Loading-effects reduction using a voltmeter in series and an ammeter in parallel

Ferran Reverter

***Abstract*— This article proposes a method for reducing the loading effects when a DC voltage or current is measured in a linear circuit with a digital multimeter (DMM). In the proposed method, the voltmeter is placed in series to estimate a current, and the ammeter is placed in parallel to estimate a voltage, which is the opposite of the conventional approach. Its application is particularly of interest when the equivalent resistance between the nodes of the DC voltage (current) under measurement is high (low). In comparison with the conventional method, the relative error is up to a factor of 10^4 lower if the equivalent resistance equals the shunt (input) resistance of the DMM when a DC current (voltage) is measured.**

Keywords*— *DC electrical measurement, input resistance, loading effects, multimeter, shunt resistance.

I. INTRODUCTION

Electrical measurements in circuits are very common to measure the operating point and to improve parameters such as the efficiency [1], consumption [2], safety [3], and fault detection [4]. These measurements can be implemented with either a specific built-in subcircuit or with external instrumentation such as a digital multimeter (DMM). Further, external measurements can be carried out automatically using instrumentation controlled by a computer.

In the testing/calibration phase of a circuit, it is essential to measure how much is the DC voltage (or current) at some critical nodes (paths) [5]. Typically, a DMM acting as a voltmeter (ammeter) is connected in parallel (series) to perform the voltage (current) measurement. This DMM offers an input (shunt) resistance for the voltage (current) measurement that should be, at ideal conditions, equal to infinite (zero) [6], [7]. However, a standard DMM has an input resistance of 10 MΩ, and a shunt resistance from units to hundreds of ohm depending on the range selected. As a consequence of these non-ideal values, the connection of the DMM to the circuit under test can cause loading effects [8], thus resulting in an erroneous measurement subjected to a loading-effect error. This error becomes critical when the equivalent resistance between the nodes of the voltage (current) under measurement is high (low).

In order to reduce the loading-effect error in the voltage measurement, a 10-GΩ input resistance (rather than 10 MΩ) can be selected in high-performance expensive DMMs. Nevertheless, such a very high input resistance usually is only selectable for low-value voltage ranges (say, lower than 10 V). On the other hand, the reduction of the loading-effect error in the current measurement can be done by selecting an input range higher than that needed. In such conditions, the value of the shunt resistance becomes lower, but the uncertainty of the instrument is higher. Loading effects in current measurements can also be avoided by using techniques that do not require the breaking of the current path. For example, some DMMs offer the in-circuit current measurement technique [9]. In such a method, the DMM is connected in parallel between two points of a printed circuit trace, and carries out first a 4-wire resistance measurement and then a voltage measurement. Such results are then employed to indirectly estimate the current without breaking the path. This method, however, is only applicable if the trace resistance is within a given range (e.g. from 1 mΩ to 10 Ω). Also with the idea of not breaking the current path, one can use a clamp-on current probe, but this generally provides a medium accuracy.

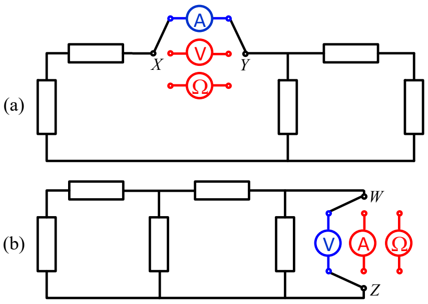

Fig. 1. Generic measurement of (a) a current, and (b) a voltage, applying the conventional (in blue) and novel (in red) approaches.

Considering the limitations indicated before, this article proposes a novel approach to reducing the loading effects when a DC voltage or current is measured with a DMM. The proposed approach puts into practice the statements formulated in [10], which were originally thought to analyze linear circuits but are also useful to improve the accuracy of DC measurements under certain conditions. Very preliminary experimental results applying such a novel approach were reported in [11], although the resulting values of error were, in the critical scenarios, between 50 and 100 times higher and, hence, it was not possible to prove there the theoretical model.

II. NOVEL APPROACH

Following the statements formulated in [10], an alternative way to measure a DC voltage or current in a linear circuit is feasible, as explained next. In the conventional current measurement, the ammeter is connected in series, e.g. between points X and Y of the generic circuit shown in Fig. 1(a). However, here it is suggested to first employ a voltmeter in series, between the same X-Y points, to measure an equivalent voltage (V_{eqA}), and then an ohmmeter also in series to determine an equivalent resistance (R_{eqA}). Using these two measurements, the current can be determined as $V_{\mathrm{eqA}}/R_{\mathrm{eqA}}$.

In the conventional voltage measurement, the voltmeter is connected in parallel, e.g. between points W and Z of the circuit shown in Fig. 1(b). Instead, here it is proposed to first employ an ammeter in parallel, between the same W-Z points, to measure an equivalent current (I_{eqB}), and then an ohmmeter also in parallel to determine an equivalent resistance (R_{eqB}). Using these, the voltage can be estimated as $I_{\mathrm{eqB}}\cdot R_{\mathrm{eqB}}$.

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Ferran Reverter is with the Department of Electronic Engineering, Universitat Politècnica de Catalunya – BarcelonaTech, Castelldefels (Barcelona), 08860, Spain (e-mail: ferran.reverter@upc.edu).

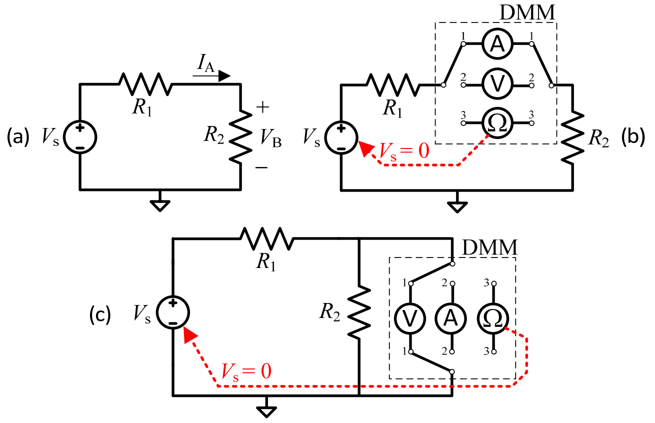

Fig. 2. (a) Circuit under test. (b) DMM connected to measure I_A. (c) DMM connected to measure V_B.

In comparison with the conventional method, the novel technique has the main advantage that loading effects are expected to be much lower under certain conditions, i.e. when the equivalent resistance between the nodes of the voltage (current) under measurement is high (low). This is proven theoretically in Section III, and experimentally in Section IV.

A disadvantage of the proposed method is that the current/voltage is indirectly determined through two previous measurements. By applying the law of propagation of uncertainties [12] and assuming that the two measurements are independent, the (indirect) measurement of current/voltage has a relative uncertainty (due to the instrument limitations) that equals the quadratic sum of the relative uncertainties of the two measurements. Therefore, in principle, the uncertainty is expected to be higher when the novel measurement technique is applied. However, in many commercial DMMs (e.g. 6 ½-digit DMMs from Keysight, Keithley, and Fluke), the uncertainty in the current measurement is usually higher (e.g. a factor of 10) than that in the voltage or resistance measurement. Consequently, in such conditions, estimating a current by measuring a voltage and a resistance should offer a lower uncertainty, although two measurements are involved. Another drawback is that the independent sources of the circuit must be turned off when the equivalent resistance (R_{eqA} and R_{eqB} according to the previous explanation) is measured, but this is quite easy to be implemented when using automatic instrumentation controlled by a computer.

III. THEORETICAL ANALYSIS

As a proof-of-concept, the analysis is carried out in the simple DC voltage divider with two resistors (R_1 and R_2) represented in Fig. 2a, where the current I_A and the voltage V_B are under measurement. Loading effects are quantified through the relative error, which is calculated as:

$$\varepsilon = \frac{M_{\mathrm{w/LE}} - M_{\mathrm{w/oLE}}}{M_{\mathrm{w/oLE}}} \qquad (1)$$

where $M_{\mathrm{w/LE}}$ is the estimated value of the variable (either current or voltage) with an error due to loading effects, and $M_{\mathrm{w/oLE}}$ is the value of that variable without loading effects (i.e. assuming ideal instrumentation). The subscripts c, n, i, and v employed next for ε correspond to "conventional", "novel", "current", and "voltage", respectively.

In the conventional approach, I_A is measured using the ammeter in series, as represented in Fig. 2b with the DMM at position 1. The analysis of this circuit, assuming a shunt resistance (R_s) of the ammeter, shows that the relative error caused by loading effects is:

$$\varepsilon_{\mathrm{c,i}} = -\left(1 + \frac{R_1 + R_2}{R_s}\right)^{-1} \qquad (2)$$

According to (2), for a given value of R_s, the lower the value of $R_1 + R_2$, the higher the error (in absolute terms, since there is a minus sign). On the other hand, the conventional measurement of V_B involves the connection of the voltmeter in parallel, as represented in Fig. 2c with the DMM at position 1. Taking into account that this voltmeter has an input resistance (R_{in}), the resulting relative error is:

$$\varepsilon_{\mathrm{c,v}} = -\left(1 + \frac{R_{\mathrm{in}}}{R_1 \parallel R_2}\right)^{-1} \qquad (3)$$

From (3), for a given value of R_{in}, the error increases with increasing the factor $R_1 \parallel R_2$, where the symbol "||" corresponds to the parallel combination of resistances.

In the novel approach, I_A is determined by connecting first the voltmeter and then the ohmmeter in series, as shown in Fig. 2b with the DMM at position 2 and 3, respectively. In such conditions, the measurement of current is affected by the input resistance of the voltmeter, thus causing:

$$\varepsilon_{\mathrm{n,i}} = -\left(1 + \frac{R_{\mathrm{in}}}{R_1 + R_2}\right)^{-1} \qquad (4)$$

According to (4) and unlike (2), for a given value of R_{in}, the lower the value of $R_1 + R_2$, the lower the error. For the determination of V_B using the novel approach, the ammeter and then the ohmmeter are successively connected in parallel, as illustrated in Fig. 2c with the DMM at position 2 and 3, respectively. In these conditions, the measurement of voltage is affected by the shunt resistance of the ammeter, thus generating the following relative error:

$$\varepsilon_{\mathrm{n,v}} = -\left(1 + \frac{R_1 \parallel R_2}{R_s}\right)^{-1} \qquad (5)$$

According to (5) and unlike (3), for a given value of R_s, the higher the value of $R_1 \parallel R_2$, the lower the error.

Taking into account the previous analysis, one realizes that the critical scenarios in the conventional approach are the optimal ones in the novel approach. Actually, it is possible to find the condition that makes the novel method better than the conventional one in terms of error by solving the inequalities $|\varepsilon_{\mathrm{n,i}}| < |\varepsilon_{\mathrm{c,i}}|$ and $|\varepsilon_{\mathrm{n,v}}| < |\varepsilon_{\mathrm{c,v}}|$ for the measurement of current and voltage, respectively. Using (2) and (4), the condition for the current measurement that makes the novel technique better is:

$$R_1 + R_2 < \sqrt{R_{\mathrm{in}} R_s} \qquad (6)$$

On the other hand, applying (3) and (5), the condition for the voltage measurement is:

$$R_1 \parallel R_2 > \sqrt{R_{\mathrm{in}} R_s} \qquad (7)$$

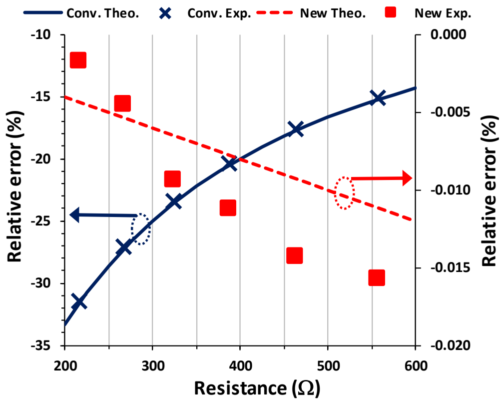

Fig. 3. Relative errors in the measurement of I_A in Fig. 2a. The DMM current/voltage/resistance ranges were 1 mA/1 V/1 kΩ, and V_s = 400 mV.

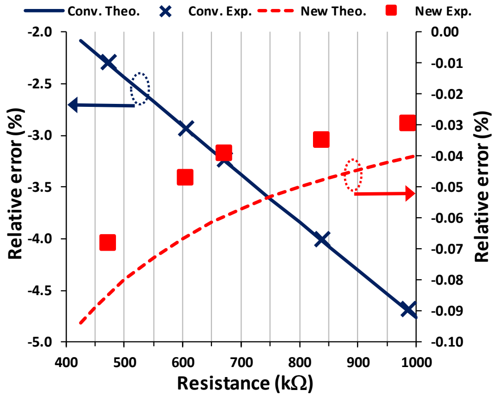

Fig. 4. Relative errors in the measurement of V_B in Fig. 2a, when $R_1 < 1$ MΩ. The DMM current/voltage/resistance ranges were 100 μA/100 V/1 MΩ, and V_s = 25 V.

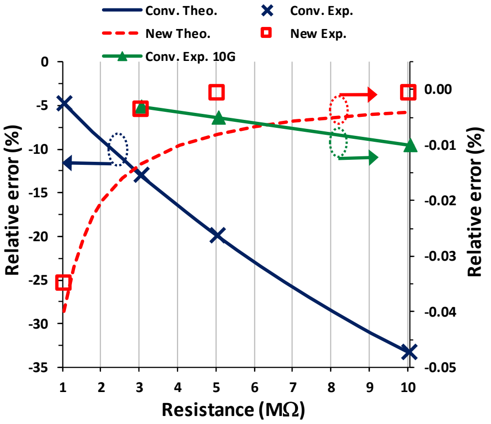

Fig. 5. Relative errors in the measurement of V_B in Fig. 2a, when $R_1 > 1$ MΩ. The DMM current/voltage/resistance ranges were 100 μA/100 V/10 MΩ, and V_s = 42 V, except for the 10-GΩ case where the range and V_s were 10 V.

Assuming R_{in} = 10 MΩ and R_s = 200 Ω (see Section IV), then $\sqrt{R_{in}R_s}$ = 44.7 kΩ. Therefore, the novel technique offers a lower error when $R_1 + R_2 < 44.7$ kΩ and $R_1 \parallel R_2 > 44.7$ kΩ in the current and voltage measurement, respectively.

IV. EXPERIMENTAL RESULTS

The circuit in Fig. 2a with $R_1 = R_2$ was built and both conventional and novel techniques were applied to measure I_A and V_B. The DC voltage source (V_s) was supplied by a precision source/measure unit (Agilent B2901A). The measurement of voltage, current, and resistance was carried out by a 6 ½-digit commercial DMM (Agilent 34410A), which was configured to have a 10-MΩ input resistance and a 200-Ω shunt resistance. In addition, the longest integration time (2 s) was set to filter out any potential noise effect.

Fig. 3 shows the theoretical and experimental results related to the measurement of I_A. When the conventional technique was applied (in blue in Fig. 3), the relative error increased (in absolute terms) with decreasing R_1, to be precise: from −15% at 560 Ω to −31% at 220 Ω. Alternatively, when the novel technique was applied (in red in Fig. 3), the relative error decreased with decreasing R_1. The relative error was at least a factor of 1000 (at R_1 = 560 Ω) but it could be up to 20000 times (at R_1 = 220 Ω) lower than that obtained in the conventional approach.

The measurement results of V_B are represented in Figs. 4 and 5 for resistances lower and higher than 1 MΩ, respectively. In both cases, the relative error increased with increasing R_1 in the conventional approach, but decreased with increasing R_1 in the novel approach. In Fig. 4, the relative error was at least a factor of 34 (at R_1 = 470 kΩ) but it could be up to 160 times (at R_1 = 1 MΩ) smaller than that obtained in the conventional approach. The degree of improvement was considerably higher in Fig. 5: the relative error was between 143 and 33000 times lower. The errors obtained using the conventional method but with the 10-GΩ input resistance, which was actually of 50 GΩ, are also shown in green in Fig. 5. Even in that case, the novel method provided a relative error that was up to 10 times lower.

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