, and the skewness of investment. These moments are not only affected by the parameters α, σ_z, and ρ_z, but also help pin down the capital adjustment cost parameters ψ_0^+, ψ^+, ψ_0^-, and ψ^-. Higher values of ψ_0^-, and ψ_0^+ lead to more volatile, less autocorrelated, and more skewed investment. Higher ψ^+ and ψ^- result in less volatile, and more serially autocorrelated investment. Also, the debt capacity parameter θ has an impact on investment variance and skewness, because financing and investment are linked through state-contingent collateral constraints. Finally, average Tobin's Q is affected by all the parameters in the models, especially by σ_z and ρ_z, by the adjustment cost parameters, and by the fixed operating costs f.

On the financing side, we consider mean, average, and serial correlation of leverage, average equity issues, and their variance. The leverage moments are affected by all parameters in the model, and especially by θ. The mean and variance of equity issues help identify λ_0 and λ_1. The remaining moments pertain to the conditional, and unconditional hedging policy. We choose to match mean, variance and serial correlation for both cash holdings, and undrawn credit from firms' credit lines. As we illustrate in section 4.4, all these moments are affected by the dynamics of the shock process, and by the capital adjustment cost parameters. Moreover, the agency parameter γ affects average cash holdings. Finally, θ plays a very important role for the tradeoff between conditional, and unconditional liquidity management. Higher values of θ imply that the amount of liquid funds which can be transferred conditionally is higher. As a consequence, the higher θ, the higher the average undrawn debt capacity, and the lower the average cash holdings.

The calibrated parameters in table 4.2 are comparable to those of existing studies. The curvature of the profit function α is close to the estimated values in Hennessy and Whited [2005], and Hennessy and Whited [2007]. The fixed cost parameter f, on an annual basis, is in line with the calibration of Gomes and Schmid [2010]. The parameters σ_z, and ρ_z, that govern the shock dynamics, are less than one standard error from the estimates in Hennessy and Whited [2005]. The external equity cost parameters λ_0 and λ_1 are also very close to the point estimates of Hennessy and Whited [2005], who use the same functional form. The value of the cash hoarding cost parameter γ is similar to the one in DeAngelo et al. [2011]. Our values for the capital adjustment cost parameters exhibit

Finally, we set the unconditional mean μ_z of the shock process such that the steady-state stock of capital is normalized to the value of two. This choice allows to obtain a sufficient precision on the grid for capital without significantly increasing the computational burden with a finer grid choice.

a similar patterns to Cooper and Haltiwanger [2006], and DeAngelo et al. [2011] as far as the relative magnitude of the fixed and convex component is concerned. Different from these studies, we also allow for asymmetries in capital adjustment costs for investment and disinvestment. Our parameters provide support to the calibration in Zhang [2005], who requires that disinvestment is by far more costly than investment to rationalize the value premium. Finally, to the best of our knowledge there is no direct quantitative term of comparison for the parameter θ in state-contingent collateral constraints. However, our calibrated value is extremely close to the share of pledgeable steady-state capital estimated by DeAngelo et al. [2011].

Table 4.2 shows that, overall, the model provides a good fit to the data. Remarkably, with only one exogenous shock process, the model manages to endogenously generate very different variances for operating income on one hand, and investment, leverage, and undrawn debt capacity on the other hand. In contrast, in existing models (e.g. Hennessy and Whited [2007], DeAngelo et al. [2011], Nikolov and Schmid [2012]) simulated variances are typically much lower than real data variances. This leads to the need to either remove firm and time fixed effects from the data, or to add noise to the simulation, in order to make volatilities of simulated and actual moments comparable. We attribute this result to the presence of additional frictions in comparison to these models, and in particular to state-contingent collateral constraints, and to our flexible adjustment cost function for physical capital. In addition, the model is able to replicate fairly well the relative differences in serial correlations for operating income, investment, leverage, cash, and undrawn credit that are observed in the data. Specifically, data moments for these variables are approximately 0.79, 0.37, 0.91, 0.89, and 0.63, while their simulated counterparts are around 0.63, 0.22, 0.68, 0.72, and 0.68.

The model appears to be slightly on the variance of cash holdings, and on the mean of undrawn debt capacity. The former is too high because in our model the only motive for which firms hold cash is hedging. Therefore, firms with no hedging needs, or firms that can satisfy all their hedging needs with conditional liquidity only, hold exactly zero cash. In reality, firms also hold cash for other reasons, for example for operating purposes. The lower mean of undrawn debt capacity with respect to the data is the result of the assumption of relative impatience of managers because of tax benefits of debt. As in Rampini and Viswanathan [2012a], firms are never completely unconstrained, and even large unconstrained firms issue debt. The fit may be probably further improved by introducing additional frictions. However, we do not include them to make the trade-off between unconditional and conditional hedging clearly driven by limited conditional hedging in presence of collateral constraints, and investment adjustment costs.

[Insert Table 4.2 Here]

4.6 Empirical Implications

4.6.1 Stylized evidence under the baseline calibration

In this section, we evaluate the model performance by reproducing the stylized empirical evidence on corporate liquidity we revisit in section 4.2. Table 4.3 is a replica of table 4.1 with a simulated panel of observations from our model. All parameters are set to the baseline values in table 4.2, and data are simulated using the same procedure.

A comparison of tables 4.1 and 4.3 shows that the model conforms with the key patterns that are observed in the data, and that we summarize in section 4.2. The patterns of simulated evidence are generally sharper that those in actual data. This is primarily because of the higher variance of cash, and the lower undrawn debt capacity, as we discuss in section 4.5.

Panel A of table 4.3 reports simulated evidence for one-way sorts on net worth and capital. The row labeled "Cash Holdings" shows that smaller and more constrained firms hoard more cash, as Almeida et al. [2004], and Denis and Sibilkov [2009] document. The "Leverage" row reproduces the well-known positive relation between size and leverage. In addition, firms with low net worth are more levered than firms with high net worth. Finally, the row labeled "Undrawn Credit" reproduces the finding that unconstrained firms are more slack on their credit lines. While the evidence in panel A provides a crude assessment of the model, the two-way sorts in panels B and C are definitely more informative. Indeed, they allow to effectively interpret empirical patterns within our framework of our model, and better understand why these patterns are observed in actual data.

The sub-panel labeled "Cash Holdings" emphasizes that the main variable that drives firms cash policy is capital, rather than net worth. This can be rationalized within our model, and is consistent with the graphical representation in figure 4.1. Conditional on some level of net worth, hence on some *total* liquidity need, capital essentially determines the optimal mix between conditional and unconditional liquidity. Transferring resources in a state-contingent way is more efficient, but a firm's ability to implement conditional hedging is limited by collateral constraints. As a consequence, smaller firms also need to transfer resources unconditionally, to all future states, and hoarding more cash than large firms. In addition, consistent with net worth being the main determinant of total corporate liquidity (figure 4.1), less constrained firms appear to have more cash than more constrained firms after controlling for capital. As in the data, the pattern is less pronounced than on the capital dimension. Unconstrained firms implement more total hedging and, *ceteris paribus*, also hoard more cash. This piece of evidence relates to the result in Denis and Sibilkov [2009] that some constrained firms have surprisingly low cash holdings.

The "Undrawn Credit" sub-panel replicates the stylized fact that firms with high net worth implement more total and, consequently, more conditional hedging. At a first glance, this result may look at odds with our key message that capital is the main determinant of the composition of corporate liquidity as conditional versus unconditional, as figure 4.1 shows. However, an important caveat is needed in interpreting this reduced-form evidence. In table 4.3, we compute undrawn credit as a fraction of debt capacity, while the mix of conditional and unconditional liquidity must account for how much cash firms hoard. Large firms have also less cash than small firms, and the ratio of conditional-to-unconditional liquidity is higher for more capitalized, hence more collateralized, firms. Panel C addresses this point and provides additional evidence by computing the ratio of conditional-to-total liquidity for simulated data, and the ratio of undrawn credit to the sum of undrawn credit and cash for the sample of table 4.1. Clearly, panel C shows that capital determines the mix of conditional and unconditional liquidity as the model predicts, and empirical proxies support this prediction.

Finally, the "Leverage" sub-panel in panel B provides substantial support for the hedging view of capital structure in Rampini and Viswanathan [2012a]. Similar to them, in our model capital stucture and conditional hedging are intimately related. For the same level of capital, the more a firm raises debt, the less resources it allocates to risk management. For this reason, *within every capital group*, we observe an opposite pattern with respect to the "Undrawn Credit" panel. Conditional on capital, which is determined endougenously, the more a firm keeps slack on its collateral contraints, the higher observed leverage is. Because in practice one important way to tranfer conditional liquidity is based on loan commitments (Rampini and Viswanathan [2010]), this pattern is also reflected in data on credit lines. Undrawn credit therefore appears to be a good proxy for conditional hedging.

We believe these results are informative in three ways. First, our dynamic model of corporate liquidity provides a unified framework to rationalize and interpret existing empirical evidence on cash, risk management, leverage, and lines of credit.

Second, our simulated results have implications for empirical work, and specifically for how to proxy financial constraints. Our model shows that net worth, that we proxy as the book value of equity, and capital, capture different aspects of financial constraints for corporate liquidity. A common practice in empirical studies is to use both capital and book value of equity as proxies for how a firm is constrained. In constrast, recognizing that net worth is a theoretically grounded state variable in models of financial constraints, and that it plays a different role from capital for liquidity and risk management decisions, appears to be a necessary condition for most empirical studies to be informative.

Third, our findings suggest an empirical proxy for hedging, namely undrawn credit from credit lines. As we discuss in section 4.2, empirical studies on risk management are plagued because hedging is unobservable. Despite there is not a one-to-one mapping between undrawn credit and conditional hedging, our results suggest that the former is a reasonable proxy for the latter. This appears plausible if one considers the widespread use of lines of credits, as Sufi [2009] points out. Data on credit lines are nowadays available for large cross sections of firms in commercial datasets. Therefore, they may help extend and complement existing studies that, while based on specific data that are more closely mapped into hedging, are limited in scope.

[Insert Table 4.3 Here]

4.6.2 Comparative Statics: Debt Capacity

Table 4.4 summarizes the predicted impact of variations in the fraction of collateralizable capital θ on firms' policy. The rows of the table refer to investment, leverage, equity issues, cash holdings, and hedging through conditional liquidity. The columns report average values for all firms, and for firms that differ in terms of the two state variables of our model, namely net worth and capital.

Panel A refers to low values of θ, panel B to moderate values, and panel C to high values. Different levels of θ can be interpreted as cross-industry predictions. Intuitively, the information technology industry relies on more intangible assets, that cannot usually be pledged as collateral.[11] In contrast, steel manufacturing companies typically operate with collateralizable capital such as properties, plants, and equipments. In our framework, industries with less pledgeable assets can be associated to lower values of θ.

Table 4.4 illustrates the hedging view of capital structure of Rampini and Viswanathan [2012a], and the tradeoff between conditional and unconditional liquidity management. Firms with a low fraction of collateralizable assets have both lower leverage, and residual debt capacity. Our model predicts that firms with θ equal to 10% have a debt-to-asset ratio of 4.7%, compared to 30.7% for firms with θ equal to 90%. Residual debt capacity is ranging from 43.8% for firms with a low fraction of pledgeable capital, to almost 60% for firms with a high fraction. The latter can implement more conditional liquidity management, and therefore face less needs to resort to cash hoarding to hedge against income shortfalls. In addition, firms with lower debt capacity have less needs for costly external equity financing.

Consistent with the patterns we illustrate in section 4.6.1, firms that differ in terms of the endogenous state variables of our model have a different expected leverage and liquidity management policy across different levels of θ. In particular, since debt capacity is a fraction of capital, the latter is the variable that interacts more with θ to determine the firm's policy. Smaller firms hoard more cash and implement less conditional liquidity management in all panels A, B, and C, but their liquidity is disproportionally more

[11] However, Amable et al. [2010] argue that a recent common practice is to pledge patents as collateral.

state-contingent for high values of θ. For instance, small firms in panel A have a 91.7% cash-to-assets ratio, and 53.6% undrawn debt capacity. Panel C instead predicts a cash-to-asset ratio of 38.2%, and a fraction of undrawn credit equal above 75% when $\theta = 0.9$. In addition, our model predicts that the positive relationship between leverage and capital is steeper in industries with more tangible assets, reflecting higher opportunities to secure debt financing with collateral.

[Insert Table 4.4 Here]

4.6.3 Comparative Statics: Capital Adjustment Costs

As we discuss in section 4.4 and illustrate in figures 4.4 through 4.7, the presence of investment and disinvestment adjustment costs has a qualitative and quantitative impact on the type of liquidity management firms implement. In this section, we examine how predicted liquidity management and financing policy vary across firms with different magnitudes for adjustment costs of physical capital.

Table 4.5 examines the case of convex disinvestment adjustment costs. As figure 4.6 illustrates, firms with higher smooth adjustment costs of disinvestment have more liquidity needs for bad states. These needs reflect the necessity to bear these expected costs in future periods, and to be able to gradually adjust their capital stock. Panels A to C show how firms with higher adjustment cost of disinvestment implement more liquidity management, both conditionally and unconditionally. Average cash holdings vary from about 10% to over 20% if disinvestment adjustment costs increase from low to high values. Analogously, undrawn debt capacity approximately ranges from 48% to 60%. As a consequence, firms with lower values for ψ^- need to save less debt capacity, and are more levered.

[Insert Table 4.5 Here]

Table 4.6 performes counterfactual analysis for firms that are associate to different smooth investment adjustment costs ψ^+. Firms with higher values for ψ^+ are more levered, invest less, and implement less liquidity management. Intuitively, investment is less profitable if associated to higher costs, and companies that are more exposed to these costs raise more debt finance to pay out more dividends, and take advantage of the tax benefits of debt. As figure 4.3 illustrates, the persistence in investment opportunities creates a need to transfer liquidity to good states. However, when adjustment costs are too high, investment needs decrease in such states, and so do liquidity needs. As a consequence, firms with high ψ^+ hoard less cash than firms with low ψ^+ (2% versus 31%), and save less debt capacity (32% versus 60%).

[Insert Table 4.6 Here]

4.6.4 Impulse Response Functions

In this section, we investigate the dynamics of investment, leverage, conditional, and unconditional liquidity management for firms that differ in terms of net worth and capital. To this end, figures 4.8 to 4.11 depict impulse response functions for the model to a positive shock (dashed lines), and to a negative shock (dashed-dotted lines). In all panels, the solid lines represents the benchmark case, that is the case in which the representative firm is exposed to neutral shocks.

Firms are classified as relatively constrained/unconstrained, and relatively small/large on the basis on their initial values for the two endogenous state variables of the models. Accordingly, figure 4.8 plots impulse response functions for firms with initial low net worth and median capital stock (constrained), figure 4.9 refers to firms with with initial high net worth and median capital stock (unconstrained), figure 4.10 refers to firms with with initial low capital stock and median net worth (small), and figure 4.11 refers to firms with with initial high capital stock and median net worth (large).

Because the model is nonlinear, we construct generalized impulse response functions following Potter [2000], to which we refer for an exhaustive treatment. Effectively, impulse response functions are computed as the averages of 5000 draws of sequences of shocks from $z_{i,t}$ for 30 periods under the baseline parametrization of table 4.2. In the benchmark case, the shock process is initalized to the mean shock μ_z for all draws, while for the positive (negative) response cases the process is initialized to values above (below) μ_z. The exact definitions of positive and negative shocks, small and high initial net worth and capital, and of the variables on the graphs are provided in the caption of the figures.

A comparison of figure 4.8 and figure 4.9 highlights how the dynamics of investment, leverage, and hedging differ between relatively constrained and unconstrained firms in response to positive and negative shocks to investment opportunities. Panel A shows that both types of firms increase investment when a positive shock occurs, and decrease investment when a negative shock occurs. This result is due to the high persistence of exogenous shock process. However, the dynamics of both unconditional and conditional hedging deeply differ, as panels D, E, and F depict. Specifically, constrained firms have less resources to allocate to risk management when the shock realizes. Therefore, their adjustment to cash holdings (panel D), hedging for good states (panel E), and hedging for bad states (panel F) are low than for unconstrained firms. This implies that the dynamics of leverage, net worth, and capital differ in that the effects of the shocks are more persistent for more constrained firms. Remarkably, the response is asymmetric. After a negative shock constrained firms become even more constrained, as the dynamics of net worth in panel C show. They can allocate little resources to conditional downstate hedging and to cash hoarding for future bad states, that are more likely to occur due to the persistence of the shock process. After a positive shock, instead, constrained firms benefit from additional cash flow, and have more net worth to tranfer to future states in the form of both conditional and unconditional liquidity. To sum up, relatively constrained firms have a lower capacity to implement total hedging the relatively unconstrained firms, and they are sluggish in reacting to negative shocks.

Figures 4.10 and 4.11 illustrate how the dynamics of corporate policy differs between small and large firms. As figure 4.1 indicates, the capital stock primarily affects the composition of corporate liquidity (conditional versus unconditional), rather than the amount of total liquidity. The hump-shaped investment dynamics in panel A suggest that small firms are slower in adjust their capital stock after the shocks. Indeed, large firms can transfer larger amounts of state-contingent liquidity (panels E and F), and be more efficient in boosting their investment in good times, and reducing their capital stock in bad times. Small firms, because on collateral constraints, can pledge less capital and need to hedge by hoarding cash (panel D). As a consequence they are forced to tranfer net worth to all future states and, *ceteris paribus*, they can tranfer less resources to the states where they are needed the most. Overall, large firms can take advantage of more pleadgeable capital for conditional liquidity management, and be more efficient in adjusting their investment policy. As a consequence, they benefit more than small firms from improved investment opportunities, and they reduce the impact of bad shocks on their value.

[Insert Figure 4.8 Here]

[Insert Figure 4.9 Here]

[Insert Figure 4.10 Here]

[Insert Figure 4.11 Here]

4.7 Conclusions

In the presence of capital market imperfections expectations of future investment opportunities or cash shortfalls provide a rationale for dynamic liquidity management. We develop a quantitative model to examine the cross-sectional and time-series determinants of corporations' liquidity management. The result is a quantitative theory of optimal liquidity management based on the trade-off between conditionali liquidity subject to collateral constraints and unconditional, unconstrained liquidity. Our model identifies unconditional liquidity management using cash and conditional liquidity management by means of drawing on credit lines as important instruments of corporate policy. In particular, our model predicts substantial cross-sectional variation in the relative usage of these instruments for liquidity purposes across firms, for which we find strong empirical support.

Similarly, the model successfully rationalizes time-series patterns in corporations' liquidity management. Overall, the model thus provides a quantitatively and empirically successful framework explaining corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

A large literature has recently attempted to rationalize the apparent secular trend in firms' cash holdings. It has been widely documented that in the US, firms' cash-to-asset ratios have increased dramatically since the 1970's. While in this paper we focus on stationary properties of firms' liquidity policies, we think it would be interesting to examine the possible determinants of this trend through the lens of our model. We leave this important question for future research.

Figure 4.1. Dynamic Corporate Liquidity

The figure illustrates the relationship between the different types of corporate liquidity, and the state variables of the model. In every period, firms are sorted independently by net worth $w_{i,t}$, capital $k_{i,t}$, and productivity $z_{i,t}$. Firms whose net worth is above the median of the cross-sectional distribution are labeled as unconstrained ('Unc'), and firms whose net worth is below the median of the cross-sectional distribution are labeled as constrained ('Con'). Firms whose capital is above the median of the cross-sectional distribution are labeled as large ('Lar'), and firms whose capital is below the median of the cross-sectional distribution are labeled as small ('Sm'). Firms whose realized productivity is above the middle state are labeled as profitable ('Pr'), and firms whose productivity is below the middle state are labeled as unprofitable ('Unp'). For each bin, we compute total hedging, the fraction of conditional to total hedging (on the horizontal axis), and the fraction of upstate to total hedging (on the vertical vertical axis). In the figure, the radious of the circle is proportional to total hedging. Data are simulated from the model with the baseline parametrization in Table X, for a panel of 1000 firms and 100 time periods.

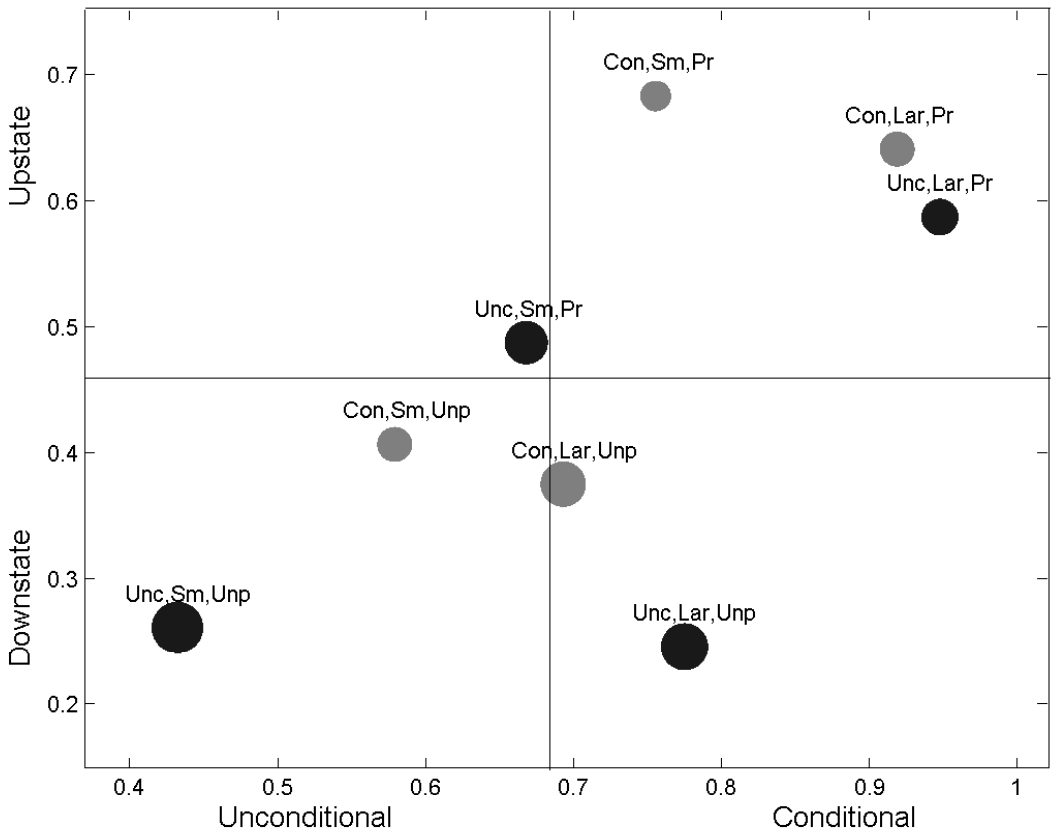


Figure 4.2. Firm's policy with no persistence and no adjustment costs

The figure illustrates the investment, financing, and risk management policy of the firm as a function of current net worth $w_{i,t}$. For illustrative purposes, the model is solved with a number of states equal to three, with uniform transition probabilities, and with all adjustment costs parameters set to zero. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.3000 for the low state, to 0.5000 for the middle state, and to 1.7000 for the high state. Panels A through F show: the future capital stock $k_{i,t+1}$, the net equity payout $e_{i,t}$ the equity value $V_{i,t}$, the observed debt stock $E[b_i(s(t+1))]$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$. In panel F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.5000$, $\gamma = 0.0010$, $r = 0.0100$.

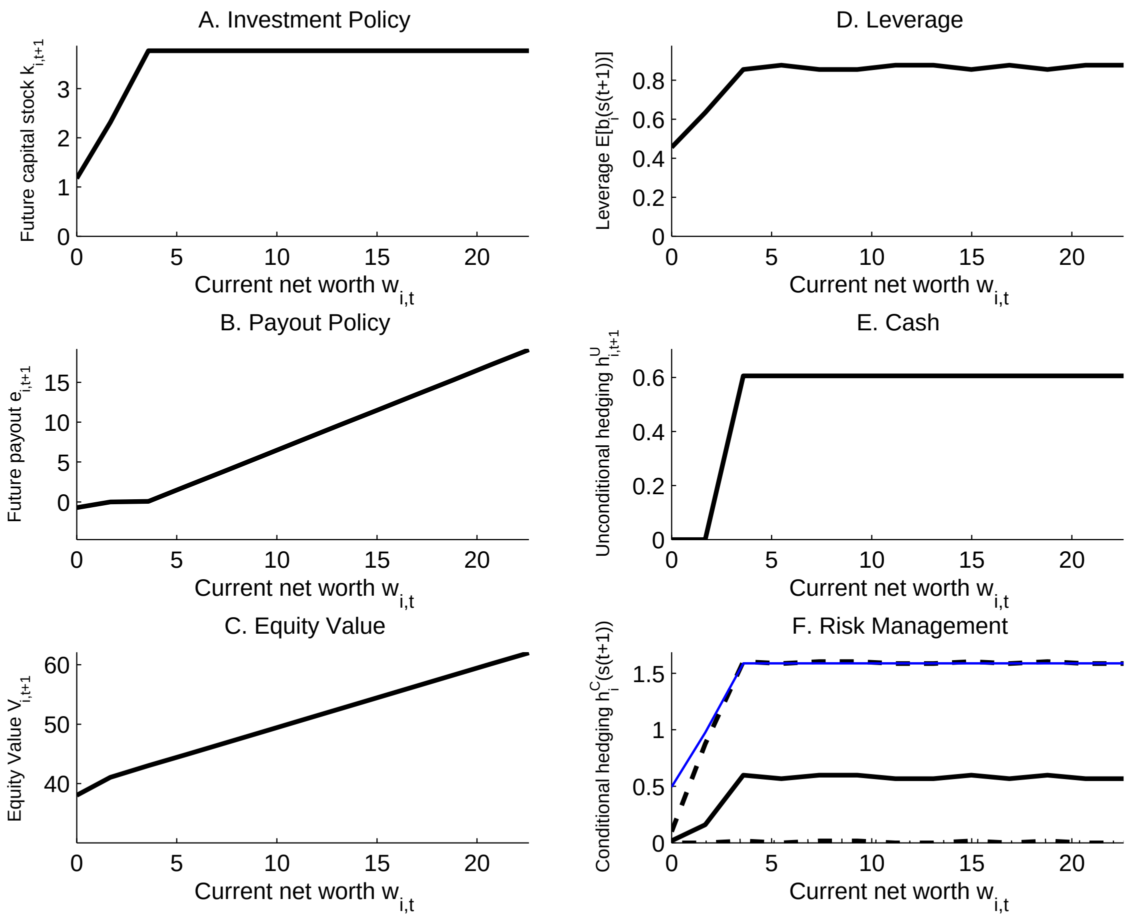


Figure 4.3. Firm's policy with persistence and no adjustment costs

The figure illustrates the investment, financing, and risk management policy of the firm as a function of current net worth $w_{i,t}$. For illustrative purposes, the model is solved with a number of states equal to three, and with all adjustment costs parameters set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.2000 for the low state, to 0.5000 for the middle state, and to 0.8000 for the high state. Panels A through F show: the future capital stock $k_{i,t+1}$, the net equity payout $e_{i,t}$ the equity value $V_{i,t}$, the observed debt stock $E[b_i(s(t+1))]$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$. In panel F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.6000$, $\gamma = 0.0010$, $r = 0.0100$.

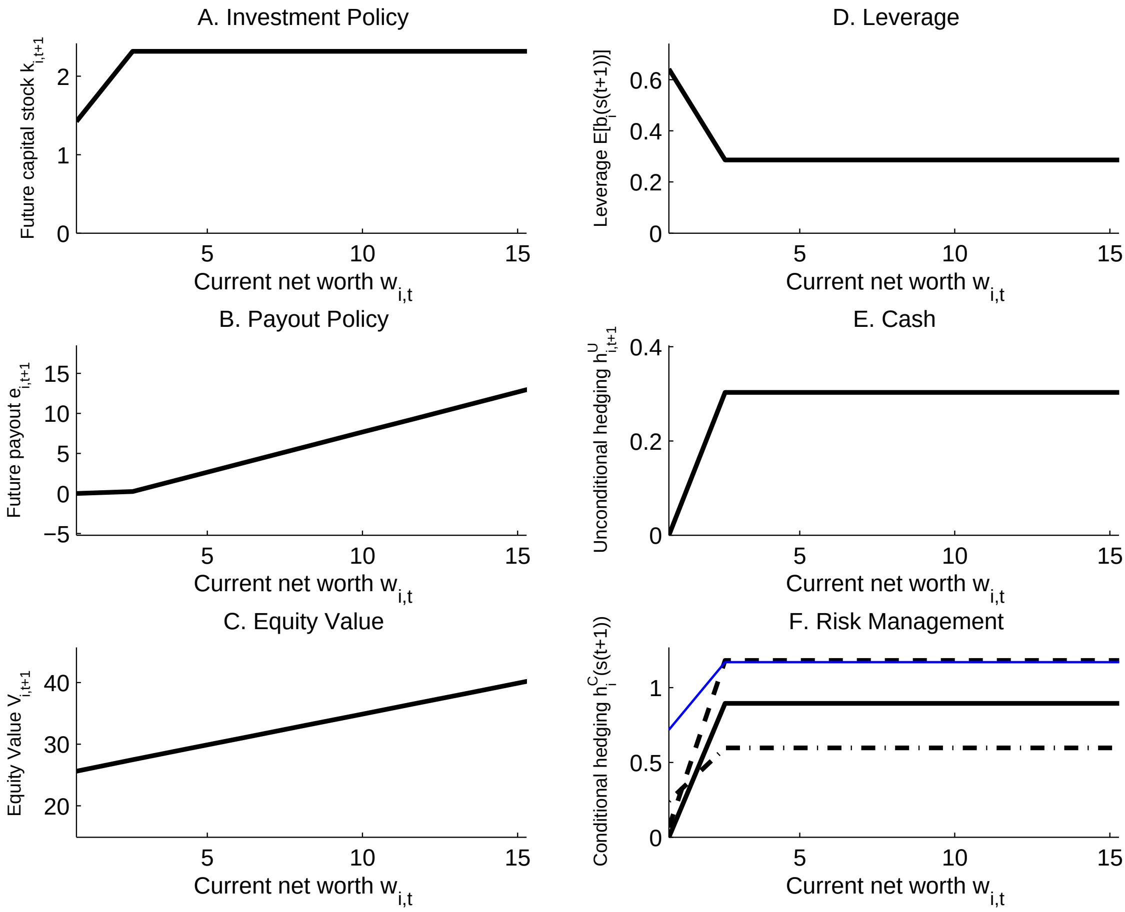


Figure 4.4. Firm's policy with convex investment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter ψ^+ is set to 1.0000. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.3000 for the low state, to 0.7000 for the middle state, and to 1.1000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of current net worth. Panels D through F show: the investment-to-capital ratio $i_{i,t}/k_{i,t}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of the current capital stock. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.6000$, $\gamma = 0.0010$, $r = 0.0100$.

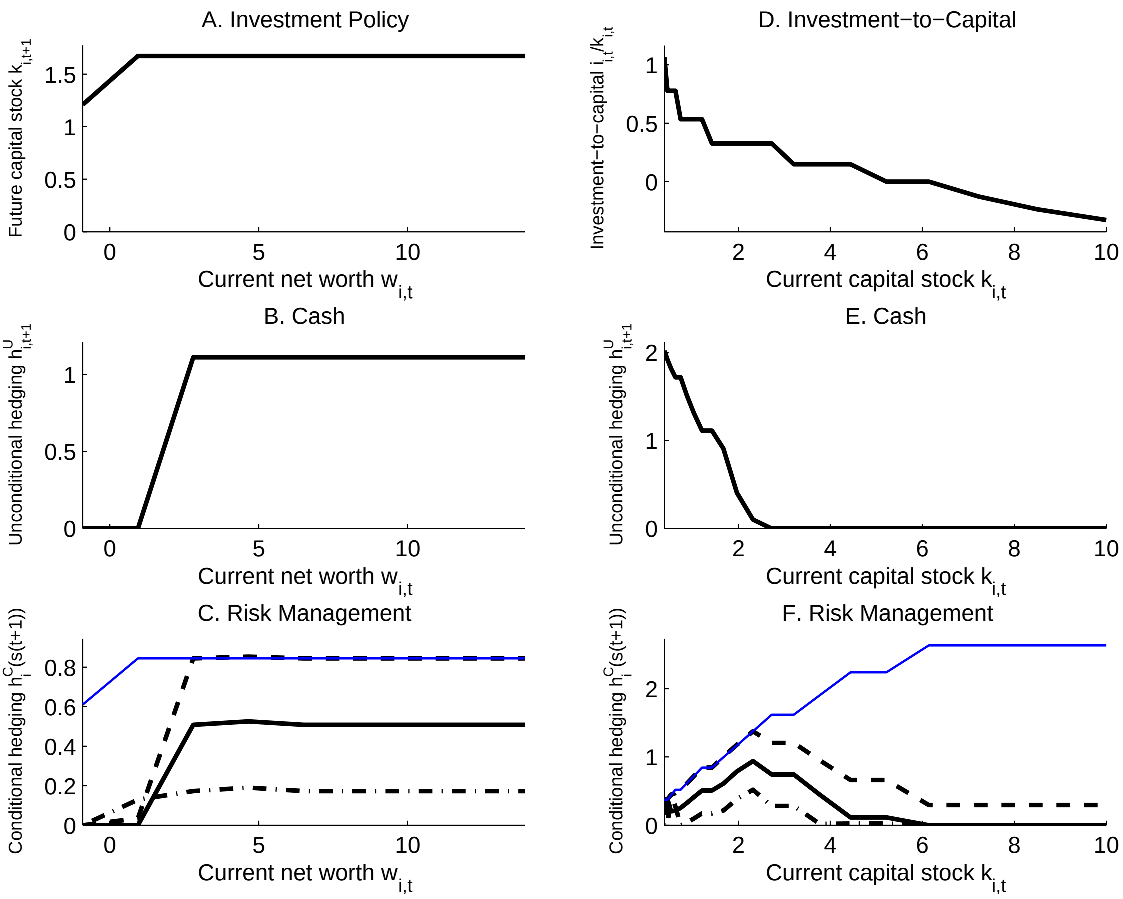


Figure 4.5. Firm's policy with fixed investment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter ψ_0^+ is set to 0.0750. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.3000 for the low state, to 0.7000 for the middle state, and to 0.9000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of current net worth. Panels D through F show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of the current capital stock. In panel D, k^* denotes the "frictionless" level of capital with $\psi_0^+ = 0$, while k^D and k^U are the bounds of the inaction region. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.3500$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.4000$, $\gamma = 0.0010$, $r = 0.0100$.

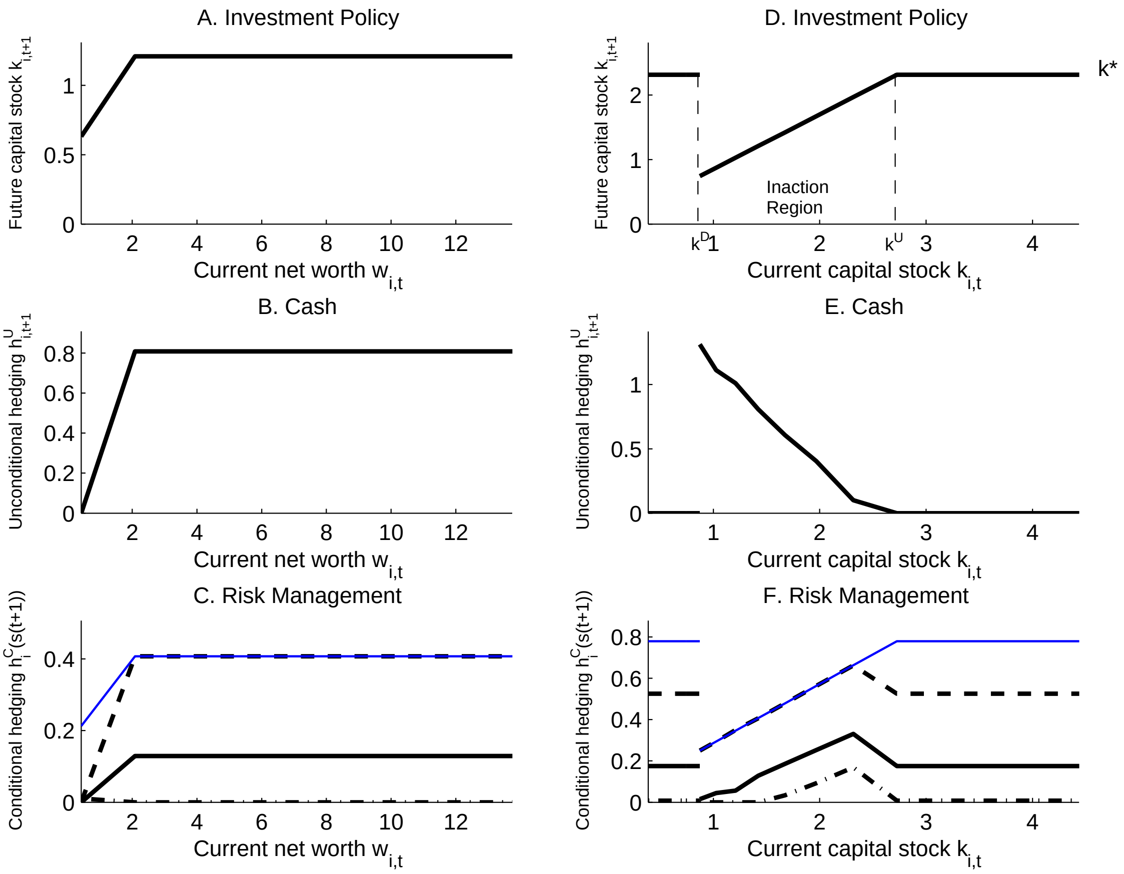


Figure 4.6. Firm's policy with convex disinvestment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex disinvestment adjustment cost parameter ψ^{-} is set to 0.4000. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.1000 for the low state, to 0.5000 for the middle state, and to 0.6000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of current net worth. Panels D through F show: the investment-to-capital ratio $i_{i,t}/k_{i,t}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of the current capital stock. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.3750$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.4000$, $\gamma = 0.0010$, $r = 0.0100$.

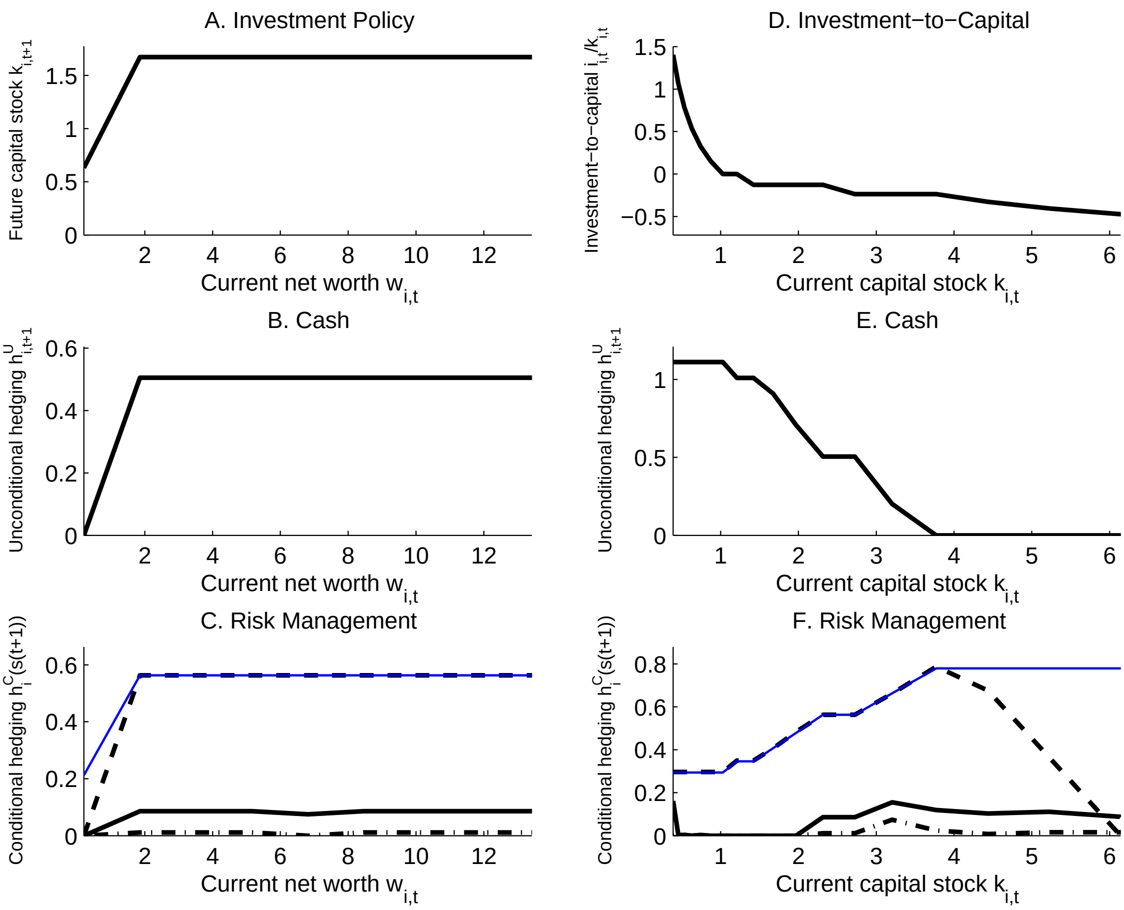


Figure 4.7. Firm's policy with fixed disinvestment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter ψ_0^- is set to 0.0250. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.1000 for the low state, to 0.6000 for the middle state, and to 0.7000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h_i^C(s(t+1))$ as a function of current net worth. Panels D through F show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h_i^C(s(t+1))$ as a function of the current capital stock. In panel D, k^* denotes the "frictionless" level of capital with $\psi_0^- = 0$, while k^D and k^U are the bounds of the inaction region. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.4000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.4000$, $\gamma = 0.0010$, $r = 0.0100$.

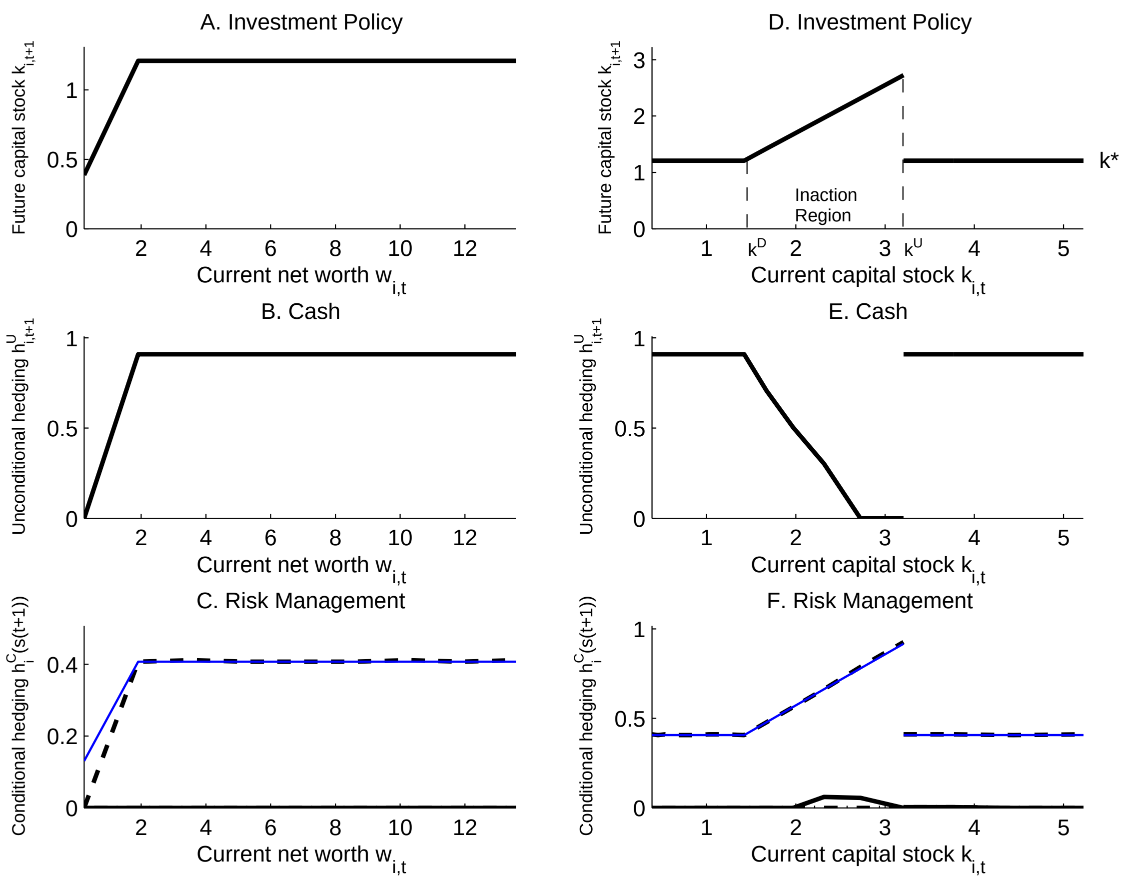


Figure 4.8. Impulse Response Functions for Relatively Constrained Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for capital are set to the corrisponding median point on the grid for $k_{i,t}$, and the initial values for net worth are set to the corresponding value for the point on the grid for $w_{i,t}$ that leaves one fifth of grid points to its left. All parameters values are set to the baseline values reported in table 4.2.

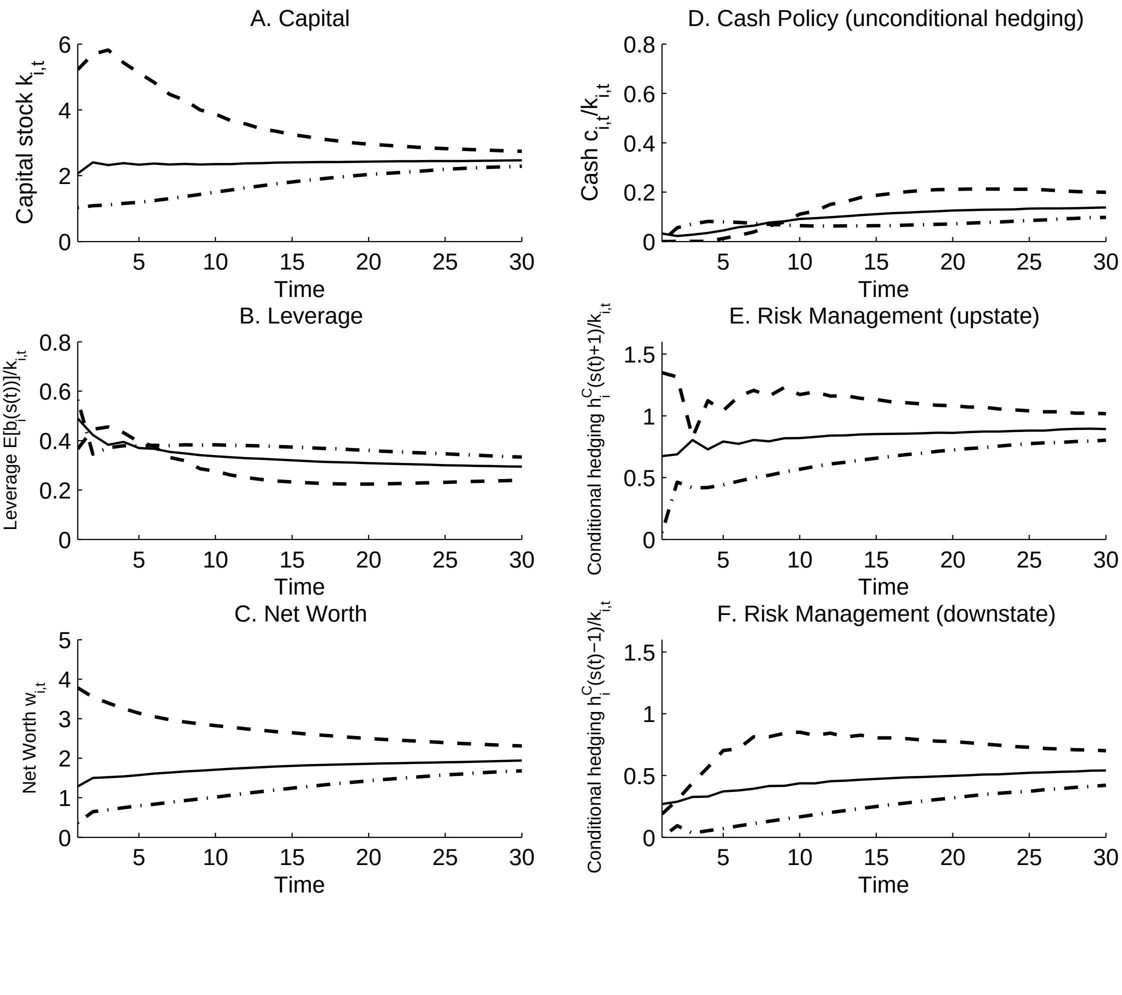


Figure 4.9. Impulse Response Functions for Relatively Unconstrained Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for capital are set to the corrisponding median point on the grid for $k_{i,t}$, and the initial values for net worth are set to the corresponding value for the point on the grid for $w_{i,t}$ that leaves one fifth of grid points to its right. All parameters values are set to the baseline values reported in table 4.2.

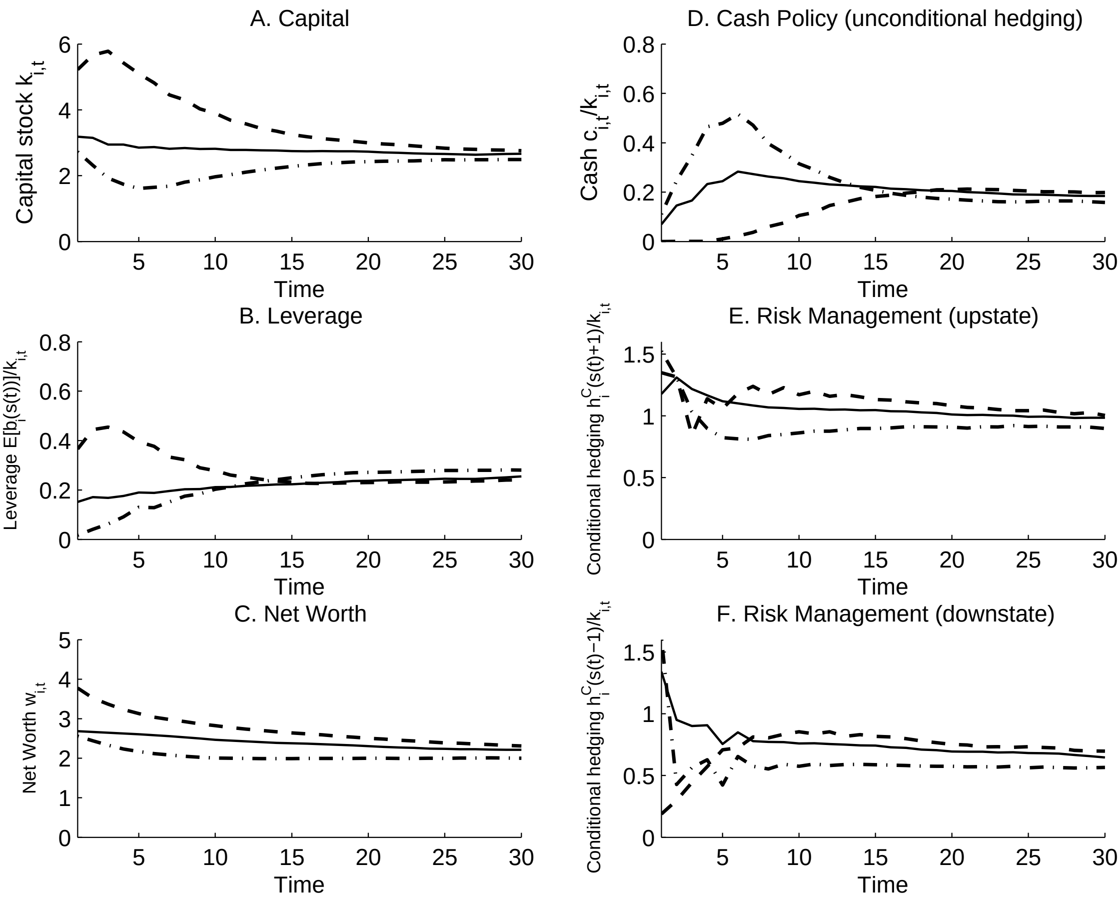


Figure 4.10. Impulse Response Functions for Relatively Small Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for net worth are set to the corrisponding median point on the grid for $w_{i,t}$, and the initial values for capital are set to the corresponding value for the point on the grid for $k_{i,t}$ that leaves one fifth of grid points to its left. All parameters values are set to the baseline values reported in table 4.2.

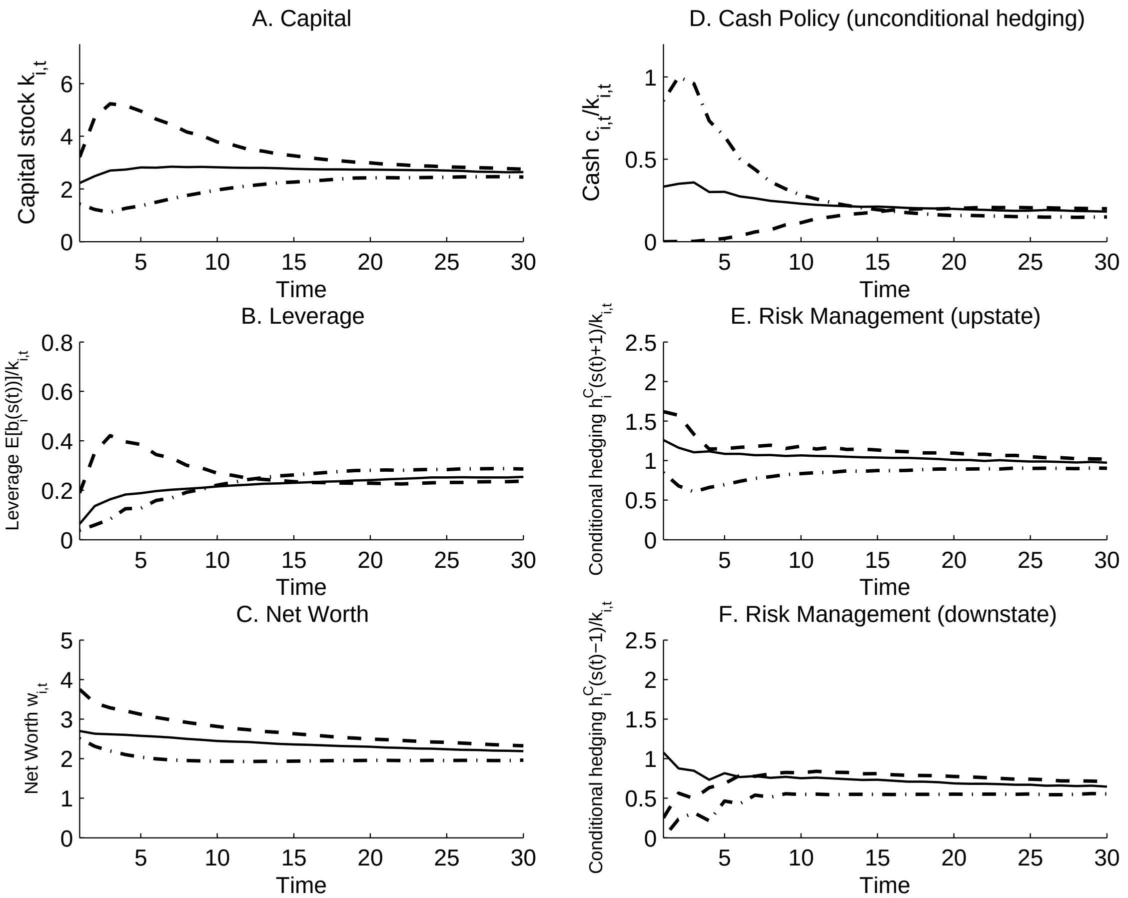


Figure 4.11. Impulse Response Functions for Relatively Large Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for net worth are set to the corrisponding median point on the grid for $w_{i,t}$, and the initial values for capital are set to the corresponding value for the point on the grid for $k_{i,t}$ that leaves one fifth of grid points to its right. All parameters values are set to the baseline values reported in table 4.2.

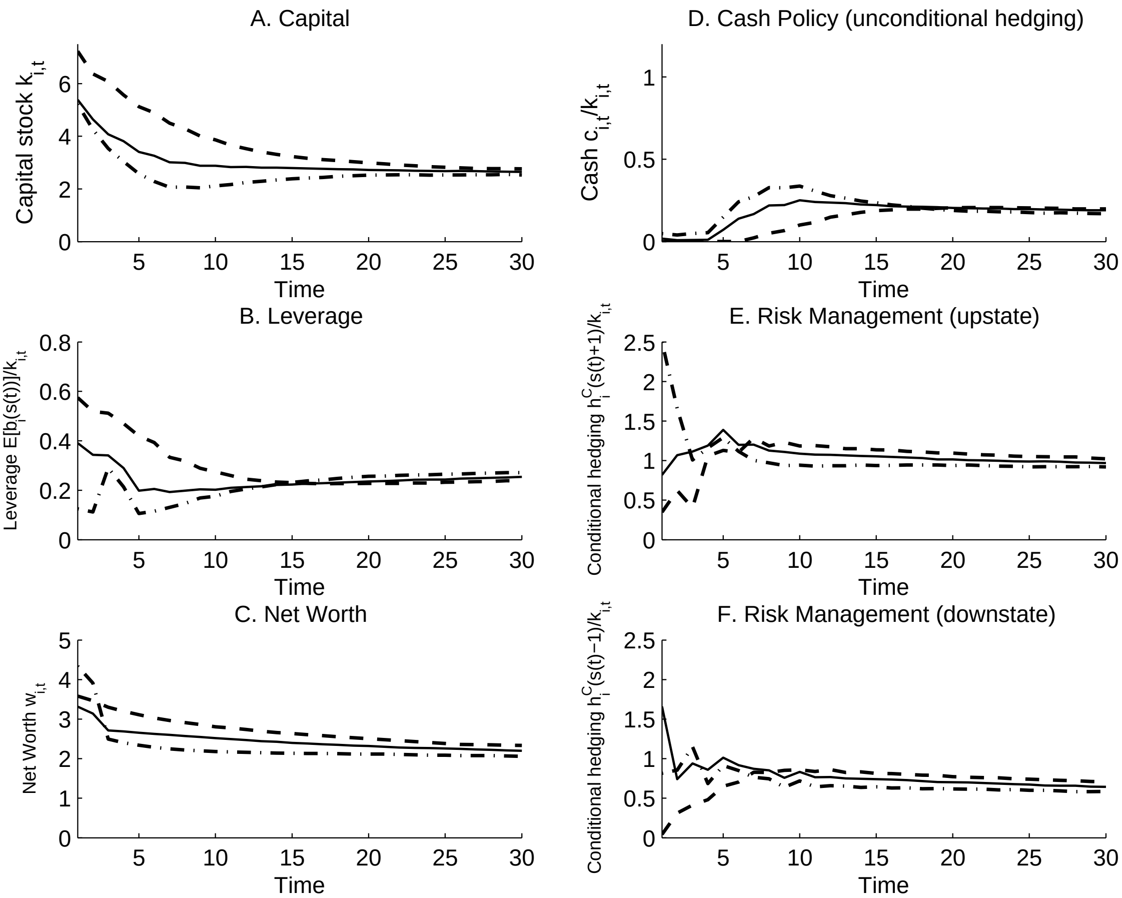


Table 4.1. Leverage, Cash, and Conditional Liquidity: Stylized Evidence.

The table reports stylized evidence from sorts of companies by net worth and capital (the state variables of our model). Data are from Compustat and Capital IQ for the period 2001-2011. Net worth is measured as the book value of equity, in line with Rampini et al. [2012], and capital is the book value of property, plant and equipment. The breakpoints for defining relative constrained and unconstrained firms for the sorts on net worth, and relatively small and large firms for the sorts on capital are the 33th and the 66th percentile of the cross-sectional distribution for every fiscal year. We exclude financials (SIC 4900-4099), utilities (SIC 6000-6999), and firms from other regulated industries (SIC greater than 9000). The final sample consists of 14220 firm-year observations. Panel A reports average cash holdings and debt as a fraction of total assets, and the fraction of undrawn credit from credit lines for one-way sorts, while panel B reports the same variables for two-way sorts.

Panel A: Univariate Sorts

	Net Worth			Capital		
	Constr.	2	Unconstr.	Small	2	Large
Cash Holdings	0.187	0.178	0.126	0.240	0.153	0.098
Leverage	0.216	0.192	0.228	0.118	0.220	0.296
Undrawn Credit	0.754	0.811	0.861	0.793	0.793	0.842

Panel B: Bivariate Sorts

		Cash Holdings			Leverage			Undrawn Credit		
	Capital	Small	2	Large	Small	2	Large	Small	2	Large
	Constr.	0.220	0.103	0.085	0.128	0.385	0.697	0.770	0.712	0.748
Net Worth	2	0.297	0.156	0.075	0.089	0.180	0.378	0.864	0.810	0.761
	Unconstr.	0.269	0.193	0.105	0.109	0.161	0.249	0.878	0.833	0.867

Table 4.2. Model Calibration.

The table reports actual and simulated moments, together with the corresponding choice of structural parameters. Calculations of data moments are based on a sample of nonfinancial, unregulated firms from the annual 2011 Compustat Industrial database merged to the Capital IQ dataset. The sample period is from 2001 to 2011. Panel A reports the moments from a simulated panel of firms, and the corresponding moments from the data. Operating income is defined as $(zk^{\alpha} - f)/k$, investment as $i = k_{t+1} - (1 - \delta)k_t$, leverage as $E[b(z_{t+1})]/k$, equity issues as $|min(d, 0)|/k$, cash holdings as c/k, undrawn debt capacity as $h^c(z_{t+1})/(\theta(1-\delta)k)$, and Tobin's Q as $(V + E[b(z_{t+1})])/k$. Panel B reports the chosen values for structural parameters. α is the curvature of the production function, f is the per-period fixed production cost, θ is the collateralizable fraction of assets, γ is the agency cost cash parameter, ψ_0^+ and ψ^+ are the fixed and convex investment adjustment costs parameters, ψ_0^- and ψ^- are the fixed and convex disinvestment adjustment costs parameters, ρ_z and σ_z are the serial correlation and the standard deviation to innovations of $ln(z)$, where z is the shock to the revenue function, λ_0 and λ_1 are the fixed and the proportional equity flotation costs. The remaining parameters are r, the interest rate, τ, the tax rate, and δ, the depreciation rate. They are set to 0.015, 0.35, and 0.15, to be approximately equal to the real interest rate in the 20th century, to the statutory tax rate in the United States, and to the average depreciation in our sample.

Panel A: Moments		
	Simulated Moments	Data Moments
Mean of operating income	0.1201	0.1387
Variance of operating income	0.0056	0.0068
Serial correlation of operating income	0.6270	0.7920
Mean of investment	0.1723	0.2018
Skewness of investment	1.3465	1.9872
Variance of investment	0.0531	0.0516
Serial correlation of investment	0.2167	0.3655
Mean of leverage	0.2965	0.2121
Variance of leverage	0.0365	0.0461
Serial correlation of leverage	0.6813	0.9173
Mean of equity issues	0.0107	0.0205
Variance of equity issues	0.0042	0.0028
Mean of cash holdings	0.1277	0.1632
Variance of cash holdings	0.0650	0.0294
Serial correlation of cash holdings	0.7298	0.8859
Mean of undrawn debt capacity	0.5278	0.8114
Variance of undrawn debt capacity	0.0926	0.0744
Serial correlation of undrawn debt capacity	0.6813	0.6344
Mean Tobin's Q	2.0666	1.5594

Panel B: Calibrated Parameters											
α	f	θ	γ	ψ_0^+	ψ^+	ψ_0^-	ψ^-	ρ_z	σ_z	λ_0	λ_1
0.6800	0.1000	0.7500	0.0020	0.0075	0.1100	0.0400	0.2000	0.7100	0.2900	0.6000	0.0080

Table 4.3. Leverage, Cash, and Conditional Liquidity: Simulated Stylized Evidence.

The table reports stylized evidence from sorts of companies by net worth and capital (the state variables of our model). Data are simulated from the model under the baseline parametrization in table 4.2 for a panel of 1000 firms and 100 time periods. The breakpoints for defining relative constrained and unconstrained firms for the sorts on net worth, and relatively small and large firms for the sorts on capital are the 33th and the 66th percentile of the cross-sectional distribution. Panel A reports average cash holdings and debt as a fraction of capital, and the fraction of undrawn debt capacity for one-way sorts, panel B reports the same variables for two-way sorts, and panel C reports the fraction of conditional to total liquidity for both simulated, and actual data.

Panel A: Univariate Sorts

	Net Worth			Capital		
	Constr.	2	Unconstr.	Small	2	Large
Cash Holdings	0.129	0.122	0.133	0.334	0.051	0.000
Leverage	0.303	0.282	0.305	0.162	0.281	0.450
Undrawn Credit	0.518	0.550	0.515	0.742	0.553	0.283

Panel B: Bivariate Sorts

		Cash Holdings			Leverage			Undrawn Credit		
	Capital	Small	2	Large	Small	2	Large	Small	2	Large
	Constr.	0.228	0.025	0.000	0.211	0.357	0.560	0.664	0.432	0.108
Net Worth	2	0.310	0.035	0.000	0.119	0.291	0.531	0.811	0.537	0.155
	Unconstr.	0.872	0.132	0.000	0.072	0.120	0.404	0.885	0.808	0.357

Panel C: Liquidity Composition

		Conditional Liquidity (model)			Conditional Liquidity (data)		
	Capital	Small	2	Large	Small	2	Large
	Constr.	0.681	0.922	1.000	0.377	0.551	0.497
Net Worth	2	0.679	0.916	1.000	0.258	0.479	0.621
	Unconstr.	0.430	0.824	1.000	0.250	0.390	0.534

Table 4.4. Comparative Statics: Debt Capacity.

The table reports simulated evidence from the model for the same panel of table 4.2. All parameters except θ are set to the baseline values of table 4.2. In each panel, we report average investment, leverage, equity issses, cash, and residual debt capacity (conditional liquidity). Averages are reported both for all firms, and for firms with low, medium, and high capital and net worth. Breakpoints are set as in panel A of tables 4.1 and 4.3. All variables are measured as in table 4.2. Panel A reports simulated moments for $\theta = 0.1$, panel B for $\theta = 0.5$, and panel C for $\theta = 0.9$.

A. $\theta = 0.1000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.170	0.215	0.173	0.123	0.191	0.166	0.153
Leverage	0.047	0.039	0.046	0.056	0.044	0.048	0.049
Equity Issues	0.007	0.016	0.003	0.001	0.017	0.002	0.002
Cash	0.513	0.917	0.492	0.129	0.547	0.702	0.282
Residual Debt Capacity	0.438	0.536	0.452	0.327	0.472	0.425	0.419

B. $\theta = 0.5000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.169	0.210	0.179	0.119	0.195	0.166	0.148
Leverage	0.195	0.121	0.157	0.309	0.176	0.158	0.253
Equity Issues	0.007	0.012	0.004	0.006	0.013	0.005	0.005
Cash	0.159	0.409	0.069	0.003	0.239	0.215	0.022
Residual Debt Capacity	0.534	0.711	0.624	0.263	0.580	0.622	0.397

C. $\theta = 0.9000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.171	0.216	0.172	0.124	0.213	0.179	0.120
Leverage	0.307	0.174	0.234	0.514	0.350	0.202	0.372
Equity Issues	0.005	0.009	0.005	0.002	0.010	0.004	0.002
Cash	0.134	0.382	0.020	0.003	0.198	0.190	0.011
Residual Debt Capacity	0.593	0.769	0.690	0.318	0.535	0.733	0.507

Table 4.5. Comparative Statics: Smooth Disinvestment Adjustment Costs.

The table reports simulated evidence from the model for the same panel of table 4.2. All parameters except ψ^- are set to the baseline values of table 4.2. In each panel, we report average investment, leverage, equity issses, cash, and residual debt capacity (conditional liquidity). Averages are reported both for all firms, and for firms with low, medium, and high capital and net worth. Breakpoints are set as in panel A of tables 4.1 and 4.3. All variables are measured as in table 4.2. Panel A reports simulated moments for $\psi^- = 0$, panel B for $\psi^- = 0.5$, and panel C for $\psi^- = 1$.

A. $\psi^- = 0.0000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.174	0.219	0.186	0.116	0.217	0.159	0.146
Leverage	0.310	0.231	0.250	0.450	0.269	0.328	0.333
Equity Issues	0.015	0.031	0.012	0.003	0.035	0.008	0.002
Cash	0.095	0.251	0.035	0.000	0.171	0.044	0.071
Residual Debt Capacity	0.479	0.611	0.579	0.243	0.547	0.449	0.441

B. $\psi^- = 0.5000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.172	0.228	0.174	0.115	0.233	0.148	0.137
Leverage	0.300	0.198	0.257	0.445	0.279	0.289	0.331
Equity Issues	0.011	0.022	0.008	0.002	0.027	0.003	0.003
Cash	0.108	0.276	0.050	0.001	0.147	0.093	0.086
Residual Debt Capacity	0.496	0.666	0.568	0.252	0.531	0.514	0.443

C. $\psi^- = 1.0000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.172	0.227	0.166	0.121	0.195	0.186	0.134
Leverage	0.241	0.147	0.197	0.391	0.250	0.202	0.274
Equity Issues	0.007	0.015	0.005	0.002	0.016	0.003	0.003
Cash	0.212	0.529	0.102	0.003	0.179	0.323	0.130
Residual Debt Capacity	0.594	0.753	0.668	0.342	0.580	0.660	0.540

Table 4.6. Comparative Statics: Smooth Investment Adjustment Costs.

The table reports simulated evidence from the model for the same panel of table 4.2. All parameters except ψ^+ are set to the baseline values of table 4.2. In each panel, we report average investment, leverage, equity issses, cash, and residual debt capacity (conditional liquidity). Averages are reported both for all firms, and for firms with low, medium, and high capital and net worth. Breakpoints are set as in panel A of tables 4.1 and 4.3. All variables are measured as in table 4.2. Panel A reports simulated moments for $\psi^+ = 0$, panel B for $\psi^+ = 0.5$, and panel C for $\psi^+ = 1$.

A. $\psi^+ = 0.0000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.224	0.422	0.197	0.050	0.275	0.226	0.172
Leverage	0.240	0.140	0.164	0.419	0.251	0.150	0.322
Equity Issues	0.007	0.010	0.007	0.005	0.014	0.006	0.003
Cash	0.314	0.814	0.123	0.001	0.412	0.457	0.070
Residual Debt Capacity	0.596	0.765	0.724	0.295	0.578	0.747	0.458

B. $\psi^+ = 0.5000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.158	0.177	0.158	0.134	0.157	0.169	0.147
Leverage	0.321	0.353	0.211	0.458	0.386	0.242	0.338
Equity Issues	0.016	0.040	0.004	0.005	0.040	0.006	0.002
Cash	0.069	0.148	0.047	0.003	0.090	0.095	0.021
Residual Debt Capacity	0.461	0.407	0.646	0.230	0.351	0.594	0.432

C. $\psi^+ = 1.0000$

		Capital			Net Worth		
	All	Low	2	High	Low	2	High
Investment	0.155	0.183	0.140	0.142	0.094	0.195	0.175
Leverage	0.403	0.424	0.483	0.301	0.514	0.444	0.251
Equity Issues	0.054	0.120	0.036	0.006	0.161	0.001	0.001
Cash	0.022	0.005	0.024	0.036	0.004	0.017	0.045
Residual Debt Capacity	0.322	0.286	0.188	0.495	0.135	0.254	0.578

4.A Proofs of Propositions

Proof of Lemma 6. From the definition of $h_i^C(s(t+1))$ we obtain:

$$b_i(s(t+1)) = \frac{\theta(1-\delta)k_{i,t+1} - h_i^C(s(t+1))}{1+r(1-\tau)} \tag{4.A.1}$$

Substituting (4.A.1) and the definition of $h_{i,t+1}^U$ into the original problem yields the result. ∎

Proof of Proposition 4. Denote the total number of states by S. The Lagrangian function for the constrained optimization problem is:

$$\mathcal{L} \equiv e_{i,t} - \Lambda(e_{i,t}) + \tfrac{1}{1+r}E_t[V(w_{i,t+1}, z_{i,t+1})] + \lambda^w(w_{i,t} - e_{i,t} - E_t\left[\tfrac{h_i^C(s(t+1))}{1+r(1-\tau)}\right] - \tfrac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma} - Pk_{i,t+1} - \Psi(k_{i,t}, k_{i,t+1})) +$$
$$+\sum_{s=1}^S \tfrac{\pi(s(t),s)\lambda_s^w}{1+r}((1-\tau)\Pi(k_{i,t+1}, z_{i,s}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s) - w_i(s)) +$$
$$+\sum_{s=1}^S \tfrac{\pi(s(t),s)\underline{\lambda}_s^C}{1+r}(h_i^C(s)) + \sum_{s=1}^S \tfrac{\pi(s(t),s)\overline{\lambda}_s^C}{1+r}(\theta(1-\delta)k_{i,t+1} - h_i^C(s)) + \underline{\lambda}^U(h_{i,t+1}^U)$$

Differentiating the Lagrangian with rispect to $e_{i,t}$, $k_{i,t+1}$, $h_{i,t+1}^U$, $\{h_i^C(s(t+1))\}$, and $\{w_i(s(t+1))\}$ yields equations (4.19a), (4.19b), (4.19c), (4.19d), (4.21b) after some algebraic manipulation. Because the Slater condition holds, the envelope theorem can expressed as:

$$\frac{\partial V(w_{i,t}, z_{i,t})}{\partial w_{i,t}} = \frac{\partial e_{i,t} - \Lambda(e_{i,t})}{\partial w_{i,t}} + \lambda^w \frac{\partial(w_{i,t} - e_{i,t} - E_t\left[\frac{h_i^C(s(t+1))}{1+r(1-\tau)}\right] - \frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma} - Pk_{i,t+1} - \Psi(k_{i,t}, k_{i,t+1}))}{\partial w_{i,t}} +$$
$$+\sum_{s=1}^S \frac{\pi(s(t),s)\lambda_s^w}{1+r}\frac{\partial((1-\tau)\Pi(k_{i,t+1}, z_{i,s}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s) - w_i(s))}{\partial w_{i,t}} + \sum_{s=1}^S \frac{\pi(s(t),s)\underline{\lambda}_s^C}{1+r}\frac{\partial(h_i^C(s))}{\partial w_{i,t}} +$$
$$+\sum_{s=1}^S \frac{\pi(s(t),s)\overline{\lambda}_s^C}{1+r}\frac{\partial(\theta(1-\delta)k_{i,t+1} - h_i^C(s))}{\partial w_{i,t}} +$$
$$\underline{\lambda}^U \frac{\partial(h_{i,t+1}^U)}{\partial w_{i,t}}$$

which immediately yields (4.21a). The Euler equation (4.22) can be simply obtained, by dividing both sides of (4.19b) by λ^w. The division is well-defined because the resource constraint at time t is always binding. Finally, equation (4.23) can be derived by substituting $\lambda_{s(t+1)}^w$ from (4.19d) into the definition of $M^w(s(t), s(t+1))$. ∎

Proof of Proposition 5. To prove the claim, we proceed in two steps. First, we show that the payoff $b_i(s(t+1))$can be replicated with the combination of securities described above. Second, we verify that the recursive problem with the new securities is equivalent to the original one in terms of constraints. First, in the recursive problem, at time $t+1$ in each state $s(t+1)$the firm pays back $D_{i,t+1} - C_i^L(s(t+1))$Therefore, using (4.31) and (4.32) we obtain

$$D_{i,t+1} - C_i^L(s(t+1)) = (1+r(1-\tau))b_i(s(t+1)) \tag{4.A.2}$$

from which the replication result follows:

$$b_i(s(t+1)) = \frac{D_{i,t+1} - C_i^L(s(t+1))}{1+r(1-\tau)} \tag{4.A.3}$$

Because state-contingent debt can be directly expressed as the combination in (4.A.3) of state-uncontingent debt and the credit line, the replicating strategy is trivially budget feasible at time $t+1$. The resource constraint at time t is also unchanged, because

$$w_{i,t} + L_{i,t} \geq e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

can be rewritten as

$$w_{i,t} + E_t[b_i(s(t+1))] \geq e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

using (4.30) and (4.A.3). Finally, we shall show that the limits for the feasible set for $b_i(s(t+1))$ implied by collateral and debt positivity constraints are preserved by the replicating portfolio of debt and lines of credit, namely that:

$$0 \leq \frac{D_{i,t+1} - C_i^L(s(t))}{1 + r(1-\tau)} \leq \theta(1-\delta)k_{i,t+1}$$

The debt positivity constraints can be rewritten as:

$$C_i^L(s(t+1)) \leq (1 + r(1-\tau))E_t[b(s(t+1))]$$

which is consistent with the feasible set for $C_i^L(s(t+1))$ because:

$$\max C_i^L(s(t+1)) = \overline{C_{i,t+1}}$$

$\blacksquare$

Bibliography

Árpád Ábrahám and Kirk White. The dynamics of plant-level productivity in us manufacturing. *Center for Economic Studies Working Paper*, 6:20, 2006.

Dilip Abreu, David Pearce, and Ennio Stacchetti. Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica*, 58(5):1041–1063, 1990.

Viral V. Acharya, Heitor Almeida, and Murillo Campello. Is cash negative debt? - a hedging perspective on corporate financial policies. *Journal of Financial Intermediation*, 16(4):515–554, 2007.

Seung C. Ahn and Christopher Gadarowski. Small sample properties of the GMM specification test based on the Hansen–Jagannathan distance. *Journal of Empirical Finance*, 11(1):109–132, 2004.

Rui Albuquerque and Hugo A. Hopenhayn. Optimal lending contracts and firm dynamics. *The Review of Economic Studies*, 71(2):285–315, 2004.

Charalambos D. Aliprantis and Kim C. Border. *Infinite dimensional analysis: a hitchhiker's guide*. Springer, 2006.

Heitor Almeida, Murillo Campello, and Michael S. Weisbach. The cash flow sensitivity of cash. *The Journal of Finance*, 59(4):1777–1804, 2004.

Bruno Amable, Jean-Bernard Chatelain, and Kirsten Ralf. Patents as collateral. *Journal of Economic Dynamics and Control*, 34(6):1092–1104, 2010.

Badi H. Baltagi. *Econometric Analysis of Panel Data*. John Wiley and Sons, West Sussex, 2008.

Santiago Bazdrech, Frederico Belo, and Xiaoji Lin. Labor hiring, investment and stock return predictability in the cross section. *Journal of Political Economy*, 2013. forthcoming.

Frederico Belo. Production-based measures of risk for asset pricing. *Journal of Monetary Economics*, 57(2):146–163, 2010.

Jonathan B. Berk. A critique of size-related anomalies. *Review of Financial Studies*, 8 (2):275–286, 1995.

Jonathan B. Berk, Richard C. Green, and Vasant Naik. Does the stock market overreact? *The Journal of Finance*, 54(5):1513–1607, 1999.

Laxmi Chand Bhandari. Debt/equity ratio and expected common stock returns: Empirical evidence. *The Journal of Finance*, 43(2):507–528, 1988.

Richard Blundell and Stephen Bond. Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics*, 87(1):115–143, 1998.

Patrick Bolton, Hui Chen, and Neng Wang. A unified theory of Tobin's Q, corporate investment, financing, and risk management. *The Journal of Finance*, 66(5):1545–1578, 2011.

Patrick Bolton, Hui Chen, and Neng Wang. Market timing, investment, and risk management. Working Paper, 2 2012.

Michael W. Brandt and David A. Chapman. Linear approximations and tests of conditional pricing models. Working Paper, 2006.

Craig Burnside. Identification and inference in linear stochastic discount factor models. Working Paper, 2010.

Craig A. Burnside, Martin S. Eichenbaum, and Sergio T. Rebelo. Sectoral Solow residuals. *European Economic Review*, 40(3):861–869, 1996.

Ricardo J. Caballero and Eduardo M. R. A. Engel. Explaining investment dynamics in u.s. manufacturing: A generalized (s,s) approach. *Econometrica*, 67(4):784–826, 1999.

Ricardo J. Caballero, Eduardo M. R. A. Engel, and John C. Haltiwanger. Plant-level adjustment and aggregate investment dynamics. *Brookings Papers on Economic Activity*, 1995(2):1–54, 1995.

John Y. Campbell and Tuomo Vuolteenaho. Bad beta, good beta. *American Economic Review*, pages 1249–1275, 2004.

Mark M. Carhart. On persistence in mutual fund performance. *The Journal of Finance*, 52(1):57–82, 1997.

Murray Carlson, Adlai Fisher, and Ron Giammarino. Corporate investment and asset price dynamics: Implications for the cross-section of returns. *The Journal of Finance*, 59(6):2577–2603, 2004.

BenoÃ®t Carmichael and Alain Coen. Asset pricing models with errors-in-variables. *Journal of Empirical Finance*, 15(4):778–788, 2008.

Louis K. C. Chan, Jason Karceski, and Josef Lakonishok. The risk and return from factors. *Journal of Financial and Quantitative Analysis*, 33(2):160–188, 1998.

Xin Chang and Sudipto Dasgupta. Target behavior and financing: How conclusive is the evidence? *The Journal of Finance*, 64(4):1767–1796, 2009.

John H. Cochrane. Production-based asset pricing and the link between stock returns and economic fluctuations. *The Journal of Finance*, 46(1):209–237, 1991.

John H. Cochrane. Rethinking production under uncertainty. *Manuscript, University of Chicago*, 1993.

John H. Cochrane. A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy*, 104(3):572–621, 1996.

John H. Cochrane. *Asset Pricing*. Princeton University Press, Princeton, NJ, 2001.

John H. Cochrane. Presidential address: Discount rates. *The Journal of Finance*, 66(4): 1047–1108, 2011.

Paolo Colla, Filippo Ippolito, and Kai Li. Debt specialization. *The Journal of Finance*, 2013. forthcoming.

William J. Cook, William H. Cunningham, William R. Pulleyblank, and Alexander Schrijver. *Combinatorial optimization*, volume 33. Wiley. com, 2011.

Thomas F. Cooley and Edward C. Prescott. Economic growth and business cycles. *Frontiers of business cycle research*, pages 1–38, 1995.

Russell W. Cooper and John C. Haltiwanger. On the nature of capital adjustment costs. *Review of Economic Studies*, 73(3):611–633, 2006.

Francisco Covas and Wouter J. Den Haan. The role of debt and equity finance over the business cycle. *The Economic Journal*, 122(565):1262–1286, 2012.

Aswath Damodaran. *Damodaran on valuation*. Wiley. com, 2008.

Kent Daniel and Sheridan Titman. Market reactions to tangible and intangible information. *The Journal of Finance*, 61(4):1605–1643, 2006.

Kent Daniel and Sheridan Titman. Testing factor-model explanations of market anomalies. *Critical Finance Review*, 1(1):103–139, 2012.

Russell Davidson and James G. MacKinnon. *Estimation and inference in econometrics.* Oxford University Press, New York, 1993.

Harry DeAngelo, Linda DeAngelo, and Toni M. Whited. Capital structure dynamics and transitory debt. *Journal of Financial Economics*, 99(2):235–261, 2011.

Eric V. Denardo. On linear programming in a Markov decision problem. *Management Science*, pages 281–288, 1970.

David J. Denis and Valeriy Sibilkov. Financial constraints, investment, and the value of cash holdings. *Review of Financial Studies*, 23(1):247–269, 2009.

Andrea L. Eisfeldt and Tyler Muir. Aggregate issuance and savings waves. Working Paper, 1 2013.

Antonio Falato, Dalida Kadyrzhanova, and Jae Sim. Intangible capital and corporate cash holdings. Working Paper, 6 2013.

Eugene F. Fama and Kenneth R. French. The cross-section of expected stock returns. *The Journal of Finance*, 47(2):427–465, 1992.

Eugene F. Fama and Kenneth R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56, 1993.

Eugene F. Fama and Kenneth R. French. Dissecting anomalies. *The Journal of Finance*, 63(4):1653–1678, 2008.

Eugene F. Fama and James D. MacBeth. Risk, return and equilibrium: Empirical tests. *Journal of Political Economy*, 81(3):607–636, 1973.

Michael Faulkender, Mark J. Flannery, Kristine Watson Hankins, and Jason M. Smith. Cash flows and leverage adjustments. Working Paper, 8 2010.

John Fernald. A quarterly, utilization-adjusted series on total factor productivity. Working Paper, 2009.

Wayne E. Ferson and Stephen R. Foerster. Finite sample properties of the Generalized Method of Moments in tests of conditional asset pricing models. *Journal of Financial Economics*, 36(1):29–55, 1994.

Edwin O. Fischer, Robert Heinkel, and Josef Zechner. Dynamic capital structure choice: Theory and tests. *The Journal of Finance*, 44(1):19–40, 1989.

Mark J. Flannery and Kristine Watson Hankins. Estimating dynamic panel models in corporate finance. Working Paper, 2 2010.

Mark J. Flannery and Ozde Oztekin. Institutional determinants of capital structure adjustment speeds. *Journal of Financial Economics*, 103(1):88–112, 2012.

Mark J. Flannery and Kasturi P. Rangan. Partial adjustment toward target capital structures. *Journal of Financial Economics*, 79(3):469–506, 2006.

Murray Z. Frank and Vidhan H. Goyal. Trade-off and pecking order theories of debt. In BjÃ¸rn Espen Eckbo, editor, *Handbook of corporate finance: empirical corporate finance*, pages 135–202. North Holland, 2008.

Douglas Gale and Martin Hellwig. Incentive-compatible debt contracts: The one period problem. *Review of Economic Studies*, 52(5):647–663, 1985.

Andrea Gamba and Alexander J. Triantis. The value of financial flexibility. *The Journal of Finance*, 63(5):2263–2296, 2008.

Lorenzo Garlappi and Hong Yan. Financial distress and the cross-section of equity returns. *The Journal of Finance*, 66(3):789–822, 2011.

Thomas J. George and Chuan-Yang Hwang. A resolution of the distress risk and leverage puzzles in the cross section of stock returns. *Journal of Financial Economics*, 96(1): 56–79, 2010.

Mark L. Gertler and Peter Karadi. A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34, 2011.

Mark L. Gertler and Nobuhiro Kiyotaki. Financial intermediation and credit policy in business cycle analysis. In Benjamin M. Friedman and Michael Woodford, editors, *Handbook of Monetary Economics, Chaper 11*. Elsevier, 2010.

Eric Ghysels, Pedro Santa-Clara, and Rossen Valkanov. The MIDAS touch: mixed data sampling regression models. Working Paper, 2004.

Eric Ghysels, Pedro Santa-Clara, and Rossen Valkanov. There is a risk-return trade-off after all. *Journal of Financial Economics*, 76(3):509–548, 2005.

Michael R. Gibbons, Stephen A. Ross, and Jay Shanken. A test of the efficiency of a given portfolio. *Econometrica*, 57(5):1121–1152, 1989.

Robert Goldstein, Nengjiu Ju, and Hayne Leland. An ebit-based model of dynamic capital structure. *The Journal of Business*, 74(4):483–512, 2001.

Joao F. Gomes. Financing investment. *The American Economic Review*, 91(5):1263–1285, 2001.

Joao F. Gomes and Lukas Schmid. Levered returns. *The Journal of Finance*, 65(2): 467–494, 2010.

William H. Greene. *Econometric Analysis, 6th Edition*. Prentice Hall, New Jersey, 2008.

Wayne Guay and S.P. Kothari. How much do firms hedge with derivatives? *Journal of Financial Economics*, 70(3):423–461, 2003.

Lars P. Hansen. Large sample properties of Generalized Method of Moments estimators. *Econometrica*, 50(4):1029–1054, 1982.

Lars Peter Hansen and Ravi Jagannathan. Assessing specification errors in stochastic discount factor models. *The Journal of Finance*, 52(2):557–590, 1997.

Lars Peter Hansen and Scott F. Richard. The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica*, pages 587–613, 1987.

Lars Peter Hansen and Kenneth J Singleton. Generalized instrumental variables estimation of nonlinear rational expectations models. *Econometrica: Journal of the Econometric Society*, pages 1269–1286, 1982.

Zhiguo He and Arvind Krishnamurthy. Intermediary asset pricing. *American Economic Review*, 103(2):732–770, 2013.

Christopher A. Hennessy and Toni M. Whited. Debt dynamics. *The Journal of Finance*, 60(3):1129–1165, 2005.

Christopher A. Hennessy and Toni M. Whited. How costly is external financing? Evidence from a structural estimation. *The Journal of Finance*, 62(4):1705–1745, 2007.

Armen Hovakimian, Tim Opler, and Sheridan Titman. The debt-equity choice: an analysis of issuing firms. *Journal of Financial and Quantitative Analysis*, 36(1):1–24, 2001.

Ronald A. Howard. *Dynamic Programming and Markov Processes*. John Wiley, 1960.

Cheng Hsiao. *Analysis of Panel Data*. Cambridge University Press, Cambridge, UK, 2003.

Rongbing Huang and Jay R. Ritter. Testing theories of capital structure and estimating the speed of adjustment. *Journal of Financial and Quantitative Analysis*, 44(2):237–271, 2009.

Peter Iliev and Ivo Welch. Reconciling estimates of the speed of adjustment of leverage ratios. Working Paper, 10 2010.

Dwight M. Jaffee and Thomas Russell. Imperfect information, uncertainty, and credit rationing. *The Quarterly Journal of Economics*, 90(4):651–666, 1976.

Ravi Jagannathan and Zhenyu Wang. The conditional CAPM and the cross-section of expected returns. *The Journal of Finance*, 51(1):3–53, 1996.

Urban J. Jermann. The equity premium implied by production. *Journal of Financial Economics*, 98(2):279–296, 2010.

Takashi Kamihigashi. Elementary results on solutions to the Bellman equation of dynamic programming: Existence, uniqueness, and convergence. Technical report, 2012.

Dongcheol Kim. Issues related to the errors-in-variables problems in asset pricing tests. In Cheng-Few Lee, Alice C. Lee, and John Lee, editors, *Handbook of Quantitative Finance and Risk Management*, pages 1091–1108. Springer, 2010.

Robert A. Korajczyk and Amnon Levy. Capital structure choice: Macroeconomic conditions and financial constraints. *Journal of Financial Economics*, 68(1):75–109, 2003.

Arthur Korteweg. The net benefits to leverage. *The Journal of Finance*, 65(6):2137–2170, 2010.

Mark T. Leary and Michael R. Roberts. Do firms rebalance their capital structures? *The Journal of Finance*, 60(6):2575–2619, 2005.

Hayne E. Leland. Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, 49(4):1213–1252, 1994.

Michael L. Lemmon, Michael L. Roberts, and Jaime F. Zender. Back to the beginning: Persistence and the cross-section of corporate capital structure. *The Journal of Finance*, 63(4):1575–1608, 2008.

Martin Lettau and Sydney Ludvigson. Consumption, aggregate wealth, and expected stock returns. *the Journal of Finance*, 56(3):815–849, 2001.

Kenneth Levenberg. A method for the solution of certain problems in least squares. *Quarterly of Applied Mathematics*, 2:164–168, 1944.

Jonathan Lewellen, Stefan Nagel, and Jay Shanken. A skeptical appraisal of asset pricing tests. *Journal of Financial Economics*, 96(2):175–194, 2010.

Erica X.N. Li, Dmitry Livdan, and Lu Zhang. Anomalies. *Review of Financial Studies*, 22(11):4301–4334, 2009.

Shaojin Li and Toni Whited. Endogenous financial constraints, taxes, and leverage. Working Paper, 7 2013.

Dmitry Livdan, Horacio Sapriza, and Lu Zhang. Financially constrained stock returns. *The Journal of Finance*, 64(4):1827–1862, 2009.

Brandon G. Lockhart. Adjusting to target capital structure: the effect of credit lines. Working Paper, 9 2010.

Donald W. Marquardt. An algorithm for least-squares estimation of nonlinear parameters. *Journal of the Society for Industrial and Applied Mathematics*, 11(2):431–441, 1963.

Ellen R. McGrattan. *Application of weighted residual methods to dynamic economic models*. Federal Reserve Bank of Minneapolis, Research Department, 1997.

Enrique G. Mendoza. Sudden stops, financial crises, and leverage. *American Economic Review*, 100(5):1941–1966, 2000.

Robert C. Merton. An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, pages 867–887, 1973.

Franco Modigliani and Merton Miller. The cost of capital, corporation finance and the theory of investment. *American Economic Review*, 48(3):655–669, 1958.

Erwan Morellec and Boris Nikolov. Cash holdings and competition. Working Paper, 10 2009.

George L. Nemhauser and Laurence A. Wolsey. *Integer and combinatorial optimization*, volume 18. Wiley New York, 1988.

Whitney K Newey and Daniel McFadden. Large sample estimation and hypothesis testing. *Handbook of econometrics*, 4:2111–2245, 1994.

Boris Nikolov and Lukas M. Schmid. Testing dynamic agency theory via structural estimation. Working Paper, 3 2012.

Boris Nikolov and Toni M. Whited. Agency conflicts and cash: Estimates from a structural model. Working Paper, 11 2009.

Boris Nikolov, Lukas M. Schmid, and Roberto Steri. Dynamic corporate liquidity. Working Paper, 8 2013.

Iulian Obreja. Book-to-market, financial leverage, and the cross-section of stock returns. *Review of Financial Studies*, 2013. forthcoming.

Iulien Obreja. Book-to-market equity, financial leverage and the cross-section of stock returns. Working Paper, 5 2010.

Ali K. Ozdagli. Financial leverage, corporate investment, and stock returns. *Review of Financial Studies*, 4:1033–1069, 2012.

Stephen H. Penman, Scott A. Richardson, and Irem Tuna. The book-to-price effect in stock returns: Accounting for leverage. *Journal of Accounting Research*, 45(2):427–467, 1992.

Simon M. Potter. Nonlinear impulse response functions. *Journal of Economic Dynamics and Control*, 24(10):1425–1446, 2000.

William E. Pruitt. Summability of independent random variables. *Journal of Mathematical and Mechanics*, 15:769–776, 1966.

Daniela Pucci de Farias and Benjamin Van Roy. The linear programming approach to approximate dynamic programming. *Operations Research*, 51(6):850–865, 2003.

Matthew Rabin. Risk aversion and expected-utility theory: A calibration theorem. *Econometrica*, 68(5):1281–1292, 2000.

Adriano A. Rampini and S. Viswanathan. Collateral, risk management, and the distribution of debt capacity. *The Journal of Finance*, 65(6):2293–2322, 2010.

Adriano A. Rampini and S. Viswanathan. Collateral and capital structure. Working Paper, 2 2012a.

Adriano A. Rampini and S. Viswanathan. Household risk management. Working Paper, 9 2012b.

Adriano A. Rampini and S. Viswanathan. Collateral and capital structure. *Journal of Financial Economics*, 109(2):466–492, 2013.

Adriano A. Rampini, Amir Sufi, and S. Viswanathan. Dynamic risk management. Working Paper, 8 2012.

Adriano A. Rampini, Amir Sufi, and S. Viswanathan. Dynamic risk management. *Journal of Financial Economics*, 2013. forthcoming.

V.K. Rohatgi. Convergence of weighted sums of independent random variables. In *Mathematical Proceedings of the Cambridge Philosophical Society*, volume 69, pages 305–307. Cambridge Univ Press, 1971.

Sheldon M. Ross. *Recursive Methods in Economic Dynamics.* Academic Press, New York, NY, 1983.

Stephen A. Ross. The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, 13:341–360, 1976.

John Rust. Numerical dynamic programming in economics. *Handbook of computational economics*, 1:619–729, 1996.

John P. Rust. Using randomization to break the curse of dimensionality. *Econometrica*, 65(3):487–516, 1997.

Lukas M. Schmid and Roberto Steri. A quantitative dynamic agency model of financing constraints. Working Paper, 10 2013.

Alexander Schrijver. *Theory of linear and integer programming.* Wiley. com, 1998.

Stephen E. Spear and Sanjay Srivastava. On repeated moral hazard with discounting. *Review of Economic Studies*, 54(4):599–617, 1987.

Nancy L. Stokey. *The Economics of Inaction: Stochastic Control Models with Fixed Costs.* Princeton University Press, Princeton, NJ, 2008.

Nancy L. Stokey and Robert E. Lucas. *Recursive Methods in Economic Dynamics.* Harvard University Press, Cambridge, MA, 1989.

Ilya A. Strebulaev. Do tests of capital structure theory mean what they say? *The Journal of Finance*, 62(4):1747–1787, 2007.

Ilya A. Strebulaev and Toni M. Whited. Dynamic models and structural estimation in corporate finance. *Foundations and Trends in Finance*, 6(1–2):1–163, 2012.

Amir Sufi. Bank lines of credit in corporate finance: An empirical analysis. *Review of Financial Studies*, 22(3):1057–1088, 2009.

Alfred Tarski. A lattice-theoretical fixpoint theorem and its applications. *Pacific journal of Mathematics*, 5(2):285–309, 1955.

Jonathan Thomas and Tim Worrall. Foreign direct investment and the risk of expropriation. *The Review of Economic Studies*, 61(1):81–108, 1994.

Michael A. Trick and Stanley E. Zin. A linear programming approach to solving stochastic dynamic programs. Working Paper, 8 1993.

Michael A. Trick and Stanley E. Zin. Spline approximations to value functions: A linear programming approach. *Macroeconomic Dynamics*, 1(1):255–277, 1997.

Peter Tufano. Who manages risk? an empirical examination of risk management practices in the gold mining industry. *The Journal of Finance*, 51(4):1097–1137, 1996.

Juan Pablo Vielma. Mixed integer linear programming formulation techniques. Working Paper, 2013.

Motohiro Yogo. A consumption-based explanation of expected stock returns. *The Journal of Finance*, 61(2):539–580, 2006.

Lu Zhang. The value premium. *The Journal of Finance*, 60(1):67–103, 2005.