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Topological vortices for sound and light

Localized zero-energy fermionic states can bind to topological defects such as two-dimensional vortices, which can be realized in the bulk of artificial acoustic and optical lattices.

Penglin Gao and Johan Christensen

Topological defects are local kinks or obstructions in an order parameter field where domain walls, superconductor vortices or dislocations are few of many prominent examples¹. Topological bound states can form around these defects much in the same way edge and surface states bind to one- and two-dimensional (1D and 2D) interfaces, respectively. Originally, the Jackiw–Rossi model² described topological defects in the form of vortices in quantum field theory showing that localized zero-energy fermionic states bind at the vortex core. Interestingly, the binding mechanism of such localized defects is identical to Majorana bound states in p -wave superconductors³. Similarly, the distortion of the hexagonal lattice of graphene by a so-called Kekulé distortion field induces an equivalent zero mode, that is, a topological mid-gap state in the Dirac spectrum⁴. Beyond advances made in intricate topological quantum systems, the latter, theoretical finding suggests that single-particle wave functions, and therefore, by the same token, sound, light or vibrations could bind to an analogous classical vortex.

Writing in *Nature Nanotechnology*, two independent research teams report on classical photonic⁵ and phononic⁶ nanoscale implementations of the Jackiw–Rossi vortex, and have each targeted interesting applications in the form of surface-emitting lasers and electromechanical circuits. These findings constitute a first glance of the prospect of unconventional wave control encompassing nanofabricated Dirac-vortex textures that are hardly attainable if not impossible in electronic systems.

There are two steps towards the Jackiw–Rossi model: first, classical waves should be able to mimic the four-dimensional Dirac system, and second, a spatially modulated mass term $\mathbf{m}(\mathbf{r})$ is required to form a vortex incorporating an angular phase winding. Mathematically, the so-called Kekulé distortion field can be formulated as

$$\mathbf{m} = m_0 \tanh\left(\left|\frac{\mathbf{r}-\mathbf{r}_0}{R}\right|^\alpha\right) \times \exp(iw\arg(\mathbf{r}-\mathbf{r}_0)) \quad (1)$$

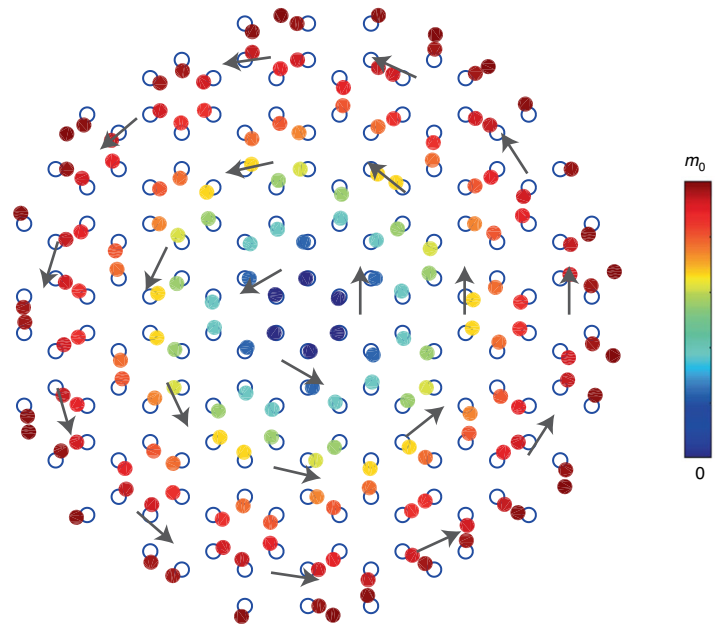


Fig. 1 | Schematic of the Jackiw–Rossi vortex. The hollow circles form a regular graphene-like lattice. The modulation of Eq. (1) distorts this periodic lattice into an aperiodic one (solid dots). This vortex lattice encompasses a phase winding process as rendered by the swirling arrows. Colours are used to show the non-uniform distortion strength $|\mathbf{m}|$ of the displaced lattice sites.

where \mathbf{r} , \mathbf{r}_0 and w denote the lattice sites, vortex centre and winding number, respectively, and parameters m_0 , α and R are introduced to provide additional tunability. Figure 1 illustrates the resulting paradigmatic vortex model whose lattice sites (coloured solid dots) are spatially distorted from the original primitive honeycomb cells (hollow circles). The vortex then originates from a phase winding process around its core that is depicted through an overlaid order-parameter vector field (grey arrows). Additionally, the radial component of the adiabatic modulation embodies a hyperbolic tangent dependency that increases from null toward m_0 at the perimeter, as indicated by the colour bar⁷. These two polar components, when combined, contribute to the final aperiodic lattice including a position-dependent site distortion.

Jingwen Ma, Xiang Xi and co-workers implement a mechanical version of the vortex, that is, for elastic waves⁶. The vortex hosts mid-gap topological states in the MHz regime. The researchers fabricated a microscale structure with a lattice constant of 21 μm on a silicon nitride membrane. Furthermore, they etched nanometre-sized holes into the membrane, forming an angularly modulated pattern, which induces the phase winding process. The authors interpret the additional degree of freedom as an orbital polarization, which introduces a topological phase modulation and was hitherto largely unexplored in phononic systems. Their finding also expands the conventional Kekulé scheme to a more generalized space supporting all possible polarization states on the entire Poincaré sphere. Beyond the trapping of topological vibrations, Ma et al. demonstrate more

functionalities of this man-made mechanical vortex. They observe both second- and third-order nonlinear interactions including power-dependent resonant frequency shifts. Their results establish topological phononic micro-lattices as a useful platform to study nonlinear physics.

At scales slightly smaller than the mechanical implementation, Xiaomei Gao, Lechen Yang and team fabricated a photonic crystal counterpart with a lattice constant of 490 nm to target applications at telecommunication wavelengths⁵. They devised the photonic analogue of the Jackiw–Rossi vortex through electron-beam lithography and dry-etched Kekulé-textured triangular perforations in a silicon layer. The exact texture follows the generalized Kekulé modulation (Eq. (1)), which includes a shifting of those triangular holes from their original position that gives rise to the photonic topological bound state. This new found state of light is a single mid-gap mode and is the most important attribute in association with the 2D topological vortex. It is highly distinctive compared to conventional cavity features resting on Fabry–Pérot, whispering-gallery and photonic crystal resonances. Since this topological single mid-gap mode has a

remarkably high free spectral range with magnitude one to two orders larger than that of conventional cavities, the Dirac-vortex cavity appears appealing in lasing applications; its topological design facilitates stable operation and wide spectral tunability.

These two contributions follow a growing movement of efforts emulating quantum properties with classical waves. Generally, the tailored phononic or photonic lattices, compared to their electronic counterparts, are much simpler both in terms of fabrication and tunability. These metamaterials have therefore garnered significant interest. Along this frontier, topological defects and their ability to trap concentrated sound or light have recently emerged as a new direction and have already been studied for audible sound and vibrations^{8,9} and near-infrared light¹⁰. The possibility to trap and enhance topological waves in the bulk of artificial crystals as opposed to their interfaces or intersections to other crystals enlarges the range of potential technological applications. In this respect, we foresee that resilient single-mode behaviour will be very useful in photonic crystal surface-emitting lasers and topological photonic circuits and waveguides in

general. On the acoustic and mechanical front, we predict that highly concentrated and localized phononic energy may spark efforts for sensing applications and when scaled further down in size, a phononic vortex may enable on-chip filtering possibilities in the 5G band.

Penglin Gao  and Johan Christensen  

Department of Physics, Universidad Carlos III de Madrid, Madrid, Spain.

 e-mail: johan.christensen@uc3m.es

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Competing interests

The authors declare no competing interests.