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A hybrid SBM-MFS methodology to deal with wave propagation

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Abstract

In this paper, a novel hybrid 2.5D SBM-MFS approach is formulated and developed in the frequency domain. This approach inherits the accuracy of MFS while keeping the robustness presented by the SBM. The MFS is employed to study the smooth portion of the boundary, while the complex segments are analysed through the SBM. For the sake of presenting the potential of the proposed hybrid approach, a square-shaped boundary excited by a unit point load is considered. The performance of the hybrid method is thoroughly assessed against 2.5D BEM, MFS, and SBM methods, in terms of convergence error analysis. Since the considered problem does not have a known analytical solution, the 2.5D FEM-BEM approach with a highly refined mesh is taken as the reference in the error analysis. The convergence error is calculated in terms of receptances at two circular distributions of evaluation points. In the hybrid method, 70 percent of the virtual sources are allocated on an auxiliary virtual boundary (MFS sources) while the remaining 30 percent are allocated on the physical boundary (SBM sources). The convergence plots obtained by four methods show that the accuracy of the hybrid method is significantly higher than the one of MFS and, in some cases, even higher than the one of BEM. While MFS requires a large number of nodes per wavelength to achieve acceptable results, the 2.5D SBM-MFS presents a high convergence rate, even for a small number of nodes per wavelength. The main benefit of the hybrid method is not solely

its accuracy, compared with the BEM and SBM methods, but also its computational efficiency is another achievement. Moreover, in contrast to integration-based methods, such as BEM, the implementation of the new procedure is quite simple. It can be concluded that the hybrid 2.5D SBM–MFS is an adequate alternative prediction tool for elastodynamic problems.

Keywords: elastic wave propagation, singular boundary method, method of fundamental solutions, meshless, origin intensity factor, hybrid method

1 Introduction

Over the years, different numerical schemes have been proposed to investigate the propagation of waves in unbounded elastic media. The Boundary Element Method (BEM), the Finite Element Method (FEM), and the Perfectly Matched Layer approach (PML) are examples of these numerical approaches that are mesh-based and also, in the case of the BEM, integration-based. To avoid the constraints due to mesh-based methods and improve the computational efficiency, meshless methods have been proposed to model the soil medium in the framework of soil-structure interaction problems. In this context, the Method of Fundamental Solutions (MFS) is a mesh-free and integration-free approach that was introduced by [1]. This method assumes that the displacement and traction fields can be constructed by a linear combination of fundamental solutions of the governing equations and employs a set of virtual sources to model the wave propagation in the medium. The extremely high accuracy of the MFS for regular geometries, such as circular or ellipsoidal shapes, has been proven in various investigations [2, 3]. However, to be able to analyse complex geometries, this approach requires a proper optimisation process to determine a proper location of the virtual sources to reach acceptable rates of accuracy [4, 5]. Singular boundary method (SBM) was proposed [6] as an alternative to the MFS. This approach avoids the difficulties associated with the positioning of virtual sources by locating them on the boundary domain. In the SBM, the singularities of the fundamental solutions due to the overlapping between the domains of the collocation and source points are eliminated through the concept of origin intensity factors (OIF) [7].

The MFS and the SBM can be also used for modelling the dynamic response of structures that can be assumed to be longitudinally invariant, as it is the case of railway tracks, tunnels, roads, and pipelines, among others. Thus, the wave propagation for these cases can be assessed by two-and-a-half-dimensional (2.5D) modelling approaches [8].

In this paper, a novel hybrid 2.5D SBM-MFS approach formulated in the frequency domain is presented. The method has been tested for a square cavity case. Results demonstrate two important features of the new method with respect to pure MFS and SBM approaches. On the one hand, the hybrid method is more accurate than both BEM and MFS for non-smooth boundaries and provides a significantly higher robustness than pure MFS. On the other hand, the efficiency of the proposed approach overcomes the SBM method.

2 Methods

The proposed hybrid methodology assumes two sets of virtual sources. The first set is located within the physical boundary (Γ), while the other is outside the domain, located within a defined virtual boundary. The proposed approach is presented in Figure 1. It should be mentioned that the bar notation represents variables in the wavenumber-frequency domain, where the dynamic Green's functions are represented with bar notation and static Green's functions are represented without bar notation.

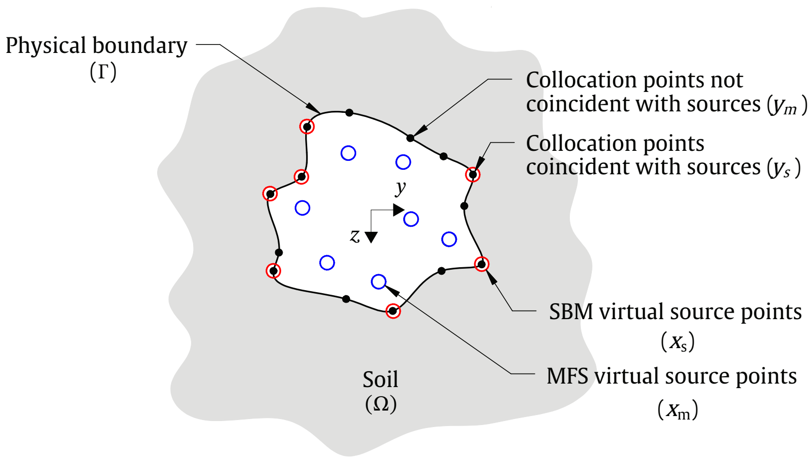

Figure 1: General description of the proposed hybrid methodology. Collocation points are denoted by black solid circle and virtual sources associated with the MFS and SBM are presented by blue and red circles, respectively.

Based on radial basis function interpolation, the displacement and traction of the soil are approximated throughout the domain using the following linear combination of the fundamental solution of the governing equations:

$$\bar{\boldsymbol{U}}(\boldsymbol{y}) = \sum_{n=1}^{N_M} \bar{\mathbf{H}}(\boldsymbol{y}, \boldsymbol{x}_M^n)\bar{\boldsymbol{S}}_{M,n} + \sum_{n=1}^{N_S} \bar{\mathbf{H}}(\boldsymbol{y}, \boldsymbol{x}_S^n)\bar{\boldsymbol{S}}_{S,n}, \tag{1a}$$

$$\bar{\boldsymbol{T}}(\boldsymbol{y}) = \sum_{n=1}^{N_M} \bar{\mathbf{H}}^{\tau}(\boldsymbol{y}, \boldsymbol{x}_M^n)\bar{\boldsymbol{S}}_{M,n} + \sum_{n=1}^{N_S} \bar{\mathbf{H}}^{\tau}(\boldsymbol{y}, \boldsymbol{x}_S^n)\bar{\boldsymbol{S}}_{S,n}, \tag{1b}$$

where $\bar{\mathbf{H}}(\boldsymbol{y}, \boldsymbol{x}_{M/S}^n)$ and $\bar{\mathbf{H}}^{\tau}(\boldsymbol{y}, \boldsymbol{x}_{M/S}^n)$ represent the displacement and traction Green's functions of the soil considering a point load applied at $\boldsymbol{x}^n_{M/S}$. The terms $\bar{\boldsymbol{S}}_{M,n}$ and $\bar{\boldsymbol{S}}_{S,n}$ represent the virtual sources strengths, associated with the SBM and MFS sources points, respectively, and $\bar{\boldsymbol{U}}(\boldsymbol{y})$ and $\bar{\boldsymbol{T}}(\boldsymbol{y})$ are the displacements and tractions of the soil, at an arbitrary field point located at y. N_M and N_S denote the number of MFS and SBM sources points, respectively. Also it should be noted that the terms $\boldsymbol{x}^m_M$ and $\boldsymbol{x}^m_S$ are the location of the mth source point associated with the MFS and SBM, respectively.

To avoid the singularities that arise when Eqs. (1a) and (1b) are employed to evaluate the solution on collocation points geometrically coincident with virtual sources (y_S), the equations are rewritten as follows [6, 8]

$$\bar{\boldsymbol{U}}(\boldsymbol{y}^m_S) = \sum_{n=1}^{N_M} \bar{\mathbf{H}}(\boldsymbol{y}^m_S, \boldsymbol{x}^n_M)\bar{\boldsymbol{S}}_{M,n} + \sum_{n=1,n\neq m}^{N_S} \bar{\mathbf{H}}(\boldsymbol{y}^m_S, \boldsymbol{x}^n_S)\bar{\boldsymbol{S}}_{S,n} + \bar{\mathbf{H}}_{mm}\bar{\boldsymbol{S}}_{S,m}, \tag{2a}$$

$$\bar{\boldsymbol{T}}(\boldsymbol{y}^m_S) = \sum_{n=1}^{N_M} \bar{\mathbf{H}}^\tau(\boldsymbol{y}^m_S, \boldsymbol{x}^n_M)\bar{\boldsymbol{S}}_{M,n} + \sum_{n=1,n\neq m}^{N_S} \bar{\mathbf{H}}^\tau(\boldsymbol{y}^m_S, \boldsymbol{x}^n_S)\bar{\boldsymbol{S}}_{S,n} + \bar{\mathbf{H}}^\tau_{mm}\bar{\boldsymbol{S}}_{S,m}, \tag{2b}$$

where $\bar{\mathbf{H}}_{mm}$ and $\bar{\mathbf{H}}^\tau_{mm}$ are defined as the origin (or source) intensity factors (OIF) in the SBM literature and the $\boldsymbol{y}^m_M$ and $\boldsymbol{y}^m_S$ are the location of the mth collocation point associated with the MFS and SBM, respectively. The detailed derivation of formulation to compute the OIFs in Dirichlet and Neumann boundary conditions can be found in [8]. The responses on the MFS collocation points are given by Eqs. (3a) and (3b), which do not have any singularity.

$$\bar{\boldsymbol{U}}(\boldsymbol{y}^m_M) = \sum_{n=1}^{N_M} \bar{\mathbf{H}}(\boldsymbol{y}^m_M, \boldsymbol{x}^n_M)\bar{\boldsymbol{S}}_{M,n} + \sum_{n=1}^{N_S} \bar{\mathbf{H}}(\boldsymbol{y}^m_M, \boldsymbol{x}^n_S)\bar{\boldsymbol{S}}_{S,n}, \tag{3a}$$

$$\bar{\boldsymbol{T}}(\boldsymbol{y}^m_M) = \sum_{n=1}^{N_M} \bar{\mathbf{H}}^\tau(\boldsymbol{y}^m_M, \boldsymbol{x}^n_M)\bar{\boldsymbol{S}}_{M,n} + \sum_{n=1}^{N_S} \bar{\mathbf{H}}^\tau(\boldsymbol{y}^m_M, \boldsymbol{x}^n_S)\bar{\boldsymbol{S}}_{S,n}, \tag{3b}$$

by solving the above equations, the strength of the virtual sources can be determined. Finally, using these known strengths and Eqs. (1a) and (1b), the responses at desired position can be computed.

3 Results

The performance of the hybrid approach is investigated in the framework of convergence error analysis. For the sake of presenting the potential of the proposed hybrid approach, the chosen geometry consists of both smooth and sharp edges. As illustrated in Figure 2, a square shape is selected for the analysis. The length of each side of the square is equal to 6 m. The system is excited by a vertical unit load applied at the bottom of the system. The considered material properties for the medium are Young's modulus of 108 MPa, a density of 1800 $\mathrm{kg/m^3}$, a Poisson's ratio of $1/3$, and a material damping ratio of 0.05.

The convergence analysis is performed by computing the root mean square error (RMSE) which is calculated as

$$\mathrm{RMSE} = \frac{\sqrt{\frac{1}{3N}\sum_{j=1}^{N}\left|\sum_{i=1}^{3}U_b^{ij} - \sum_{i=1}^{3}U_{br}^{ij}\right|^2}}{\sqrt{\frac{1}{3N}\sum_{j=1}^{N}\left|\sum_{i=1}^{3}U_{br}^{ij}\right|^2}}, \tag{4}$$

where i is the index associated with the Cartesian coordinate components (x, y and z), j is the index associated with the collocation points, and N refers to the total number of collocation points considered. Moreover, U_b^{ij} and U_{br}^{ij} represent the receptances in the frequency domain on the evaluation points obtained by the selected method and by a reference method (in this work the reference method is the 2.5D FEM-BEM approach), respectively. The procedure to compute the receptances can be found in [8]. The RMSE is computed at two distinct sets of test points, both distributed on circles centred at the cylinder axis and with radius 7 m and 20 m, representing near-field and far-field positions, respectively. The formulation to compute the receptances is presented comprehensively in [8]. For both sets, the evaluation points are distributed uniformly along the circle perimeter. Two frequencies are chosen for the present convergence error analysis: 20 Hz and 80 Hz. The results have been obtained for values of the number of nodes per wavelength (NpW) between 5 and 25, based on the mentioned soil characteristics and a maximum frequency of 100 Hz. In this example, 30 percent of the virtual sources are SBM sources and the remaining 70 percent are MFS sources.

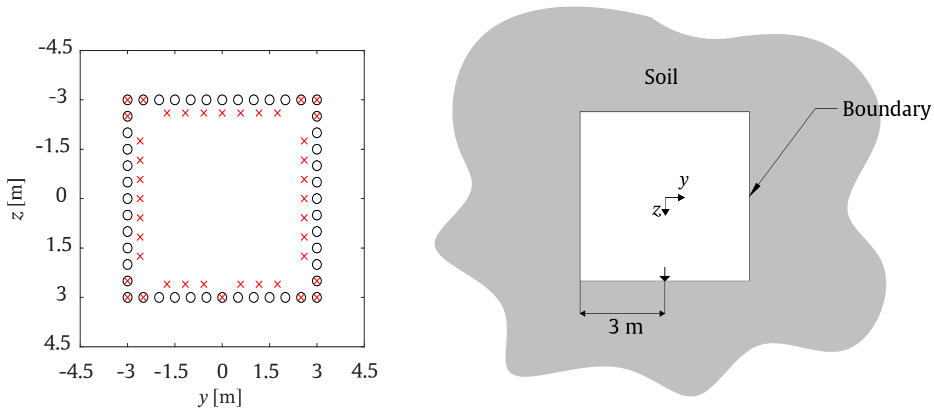

Figure 2: The square geometry and position of the collocation points and virtual sources in the hybrid method.

As shown in Figure 3, the SBM method is the most accurate approach for a frequency of 20 Hz, while the SBM and hybrid methods present approximately the same accuracy for a frequency of 80 Hz. At frequency 80 Hz, the SBM and hybrid approaches are the most accurate methods, from 5 to 15 NpW. Also, it should be noted that the MFS does not presents accurate results in any of the cases. Generally, the performance of the proposed hybrid approach in terms of accuracy and stability of the results is almost as good as the one presented by the SBM approach.

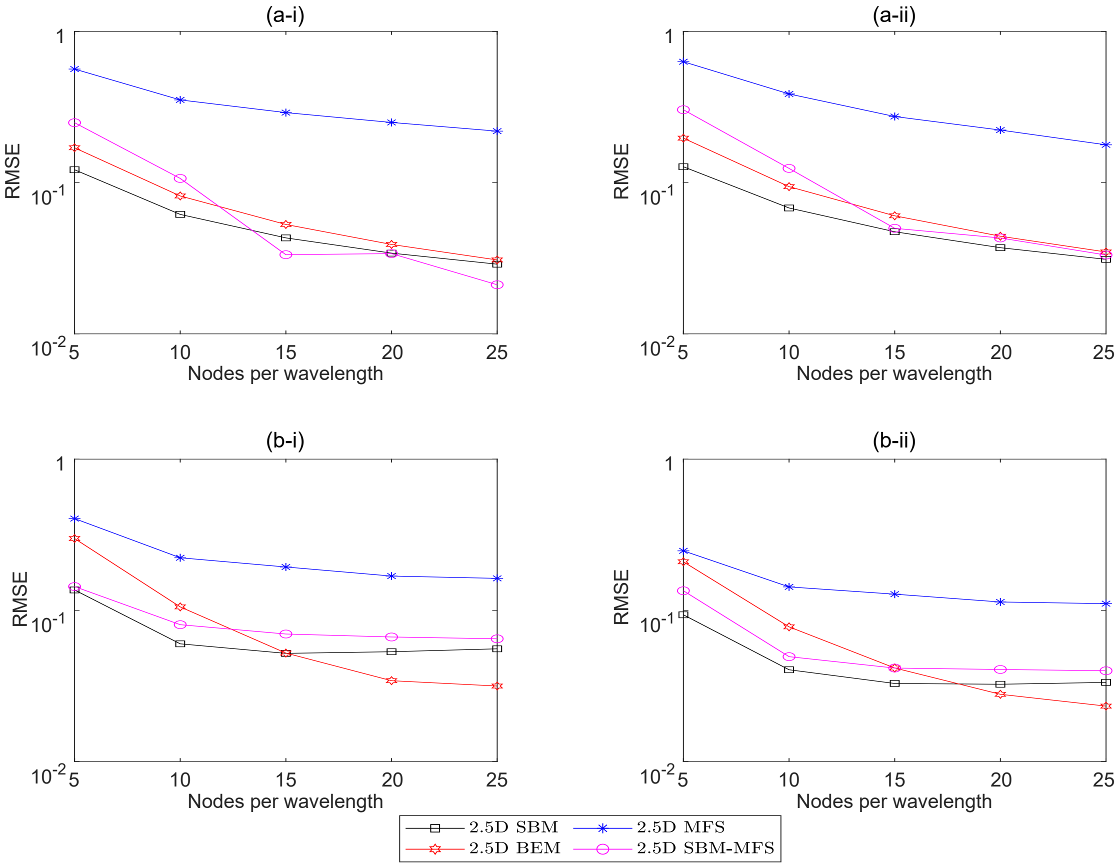

Figure 3: RMSE for different numerical strategies. Two sets of evaluation points at radii of 7 m (i) and 20 m (ii) are considered. Calculation frequencies: 20 Hz (a) and 80 Hz (b).

4 Conclusions & Contributions

This paper proposes a novel hybrid methodology to model wave propagation in elastodynamic problems. In this method, the 2.5D MFS is used to deal with smooth sections of the boundary, while the complex segments are analysed through the SBM method. The performance of the new method is compared to other numerical modelling techniques for a specific example. The following conclusions can be drawn from the numerical analyses presented in this work:

- The accuracy of the hybrid method is higher than the one of the MFS (in all the assessed cases), and to the one of the BEM (at high NpW for the 20 Hz case and low NpW for the 80 Hz case).
- At frequency 20 Hz, the accuracy of the 2.5D SBM-MFS is similar to the SBM approach at 15 nodes per wavelength and above.
- The inaccurate results obtained by the 2.5D MFS indicate that it is not a suitable approach for the considered geometry.
- The hybrid SBM-MFS method is much more efficient than SBM since fewer singular terms exist in the coupled SBM-MFS which means less computational time is needed to compute the OIFs. In the presented example, 70 percent of the sources are MFS sources and 30 percent are SBM sources
- Analogous to SBM and MFS, the implementation procedure is simpler than the one required by integration-based approaches, such as BEM.

To conclude, the hybrid 2.5D SBM–MFS is found to be an adequate prediction tool for the wave propagation in elastodynamic problems since it inherits the computational efficiency of MFS while keeping the robustness and accuracy presented by the SBM.

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