A 2.5D automatic FEM-SBM method for the evaluation of free-field vibrations induced by underground railway infrastructures

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ABSTRACT

This paper presents an efficient method to predict underground railway-induced vibrations. The method uses the finite element method (FEM) to model the railway tunnel structure and the singular boundary method (SBM) to model the wave propagation in the surrounding soil. The FEM mesh and the distribution of SBM collocation points at the tunnel/soil interface are generated using an automatic meshing strategy. The presented method is one of the main components of VIBWAY, a user-friendly prediction tool to address railway-induced vibration problems. This paper presents three calculation examples in which the soil response due to forces applied on the tunnel structure are computed in terms of transfer functions. The results obtained for each one of the calculation examples are compared with those computed using a model based on a 2.5D FEM-BEM approach. The presented comparisons show that the proposed approach is a suitable strategy for predicting underground railway-induced vibrations, both in terms of accuracy and computational efficiency. Moreover, the use of an automatic meshing strategy and the SBM formulation not only eases the implementation of the approach but it also makes it easier to use, which is one of the key features of the VIBWAY tool.

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1. INTRODUCTION

Railway traffic induces ground-borne vibrations that can cause a negative impact on inhabitants living in the vicinity of railway lines, limit the performance of sensitive high-tech equipment or even cause structural damages on historical structures. These problems can be reduced by the implementation of effective vibration abatement solutions. However, to design cost-effective countermeasures, engineers need prediction tools capable of capturing the complexity of the addressed problem.

Over the last two decades, several types of models have been proposed to predict railway-induced vibrations. Among them, two-and-a-half-dimensional (2.5D) models, which consider that the railway structure is invariant in the traveling direction, have been found to be accurate for performing predictions in many practical applications. A 2.5D hybrid methodology that considered the Finite Element Method (FEM) for modelling the track structure and the Boundary Element Method (BEM) to model the wave propagation through the soil was presented in [1]. The same approach was used by Ghangale et al. [2], who developed a computationally efficient energy flow study that considered the 2.5D FEM-BEM model for computing the tunnel-soil interaction and analytical cavity solutions for modelling the wave propagation through the soil. An alternative approach was considered in [3], where the FEM is combined with the use of Perfectly Matched Layers (PML) to develop a hybrid 2.5D FEM-PML method for predicting railway-induced vibrations. Despite being much more efficient than their 3D counterparts, one of the drawbacks of the previous methods is that they still require significant engineering and computational times.

More efficient modelling strategies can be developed by using the so called meshless methods, numerical methods that do not require to define a geometrical mesh as the one used by FEM or BEM approaches. One of the most well-known meshless methods is the Method of Fundamental Solutions (MFS), which has been extensively used in many different engineering problems. Godinho et al. [4] developed a 2D hybrid FEM-MFS approach for predicting dynamic soil-structure interactions. Their method was later extended to develop a 2.5D FEM-MFS methodology for predicting railway-induced vibrations [5]. More recently, Liravi et. al. [6] proposed the use of a hybrid 2.5D FEM-BEM-MFS methodology. In their approach the railway structure is modelled using FEM, the soil surrounding the structure using BEM and wave propagation through the soil using MFS. Although the MFS allows to develop models that are simple to implement and computationally efficient, they usually face an important drawback: the accuracy of the method is highly sensitive to the location and number of virtual sources used for representing the geometry of interest, specially when this geometry is complicated [7]. The Singular Boundary Method (SBM) is an alternative type of meshless method that is capable of overcoming the main limitations of the MFS. In contrast with the MFS, the SBM does not require to define an auxiliary boundary in which the virtual sources are located, which increases its robustness. Moreover, the method inherits the computational efficiency and implementation simplicity of the MFS. Both features make it a suitable approach for performing railway-induced vibration predictions.

This paper presents several application examples of an efficient method to predict underground railway-induced vibrations. The method considers a 2.5D formulation of the problem and combines the use of the FEM for modelling the tunnel structure and the SBM for representing the soil where it is embedded. The method is one of the main components of VIBWAY, a user-friendly prediction tool to address railway-induced vibrations problems for different types of tracks. In order to simplify the use of the tool, the FEM mesh of the tunnel structure is generated using the Delaunay-based mesh generation algorithm presented [8]. The 2.5D SBM approach implemented in the tool is based on the formulation presented in [9]. The proposed methodology has been applied to three different calculation examples: two of them consider a circular tunnel and the other a cut-and-cover tunnel. In the following sections, the performance of the method is evaluated by comparing the results obtained using it with those computed using other well-established methods. More precisely, the results are compared with those obtained using a 2.5D FEM-BEM model of the system and, for the first calculation case, with the ones obtained using the Pipe-in-Pipe (PiP) model [10].

2. DESCRIPTION OF THE CALCULATION EXAMPLES

The aim of this paper is to present the application of the 2.5D FEM-SBM methodology presented in [9] to several calculation examples that are of interest in the type of railway-induced ground-borne vibration problems that VIBWAY prediction tool aims to address. The calculation examples considered in this work are:

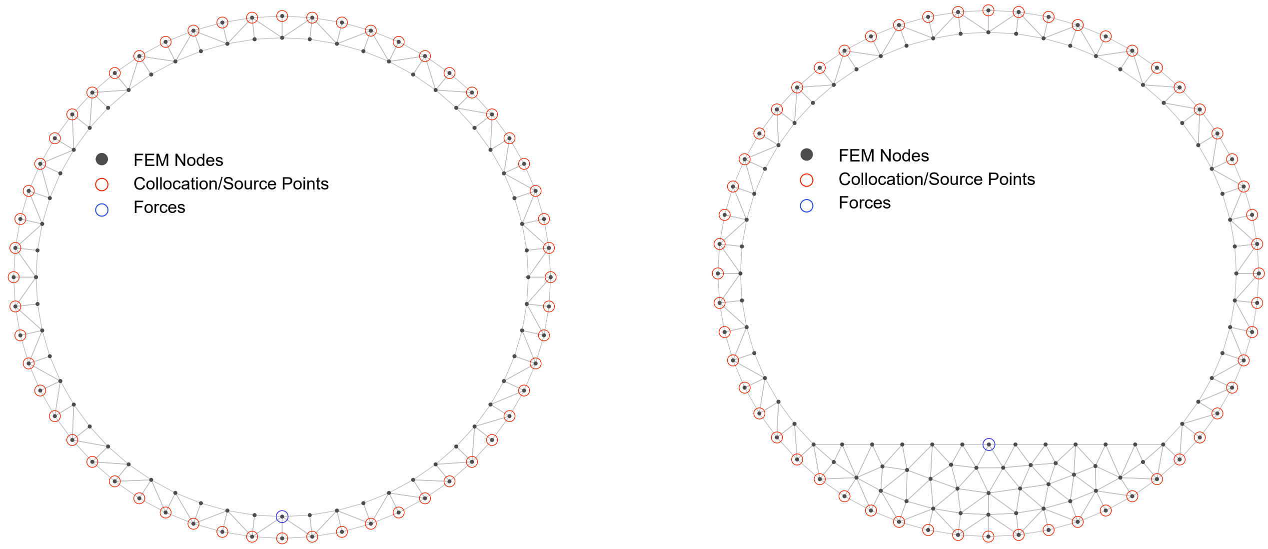
- A circular tunnel embedded in a homogeneous full-space.
- A circular tunnel embedded in a homogeneous half-space. The results obtained for this case are compared with those obtained for the following case.
- A circular tunnel with a slab embedded in a homogeneous half-space.
- A cut-and-cover tunnel embedded in a layered half-space.

For the sake of brevity, the proposed methodology will be only briefly outlined here but the interested reader can find more detailed explanations in [9].

On the one hand, the FEM is used to model the tunnel structure. For each calculation example, a unique mesh has been used for the whole range of frequencies of interest. The element size has been defined considering the criteria that, for the maximum frequency of interest, there are at least six boundary nodes per wavelength (the soil shear wave wavelength has been considered for this calculation). On the other hand, the soil response is modelled using the SBM. In brief, the SBM method assumes that the response of the modelled system (in this work an elastic space with a cavity in it) can be computed using fundamental solutions (in this work the response of the elastic space without the cavity) to a set of virtual sources distributed along the boundary of the system. The strength of the virtual sources is determined solving the equations obtained by considering the response of the system at each source position. Therefore, in contrast with what it is considered in the MFS, the set of source points and the set of collocation points of the SBM method is the same. Finally, the coupling between the tunnel structure and the soil is performed imposing also that the collocation/source points used in the SBM formulation coincide with the FEM nodes that are located on the structure's outer boundary (interaction interface).

Figure 1 illustrates the geometrical information used for the cases of a circular tunnel with and without slab. The black dots indicate the FEM nodes, the red dots indicate the SBM source/collocation points and the blue dot indicates where the external vertical harmonic point load is applied. It should be noted that the SBM points are used to model the soil response but they are included in this figure to illustrate their coincidence with the boundary FEM nodes.

The accuracy of the 2.5D FEM-SBM method is evaluated by comparing it with two other modelling strategies: the FEM-BEM approach and, only for the first calculation example, with the PiP method. The comparison is performed considering the receptance at different field points for harmonic loads applied on the tunnel structure. The frequency range of interest is 1-100 Hz. The receptanes are computed at $x = 0$, being x the direction along which the system is invariant. As detailed in [9], the receptance is computed from the response of the system in the wavenumber-frequency domain.



(a) Circular tunnel without a slab *(b) Circular tunnel with a slab*

Figure 1: The FEM mesh, the collocation/source points and the force position for the circular tunnel cases.

3. RESULTS

This work presents three calculation examples that use the proposed 2.5D FEM-SBM methodology. The aim of the first example, which considers the case of a circular tunnel without slab (i.e. a thin cylindrical shell) embedded in a homogeneous full-space, is to assess the accuracy of the method. The second example, which considers circular tunnels with and without slab embedded in a homogeneous half-space, aims to assess the effect that the slab has on the soil response. Finally, example three, which considers a cut-and-cover tunnel embedded in a layered half-space, aims to highlight the capabilities of the proposed strategy for a more complex scenario. The meshes and collocation points used in all the calculation examples are generated using an automatic meshing strategy, an approach that greatly simplifies the development of the hybrid FEM-SBM models of each case.

3.1. Example 1: Circular tunnel embedded in a full-space

The proposed methodology is used to compute the response of a circular tunnel embedded in a homogeneous full-space. The FEM mesh of the tunnel and the excitation point have been presented in Figure 1a. The geometrical and mechanical parameters used are those considered in [11]. The tunnel is made of concrete and has a radius of 3 m and a thickness of 0.25 m. The concrete has a Young's modulus of 50 GPa, a density of 2500 kg/m^3 and a Poisson's ratio of 0.3. The surrounding soil has a Young's modulus of 550 MPa, a density of 2000 kg/m^3, a Poisson's ratio of 0.44 and a material damping ratio of 0.03. The tunnel is excited by a harmonic load applied on the tunnel invert and the soil receptance is obtained at four field points: A (x = 0 m; y = 10 m; z = 0 m), B (x = 0 m; y = 20 m; z = 0 m), C (x = 0 m; y = 0 m; z = 10 m) and D (x = 0 m; y = 0 m; z = 20 m). The origin of the rectangular system of coordinates used for this example is the centre of the tunnel (in later examples this origin will be placed on the ground surface). The receptances are computed considering a linear wavenumber sampling with 1024 sampling points from 0 to 2π rad/m.

Figure 2 compares the receptances obtained using the 2.5 FEM-SBM method (black) with those obtained using the FEM-BEM method (red) and the PiP model (blue). The results are presented in dB considering a 1 m/N reference. As the excitation has been applied at x=0, the responses of the field points do not have any component in the x-direction. The results highlight that the accuracy of the proposed approach is similar to the one obtained using a FEM-BEM approach, and that both agree well with the PiP model predictions. The small discrepancies observed at high frequencies can be attributed to an insufficient number of nodes per wavelength.

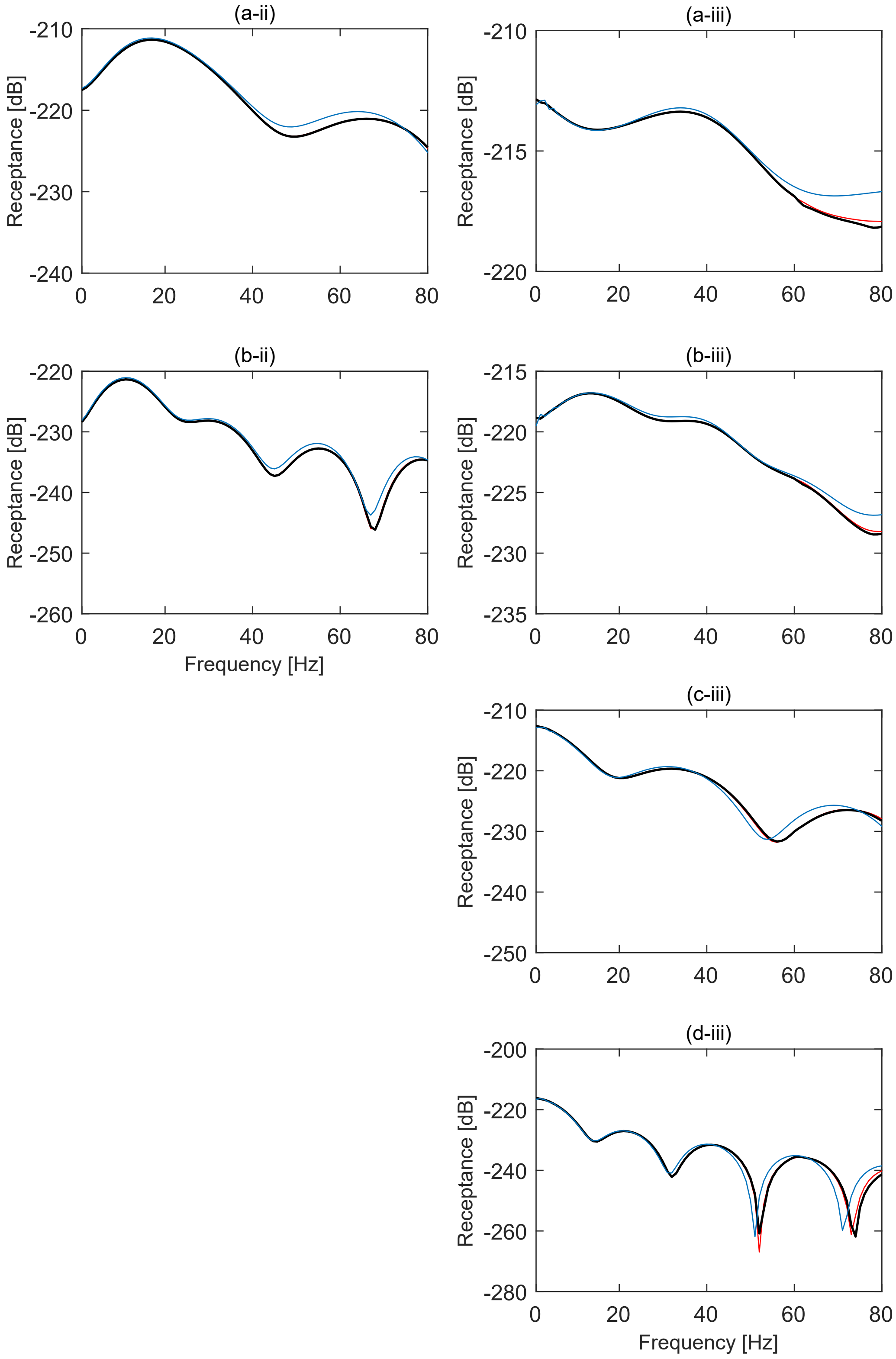


Figure 2: Receptances at field points A (a), B (b), C (c) and D (d) for y (ii) and z (iii) directions. Methods: 2.5D FEM-BEM (red), 2.5D FEM-SBM (black) and PiP (blue). Results presented in dB considering 1 *m/N as the reference value.*

This example has also been used as a framework for comparing the computational cost of the 2.5D

FEM-SBM approach against the 2.5D FEM-BEM one. Both methodologies have been implemented in MATLAB and have been executed using a single core of a high-performance cluster with 2 GHz Intel® Xeon® Gold 6138 CPU (with 40 cores). The computational time of the methods has been evaluated for two different case scenarios. In the first one, the receptance has been computed at a single field point, for a specific frequency and considering 6, 10, 17 and 24 boundary nodes per wavelength. In the second case, the soil responses have been computed for one frequency, for 24 boundary nodes per wavelength and considering different values for the number of evaluated field points. The results obtained for both scenarios are presented in Tables 1 and 2. In both cases, the computational time required by the 2.5D FEM-SBM approach is presented relative to the 2.5D FEM-BEM one (i.e, a 100% value would indicate that both times are equal). The comparison shows that the 2.5D FEM-SBM approach efficiency is significantly higher than the one of the 2.5D FEM-BEM approach, specially when the response to a large number of field points is required.

Number of boundary nodes per wavelength	6	10	17	24
2.5D FEM-SBM computational time [%]	78	77	76	77

Table 1: Computational time comparison for different number of boundary nodes per wavelength.

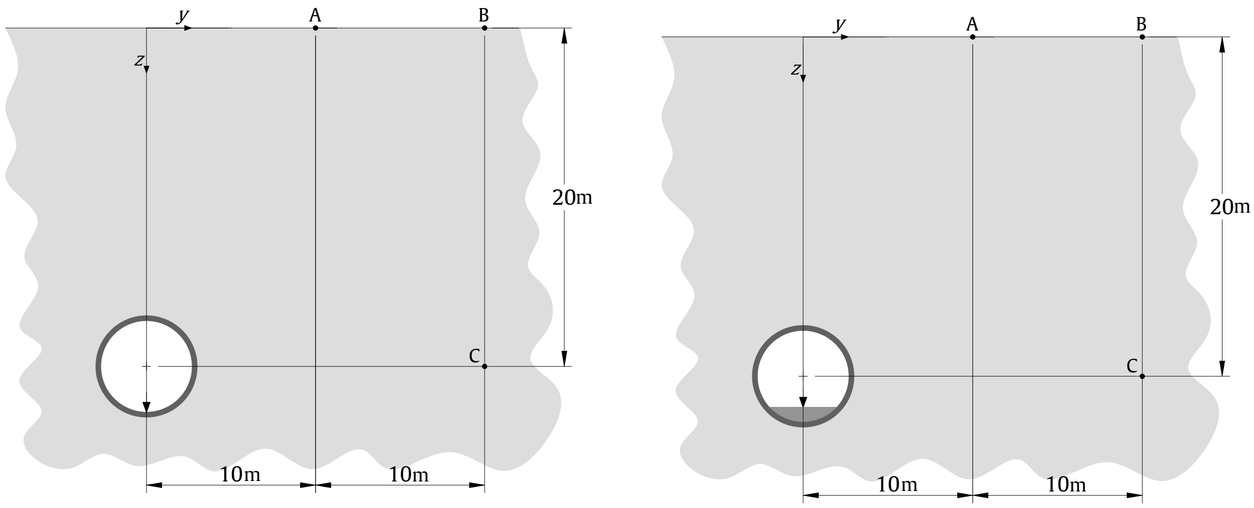
Number of field points	5	25	60	100	160	200
2.5D FEM-SBM computational time [%]	81	61	41	31	22	19

Table 2: Computational time comparison for different number of field points.

3.2. Example 2: Circular tunnels embedded in a homogeneous half-space

The proposed FEM-SBM methodology is used in this example to, on the one hand, test the accuracy of its predictions for a case involving an elastic half-space and, on the other hand, to compare the soil response for two different configurations of the circular tunnel: one with a concrete slab at the tunnel invert and the other without it (as in Example 1). The FEM mesh of the tunnel and the excitation points for each case are the ones that have been presented in Figure 1.

The tunnel geometry and mechanical properties are the same than those used in Example 1. The thickness of the slab is 0.85 m at its centre. The soil considered for this case is softer than the one used in Example 1. It has a Young's modulus of 192 MPa, a density of 1800 kg/m^3, a Poisson's ratio of $1/3$ and a material damping ratio of 0.02. The centre of the tunnel is assumed to be buried at a depth of 20 m. The results are computed at three field points located on the soil: two on the ground surface (A, B) and the other inside the soil (C). The geometry of the circular tunnel and the location of the field points are shown in Figure 3.



(a) Tunnel without slab. *(b) Tunnel with slab.*

Figure 3: Scheme of the tunnel-soil system for the case of a circular tunnel embedded in a half-space. The location of the evaluated field points (A,B and C) is also detailed.

Figure 4 presents the comparison of the response obtained at the field points of interest for the two tunnel configurations considered in this example: with (dashed lines) and without a slab (solid lines). The results obtained with the 2.5D FEM-SBM methodology (black) are also compared with the ones obtained using a 2.5 FEM-BEM approach (red). In this example, the results are presented in dB considering a 10^{-12} m/N reference.

The comparison between both modelling strategies highlights that, at low frequencies, the accuracy of the proposed approach is similar to the one obtained using a FEM-BEM approach. However, some discrepancies can be observed for frequencies larger than 50 Hz. These discrepancies can be again attributed to the number of collocation points used, as it is expected that, when the number of nodes per wavelength is reduced, the accuracy of the FEM-SBM method decays faster than that of the FEM-BEM method. A more detailed study of these decays can be found in [9].

The comparison between tunnel configurations shows that the inclusion of a concrete slab on the tunnel structure has a clear impact on the dynamic response of the soil-structure system. The effect is specially significant in the comparisons of the horizontal (y-direction) components, where the addition of the slab reduces significantly the soil response in all the range of frequencies considered. In contrast, the change caused by this added stiffness is less clear in the comparisons of the vertical (z-direction) components. These results highlight that a suitable vibration prediction model for this type of tunnel configuration should take into account the slab geometrical and mechanical characteristics.

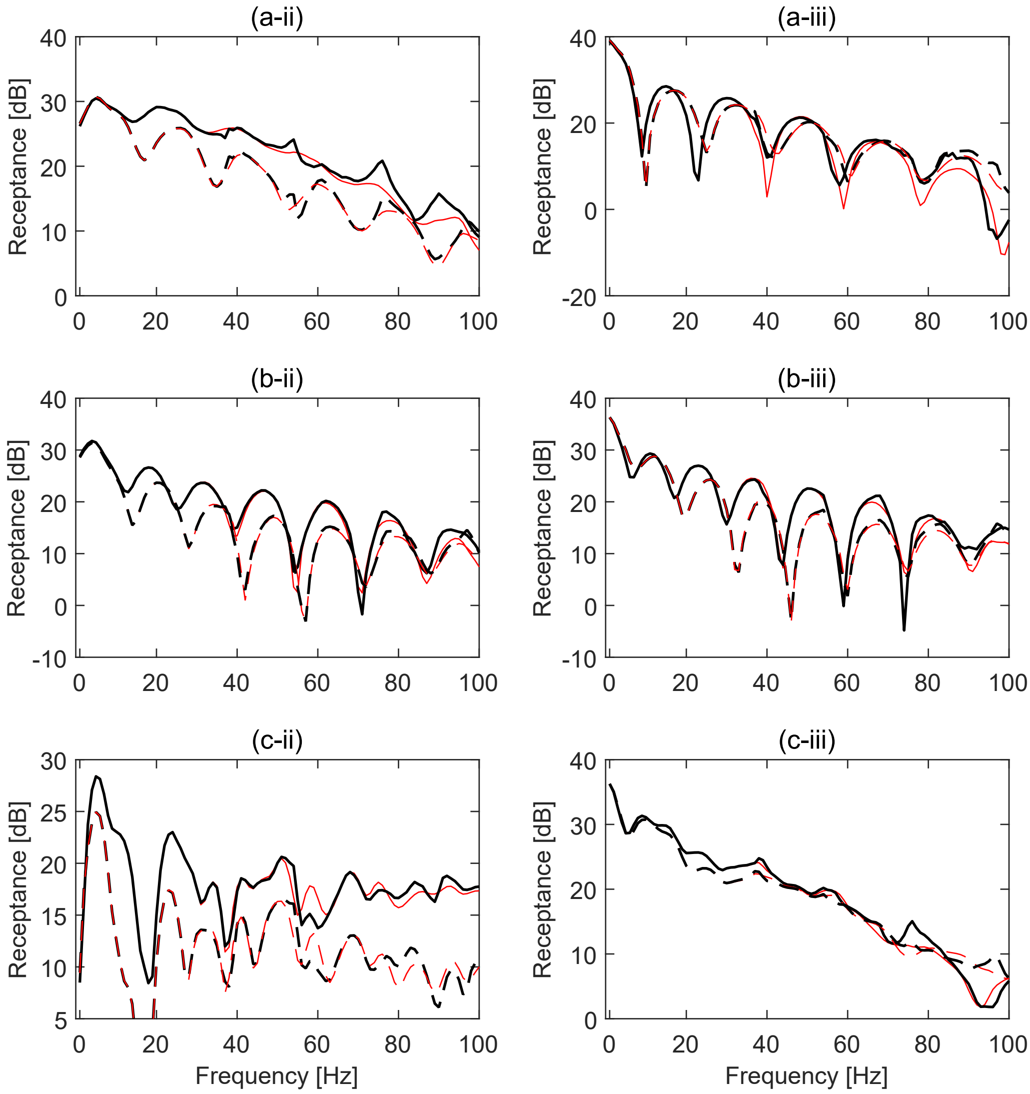


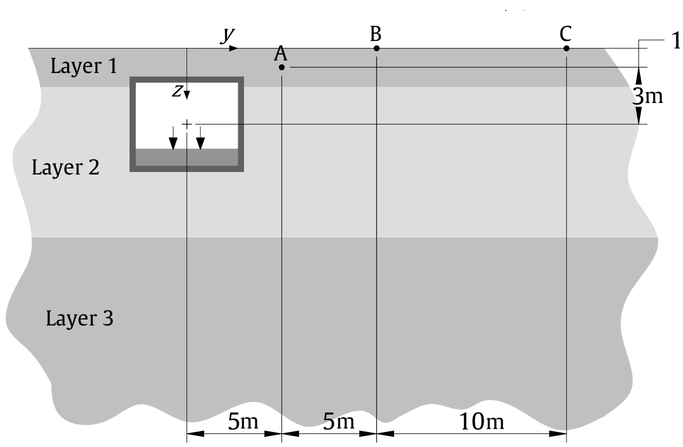
Figure 4: Receptances at field points A (a), B (b) and C (c) for y (ii) and z (iii) directions; Tunnel configurations: With slab (dashed lines) and without slab (solid lines); Methods: 2.5D FEM-BEM (red lines) and 2.5D FEM-SBM (black lines). Results presented in dB considering 10^{-12} *m/N as the reference value.*

3.3. Example 3: Cut-and-cover tunnel embedded in a layered half-space

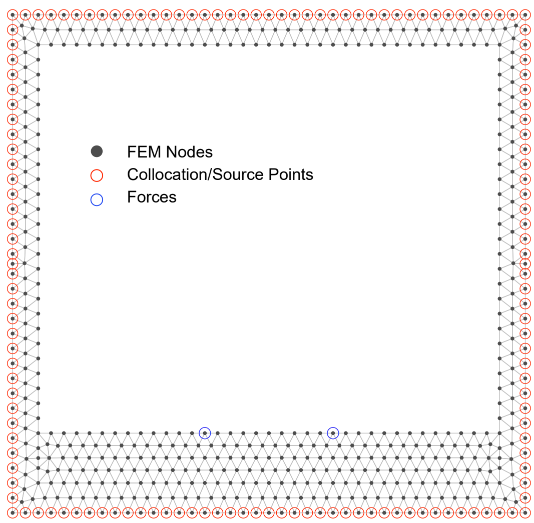
This last example aims to assess the performance of the 2.5D FEM-SBM methodology for a more challenging case: a cut-and-cover tunnel that is embedded in a horizontally layered half-space. The main differences between this case and the previous ones are that, first, the soil-structure interface is no longer a smooth boundary, and second, that the computation of the soil response needs to take into account the soil stratification. In this example it is assumed that the tunnel shape is a rectangle with an outer height of 5 m and width of 6 m. The tunnel thickness is 0.25 m and the slab attached to the tunnel invert is 0.55 m thick. The centre of the tunnel is located at a depth of 4 meters from the ground surface. The slab is excited by two harmonic vertical point loads separated 1.5 meters and situated at the same distance of both tunnel walls. It is assumed that the soil is composed of three horizontal layers and that the tunnel structure is embedded between the two upper layers. The soil receptance is evaluated at three field points: one close to the tunnel structure (A) and two on the ground surface (B and C). The geometrical characteristics of this example and the location of the field points are shown in Figure 5a. The mechanical parameters considered in the calculations for the slab, tunnel lining and each one of the soil layers can be found in Table 3. As in the previous examples, a unique mesh has been used for the whole range of frequencies of interest. In this case, it has been ensured that there are at least ten boundary nodes per wavelength at the maximum frequency of interest. The FEM mesh of the tunnel structure, the SBM source/collocation points and the excitation points have been presented in Figure 5b. As in the previous cases, the model has been developed using an automatic meshing strategy.

Type	E [MPa]	ρ [kg/m^3]	ν	Damping [-]	Thickness [m]
Tunnel lining	31000	2500	0.2	0.001	-
Slab	28000	2400	0.2	0.001	-
Soil layer 1	108	1800	0.333	0.05	2
Soil layer 2	150	1900	0.333	0.05	8
Soil layer 3	350	2000	0.333	0.05	∞

Table 3: Mechanical parameters used for the cut-and-cover calculation example.



(a) Scheme of the tunnel-soil system. The location of the evaluated field points (A,B and C) is also included.



(b) The FEM mesh, the SBM points and the force positions.

Figure 5: Geometry and mesh used for the cut-and-cover tunnel embedded in a layered half-space.

The soil receptances obtained for this case are presented in Fig. 6. As before, the results of the three evaluation points obtained with the 2.5D FEM-SBM methodology (black) are compared with the ones obtained using the 2.5 FEM-BEM approach (red). The results are again presented in dB considering a 10^{-12} m/N reference.

The comparison shows a good agreement between the receptances computed with both methods, even at high frequencies. This better performance can be justified by the larger number of nodes/points used in the calculations. The results allow to conclude that the FEM-SBM approach can deal with soil-structure interaction problems that involve horizontally layered soils. The results also highlight that, in contrast with what happens when using the MFS, the SBM approach can easily deal with non-smooth geometries (i.e. containing sharp edges).

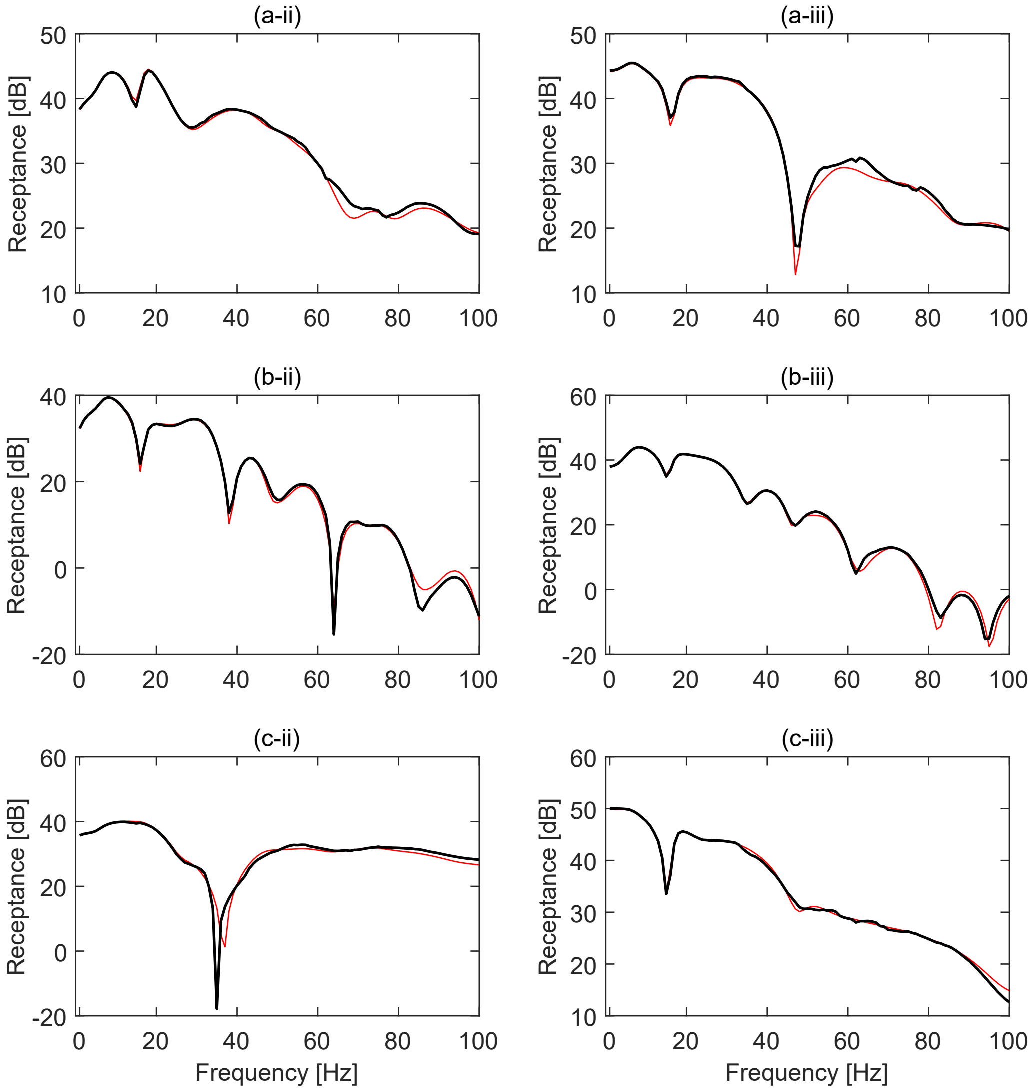


Figure 6: Receptances at the field points A (a), B (b) and C (c) for y (ii) and z (iii) directions. Methods: 2.5D FEM-BEM (red) and 2.5D FEM-SBM (black). Results presented in dB considering 10^{-12} *m/N as the reference value.*

4. CONCLUSIONS

This paper presents several application examples of a novel hybrid 2.5D FEM-SBM method for predicting underground railway-induced vibrations. The cases include tunnel structures with circular and rectangular shapes, and with an without tunnel invert slabs. Regarding the embedding soils, both full-space and (homogeneous or layered) half-space cases have been considered. The performance of the method has been investigated by comparing the results obtained at different soil locations with those obtained using two other well-established approaches: A 2.5D FEM-BEM methodology and, for the case of a circular tunnel embedded in a full-space, the PiP model. In general, a good agreement has been observed between the different methods, justifying the use of a FEM-SBM approach for dealing with railway-induced vibration problems. The main benefits of the approach are that, first, it is more efficient and easier to implement than a FEM-BEM approach and, second, it can deal with structures that have non-smooth geometries and/or complex soil stratifications. Moreover, the fact that the FEM meshes used in the presented examples have been generated using an automatic meshing strategy also simplifies the development of the models. This approach can be very convenient in many practical situations and it is the one considered in VIBWAY, a user-friendly prediction tool to address railway-induced vibration problems.

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