Critical Current Distribution in Composite Superconductors

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***Abstract*—It has long been known that for the description of the E(I)-characteristics of composite superconductors a distribution function can be used instead of a power law relation. With this approach, the conductor is represented by a serial connection of short subsections where the electrical resistance of each subsection is given by a parallel connection of the flux flow resistance of the superconductor and the resistance of the normal conducting matrix. Furthermore the local critical currents are assumed to be normally distributed around a mean value μ with a standard deviation σ. If the local critical current in a subsection is exceeded a voltage is generated. The current distribution function is then given by the second derivative of the E (I)-characteristics divided by R/L with R being the overall resistance and L the measuring length. In general only the lower part of the distribution function is apparent. By soldering the conductor in a copper bar the whole distribution can be made visible.**

In this paper we will give examples of the suitability of the description with Gaussian distribution functions for the low temperature superconductors NbTi and Nb_3Sn as well as for a Bi2223 tape. A comparison will be made between measurements with and without additional copper.

***Index Terms*—Bi2223, critical current, distribution function, Nb_3Sn, NbTi, n-value.**

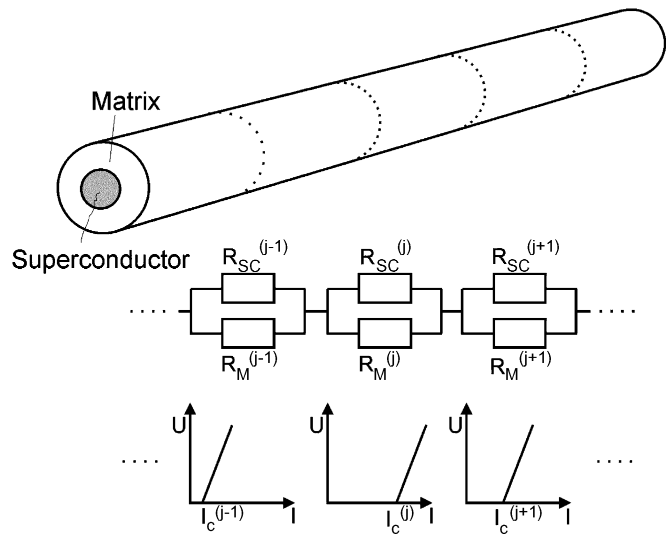

Fig. 1. Principle of distribution function model. The conductor is divided into subelements. Each element has beyond its individual i_c a resistivity which is given as a parallel connection of matrix and flux flow resistance. The i_cs themselves are assumed to be normally distributed.

I. Introduction

A THOROUGH understanding of the E(I) characteristic is a crucial point for magnet design. This is especially true in the case of persistent mode operation where the critical current values have to be extrapolated down to electrical fields several orders of magnitudes below the usual criterion for the critical current. As well as the phenomenological description with a power law other approaches have been tried. The use of a distribution function was first introduced by Baixeras and Fournet [1] and has been used by several authors [2]–[5].

II. Theoretical Model

The main assumption of the model is that the conductor can be longitudinally divided in short subelements. Each of these has its own critical current $i_c^{(j)}$ and for each one ideal flux flow behavior is assumed. So the voltage drop for $I > ic$ of the subelement j is given by

$$U^{(j)}(I,B,T) = R^{(j)}(B,T)\left(I - i_c^{(j)}(B,T)\right) \tag{1}$$

with $R^{(j)}$ the resistivity of the element and I the transport current through the wire. In the case of technical superconductors there is always a stabilizing matrix present and therefore $R^{(j)}$ is a parallel connection of the flux flow resistance of the superconductor and the resistance of the normal conducting matrix as is shown in Fig. 1.

In the model the individual subelements are connected in series and therefore the overall resistance R(B,T) is given as the sum of all the individual resistances $R^{(j)}$.

In addition it is assumed that the variation of the i_cs of the subelements can be described with a distribution function $\phi(i_c)$. Then the voltage over the whole length of the conductor is given by

$$U(I,B,T) = R(B,T)\int_0^I (I - i_c(B,T))\,\phi\,(i_c(B,T))\,di_c \tag{2}$$

or by dividing by the measuring length L to get the electrical field

$$E(I,B,T) = \frac{R(B,T)}{L}\int_0^I (I - i_c(B,T))\,\phi\,(i_c(B,T))\,di_c. \tag{3}$$

Double differentiation yields

$$\frac{d^2E(I,B,T)}{dI^2} = r(B,T)\phi(I) \tag{4}$$

with $r = R/L$.

The local i_cs should be affected by a superposition of many independent influences and therefore, according to the central limit theorem, the distribution function should be of Gaussian

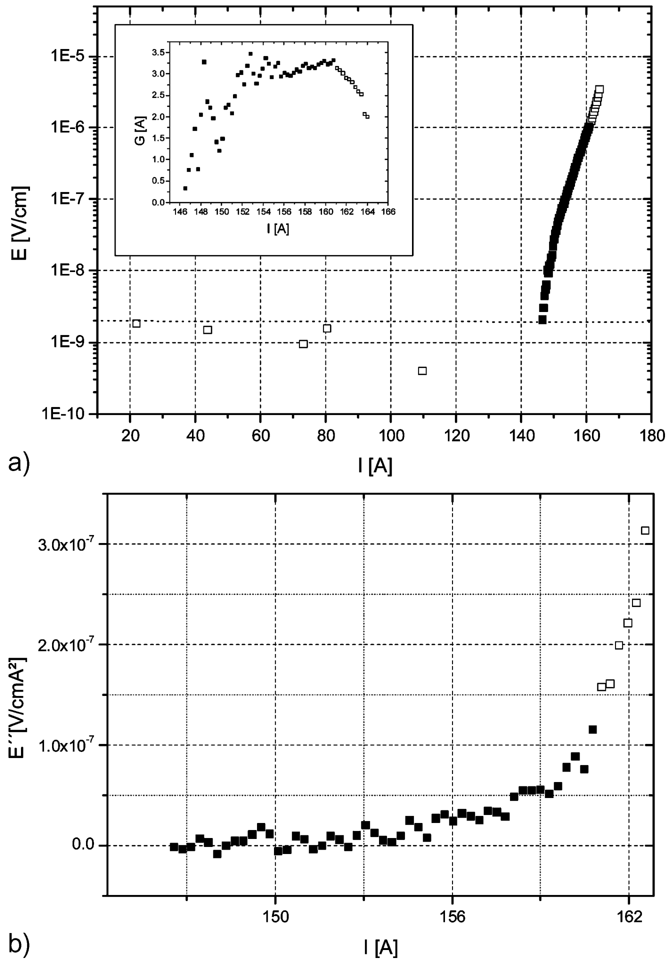

Fig. 2. (a) E(I)-characteristic of a bronze route $\mathrm{Nb_3Sn}$ conductor at an external magnetic field $\mathrm{B} = 15\ \mathrm{T}$. The lower limit of the usable part (black dots) is given by the resolution limit, the upper by deviations of G from a monotonically increasing behavior (see insert). (b) The corresponding second derivative $\mathrm{E}'' = \mathrm{d^2E/dI^2}$. No Gaussian distribution function can be observed.

shape with the mean value μ and the standard deviation σ, which leads to

$$\frac{\mathrm{d^2E(I)}}{\mathrm{dI^2}} = \frac{\mathrm{r}}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(\mathrm{I}-\mu)^2\right). \tag{5}$$

So instead of $\mathrm{I_c}$ and n of the power law description the parameter set $\{\mathrm{r}, \mu, \sigma\}$ describes the transition to the flux flow regime. In this description r represents the area under the curve.

For standard conductors only a part of the distribution function is accessible [5] as can be seen in Fig. 2 for a bronze route $\mathrm{Nb_3Sn}$ conductor. Due to the resolution of the voltmeters there is a lower voltage limit for the usable data points. For large voltages the E(I)-characteristic deviates from a distribution function behavior before all the elements are in the flux flow regime. This can be seen with help of the function G which is defined as

$$\mathrm{G} = \frac{1}{\frac{\mathrm{d}}{\mathrm{dI}}\ln(\mathrm{E(I)})} = \frac{\mathrm{E(I)}}{\frac{\mathrm{d}}{\mathrm{dI}}\mathrm{E(I)}}. \tag{6}$$

TABLE I
PROPERTIES OF THE CONDUCTORS

Material	Conductor	A [mm^2]	Co/Ag frac. [%]	No. of Fil.
NbTi	EAS-S1	0.19	60	1
NbTi +Cu	EAS-S1 soldered in Cu	10	99	1
(NbTaTi)3Sn	EAS NSTT 10000	0.64	17	10000
(NbTa)3Sn	EAS-HNST13000	1.24	17	13000
(NbTa)3Sn +Cu	EAS-HNST13000 soldered in Cu	10	90	13000
Bi2223	Sumitomo CT-OP	0.94	69	≈100

G has been introduced by Kimmich [5] and has to be monotonically increasing. Any deviations from such behavior are a sign that the description with a Gaussian distribution function is not valid any more. From the insert in Fig. 2(a) one can see that the slope of G gets negative in the vicinity of the quench point.

With only a small part of the distribution function accessible, it is nearly impossible to find a stable solution of the 3-parameter fit to (5). Kimmich developed a method where r is temporarily eliminated from the equations and a solution is found for μ and σ alone. Later r is introduced back again and fitted separately. This procedure results in a large scattering of r of several orders of magnitude for nearly identical measurements.

Two methods were used to get a reliable value of r and hence the full set of parameters $\{\mathrm{r}, \mu, \sigma\}$ of the distribution function.

The first is to reduce the resistance by enhancing the copper area by soldering the superconducting wire into a copper bar.

The second method is to determine r in a separate measurement and so reducing the number of parameters for the fit

III. Experimental and Results

All experiments were carried out in the JUMBO magnet system. In this facility measurements up to transverse magnetic fields of 15 T are possible in a He-bath of 4.2 K. The wires are wound into test coils on glass fiber mandrels with diameters ranging from 33 mm up to 90 mm. The E(I)-characteristic of the wire is determined by 4 point measurements where the current is increased in discrete steps.

Measurements on standard conductors were carried out for HNST 13000 and NSTT 10000 bronze route $\mathrm{Nb_3Sn}$ conductors, a S1 monofilament NbTi conductor, all manufactured by EAS, and a CT-OP Bi2223 conductor manufactured by Sumitomo. Further details of the conductors can be found in Table I.

A. Cu-Stabilized Conductors

Measurements with additional copper were performed only for the low temperature superconductors. The wires were soldered in a U-shaped copper bar of about 5 mm width and 2 mm

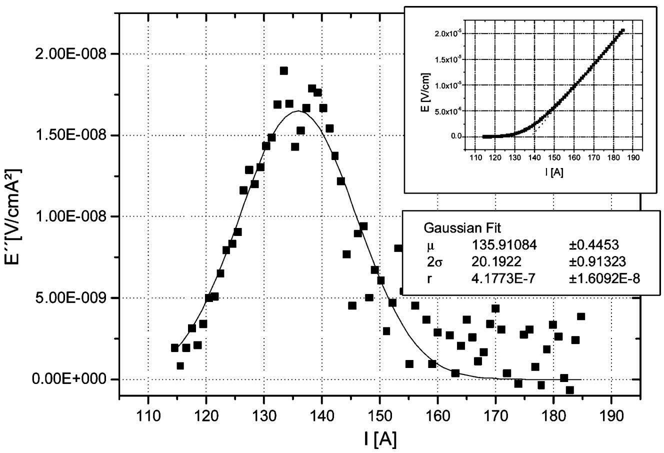

Fig. 3. $E''(I)$ at $B = 15$ T of a Nb_3Sn conductor soldered in a copper bar. The whole distribution function is accessible. As can be seen from the insert a homogenous flux flow state is reached when all individual subelements are in the flux flow regime.

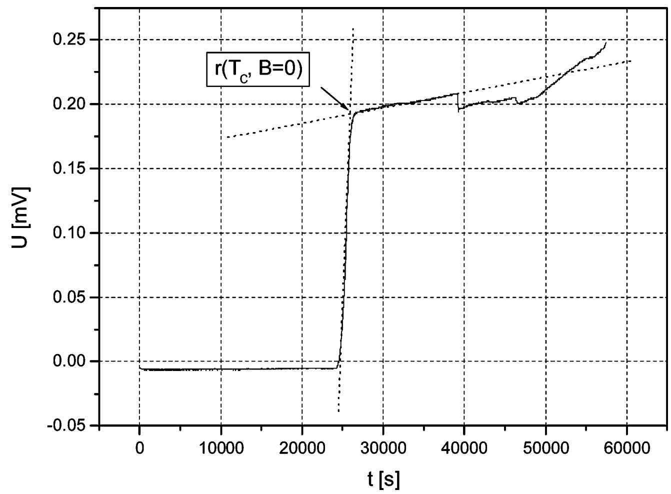

Fig. 4. Measurement of U(t) at a constant current I during heating up of the cryostat. The upper point of the sharp transition corresponds to the resistance of the sample at $T = T_c$ and $B = 0$.

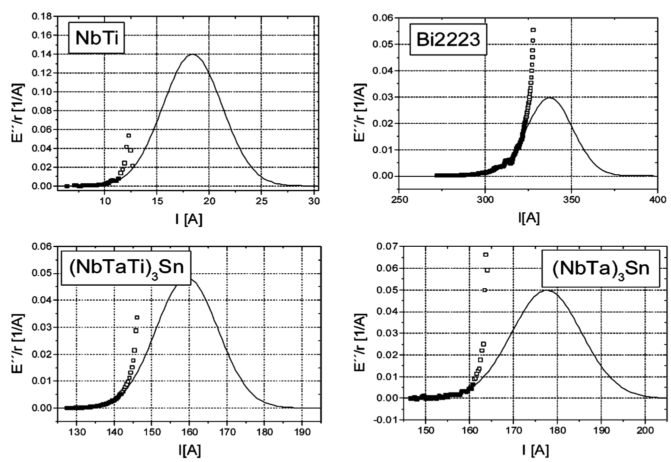

Fig. 5. Fittings of Gaussian distribution functions to $E''/r(I)$ of NbTi, two differently alloyed Nb_3Sn and a Bi2223 conductor. For the fit only the data points with increasing G have been considered.

height and E(I) was measured. No quench of the conductors occurred and the experiment had to be stopped manually.

The result for the Nb_3Sn wire at a magnetic field of 15 T is given in Fig. 3. The second derivative shows a normal distribution which now can be fitted easily. In addition, an overall flux flow behavior of E(I) is observable when all single elements are in the flux flow state. For the NbTi wire with additional copper the whole Gaussian distribution can be made visible as well.

B. Experimental Determination of r

For the determination of r, the samples were heated up by overnight evaporation of LHe from the magnet system and the voltage drop at three different measuring lengths was measured for a given current. From the upper edge of the sharp voltage per length increase at the phase transition from superconducting to normal state a value $r(T = T_c, B = 0)$ can be measured quite accurately, as can be seen in Fig. 4. These values have to be extrapolated to 4.2 K and to the background fields where the actual E(I) is measured.

Due to the low T_c of NbTi and Nb3Sn, the temperature dependent part of r is negligible for these conductors. The magnetic field dependence of the resistivity of copper can be described by

$$\rho(B) = \rho_0 + \rho_B B. \tag{7}$$

The copper resistance is about 2–3 orders of magnitude smaller than the flux flow resistance, therefore the overall resistance is nearly identical to the value for copper alone. Only the cross sectional area of the matrix has to be known to calculate $r(T = 4.2\ K, B = B_{ext.})$ for the stabilized wires and the NbTi conductor. For these measurements the accuracy is high and for the additionally stabilized wires the values are also in very good agreement to the r obtained by the three parameter fit on the Gaussian as in Fig. 3.

For Nb_3Sn, one has to take into account the presence of bronze, unreacted Nb and Ta from the diffusion barrier which contribute as additional terms to the matrix resistance. Therefore only upper and lower limits for the resistivity ρ_0 can be given, taking into account on one hand only the actual area of copper and assuming on the other hand the whole non-superconducting area being made of copper. The mean value of these two estimates has been taken for the calculation of $\rho(B)$ with (7). Since the critical temperature of Nb_3Sn is about 18.5 K, the error due to the temperature dependence of the resistivity is negligible.

For HTS the resistivity is measured at about 90 K. The data were extrapolated to 4.2 K using the Wiedemann- Franz law. Due to a lack of data concerning the magnetic field dependence of the Ag-AgMg matrix the values for $B = 0$ have been taken as an estimate for r.

With r known, the fitting of μ and σ to the data is straightforward for all types of conductor as can be seen in Fig. 5. To compare the different wires the normalized value E''/r is plotted and only data points where G is increasing were used for the fit. The values of r, μ and σ are listed in Table II, as well as values of two conductors embedded in a copper bar.

TABLE II
PARAMETERS OF MEASUREMENTS

Material	Conductor	B [T]	r [Ω/cm]	μ [A]	σ
NbTi	EAS-S1	10	6.6e-5	18	6
NbTi +Cu	EAS-S1 soldered in Cu	10	6.9e-7	14	6
(NbTaTi)3Sn	EAS NSTT 10000	15	4.3e-5	159	17
(NbTa)3Sn	EAS-HNST13000	15	1.7e-5	179	16
(NbTa)3Sn +Cu	EAS-HNST13000 soldered in Cu	15	4.4e-7	137	11
Bi2223	Sumitomo CT-OP	10	6.7e-6	337	27

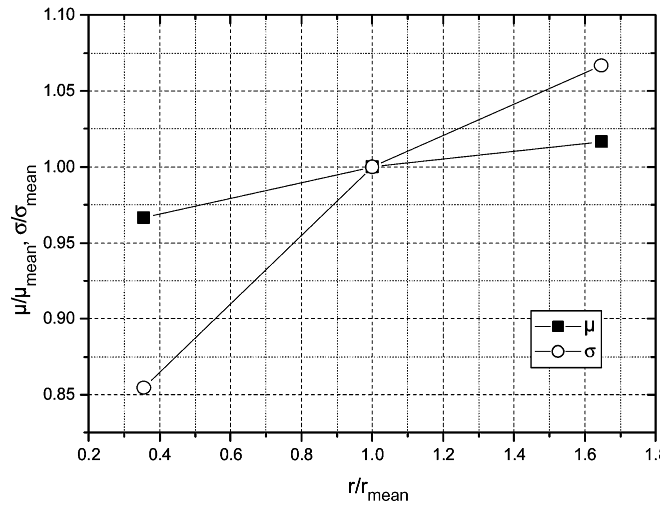

Fig. 6. Standardized deviation of μ and σ from the value at $\mathrm{r} = \mathrm{r}_{\mathrm{mean}}$, the mean value between maximum and minimum possible r, for the $\mathrm{Nb_3Sn}$ conductor HNST 13000.

IV. DISCUSSION

For the conductors which are embedded in additional copper the whole Gaussian distribution is visible and the determination of the three parameters r, μ and σ is straightforward. For partial distribution functions obtained with standard conductors, the quality of the fit of μ and σ depend on the accuracy of the experimental determination of r.

As mentioned earlier the determination of r for NbTi can be done quite accurately since only two materials are present and the respective areas are known.

For $\mathrm{Nb_3Sn}$ the r was calculated using a mean value for ρ_0. To check the influence of the error the fitting have been done with the maximum and minimum values for r as well. The results, standardized to the mean values, are shown in Fig. 6. As can be seen the error for μ is less than 5%, the deviations of σ are about 7% for the maximum r and about 15% for the minimum r.

For the Bi-tape the determination of $\mathrm{r}(\mathrm{B} = 0)$ with the Wiedemann-Franz law should be quite correct. For the magnetic field dependence no data could be found. This magnetic field dependence is maybe not negligible since for copper the values at $\mathrm{B} = 0\ \mathrm{T}$ and $\mathrm{B} = 15\ \mathrm{T}$ differ by more than half an order of magnitude. Up to now the results for Bi2223 should be taken as an estimate that shows that the introduced method works also for high temperature superconductors.

It is known from literature [2], [5] that Ic is correlated with μ are as well as the n-value with μ/σ. These correlations could be observed for all conductors investigated.

V. CONCLUSION

One way to describe the E(I) characteristic of technical superconductors is to assume flux flow behavior for individual subelements along the conductor and a normal distribution of the local $\mathrm{i_c}$s of these elements.

For standard conductors only a small part of the distribution function can be measured.

The use of the function G provides a way of observing deviations from the normal function distribution.

The whole distribution function is visible when the superconductor is embedded in additional copper, which reduces the resistance by an order of magnitude.

Stable solutions of the parameters μ and σ can be obtained when r is determined independently before the fit.

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