# Multi-quasiparticle structures up to spin $\sim 44 \hbar$ in the odd-odd nucleus ${ }^{168} \mathbf{T a}$ 

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#### Abstract

High-spin states in the odd-odd nucleus ${ }^{168} \mathrm{Ta}$ have been populated in the ${ }^{120} \mathrm{Sn}\left({ }^{51} \mathrm{~V}, 3 n\right)$ reaction. Two multiquasiparticle structures have been extended significantly from spin $\sim 20 \hbar$ to above $40 \hbar$. As a result, the first rotational alignment has been fully delineated and a second band crossing has been observed for the first time in this nucleus. Configurations for these strongly coupled rotational bands are proposed based on signature splitting, $B(M 1) / B(E 2)$ ratio information, and observed rotation-alignment behavior. Properties of the observed bands in ${ }^{168} \mathrm{Ta}$ are compared to related structures in the neighboring odd- $Z$, odd- $N$, and odd-odd nuclei and are discussed within the framework of the cranked shell model.


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## I. INTRODUCTION

Collective structures at high spin in the rare-earth nuclei (e.g., ${ }_{68} \mathrm{Er},{ }_{69} \mathrm{Tm},{ }_{70} \mathrm{Yb},{ }_{71} \mathrm{Lu},{ }_{72} \mathrm{Hf},{ }_{73} \mathrm{Ta}$ ) of the mass $\sim 160-170$ region have become a focus of recent studies because a number of these are associated with rather exotic excitation modes [1-7]. More specifically, wobbling bands [8], characteristic of triaxial deformation, have drawn much attention. The latter have been interpreted as structures built on the $i_{13 / 2}$ proton orbital. However, it remains puzzling that this excitation mode has only been confirmed experimentally thus far in the odd-mass Lu isotopes [1,9-12] and in ${ }^{167} \mathrm{Ta}$ [13]. In fact, evidence for wobbling in ${ }^{167} \mathrm{Ta}$ was found as part of a systematic search for this phenomenon in Ta nuclei (having two more protons than Lu ) $[3,14,15]$. Owing to the power of Gammasphere, a considerable amount of data on ${ }^{168} \mathrm{Ta}$ was also collected as a byproduct of this search. Although these data do not provide evidence for wobbling in ${ }^{168} \mathrm{Ta}$, they enabled new insight in the high-spin structure of this nucleus about which little was known.

Compared to even-even and odd-mass nuclei, the properties of odd-odd systems are more challenging to study owing to the complexity of level structures associated with contributions from both valence protons and neutrons. Consequently, only limited information is usually available for odd-odd nuclei

[^0]throughout the chart of nuclides, even though these systems can aid significantly in our understanding of the dependence of band-crossing frequencies and aligned angular momenta on the occupation of specific orbitals. The light odd-odd Ta nuclei are characterized by fairly small quadrupole deformations and provide a good opportunity to investigate this issue [16]. A recent high-spin study on ${ }^{170} \mathrm{Ta}$ [15] by the present collaboration revealed 11 bands and allowed for a detailed investigation of these crossings. The data on ${ }^{168} \mathrm{Ta}$ discussed here can be viewed as an extension of this work.

Thus far, only two bands with spins up to $\sim 20 \hbar$ are known in ${ }^{168} \mathrm{Ta}$ [16]. The purpose of the present work was threefold: first, to confirm the previously known states at low spin and verify their properties (configuration, spin, and parity); second, to expand the level scheme by searching for new levels and structures; and third, to obtain a satisfactory interpretation of the experimental observations.

## II. EXPERIMENTAL DETAILS

High-spin states in ${ }^{168} \mathrm{Ta}$ were populated with a $235-\mathrm{MeV}{ }^{51} \mathrm{~V}$ beam delivered by the ATLAS facility at Argonne National Laboratory. The beam bombarded a stacked target of two self-supporting ${ }^{120} \mathrm{Sn}$ foils, each with a thickness of $500 \mu \mathrm{~g} / \mathrm{cm}^{2}$. The $\gamma$ rays emitted in the reaction were collected by the Gammasphere array [17], which consisted of 101 Compton-suppressed Ge detectors. Approximately $2 \times 10^{9}$ coincidence events with fold 4 and higher were accumulated during 5 days of beam time. Although the $4 n$ (leading to ${ }^{167} \mathrm{Ta}$ ) and $5 n\left({ }^{166} \mathrm{Ta}\right)$ reaction channels dominated, a useful amount of data on ${ }^{168} \mathrm{Ta}$ ( $3 n$ channel)
were also acquired. The off-line data analysis was performed with the Radware software package [18]. Hence, cubes and hypercubes were generated to establish the $\gamma$-ray coincidence relationships.

## III. RESULTS

The states at low spin in ${ }^{168} \mathrm{Ta}$ were first observed by Meissner et al. [19] in the $\beta^{+}$decay of ${ }^{168} \mathrm{~W}$. In subsequent work by Theine et al. [16], two coupled $\Delta I=1$ rotational bands with spins up to $22 \hbar$ and $20 \hbar$ were identified and configurations were proposed based upon the fact that (i) the yrast band in the odd- $Z$ neighbor ${ }^{167} \mathrm{Ta}$ (also studied in Ref. [16]) is built on a $h_{11 / 2}$ proton and that (ii) a $v i_{13 / 2}$ structure is yrast in ${ }^{167} \mathrm{Hf}$ (odd- $N$ neighbor) [20]. In ${ }^{167} \mathrm{Ta}$, a strong $\pi d_{5 / 2}$ band has also been observed. The concept of the additive coupling of quasiproton and quasineutron states in neighboring odd-mass nuclei [21] was employed to propose tentative spin and parity assignments to the bands observed in ${ }^{168} \mathrm{Ta}$ [16].

It is worth pointing out that, in the previous high-spin work of Ref. [16], the transitions belonging to ${ }^{168} \mathrm{Ta}$ were unambiguously identified by combining the following criteria: (i) all strong transitions were found to be in coincidence with Ta characteristic x rays; (ii) many of the strong transitions were also found to be in coincidence with $A=168$ evaporation residues identified in another experiment where a recoil mass separator was used [22]; and (iii) all transitions at high spin were in coincidence with known low-lying transitions. Because the main purpose in the present work was to search for weakly populated states and bands at high spin, the expansion of level structures in ${ }^{168} \mathrm{Ta}$ was based solely on coincidence relationships between newly observed $\gamma$ rays and known transitions of Ref. [16].

In the level scheme proposed in Fig. 1, many $\gamma$ rays are degenerate or very close in energy to others. The placement of these complex transitions and the expansion of the level scheme were challenging as a result. However, owing to the large resolving power and efficiency of Gammasphere for high-multiplicity $\gamma$-ray events, the triple-gating technique ( $\gamma^{4}$ format) could be used to delineate the band structures of interest, rather than the single-gating method ( $\gamma^{2}$ format) of Ref. [16]. This higher-order gating method proved beneficial, as illustrated by the sample spectra of Fig. 2. All of the tentative transitions proposed earlier [16] have been confirmed, and, in addition, a large number of new $\gamma$ rays were observed and placed in the level scheme. As a result, the two structures have now been extended significantly, both to lower and higher spins. It was also found that the energies of some transitions at the bottom of band 1 were improperly assigned earlier [16]. These $\gamma$ rays and, hence, the associated states have been rearranged here (Fig. 1), based on the inspection of coincidence relationships ( $12^{-} \rightarrow 10^{-}, 285$ changed to $267 \mathrm{keV} ; 11^{-} \rightarrow$ $10^{-}, 121$ changed to $104 \mathrm{keV} ; 10^{-} \rightarrow 9^{-}, 104$ changed to 121 keV ). In addition, the $198-\mathrm{keV}$ transition at the bottom of band 1 appears to be much stronger than the neighboring $267-\mathrm{keV}$ line, as can be seen clearly in the top panel of Fig. 2. To the best of our knowledge, this is an abnormal situation and does not support the view that the $198-\mathrm{keV}$ transition belongs


 ( ${ }^{\left(40^{-}\right)}$


(36-)

$4_{977}^{- \text {-) }}$ (32-) 1


Band 1
$\pi[514] 9 / 2 \nu[642] 5 / 2$


Band 2
$\pi[402] 5 / 2 \nu[642] 5 / 2$

FIG. 1. Level scheme of ${ }^{168} \mathrm{Ta}$ resulting from the present work. All spin and parity assignments are tentative (labels are put in parentheses) and are based on the proposed configurations. The states and transitions drawn as dashed lines are tentative. See text for details.
to band 1. Perhaps it could correspond to a transition linking band 1 to other unidentified structures in ${ }^{168} \mathrm{Ta}$. Therefore, the placement of this transition in band 1 should be viewed as


FIG. 2. (Color online) Coincidence spectra representative of bands 1 (top) and 2 (bottom) in ${ }^{168} \mathrm{Ta}$. These summed spectra were obtained by triple gating on selected in-band transitions, $267 \rightarrow 1017$ and $472 \rightarrow 1033$ of band 1 and $381 \rightarrow 1117$ and $317 \rightarrow 973$ of band 2 (in keV). The highest-energy $\gamma$ rays observed in these bands are given in the insets. Transitions are labeled by their energies in keV . The labels with the "number(number)" format denote $\gamma$-ray multiplets, such as " $479(477)$ " pointing to the $479-\mathrm{keV}$ transition mixed with the $477-\mathrm{keV}$ line, for example.
tentative, and the associated $8^{-}$level is drawn as a dashed line in Fig. 1.

In the previous high-spin study [16], none of the $\gamma$ rays connecting the high-spin sequences to the low-spin states established in the $\beta^{+}$decay of ${ }^{168} \mathrm{~W}$ [19] were observed. Thus, spin and parity information could not be deduced directly for the observed bands, and possible $I^{\pi}$ quantum numbers and other intrinsic properties were assigned tentatively, based solely on the configurations assumed for the bands. Here, an extensive study of the configurations, spins, and parities associated with the bands was carried out. As a result, general agreement was found with the configurations proposed in Ref. [16]; that is, for both bands the parity remains the same, but the spin values were reduced by $1 \hbar$, compared to the previous assignments [16]. In fact, this change in spin is important because it provides consistency in the discussion of energy staggering patterns presented in what follows. The relevant details on the assignment of configuration, spin, and parity are in the next section. Despite an extensive search, no
linking transitions between bands 1 and 2 or any new structure connected to the known bands were found. Hence, the relative spins and energies of the two bands cannot be determined experimentally.

## IV. DISCUSSION

To understand band structures in odd-odd nuclei, it is useful to study the additive coupling of quasiproton and quasineutron states in neighboring odd-mass nuclei [21]. This concept has been successfully employed in the $A \sim 160-170$ region (see Refs. $[15,16,23]$, for example). In the previous high-spin study of ${ }^{168} \mathrm{Ta}$ [16], the two observed bands were interpreted as structures resulting from the coupling of proton states in ${ }^{167} \mathrm{Ta}$ and neutron states in ${ }^{167} \mathrm{Hf}$. More specifically, bands 1 and 2 were proposed to have the configurations $\pi h_{11 / 2}[514] 9 / 2 \otimes$ $\nu i_{13 / 2}[642] 5 / 2$ and $\pi d_{5 / 2}[402] 5 / 2 \otimes v i_{13 / 2}[642] 5 / 2$, respectively. However, this assignment was largely based on the relative intensity of the two bands, which may not always be reliable.


FIG. 3. (Color online) The energy staggerings $E(I)-[E(I+1)+E(I-1)] / 2$ as a function of spin $I$ for the available $\pi h_{11 / 2}[514] 9 / 2 \otimes$ $\nu i_{13 / 2}[642] 5 / 2$ bands in the odd-odd nuclei neighboring ${ }^{168} \mathrm{Ta}$ and band 1 in ${ }^{168} \mathrm{Ta}$. The $\alpha=0$ and $\alpha=1$ states are denoted with solid and open symbols, respectively. Inversion points are marked with red arrows. See text for details.

In the present work, the configuration assignments are discussed by considering information on energy staggerings, $B(M 1) / B(E 2)$ ratios, and rotation-induced alignments.

As described above, the bands in ${ }^{168} \mathrm{Ta}$ can be understood in terms of coupling the single-quasiparticle structures in ${ }^{167} \mathrm{Ta}$ and in ${ }^{167} \mathrm{Hf}$. The band of $v i_{13 / 2}$ parentage is the yrast and most strongly populated band in ${ }^{167} \mathrm{Hf}$. Further, this $v i_{13 / 2}$ band is characterized by a large signature splitting ( $\sim 240 \mathrm{keV}$ ) at $\hbar \omega=0.175 \mathrm{MeV}$ ) $[24,25]$ and all signature splittings observed in ${ }^{168} \mathrm{Ta}$ are far smaller. Hence, it is the favored $\alpha=+1 / 2$ sequence of this $v i_{13 / 2}$ structure that couples to both signature partners of a single-quasiproton structure to form the strongly coupled bands in ${ }^{168} \mathrm{Ta}$. In the most recent ${ }^{167}$ Ta data [13], six different single-quasiproton structures were observed. The bands of $\pi h_{9 / 2}$ and $\pi i_{13 / 2}$ parentage were found to be decoupled (only a single sequence was observed for each band); therefore, they are unlikely to be involved in forming the bands in ${ }^{168} \mathrm{Ta}$. The $\pi d_{3 / 2}$ band possesses a large signature splitting ( $\sim 100 \mathrm{keV}$ ) at low spin. The coupling of the two signature partners of $\pi d_{3 / 2}$ to the $\alpha=+1 / 2$ signature of $v i_{13 / 2}$ would result in a structure having a comparably large signature splitting. This is not the case for the bands in ${ }^{168} \mathrm{Ta}$ and, therefore, this quasiproton is ruled out as well.

The energy staggering between signature partners of a rotational band can be studied by comparing the energy of a given level with the average of the energies of the signature-partner levels with one unit of spin change (higher and lower). The energy staggerings for the two bands of ${ }^{168} \mathrm{Ta}$ are compared in Figs. 3 and 4 with the relevant structures in
the neighboring odd-odd nuclei. In Fig. 3, a sharp signature inversion is observed at spin $\sim 19 \hbar$ in band 1 of ${ }^{168} \mathrm{Ta}$. This phenomenon is understood as one of the characteristics of the $\pi h_{11 / 2} v i_{13 / 2}$ structures in odd-odd nuclei in this mass region [26]. In Fig. 3, the systematic trends of signature inversion for this configuration [26] are given for the odd-odd $\mathrm{Ta}, \mathrm{Lu}$, and Tm nuclei with $N=93,95$, and 97, respectively. In a chain of isotopes, the inversion point moves to lower spin with increasing neutron number, and in a chain of isotones the inversion point shifts to higher spin with increasing proton number. These systematic observations are consistent with the $\pi h_{11 / 2} v i_{13 / 2}$ configuration assignment to band 1 in ${ }^{168} \mathrm{Ta}$. Additional supporting arguments are discussed in what follows.

For band 2 in ${ }^{168} \mathrm{Ta}$, it is more difficult to assign a firm configuration based on the energy-staggering information. In the energy-staggering plot (Fig. 4), the $\alpha=1$ sequence of band 2 is lower in energy than its partner $(\alpha=0)$ up to spin $\sim 21 \hbar$. By investigating the data available for all $\pi g_{7 / 2} v i_{13 / 2}$ and $\pi d_{5 / 2} \nu i_{13 / 2}$ bands in the neighboring odd-odd nuclei, band 2 in ${ }^{168} \mathrm{Ta}$ was found to be more consistent with a $\pi d_{5 / 2} v i_{13 / 2}$ configuration, based on the observed energy-staggering pattern at low spin; that is, a larger energy difference between the signature partners (as found in band 2) is associated with the $\pi d_{5 / 2} v i_{13 / 2}$ rather than the $\pi g_{7 / 2} v i_{13 / 2}$ configuration.

It should be noticed that, at the highest spins ( $I \geqslant 30$ ), it is the $\alpha=0$ signature that is favored in band 2 of ${ }^{168} \mathrm{Ta}$. This signature inversion is not found in any other example for the $\pi d_{5 / 2} \nu i_{13 / 2}$ configurations in Fig. 4. This inversion may


FIG. 4. (Color online) The energy staggerings $E(I)-[E(I+1)+E(I-1)] / 2$ as a function of spin $I$ for the available $\pi d_{5 / 2}[402] 5 / 2 \otimes$ $\nu i_{13 / 2}[642] 5 / 2$ and $\pi g_{7 / 2}[404] 7 / 2 \otimes \nu i_{13 / 2}[642] 5 / 2$ bands in the odd-odd nuclei neighboring ${ }^{168} \mathrm{Ta}$ and band 2 in ${ }^{168} \mathrm{Ta}$. The $\alpha=0$ and $\alpha=1$ states are denoted with solid and open symbols, respectively. See text for details.
result from complex band crossings at higher spin (see below). However, similar signature inversions have also been observed at lower spin in $\pi g_{7 / 2} v i_{13 / 2}$ bands in the odd-odd neighbors, for example, ${ }^{162} \mathrm{Tm}$. This indicates that the alternative assignment, $\pi g_{7 / 2} \nu i_{13 / 2}$, for band 2 in ${ }^{168} \mathrm{Ta}$ cannot be ruled out based on the present data. Such an observation merits further investigation, especially once higher-spin data in the neighboring odd-odd nuclei become available.

To investigate the configuration assignments for bands 1 and 2 further, $B(M 1) / B(E 2)$ ratios were extracted from the measured $\gamma$-ray intensities. In Fig. 5, these ratios are compared with the calculated values for specific configurations. The geometric model of Dönau [27] was used. Calculations of the $B(M 1)$ values assumed $g_{R}=Z / A$ and the $g_{\Omega}$ values of Table II in Ref. [3]. The standard rotational form of the $B(E 2)$ strength [8] was employed, assuming a quadrupole moment of 6.0 eb deduced from the predicted deformation of $\beta_{2}=0.24$ [28]. Data for some ratios are missing because the associated intraband and/or interband transitions are contaminated by other coincident $\gamma$ rays. In the top panel, for band 1, there is good agreement between the data and the calculations for the $\pi h_{11 / 2} \nu i_{13 / 2}$ configuration. The calculations with the configurations $\pi g_{7 / 2} v i_{13 / 2}$ and $\pi d_{5 / 2} v i_{13 / 2}$ do not reproduce the measured $B(M 1) / B(E 2)$ ratios in band 2 as well (see bottom panel in Fig. 5). However, overall the $\pi d_{5 / 2} \nu i_{13 / 2}$ calculations are a closer match to the data.

From the preceding analyses, it is concluded that the configuration assignments proposed previously [16],
that is, $\pi h_{11 / 2}[514] 9 / 2 \otimes v i_{13 / 2}[642] 5 / 2$ for band 1 and $\pi d_{5 / 2}[402] 5 / 2 \otimes v i_{13 / 2}[642] 5 / 2$ for band 2 , are most likely correct.

In the literature [16], the spins of the levels in ${ }^{168} \mathrm{Ta}$ were proposed based on an analysis of the additivity of alignments. According to the Gallagher-Moszkowski rule [29], the lowest states correspond to the parallel case of a quasiparticle coupling; therefore, $K=7$ and $K=5\left(K=\Omega_{p}+\Omega_{n}\right)$ were assigned to bands $1\left(\pi h_{11 / 2}[514] 9 / 2 \otimes \nu i_{13 / 2}[642] 5 / 2\right)$ and 2 $\left(\pi d_{5 / 2}[402] 5 / 2 \otimes v i_{13 / 2}[642] 5 / 2\right)$, respectively. In Ref. [16], when plotting the alignments for the bands, different Harris parameters were used for each nucleus. However, in the present work, a common set of Harris parameters from Ref. [14] ( $J_{0}=28 \hbar^{2} \mathrm{MeV}^{-1}$ and $J_{1}=58 \hbar^{4} \mathrm{MeV}^{-3}$ ) was adopted such that the ground-state band in the neighboring even-even ${ }^{168} \mathrm{Hf}$ nucleus has nearly zero initial alignment and a constant alignment above the first crossing. The extracted aligned spins for bands of interest in ${ }^{167} \mathrm{Ta},{ }^{167} \mathrm{Hf}$, and ${ }^{168} \mathrm{Ta}$ are plotted in Figs. 6 and 7. As in Ref. [16], the spin was assigned such that the alignment additivity rule is realized for each band; that is, $i_{p n}\left({ }^{168} \mathrm{Ta}\right)=i_{p}\left({ }^{167} \mathrm{Ta}\right)+i_{n}\left({ }^{167} \mathrm{Hf}\right)$. It was found that the best quantitative agreement is reached if $I=8 \hbar$ is assigned to the lowest state in each of the bands 1 and 2, as seen in Figs. 6 and 7. In other words, the spins of states in both bands have been reduced by $1 \hbar$ relative to the assignments proposed in Ref. [16] (see the level scheme in Fig. 1). These updated spin assignments for the two bands in ${ }^{168} \mathrm{Ta}$ can be viewed as further evidence for the validity of the energy-staggering patterns emerging in Figs. 3 and 4 (see


FIG. 5. (Color online) Measured $B(M 1) / B(E 2)$ ratios as a function of spin for bands 1 (top) and 2 (bottom) in ${ }^{168} \mathrm{Ta}$. Solid (open) symbols denote the signatures $\alpha=1(0)$. Theoretical calculations are displayed as lines with the following configurations in the top panel: (1) $\pi[514] 9 / 2 \otimes \nu[642]$ $5 / 2$, (2) $\pi[514] 9 / 2 \otimes \nu[642] 5 / 2 B C$; and in the bottom panel: (1) $\pi[402] 5 / 2 \otimes \nu[642] 5 / 2$, (2) $\pi[402] 5 / 2 \otimes \nu[642] 5 / 2 B C$, (3) $\pi[404] 7 / 2 \otimes v[642] 5 / 2$, (4) $\pi[404] 7 / 2 \otimes v[642] 5 / 2 B C$. See Table I (and Fig. 8) for the definition of $B C$. See text for details.
preceding discussion). Spins were then proposed for all new, higher-lying levels observed in the present work by assuming that the rotational behavior persists throughout the bands. The parity of the states has not been measured directly. Instead, it is based on the proposed configurations associated with the bands. Consequently, bands 1 and 2 are assumed to have negative and positive parity, respectively, again in agreement with Ref. [16].

To interpret the variations in alignment properties of the rotational sequences as a function of rotational frequency, cranked-shell-model (CSM) calculations have been performed for both quasiprotons and quasineutrons in ${ }^{168} \mathrm{Ta}$. In the calculations, deformation parameters of $\beta_{2}=0.24, \beta_{4}=0$, and $\gamma=0$ were adopted, as suggested in Ref. [28]. The pairing energies were estimated from the mass differences of neighboring nuclei, using the five-point fit defined in Ref. [30]. The resulting quasineutron and quasiproton Routhians are displayed in Figs. 8 and 9, respectively. Quasiparticle orbitals


FIG. 6. (Color online) Experimental alignments for band 1 in ${ }^{168} \mathrm{Ta}$, the $\nu i_{13 / 2}$ band in ${ }^{167} \mathrm{Hf}$, and the $\pi h_{11 / 2}$ band in ${ }^{167} \mathrm{Ta}$ as a function of the rotational frequency. A reference with the common $J_{0}$ and $J_{1}$ Harris parameters indicated in the figure was subtracted for each band. Solid (open) symbols denote the signatures $\alpha=+\frac{1}{2}\left(-\frac{1}{2}\right)$ or $1(0)$. Tentative transitions are excluded from the figure. See text for details.
close to the Fermi surface in these figures have been labeled using the Stockholm convention [31] and are summarized for ${ }^{168} \mathrm{Ta}$ in Table I.

As seen in the alignment plot (Fig. 6), the first discontinuity (upbending or backbending) for both signatures of band 1 in ${ }^{168} \mathrm{Ta}$ occurs at a frequency of $\hbar \omega \sim 0.31 \mathrm{MeV}$. Based on


FIG. 7. (Color online) Experimental alignments for band 2 in ${ }^{168} \mathrm{Ta}$, the $\nu i_{13 / 2}$ band in ${ }^{167} \mathrm{Hf}$, and the $\pi d_{5 / 2}$ band in ${ }^{167} \mathrm{Ta}$ as a function of the rotational frequency. A reference with the common $J_{0}$ and $J_{1}$ Harris parameters given in the figure was subtracted for each band. Solid (open) symbols denote the signatures $\alpha=+\frac{1}{2}\left(-\frac{1}{2}\right)$ or $1(0)$. Tentative transitions are excluded from the figure. See text for details.


FIG. 8. (Color online) Quasineutron Routhians for ${ }^{168} \mathrm{Ta}$ calculated in the CSM with deformation parameters of $\beta_{2}=0.24, \beta_{4}=0$, and $\gamma=0$. The four line types denote four combinations of (parity, signature): solid $(+,+1 / 2)$, $\operatorname{dot}(+,-1 / 2)$, dash-dot $(-,+1 / 2)$, dash $(-,-1 / 2)$. See text for details.
the frequency and the gain in angular momentum ( $\sim 5 \hbar$ ), this alignment can be understood as resulting from the $B C$ neutron crossing, which is predicted to occur at $\hbar \omega \sim 0.31 \mathrm{MeV}$ in the CSM calculations with a calculated alignment of $5-6 \hbar$ (see Fig. 8). The $v i_{13 / 2}$ band observed in ${ }^{167} \mathrm{Hf}$ exhibits a similar first alignment (Fig. 6) and is consistent with this interpretation. A similar alignment (with the same gain, but at slightly lower frequency) is observed in band 2 (Fig. 7). It can also be interpreted as associated with the $B C$ neutron crossing. The fact that the backbendings observed at $\hbar \omega \sim 0.24 \mathrm{MeV}$ ( $A B$ crossing) in both the $\pi h_{11 / 2}$ (Fig. 6) and $\pi d_{5 / 2}$ (Fig. 7)


FIG. 9. (Color online) Quasiproton Routhians for ${ }^{168} \mathrm{Ta}$ calculated in the CSM with deformation parameters of $\beta_{2}=0.24, \beta_{4}=0$, and $\gamma=0$. The convention for drawing the lines is same as that used in Fig. 8. See text for details.

TABLE I. The convention used to label the quasiparticle orbitals in the Routhian figures and in the discussion in the text.

| Shell-model state | Nilsson label | Label used $\left(\alpha=+\frac{1}{2},-\frac{1}{2}\right)$ |
| :--- | :---: | :---: |
| $\nu i_{13 / 2}$ | $\nu[642] 5 / 2$ | $A, B$ |
| $\nu i_{13 / 2}$ | $\nu[633] 7 / 2$ | $C, D$ |
| $\pi d_{5 / 2}$ | $\pi[402] 5 / 2$ | $A_{p}, B_{p}$ |
| $\pi g_{7 / 2}$ | $\pi[404] 7 / 2$ | $C_{p}, D_{p}$ |
| $\pi h_{11 / 2}$ | $\pi[514] 9 / 2$ | $E_{p}, F_{p}$ |
| $\pi h_{9 / 2}$ | $\pi[541] 1 / 2$ | $G_{p}, H_{p}$ |

bands of ${ }^{167} \mathrm{Ta}$ are Pauli blocked in the two bands of ${ }^{168} \mathrm{Ta}$, due to the occupation of the single neutron $A\left(i_{13 / 2}\right)$ orbital, is consistent with the interpretation. The small difference in the first band-crossing frequencies between the two bands in ${ }^{168} \mathrm{Ta}$ may possibly reflect a slight difference in deformation or a small change in the value of the proton-neutron residual interaction between the odd particles.

Owing to the significant extension of the rotational sequences achieved in the present analysis, a second band crossing is revealed for the first time in ${ }^{168} \mathrm{Ta}$. As can be seen in Fig. 6, the two signature partners of band 1 split above $\hbar \omega \sim 0.4 \mathrm{MeV}$. In the $\alpha=0$ sequence, the second discontinuity starts at $\hbar \omega \sim 0.47 \mathrm{MeV}$, while the $\alpha=1$ partner does not display any sign of a second alignment up to the highest frequency ( $\hbar \omega \sim 0.51 \mathrm{MeV}$ ) observed. Such a difference between signatures also takes place at a similar frequency for the $\pi h_{11 / 2}$ band in ${ }^{167} \mathrm{Ta}$ (Fig. 6). This occurs in ${ }^{167} \mathrm{Ta}$ after a common alignment gain of $\sim 4 \hbar$ between 0.35 and 0.45 MeV in rotational frequency. The latter alignment is also present in the yrast band of neighboring even-even nuclei, where it is interpreted as the $C D$ neutron crossing. Such a crossing ( $C D$ ) is Pauli blocked in the $v i_{13 / 2}$ band of ${ }^{167} \mathrm{Hf}$ ( $A B C$ in the frequency range discussed) as well as in band 1 of ${ }^{168} \mathrm{Ta}$ ( $A B C E_{p}$ and $A B C F_{p}$ in the frequency range discussed). For the splitting in alignment of the two signatures in band 1 of ${ }^{168} \mathrm{Ta}$ at high frequency ( $\hbar \omega \geqslant 0.4 \mathrm{MeV}$ ), inspection of the calculated quasineutron Routhians (Fig. 8) reveals that there are no suitable neutron crossings close to this frequency ( $\hbar \omega \sim 0.5 \mathrm{MeV}$ ) that could account for the observation. This suggests that the second alignment observed in the $\alpha=0$ signature of band 1 in ${ }^{168} \mathrm{Ta}$ may arise from a proton crossing.

Quasiproton Routhians calculated using the CSM (Fig. 9) exhibit a complex set of crossings at frequencies near $\hbar \omega \sim 0.5 \mathrm{MeV}$, due in part to the presence of a favored $h_{9 / 2}$ orbital $\left(G_{p}\right)$. The crossings $E_{p} F_{p}, G_{p} H_{p}, F_{p} G_{p}$, and $E_{p} H_{p}$ are all calculated to occur approximately at the frequency of interest. In fact, the $\pi h_{11 / 2}$ and $\pi h_{9 / 2}$ orbitals are well known to be strongly mixed at high rotational frequencies [3,14,24,25]. It is tentatively proposed that the second discontinuity in the aligned spin observed in band 1 is caused by the mixed $E_{p} H_{p}$ proton crossing. Such mixed crossings have previously been discussed in the osmium-iridium region [32]. Furthermore, the same orbitals have been proposed to explain similar high-frequency alignments in neighboring ${ }^{169} \mathrm{Hf}$ [33].

The single quasiproton $F_{p}$ in the even-spin sequence ( $\alpha=0$ ) of band 1 in ${ }^{168} \mathrm{Ta}$ would block the $E_{p} F_{p}$ crossing, but not the $E_{p} H_{p}$ one. Thus, in the context of the present
interpretation of a mixed $E_{p} H_{p}$ proton crossing, it is expected that, at a frequency of $\sim 0.5 \mathrm{MeV}$, the second alignment occurs in the sequence with signature $\alpha=0$, while it is not present in the $\alpha=1$ partner. This is consistent with the observations in band 1 of ${ }^{168} \mathrm{Ta}$ (Fig. 6). However, the CSM calculations do not reproduce some of the associated proton orbitals, as can be seen in Fig. 9. The $G_{p}$ orbital is positioned low in energy and becomes yrast at $\hbar \omega>0.3 \mathrm{MeV}$. This results in the prediction that the $G_{p} H_{p}$ and $F_{p} G_{p}$ crossings should also occur at frequencies between 0.5 and 0.6 MeV , similar to the $E_{p} H_{p}$ one. If the $G_{p} H_{p}$ or $F_{p} G_{p}$ crossing or both were involved, the two signatures in band 1 in ${ }^{168} \mathrm{Ta}$ would encounter the second alignment in a similar way, in contradiction with the experimental observations. The difficulty of the CSM in reproducing proton crossings at high frequencies for nuclei in this region is well documented. Such discrepancies may come, for example, from a change of deformation with increasing frequency and/or higher-order pairing terms, which are not contained in the CSM $[3,34]$.

For band 2 in ${ }^{168} \mathrm{Ta}$, the existence of the $d_{5 / 2}\left(A_{p}\right.$, $B_{p}$ ) quasiproton in its configuration would not prevent the occurrence of the mixed $E_{p} H_{p}$ (or $E_{p} F_{p}$ ) proton crossing in both signatures. However, as can be seen in Fig. 7, the $E_{p} H_{p}$ alignment is delayed up to the highest frequency observed in the $\alpha=0$ sequence ( $\leqslant 0.56 \mathrm{MeV}$ ) and the $\alpha=1$ signature $(\leqslant 0.48 \mathrm{MeV})$. It is possible that the $E_{p} F_{p}$ crossing occurs with a large interaction strength and results in a smooth gradual alignment for both signatures of band 2, which would block the subsequent $E_{p} H_{p}$ and $F_{p} G_{p}$ crossings.

From the preceding detailed discussion, it is clear that the understanding of the proton alignments involving $h_{11 / 2}$ and $h_{9 / 2}$ orbitals that occur in this mass region is far from complete and that further investigation is necessary.

## V. SUMMARY AND CONCLUSIONS

The two coupled bands previously observed in ${ }^{168} \mathrm{Ta}$ have been extended significantly to higher spin, from $\sim 20 \hbar$
to above $40 \hbar$. Quasiparticle configurations were assigned to the bands based on the observed signature splittings, $B(M 1) / B(E 2)$ ratios, and rotation-induced alignments. A $\pi h_{11 / 2} v i_{13 / 2}$ configuration has been assigned to band 1 , while band 2 is proposed to be associated with the $\pi d_{5 / 2} v i_{13 / 2}$ configuration. Revised spin values for the two bands have been proposed, based on the concepts of additivity and signature staggering. A second discontinuity in aligned spin has been observed for the first time in this nucleus and interpreted as a quasiproton band crossing. The observed alignment behaviors are found to be consistent with similar observations in the neighboring ${ }^{167,169,171} \mathrm{Ta}$ (odd- $Z$ ) and ${ }^{167} \mathrm{Hf}$ (odd- $N$ ) nuclei. In the CSM calculations, a complex pattern of alignments involving $h_{11 / 2}$ and $h_{9 / 2}$ proton orbitals is predicted at high rotational frequencies. A fully consistent picture has yet to emerge to explain the high-frequency alignments observed in ${ }^{168} \mathrm{Ta}$ and other nuclei in this mass region. A dedicated experiment optimized to populate ${ }^{168} \mathrm{Ta}$ would likely reveal additional band structures, such as those associated with the $\pi h_{9 / 2} v i_{13 / 2}$ configuration, which is expected to be yrast at the highest spins. Such a measurement could help resolve the alignment puzzles at high frequency discussed in the present work.

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