Mathematics for social integration

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Abstract. This work presents preliminary research findings from the Italian project Teenagers Experience Empowerment by Numbers (TEEN), funded by the Politecnico di Milano through the Polisocial Award 2017. The project TEEN (www.teen.polimi.it) deals with the phenomenon of young immigrants, the so-called "teen-immigrants", and aims at promoting basic mathematical literacy as another fundamental right that may significantly increase the level of autonomy of teen-immigrants resorting to activities that have a mathematics root.

Keywords: mathematics education, app, mathematical skill, geometry, out-of-school mathematics

1 Introduction

The project TEEN (Teenagers Experience Empowerment by Numbers http://www.teen.polimi.it), founded by Politecnico di Milano through the Poliscocial Award 2017, deals with the phenomenon of young immigration towards Europe. More precisely, such a phenomenon concerns the underage who leave their country without parents and relatives, we called them "teen-immigrants" [1,2]. In Italy, the number of teen-immigrants has reached a peak of 18 thousands in 2017 [3]. Among them, the 93.3% ranges within 15yo and 17yo, while the majority of teen-immigrants are male (93.2% in 2017). Fig. 1 shows the distribution of teen-immigrants towards Italy, the 69.9% of teen-immigrants come from Africa (e.g., Gambia 2202, Egypt 1807, Guinea 1752), a relatively low percentage of underage come from Albania (9.2%) and Bangladesh (4.7%).

Once teen-immigrants arrive in Italy, they are accommodated in communities for minors, that provide them for their basic needs (accommodation, food, health services and a language course). That turns out to be insufficient to deal with the requirements of the "real world" that they need to face early, considering that the protection guaranteed by the Italian legislation to unaccompanied minors ends on the day of their 18th birthday. The TEEN project aims at promoting basic mathematical literacy as another fundamental right that may significantly increase the level of autonomy of teen-immigrants resorting to activities that have a mathematics root.

The strength of the project is the idea that mathematics can be pivotal in the social integration of disadvantaged students [4-8]. The successful experiences in different contexts inform that a part of considering the social and ethical implications of teaching mathematics to a “minority” [6], students learn better when mathematics is related to their identity and their attitudes [5,8] and students turn out to be more aware of the role of mathematics when teachers exploit the students' language and the way they interact to design the activities [7]. These two considerations reinforce the claim that it is necessary to promote a new idea of mathematics as valuable skills for social integration and to create alternative curricula with the purpose of showing mathematics as accessible (and useful) [9].

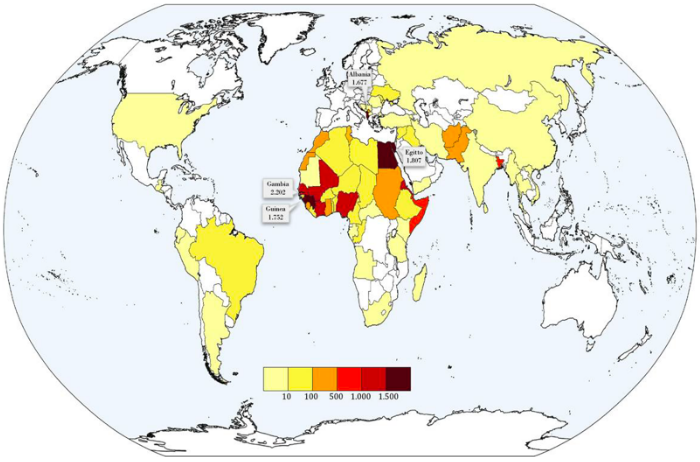


Fig.1 Distribution of the origin countries of teen-immigrants in 2017. Source [3]

On the path of these previous experiences, the TEEN project aims at developing the mobile app *StreetMath* that can be used by teen-immigrants, outside the classroom and without the mathematics teacher. Moreover, in order to make the discipline more accessible and attractive, the app is designed to deal with everyday real problems, with non-academic language and, to be inclusive, with the least possible amount of written words through the extensive use of graphical support.

In this work, we present examples of activities focusing on geometry and preliminary research findings of the project from the experimental learning session with small groups of immigrants organized in the communities where they live.

2 StreetMath: the learning environment

StreetMath is the web app developed by the interdisciplinary team of the project TEEN. StreetMath is available for Android and can be also accessed by browser from any device. The activities proposed address situations that are familiar for teen-immigrants because connected to their identities and experiences. We called such situations as realistic scenarios, that are the units of the proposed educational project.

On one hand, the scenarios allow learners to recognise mathematics as an accessible and valuable skills [9], on the other hand, learners resort to their own experiences to deal with it [5,7-8]. In particular, teen-immigrants can exploit previous knowledge (scholar and not) and attitudes towards a scenario, for instance, the motivation in dealing with a task.

The app contains two types of units collected in two sections:

1. the challenge units in home page (see Fig. 2a), and
2. the math-recap units in the "mathematics" page (see Fig. 2b).

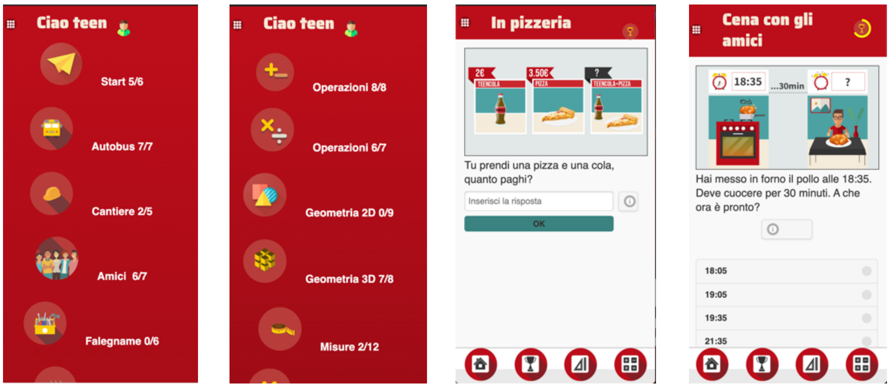
The units are composed of tasks (open-end and multiple-choice) in Italian language and are supported by images that help the understanding of the text and provide information required to answer.

The tasks are built upon realistic scenarios, but in the real-challenge are more structured with the purpose of solving a problem identified by the units: for instance, the unit called "bus" is based on the experience of reaching the workplace from home to have a job interview, so the problem consists in planning the trip. Nevertheless, also the tasks of math-recap units resort to daily-life experiences to recap the main mathematical ideas.

The unit "start", which is the first one (see at the top of Fig. 2a), was designed with the purpose of introducing learners to the environment and addressing the instrumental genesis [10] of the app, with particular focus on the utilisation scheme. The unit is composed of 6 tasks, 4 are open-end tasks concerning elementary operations (addition, subtraction, multiplication and division), while 2 tasks are multiple-choice concerning the time estimation. In Fig. 2c, we report the first task as an example of the open-end task:

1. the title (at the top) identify the task and the scenario,
2. the image supports the understanding of the task,
3. a brief text reports the question and often it may be avoided,
4. the text form, in the middle of the page, allows writing the answer using a number-keypad,
5. the button "ok" submits the answer,
6. a system of feedback and help (the "i" button) is present to support the learners both in case of wrong and right answers.

The structure of the multiple-choice task (see Fig. 2d) is similar to the open-end, except for the input-form which is replaced by the list of answers.



(a) (b) (c) (d)

Fig. 2 Screenshots from the StreetMath: (a) list of the real-challenge, (b) list of math-recap, (c) example of an open-end task, (d) example of a multiple-choice task.

The navigation bar at the bottom of the page is characterized by the three navigation buttons for (from left to right) "home", "progress", "mathematics" pages and one (the last on the right corner) that pop-ups a "calculator". We refer to "mathematics" and "calculator" buttons as tools to face with scenarios. This choice, in agreement with the idea of accessible math [9], unfolds the focus on the process and the mathematical thinking rather than the procedure and the computational skills, even though we recognise their importance in doing mathematics.

(a) (b) (c)

Fig. 3 The feedback system: example of a message of (a) "help" (b) "support" (c) "remark".

An important feature of the learning environment is the feedback system, that we intend as part of a range of practices that aid the learners' understanding [11]. The feedback system should address the goal, the progress and the quality of the performance of the learners, not only in terms of checking the correctness [1,11]. To that end, we develop a feedback system composed of three parts (see Fig. 3):

A. the aforementioned "help" (button "i" in the middle of the page, see Fig. 2c-d) and provides details and suggestion about the task,
B. the "support" that popups in case of the wrong answer and provides further suggestions accordingly to common mistakes and misconceptions, and
C. the "remark" after the correct answer is given, it provides further information about and beyond the task as well as social messages.

We note that the "support" message is composed of two parts: the first one is the suggestion after the red word "Try again", while the second one is the actual answer to the task under the button "Show answer".

We design the three parts in order to address the three notions that underlie feedback systems [11] "feed up", "feed back" and "feed forward" that work at four intertwined levels: the one the task, if they provide information about how well tasks are understood and/or performed; the level of the process; the level of self-regulation, if they concern self-monitoring, directing and regulating of actions; and the level of self, if they point to personal evaluations and positively affect learner's identities [1, 11].

Moreover, the feedback system allows learners to support their experience of *empowerment*, that roughly speaking, is the process of learning and using problem-solving skills and the achievement of perceived or actual control [2,12]. That suggests that experiences which provide opportunities to enhance perceived control help individuals to cope with stress and solve problems in their personal lives [2]. In the mathematical context, empowerment concerns the role of mathematics in daily activities and its impact both on the learning process at school and in social life [13]. Three different domains of empowerment have been identified: *mathematical*, *social*, and *epistemological*. The *mathematical* empowerment of power over the language, symbols, knowledge and skills of mathematics and the ability to confidently apply them in mathematical applications within the context of schooling, and possibly to a lesser extent, outside of this context. *Social* empowerment ranges from the straightforwardly utilitarian to the more radical 'critical mathematical citizenship'. *Epistemological* empowerment concerns the individual's growth of confidence not only in using mathematics, but also a personal sense of power over the creation and validation of knowledge [2].

StreetMath supports the empowerment thanks to its features: i) reinforcing the motivation through the realistic scenario, ii) aiding the text understanding using images, iii) boosting the mathematical process exploiting the calculator, and iv) driving towards the task resolution through the feedback system. We remark that all those features have the purpose to make learners feel as the active subject within the learning environment. In such a way teenagers can resort to the provided aids and their previous knowledge and skills in order to achieve control of the scenario as well as improve their level of mathematical, social and epistemological empowerment.

3 Examples of activities

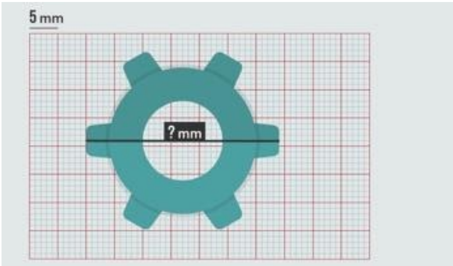
StreetMath provides activities concerning work scenarios. Among the possible jobs the teen-immigrants can access we identify the most popular that require mathematical competences and knowledge. The main ones regard two intertwined domains, such as geometry and arithmetic, more precisely the teen-immigrants are asked to

1. measure physical quantities (length, mass),
2. read measures (e.g. from blueprint),
3. operate with measures (equivalences),
4. derive quantities (area, volume),
5. identify the proportion between quantities,
6. compute the percentage of quantities,
7. estimate measures and quantities.

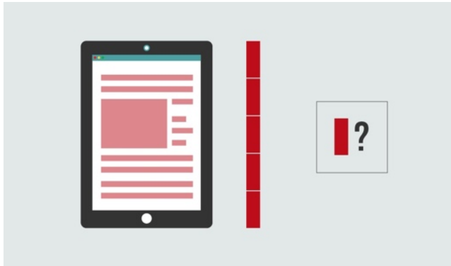
We report four tasks from four different units with the purpose of showing the approach we adopted to drive learners towards the achievement and development of such targets. More precisely, we report examples of tasks focused on geometrical aspects from real-challenge units that concern four work scenarios, coupled with four related tasks from math-recap units. The latter are meant as mathematical "tools" the learner may resort to deal with the four work scenarios.

Task 1. The first task we report, titled "Gear", is the last one (out of eight) from the unit named "Garage". This unit is mainly focused on the analysis of engines' efficiency that allows investigating the idea of percentage and proportion. Nevertheless, the task "Gears" is designed to improve the measures reading: the question is: "what is the length in mm of the gear?", the task is supported by the image in Fig. 4a which shows a gear (of 34 mm length) on the graph-paper sheet. Behind the evident request of identifying how many "big-square" (5 mm) and "small-square" (1 mm), learners need to resort their knowledge about the integer multiple and submultiple of a quantity. Once the learners have difficulties with this task, they can resort to the feedback system: 1) the "help" suggests to "look at the squares in background" and that big-square is 1 mm length. 2) the "support" provides further details on the dimension of the "small-square".

In case learners struggle with the task, they can resort to the unit named "Measuring". That unit belongs to the math-recap units, its aim is to introduce and recall the idea of measuring lengths and converting them. An example of a task is the one titled "Tablet" (see Fig. 4b) in which the learner is asked to measure the tablet length counting the numbers of red blocks. This approach allows introducing the measurement as correspondence between two quantities: the tablet (to be measured) and the red block (the measure unit). The "remark" of this task concerns the introduction of the term "measure unit", that in the following tasks will turn in meter (adopting the international standard, SI).



(a)

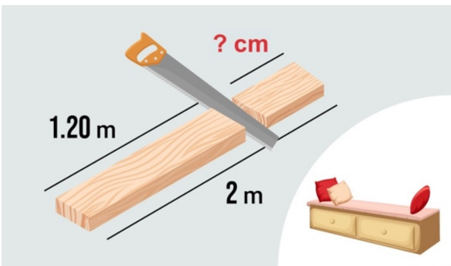


(b)

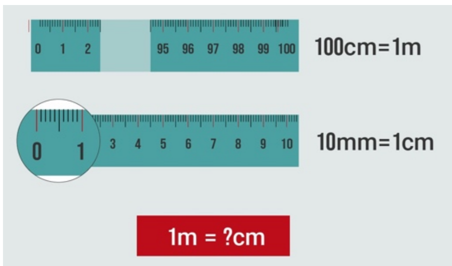
Fig. 4 Examples of images: (a) task “Gear” from “Garage” unit, (b) task “Tablet” from “Measuring” unit.

Back to the task “Gear”, we note that in the pop-up remark the measured quantity is called diameter and it is used to identify the gear dimensions in the automotive field. Those remarks are good examples of providing further information about the task, the topic and the scenario we are considering.

Task 2. The task named “Trim” is the fourth (out of six) of the unit “Joiner”. In this unit the learner is asked to deal with tasks related to making a settle whose dimensions are 1.20m x 0.40 m x 0.45 m. The main focus is on reading and converting measures, among those the task “Trim” asks to figure out what is the length of the discarded plank, more precisely the question is “You trim a piece of 1.20 m, how many cm of plank are discarded?”. The Fig. 5a is the image that supports the task understanding, it shows a plank 2 meters long trimmed at 1.20 m and the discarded piece labelled with the red text “? cm”. The learners are asked to convert the measure to centimetres then make the subtraction: 200 cm - 120 cm = 80 cm.



(a)

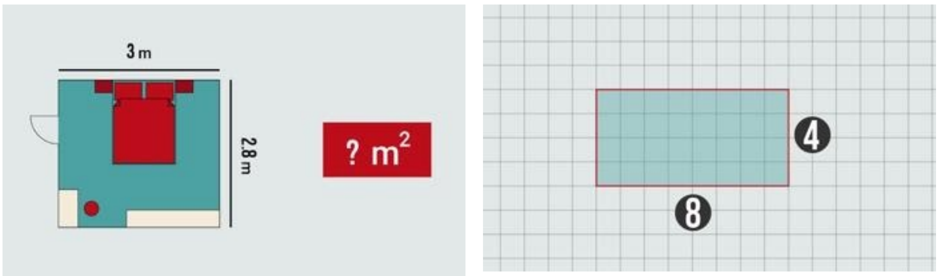


(b)

Fig. 5 Examples of images: (a) task “Trim” from “Joiner” unit, (b) task “Equivalences” from “Measuring” unit.

However, the feedback system suggests, through the help and support messages, to compute the subtraction 2 m - 1.20 m = 0.80 m, then converting it to centimetres. The reason why we drive in such direction is rooted both in the graphical representation of the task and in the way learners resort their previous knowledge and competences to convert measure units. Many teen-immigrants, as well as many Italian people, read the measure 1.20 m as “1 meter and 20”, omitting centimetres for the second part. Therefore, from the image they are suggested to compute the subtraction, even using the calculator, then they read 0.80 m as “80” without unit, assuming it is centimetre. However, in case some of them have difficulties with the equivalences, the learner may exploit the aforementioned unit “measurement” in which we propose some task with such purpose (see Fig. 5b).

Task 3. The third unit we report is the “Painter”, in which the learners are asked to compute how many litres of paint is needed to paint a room. The unit is composed of six tasks, mainly focused on computing the painting area (walls and ceiling). Here we report the second task titled “floor” whose supported image is shown in Fig. 6a. The question is “What is the dimension of your room?”.

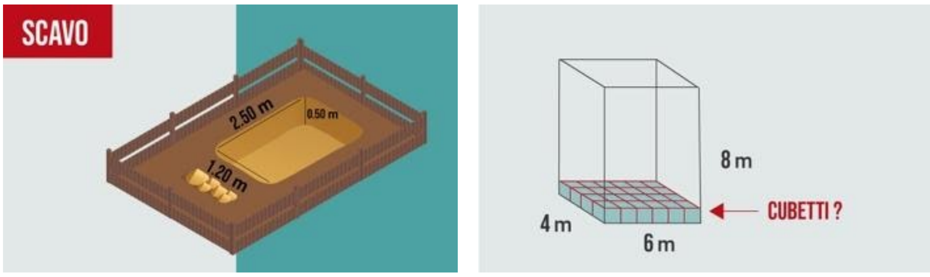


(a) (b)

Fig. 6 Examples of images: (a) task “Floor” from “Painter” unit, (b) task “Rectangle area” from “Geometry 2D” unit.

The feedback system drives the learner towards the solution: the “help” suggests to identify the shape of the room, then in the “support” we refer to the rectangle shape and its area. At that point, the learners may visit the math-recap unit labelled “Geometry 2D” where we design tasks focused on the area of rectangles (even square), triangles and circle. Fig. 6b shows one of such tasks (the fourth out of nine) where it is required to compute the area of a rectangle exploiting the idea that the measure of the area is the number of small-square within the rectangle. The remark of the task unfolds that the measure unit is the square metre (m^2). A similar message is provided by the remark of the task “Floor” that also reports the use of the term “M squared” (“mq” in Italy).

Task 4. The last task is part of the unit called “Building site” composed of 5 tasks. The scenario concerns the preparation of the concrete for casting into a digging. We report the second task that concerns the employing of the digging volume. Its dimensions are 2.50 m x 1.20 m x 0.50 m, that are provided by the image in Fig. 7a, that is coupled with the question: “how much cubic metre of concrete is needed to fill the digging in?”. We stress that the question is realistic, because it is similar to the one the supervisor may ask the worker. The feedback system suggests to compute the volume (the message in the help pop-up), then provides hints on how to do that. To support learners in this computation the unit “Geometry 3D” is provided within the math-recap section. Similar to the "Geometry 2D" we construct the idea of volume and its measurement starting from the idea of "cubes" (see Fig. 7b) inside the volume.



(a) (b)

Fig. 7 Examples of images: (a) task "Digging" from "Building site" unit, (b) task "Cubes" from "Geometry 3D" unit.

The StreetMath contains about 200 tasks so that the presented tasks are just a flavour of the activities available. However, from those, the reader can figure out the nature of the activities and our attitude in design and produce that. In particular, we want to highlight that each unit is the result of months of co-design with the teen-immigrants and the educator of the communities involved in the project. In the next section, we describe two episodes that contribute to achieving such goals.

4 Findings and conclusions

The research project follows a design-based research methodology, which focuses on examining a particular intervention by continuous iteration of design, enactment, analysis, and redesign [1, 14]. To that end, the two-years long TEEN project planned a design phase composed of three main moments, that involved about 60 teen-immigrant hosted in six communities.

In the design phase of the TEEN project, we worked with small groups of teen-immigrants who volunteered to spend some time with us. The phase consists of different meetings that lasted 45 minutes on average (a minimum of 10 minutes to a maximum of 3 hours), for a total of 30 hours. The meetings provided us with important information about the learners, the scenarios and the learning environment. In the following, we report two episodes related to the tasks above, describing the process that emerged during the interaction with the app and the mathematics within the app, and focusing on the way(s) teen-immigrants empowerment increased.

In the first episode, Ali and Ken (fictional names), 17yo from Gambia and 18yo from Turkey respectively, were working on the "Joiner" scenario. After about 10 minutes, they approached Task 2 (titled "Trim"). Ken and Ali looked at the task for a while, then Ken said: "[it is] eighty!". The tutor (one of the authors) asked why then the dialogue continues as follows:

- Ken: “because two minus one meter and twenty is eighty”
- Ali: “zero point eighty” *(showing the pop-up calculator)*
- Ken: “yes! eighty” *(pointing the pop-up calculator)*
- Ali: “fine! put the number”. *(pointing the app)*

Ken and Ali inserted the answer and got the remark, then said “nice!”. At that point, the tutor explored the possibility of resorting to a different strategy, namely converting in centimetres (1.20 m = 120 cm, 2 m =200 cm) and then computing the difference (200 cm - 120 cm = 80 cm). Ali and Ken did not object to the request of using a different strategy, however, at the first attempt they did not manage to employ by heart the conversion of the two measures contemporary. Somehow, the numbers “200” and “120” were too big and not natural for them: they forgot the first one (200) once they had computed the second one (120), so Ali wrote 120 in the calculator and then converted the first one again. But, at that point, they computed the difference 120-200 = -80. Ali and Ken were confused and started to struggle with the task, employing different computation (120+200, 120-2, etc…). Eventually, they provided the correct answer complaining that the task was very hard for them.

From this episode, we are informed by two main findings. Firstly, the feedback system for this task mirrors the “natural” process teenagers enact to answer this type of task, which resorts to the natural language. The reader can argue that the second strategy would be better to unfold difficulties and misconceptions. However, the main purpose of StreetMath is to support teen-immigrants empowerment through mathematics. That leads us toward the second findings: the nature of the task motivates the learners making them “active”, the “natural” process drives the learners towards the correct solution, making them “happy”. Such dynamics can be read in terms of empowerment: Ali and Ken were able by their own to manage the task recognizing mathematics as useful and feeling the control on the scenario. On the other hand, the same guys, after they were asked to change strategy, felt powerless: they did not feel the sense of control on the “new” task and recognized the mathematics as “hard”.

The second episode concerns the “Building site” scenario (Task 4 titled “Digging”). We report one of the earliest experiences when the app contained the first draft of such activity and the feedback system was not well developed at all. The protagonists are two 17-yo guys, fictitiously named Ibra (from Ivory Coast) and Drissa (from Mali), Ibra and Drissa were stuck on Task “Digging” (see Fig. 7a): they did not figure out what was the request, so they asked for help. So the tutor (one of the authors of this paper) clarified that cubic meter is related to the volume of the digging, Drissa has started to struggle with the numbers writing on the paper sheet some computations while Ibra looked at him silently. After a couple of minutes, Drissa looked at Ibra to have feedback by him, but vainly. The tutor got that something missed, so asked them what is a volume, Ibra remained silent whilst Drissa replied: “the volume is measured in cubic meter, centimetre. It is a solid: length, width and height”. But they did not know how to compute it, so the tutor provided further information constructing with them the idea of volume and its measure. After twenty minutes, Ibra and Drissa, using paper and pencil, addressed the task computing with the calculator the volume of the digging: 2.50 m x 1.20 m x 0.50 m = 1.5 m^3. Finally, Ibra inserted the right number and Drissa exclaimed: “This was very difficult, but I think it is important because the technical jobs require this”.

This episode is relevant for two main reasons. On one hand, the discussion with Ibra and Drissa informed us about the need for developing both the multilayer feedback system and the math-recap units, that were not planned at the beginning of the project. Moreover, the feedback system for the whole unit (“help”, “support” and “remark”) and the “Geometry 3D” unit were designed upon this episode in order to reflect the learning trajectory with Drissa and Ibra. On the other hand, the episode shed a light on the affective dimension: despite the starting stuck, Drissa and Ibra were motivated because they recognized the value of that task, they wanted to control that moment and with the right feedback they achieved the goal of empowering themselves.

We recall that StreetMath was the result of months of collaboration with the educators and the teen-immigrants involved in TEEN projects. These two episodes illustrate the co-design process [14] as well as the identification of the scenarios embedded and the way they were delivered. This coaction is the strength of the project.

In conclusion, we claim that this synergy makes StreetMath a learning environment in which learners can have experience of empowerment (mathematical, social and epistemological) thanks to everyday mathematics they are exposed through realistic scenarios.

ACKNOWLEDGEMENT

Project funded by Politecnico di Milano through the 5x1000 Polisocial Award 2017 in collaboration with Fondazione Politecnico di Milano.

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