PROMOTING ENGINEERING STUDENTS' LEARNING WITH MATHEMATICAL MODELLING PROJECTS

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ABSTRACT

Mathematics constitutes a key component in engineering education. Engineering students are traditionally offered a number of mathematics courses which provide the knowledge needed at the workplace. Unfortunately, many students perceive mathematics as a discipline that teaches mostly procedures not relevant to their

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future careers and often view it as one of the main obstacles on their way to an engineering degree. In this paper, we discuss how introducing university students in a standard Differential Equations course to mathematical modelling (MM), a powerful strategy for solving real-life problems, contributes to the development of their mathematical competencies, motivates their interest to mathematics, promotes the use of advanced mathematical thinking, methods of applied mathematics, and digital computational tools.

1 INTRODUCTION

1.1 Mathematics in engineering education

Mathematics is an integral part of engineering education; it helps to establish connections between different physical quantities and furnishes powerful tools to model the behaviour of complex engineering and industrial systems. Methods of applied mathematics are efficiently used to develop new technologies and simulate various real-world phenomena. Analysis of mathematical models reduces the costs for setting experiments '*in vivo*', allows us to evaluate different scenarios, explore feasibility of solutions, and make comprehensive decisions.

The increasing computational difficulty of the engineering tasks and wider implementation of technology in teaching and learning inevitably lead to a more extensive use of computer algebra systems (CAS) in mathematics courses for engineering students. Kent and Noss argued that "advances in the use of information technology and computers have transformed engineering analytical techniques, and production and management processes" [3, p. 4]. Nowadays, students are expected to combine the power of theoretical knowledge with that of modern digital technologies, and the efficient use of both requires advanced mathematical thinking. Not all engineering students are prepared to meet high demands set in mathematics courses and often view them as an obstacle - "some see mathematics as the gateway to engineering, paving the way to sound design; others see mathematics as a gatekeeper, denying entry to otherwise talented would-be engineers" [13, p. 305].

Faulkner et al. [2] emphasised that engineering faculty want students to acquire "mathematical maturity" rather than calculus skills as the learning outcome from mathematics courses. Although "any single construct will provide a complete view of mathematical maturity" [2, p. 100], it is often used by mathematics faculty to describe students "who have achieved a certain combination of technical skills, habits of investigation, persistence, and conceptual understanding" [1]. Faulkner et al. argued that "these engineering faculty believed that the mathematically mature student would have strong mathematical modelling skills supported by the ability to extract meaning from symbols and the ability to use computational tools as needed" [2, p. 97] concluding that a successful mathematics course should be situated on the cross-roads between engineering, mathematics, and programming. The reform of engineering mathematics education requires timely adjustment of course curricula to the changing demands set by the employers. According to Niss and Højgaard, the three main components in the curriculum of a mathematics course are: (a) purpose of the teaching, (b) syllabus, that is, mathematical content, and (c) assessment including instruments to estimate the extent to which the students have learned the mathematical content [5, p. 45].

1.2 Current trends in mathematics education of future engineers

In response to new trends in engineering education, several efforts have been recently made to include modelling tasks and promote the use of CAS in mathematics courses [4, 8-10]. However, time limitations make the use of CAS in the classroom quite demanding, many difficulties also arise with the assessment of students' learning in courses heavily relying on digital tools. Finding a suitable form of assessment in a mathematics course has always been challenging for teachers. The traditional form of assessment in service mathematics courses in engineering departments is a graded written final examination after which the grade for the entire course is assigned. Quite often, restrictions on the time allocated for the exam and associated stress negatively affect students' performance also impacting the assessment results. To avoid this unpleasant situation, efforts have been made to develop different assessment forms, including several forms of embedded assessment and continuous assessment [12].

In order to develop students' mathematical maturity, some changes in teaching practices should be made. Schoenfeld explained that "mathematics is an inherently social activity in which a community of trained practitioners (mathematical scientists) engages in the science of patterns – systematic attempts based on observations, study; and experimentation to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics")" [11, p. 335]. Engineering students often think about mathematics as a set of formal rules which should be used to solve a problem and may apply them even without attempting to understand the logic behind the actions taken. The important role of mathematics lecturers is to motivate students to learn how to "do mathematics" and how to "think mathematically", thus enculturating students into the world of advanced mathematical thinking. A social component is especially important in the process of the development of abstract mathematical reasoning. Collaborative work in groups develops students' use of mathematical language, helps to improve both the mathematical argumentation and communication skills. The importance of the project work in engineering education was emphasised in the recent evaluation of engineering degree programs in Norway: "Making the teaching more project-based offers a means of exercising and evaluating the students' communication skills, of participating in interdisciplinary collaboration and demonstrating professional and ethical practices" [7, p. 10].

In this paper, we discuss how mathematical modelling (MM) projects were introduced in a Differential Equations course for graduate students in mechatronics. The first author modified the traditional course curriculum including a small group project work with MM tasks offered in the format of graded course projects counting towards the final grade. Engaging students into solving applied problems relevant for engineering, we connect their knowledge gained in mathematics, physics, and engineering courses. Furthermore, our MM projects promote students' conceptual understanding of differential equations and show how abstract mathematical ideas can be efficiently combined with the possibilities offered by the modern computer algebra systems. The organisation of students' work in small groups introduces essential elements of collaborative learning and enhances students' social skills. Last but not least, the use of graded projects in the assessment distributes students' work more evenly through the semester and reduces the exam stress.

The research question we address in this paper is: *How modelling projects in a mathematics course contribute to the development of engineering students' mathematical maturity and integration of mathematics and engineering?*

2 METHODOLOGY

2.1 Mathematical competency framework

For the data analysis, we use the concept of mathematical competency defined by Niss and Højgaard as "a well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge." [5, p. 49]. An overall mathematical competence is composed of eight competencies which are defined separately but may overlap. These are: (1) thinking mathematically, (2) reasoning mathematically, (3) posing and solving mathematical problems, (4) modelling mathematically, (5) representing mathematical entities, (6) handling mathematical symbols and formalism, (7) communicating in, with, and about mathematics, and (8) making use of aids and tools [5, p.14]. Designing the tasks that contribute to the development of all mathematical competencies at the same time is challenging. However, we argue that project-based MM assignments not only provide an opportunity to improve mathematical competencies of engineering students, but also make the assessment in a mathematics course more efficient.

An important learning outcome of a mathematics course for engineers is the ability to transform a physical or, more generally, an engineering problem into a mathematical problem (mathematisation) and then solve it using appropriate mathematical and computational tools. To achieve this goal in our standard course in Differential Equations, changes to parts of the course introducing theory and methods of solving differential equations were made. These modifications had the following objectives: (i) development of conceptual understanding of differential equations, (ii) meaningful application of differential equations in practical problems, (iii) improvement of students' computational and analytical skills, including the use of programming for solution and simulation in modelling problems, (iv) improvement of students' communication skills and acquaintance with main principles of collaborative work in groups typical for practising engineers, (v) improved, fair, and less stressful assessment of students' work more evenly distributed throughout the semester.

2.2 Teaching experiment and data collection

The teaching experiment was organised in a one-semester course “Mathematics for Mechatronics” for the class of senior students in mechatronics. Three project-oriented assignments, accounting for 30% of the total course grade, were distributed to the students. Each assignment was related to one of the main topics in the course: (a) first order linear differential equations, (b) higher order linear differential equations, and (c) systems of linear differential equations. The class was divided into small groups of 2-3 students, according to their own choice. Students' written reports for each of the projects were collected and graded during the course. The data analysis was conducted after the final grades were assigned to all students in the course.

3 DATA ANALYSIS

3.1 Sample modelling task

In one of the tasks students were asked to conduct a small experiment with a falling chain and observe how the physical parameters affect the result exploring, for example, what is needed for the chain to start moving. For the mathematical analysis of the problem, it is necessary to describe physical forces acting on the chain and analyse the impact of friction by considering the type of the surface.

Modelling task. Lay a uniform chain on a horizontal surface (say, a table) with a part of it hanging down the table. Release it so that it starts moving down. Find the time when it slips off the table.

(a) What conditions and parameters affect the experiment outcome?

(b) Consider cases without and with friction. What are the conditions for the chain to start moving (for both cases)?

(c) Give the explanation of the physical problem and set it in mathematical terms (for both cases).

(d) Set the differential equation describing your model (for both cases).

(e) Solve the problem mathematically (for both cases).

(f) Use Maple/MATLAB to solve the problems numerically, plot and analyse the solution (for both cases).

(g) Validate your model in practice: conduct an experiment and present a small video showing the timer. Do the experimental data correspond to your analytic solution? Which of the two solutions describe your experiment better?

(h) Submit your video as a media file and your report in a PDF format.

3.2 Analysis of students’ solution

Students were asked to experiment with the chains and table surfaces of their own choice. The weight of the chain, its initial position, and the smoothness of the surface had an effect on the time required for the chain to fall off the table. In this paper, we discuss the solution produced by one group of students only. Students illustrated the experiment with Figure 1. They used a necklace to make a chain with the length $L = 0.46\ m$ and the mass $m = 22\ g$.

After several experiments, the students concluded that for the chain to start sliding down the initial length h (see Figure 1) has to be about $0.11\ m$. At this stage, they used the competencies to *pose a problem and ask questions*, and to *describe the relations* between various parameters. The possibility of choosing their own experiment settings allowed students to conduct practical tests prior to designing a mathematical model.

Fig. 1. Group's illustration to a model and snapshots from the experiment video

Next, the students considered the distribution of forces for the frictionless case. They modelled the chain representing it as two masses connected with a massless rope and a frictionless pulley. At this stage, students were engaged in the process of *mathematising*, that is translating a physical problem into a mathematical problem. In the modelling process, one does not need to reinvent a wheel. Usually, if we want to model with differential equations, the motion of bodies is described by Newton's laws, in particular, by the second Newton's law. We emphasise that it is important for the students in advanced mathematics courses to synthesise relevant knowledge from different courses and apply it for solving the problem.

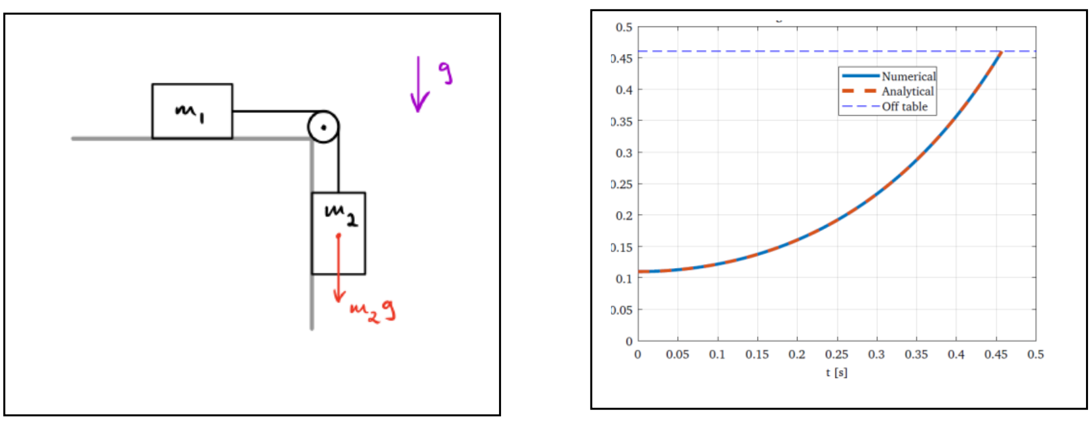

Fig. 2. Illustration of the frictionless model and results of numerical computation

Mathematically the system can be described by the second Newton's law which leads to the differential equation of motion, $m_2\, g = (m_1 + m_2)a$, so the acceleration

$$a = \frac{m_2\, g}{m_1 + m_2}.$$

Using the chain's linear mass density $\frac{m}{L} = \lambda$, we have $m_1 = (L-h)\lambda$, $m_2 = h\lambda$. Then the equation of motion assumes the form

$$a = \ddot{h} = \frac{h\frac{m}{L}g}{(L-h)\frac{m}{L} + h\frac{m}{L}} = \frac{h\frac{m}{L}g}{m} = h\frac{g}{L} \quad \text{or} \quad \ddot{h} - h\frac{g}{L} = 0.$$

The students formulated a mathematical problem by *thinking and reasoning mathematically*, they relied on their previous knowledge in physics for solving the problem. Interestingly, they used the notation "double dot" for the second derivative, as it is used in a physics course, not in mathematics courses. Students identified the differential equation as a second order linear differential equation, and found its general solution in the form

$$h(t) = C_1 e^{\sqrt{\frac{g}{L}}t} + C_2 e^{-\sqrt{\frac{g}{L}}t}.$$

Given that at the initial moment the chain velocity equals zero, the particular solution assumes the form:

$$h(t) = \frac{h_0}{2} e^{\sqrt{\frac{g}{L}}t} + \frac{h_0}{2} e^{-\sqrt{\frac{g}{L}}t} \quad \text{or} \quad h(t) = h_0 \cosh\left(\sqrt{\frac{g}{L}}\, t\right).$$

In the process of solving the mathematical problem, students demonstrated the *ability to represent mathematical entities* and *handle mathematical symbols and formalism*. Solving the problem numerically, they plotted the graph of the solution with the help of CAS MATLAB demonstrating good skills in using technology.

Finding the time for $h(t) = L$, they obtained $t = \sqrt{\frac{L}{g}}\cosh^{-1}\left(\frac{L}{h_0}\right)$. Using the values of the parameters L, g, h_0, the students concluded that

$$t = \sqrt{\frac{0.46}{9.81}}\cosh^{-1}\left(\frac{0.46}{0.11}\right) \approx 0.457 \text{ sec.}$$

Including additional forces in the diagram, the students discussed the second scenario with the friction. The condition for the chain to start moving should depend on the length of the chain hanging down from the table. Compared to the previous scenario, the start of the motion was only dependent on whether a part of the chain was hanging from the edge of the table or not, regardless of how long it was. The reason for this change in conditions is that now a friction force acts in the direction opposite to that of the chain's motion. The strength of this friction force depends on the mass which is in the contact with the horizontal surface of the table, m_1. As the chain starts moving, m_2 increases, and so does the gravitational force. At the same time, the mass of the chain remaining on the table decreases proportionally and leads to a smaller friction force. This reasoning requires the change in the second Newton's law, $m_2\, g - F_{fr} = (m_1 + m_2)a$.

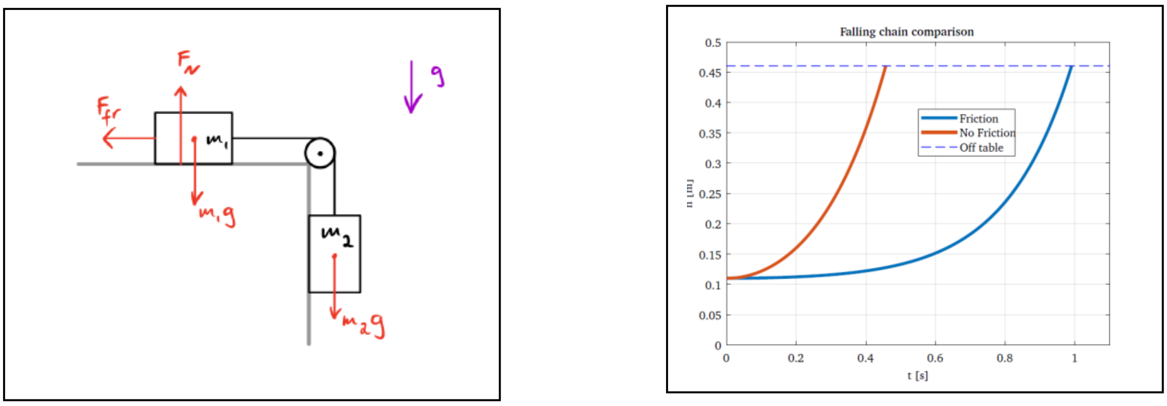

Fig. 3. Illustration of model with friction and results of numerical computation

Then $F_{fr} = \mu F_N$, $F_N = m_1 g$, and $a = \frac{m_2 g - \mu m_1 g}{m_1 + m_2}$. The equation of motion assumes the form

$$a = \ddot{h} = \frac{h\frac{m}{L}g - \mu(L-h)\frac{m}{L}g}{(L-h)\frac{m}{L} + h\frac{m}{L}} = \frac{hg + \mu hg}{L} - \mu g, \quad \text{or} \quad \ddot{h} - h\frac{(\mu+1)g}{L} = -\mu g.$$

Denoting $k_1 = \frac{(\mu+1)g}{L}$, $k_2 = -\mu g$, and solving the equation, students obtained the following solution:

$$h(t) = \frac{1}{2}\left(h_0 + \frac{k_2}{k_1}\right)e^{\sqrt{k_1}t} + \frac{1}{2}\left(h_0 + \frac{k_2}{k_1}\right)e^{-\sqrt{k_1}t} - \frac{k_2}{k_1} = \left(h_0 + \frac{k_2}{k_1}\right)\operatorname{csch}\left(\sqrt{k_1}t\right) - \frac{k_2}{k_1}.$$

Substitution of $k_1 = \frac{(\mu+1)g}{L}$ and $k_2 = -\mu g$ yields

$$h(t) = \left(h_0 - \frac{\mu L}{\mu + 1}\right)\cosh\left(\sqrt{\frac{(\mu+1)g}{L}}t\right) + \frac{\mu L}{\mu + 1}.$$

Finding the fall time $t \approx 0.990\ sec$, students compared the two cases in Figure 3. From the video snapshot in Figure 1, one can see that the fall time in the experiment was 0.925 seconds. After the validation of both models against experimental data, students concluded that the model with friction describes the real system better.

4 DISCUSSION AND CONCLUSIONS

Discussing the role of mathematics in contemporary engineering education, Nethercot and Lloyd-Smith emphasised that it "should be seen as a medium for communicating concepts, ideas, and information in a parallel way to text – and not as the mastery of a series of abstract processes with little or no linkage to physical understanding and application. It should instil disciplined thinking and rigour in the development of arguments based on assumption and simplification in modelling, should teach the importance of controlled approximation, and, above all, impress upon students its value as a tool to be invoked when quantitative evidence is needed to underpin assertion, hypothesis, or sheer physical intuition" [6, p. 14]. We believe that the use of mathematical modelling projects in a standard mathematics course for senior mechatronics students presents mathematics as a medium for communicating ideas and concepts much better.

The research question we asked is *How modelling projects in a mathematics course contribute to the development of engineering students' mathematical maturity and integration of mathematics and engineering?* Using as an example only one of many modelling projects offered to our students, we illustrate how a standard mathematics course can be modified to bring together knowledge and skills from engineering, physics, mathematics, and programming in a meaningful way, cf. [2]. Working on mathematical modelling projects, students combine advanced mathematical thinking with physical intuition, they set experiments, make assumptions, suggest, test, and validate models – all these components contribute to the acquisition of mathematical maturity so much wanted by engineering faculty and employers. At different stages of all projects, eight mathematical competencies [5] are employed and tested without additional exam stress. The work in small groups also provides useful social experience needed for future collaboration in communities of practitioners, cf. [11].

Our experience with the use of modelling projects in a course was positive and encouraging - projects engaged students in collaborative work, contributed to their learning, stimulated exploration, and creativity, and brought positive emotions. Students encountered many practical problems relevant to their engineering specialisation where the theoretical knowledge of linear differential equations and systems of linear differential equations was applicable. Last but not least, students experienced much less stress at the exam since project work contributed to the total grade, and an overall performance in the course improved.

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