EFFECT OF CREEP AND SHRINKAGE ANALYSIS ON DEFLECTIONS OF CONTINUOUS COMPOSITE BEAMS

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Abstract. Creep and shrinkage of concrete affect the behavior of steel-concrete composite beams and should be taken into account in the analysis of these structures. A number of methods which have different level of accuracy are available for the time analysis of composite structures. Eurocode 4, the contemporary European code for design of steel-concrete composite structures, recommends simple methods for calculation of creep and shrinkage effects. In this paper, the deflections of continuous composite beams calculated with method proposed by Eurocode 4 and the more accurate Age Adjusted Effective Modulus Method (AAEM) are compared. Cracking of concrete is also considered. Through a set of numerical examples, accuracy of methods proposed by Eurocode 4 is discussed.

1. Introduction

Creep and shrinkage of concrete affect behavior of continuous composite steel-concrete beams. Due to these viscous deformations, the redistribution of stresses and change in deformations of composite beam occur in time. Therefore, the analysis should include these effects [1].

There are several methods that with different accuracy take into account the viscous deformations. In this paper, two methods are explained and studied: method proposed by Eurocode 4 [2] which is based on the Effective Modulus Method (EM) and the Age Adjusted Effective Modulus Method (AAEM) [3].

2. Creep and shrinkage effects according to Eurocode 4

Eurocode 4 [2], the contemporary European code for design of steel-concrete composite structures, recommends a simple method for including viscous deformations of concrete into analysis. According to this code, creep and shrinkage effects can be taken into account using the following modular ratio n_L for the concrete:

$$n_L = n_0\left(1+\psi_L\varphi_t\right) \tag{1}$$

with:

n_0	the modular ratio between modulus of elasticity of steel and concrete for short term loading E_a/E_{cm},
E_{cm}	the secant modulus of elasticity of concrete for short-term loading
φ_t	the creep coefficient determined according to Eurocode 2 [4]
ψ_L	the creep multiplier which depends on the type of loading and is equal to 1.1 for permanent loading, 0.55 for effects of shrinkage and 1.5 for prestressing by imposed deformations.

This method is based on the well-known Effective Modulus Method (EM) [3] with the effective modulus of concrete $E_{c,eff}$ given by the expression:

$$E_{c,eff} = \frac{E_{cm}}{1+\psi_L\varphi_t}\,. \tag{2}$$

The stress-strain relation for concrete in time *t* is linear:

$$\sigma_c(t) = E_{c,eff}\,(\varepsilon_c - \varepsilon_{sh}) \tag{3}$$

where ε_c is the concrete strain and ε_{sh} is the shrinkage strain.

Therefore, according to this method, analysis of composite beam under long-term loading is equivalent to the analysis of composite beam under short-term loading, with the difference that instead of modulus of elasticity of concrete E_{cm}, the effective modulus of concrete $E_{c,eff}$ is used.

3. AAEM method

The assumed stress-strain relation for concrete in time *t* according to the AAEM method is:

$$\sigma_c(t) = E_{c,aeff}(\varepsilon_c - \varepsilon_{sh}) - \rho_c\sigma_{co} \tag{4}$$

where

$E_{c,aeff}$	is the age-adjusted effective modulus of concrete equal to:

$$E_{c,aeff} = \frac{E_{co}}{1+\chi\varphi_t} \tag{5}$$

E_{co}	is the modulus of concrete at the time of loading t_0;
χ	is the aging coefficient with values in the range (0.6-0.9);
ρ_c	the coefficient equal to $(1-\chi)\varphi_t / (1+\chi\varphi_t)$

Analysis according to this method is, because of the accepted stress-strain relation (4), more complex than EC4 analysis, but, due to consideration of the aging of concrete, it is also more accurate.

4. Numerical study

In order to compare the maximal deflections of continuous composite beams obtained with taking into account creep and shrinkage effects through the two previously explained methods, the following numerical examples are studied.

Four continuous composite beams with spans commonly used in building structures are analyzed with the computer program “Kontinualac” [5]. The study is limited to girders applied in buildings, not mainly intended for storage, not pre-stressed by controlled deformations and constructed as propped. A brief description of beams follows.

Beam 1 is the two span continuous composite beam (Fig. 1) with equal span lengths $L_1=L_2=10$m. Dead loading is 16 kN/m and live loading is 24 kN/m.

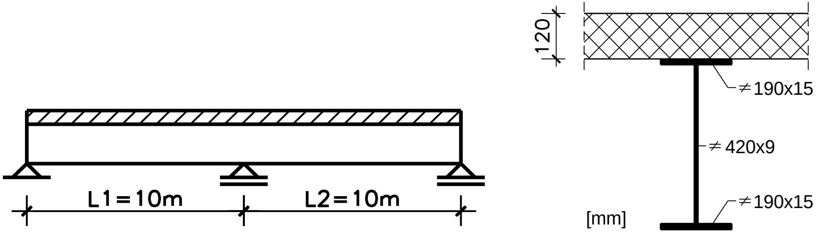

Figure 1. Beam 1 and its cross-section.

Spacing between adjacent beams is 5m, reinforcement in the slab is Ø10/12.5cm with $f_{sk}=42$kN/cm^2 and $E_s=210$kN/mm^2, constructional steel grade is S235 and concrete class is C25/30.

Beam 2 is the two span continuous composite beam (Fig. 2) with equal span lengths $L_1=L_2=12$m. Dead loading is 17.1 kN/m and live loading is 25.6 kN/m.

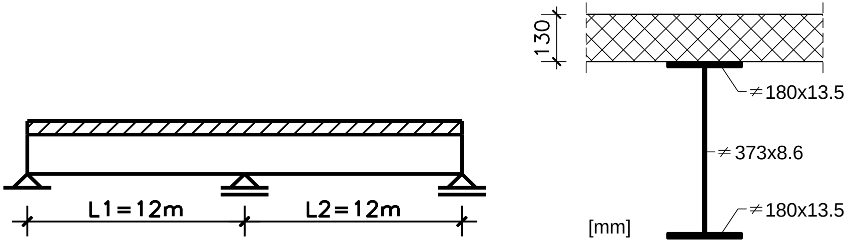

Figure 2. Beam 2 and its cross-section.

Spacing between adjacent beams is 3m, reinforcement in the slab is Ø12/15cm with $f_{sk}=50$kN/cm^2 and $E_s=210$kN/mm^2, constructional steel grade is S355 and concrete class is C25/30.

Beam 3 is the two span continuous composite beam (Fig. 3) with equal span lengths $L_1=L_2=9.5$m. Dead loading is 21.8 kN/m and live loading is 20 kN/m.

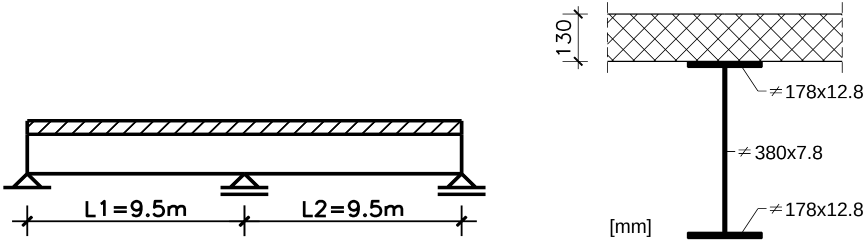

Figure 3. Beam 3 and its cross-section.

Spacing between adjacent beams is 4m, reinforcement in the slab is Ø8/10cm with f_{sk}=46kN/cm^2 and E_s=210kN/mm^2, constructional steel grade is S355 and concrete class is C25/30.

Beam 4 is the three span continuous composite beam (Fig. 4) with equal span lengths L_1=L_2= L_3=9m. Dead loading is 22 kN/m and live loading is 25 kN/m.

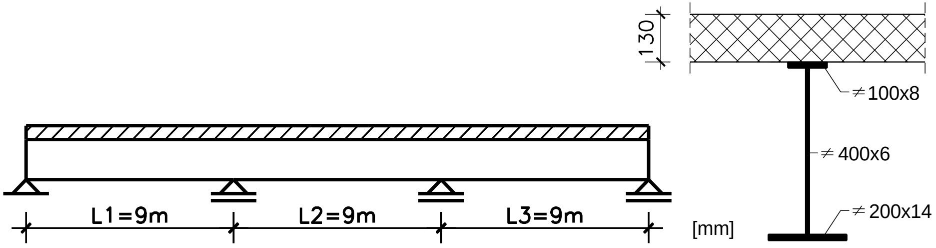

Figure 4. Beam 4 and its cross-section.

Spacing between adjacent beams is 3m, reinforcement in the slab is Ø12/10cm with f_{sk}=42kN/cm^2 and E_s=210kN/mm^2, constructional steel grade is S235 and concrete class is C25/30.

4.1. Type of analyses

According to EC4 [2], for verification of serviceability limit states, the creep effects need to be taken into account. The effects of shrinkage of concrete in some cases can be neglected [6]. Also, cracking of concrete in the area of intermediate supports has to be included in the analysis. EC4 [2] allows two simple methods for taking into account this effect: “15% cracked” analysis and “cracked analysis”.

In the “15% cracked” analysis, the following variation of flexural stiffness over the length of a beam is assumed: within the support range, over 15% of the field length, flexural rigidity is EaI_2 of the cracked section (Ea is modulus elasticity of steel, I_2 is second moment of area of the equivalent steel section neglecting concrete in tension but including reinforcement); in the remaining regions, flexural rigidity is the “uncracked” section rigidity, EaI_1 (I_1 is second moment of area of the effective equivalent steel section assuming that concrete in tension is uncracked). Flexural rigidity EaI_2 does not include the tension stiffening effect.

The “cracked” analysis is an iterative procedure. In the first iteration, internal forces and moments are calculated assuming uncracked concrete slab (stiffness EaI_1). In the areas where concrete tensile stresses exceed prescribed value ($2.0f_{ctm}$, f_{ctm} is the mean value of the axial tensile strength of concrete), flexural rigidity is reduced to the value of the cracked section (EaI_2). The second iteration starts with this new distribution of flexural rigidities, and so on.

Referring to the shear lag effect, the constant effective width is assumed over each span with widths found from the expressions given in EC4 [2].

Considering all mentioned effects, the following 4 analyses suitable for serviceability limit state verifications are performed in order to found maximal deflections:

Analysis 1: The effects of shrinkage are neglected; creep effects are taken into account by using two different modular ratios n_o and n_L for short-term and long-term loadings. Variations of this method, related to cracking of concrete effect, are explained below:

a) “15% cracked” analysis is applied
b) “cracked” analysis is applied

Analysis 2: The creep effects are taken into account by using two different modular ratios n_o and n_L for short-term and long-term loadings. Shrinkage effects are calculated using the appropriate modular ratio n_s. The same variations of this method a) and b), related to cracking of concrete effect, are performed.

Analysis 3: The effects of shrinkage are neglected. Separate analyses for long-term and short-term loadings are done and long-term effects are calculated according to AAEM method. The a) and b) variations of this method, related to cracking of concrete effect, are performed.

Analysis 4: The effects of shrinkage are included. Separate analyses for long-term and short-term loadings are done and long-term effects are calculated according to AAEM method. The a) and b) variations of this method, related to cracking of concrete effect, are performed.

4.2. Results

Results of the analysis are given in Table 1 and Table 2.

Table 1 Maximal deflection in [m], Analysis 1 and 2

Beam type:	Analysis 1		Analysis 2	
	a)	b)	a)	b)
1	0,0183	0,0171	0,0223	0,0199
2	0,0344	0,0336	0,0405	0,0390
3	0,0212	0,0199	0,0255	0,0230
4	0,0191	0,0180	0,0233	0,0213

Table 2 Maximal deflection in [m], Analysis 3 and 4

Beam type:	Analysis 3		Analysis 4	
	a)	b)	a)	b)
1	0,0184	0,0172	0,0222	0,0198
2	0,0347	0,0340	0,0405	0,0390
3	0,0214	0,0200	0,0255	0,0230
4	0,0192	0,0181	0,0232	0,0212

According to the results of Analysis 1 and Analysis 3, the deflections found by the AAEM method are very slightly greater than found by EC4 method. However, these differences between the results are negligible.

According to the results of Analysis 2 and Analysis 4, when shrinkage effects are also included, differences between results of the two analyses completely disappear.

Therefore, the application of simple method proposed by EC4 is fully legitimate for deflection analysis of continuous composite beams.

The effects of shrinkage increase maximal deflections by approximately 20%. Also, in all studied cases the “15% cracked” analysis gave larger deflections than “cracked” analysis, which is on the safe side.

5. Conclusions

EC4 allows use of simple method based on the EM method for taking into account the creep and shrinkage effects. Results for maximal deflections for set of four continuous composite girders obtained with this analysis are compared with the results obtained with more accurate AAEM method. The results have shown that the differences in results are negligible, and that is, therefore, EC4 method suitable for practical applications due to its simplicity.

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