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A Decision Support System for Improving the Inconsistency in AHP

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ABSTRACT

This paper presents a DSS aimed at helping decision makers reduce and improve their inconsistency in eliciting their judgements when using the analytic hierarchy process (AHP). The DSS is designed for revising the judgements of a pairwise comparison matrix when the row geometric mean (RGM) is used as the prioritisation procedure and the geometric consistency index (GCI) as the inconsistency measure. The procedure employed guarantees that both the judgements and the derived priority vector will be close to the initial values. The DSS allows different degrees of participation of the decision maker in the review/modification of the judgements: no participation (automatic mode); prior participation (semi-automatic mode); and ongoing participation (interactive mode). The DSS also includes options to incorporate other requirements of the decision maker, such as limiting the modified values to an interval or improving inconsistency by modifying the lowest number of judgements, among others.

KEYWORDS

Analytic Hierarchy Process, Decision Support System, Geometric Consistency Index, Inconsistency Improvement, Row Geometric Mean

INTRODUCTION

The analytic hierarchy process (AHP), proposed by Thomas L. Saaty at the end of the 1970s, is a multi-criteria decision technique that has become one of the most commonly employed approaches to the resolution of complex problems (Subramanian and Ramanathan, 2012; Zyoud and Fuchs-Hanusch,

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2017). Decision makers incorporate their preferences using pairwise comparisons and some degree of inconsistency is allowed when eliciting their judgements. Consistency is a particularly important issue as it is a requirement for the validity of the derived priority vector (Grzybowski, 2016).

Given a pairwise comparison matrix (PCM), $A = (a_{ij})_{n\times n}$ with $a_{ij}\cdot a_{ji} = 1$ and $a_{ij} > 0$, Saaty (1980) established that the matrix A is consistent if $a_{ij}\cdot a_{jk} = a_{ik} \forall i, j, k = 1,\ldots, n$. This is a desirable property that reflects a certain rationality, logic, or formal coherence. There are many factors that may cause inconsistencies in the judgements elicitation process, such as (Aguarón et al, 2020): (1) the ambiguity and complexity of the problem; (ii) the knowledge of the actors in the matter under consideration; (iii) the affective aspects (mood, emotions, personality features, attitudes and motivations) that condition the behaviour of the actors; (iv) the level of attention (errors in the response) during the assessment process; and (v) the rationality of the procedure followed when incorporating preferences, especially when working with subjective aspects.

To measure the inconsistency different indicators have been proposed in the AHP literature. Two of the most widely used are the Consistency Ratio (CR) associated with the eigenvector (EV) prioritisation procedure and the Geometric Consistency Index (*GCI*) associated with the row geometric mean (RGM) prioritisation procedure. Other inconsistency measures for pairwise comparisons were proposed in the literature. Brunelli (2018) presents a survey of them as well as a study of their properties and relations. With regards to the improvement of inconsistency in AHP, different procedures have also been described in the literature. An overview of these approaches can be found in Khatwani and Kar (2017).

Aguarón et al. (2021) proposed, for the first time in the literature, a procedure for improving the inconsistency when the Row Geometric Mean (RGM) is used to derive the priorities and the Geometric Consistency Index (*GCI*) is employed to measure the inconsistency. This is a sequential procedure that, at each iteration, identifies the judgement that would improve the *GCI* faster and with greater intensity. In the proposed procedure the decision maker intervenes at the beginning indicating its permissibility threshold, that is, the maximum variation, in relative terms, that they would accept to modify the initial judgements. Limiting the variations of the judgements by the permissibility threshold guarantees that both the final judgements and the derived priority vector will be close to the initial values, as recommended by Saaty (2003).

The objective of the paper is to present a DSS that implements the Aguarón et al. (2021)'s procedure proposed for reducing the inconsistency in AHP by adapting it to be used interactively. The DSS also calculates the minimum permissibility necessary to achieve an allowable inconsistency level (below the required threshold). The value of this parameter (minimum permissibility) provides relevant information about the decision problem, in line with the cognitive multicriteria decision making paradigm (Moreno-Jiménez and Vargas, 2018), that can be used by the decision maker as a starting point to set their own permissibility.

The DSS can then be used to obtain the final values of the judgements and the derived priorities in three different ways: automatically (without personal participation of the decision maker in the resolution process), semi-automatically (prior participation of the decision maker fixing the permissibility) or interactively (personal participation throughout the resolution process). In the last case, the decision maker intervenes more actively at each iteration of the algorithm implemented in the DSS. Participation refers to the selection of new values for the judgements that the decision-maker decides to modify (guided by the values suggested by the DSS) and the acceptance of the values in the final matrix. The greater the degree of participation of the decision maker, the greater the cost in terms of time and effort spent on applying the process, but also the greater the learning about the decision problem. The DSS is also designed to meet other possible requirements of decision makers.

The rest of the paper is organised as follows. Next section (Background) summarises the main theoretical results on which the DSS is based. The following section (A DSS for Improving the Inconsistency in AHP) presents the DSS, its modules and the different modes in which it can be employed to reduce and improve inconsistency. Section ‘Numerical example’ applies the DSS to a numerical example to illustrate how it works, and finally, ‘Conclusions’ Section summarises the main contributions of this paper.

BACKGROUND

Let $A = (a_{ij})_{n \times n}$ be a PCM. The inconsistency measure proposed for the RGM method is (Aguarón and Moreno-Jiménez, 2003) the Geometric Consistency Index (*GCI*):

$$GCI = \frac{2}{\left(n-1\right)\left(n-2\right)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \log^2(e_{ij}) \tag{1}$$

where $e_{ij} = a_{ij} \frac{w_j}{w_i}$ and $w = \left(w_i\right)$ is the priority vector obtained with the RGM method (e_{ij} is the error obtained when the ratio of priorities w_i / w_j is approximated by a_{ij}). These authors established thresholds for the *GCI* with an interpretation analogous to the 10% of Saaty’s CR. These values are $GCI = 0.31$ for $n = 3$, $GCI = 0.35$ for $n = 4$ and $GCI = 0.37$ for $n > 4$.

Following a similar approach to that employed by Dadkhah and Zahedi (1993) for the EV method, Aguarón et al. (2021) calculated the derivatives of the *GCI* with respect to the judgements to identify which entries further improve the inconsistency. These authors proved that the judgement a_{rs} that must be considered for reducing the *GCI* is the one for which $\log e_{rs}$ is the largest in absolute terms. This is the judgement that most rapidly decreases the value of the *GCI* and it is also the one that allows the greatest reduction in absolute terms. Expression (2) provides the relative variation of judgement a_{rs} that produces the greatest decrease of the *GCI* and, therefore, correspond to the limit of the variation for this judgement. A modification beyond this value will produce an increase in the *GCI*.

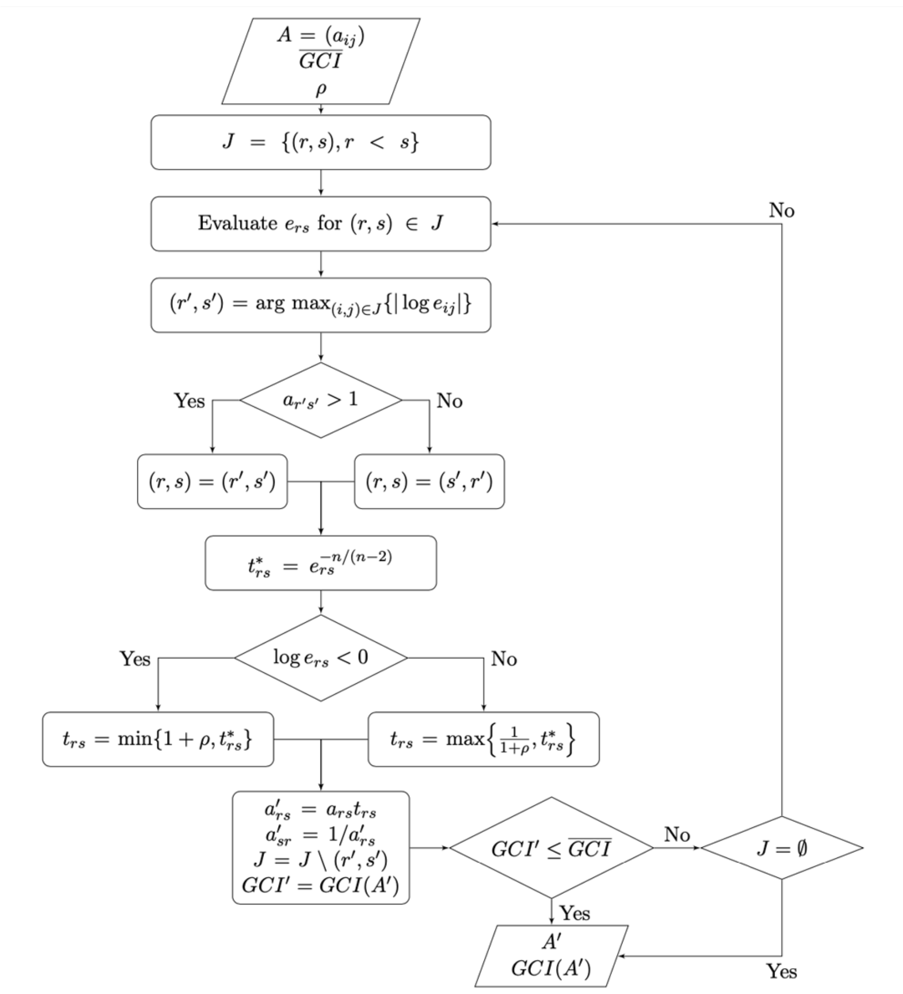
$$t_{rs}^* = a_{rs}^* / a_{rs} = e_{rs}^{-n/\left(n-2\right)} \tag{2}$$

Using these results, Aguaron et al (2021) designed an algorithm for improving the inconsistency. The authors introduce the concept of permissibility in oder to avoid major modifications of the judgementes. The permissibility parameter indicates the maximum relative variation permitted by the decision maker for the modifications of any judgement considered. In this way, the procedure follows Saaty’s suggestion of improving the inconsistency by slightly modifying the judgements that further improve the inconsistencty measure.

The sequential procedure proposed by Aguarón et al. (2021) for improving the *GCI* is summarised in the flowchart shown in Figure 1. The inputs for the algorithm are a PCM, $A = (a_{ij})_{n \times n}$, for which it is desired to reduce its inconsistency; the permissibility allowed in relative terms for the modification of judgements, ρ; and the desired threshold for the *GCI*, $\overline{GCI}$. The outputs are the updated PCM A', where the new judgements a_{rs}' have been incorporated, and the associated value $GCI' = GCI\left(A'\right)$. The judgements are selected sequentially from the highest to the lowest value of $\log(e_{rs})$ in absolute terms. *J* denotes the set of pairs associated to judgements that have not yet been considered. At the beginning it contains all the elements of the PCM that are above the principal diagonal. At the end of each iteration the pair corresponding to the judgement modified in that iteration is removed from the set J. The algorithm ends when J is empty or when the desired inconsistency threshold is reached ($GCI \leq \overline{GCI}$). Modifications will be made to judgements that are above 1 and associated with these, the corresponding reciprocal ones will be modified. Therefore, for the selected pair, it is evaluated whether it corresponds to the judgement that is greater than 1 or is its reciprocal, in order to use the one that fulfils the condition. Then, the optimal value for the variation of this judgement, t^*_{rs} is calculated. The maximum relative modification will be limited by the permissibility (ρ) and the range of improvement (t^*_{rs}). If $\log\left(e_{rs}\right) < 0$, the value of the judgement a_{rs} should be increased (t_{rs} = min{1+ ρ, t^*_{rs} } ³ 1) in order to reduce the *GCI*. If $\log\left(e_{rs}\right) > 0$, the value of the judgement a_{rs} should be reduced (t_{rs} = max{(1+ ρ)$^{-1}$, t^*_{rs} } £ 1). In this situation, the permissibility is incorporated as 1/(1+ρ) to keep the property of reciprocity. Then, the values of the associated judgements are updated ($a'_{rs} = a_{rs} t_{rs}$ and $a'_{sr} = 1 / a'_{rs}$) and the *GCI* of the updated matrix is calculated.

Note that the modified values of the PCM do not have to belong to the discrete set {1/9,...,9}, so in this paper real values will be allowed for the judgements.

Figure 1. Flowchart of Aguarón et al. (2021)'s algorithm



A DSS FOR IMPROVING THE INCONSISTENCY IN AHP

DSS Design

The DSS implements the algorithm proposed in Aguarón et al. (2021) and permits to adapt it to different situations. In addition to reducing inconsistency and allowing the consideration of particular situations, the DSS facilitates, by following a cognitive orientation (Moreno-Jiménez & Vargas, 2018) in solving the problem, the extraction of relevant knowledge about the problem, the resolution process and the people involved in its resolution.

The DSS calculates the minimum permissibility (ρ_{min}) necessary to get to an acceptable inconsistency (the required threshold $\overline{GCI}$). The value of ρ_{min} is obtained by repeatedly applying the algorithm described in Figure 1. The bisection method is applied to suggest ρ values to be inspected, for which the algorithm is applied to calculate the final *CGI* that would be achieved. In this way, it is possible to find the ρ_{min} value with the desired precision.

There are three possible ways to use the DSS, depending on the degree of participation of the decision maker in the review of the judgements:

1. Automatic mode: the DSS applies the algorithm described in Figure 1 employing the minimum permissibility (ρ_{min}) value at each iteration to modify the judgements. There is no intervention of the decision maker at any stage of the process.
2. Semi-automatic mode: the decision maker intervenes at the beginning of the process by indicating their own permissibility (ρ). The minimum permissibility provided by the DSS (ρ_{min}) may help the decision maker to set their own value of this parameter. Then, the DSS applies the algorithm described in Figure 1 to modify the judgements without exceeding the permissibility established by the decision maker.
3. Interactive mode: the DSS informs the decision maker which is the judgement that will be modified at each iteration of the algorithm and the maximum change that can be applied to it to get the maximum possible decrease in the inconsistency value (is the one that the algorithm would use if the automatic mode were used with the largest possible permissibility). Then, the DSS permits an active participation of the decision maker who will decide what value to give exactly to each judgement at each step/iteration. The DSS also allows the user to go back and revise the values that have been set before. This feature provides the decision maker with valuable learning about the behaviour of the problem when faced with the different changes being considered.

Obviously, greater participation requires more time and effort from the decision maker, but greater is the knowledge extracted from the resolution process. It should be the decision maker who decides which way they prefer. In the interactive mode the decision maker validates the results that are achieved at the end of the process. In the semi-automatic mode, the decision maker fixes their permissibility and assumes the matrix resulting from the application of the process. In the automatic mode, the decision maker does not validate the final matrix. Nevertheless, the automatic mode can be useful to know the frame where the judgements are going to move. In short, the DSS has the potential both to help the decision maker reduce their inconsistency and to increase their own knowledge of the problem.

The DSS also incorporates other complementary options that can be applied to meet other possible requirements of decision makers, such as limiting the modified values to the interval [1/9,9] as usual in the context of AHP, allowing several modifications for each judgement, modifying the lowest number of judgements by eliminating the permissibility, or getting the maximum possible reduction of the inconsistency measure. Some of these options could lead to changes in the judgements of remarkable intensity, providing values for judgements outside the priority stability intervals (Aguarón and Moreno-Jiménez, 2000). If this happens, it will be even more necessary for the decision maker to validate the results and be aware of the level of change achieved.

The different options and procedures have been implemented into different modules that integrate the DSS. The modular structure of the DSS is shown in Figure 2, where the modules that correspond explicitly to the improvement of the inconsistency are those framed on the left. These modules are integrated into an AHP software called PRIOR (Aguarón et al., 2010), developed in Delphi, which includes the standard input modules for judgements, basic AHP calculations and other modules such as multiactor decision making, web-DSS, or model exploitation.

DSS Software

In what follows, we can see some screenshots of the DSS software. Figure 3 shows the main window of the application where the user will first select if introducing a new problem of loading a problem previously saved. Figure 4 shows the interface for introducing the pairwise comparison values or editing them.

Figure 2. Modules of the DSS integrated in the PRIOR software

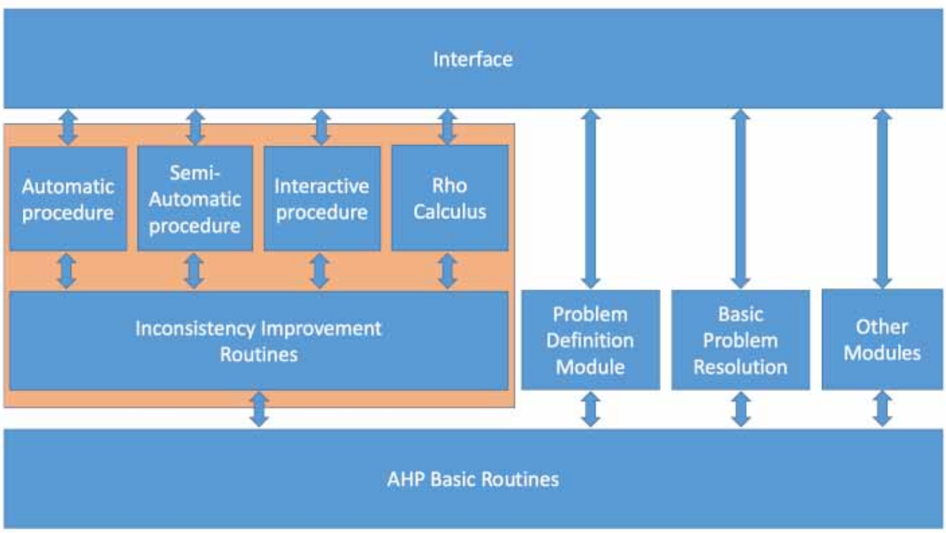


Figure 3. Main window of the DSS

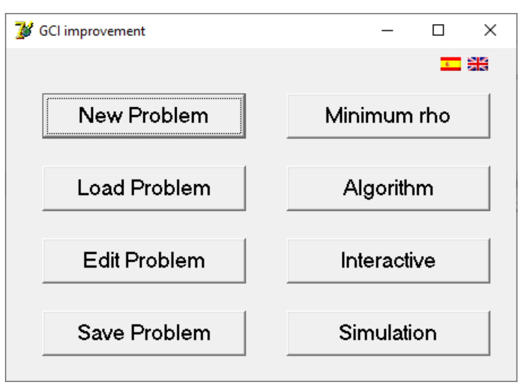
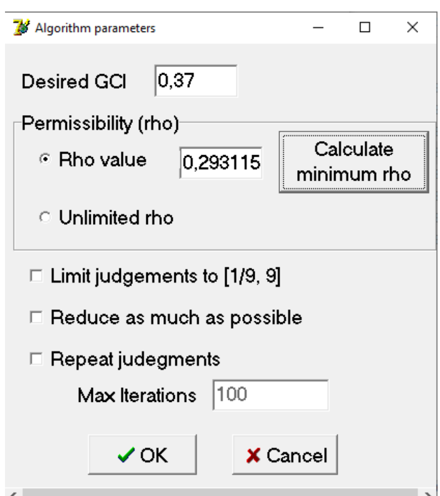


Figure 4. Data entry interface

Once the PCM has been introduced, the option 'Minimum rho' will inform the user about the value of the minimum permissibility to get an acceptable inconsistency. Then, the user will select at the main window which option to use to reduce the inconsistency, the option 'Algorithm' if they want to use the automatic or semi-automatic mode and the option 'Interactive' if they want to use the interactive mode. If 'Algorithm' is selected, the decision maker will then be able to enter the desired value for the *GCI* (by default the inconsistency threshold is suggested) and to provide their own value of the permissibility ('Rho value'). If button 'Calculate minimum rho' is used, then the permissibility will take the value of the minimum permissibility to get the desired *GCI*. The option 'Unlimited rho' will apply the algorithm with no limitation for the variations of the judgements ($\rho =$ ¥). Different options are available to customize the application of the algorithm, which can be seen in Figure 5. If none is selected, the algorithm is applied as described in the previous section (see flowchart in Figure 1). The option 'Simulation' carries out a simulation study for different matrix sizes and permissibility values according to the different customisation options available.

Finally, if the user selects the 'Interactive' option a window like that shown in Figure 6 will appear for each iteration. The initial matrix is shown on the left and the modified matrix for that iteration is shown on the right. The corresponding *GCI* values associated with these matrices are

Figure 5. Options to customize the application of the algorithm



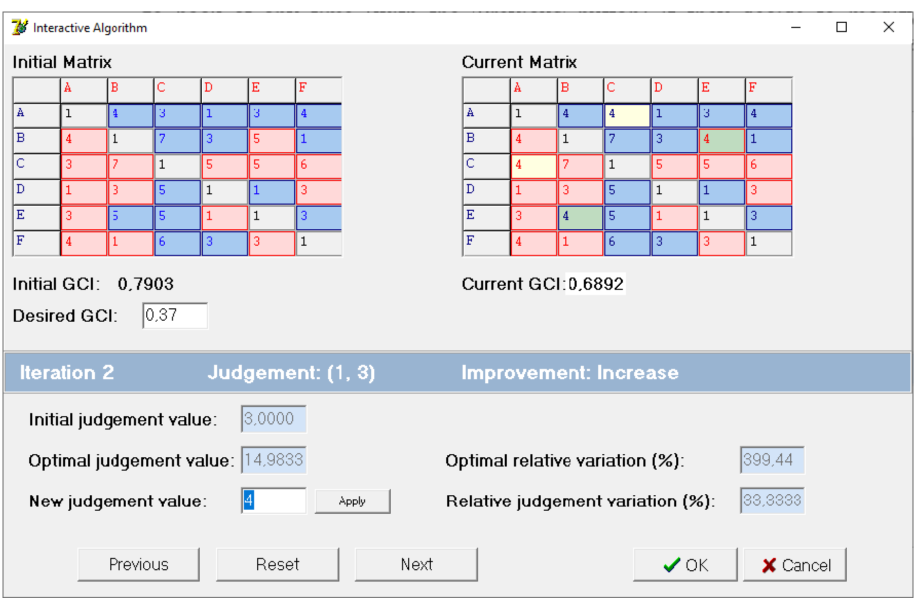
shown below each of them (the current *GCI* in red if it is above the desired level or in green if it is below). The matrix on the right shows in green the judgements already modified and in yellow the judgement that can be modified in such iteration. At the bottom, the user is informed of the initial value of the judgement (Initial judgement value), the value it should take to achieve the maximum reduction in inconsistency (Optimal judgement value) and what percentage of relative variation such a change would imply (Optimal relative variation). The decision maker can enter the new value for that judgement (New judgement value) and will automatically be informed of the relative variation that such a change represents (Relative judgement variation). Once the decision-maker determines the new value for that judgement, they will go on to the next judgement (with the 'Next' button), although they can go back at any time (with the 'Previous' button) if they decide to modify any previous judgement again. When the desired level of inconsistency is achieved, the decision maker will finish the process ('Ok' button).

NUMERICAL EXAMPLE

In what follows, we present the application of the DSS to a numerical example included in Saaty (2000). It has been selected since it presents a somewhat high degree of inconsistency. The PCM ($n = 6$) is:

$$A = \begin{pmatrix} 1 & 4 & 3 & 1 & 3 & 4 \\ 1/4 & 1 & 7 & 3 & 1/5 & 1 \\ 1/3 & 1/7 & 1 & 1/5 & 1/5 & 1/6 \\ 1 & 1/3 & 5 & 1 & 1 & 1/3 \\ 1/3 & 5 & 5 & 1 & 1 & 3 \\ 1/4 & 1 & 6 & 3 & 1/3 & 1 \end{pmatrix}$$

Figure 6. Windows for interactive modification of judgements



The associated priority vector obtained by using the RGM is: $w = \left(0.316,\quad 0.139,\quad 0.036,\quad 0.125,\quad 0.236,\quad 0.148\right)$ and the corresponding value of the inconsistency measure is $GCI = 0.790$ that far exceeds the maximum inconsistency threshold ($\overline{GCI} = 0.37$) for a matrix of size 6.

The DSS calculates first the minimum permissibility that is necessary to be able to get a *GCI* below the inconsistency threshold. This value is 29.32%, a high amount as the initial inconsistency value is far from the inconsistency threshold. The application of the DSS to the previous example is presented below, using each of the three modes contemplated, the automatic, the semi-automatic and the interactive.

Automatic mode

Using the DSS in the automatic mode with no personal intervention of the decision maker consists of applying the procedure using the minimum permissibility value determined before (29.32%). 15 iterations are necessary to get an acceptable inconsistency ($GCI = 0.37 = \overline{GCI}$). The resulting final matrix and the corresponding priority vector are the following ones:

$$A_{\rho min} = \begin{pmatrix} 1 & 3.09 & 3.88 & 1.29 & 2.32 & 3.09 \\ 1/4 & 1 & 5.41 & 2.32 & 1/3.87 & 1/1.09 \\ 1/3 & 1/7 & 1 & 1/3.87 & 1/6.47 & 1/4.64 \\ 1 & 1/3 & 3.87 & 1 & 1/1.29 & 1/2.32 \\ 1/3 & 3.87 & 6.47 & 1.29 & 1 & 2.32 \\ 1/4 & 1.09 & 4.64 & 2.32 & 1/2.32 & 1 \end{pmatrix}$$

$$w_{\rho min} = \left(0.305 \quad 0.138 \quad 0.038 \quad 0.121 \quad 0.248 \quad 0.151\right)$$

The amount of the changes, in relative terms, between initial and final values for either the judgements of the matrix and the corresponding priorities are shown in Table 1. The priorities change in relative terms by a maximum of 5.3%.

If the procedure is applied automatically with no permissibility parameter but limiting the judgements to the interval [1/9, 9] only three iterations are necessary to get an acceptable inconsistency ($GCI = 0.275 < \overline{GCI}$), and the final results are:

Table 1. Relative changes (%) in judgements and priorities after 15 iterations for ρ = 29.32%

	A1	A2	A3	A4	A5	A6	Priorities
A1	0	22.67	29.32	29.32	22.67	22.67	3.6
A2	29.32	0	22.67	22.67	29.30	8.53	0.8
A3	22.65	29.25	0	29.3	22.65	29.27	5.3
A4	22.67	29.34	22.67	0	22.67	29.34	3.5
A5	29.34	22.67	29.32	29.32	0	22.67	5.0
A6	29.32	9.33	22.67	22.67	29.34	0	2.2

$$A_{\rho\_inf} = \begin{pmatrix} 1 & 4 & 9 & 5.28 & 3 & 4 \\ 1/4 & 1 & 7 & 3 & 1.01 & 1 \\ 1/9 & 1/7 & 1 & 1/5 & 1/5 & 1/6 \\ 1/5.28 & 1/3 & 5 & 1 & 1 & 1/3 \\ 1/3 & 1/1.01 & 5 & 1 & 1 & 3 \\ 1/4 & 1 & 6 & 3 & 1/3 & 1 \end{pmatrix}$$

$$w_{\rho\_inf} = \begin{pmatrix} 0.441 & 0.161 & 0.026 & 0.084 & 0.159 & 0.130 \end{pmatrix}$$

When calculating the relative changes in judgements and priorities (see Table 2), only 3 judgements have been modified to obtain the previous matrix, but the relative change in one of the judgements reaches 428.07%. In the case of the priorities, these variations range from 12% to 39.5%. From a practical point of view, it does not make much sense to allow such high variations, but the DSS can be applied to extract information for such an extreme case.

Semi-automatic mode

In the semi-automatic mode, the decision maker establishes its own permissibility at the beginning of the resolution procedure. This value must be higher than the minimum permissibility previously calculated by the DSS ($\rho_{min} = 29.32\%$), which the decision maker can consult. Let's assume that, at the sight of this minimum permissibility value, the decision maker established a permissibility threshold of $\rho = 35\%$, that is to say, they would admit variations up to a 35% in their judgements. For this situation we will see present the iterations that would be followed when applying the procedure to the case of $\rho = 35\%$.

At the first iteration, the DSS indicates that the judgement that will reduce faster the inconsistency is a_{52}. The optimal value for this judgement to reduce the inconsistency most would be 0.989, but in order not to exceed the permissibility established by the decision maker the new value, accepted by the decision maker, is 3.704. Then, the PCM after the first iteration is:

$$A^{(1)}_{=35} = \begin{pmatrix} 1 & 4 & 3 & 1 & 3 & 4 \\ 1/4 & 1 & 7 & 3 & 1/3.70 & 1 \\ 1/3 & 1/7 & 1 & 1/5 & 1/5 & 1/6 \\ 1 & 1/3 & 5 & 1 & 1 & 1/3 \\ 1/3 & 3.70 & 5 & 1 & 1 & 3 \\ 1/4 & 1 & 6 & 3 & 1/3 & 1 \end{pmatrix}$$

Table 2. Relative changes (%) in judgements and priorities after 3 iterations with ρ = infinite

	A1	A2	A3	A4	A5	A6	Priorities
A1	0	0	200	428.07	0	0	39.5
A2	0	0	0	0	406.15	0	15.4
A3	66.67	0	0	0	0	0	26.7
A4	81.06	0	0	0	0	0	33.3
A5	0	80.24	0	0	0	0	32.8
A6	0	0	0	0	0	0	12.0

The associated value of the GCI for this matrix is $GCI = 0.731$, which continues to be greater than the inconsistency threshold ($\overline{GCI}$). Table 3 summarises the iterations that were followed by the DSS until reaching a GCI value under the allowable inconsistency threshold:

It took 11 iterations (out of 15 possible, as the algorithm does not repeat judgements) to reach a GCI under the allowable inconsistency threshold ($GCI = 0.354 < \overline{GCI}$). The PCM that is obtained after all these iterations as well as its corresponding priority vector are:

$$A^{(11)}_{=35} = \begin{pmatrix} 1 & 2.96 & 4.05 & 1.35 & 2.22 & 2.96 \\ 1/2.96 & 1 & 5.19 & 2.22 & 1/3.70 & 1 \\ 1/4.05 & 1/5.19 & 1 & 1/5 & 1/5 & 1/6 \\ 1/1.35 & 1/2.22 & 5 & 1 & 1/1.35 & 1/2.22 \\ 1/2.22 & 3.70 & 5 & 1.35 & 1 & 2.22 \\ 1/2.96 & 1 & 6 & 2.22 & 1/2.22 & 1 \end{pmatrix}$$

$$w^{(11)}_{=35} = \begin{pmatrix} 0.303, & 0.140, & 0.036, & 0.126, & 0.238, & 0.157 \end{pmatrix}$$

The amount of the changes, in relative terms, for either the judgements of the matrix and the priorities are shown in the Table 4.

It can be noticed that the changes in the priorities are below 6%. This means that although it has been necessary to modify up to a 35% the values of most of the judgements of the matrix, the priorities do not need such big changes.

Aguarón et al. (2021) provide a simulation study to validate the algorithm followed in the DSS. Generating 100,000 matrices for different combinations of n (3 to 9) and initial values of $GCI(A)$ (up to the value GCI^*=0.75), proved that the algorithm was able to provide acceptable levels of

Table 3. Iterations of the algorithm with ρ = 35%

Iter #	GCI	(r,s)	a_{rs}	ρ limit	t^*_{rs}	a'_{rs}	GCI'
1	0.790	(5,2)	5	0.741	0.198	3.704	0.731
2	0.731	(1,3)	3	1.35	4.994	4.05	0.673
3	0.673	(1,4)	1	1.35	4.325	1.35	0.621
4	0.621	(2,4)	3	0.741	0.262	2.222	0.573
5	0.573	(6,4)	3	0.741	0.247	2.222	0.523
6	0.523	(1,5)	3	0.741	0.373	2.222	0.490
7	0.490	(4,5)	1	0.741	0.416	0.741	0.460
8	0.460	(2,3)	7	0.741	0.441	5.185	0.434
9	0.434	(5,6)	3	0.741	0.452	2.222	0.408
10	0.408	(1,6)	4	0.741	0.422	2.963	0.379
11	0.379	(1,2)	4	0.741	0.461	2.963	0.354

Table 4. Relative changes (%) in judgements and priorities after 11 iterations for ρ = 35%

	A1	A2	A3	A4	A5	A6	Priorities
A1	0	25.9	35	35	25.9	25.9	4.1
A2	35	0	25.9	25.9	35	0	0.8
A3	25.9	35	0	0	0	0	0.8
A4	25.9	35	0	0	25.9	35	0.8
A5	35	25.9	0	35	0	25.9	0.8
A6	35	0	0	25.9	35	0	6.0

inconsistency if the permissibility of the decision maker was large enough. For example, permissibility values of 25% and 41% would be needed to reach an acceptable inconsistency for an initial *GCI* of 0.60 and 0.75, respectively.

Interactive Mode

Finally, let us see how the DSS works when the decision maker prefers to participate more actively. At each iteration, the DSS informs the decision maker which is the judgement that is going to be modified and the value that would provide the maximum reduction of the inconsistency.

In the numerical example, at the first iteration the DSS selects the judgement (5,2) whose initial value was 5. The DSS informs that the maximum reduction of the inconsistency will be achieved giving this judgement the value 0.9879, with an associated relative variation of 406.14%. The decision maker decides to change this judgement to 4, which represents a relative change of 25%.

The following judgement (second iteration) considered is judgement (1,3). The initial value was 3 and the DSS informs that the value that would reduce the inconsistency most is 14.9833 (399.44% of relative variation). The decision maker decides to change it to 4, which corresponds to a relative change of 33.3%.

The interactive process will follow in a similar manner until a value of the *GCI* below the inconsistency threshold is achieved. Table 5 summarises the iterations that the decision maker may have followed, selecting at each iteration new values for the judgements selected. All the new values that the decision maker has provided in this case are integer, but the DSS would allow them to be continuous. It can be observed that the relative variations associated with the changes (ρ_{rs}) are different depending on the judgement (iteration) considered. Obviously, if in some iterations (for their corresponding judgements) the decision maker allows greater relative variations than the minimum permissibility initially determined (ρ_{min} = 29.32% in this case), in others they may be more restrictive and limit these relative variations more and vice versa.

The final matrix that is obtained after 10 iterations of the interactive mode (with an associated value of the $GCI = 0.360 < \overline{GCI}$) and the corresponding priority vector are:

$$A_{interactive}^{(10)} = \begin{pmatrix} 1 & 4 & 4 & 1.5 & 2 & 3 \\ 1/4 & 1 & 5.5 & 2 & 1/4 & 1 \\ 1/4 & 1/5.5 & 1 & 1/5 & 1/5 & 1/6 \\ 1/1.5 & 1/2 & 5 & 1 & 1/1.33 & 1/2 \\ 1/2 & 4 & 5 & 1.33 & 1 & 2.5 \\ 1/3 & 1 & 6 & 2 & 1/2.5 & 1 \end{pmatrix}$$

Table 5. Iterations of the algorithm using the interactive mode

Iter #	GCI	(r,s)	a_{rs}	a'_{rs}	ρ_{rs}	GCI'
1	0.790	(5,2)	5	4	25%	0.745
2	0.745	(1,3)	3	4	33.3%	0.689
3	0.689	(1,4)	1	1.5	50%	0.621
4	0.621	(2,4)	3	2	50%	0.560
5	0.560	(6,4)	3	2	50%	0.495
6	0.495	(1,5)	3	2	50%	0.453
7	0.453	(4,5)	1	0.75	33.3%	0.425
8	0.425	(2,3)	7	5.5	27.27%	0.401
9	0.401	(5,6)	3	3	33.3%	0.377
10	0.377	(1,6)	4	2.5	20%	0.360

$$w_{interactive}^{(10)} = \left(0.314, \quad 0.129, \quad 0.036, \quad 0.127, \quad 0.246, \quad 0.148\right)$$

The amount of the changes, in relative terms, for either the judgements of the matrix and the priorities are shown in the Table 6. In this application, the maximum relative change in a priority is 7.3% that corresponds to the priority of alterative 2.

CONCLUSION

AHP allows some level of inconsistency when the decision maker elicit their judgements. Consistency is a particularly important issue as it is a requirement for the validity of the derived priority vector. Saaty (2003) recommends that "*to improve the validity of the priority vector, we must transform a given reciprocal judgement matrix to a near consistent matrix.*"

It can be complex for the decision maker to identify how to modify judgements to achieve acceptable levels of inconsistency. It is therefore important to develop mechanisms to support the

Table 6. Relative changes (%) in judgements and priorities after 10 iterations in the interactive mode

	A1	A2	A3	A4	A5	A6	Priorities
A1	0	0	33.33	50	33.33	25	0.6
A2	0	0	21.43	33.33	25	0	7.3
A3	24.99	27.27	0	0	0	0	1.1
A4	33.33	50	0	0	25	50	1.4
A5	50.02	20	0	33.33	0	16.67	4.3
A6	33.33	0	0	33.33	20	0	0.5

decision maker in this task. To address this need, this paper has presented a DSS whose purpose is to reduce the inconsistency of a pairwise comparisons matrix to an acceptable level of inconsistency.

Based on the theoretical results and the algorithm provided in Aguarón et al. (2021), the DSS is designed for revising the judgements of a PCM when the Row Geometric Mean method is employed to derive the priorities and the *GCI* is the inconsistency measure. The procedure implemented selects at each iteration the judgement that improve the *GCI* faster and with greater intensity and it guarantees that both the judgements and the derived priority vector will be close to the initial values.

The DSS has also incorporated the calculation of the minimum permissibility necessary to get an allowable inconsistency. Using this value provides the decision maker with the information of where the judgements are going to move and can be used as a starting point to set their own permissibility. Then, the DSS allows different degrees of participation to the decision maker, depending on their interests. In the more interactive option, the decision-maker will have to confirm or modify the specific values suggested to them at each iteration of the algorithm.

Therefore, the DSS not only helps to reduce inconsistency. The exploitation of the model (performance when faced with changes in parameters – judgements, permissibility, ... –) provides relevant knowledge about the problem and the resolution process. In short, the DSS has the potential both to help the decision maker reduce their inconsistency and to increase their own knowledge of the problem.

Extensions to this paper include adapting the software to work with values within Saaty's fundamental scale and with incomplete pairwise comparison matrices.

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