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S&P 500 Options Returns: Bull and Bear Markets

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Resumo

Uma opção consiste num produto financeiro estabelecido entre duas partes, um comprador e um vendedor. Este tipo de contrato providencia ao seu comprador o direito (não a obrigação) de comprar ou vender uma quantidade do subjacente com o preço de exercício e data de maturidade pré-estabelecidas no contrato. Por sua vez, o vendedor deste contrato terá a obrigação de, caso o comprador decida exercer os seus direitos, de entregar ou adquirir o ativo subjacente.

Existem dois tipos de opções standard, aquelas que providenciam ao seu comprador o direito de comprar na data de maturidade do contrato são as opções Call e aquelas que providenciam ao seu comprador o direito de vender na data de maturidade são as opções Put.

Em 1900, o primeiro modelo de *pricing* de opções foi introduzido por Louis Bachelier [1] na sua tese de doutoramento *Théorie de la Spéculation*.

Em 1973, foi transacionada a primeira opção nos mercados financeiros em Wall Street. Neste mesmo ano foi publicado o modelo de *pricing* de opções por Black e Scholes [2].

Nos últimos anos este tipo de contratos tem vindo a ganhar elevada atração por parte dos investidores. Estes têm sido utilizados tanto para especulação nos mercados financeiros como para cobertura e redução das suas exposições aos riscos inerentes a cada estratégia e carteiras de investimento aplicada pelos investidores.

Em 2001, Coval e Shumway [5] publicaram o primeiro paper focado nos retornos dos contratos de opções. Até à data a vasta maioria dos papers publicados analisavam novos modelos de *pricing* destes produtos financeiros, o *misprice* das opções, nomeadamente das opções Put.

Coval e Shumway [5] foram os primeiros a focar a sua atenção nos retornos esperados das opções.

Nesta tese visamos responder às seguintes questões: Será que os seus métodos de cálculo/ proposições indicadas em Coval e Shumway [5] ainda se aplicam quando analisamos uma base de dados mais atual? Será que os retornos das opções reagem de forma diferente em momentos em que os mercados estão a crescer (períodos Bull) ou a cair (períodos Bear)?

Com o intuito de dar resposta a estas questões procedemos à análise dos preços das opções sobre o índice S&P 500 ao longo de um período de 15 anos, desde janeiro de 2004 até abril de 2019.

O índice S&P 500 é um benchmark composto por 500 ações individuais das maiores e mais diversificadas empresas dos Estados Unidos da América que se encontram cotadas nas bolsas NASDAQ e NYSE. Algumas empresas que compõe este índice são a Apple Inc., Microsoft Corp., Amazon.com Inc e Facebook Inc. que representam cerca de 20% da composição do mesmo.

Como análise numérica nesta tese analisamos os retornos das opções de compra (Call) e opções de venda (Put) durante todo o período da base de dados. Adicionalmente, procedemos a uma análise mais detalhada de duas subamostras, que correspondem aos períodos Bull e aos períodos Bear dos mercados financeiros.

Num período Bull o mercado financeiro caracteriza-se por estar num momento de crescimento e num período Bear o mercado caracteriza-se por estar em queda e descidas de preços, com descidas que podem ultrapassar os 20%.

Através da análise da evolução dos preços do índice S&P 500, conseguimos identificar três períodos Bull (que são de 02/01/2004 até 09/10/2007; 09/03/2009 até 29/04/2011; e 03/10/2011 até 20/09/2018) e três períodos Bear (que são de 09/10/2007 até 09/03/2009; 29/04/2011 até 03/10/2011; e 20/09/2018 até 21/12/2018) nos mercados financeiros. Nos períodos em que o preço do índice sobe assumimos que o mercado está num ciclo Bull, e caso o preço do índice estejam a cair estamos num ciclo Bear.

Nesta tese, implementamos a metodologia de cálculo dos retornos das opções utilizada por Coval e Shumway [5] e analisamos os retornos diários sobre o índice para cada amostra dos dados.

Para o cálculo dos retornos procedemos da seguinte forma:

- Identificamos todas as roots que correspondem às opções standard de estilo europeu;
- Para cada opção identificamos os preços *bid-ask* observado às 15h45 ET;
- Identificamos e apenas consideramos as opções que expiram no mês seguinte. Desta forma todas as opções terão data de maturidade nos próximos 20 a 50 dias após a data da observação;
- Utilizamos os preços *bid* e *ask* para cada opção e calculamos o preço *mid*, que representa o preço médio entre os preços *bid* e *ask*;
- Com o mid price calculamos os retornos diários para cada opção (assumindo uma estratégia em que compramos uma opção num dia e vendemos imediatamente no dia seguinte);
- Classificamos as opções em cinco grupos tendo em atenção a relação entre o preço de exercício (*strike price*) e os preços do ativo subjacente, neste caso o índice S&P 500. Cada grupo terá um incremento de 5 pontos e é calculado através da subtração entre o preço do subjacente e o preço de exercício da opção.

Na análise dos resultados calculamos algumas métricas estatísticas como a média, mínimo, máximo e o desvio padrão.

A implementação do método de Coval e Shumway [5] na nossa base de dados mais atual apresenta resultados similares aos anteriormente observados em 2001 pelos autores, quer para o total da nossa amostra, quer para a subamostra dos períodos Bull dos mercados.

Verificamos que para as opções Call os retornos médios são positivos e para as opções Put os retornos médios são negativos. Para ambos os tipos de opções, opções Call ou Put, verificamos também que os retornos crescem com o preço de exercício (*strike price*) das mesmas.

Apesar das opções Call apresentarem retornos positivos, verificamos que a mediana dos retornos é negativa, o que pode indicar que um enviesamento positivo dos dados. O que significa que para a estratégia em análise apresenta um maior número de casos em que o retorno destas opções é negativo e não positivo como indica a média dos retornos.

Tendo em atenção os resultados da amostra nos períodos Bull, ou seja, nos períodos em que o preço do índice S&P 500 sobe, podemos verificar que as conclusões a retirar são muito semelhantes às apresentadas na análise da amostra total da base de dados. No entanto, podemos observar que os retornos das opções Call são superiores (passando de cerca de 50 pontos base na amostra total para 100 pontos base na subamostra) e os retornos das opções Put são mais negativos do que os observados na amostra total (passando de cerca de -150 pontos base na amostra total para -200 pontos base na subamostra).

Tendo em atenção os resultados da amostra nos períodos Bear, ou seja, nos períodos em que o preço do índice S&P 500 desce, podemos verificar que as conclusões a retirar são opostas às anteriormente indicadas na análise da amostra total. Uma vez que, observamos retornos positivo para as opções Put (passando de cerca de -150 pontos base na amostra total para 300 pontos base na subamostra) e retornos negativos para as opções Call (passando de cerca de 50 pontos base na amostra total para -300 pontos base na subamostra). Os retornos das opções Put são bastante superiores aos retornos proporcionados pelas opções Call em períodos Bear.

Em suma, verificamos que em períodos Bull os investidores acreditam mais no mercado e têm altas expectativas, pelo que tenderão a investir mais em opções Call, uma vez que as mesmas proporcionam, em média, retornos mais vantajosos nestas situações.

Em períodos Bear os investidores tenderão a investir mais em opções Put, visto serem estas as que proporcionam maiores retornos.

Algum trabalho futuro para esta tese passaria pela implementação de diferentes modelos e métodos, com os aplicados por Bondarenko [3], Broadie et al. [4] e Jones [7], para o cálculo dos retornos das opções sobre o índice S&P 500 e também sobre outros índices e verificar qual seria o comportamento dos retornos em períodos Bull e Bear.

Keywords: Retornos de opções; Mercados Bull; Mercados Bear; Opções sobre S&P500

Abstract

An option consists in a financial contract between two parties, the buyer and the seller. These types of contracts provide its holder the right (but not the obligation) to buy or to sell a certain pre-established quantity of the underlying, under a pre-established price condition within a determined period, as described in Black and Scholes [2]. As for the seller of the option, if the buyer decides to exercise the contract, he must deliver or acquire the underlying asset at the pre-established conditions of the contract.

Back in 2001, Coval and Shumway [5] provided the first ever published paper focused on the returns of option contracts. But does their methodology/propositions still hold with a more up-to-date database? Do option returns react differently if the market is growing or crashing?

In this thesis we implement the method of calculation for options returns from Coval and Shumway [5] on the S&P500 index options over a 15-year period, from January 2004 to April 2019. During this period we identify three Bull cycles and three Bear cycles on the financial markets by analysing the evolution of the S&P500 prices.

We find similar results as the ones found in Coval and Shumway [5] and analyse the daily option returns under the index. We also make a more detailed analysis in what are the main discrepancies between the returns of Call and Put options in Bull vs Bear cycles.

Keywords: Option returns; Bull markets; Bear markets; S&P500 options

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Nomenclature

ATM At-the-money

CAPM Capital Asset Pricing Model

CBOE Chicago Board Options Exchange

CST Central Standard Time

ETF’s Exchange-traded funds

GBM Geometric Brownian Motion

ITM In-the-money

NASDAQ National Association of Securities Dealers Automated Quotations

NYSE The New York Stock Exchange

OTM Out-of-the-money

1. Introduction

Back in 1973 the first option contract was officially traded in Wall Street. Although, it's roots come from way back in time. In fact, the first "option contract" was used in ancient Greece to speculate on the olive prices.

Over the years, option contracts have become more traded due to their capability to tailor investor's risks to their preferences and aversions.

This type of contracts represents a very versatile financial product which can be used for very different reasons by traders and investors, for example, for speculation, hedging or reduce of the exposure to risk on a portfolio of assets.

In this work, we present the option returns over the S&P500 index during an analysis period of 15 years, from January of 2004 to April of 2019. During this period we were able to identify three Bull market periods and also three Bear market periods by analysing the price evolution of the index.

Throughout this thesis we try to answer the following questions. Does Coval and Shumway [5] methodology/propositions still apply with more up-to-date data? How do option returns behave in different environments of growth in the financial markets? What are the main differences on the returns when in a Bull market vs a Bear market?

In order to attempt to answer these questions we divided this thesis as follows:

In Chapter 2 we provide a brief explanation on what are option contracts (in more detail index options); what can be understood by Bull and Bear markets and provide a brief literature review on related work to this topic.

Chapter 3 presents with more details the paper we used as base to the calculation method of the returns.

Chapter 4 describes the data, provides a step by step method of calculation for the returns and describes some problems encountered throughout the thesis with the data and the solutions implemented to overcome them.

Chapter 5 describes and analyses the results obtained comparing them to the ones obtained on the paper from Coval and Shumway [5].

2. Literature Review

2.1. Basic Concepts

2.1.1. Options

An option is a financial derivative contract between two parties, the buyer and the seller. These types of contracts provide its holders, i.e. the buyer, the right (but not the obligation) to buy or to sell a certain pre-established quantity of the underlying asset (S), under a pre-established price condition (which is called the strike price - X) within a determined period, as described in Black and Scholes [2]. As for the seller of the option, if the buyer decides to exercise the contract, he is obligated to deliver or acquire the underlying asset at the pre-established conditions of the contract.

There are two types of standard options, the ones that give its holder the right to buy are called "Call options" and the ones that provide the right to sell are called "Put options".

Options can be identified by their moneyness. Moneyness describes the relation between the strike price and the underlying asset price of the derivative product, which in this case is the option contract.

Throughout this work we refer to these different moneyness as:

- **At-The-Money (ATM) -** when the strike price is equal to the underlying asset price.
- **In-The-Money (ITM) -** when the strike price is below (above) the underlying asset price for Call (Put) options.
- **Out-of-The-Money (OTM) -** when the strike price is above (below) the underlying asset price for Call (Put).

Each option contract has an associated premium, which refers to the current market price of the contract established. The buyer of the option needs to pay the seller this premium for the rights that are guaranteed by the contract itself. This premium depends on the option expiration date, strike price, the underlying volatility, the risk-free interest rate and others.

It is also important to refer that there are "European style options" that can only be exercised at the pre-established maturity date, and "American style options" that can be exercised any time until the maturity date.

We can also distinguish these contracts in accordance to their settlement form. We can have options with a cash settlement or a physical settlement. In the physical settlement, at the maturity date, the underlying asset of the option is transferred from the seller to the buyer. On the other side, the cash settlement, usually used in European style options, simply consists of a cash payment at the maturity date.

Options can have the most diversified underlying. It can vary from stocks, indexes, bonds, currencies, commodities or even other financial derivative products.

2.1.2. Index Options

An index measures the performance of a sub-group of assets that represents and replicates some area of the markets.

Since there is no way to purchase indexes directly (due to the fact that these are just benchmarks) there are some financial contracts that allow the investor to own the index in an indirect way. This can be done using index derivatives (such as option or future contracts) or by using Index funds and Exchange-Traded Funds (ETF's).

The trading of this kind of derivative products under the market indexes started in 1982 with the first ever traded stock index future. At the beginning options had as underlying the stock index futures, only then options on indexes started to be traded in the stock markets, which is the case we analyse on this thesis.

An index option provides its holder with the right (but not the obligation) to buy or sell the value of the underlying index at the pre-established price.

These options are typically cash-settled and European style options. This means that, at the maturity date, there occurs no transfer of the underlying. Instead, a cash payment, in accordance to the value of the index, is transferred.

In this work we focus on the S&P500 index option returns. The underlying on these options is the S&P500 index, which is composed by the 500 individual stocks from the biggest companies in the USA, from multiple industries, quoted in the NASDAQ and NYSE exchanges.

Many of the top companies that compose the index are technological firms and financial businesses, such as, Apple Inc. (AAPL), Microsoft Corp. (MSFT), Amazon.com Inc. (AMZN) and Facebook Inc. (FB). These are the top four and they represent almost 20% of the index.

The options over the S&P500 started to get traded more heavily after the crash of 1987 (known as the "Black Monday") where the Dow Jones index crashed over 22%. The Dow Jones index is one of the other indicators besides the S&P500 for the growth and volatility of the American market. It was created in 1896 by the "The Wall Street Journal" editor and founder of the Dow Jones Company, Charles Dow. The Dow Jones is calculated by considering the 30 stocks of the biggest and more important companies in the US.

2.1.3. Bull and Bear markets

Throughout history, it is possible to identify that stock markets can be in one of two states: "Bull" or "Bear". These two terms describe the market's health and what are the main expectations from investors against the market.

The "Bull" state is known for being a positive growth moment in the market.

On the other hand, in the "Bear" state, the market declines. A true "Bear" market usually expects at least 20% or more drops in the stock prices.

These different states tend to occur in cycles. Therefore, the "Bull" market is described by an expected 20% (or more) stock price rise. This period is always followed and preceded by a "Bear" market also with an expected 20% or more drop in the prices.

These cycles can be linked to the business cycles defined by Schumpeter [9] in 1939, where he defines the existence of four stages:

- **Expansion** - increase in prices and in the levels of productions and low interest rates;
- **Crisis** - crash of stock exchanges and bankruptcies;
- **Recession** - drops in prices and high interest rates;
- **Recovery** - period of economic adaptation and changes to adapt to new circumstances.

A bear market, on one side, is going to be linked to the period in which the market starts to fall until it reaches the bottom, therefore, it includes the Crisis and Recession stages of the business cycles.

A bull market, on the other side, is going to be linked to the period in which the market starts to rise back up from the bear market, therefore, it includes the Recovery and Expansion stages of the business cycles.

2.2. Related work

In 1973, the first option was officially traded in the Chicago Board Options Exchange (CBOE). In the same year, the model for the pricing of this derivative product developed by Black and Scholes [2] which assumed constant interest rates and volatility and that the underlying price follows a Geometric Brownian motion (GBM), was published.

Over the last few decades, the trading of these contracts has been rising mostly due to the ability they provide the investors to tailor their risks to their preferences. Literature has been focusing not only on the pricing of the options, but also on the understanding of the expected option returns.

Back in 2001, Coval and Shumway [5] published the first paper focused on the option returns for different option positions - calls, puts and straddles. They propose three main reasons to focus their attention on this topic:

- The test results based on the option returns allow to directly analyse if the **leverage effect** is priced without imposing any particular model;

- The analysis of risk-return in options allows any violations of market efficiency to be directly measured in economic terms;

- By examining the option returns it can be considered the economic payoff for taking very particular risks.

Under very general conditions, Coval and Shumway [5] find that Call options have positive expected returns, Put options have negative expected returns, and both Call and Put option returns are expected to increase with the strike price. These conditions are explained in more detail in Chapter 3.

Options are known to be mispriced, in the sense that their returns tend to exceed their risk, therefore they can have larger Sharpe ratios than those of the underlying index. The Sharpe ratio represents the average return earned in excess of the risk-free rate per unit of total risk, measured by the price fluctuations. This effect is particularly noticeable in strategies that involve selling Put options over the S&P500 index as found in Coval and Shumway [5], Bondarenko [3] and Jones [7].

Bondarenko [3] focused more on the Put option monthly returns. In the paper, he reported that ATM Put option returns result in losses of -39% per month and OTM Put options of -95% per month by using a test based on equilibrium models.

Selling these "unprotected" Put options can lead to extremely high profits, in the sense that, since the returns are negative, the holder of the option would not exercise it, but would have been required to pay a big premium upfront when the contract was established.

Regarding Put option returns, it is known that:

- Jensen's alpha for ATM puts is -23% per month and highly significant. Other measures (such as, the sharpe ratio) also indicate that the Put option prices have been very high;

- For Put options to have an average excess return of zero, crashes like the well known "Black Monday" would have to happen 1.3 times per year;

- The mispricing of specially Put options has substantial economic impact due to the high level of cumulative wealth transfers between the respective buyers and sellers.

Considering the CAPM [6] and Rubinstein [8] asset pricing models, Bondarenko [3] stated that Put prices are higher than the ones that these models predict. Therefore, puts are mispriced and thus there exists a "Put pricing anomaly". In his work, the author explored three possible explanations for this overprice: the risk premium, the peso problem and the biased beliefs.

In his analysis, Bondarenko [3] was particularly focused on the pricing of the options over the S&P500 futures. Instead of the underlying being the index itself, as in Coval and Schumway [5], he considered options under another derivative product, in this case futures with an index as underlying. He analysed a period of 13 year, from 1987 to 2000 of these options, and calculated their returns adjusted to risk by applying the CAPM asset pricing framework to the settlement prices of the options.

The author assumes that once one maturity has reached its end, they use the following available maturity and maintain the position until the expiry of the option.

In his paper he classifies the options by their strike-to-underlying ratio, over equally spaced bins with centers in k=0.94, k=0.96, ..., k=1.06. Each bin is only composed by one unique strike, which is the one that is closer to the center of the bin itself.

The work of Jones [7] focused more on the average returns of S&P500 future options and on the reasons for the absurd expected returns of these contracts. To accomplish this, the author uses a nonlinear multi-factor model to calculate the returns of the options. He found that priced factors such as the levels of volatility (given by the VIX index) and jump risk premia are insufficient to explain the magnitude of, particularly, short-term deep OTM puts returns.

In this analysis the author calculated the returns by considering the transaction prices at 3pm every day from the Chicago Mercantile Exchange, and considers only the options with at least 10 days to the maturity.

The author analyses only OTM options by classifying them in accordance to their moneyness. Therefore, by dividing the strike by the underlying, Put options have moneyness below 1, and Call options have moneyness above 1.

In some more recent work, in 2009, Broadie et al. [4] compared the observed returns with those generated by standard option pricing models such as Black-Scholes and also extend the analysis by incorporating jumps or stochastic volatility by using Monte-Carlo simulations.

In his work he states that there are three obvious factors to consider when analysing the results found in papers, such as, Bondarenko [3]:

- Options returns are non-normal and metrics that assume normality, like CAPM alphas or Sharpe ratios, are inappropriate;
- Average Put returns or CAPM alphas are significantly different from zero;
- Options contracts have a short period of existence in time, therefore it is difficult to assess the statistical significance of option returns.

He analyses monthly returns over the S&P500 future options, and for each month he chooses the settlement price for the option with exactly one month until the maturity date. As done in Jones [7], Broadie et al. [4] also classifies the options in accordance to their moneyness, but with intervals from 0.94 to 1.02, with increments of 2% between intervals.

3. Method Selection

In this chapter we describe in more detail the Coval and Shumway [5] methodology, propositions and the main conclusions found in their work. This was the first paper that looked into the options returns and not just the pricing of the options.

We use their methodology to calculate the S&P500 options returns.

We chose this method in order to see if the first and most simple method for the calculation of the S&P500 option returns would still provide the same conclusions as it did back in 2001 and to check how it would act in the Bear and Bull periods throughout our data base.

3.1. Coval and Shumway method

According to the Black-Scholes/CAPM asset-pricing framework, Call options are expected to have positive returns greater than those of the underlying asset, either it is a market index or other asset. On the other side, Put options expected returns are below the risk-free rate. Both returns are expected to increase with the strike price.

In this paper, Coval and Shumway [5] provide two more generic propositions when compared to the ones stated in the previous paragraph:

Proposition 1: "If the stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any Call option on that security will have a positive expected return that is increasing in the strike price."

Proposition 2: "If a stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any Put option on that security will have an expected return below the risk-free rate that is increasing in the strike price."

These propositions are stated in terms of standard returns (not log returns) and admit the existence of a well-behaved stochastic discount factor that prices all assets in accordance to the following:

$$E[R_i \cdot m] = 1, \tag{3.1}$$

where E denotes the expectation operator, R_i is the gross returns of the asset and m is the strictly positive stochastic discount factor. This discount factor only exists in the absence of arbitrage.

For their testing they used the Berkeley Options Data base of Chicago Board Options Exchange (CBOE) bid-ask quotes. Their data base on the European S&P500 option prices was from January 1990 to October 1995 and contained weekly observations over this period of high market liquidity.

The time of observation was every Tuesday (or Wednesdays when there was any holiday on the observation day) after 9 a.m. Central Standard Time (CST).

They take options that expire on the following calendar month, therefore, the options used have between 20 to 50 days until expiration and use the bid-ask quotes to calculate the mid price.

In this work options were classified into five groups in accordance to the difference between the strike price and the underlying price and used a five-point increment between groups (groups are -15 to -10; -10 to -5;....; 5 to 10).

Instead of calculating the logarithmic option returns, as in previous work presented by Sheik and Ronn [10], they calculated the raw weekly net returns. The main explanation for this calculation method is that options that are held until expiration date often have a net return of -1 (worthless) and the log-transformation of the returns would be significantly lower than the raw net returns.

$$Return = \frac{P_{t+1}}{P_t} - 1. \tag{3.2}$$

We use their method to calculate the option returns over a different period of time and instead of weekly returns we analyse daily returns. More details are provided into the following chapter.

4. Data

On this chapter we describe the data base of the S&P500 index options. We also explain on how we proceed in order to calculate the daily option returns for Put and Call options over the index throughout our period of analysis.

4.1. Data description

The data base consists of the daily bid and ask prices for the S&P500 options throughout a 15 year period of time, since January 2004 to April 2019. The data was retrieved from the CBOE daily price reports.

The period of observation we analyse include the impact of the financial crisis, especially the "Subprime mortgage crisis" from 2008. As well as it includes periods of great growth in the economy.

For each day there exists a corresponding csv file from which we extract, using Matlab, the following information in order to analyse the European style S&P500 options returns:

- **Quote date** - corresponds to the date of the file we are analysing, we need this information because we compare quote dates to obtain the returns;
- **Root** - corresponds to the root of each option incorporated in each file, it contains information related to the style of the option;
- **Expiration date** - corresponds to the maturity date of the option, it is the last day that the option contract is valid;
- **Strike** - corresponds to the pre-established price at which the contract can be executed by the maturity date;
- **Option type** - corresponds to the type of option, indicating if the option is a Call (the holder has the right to buy at the maturity date) or if it is a Put option (the holder has the right to sell at the maturity date);
- **Ask price** - corresponds to the minimum price a seller is willing to accept for the option;

- **Bid price** - corresponds to the maximum price a buyer is willing to pay for the same option as the seller;

- **Underlying price** - corresponds to the spot price of the underlying asset of option, in our case is corresponds to the price of the S&P500 index;

4.2. Calculation method

Below we provide a step-by-step description of how the returns for the S&P500 index options observed from 2004 to 2019 were calculated. The calculation method goes as follows:

1. Identify all the roots that correspond to the monthly standard European-style options;

2. For each option identify the bid-ask quote at the time of day 15h45 ET;

3. Identify and only consider options that are to expire during the following calendar month, therefore their maturity date is within 20 to 50 days;

4. Take the bid and ask prices for each option and calculate the mid price - this represent the average price between the bid and ask;

$$Mid\ Price = \frac{Bid\ Price + Ask\ Price}{2}. \tag{4.1}$$

5. Use the mid price to calculate the daily holding-period returns for each option (assuming a strategy where you buy one option contract on one day and you sell it immediately on the following day);

6. Classify the options into five groups according to their strike prices relative to the S&P500 index prices with a 5 point increment within each group. Each group is calculated by subtracting the underlying price (S) to the strike price for each option (X).

The intervals are defined with the strike and underlying prices of time $t-1$ and then to calculate the returns for each group for each day we use the options in t with the same strikes as the ones identified in $t-1$.

Therefore, options are grouped according to their strikes and not their moneyness levels. This is done to ensure that there is at least one option for each group at every determined point in time.

4.3. Problems with the data base

Throughout the analysis of the data base, we faced some issues with the data itself. These problems were leading to unrealistic results. In this section we focus on these problems we were able to identify and describe them.

We also explain how we proceed to fix these and the type of exceptions we had to incorporate on our code in order to overcome them.

First issue: There were two days in the data base with no data related to the S&P500 option prices. One of the days was 01-06-2016, which only had data related to the options under VIX index, not the S&P500 index. The other day was 24-12-2018, since this day refers to Christmas eve, the exchange was only opened part of the day, therefore we had all prices as zero, which would have had a huge incorrect impact on the daily returns.

Solution: We exclude these two days from our data base.

Second issue: On the following quote dates almost all the options have price of zero: 17/09/2004, 01/07/2005 and 05/10/2007.

Solution: We exclude these returns from our data base.

Third issue: For quote date 25-10-2005 in our data base there were two bid and ask prices for the same option (they had the same strike, maturity, root and type (C/P)), one of them was priced as zero for both Call and Put option, and the other was priced above zero.

Solution: We implemented some conditions in the code that check if is there any duplicate keys (our keys are composed by the root+strike+option type), and if true we get the one which has the bid price above zero.

Fourth issue: On quote dates 16/01/2009, 13/02/2009 and 10/07/2009 the options with strikes 850, 825 and 875 respectively did not exist on the following quote date, therefore, it was not possible to calculate the returns for these strikes.

Solution: We exclude these returns from our data base.

5. Numerical Results

This chapter presents the numerical results using Coval and Shumway [5] method of calculation explained on the previous chapter. The results are separated by option type: Call and Put options. There is also an analysis of the returns for these option types when in Bull and Bear Markets.

For this analysis we calculate the following statistical measures as in the Coval and Shumway [5]:

- **Mean:** The arithmetic average of a set of numbers, or distribution;

$$Mean = \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{5.1}$$

- **Median**: The numeric value separating the higher half of a certain population or distribution, from the lower half;
- **Minimum**: The number that is less than or equal to all other values in the data set;
- **Maximum**: The number that is greater than or equal to all other values in the date set;
- **Standard Deviation**: The average distance between the values of the data set and the mean.

$$SD = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}} \tag{5.2}$$

The resume tables on the following sections presents the option groups in accordance to their strike prices relative to the underlying index price. These range from options with strikes 15 to 10 points below the underlying price of these options to the ones with strikes 5 to 10 points above the underlying price.

5.1. Total sample

Tables 5.1 and 5.2 provide us different statistics for the daily S&P500 options returns over a period of 15 years, from 2004 to 2019.

Table 5.1.: Daily S&P 500 Call Option Returns

X-S	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Mean return	0.52%	0.57%	0.55%	0.58%	0.69%
Median	-0.34%	-0.41%	-0.69%	-0.94%	-1.30%
Minimum	-67.82%	-72.05%	-72.32%	-74.01%	-75.86%
Maximum	98.56%	106.72%	117.21%	125.59%	136.47%
Std deviation	20.59%	21.57%	22.70%	23.95%	25.31%

Table 5.2.: Daily S&P 500 Put Option Returns

X-S	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Mean return	-1.51%	-1.44%	-1.32%	-1.20%	-1.18%
Median	-6.00%	-5.71%	-5.31%	-5.17%	-4.90%
Minimum	-78.13%	-76.98%	-75.78%	-74.78%	-73.82%
Maximum	430%	376.79%	328.99%	289.52%	260.14%
Std Deviation	29.07%	28.17%	27.40%	26.72%	25.98%

Focusing on Table 5.1, as expected, the Call option mean returns are, on average, positive. From our sample, the Call option daily average returns vary from 52 basis points to 69 basis points.

Near ATM Call options tend to earn between 55 and 58 basis points per day.

It can also be noticed that the returns increase with the strike price, therefore, an OTM call option within the price range of 5 to 10 points above the underlying price earn on a daily basis 12 basis points more than the ITM option within the same price range.

Even though the discrepancies in the average returns are not very statistically significant, by analyzing the remaining statistics it is observed a positive skewness on the median, minimum and maximum, which is increasing with the strike prices.

Focusing on Table 5.2, the Put option mean returns are negative and vary from -151 basis points to -118 basis points.

Near ATM Put options are expected to lose between -132 basis points and -120 basis points. For Put options it is observed that the returns also increase with the strike price, even though they are still negative expected returns.

ITM Put options within the price range of 5 to 10 points above the underlying price have a loss of less than 26 basis points than the OTM Put option within the same price range.

We can also observe that the values for the median and mean are different. For call options we can see that despite the mean daily returns being positive the median is negative, which indicates again the positive skewness on the data. Meaning that there are much more cases in which this strategy, even for Call options lead to a negative return. For the Put options, we also have a discrepancy between these two statistics, but both with negative signs, the median indicates that the returns are actually more negative than those that the mean shows.

Finally, looking into the standard deviation it is noticed that the data is dispersed, meaning that there are a lot of days with returns that are very different from the mean returns. This also leads to the conclusion that the returns on the S&P500 options do not follow a normal distribution.

5.2. Bull Market sample

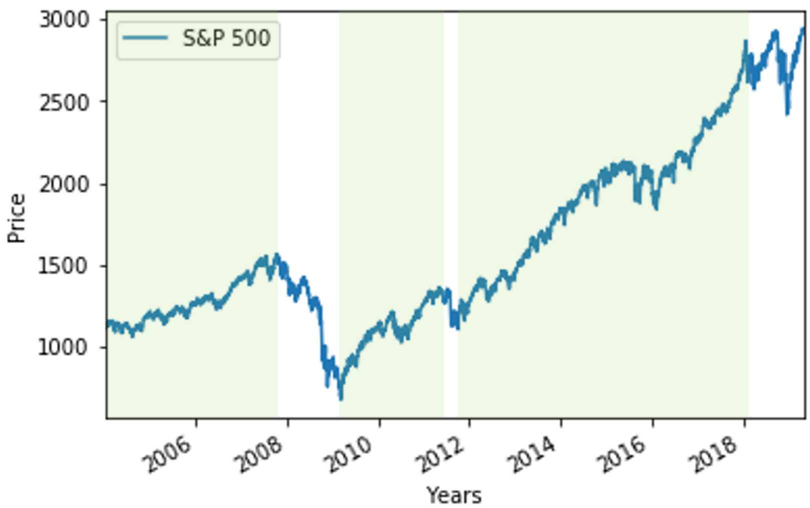

Figure 5.1.: Price evolution on S&P 500 index - Bull Markets

Now we focus on the returns during financial Bull cycles. During these periods the market is expected to have a positive growth and the stock prices normally go up.

To identify if we are on a Bull cycle, we analyse the prices of the S&P500 and consider it as a good representation of the financial market itself. Therefore, if the prices of the index are growing we assume we are in a Bull cycle, otherwise, we are facing a Bear cycle.

In Figure 5.1, we have the time series of the S&P 500 index prices, we have highlighted the three Bull cycles during our time analysis from 2004 to 2019. These are characterized by a positive variation in the S&P500 prices and upward slope.

These periods are 02/01/2004 to 09/10/2007; 09/03/2009 to 29/04/2011; and 03/10/2011 to 20/09/2018.

In Tables 5.3 and 5.4 we analyse in more detail the returns on the S&P500 options when the market is in Bull periods.

The S&P500 prices had positive price variation that vary up to 62.5%.

Table 5.3.: Daily S&P 500 Call Option Returns - Bull Markets

X-S	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Mean return	1.03%	1.09%	1.07%	1.12%	1.27%
Median	0.015%	-0.00%	-0.26%	-0.43%	-0.68%
Minimum	-67.82%	-72.05%	-72.32%	-74.01%	-75.86%
Maximum	98.56%	106.72%	117.21%	125.59%	136.47%
Std deviation	20.02%	21.05%	22.27%	23.56%	25.03%

Table 5.4.: Daily S&P 500 Put Option Returns - Bull Markets

X-S	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Mean return	-2.23%	-2.14%	-1.97%	-1.84%	-1.81%
Median	-6.38%	-6.10%	-5.62%	-5.51%	-5.27%
Minimum	-78.13%	-76.98%	-75.78%	-74.78%	-73.82%
Maximum	430.00%	376.79%	328.99%	289.52%	260.14%
Std deviation	27.78%	27.25%	26.73%	26.16%	25.56%

For Call options, the returns exceed the ones we analysed in the total sample. This emphasises the idea that in a Bull cycle, Call options will have higher returns. Call options are expected to earn between 103 basis points to 127 basis points per day. OTM Call option within the price range of 5 to 10 points above the underlying price earn on a daily basis 18 basis points more than the ITM option within the same price range.

As for Put options, the mean returns are still negative and vary from a range of -223 basis points from the most OTM to -181 for the most ITM Put options. In these periods Put option returns are more negative than what is expected.

In these tables we are able to clearly spot the propositions of Coval and Shumway [5], mean returns increase with the strike and Call have positive expected returns and puts have negative expected returns.

Near ATM calls have returns in the range of 107 to 112 basis points, and near ATM puts have negative returns in the range from -184 to -197 basis points.

OTM options are the ones which provide a higher maximum returns, and ITM options are the ones with lower maximum returns.

5.3. Bear Market sample

Finally focusing on the returns during financial Bear cycles. During these periods the market is expected to have a negative growth and the stock prices decline.

Whenever the financial markets are not in a Bull cycle, it is said they are in a Bear cycle.

In Figure 5.2, we identify the periods in the gap between the different Bull cycles identified previously, which will correspond to the Bear cycles, where the index prices decline.

These periods are 09/10/2007 to 09/03/2009; 29/04/2011 to 03/10/2011; and 20/09/2018 to 21/12/2018.

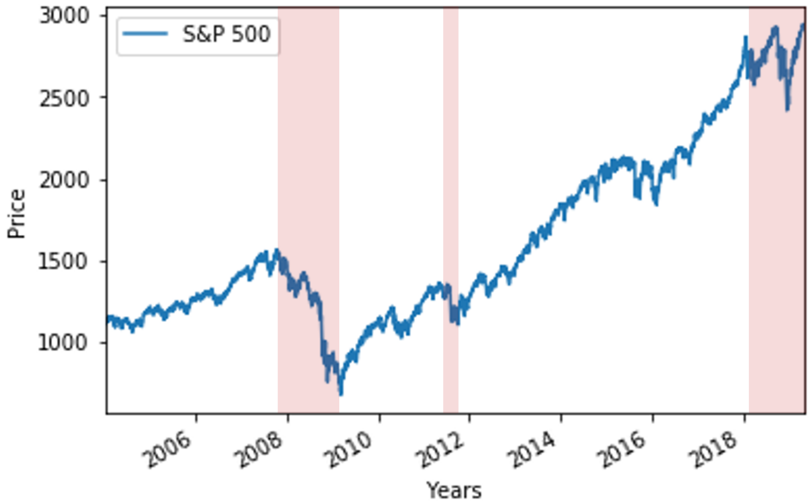

Figure 5.2.: Price evolution on S&P 500 index - Bear Markets

Table 5.5.: Daily S&P 500 Call Option Returns - Bear Markets

X-S	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Mean return	-3.34%	-3.33%	-3.39%	-3.48%	-3.56%
Median	-3.92%	-3.92%	-4.11%	-4.18%	-4.86%
Minimum	-58.04%	-58.75%	-61.18%	-64.68%	-67.27%
Maximum	76.25%	79.94%	87.54%	111.47%	115.70%
Std deviation	23.39%	24.11%	24.89%	25.94%	26.73%

Table 5.6.: Daily S&P 500 Put Option Returns - Bear Markets

X-S	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Mean return	3.86%	3.79%	3.59%	3.68%	3.57%
Median	-1.15%	-0.83%	-0.75%	-0.51%	-0.64%
Minimum	-61.59%	-61.22%	-60.71%	-60.62%	-59.97%
Maximum	147.98%	146.87%	142.53%	138.29%	130.98%
Std deviation	32.75%	31.90%	31.18%	30.79%	30.03%

In Tables 5.5 and 5.6 provide us the daily option returns when the market is in Bear periods.

These periods include one of the biggest recessions of all times that was characterized by the subprime mortgage crisis provoked by high risky loans which led to huge levels of debt and bankruptcy of numerous banks.

During these periods Put options have positive returns and the Call options have negative returns.

Deeply ITM Call options are the ones providing smaller losses, with mean returns of -334 basis points. Deeply OTM Put options are the ones providing the higher returns, with mean returns of positive 386 basis points.

Call options are expected to lose between -334 basis points to -356 basis points per day. OTM Call option within the price range of 5 to 10 points above the underlying price lose on a daily basis 23 basis points more than the ITM option within the same price range.

Put options are expected to earn between 357 basis points from deeply ITM options to 386 basis points for the deeply OTM options.

Near ATM calls have returns that range from -348 to -339 basis points, and near ATM Put options have returns in the range from 359 to 368 positive basis points.

Maximum returns occur in the most deeply OTM groups for both Call and Put options.

In Bear cycles we find that the two propositions of Coval and Shumway [5] do not apply, since the returns are not increasing with the strike prices, in fact, the opposite occurs for Call and Put options, there is a decrease in returns as the strike goes up. Also, Call options provide negative expected returns and Put options provide positive expected returns, which is the opposite as propositions 1 and 2.

6. Conclusion

In this thesis, we have presented the results using the Coval and Shumway [5] methodology for the calculations of the daily returns on the S&P500 options returns.

As explained in section 5.1, we were able to reach similar results to the ones reached by Coval and Shumway [5] had back in 2001 in our database from January 2004 to April 2019. The two propositions from their work are still reflected in a more up-to-date database:

- Call options have positive expected mean returns;
- Put options have negative expected mean returns;
- Returns for Call and Put options are increasing with the strike price.

Throughout the different cycles of the economy, we can conclude that expected option returns also vary from Bull to Bear periods.

When the market is growing, Call options seem to be the best investment option since they provide positive mean returns. When the market is declining, Put options seem to be the best investment option.

Investors are moved by their preferences, but also by their fears. When the expectations in the market are that it will decline and prices are expected to drop, investors are moved to invest in the kind of product that possibly provides them the returns on a bad state of the economy. This leads to investments "against" the market in Bear cycles, which can be done using Put options.

On the other hand, when the market is expected to grow and prices to increase, investors believe in the market and are not afraid, therefore, invest more in products such Call options.

Some future work for this thesis would be to implement different methodologies for the calculation of the options returns and see how the measurement of the returns would be impacted by the Bull vs Bear cycles.

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