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Sensitivity analysis and Bayesian calibration of a dynamic wind farm control model: FLORIDyn

Vinit V. Dighe[1], Marcus Becker[1], Tuhfe Göçmen[2], Benjamin Sanderse[3] and Jan-Willem van Wingerden[1]

[1]Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628CD Delft, The Netherlands.
[2]Technical University of Denmark, Department of Wind Energy, Risø campus, Roskilde, Denmark.
[3]Centrum Wiskunde & Informatica, Science Park 123, 1098 XG Amsterdam, The Netherlands.

E-mail: `v.v.dighe@tudelft.nl`

Abstract.
FLORIDyn is a parametric control-oriented dynamic model suitable to predict the dynamic wake interactions between wind turbines in a wind farm. In order to improve the accuracy of FLORIDyn, this study proposes to calibrate the tuning parameters present in the model by employing a probabilistic setting using the UQ4WIND framework. The strategy relies on constructing a surrogate model (based on polynomial chaos expansion), which is then used to perform both global sensitivity analysis and Bayesian calibration. For our analysis, a nine wind turbine configuration in a yawed setting constitutes the test case. The results of sensitivity analysis offer valuable insight into the time-dependent influence of the model parameters onto the model output. The model parameter tied to the turbine efficiency appear to be the most sensitive parameter affecting the model output. The calibrated FLORIDyn model using the Bayesian approach yield predictions much closer to the measurement data, which is equipped with an uncertainty estimate.

1. Introduction

The global wind industry will focus on scaling up of wind energy projects contributing towards reaching the net zero targets (carbon neutrality) by 2050. To this aim, the European Commission plans to install 30 GW of new wind farms every year between now and 2030 [1]. Wind turbines in modern wind farms are grouped together with close spacing to reduce the maintenance and deployment costs. The close spacing, however, reduces the wind speed and increases the turbulence on the downstream turbines, thus reducing the overall efficiency of the wind farm [2]. Wind farm flow control (WFFC) technology using wake steering strategy have demonstrated overall increased wind farm efficiency [3]. This technique implies that the upstream turbines in a wind farm operate with a certain yaw offset/misalignment to deflect their wakes away from downstream turbines.

A primary component of a closed-loop WFFC framework is a computationally inexpensive computer model that predicts the power production of the wind farm ahead in time, on a time-scale relevant for real time control. A commonly used WFFC model is FLORIS [4], which is a parametric steady-state model that neglects all temporal dynamics. As indicated in an expert elicitation by van Wingerden et. al. [5] this leaves a scientific gap. Gebraad et al. [6] presented a reduced-order parametric control-oriented wake model named FLORIDyn (FLOw Redirection and Induction Dynamics model). The model approximates the dynamic wake interactions between wind turbines in a wind farm with a computational cost in the same orders of magnitude as the steady-state FLORIS model. Subsequently, Becker et al. [7] presented an improved version of FLORIDyn to include spatially heterogeneous wind conditions. FLORIDyn trades off accuracy with computational cost when compared with high-fidelity computational fluid dynamics (CFD) models. For instance, the FarmConners benchmark study [8] revealed large differences between FLORIDyn and high-fidelity LES (SOWFA) results for a three turbine and nine turbine configurations under wake steering. Literature suggests that the values of the tuning parameters used within the FLORIDyn model vary depending on the end user despite the fact that the same operating conditions are chosen [4, 7]. Changing these tuning parameters showed noticeable differences in the FLORIDyn model output. Therefore, in order to improve the accuracy of the FLORIDyn results and supplement them with a quantified level of uncertainty, it is important to calibrate these tuning parameters in a framework that includes uncertainty estimates.

Many calibration methods are essentially regression techniques that estimate the model parameters based on the outputs, and eventually on the input, by means of various optimization algorithms. Some of the classical approaches of parameter estimation include weighted least-squares estimation [9] and best linear unbiased estimation [10]. Basically, they correspond to optimization problems that seek to minimize the difference between computed model output and the measurement data. A major drawback of such deterministic approaches is that the prior information of the model parameters is not naturally included during calibration. This may lead to systematic bias and underestimated uncertainty in the output for a calibrated model, which becomes significant especially when few measurement data are available; a common situation observed in WFFC-oriented model calibration [11]. This leads us to instead consider the calibration problem from a probabilistic perspective. The recently developed UQ4WIND framework [12] has been applied to calibrate the model parameters of FLORIDyn using Bayesian inversion method. This approach provides the full multivariate distribution of the calibrated parameters and insights into a variety of quantities of interest (e.g., mean, maximum a posteriori estimates and confidence intervals). The main downside of the Bayesian approach is the associated Markov Chain Monte Carlo (MCMC) sampling step, which is computationally typically very expensive. Polynomial chaos expansions (PCE) will therefore be used used to train a surrogate model, which is constructed using a relatively small number of FLORIDyn model runs. The trained surrogate model offers simplicity and faster convergence in comparison to a full MCMC simulation based on the original model. Additionally, the trained surrogate model allows us to conduct a variance-based global sensitivity analysis at no additional cost using Sobol' indices.

The paper is organized as follows. Section 2 describes the FLORIDyn model and its uncertain model parameters. Section 3 introduces the dataset used to perform Bayesian calibration. The UQ study is executed using the UQ4WIND framework in order to perform sensitivity analysis and Bayesian calibration, both accelerated by constructing a PCE-based surrogate model, and is described in section 4. Finally, the results of sensitivity analysis and Bayesian calibration are presented and discussed in section 5 followed by conclusions drawn in section 6.

2. FLORIDyn

FLORIDyn is an extensions of the steady-state FLORIS model and was originally described in [6]. The central idea was to make use of its low computational cost to approximate the dynamic wake behaviour of wind turbines in a wind farm by piece-wise updating the steady state flow field with a new steady state description which fits the new states. In contrast to the

Table 1. Assumed intervals for the model parameters with the aspects they influence.

	Near wake length		Wake expansion		Added turbulence				Advection	Power	
	α^*	β^*	k_a	k_b	$k_{f,a}$	$k_{f,b}$	$k_{f,c}$	$k_{f,d}$	d	η	p_p
θ_{min}	0.326	0.066	0.161	-0.0014	2.352	1.371	0.129	-0.4182	0.3	0.324	0.45
θ_{max}	1.848	0.377	0.912	-0.0002	13.328	7.769	0.731	-0.0738	1.7	1.836	2.55

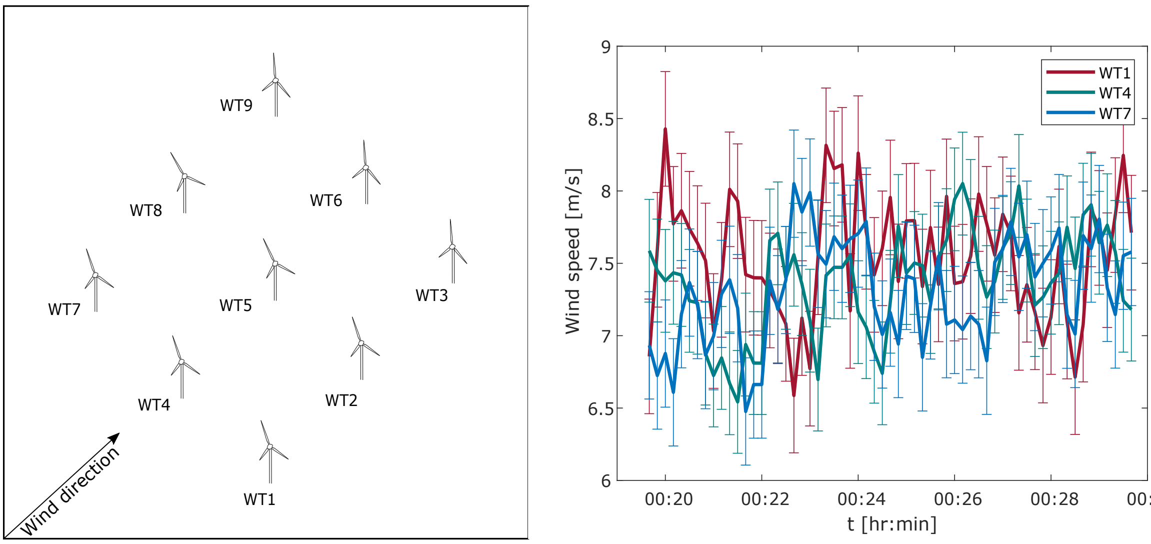

Figure 1. Schematic of the 9WT FarmConners case (left). On the right can be seen the velocity measured at hub height for the 1st row of turbines over a period of 10 minutes.

steady-state FLORIS model [4], FLORIDyn includes time-dependent wake propagation. This update is driven by Observation Points (OPs), which serves the purpose to describe the local wake characteristics at its respective location. A detailed description of FLORIDyn is beyond the scope of the current discussion; interested readers should refer to [7].

In the current study, we will perform uncertainty quantification and model calibration by using 11 model parameters of FLORIDyn. In mathematical notation, we denote the vector of model parameters by $\theta_M = (\theta_{M_1}, ..., \theta_{M_{11}})$. Having said that, a FLORIDyn model evaluation, denoted by $\mathcal{M}$, returns a vector of outputs $Y_i(t)$ at discrete time steps t, depending on the (uncertain) model parameters θ_M and the (given) scenario $\mathcal{S}$ (containing for example the inflow conditions):

$$Y_i(t) = \mathcal{M}(\theta_M, \mathcal{S}) \qquad \text{with} \qquad i = 1, \ldots, N_t, \tag{1}$$

where i is the turbine under consideration and N_t is the number of turbines. Each of the model parameters in θ_M follows a uniform distribution $\mathcal{U}(\theta_{min}, \theta_{max})$ where the mean values for the model parameters obtained from [4] are perturbed by $\pm 70\%$ in order to obtain lower and upper bounds; the values are shown in Table 1.

3. FarmConners benchmark CL-Windcon LES test case

The data used to perform FLORIDyn model calibration is obtained from high-fidelity simulations performed in the context of the European R&D project CL-Windcon [13]. The data is made available for the FarmConners benchmark study [8] in order to validate different engineering

WFFC models. Simulations consisting of different scenarios ($\mathcal{S}$) were performed with NREL's open-source SOWFA simulation package [14], which is a set of computational fluid dynamics (CFD) solvers based on OpenFOAM tool-kit [15], boundary conditions, and turbine models represented through OpenFAST simulation tool [16].

The scenario $\mathcal{S}$ chosen for the current investigation consist of 9 wind turbines: a 3×3 configuration, with 7D spacing among the turbines. The layout is shown in Figure 1 (left). The baseline orientation of the wind turbine is set to 225° (perpendicular to the south-west wind direction). The employed yaw angle convention is that the positive value represents clock-wise rotation while negative value indicates counter-clockwise. The turbine model is the INNWIND.EU reference turbine [17]. In this simulation, the yaw angle γ for the turbines in the first row is set to -20°, on the second row, from bottom to top, -10°, -20° and -30°, and the third row to 0° relative to the baseline orientation. In Figure 1 (right) the velocity for the first row of the turbines is shown, over a 10 minutes time window and measured at hub-height. This velocity field together with turbulence intensity of 5.39%, and roughness length of 0.001 m forms part of $\mathcal{S}$ and is used for initializing the SOWFA and FLORIDyn simulations. It is important to note that measurement uncertainty, i.e. standard deviations, shown by error bars in Figure 1 (right), will be ignored while specifying $\mathcal{S}$.

In order to perform Bayesian calibration, the measurement data consists of time-dependent individual turbine power output gathered in the data matrix y:

$$y = \begin{pmatrix} y_1(t_1) & \cdots & y_1(t_{end}) \\ \vdots & & \vdots \\ y_9(t_1) & \cdots & y_9(t_{end}) \end{pmatrix}, \tag{2}$$

where $t_{end} = 600$, so that 60 discrete time steps are stored when sampling every 10 seconds.

4. UQ4WIND framework

The UQ4WIND framework (available from `https://github.com/bsanderse/uq4wind`) is built around the UQLab software package [18], version 1.4. For the sake of brevity, we describe the framework only briefly here; a more detailed description is available in [12].

4.1. PCE-based surrogate model

In order to perform the UQ study, comprising of sensitivity analysis and Bayesian calibration, typically a high number of FLORIDyn model evaluations $\mathcal{M}$ are required; this becomes computationally expensive. To alleviate this issue, a polynomial chaos expansion (PCE) surrogate model is constructed to approximate the model output:

$$Y_i(t) = \mathcal{M}^{PC}(\theta_M, \mathcal{S}) \approx \mathcal{M}(\theta_M, \mathcal{S}), \tag{3}$$

where $\mathcal{M}^{PC}$ is the PCE surrogate model evaluation and can be defined as the weighted sum of multivariate polynomials [19]. To assess the accuracy of the obtained surrogate model, the Leave-One-Out (LOO) cross-validation error ϵ_{LOO} [19], is computed:

$$\epsilon_{LOO} = \frac{1}{N} \sum_{n=1}^{N} \left(\mathcal{M}(\theta_M^{(n)}, \mathcal{S}) - \mathcal{M}^{PC\backslash(n)}(\theta_M^{(n)}, \mathcal{S}) \right)^2, \tag{4}$$

where N represents the samples of θ_M, which we denote by $\theta_M^{(n)}$, with $n = 1 \dots N$. $\mathcal{M}^{PC\backslash(n)}$ denotes the PCE surrogate model trained by leaving the n-th sample out. As a matter of fact, the FLORIDyn model $Y_i(t) = \mathcal{M}(\theta_M, \mathcal{S})$ considered in this paper returns a matrix of size 60 × 9, and thus PCE approach requires the construction of 540 independent PCE surrogates.

4.2. Sensitivity analysis

Sensitivity analysis (SA) aims at finding which input parameters within θ_M explain at best the uncertainties or variations in $Y_i(t)$. In this work the total order Sobol' index S_i^{Total} [20] for each model output (i.e. individual turbine power and at discrete time step), corresponding to random realizations for θ_M, is computed and is given by:

$$S_i^{Total} = \frac{1}{D} \sum_{\mathbf{k} \in \mathcal{K}_i} w_{\mathbf{k}}^2 \qquad \text{with} \qquad i = 1, \ldots, N_\theta, \tag{5}$$

where $\mathcal{K}_i$ consists of a set of multivariate polynomials, $w_{\mathbf{k}}$ is the basis function coefficient, and $D = \mathcal{V}[\mathcal{M}^{PC}(\theta_M, \mathcal{S})]$ is the variance of the PCE. $N_\theta = 11$ is the number of model parameters. It should be noted that S_i^{Total} can be larger than one because the higher order interactions are accounted for in the total Sobol' indices.

4.3. Bayesian calibration

Bayesian calibration implies updating the prior distribution $\pi(\theta)$ with observed data y to end up with the posterior parameter distributions $\pi(\theta|y)$. In mathematical notation, it follows the classical Bayes' theorem [21]:

$$\pi(\theta|y) = \frac{\mathcal{L}(\theta; y)\pi(\theta)}{Z} \quad \text{with} \quad Z = \int \mathcal{L}(\theta; y)\pi(\theta)\, d\theta, \tag{6}$$

where $\mathcal{L}(\theta; y)$ is the likelihood function and Z is the normalizing factor called the evidence. In equation (6), we assume the combined parameter vector $\theta = (\theta_M, \theta_E)$:

$$\pi(\theta) = \pi(\theta_M)\pi(\theta_E), \tag{7}$$

where the prior on the model parameters θ_M and on the discrepancy parameters θ_E are independent. In the results section, we will report the Maximum a Posteriori (MAP) estimate, defined as the point value where $\pi(\theta|y)$ is maximum, i.e. $\theta_{\text{MAP}} = \arg\max_\theta \pi(\theta|y)$. We will also report the posterior predictive, which is obtained by propagating the posterior distribution, given by (6), through the model:

$$\pi(\hat{Y}|y) = \int \mathcal{L}(\theta; \hat{Y})\pi(\theta|y)\, d\theta. \tag{8}$$

Here the posterior predictive expresses the probability of observing new (calibrated) model output $\hat{Y}$ given the calibrated model parameters. The posterior predictive is computed by using the samples of the posterior and evaluating the likelihood from the PCE model evaluations (adding an independently sampled discrepancy term). The computation of the high-dimensional integral Z in equation (6) is typically intractable. This can be circumvented by using MCMC methods, which avoid the need to compute Z. In this work, we will use the so-called affine-invariant ensemble sampler (AIES) [22]. AIES algorithm is invariant to affine transformations of the target distribution, which requires little tuning and is suitable for cases where strong correlations exist between the parameters.

5. Results and discussion

In this section, the results of sensitivity analysis and Bayesian calibration of the FLORIDyn model parameters will be presented and discussed. The PCE-based surrogate model is used in place of FLORIDyn throughout the UQ analyses. In order to build the PCE surrogate model as described in equation (3), FLORIDyn is evaluated at a number of random samples using a uniform distribution with bounds indicated in Table 1. In total 600 model evaluations were sufficient to achieve a PCE surrogate model with a LOO-error (equation 4) smaller than 10^{-3}.

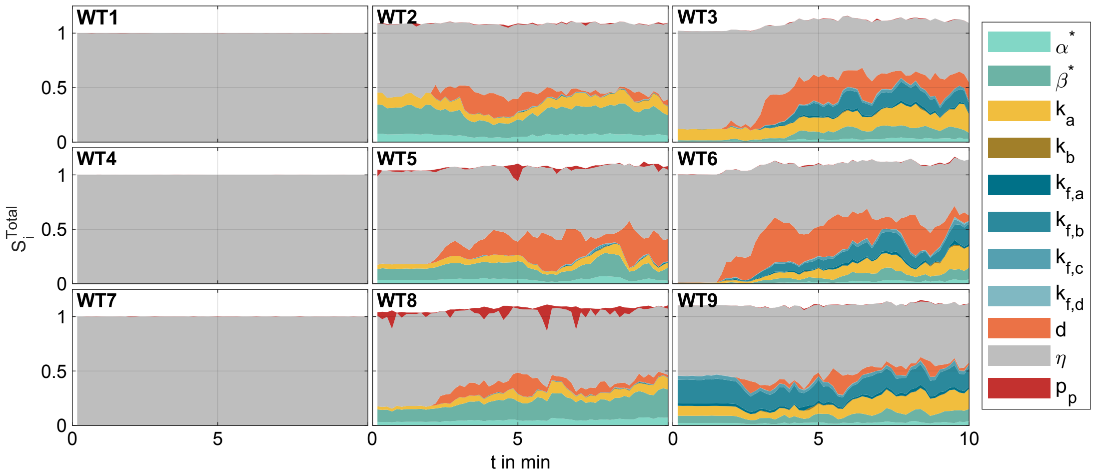

Figure 2. Sobol indices indicating the sensitivity of the time-dependent power output of each wind turbine with respect to the 11 model parameters.

5.1. Sensitivity analysis

Time dependent total order Sobol' indices S_i^{Total} are computed expressing the sensitivity of $Y_i(t)$, following equation (5), towards the perturbation in θ_M. The results are displayed in Figure 2.

For the first row of turbines (WT1, WT4 and WT7), the variation in the $Y_i(t)$ across the entire time range can be completely attributed to the variation in η. The first row of turbines, for instance, can not be influenced by the wake parameters as they are placed in the free stream and FLORIDyn inherently does not capture any possible upstream wake effects. Therefore, the parameters that the first row of turbines can perceive are the ones directly tied to the power generated, viz. η and p_p. η directly scales the power generated whereas p_p is an exponent (used for yaw angle correction in the power coefficient equation) and has almost negligible influence for the chosen $\mathcal{S}$, where the yaw angles of the turbines are held constant throughout the simulation period.

The second row of turbines (WT2, WT5 and WT8), like the first row, shows a similar result with a dominant influence of η and now also a minor influence of p_p on $Y_i(t)$. Additionally, the scaling factor of the advection d also shows a noticeable influence. This is due to the fact that the FLORIDyn wind field data is stored in the OPs and transported with the advection speed driven by parameter d. This information reaches the second row of turbines earlier or later, depending on the value of d, which causes a variation in $Y_i(t)$. Moreover, the second row of turbines will perceive the upstream wakes and the wind speed is therefore reduced. Hence, their power generated $Y_i(t)$ is also influenced by the changes of the parameter sets for the near wake length (α^*, β^*) and the wake expansion (k_a, k_b).

In the third row (WT3, WT6 and WT9), the effect of the added turbulence parameter set ($k_{f,a}$, $k_{f,b}$, $k_{f,c}$ and $k_{f,d}$) becomes visible besides the influencing parameters in row 1 and 2. The wakes of the second row will be influenced by the added turbulence levels of the first row of turbines. Therefore, the third row of turbines will be the only row where the added turbulence shows a significant effect.

As can be seen from the results of SA, all the model parameters, for different turbines at different time instances, influences $Y_i(t)$, and therefore the entire parameter set θ_M will be considered for calibration in the next section.

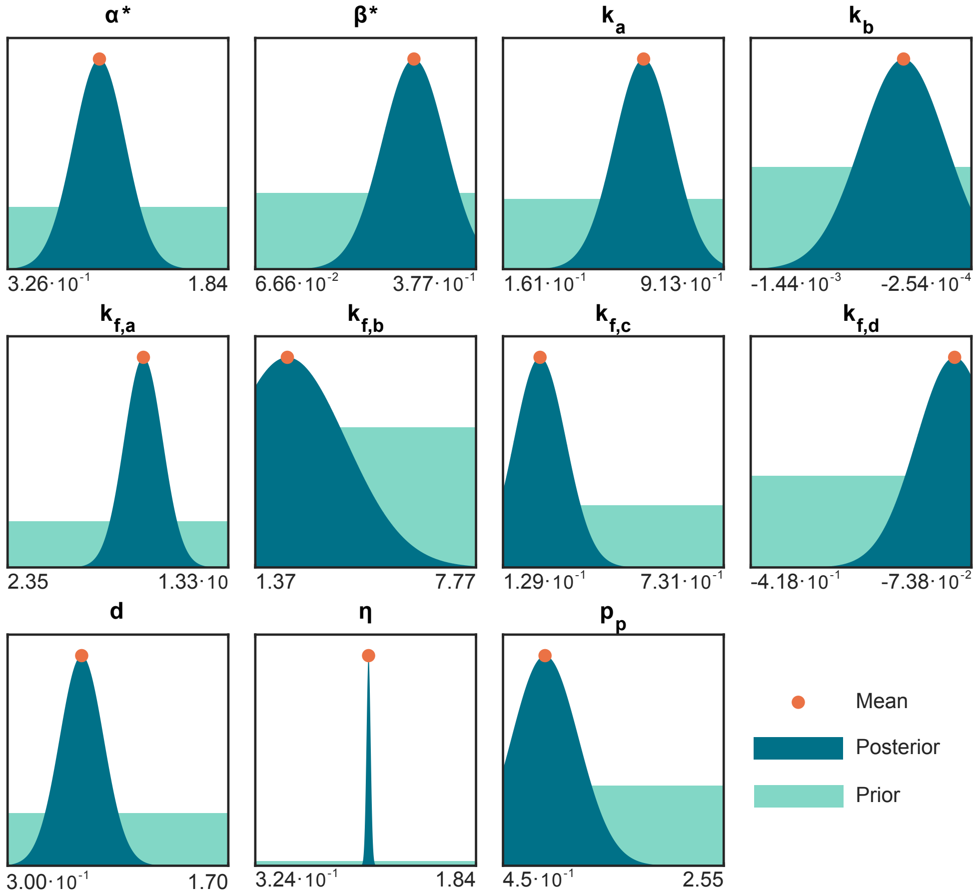

Figure 3. Samples of prior and posterior distribution of the FLORIDyn model parameters. The orange dots correspond to the mean estimates.

5.2. Bayesian calibration

The sensitivity analysis did not identify clear non-influential parameters, so all eleven model parameters will be included in the calibration process. The measurement data for the calibration consists of time-dependent individual turbine power y (see equation 2) for the nine turbine case obtained from the CL-Windcon LES data as detailed in section 3. The prior on the model parameters θ_M is taken the same as in SA. To complete the prior information, as shown in equation 7, the discrepancy parameters are not calibrated in this test case and are chosen to be fixed. We assume θ_E is equal to 1e-2, which is a rough estimate to account for the numerical approximation error between the FLORIDyn model prediction and the SOWFA simulations. AIES MCMC algorithm with 10^3 steps and 10^3 parallel chains is deployed. The posterior resulting distribution for θ_M is illustrated in in Figure 3. The posterior distributions are now Gaussian-like following uniform prior distribution. A major advantage of the posterior distribution is that it provides a full picture of the calibrated model parameter vector θ_M, and allows computation of, for example the MAP, the standard deviation and the confidence intervals of the individual model parameters.

Given the samples of the posterior distribution, the posterior predictive is computed following equation (8) and plotted along with the measurement data, the non-calibrated FLORIDyn model output, and the calibrated FLORIDyn model output evaluated at mean in Figure 4. Overall, the posterior predictive $\pi(\hat{Y}|y)$ (which expresses the probability of observing new data using uncertainty bounds) encapsulates the SOWFA simulations fairly well. The improvement

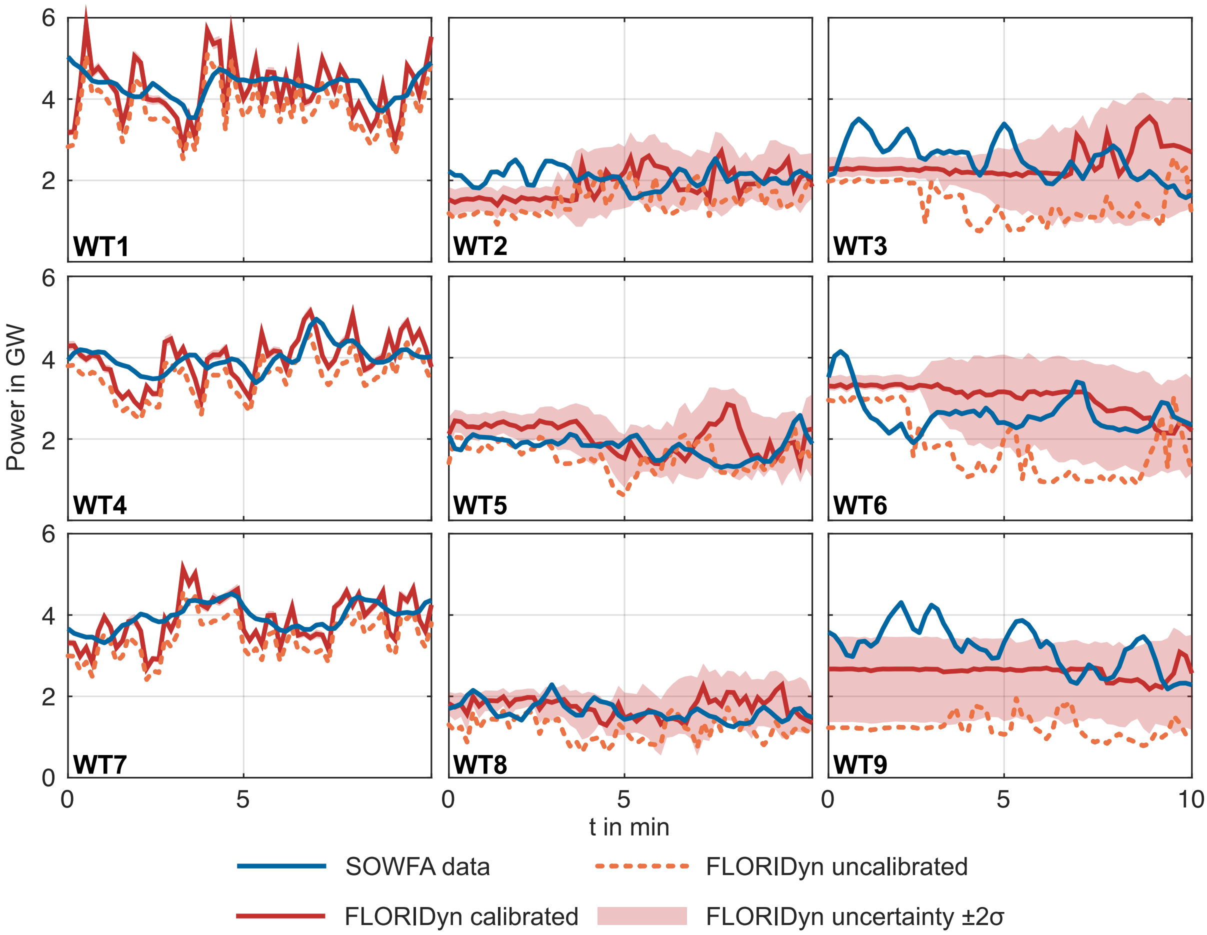

Figure 4. Comparison of non-calibrated and calibrated FLORIDyn predictions with the SOWFA simulations. The red shaded areas indicate the variance calculated for the FLORIDyn predictions using two standard deviation constructed from the samples of posterior predictive distribution.

is especially evident for the first two turbine rows, where the mean values of the calibrated FLORIDyn predictions almost overlaps with the SOWFA simulations. For the last row on average the calibrated model fits the simulations much better than the non-calibrated one, except for a few points at the beginning of the simulation (0-3 minutes), where the SOWFA results fall outside of the uncertainty bounds. This suggests that the chosen parametrization bounds (shown in Table 1) are insufficient to reproduce the given reference (SOWFA simulations) data for that particular time period, which can be improved further by increasing the parametrization bounds.

6. Conclusions

In this article, the previously developed UQ4WIND framework is applied to perform the UQ study on a reduced-order dynamic WFFC model: FLORIDyn. For the current investigation, FarmConners benchmark CL-Windcon LES test case with 9 turbine configuration is simulated in FLORIDyn in order to obtain the model output, viz. time-dependent individual turbine power. The corresponding measurement data for calibration are obtained using SOWFA simulations. In an effort to reduce the computational cost from the required repeated FLORIDyn model evaluations for UQ analysis, a surrogate model was constructed based on polynomial chaos expansions (PCE). This PCE surrogate model offered the possibility to conduct both sensitivity analysis using the time-dependent Sobol' indices and Bayesian calibration of FLORIDyn model parameters. The results of Sobol' indices highlights the influential parameters affecting the model output, which varies for different turbines at different time steps. It was found that the model parameter η, tied to the power calculation, remain the most influential to the total model output variance, while the others have a smaller influence in comparison. Bayesian calibration of model parameters was then carried out using the AIES MCMC algorithm for sampling the posterior distribution. Finally, the posterior distribution allowed the computation of posterior predictive, from which the uncertainty estimates for the calibrated FLORIDyn model output was deduced.

It is worth emphasizing that the UQ performed in this paper relies on assuming a constant value for the discrepancy parameters. Future work should consider calibration of discrepancy parameters together with the model parameters.

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