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Candidate superdeformed band in ^{28}Si

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Recent antisymmetrized molecular dynamics (AMD) calculations for ^{28}Si suggest the presence of a superdeformed (SD) band with a dominant $^{24}\mathrm{Mg}+\alpha$ clustering for its configuration, with firm predictions for its location and associated moment of inertia. This motivates a review of the experimental results reported in the literature with a particular focus on ^{24}Mg(α,γ) studies, as well as on α-like heavy-ion transfer reactions such as ^{12}C(^{20}Ne,α)^{28}Si. Combining this information for the first time leads to a set of candidate SD states whose properties point to their α-cluster structure and strong associated deformation. Analysis of data from Gammasphere allows the electromagnetic decay of these candidate states to be probed and reveals further supporting evidence for such a structure. This paper appraises this body of information and finds the evidence for an SD band is strong.

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I. INTRODUCTION

Superdeformed (SD) states in nuclei were first reported in rare-earth isotopes like ^{152}Dy [1], and were later found to exist in several mass regions, including those with $A \sim 150$, $A \sim 130$, and $A \sim 190$ [2,3]. The identification of these weakly populated, highly excited structures came about through a step-change in technology with the advent of highly segmented, high-resolution γ-ray detector arrays. These same techniques led to the discovery around ten years ago, of SD bands in the light, α-conjugate nuclei, ^{36}Ar [4] and ^{40}Ca [5]. These form fascinating examples of superdeformation since complementary descriptions can be found in terms of particle-hole excitations in the shell model [6,7], and α-clustering configurations within various cluster models [8–10]. Key theoretical questions center on whether the clustering is a real feature of the system, or whether it simply corresponds to the appearances but is not a true physical description. In addition, a major question is how such clustered configurations evolve into deformed ones. Naturally, it would be of interest to locate hyperdeformed states predicted in nuclei such as ^{36}Ar [10–13] to complement our existing knowledge of SD bands, but it is particularly important to locate SD bands in lighter, α-conjugate nuclei such as ^{32}S and ^{28}Si for which longstanding theoretical predictions exist and which continue to attract the interest of theory. Recent examples of theory initiatives in this area include AMD calculations for ^{28}Si [14] and ^{32}S [15], and macroscopic-microscopic calculations for both nuclei [16]. In addition, a paper presented in parallel to the present work [17] considers shape isomers and clustering in ^{28}Si from the perspective of the Nilsson model combined with quasidynamical SU(3) symmetry considerations. In all cases, it is predicted that the SD bands in ^{28}Si and ^{32}S should lie at high excitation energy; i.e., with bandheads around 10 MeV. This has two consequences in terms of the challenge in identifying such states experimentally: firstly, phase space favors high-energy, out-of-band transitions compared to low-energy, in-band ones despite the strong collective character of the latter. Secondly, the bandhead lies on or above the particle-decay threshold meaning that there is competition with particle emission.

Recently, Taniguchi *et al.* [14] carried out an extensive study of collective structures in ^{28}Si using the AMD model. They explore clustering degrees of freedom of the type: $^{24}\mathrm{Mg}+\alpha$ and $^{12}\mathrm{C}+{}^{16}\mathrm{O}$. These studies reveal a rich diversity of rotational behaviors. There is shape coexistence between the oblate ground-state band and a prolate [normal-deformed (ND)] band, in conformity with the known band structure of ^{28}Si. The ground state band was identified up to $J = 8$ by Ford *et al.* [18], and its oblate nature was demonstrated through Coulomb excitation measurements performed by Häusser *et al.* [19]. An excited rotational band whose bandhead lies at 6.691 MeV, was identified 30 years ago by Glatz *et al.* [20], and is assumed to correspond to the prolate (ND) band. An SD band is identified in the AMD calculations [14] with a strong $^{24}\mathrm{Mg}+\alpha$ configuration as well as some $^{12}\mathrm{C}+{}^{16}\mathrm{O}$ component. Such a cluster configuration for the

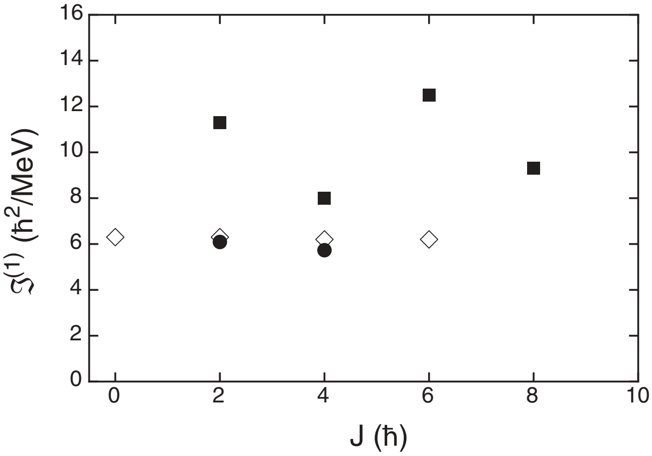

FIG. 1. Kinematic moments of inertia for the candidate SD band, evaluated as $\mathcal{J}^{(1)} = \frac{2J+3}{E(J+2)-E(J)}$. The results of the AMD calculations are the open diamonds. The filled squares correspond to the "excited prolate" band identified by Kubono *et al.* [21], while the filled circles relate to the candidate SD states discussed in the present work.

SD minimum is supported by recent macroscopic-microscopic potential-energy surface calculations for ^{28}Si [16] as well as by Nilsson model calculations presented in parallel work [17]. The AMD calculations [14] suggest that the SD band should have a moment of inertia $\mathcal{J}^{(1)} \approx 6\,\hbar^2$/MeV, related to the large associated deformation, $\beta_2 \approx 0.8$. It is difficult to identify experimental counterparts for the predicted SD states. Taniguchi *et al.* [14] compare their predictions for the SD band in ^{28}Si with the properties of a so-called "excited prolate" band identified in the early 1980s by Kubono *et al.* [21] using the ^{12}C(^{20}Ne,α)^{28}Si reaction. The experimental assignment of this "excited prolate" band rests on peaks in a charged-particle spectrum, and many of the associated states do not have well-established spin/parity. As shown by Taniguchi *et al.* [14], the states identified by Kubono *et al.* [21] do not form a smooth sequence characteristic of a rotational band even when making plausible allowance for mixing, and the suggested moments of inertia are higher than the calculated values (see Fig. 1). Moreover, γ-ray transitions between these states are not observed, and, consequently, transition strengths are unknown. Without the observation of in-band transitions, assigning candidate rotational bands is difficult and potentially ambiguous, although such an approach has been a common procedure in the past for "cluster" bands in light nuclei. In summary, the experimental counterparts for the predicted SD band in ^{28}Si are not on a firm footing and the evidence is weaker than that presented for SD bands in ^{36}Ar and ^{40}Ca [4,5], where well-developed band structures connected by $E2$ transitions are seen.

II. LOCATING THE SD BAND EXPERIMENTALLY

The fact that both recent AMD [14] and other [16,17] calculations suggest that the SD band in ^{28}Si should have a strong ^{24}Mg + α component, raises the question as to whether the ^{24}Mg(α,γ) radiative capture reaction might prove to be a favored one to selectively populate SD states in ^{28}Si. Such a possibility was not considered by Taniguchi *et al.* [14], but a detailed review of the literature suggests, in fact, that plausible candidates for SD states may already exist. In a series of articles, Brenneisen *et al.* [22] collate data from studies they carried out with the ^{27}Al(p,γ)^{28}Si and ^{24}Mg(α,γ)^{28}Si reactions. Of the large number of states identified in this systematic study, a number stand out as having unusual characteristics. In particular, a 6^+ state at 12.86 MeV is identified which is populated in the (α,γ), but not in the (p,γ) reaction. This 12.86-MeV level has decay branches to a number of states including a 4^+ state at 10.945 MeV, via a 1.921-MeV transition. The observation of a relatively intense, low-energy $E2$ transition, in competition with high-energy γ rays, immediately suggests that it must have a large transition strength. Brenneisen *et al.* [22] infer that $(2I+1)\Gamma_\gamma > 0.37$ eV for the 12.86-MeV state which means that the transition to the 10.945-MeV level has an associated $B(E2)$ value exceeding 25 Wu [22]. Comparison with the results of shell model calculations with the USD interaction leads Brenneisen *et al.* [22] to conclude that the respective 6^+ and 4^+ states as well as a 2^+ level at 9796 keV are not consistent with expected shell model states, but are more likely to correspond to intruder states. They suggest, accordingly, that the states they have identified form a $K^\pi = 0^+$ intruder band. If this set of states did form a rotational band, the kinematic moment of inertia would be $\approx 6\,\hbar^2$/MeV, in good agreement with that predicted by the AMD calculations (see Fig. 1). Extrapolating the band suggests that the 0^+ bandhead would lie near 9.3 MeV. Brenneisen *et al.* point to a possible 0^+ level at 8819 keV [22]. The existence and properties of the latter state are not clear. Moreover, the level would be somewhat displaced from its expected position. In all likelihood, the true bandhead state is yet to be identified as locating it is experimentally challenging. It would not be observable in an (α,γ) reaction as it would be expected to lie below the threshold. It would also be difficult to populate the level from above due to phase space. Appropriate techniques for identifying the 0^+ state might include inelastic scattering; e.g., (α,α'), (e,e') or (γ,γ') reactions.

The unusual character of the 10.94-MeV and 12.86-MeV states becomes clear in conjunction with other work such as the ^{12}C(^{20}Ne,α)^{28}Si reaction studied by Kubono *et al.* [21]. The reaction mechanism is not completely understood and a comparison of the particle spectra from the ^{12}C(^{20}Ne,α)^{28}Si study with those of other transfer reactions such as the ^{25}Mg(^{12}C,^{9}Be) study by Ford *et al.* [18], indicates strong selectivity of nonyrast states. In particular, the 10.94-MeV state is the most strongly populated level below 12 MeV (see Figure 1(a) of Ref. [21]), and it is populated with more than ten times the cross section of the 4^+ levels in the prolate and oblate ground-state bands. A ^{24}Mg(^{6}Li,d) reaction by Tanabe *et al.* [23] also shows a remarkably similar spectra of states with selective population. Again, the 10.94-MeV level is the most strongly populated one below 12 MeV, exceeding the cross section to the other 4^+ levels by a similar factor. The (^{6}Li,d) reaction is not completely straightforward either, but direct α transfer is expected to be the chief contribution. These observations taken together would suggest that the

10.94-MeV state has a dominant $^{24}\mathrm{Mg}+\alpha$ configuration. Indeed, it is interesting to consider this in the light of studies of the ^{32}S(^{12}C,α) reaction by Middleton *et al.* [24], where the 0^+ state attributed to the 4p-4h configuration is excited ten times more strongly than the level associated with the 0p-0h configuration. The 8p-8h level is excited 1.5 times more strongly than the 4p-4h one. Indeed, the state most strongly excited in this reaction is at 7.98 MeV in ^{40}Ca which has later been shown to correspond to the 6^+ member of the SD band based on the 8p-8h configuration [5].

A state at 12.8 MeV is strongly excited in both the ^{12}C(^{20}Ne,α)^{28}Si [21] and ^{24}Mg(^{6}Li,d) reactions [23]. Analysis of angular correlations in the ^{12}C(^{20}Ne,α)^{28}Si reaction provides a firm assignment of 6^+ to a 12.8-MeV state [21]. This level is also shown to have a direct proton branch to the $5/2^+$ ground state of ^{27}Al [21], implying $L=4$ decay and, hence, there must be an associated $g_{9/2}$ component. This is estimated by Kubono *et al.* [25] as corresponding to a spectroscopic factor for the $g_{9/2}$ component of $S=0.3$. This result is reinforced by a parallel ^{24}Mg(α,t) study by Kubono *et al.* [21,25] which also indicated a sizable $g_{9/2}$ component in the 12.82-MeV state. This is an unusually large component which may reflect a strong associated deformation. It is surprising that fragments of the $g_{9/2}$ configuration are observed so low in excitation energy, but evidence also exists for $g_{9/2}$ components in high-spin (7^+) states of ^{26}Al [26]. Despite the small mismatch between the different energies reported for the 6^+ states observed in the ^{12}C(^{20}Ne,α)^{28}Si reaction (12.8 MeV), the ^{24}Mg(α,t) study (12.82 MeV), and the (α,γ) study discussed above (12.86 MeV), it seems likely that these correspond to the same state within errors of calibration, especially given the low level density for 6^+ states in this region (less than one state per MeV according to the shell model). In this scenario, a consistent picture emerges where the candidate intruder states discussed by Brenneisen *et al.* [22] appear with unusual selectivity in the ^{12}C(^{20}Ne,α)^{28}Si reaction, and with the suggestion of strong deformation, in the case of the 12.86-MeV state.

III. PRESENT WORK

Here, we have analysed a data set related to a γ-ray spectroscopy study where ^{28}Si was one of the main channels. The original objective of the experiment was, in fact, the study of mirror symmetry in ^{31}S and ^{31}P, for which results were published some years ago [27]. The aim of the present analysis was to examine the candidate states in the highly deformed band in ^{28}Si proposed by Brenneisen *et al.* [22], to verify their placement and to seek more complete information on their decay and on the transition strengths of the respective γ rays. Excited states in ^{28}Si were populated via the ^{12}C(^{20}Ne,α) reaction using a 32-MeV, ^{20}Ne beam from the ATLAS accelerator at Argonne National Laboratory. A self-supporting ^{12}C target of 90 μg/cm^2 was bombarded with a 40 pnA ^{20}Ne beam for a period of 2 d. The resulting γ decays were detected by Gammasphere, an array of 100 Compton-suppressed germanium detectors [28]. The array was operated in stand-alone mode with a trigger condition of two or more coincident γ rays. Since evaporated α particles were not detected, γ rays associated with ^{28}Si were strongly Doppler-broadened, but the use of high-fold coincidence data still permitted a level scheme for ^{28}Si to be produced from the analysis of a γ-γ matrix and a γ-γ-γ cube. The present analysis confirms the location and decay branching of the candidate states in the intruder band identified by Brenneisen *et al.* [22]. For example, Fig. 2 provides a coincidence spectrum gated on the 3565-keV transition in the γ-γ matrix which illustrates the decay pathway down from the 6^+ state at 12865 keV via the 1919-keV transition towards the ground state. In addition, it has been possible to locate further transitions connected with the decay out of the intruder band such as the 3106-keV γ ray which connects the 9796-keV 2^+ level to the 0^+ state in the ND band at 6691 keV (see Fig. 3). We do not observe the $4^+ \rightarrow 2^+$ transition between the candidate SD states. The data are insufficiently clean to allow us to set a meaningful upper limit on its nonobservation

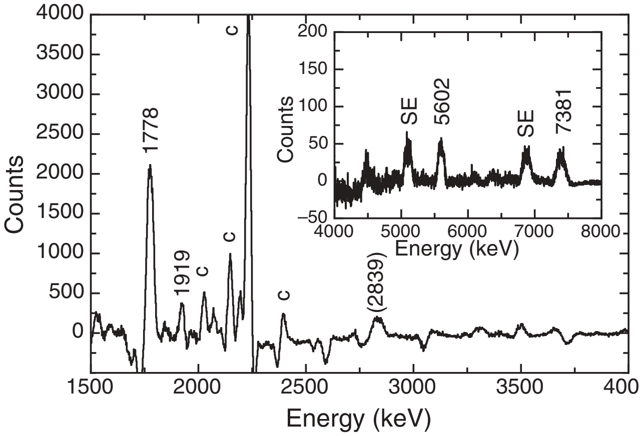

FIG. 2. Coincidence gate on the 3565-keV transition in the γ-γ matrix. Transitions in ^{28}Si are labeled with their energy, while contaminant channels, such as ^{30}Ne, are indicated with a ‘c’. The inset shows the high-energy portion of the spectrum from 4 to 8 MeV. Single escape peaks are marked “SE”.

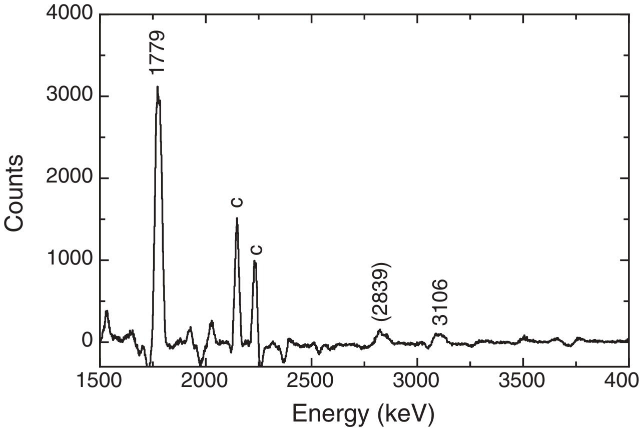

FIG. 3. Coincidence gate on the 4912-keV transition in the γ-γ matrix. Transitions in ^{28}Si are labeled with their energy, while contaminant channels, such as ^{30}Ne, are indicated with a ‘c’.

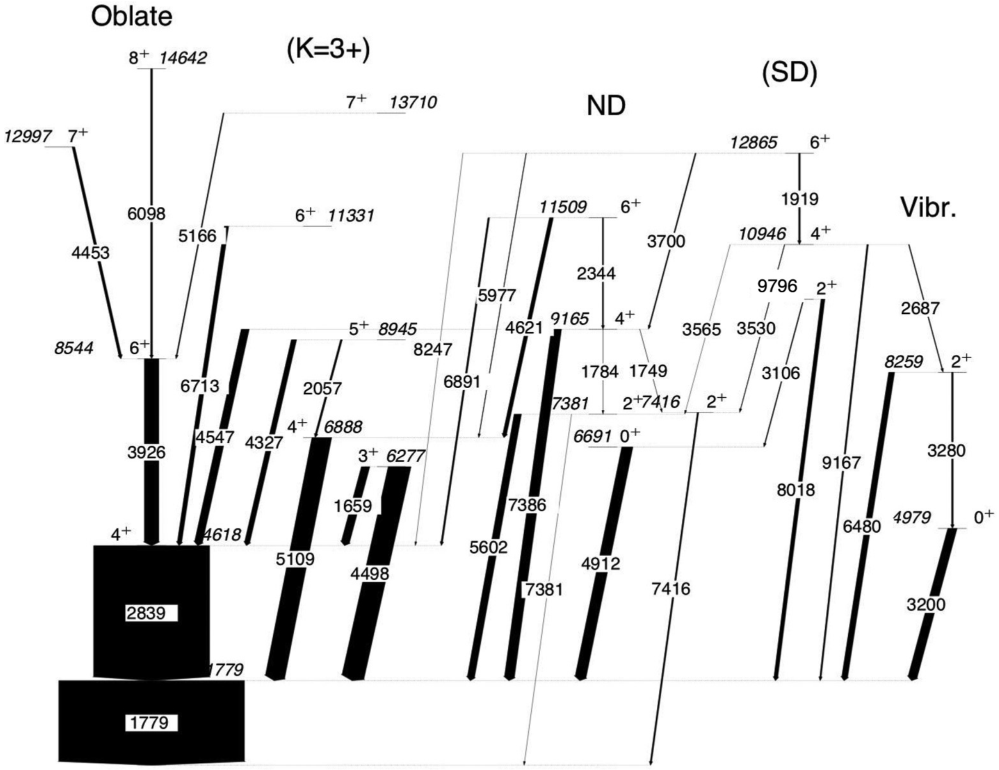

FIG. 4. Subset of positive-parity levels in ^{28}Si derived from the analysis of the Gammasphere data set in the present work. Excited states and transition energies are labeled with their energy in keV, while the width of the arrows corresponds to the relative intensity of the observed transitions in the present work. The different structures are labeled according to previous assignments as oblate, prolate (ND), vibrational and with different K values.

nor infer something related to a lower limit for the $B(E2)$ strength of such a transition. A dedicated study would be needed to search for this transition which would be challenging given the large phase space factor relative to the intraband transitions.

In the present work, we have located nearly all known levels below 10 MeV (the α-breakup threshold is 9.98 MeV), and essentially all known γ-decaying high-spin ($J > 4$) states in ^{28}Si. A relevant subset of the positive-parity states identified here is presented in Fig. 4. In addition to the information on positive-parity states, this analysis confirms the previously known negative-parity band structures which are intensely populated; discussion of these, however, is outside the scope of this paper. The in-beam data form a complement to the radiative capture and reaction data discussed above, as the reaction mechanism is entirely different and favors population of near-yrast states.

The present results have been synthesised with the tabulated data [29] and the work of Brenneisen *et al.* [22], to extract $B(E2)$ values for transitions within the main bands: oblate, prolate, and candidate SD band as well as interband transitions originating from the candidate SD band (see Fig. 5). A half-life is not available for the 9796-keV state, but the branching ratio between the transitions de-exciting it is known from the literature. It should be noted that we do not directly observe the 9796-keV transition from this state to the ground state as it is not in coincidence with any other transition and the data were taken with a γ-γ trigger. Nevertheless, taking account of the branching ratio between the unobserved 9796-keV transition and the observed 8018-keV transition, we can deduce that if

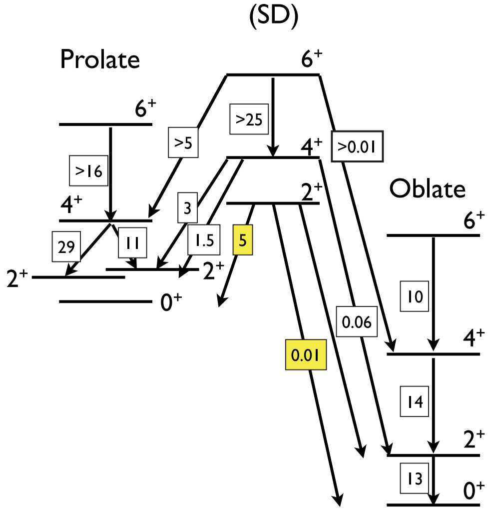

FIG. 5. (Color online) Simplified level scheme for ^{28}Si showing only the oblate ground state band, the prolate band, and the candidate SD band. Transition strengths in Wu are presented for in-band and interband transitions originating from the SD band. The values in the yellow (shaded) boxes for decay from the 2^+ state in the candidate SD band are based solely on the branching ratio of the state (see discussion in the text).

the 9796-keV transition to the ground state has $B(E2) \approx 0.01$ Wu, then the 3106-keV transition has $B(E2) \approx 5$ Wu. This is broadly in conformity with the branching observed for the 4^+ and 6^+ states at 10.945 and 12.86 MeV, respectively.

A consistent picture emerges for the candidate SD states, where a large transition strength is seen for the $6^+ \rightarrow 4^+$ transition, the transitions to the oblate ground-state band are strongly retarded (~ 0.01 Wu), while de-excitations towards the prolate band are enhanced ($\sim$5 Wu). This could reflect structural similarity between the SD and prolate configurations. Clearly, future calculations able to predict interband transitions would provide a discriminating test of the present identification of candidate SD states. It is interesting to note that in a study of the ^{12}C(^{16}O,γ)^{28}Si reaction, Collins *et al.* [30] found that radiative capture resonances had a strongly preferred decay (by more than a factor of ten) to the bandhead of the prolate band in ^{28}Si rather than to the ground state which is the bandhead of the oblate band.

IV. CONCLUSIONS

In conclusion, we have examined the evidence for an SD band in ^{28}Si. Recent calculations suggest that this band should have a strong $^{24}\mathrm{Mg} + \alpha$ configuration [14,16,17]. This suggests that the ^{24}Mg(α,γ) reaction may be a good prospect for selectively populating the SD states. Examination of the literature [22] reveals candidate states that form a sequence in good agreement with the predicted moment of inertia. The 6^+ and 4^+ states are connected by an $E2$ transition with $B(E2) > 25$ Wu. We connect this work to ^{12}C(^{20}Ne,α)^{28}Si and ^{24}Mg(^{6}Li,d) reaction studies, where strong selection of these levels is observed, and a significant $g_{9/2}$ component is identified for the configuration of the 6^+ state. This may point to a large deformation. Analysis of an in-beam study of ^{28}Si confirms the assignment of these states and also leads to the identification of additional interband transitions. The decay of the candidate SD band is highly characteristic with strongly retarded decays to the oblate ground-state band, and enhanced de-excitation to the prolate band. This poses a challenge for theory to compute such interband transition matrix elements. Further experimental work is clearly warranted to identify the missing 0^+ bandhead. For this purpose, an inelastic scattering reaction may be most appropriate. To identify even higher-lying SD states, a particle-γ methodology such as that used to identify the 10^+ state in ^{24}Mg [31] would be a good approach.

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