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Abstract: There exist capital constraints in many distribution channels. We examine a channel consisting of one manufacturer and one retailer, where the retailer is capital constrained. The retailer may fund its business by borrowing credit either from a competitive bank market or from the manufacturer, provided the latter is willing to lend. When only one credit type (either bank or trade credit) is viable, we show that trade credit financing generally charges a higher wholesale price and thus becomes less attractive than bank credit financing for the retailer. When both bank and trade credits are viable, the unique equilibrium is trade credit financing when production cost is relatively low and is bank credit financing otherwise. We also study the case where both the retailer and the manufacturer are capital constrained and demonstrate that, to improve the overall supply chain efficiency, the bank should finance the manufacturer if production cost is low and finance the retailer otherwise. Our analysis further suggests that the equilibrium region of trade credit financing shrinks as demand variability or the retailer's internal capital level increases.

Keywords: capital constraint; distribution channel; financing; trade credit

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1 Introduction

Trade credit refers to the credit extended by a seller to its buyer for the purchase of goods. The seller lending trade credit thus also acts as an investor besides its conventional role of production. According to Petersen and Rajan (1997), trade credit is the most important form of short-term financing for firms in the United States. Accounts receivable (trade credit) of listed firms in the G7 countries on average varied from 15 to 30 percent of assets during the early 1990s (Rajan and Zingales, 1995). Trade credit is also used widely in economies with less developed financial markets or weak bank-firm relationships (Biais and Gollier, 1997; Booth et al., 2001).

The existing literature on trade credit provides theories to explain why firms specialized in other activities such as production also lend credit, even in the presence of specialized financial intermediaries such as banks (see Chen and Cai, 2011; Gupta and Wang, 2009; Kouvelis and Zhao, 2008, 2011; Zhou and Groenevelt, 2007). However, the limited number of related papers seem to hold opposite views on whether trade credit or bank credit should be adopted by the capital-constrained buyers/retailer. For example, through numerical analysis Zhou and Groenevelt (2007) suggest that bank credit is preferable to trade credit. However, Kouvelis and Zhao (2008) pointed out that, "if offered an optimally structured scheme the retailer will always prefer supplier to bank financing." Although other factors, such as risk aversion, agency cost, and asymmetric information, might play a role in the coexistence of bank and trade credits, to the best of our knowledge, none of the extant models provides a unified theory to justify this economic phenomenon. In this paper, we aim to address the following research questions. First, what is the financing equilibrium when both trade and bank credits are viable for the capital-constrained buyer? Second, how does the product's demand variability or the retailer's internal capital level affect the financing equilibrium? Third, when the manufacturer is also capital constrained, how does the manufacturer's capital constraint affect equilibrium financing in the channel? We show that both trade and bank credits can emerge as the unique equilibrium financing scheme, and identify a threshold cost condition which separates the two credit types in equilibrium. This result essentially establishes the co-existence of the two credit types as cost conditions vary across firms and/or industries. This core result distinguishes our model from the extant literature.

We examine a distribution channel consisting of one manufacturer and one retailer, where the retailer is capital constrained. Our basic model is embedded in the classical single-period newsvendor framework. Demand for the product is stochastic and its prior distribution is common knowledge. The limited capital endowment of the retailer is normalized to zero without loss of generality. In addition to the product market, there is also a market of specialized financial intermediaries such as banks. Following Dammon and Senbet (1988) and Dotan and Ravid (1985), we assume the bank market to be competitive so that the interest rate on bank credit equates the expected return to its costs. Aside from production, the manufacturer may also choose to offer trade credit to fund the retailer's purchase. The retailer can always access the bank market. If the manufacturer offers trade credit, then the retailer can also opt to finance via trade credit.

At time zero, the manufacturer announces a per-unit wholesale cash price, applicable if the retailer finances its purchase with bank credit, and a (postponed) wholesale price, applicable at the end of the period if the retailer finances with trade credit. The retailer then decides between bank and trade credits and chooses a corresponding order quantity. If the retailer adopts bank credit, she borrows from one of the banks and makes a full payment for her purchase. If the retailer adopts trade credit, she makes a zero initial payment to the manufacturer. After her revenue realizes at the end of the period, the retailer repays the smaller of her revenue and the loan.

First, we consider the case where only one financing type, either bank credit financing (BCF) or trade credit financing (TCF), is viable. Analyzing each credit type in isolation helps us better understand its associated channel dynamics. Besides, studying TCF in the absence of BCF also has practical implications when the retailer is denied access to bank credit. Correspondingly, the wholesale price associated with the absent financing option is not offered to the retailer. We characterize the manufacturer's optimal wholesale price and the retailer's optimal order quantity under each credit type. When only one credit type is feasible, we show that the manufacturer generally charges a higher wholesale price under TCF than under BCF. Particularly, under TCF, the manufacturer fully extracts the retailer's profits. Therefore, TCF is less attractive than BCF for the retailer.

Next, when both credit types are available, we show that TCF (BCF) is the unique financing equilibrium when production cost is below (above) a certain threshold. The rationale is that a low production cost makes the manufacturer better able to counter the demand uncertainty risk associated with offering trade credit. When production cost is below this threshold, the manufacturer chooses the optimal postponed wholesale price when only TCF is viable but sets the wholesale cash price above that when only BCF is viable, to induce the retailer to choose trade credit. When production cost is above this threshold, however, the manufacturer prefers to have the bank market to bear the retailer's demand uncertainty risk. Thus, BCF becomes the unique financing equilibrium.

We further study the case where both the manufacturer and the retailer are capital-constrained, and find that the equilibrium is the same as only the retailer is capital-constrained. Because the lending bank makes zero profit in a competitive bank market, the capital-constrained manufacturer can raise needed capital from the bank and earn the same expected profits as if it is endowed with sufficient capital. Our analysis further reveals another threshold value of production cost such that the overall supply chain efficiency is higher under TCF than BCF when production cost is lower than this threshold; otherwise, the reverse is true. This suggests that, if the bank is a Pareto player that maximizes the overall supply chain efficiency, it will lend to the manufacturer when production cost is low and to the retailer otherwise.

We further examine how the financing equilibrium is affected by demand variability and the retailer's internal capital level. As demand variability increases, BCF occurs in equilibrium under wider conditions. Overall, for the manufacturer, TCF is more profitable than BCF when production cost and demand variability are low and is less profitable otherwise. BCF generally, although not monotonically, becomes more profitable for the manufacturer as the retailer's internal capital level increases.

Our theoretical contribution is to characterize the financing equilibrium between BCF and TCF in terms of production cost, and to study the impact of demand variability and the retailer's internal capital level on this equilibrium. Our analysis also provides guidance for financing managers at manufacturing firms. Our results indicate that the manufacturer should promote TCF when its production cost and demand variability are relatively low or the retailer's internal capital is relatively low. Otherwise, it should encourage the retailer to use bank credit.

The remainder of this paper is organized as follows. We review the existing literature in Section 2 and present the model in Section 3. Section 4 derives the optimal solutions when only one financing type, either BCF or TCF, is viable. We perform a detailed equilibrium analysis when both BCF and TCF are available in Section 5. Section 6 studies the impact of demand variability and the retailer's internal capital level on the financing equilibrium. Section 7 concludes. All proofs are given in the appendix (Online Supplements).

2 Literature Review

This paper is closely related to the extant literatures on bank and trade credits in supply chain management, and on distribution channels in the marketing-operations interface.

Recent research on supply chain management has examined a capital-constrained firm's production, inventory, capacity, and debt decisions, and demonstrates that it is important to incorporate the firm's financial concerns into its operational decisions (see Babich and Sobel, 2004; Ding et al., 2007). As one of the first to discuss capital constraints in a newsvendor model (see Lariviere and Porteus, 2001), Xu and Birge (2004) illustrate how a firm's inventory decisions are affected by its capital constraints and capital structure (i.e., debt/equity). Also in a single-period newsvendor model, Buzacott and Zhang (2004) analyze the decisions of a bank and retailers with different capital levels. They show that asset-based financing allows the retailers to enhance their return compared with using only their own capital. In a similar setting, Dada and Hu (2008) consider a supply chain with a capital-constrained retailer, and propose a mechanism to partially coordinate the supply chain under bank credit financing. Kouvelis and Zhao (2009) study the impact of different types of bankruptcy costs on the optimal order quantity of the retailer. In a multi-period inventory model, Chao et al. (2008) show that it is essential for retailers to incorporate financial considerations into their operational decisions, especially for those with capital shortage.

In Boyabatli and Toktay (2006), the firm's limited capital can be increased by borrowing from the external market (a commercial loan collateralized on physical assets), and its distribution can be altered with financial risk management (using forward contract to reduce the financial risk of tradable assets). Caldentey and Haugh (2009) compare the performance of supply chains with capital-constrained retailers with and without hedging. In their model, the retailer can dynamically trade in the financial market to adjust her capital and to make it contingent on the evolution of the index and choose the timing of executing the contract with the supplier. They clearly show how the financial market can be used as a means of financial hedging to mitigate the capital constraint. However, the above models consider only a single credit/loan type. In contrast, our current paper analyze both bank and trade credits in an integrated framework and obtain a unique financing equilibrium. We also demonstrate how this equilibrium is mediated by several factors such as production cost, demand variability, and the retailer's internal capital level.

Although few have explicitly compared bank and trade credits in a newsvendor setting as we do, there are exceptions. In a discrete time model with random demand, Gupta and Wang (2009) show that the structure of the retailer's optimal policy is not affected by credit terms, although the optimal value of the policy parameter is. In each period, the firm compares the costs of bank versus trade credit and chooses the one with a lower cost. They also model the supplier's problem and calculate the optimal credit parameters through numerical analysis.

Two other working papers, Zhou and Groenevelt (2007) and Kouvelis and Zhao (2008), deserve our special attention.[1] Zhou and Groenevelt (2007) compare bank and trade credit from the perspective of asset based financing. In their model, the bank is a monopoly and maximizes its profits, and the manufacturer pay back the interest to the bank after demand is realized. Therefore, their bank credit is also called joint supplier financing (in the jargon of Kouvelis and Zhao (2008)). In their trade credit (open account financing), the retailer pays up-front a certain percentage of the total trading amount. Under both credit types, the retailer is allowed to purchase only a portion of the desired order due to credit line limit. Zhou and Groenevelt then mostly use numerical examples to show that with a fairly priced bank loan the manufacturer weakly prefers trade credit and that bank credit is preferable for the overall supply chain and the retailer.

Different from Zhou and Groenevelt (2007), our model allow the retailer to borrow a loan as desired in a competitive bank market, which is in line with Brennan et al. (1988); Dammon and Senbet (1988); Dotan and Ravid (1985); Kouvelis and Zhao (2008); Xu and Birge (2004), thus, the bank credit interest will be fairly priced. We then explicitly compare bank and trade credits in terms of the manufacturer's production cost and demonstrate that in financing equilibrium, bank credit actually can outperform trade credit for the manufacturer, as well as the entire supply chain, when the production cost is relatively high. We further characterize the financing equilibrium from a variety of perspectives including the situation when the manufacturer is also capital constrained, the impact of demand variability, and the impact of internal capital. Our results thus analytically explain why bank and trade credits coexist in reality even if the bank loan is fairly priced, which is consistent with the fact that "70 − 80% of trade between firms is still conducted in open account (trade credit)" according

[1] The first draft of our work was available in August, 2007 and has been developed independently without knowledge of Zhou and Groenevelt (2007) and Kouvelis and Zhao (2008).

to Zhou and Groenevelt (2007).

Kouvelis and Zhao (2008) is the second notable working paper comparing both bank and trade credits. In their "supplier earlier payment discount" or "(direct) supplier financing," similar to ours, the bank market is also assumed to be perfect and both the manufacturer and the retailer might face bankruptcy, although we also consider the case where the manufacturer has sufficient capital. Differently, in their model, the retailer can choose to pay a discount wholesale price up-front or a full wholesale price after demand is realized. The supplier interest rate is then assumed to be determined by the full wholesale price divided by the discount wholesale price minus one. Technically, this setting does not allow the manufacturer to force out the bank credit, because in doing so, the interest rate for trade credit is automatically reduced to zero. In this sense, the trade credit is equivalent to the bank credit. Kouvelis and Zhao (2008) then suggest the manufacturer "should always provide financing to the retailer at rates less than or equal to the risk free rate, and if offered an optimally structured scheme the retailer will always prefer supplier (trade credit) to bank financing." This result is obviously different from the main result in Zhou and Groenevelt (2007) who indicate otherwise. Different from Kouvelis and Zhao (2008), we assume the manufacturer can actually price out the bank credit in a newsvendor setting if the trade credit is more profitable while the manufacturer can actually charge a non-zero interest rate, given that the manufacturer is the Stackelberg leader in a single channel, a common assumption in most of related papers on newsvendor (see Lariviere and Porteus, 2001). In other words, our optimal interest rate for trade credit can be greater than the risk free rate, which is supported by empirical data (see Burkart and Ellingsen, 2004; Guariglia and Mateut, 2006; Petersen and Rajan, 1997). Therefore, our paper complements both Zhou and Groenevelt (2007) and Kouvelis and Zhao (2008) by pointing out that actually, either bank credit or trade credit could be the equilibrium choice for the manufacturer, where the equilibrium domain hinges on production cost, demand uncertainty, and internal capital level. To the best of our knowledge, our paper is the first to analytically explain in a newsvendor setting why both credit types can coexist in reality at what conditions in terms of production cost. In addition, we also demonstrate when both the manufacturer and the retailer are capital constrained, to improve the overall supply chain efficiency, it is Pareto efficient for the bank to lend loans to the manufacturer when production cost is relatively low or to the retailer otherwise. Furthermore, we also demonstrate the impact of other factors, including demand variability, and internal capital level on the financing equilibrium.

3 Model

We consider a single-period product market, where manufacturing is controlled by a monopolist firm. An entrepreneur with no wealth endowment acts as the retailer. The distribution channel thus consists of the manufacturer and the entrepreneurial retailer. Demand in the retail market, D, is random and not realized until the end of the period. The *ex ante* demand distribution function is $F(D)$ and has the following properties: (1) $F(D)$ is absolutely continuous with density $f(D) > 0$ on $(0, \infty)$, (2) $F(D)$ has a finite mean, and (3) its hazard rate $h(D) \equiv f(D)/\bar{F}(D)$ is increasing in D, where $\bar{F}(D) \equiv 1 - F(D)$. Let $H(D) \equiv Dh(D)$ denote the generalized failure rate. Then $H(D)$ is monotonically increasing in D.

The retailer must decide the number of units to order at time zero. We assume that consumers hold the same reservation price for the product. Therefore, we shall take retail price as fixed and common knowledge. Without further loss of generality, we normalize the retail price to 1. Both the manufacturer and the retailer have zero fixed costs. The manufacturer has a constant marginal production cost c, with $0 \leq c \leq 1$. The retailer incurs no other variable costs besides the wholesale price w.[2] The product we consider is perishable and has zero salvage value by the end of the period. This means that the retailer can not use unsold inventory as collateral on her loan. To simplify exposition, we also ignore any goodwill loss to either channel member due to stockout.

[2]Assuming another constant component of marginal cost in addition to w for the retailer does not alter the analysis.

We assume the manufacturer is endowed with sufficient capital to cover its manufacturing expenditures.[3] However, the retailer has no capital endowment and must rely on some additional sources to finance her operations. Specifically, the retailer can borrow from either an external bank market or the manufacturer, if it is to the latter's benefit to extend trade credit. Following the convention in the bank credit literature, we assume the bank market is competitive and that risk-neutral banks have access to unlimited funds at the risk-free interest rate r_f, which is normalized to zero without loss of generality (see Brennan et al., 1988; Dammon and Senbet, 1988; Dotan and Ravid, 1985; Kouvelis and Zhao, 2008; Xu and Birge, 2004). A zero risk-free interest rate also confers the advantage of allowing us to ignore discounting.[4] Both the retailer and the manufacturer are risk neutral and maximize their expected profits.

If at time zero the retailer borrows x dollars of (bank or trade) credit at interest rate r, then she has to repay $(1 + r)x$ dollars to the creditor at the end of the period, provided that her realized revenue exceeds $(1 + r)x$. Otherwise, she will repay her entire realized revenue and default on the remaining portion of the loan. Therefore, the retailer has limited liability. The limited liability of the retailer is a common treatment in the trade credit literature (e.g., Brennan et al., 1988; Burkart and Ellingsen, 2004; Kouvelis and Zhao, 2008).

We assume that lending is exclusive. That is, the retailer may neither borrow from multiple banks simultaneously, nor borrow from the manufacturer and a bank simultaneously. In the above setting, it seems natural to assume that the manufacturer is the Stackelberg leader. The sequence of events is as follows. First, at time zero the manufacturer simultaneously announces two wholesale prices for the retailer to choose: (1) a wholesale cash price w_B, applicable at time zero if the retailer borrows bank credit and pays in cash for her entire inventory purchase; and (2) a postponed wholesale price w_T, applicable at the end of the period if the retailer borrows trade credit. Equivalently, the manufacturer may announce a

[3] We relax this assumption and consider an extended model with capital costs in Section 5.2.

[4] If a player's expected payoff at the end of the period is y, then its present value is simply $\frac{y}{1+r_f}$.

wholesale cash price w and an associated interest rate r_T under TCF. It will become clear later that in the case of trade credit the only contract variable that matters is the "postponed" wholesale price $w_T \equiv w(1 + r_T)$. Thus the manufacturer's decision variables reduce to a wholesale cash price w_B and a postponed wholesale price w_T. In the event that the manufacturer does not find it optimal to extend trade credit (bank credit), he simply sets w_T (w_B) so high that the retailer prefers bank credit (trade credit). Next, observing w_B and w_T, the retailer chooses between bank and trade credit and announces a corresponding order quantity. Lastly, if the retailer adopts bank credit, the competitive banks simultaneously announce interest rate r_B. The retailer then borrows from one of the banks and makes a full payment to the manufacturer for her purchase. If the retailer decides to use trade credit, she makes zero initial payment to the manufacturer. At the end of the period, retail revenue realizes and any repayment is made accordingly. Our model thus examines bank versus trade credit financing under the wholesale price contract.

Finally, we make the following tie-breaking rules. If the retailer is indifferent to entering and not entering this market, we assume she enters. In this sense, the retailer is a Pareto or good-willed player who chooses to order even if she makes no profit in the game, since it is good for the manufacturer. In addition, when both BCF and TCF are available and the retailer is indifferent between the two, we assume that she adopts BCF without loss of generality.

4 Financing with Bank or Trade Credit

In this section, we analyze the scenario where only one financing type, either bank or trade credit financing, is viable. In section 5, we explicitly study the interactions between BCF and TCF when they both are viable and derive the subgame perfect equilibrium.

4.1 Bank Credit Financing (BCF)

Suppose trade credit is not available and the retailer adopts BCF. At time zero, the manufacturer first chooses wholesale cash price w_B and the retailer then chooses order quantity Q_B. Observing w_B and Q_B, the banks simultaneously announce interest rate r_B. The retailer borrows $w_B Q_B$ dollars from a bank and pays this amount to the manufacturer for Q_B units of product.

We proceed backwards to derive the equilibrium in the bank market and the channel. Because the bank market is competitive, a bank makes zero expected profits by lending to the retailer. For an order quantity Q_B (or equivalently, loan size $w_B Q_B$) chosen by the retailer, the prevailing interest rate r_B^* equates the expected return from the loan to its costs. At the end of the period the retailer yields revenue $\min\{D, Q_B\}$ and her expected repayment to the lending bank is thus $E\min\{w_B Q_B(1 + r_B^*), \min\{D, Q_B\}\}$. The lending bank's costs of extending this loan are $w_B Q_B$. Its zero-profit condition is thus

$$w_B Q_B = E \min\{w_B Q_B(1 + r_B^*), \min\{D, Q_B\}\}. \tag{1}$$

We then examine the quantity decision of the retailer. Her problem is

$$\underset{Q_B \geq 0}{Max} \quad E(\min\{D, Q_B\} - w_B Q_B(1 + r_B^*))^+, \tag{2}$$

subject to the bank's zero-profit condition in Eq. (1). Here the function x^+ equals x when $x > 0$ and equals 0 otherwise. When selecting Q_B, the retailer anticipates the corresponding interest rate r_B^* as given by the zero-profit condition of the banks. At the end of the period, the retailer collects revenue $\min\{D, Q_B\}$, which is used to repay her debt $w_B Q_B(1 + r_B^*)$ (including the principal and interest of the loan).

Lemma 1 *Suppose that $(1+r_B)w_B < 1$. The retailer's problem under BCF in Eqs. (1) and (2) is equivalent to the standard newsvendor problem without capital constraint:*

$$\underset{Q_B \geq 0}{Max} \quad E\min\{D, Q_B\} - w_B Q_B. \tag{3}$$

Lemma 1 delivers the same message as Proposition 2.1 of Xu and Birge (2004) although through a different model setting. Lemma 1 shows that under BCF the capital-constrained channel is equivalent to the usual channel without capital constraint. Lemma 1 overturns the intuition that due to the retailer's limited liability, she will order more than she would if she were endowed with sufficient capital. An insight is thus that under BCF limited liability does not create any real benefit for the retailer. Instead, the retailer behaves as judiciously as when purchasing with her own funds if she had enough capital endowment.

Recall, the unit retail price is normalized to 1. Since r_B^* measures the financing cost of the retailer and her sales are not realized until the end of the period, conceptually $1/(1+r_B^*)$ is the discounted retail price. That $(1+r_B^*)w_B < 1$ ensures that the discounted retail price exceeds the wholesale cash price, making decentralized distribution viable. In the subgame perfect equilibrium r_B^* and w_B^* indeed satisfy this condition.

For a given wholesale cash price w_B, the retailer's optimal order quantity $Q_B^*(w_B)$ is uniquely given by $w_B = \bar{F}(Q_B^*)$ (from Lemma 1). The manufacturer's profit function is thus $(w_B - c)Q_B^*$ or, equivalently,

$$[\bar{F}(Q_B^*) - c]Q_B^*. \tag{4}$$

His problem of choosing w_B is thus equivalent to choosing Q_B^*. In the subgame perfect equilibrium, the retailer's order quantity is thus $Q_B^* = Q_N$, where Q_N is uniquely given by $\bar{F}(Q_N) - Q_N f(Q_N) = c$, the manufacturer's wholesale price is $w_B^* = \bar{F}(Q_N)$, and the interest rate on bank credit is given by Eq. (1). At equilibrium, the retailer's (and the channel's) expected marginal revenue is $\bar{F}(Q_N)$, and the manufacturer's expected marginal revenue is $\bar{F}(Q_N) - Q_N f(Q_N)$. The gap between the channel members' marginal revenue, $Q_N f(Q_N)$, reflects the efficiency loss due to double marginalization. Proposition 1 summarizes the market equilibrium under BCF.

Proposition 1 *Under BCF, (1) the capital-constrained retailer's order quantity is* $Q_B^* = Q_N$*, where* Q_N *is uniquely given by* $\bar{F}(Q_N) - Q_N f(Q_N) = c$*; (2) the manufacturer's wholesale* *price is* $w_B^* = \bar{F}(Q_N)$*; (3) the banks' interest rate* r_B^* *is the unique solution to* $w_B^* Q_N = E[\min\{w_B^* Q_N(1+r_B^*), \min\{D, Q_N\}\}]$*; and (4)* $(1+r_B^*)w_B^* < 1$.

The rationale behind $Q_B^* = Q_N$ and the first two parts of Proposition 1 follow directly from Lemma 1 and can be inferred from Lariviere and Porteus (2001). The lending bank earns zero expected profit because of the perfect competition in the bank credit market. The bank's break-even rate r_B^* exceeding the risk-free rate ($r_f = 0$) compensates the chance that realized revenue is low so that the retailer defaults. Because the lending bank makes zero profit, the retailer's cost of using bank credit is identical to that of using her own capital (should she have enough capital). The outcome is thus as if the retailer has sufficient capital endowment and finances with her own capital. That $Q_B^* = Q_N$ reflects that the optimal order quantity equates her marginal revenue ($\bar{F}(Q_N)$) to her marginal cost (w_B^*). From the manufacturer's point of view, the capital-constrained retailer together with a competitive bank market amounts to a retailer with sufficient capital. Because here the retailer makes a cash payment (with borrowed bank credit) for her entire order, the manufacturer's incentives are unaffected and he sets the same wholesale price (w_B^*) as when selling to a self-sufficient retailer. Since the bank makes zero profits, by maximizing her own expected profits the retailer essentially maximizes the joint profits of herself and the lending bank. This is precisely why the strategic interactions between the manufacturer and the capital-constrained retailer remain identical to those in a channel without capital constraints. Finally, note that *distribution and financing are decentralized under BCF*. The manufacturer finances production and the bank market finances retail distribution.

4.2 Trade Credit Financing (TCF)

Now, suppose that the bank market does not exist and the retailer has to adopt TCF. Let w_T denote the postponed wholesale price at the end of the period. At time zero, the retailer orders Q_T units without payment. At the end of the period, the manufacturer receives from the retailer a repayment of $w_T Q_T$ if retail revenue $\min\{D, Q_T\}$ exceeds $w_T Q_T$ and a repayment of $\min\{D, Q_T\}$ otherwise. We proceed backwards and first compute the retailer's optimal order quantity, $Q_T^*(w_T)$, for a given w_T.

The retailer's order quantity decision

For a given w_T, the retailer's problem under TCF is

$$\underset{Q_T \geq 0}{Max}\ E(\min\{D, Q_T\} - w_T Q_T)^+. \tag{5}$$

The following proposition characterizes her optimal order quantity and its basic properties.

Proposition 2 *Suppose $w_T \in [c, 1]$. Under TCF: (1) the optimal order quantity of the retailer, Q_T^*, is uniquely given by $\bar{F}(Q_T^*) = w_T \bar{F}(w_T Q_T^*)$; (2) Q_T^* is decreasing in w_T; and (3) $Q_T^* \geq Q_N$.*

Decentralized distribution requires $w_T \leq 1$. That $w_T \geq c$ ensures the marginal expected revenue of the manufacturer to exceed or match its marginal cost. As the retail price is normalized to 1, $\bar{F}(Q_T^*) = 1 \times \Pr\{D > Q_T^*\}$ represents the retailer's (expected) marginal revenue from ordering an additional unit. We have

$$\Pr\{\min\{D, Q_T^*\} > w_T Q_T^*\} = \Pr\{D > w_T Q_T^*\} = \bar{F}(w_T Q_T^*), \tag{6}$$

which is the probability that retail revenue will exceed $w_T Q_T^*$, the amount owed to the manufacturer at the end of the period. The retailer's (expected) marginal cost of ordering an additional unit is thus $w_T \bar{F}(w_T Q_T^*)$. At the optimal stocking level her marginal revenue equals her marginal cost. That Q_T^* decreases with w_T is intuitive. As the postponed wholesale price increases, using trade credit becomes more costly and the retailer lowers her order level.

The retailer's marginal costs under TCF ($w_T \bar{F}(w_T Q_T^*)$) are below those when she is not capital constrained or those under BCF. Under BCF, the retailer's marginal cost remains constant at the wholesale price. Therefore, by lowering the retailer's marginal cost TCF induces her to raise her stocking level.

The manufacturer's choice of w_T

Under TCF, at time zero the manufacturer incurs production costs of $cQ_T^*(w_T)$. It receives a repayment of $E[\min\{w_T Q_T^*(w_T), \min\{D, Q_T^*(w_T)\}\}]$ at the end of the period. The manufacturer's problem is to maximize his total expected profits

$$\underset{c \leq w_T \leq 1}{Max}\, \pi_T^M(w_T) = E \min\{w_T Q_T^*, \min\{D, Q_T^*\}\} - cQ_T^*, \tag{7}$$

subject to $\bar{F}(Q_T^*) = w_T \bar{F}\left(w_T Q_T^*\right)$, as he anticipates the retailer's optimal quantity response. Let $\eta(w_T) \equiv \frac{\bar{F}(Q_T^*)[1-H(Q_T^*)]}{c[1-H(w_T Q_T^*)]}$. We then have

$$\begin{aligned} \frac{d\pi_T^M}{dw_T} &= \frac{\partial \pi_T^M}{\partial Q_T^*}\frac{dQ_T^*}{dw_T} + \frac{\partial \pi_T^M}{\partial w_T} \\ &= \frac{c[1-H(w_T Q_T^*)]}{w_T[h(Q_T^*) - w_T h(w_T Q_T^*)]}\left[1-\eta(w_T)\right], \end{aligned} \tag{8}$$

where $h(D)$ is the hazard rate of D and $H(D)$ is the generalized failure rate. The following proposition gives the optimal solution to TCF.

Proposition 3 *Given that only TCF is viable, the manufacturer's optimal postponed wholesale price is $w_T^* = 1$. Correspondingly, the optimal order quantity for the retailer is implied by $H(Q_T^*(w_T^*)) = 1$.*

As we show in the proof of Proposition 3, the manufacturer's profit increases with the postponed wholesale price (i.e., $\frac{d\pi_T^M}{dw_T} > 0$ when $w_T \in [c, 1]$). Thus, the manufacturer has incentives to increase the postponed wholesale price to its uppder bound as long as the retailer would stay in the game. Given that the retailer is a Pareto or good-willed player as assumed, we have $w_T^* = 1$.[5] In other words, if only TCF is viable, the manufacturer will set the postponed wholesale price at the retail price and take away all profits from the retailer. This is because the manufacturer shoulders both production cost and all financing risks whenever the retailer cannot pay back the credit. In contrast to BCF, the manufacturer's

[5] If the retailer is not a Pareto or good-willed player, to have a unique $Q_T^*(w_T^*)$, we let $w_T^* = 1 - \epsilon$, where $\epsilon > 0$ is infinitesimal, such that the retailer yields a positive profit. When ϵ approaches zero, w_T^* converges towards our above solution and $Q_T^*(w_T^*) = \lim_{w_T^* \to 1} Q_T^*(w_T^*) = Q_T^*(1)$.

payoff now consists of two components: profits from product sales and return on trade credit extension. Under TCF, the banks are dormant and the manufacturer plays a dual role of vendor and financier. The retailer's costs of the purchase and financing are summarized by the postponed wholesale price w_T.

Recall, the manufacturer's marginal revenue under BCF (or when the retailer is not capital constrained) is $\bar{F}(Q_N)(1 - H(Q_N))$ according to Proposition 1. In contrast, under TCF the manufacturer's marginal revenue is $\bar{F}(Q_T^*)$, where Q_T^* solves $1 - H(Q_T^*) = 0$.

5 Financing Equilibrium between BCF and TCF

So far we have separately derived the equilibrium in the channel under bank and trade financing. However, if both BCF and TCF are viable, one naturally wonders whether TCF would always outperform BCF for the manufacturer such that TCF is the only financing equilibrium in our model. Without loss of generality, we assume the retailer chooses BCF over TCF, if she is indifferent between the two.[6] It is worth pointing out that, although the manufacturer can charge a higher wholesale price in TCF than BCF, he also inadvertently embraces the risk of potential bankruptcy of the retailer if demand is significantly lower than the order quantity. Therefore, when both BCF and TCF are viable, the manufacturer must strategically design the wholesale contract menu to maximize his profit. If TCF is more profitable for the manufacturer, he can simply raise the wholesale price under BCF to a sufficiently high level such that BCF is not attractive to the retailer. That is, the wholesale price under BCF can be higher than that described in Proposition 2. On the other hand, if BCF is more profitable than TCF for the manufacturer, he will set the wholesale price at the level stated in Proposition 1, so that both channel members benefit from BCF. Below, we will identify the conditions for BCF and TCF to obtain as equilibrium.

[6]The manufacturer can always slightly lower the postponed wholesale price or increase the wholesale price under BCF to attract the retailer to TCF, if TCF is more profitable than BCF for the manufacturer.

5.1 The Financing Equilibrium

Under both bank and trade credit financing, the marginal cost of the channel is c. Proposition 2 shows $Q_T^* \geq Q_N$. Thus, $\bar{F}(Q_T^*) \leq \bar{F}(Q_N)$ (i.e., the equilibrium marginal revenue of the channel under TCF is lower than that under BCF) and TCF yields higher total channel profits than BCF. However, as argued previously, under TCF the manufacturer also bears the risk of demand uncertainty by not collecting payment until demand is realized. This financing risk is monotonically increasing in production cost, because the higher production cost is, the more the manufacturer loses (i.e., the manufacturer would lose $cQ_T^* - \min\{D, Q_T^*\}$) if the retailer defaults. When the extra financing risk outweighs the additional revenue, the manufacturer will set the wholesale price under BCF at its optimal level, so that the retailer chooses BCF over TCF; otherwise, TCF is the equilibrium. We characterize this critical threshold value in our following main proposition.

Proposition 4 *There exists a unique* $0 < \hat{c} < 1$*, such that the unique subgame perfect financing equilibrium is TCF when* $0 \leq c < \hat{c}$*, and is BCF when* $\hat{c} \leq c \leq 1$*.*

Proposition 4 describes the strategic interplay between the manufacturer and the retailer, when both BCF and TCF are viable. As previously discussed, the manufacturer's motive to issue trade credit depends on the trade-off between the additional revenue from charging a higher wholesale price and the potential loss if the retailer defaults. This trade-off critically hinges on the manufacturer's production cost. A low production cost ($c < \hat{c}$) sufficiently reduces the manufacturer's loss caused by the retailer's possible default and makes it worthwhile to issue trade credit. Therefore, the manufacturer will set the wholesale price under TCF at its optimal level w_T^* but overstate the wholesale price under BCF to prevent the retailer from choosing BCF. TCF is thus the unique subgame perfect financing equilibrium when $0 \leq c < \hat{c}$. When $\hat{c} \leq c \leq 1$, however, the manufacturer's potential loss under TCF exceeds its additional gain, and the manufacturer sets the wholesale prices under BCF and TCF at their optimal levels w_B^* and w_T^*, respectively. Because the retailer chooses BCF over

TCF, BCF becomes the unique subgame perfect financing equilibrium in this case.

Next, to check the robustness of Proposition 4, we study the impact of the manufacturer's possible capital constraint on the financing equilibrium.

5.2 When the Manufacturer is also Capital-Constrained

Our above analysis assumes that the manufacturer has sufficient capital. However, there are situations where the manufacturer is also capital-constrained. If so, the manufacturer has to borrow from banks to finance his production. To ease exposition, we assume that the manufacturer has zero capital endowment and limited liability, consistent with our assumption on the retailer.[7]

In practice, the manufacturer can use his revenue as collateral when borrowing from banks. Assume the interest rate faced by the manufacturer is $r_s \geq 0$. Given the bank market is competitive, the lending bank chooses r_s to satisfy

$$cQ_T^* = \mathbb{E}\min[cQ_T^*(1+r_s), \min\{\min\{Q_T^*, D\}, w_T^*Q_T^*\}],$$

or equivalently

$$\mathbb{E}\min[cQ_T^*r_s, \min\{\min\{Q_T^*, D\}, w_T^*Q_T^*\} - cQ_T^*] = 0.$$

With the capital borrowed from the bank, the manufacturer fulfills the production order of the capital-constrained retailer without collecting payment up-front under TCF. The manufacturer's problem is formulated in the Appendix.

We can show (in the Appendix) that the objective function of the manufacturer is the same as that in Section 4.2. That is, TCF with a capital-constrained manufacturer is equivalent to that with a capital-sufficient manufacturer. This is intuitive, because a capital-

[7] In an alternative scenario where the manufacturer is capital sufficient but bears opportunity costs for forgoing investment in other higher-interest projects, we can show the qualitative result in Proposition 4 continues to hold, although the equilibrium region of TCF shrinks as outside projects becomes more profitable.

constrained manufacturer under TCF behaves similarly to a capital-constrained retailer under BCF. Since the bank makes zero profit, the capital-constrained manufacturer raises needed capital and earns the same expected profits as with sufficient capital.

The above observation brings about additional managerial insights. In our earlier discussion, we implicitly assume both the retailer and the manufacturer can borrow needed capital from the bank when they are capital constrained under BCF and TCF, respectively. Because the bank market is competitive, the lending makes zero profits under either financing scheme. However, the retailer's order behavior under BCF differs than under TCF, resulting in different levels of supply chain efficiency. To maximize overall supply chain efficiency, should the bank finance the retailer or the manufacturer? The next proposition answers this question.

Proposition 5 *The exists a unique $\tilde{c}$, $0 < \tilde{c} < 1$, such that (i) in terms of the overall supply chain efficiency, TCF outperforms BCF if $0 \leq c < \tilde{c}$; otherwise (if $\tilde{c} \leq c \leq 1$), BCF outperforms TCF; (ii) $\tilde{c} < \hat{c}$.*

Proposition 5 indicates the overall supply chain also performs better under TCF than BCF when production cost is low. Therefore, as a good-will gesture the bank should lend loans to the capital-constrained manufacturer instead of the retailer when production cost is low. Otherwise, it should lend to the capital-constrained retailer. There is some nuance though. Given that $\tilde{c} < \hat{c}$, the manufacturer prefers TCF to BCF as long as the overall supply chain efficiency is higher under TCF than BCF (i.e., $0 \leq c < \tilde{c}$). The reverse is true if $c > \hat{c}$. However, if $\tilde{c} < c < \hat{c}$, the manufacturer prefers TCF to BCF while the overall supply chain performs better under BCF than TCF. In other words, if $\tilde{c} < c < \hat{c}$ and the bank lends to the capital-constrained manufacturer, the overall supply chain is less efficient at the expense of the retailer.

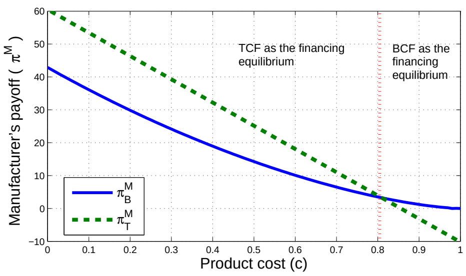

Figure 1: The Manufacturer's profits under BCF and TCF.

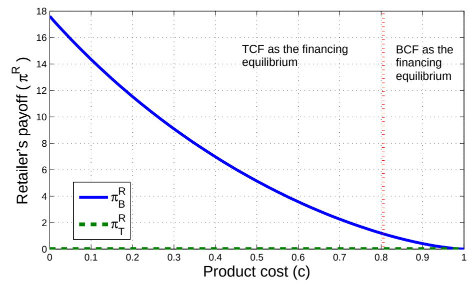

Figure 2: The retailer's profits under BCF and TCF.

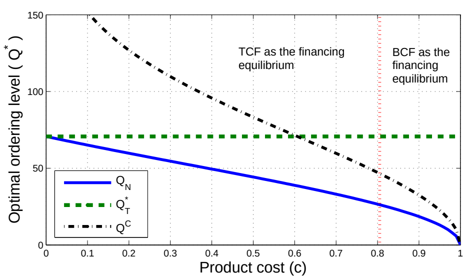

Figure 3: Order quantities under BCF, TCF, and the centralized case.

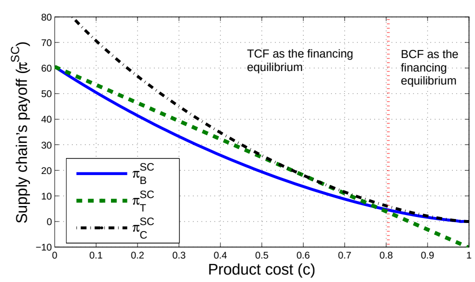

Figure 4: Overall channel profits under BCF, TCF, and the centralized case.

5.3 An Illustrating Example

We now use the Weibull distribution with tail function $\bar{F}(x) = \exp(-(0.01x)^2)$ to illustrate the financing equilibrium. To better capture the manufacturer's strategic moves, we study BCF and TCF as if only one of them is viable (as in Section 4). We then plot and compare the manufacturer's and the retailer's profits under BCF and TCF in Figures 1 and 2. As illustrated in Figures 1 and 2, all profits decrease as c grows. When $c < \hat{c} = 0.81$, the manufacturer is better off under TCF than under BCF while the retailer strictly prefers BCF to TCF. However, when both BCF and TCF are viable, the manufacturer prices out BCF, thus, TCF is the unique subgame perfect financing equilibrium. When $c > \hat{c}$, BCF becomes more profitable than TCF for the manufacturer, who sets the wholesale price to induce the retailer to opt for BCF. This pattern confirms Proposition 4. Consistent with

Proposition 2, the order quantity under TCF is greater than that under BCF, as shown in Figure 3. Interestingly, the order quantity under TCF may surpass that in the centralized channel over a certain range of c. Regarding overall supply chain efficiency, as illustrated in Figure 4, we find TCF outperforms BCF when $0 \le c \le \tilde{c} = 0.77$; otherwise, BCF has the advantage.

6 Extensions

We have so far discussed how the financing equilibrium reacts to changes in production cost. Next, we examine how demand variability, the retailer's internal capital level, and channel coordination affects the threshold value of c that separates the equilibrium domains of BCF and TCF.

6.1 Impact of Demand Variability

We use a (λ, k)-Weibull distribution to capture demand uncertainty, because it is conducive to describing the mean and variation of demand. To isolate the effect of demand variability, we keep its mean constant and vary the standard deviation.

We compare the channel efficiency under BCF and TCF and provide the threshold points $\hat{c}$ and $\tilde{c}$ in Table 1. For each financing venue we compute the percentage competition penalty

$$\mathcal{P} := \left(1 - \frac{\Pi_R + \Pi_M}{\Pi^C}\right) \times 100\%,$$

as a function of the manufacturer's production cost c and the coefficient of variation of the market demand. Let $\mathcal{P}_T$ and $\mathcal{P}_B$ denote the percentage competition penalties under TCF and BCF, respectively. Table 1 shows that when demand variability is low (e.g., $CV = 10\%$), TCF is generally more efficient than BCF (i.e., $\mathcal{P}$ is smaller under TCF) and their efficiency increases in the production cost. However, when demand variability is medium-large ($CV \ge 50\%$), the efficiency of TCF is a U-shaped function of the production cost

Percentage Competition Penalty ($\mathcal{P}_T$, $\mathcal{P}_B$)				
	Coefficient of Variation (%)			
c	10%	50%	100%	200%
0.0	(15.59, 15.59)	(31.32, 31.33)	(36.78, 36.79)	(40.21, 40.48)
0.1	(14.24, 14.98)	(24.31, 28.31)	(20.55, 30.70)	(5.29, 29.98)
0.2	(13.03, 14.59)	(18.60, 26.90)	(9.62, 28.88)	(3.42, 28.49)
0.3	(11.79, 14.29)	(13.14, 25.94)	(1.97, 27.82)	(54.76, 27.73)
0.4	(10.47, 14.03)	(7.87, 25.22)	(0.58, 27.09)	*(237.04, 27.22)*
0.5	(9.01, 13.80)	(3.14, 24.64)	(13.87, 26.54)	*(769.71, 26.86)*
0.6	(7.34, 13.60)	(0.14, 24.16)	*(65.65, 26.12)*	*(Large, 26.58)*
0.7	(5.36, 13.42)	(2.93, 23.76)	*(234.88, 25.77)*	*(Large, 26.35)*
0.8	(2.92, 13.25)	(28.55, 23.40)	*(881.37, 25.48)*	*(Large, 26.16)*
0.9	(0.24, 13.10)	*(203.67, 23.10)*	*(Large, 25.22)*	*(Large, 26.00)*
$\hat{c}$	0.99	0.82	0.58	0.30
$\tilde{c}$	0.98	0.78	0.53	0.26

Table 1: Competition penalty as a function of the manufacturers's production cost (c) and the coefficient of variation (StDev/Mean) of the market demand. Demand has a Weibull distribution with fixed mean $\mu = 100$. "Large" means a value greater than 1000.

c. In these cases, the competition penalty is maximized at intermediate values of demand variation and intermediate production costs ($CV = 50\%$ and $c = 0.6$). However, as c and CV grow, the efficiency of BCF dominates that of TCF.

In addition, $\hat{c}$ and $\tilde{c}$ both decrease in demand variability. With greater demand uncertainty, the manufacturer is more reluctant to bear the financing risk entailed by TCF for relatively high production costs, and the equilibrium region of BCF encroaches on that of TCF. The bank, as a Pareto player, will more likely lend loans to the retailer rather than the capital-constrained manufacturer. Note that the equilibrium region of TCF is above the step-wise line as demonstrated in Table 1, while the remaining area is that of BCF.

6.2 Impact of the Retailer's Internal Capital Level

To ease exposition, we have assumed the retailer's internal capital to be zero. Below, we allow the retailer to have a positive internal capital and examine how it affects the financing

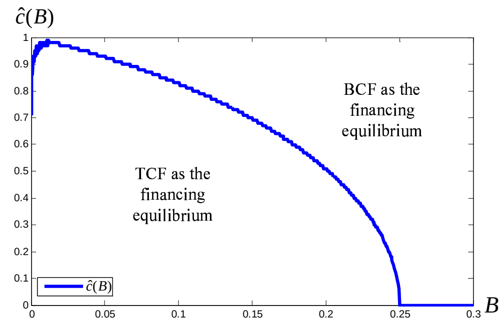

Figure 5: The Financial Equilibrium between BCF and TCF w.r.t. B.

equilibrium.

Due to the competitive bank market, Lemma 1 and Proposition 1 will continue to hold when the retailer's internal capital is positive. That is, the optimal order quantity is equivalent to that in the standard newsvendor model without capital constraint. However, under TCF the optimal order quantity and postponed wholesale price vary as the retailer's internal capital (B) grows. Given B, the retailer and the manufacturer's problems are

$$\max_{Q_T(B)\geq 0} \mathbb{E}(\min\{D, Q_T(B)\} - w_T Q_T(B) + B)^+ \tag{9}$$

and

$$\max_{c\leq w_T\leq 1} \mathbb{E}[(w_T - c)Q_T^*(B) - (w_T Q_T^*(B) - B - D)^+], \tag{10}$$

respectively.

Proposition 6 *Under TCF, for a given $B > 0$, the retailer's unique optimal order quantity, $Q_T^*(B)$, solves $\bar{F}(Q_T^*(B)) = w_T\bar{F}(w_T Q_T^*(B) - B)$; the unique optimal postponed wholesale price is given by $w_T^*(B) = \min[\tilde{w}, 1]$, where $\tilde{w}$ is implied by $\bar{F}(Q_T^*(\tilde{w}))\xi(\tilde{w}) = c$ and $\xi(w_T) = \frac{1-H(Q_T^*)}{1-w_T Q_T^* h(w_T Q_T^* - B)}$.*

Proposition 6 gives the optimal solution when the retailer has a positive internal capital. Due to the complexity of comparing BCF and TCF analytically, we numerically illustrate the financing equilibrium for $B > 0$. When demand is uniformly distributed on $[0, 1]$, we depict $\widehat{c}(B)$, the threshold cost level that separately BCF and TCF as financing equilibrium, as a function of B in Figure 5. As we can see, $\hat{c}(B)$ is not monotone in B. When B is very low, $\hat{c}(B)$ increases in B and the equilibrium domain of TCF expands. However, when B further increases, $\hat{c}(B)$ decreases with B and eventually drops to zero. Here the rationale is as follows. When B is very small, the retailer tends to place a big but risky order due to her limited liability. Consequently, as B grows, the manufacturer's potential financial loss can be significantly reduced under TCF while it remains constant under BCF. However, as B grows sufficiently large, the retailer's loss can be substantial if she defaults. Her order quantity decreases with B and finally drops to the order level of BCF at some point (i.e., $B > 0.25$ approximately). Thus, the manufacturer's marginal profit with respect to B decreases (although his marginal loss will also decrease). As a result, TCF becomes less favorable to the manufacturer than BCF when B is sufficiently high. When $B > 0.25$ in Figure 5, BCF becomes the financing equilibrium.

Overall, we find that TCF is more likely to be the financing equilibrium when the retailer's internal capital level is relatively low; otherwise, BCF is more likely to be the financing equilibrium. This finding is well supported by empirical studies. Deloof and Jegers (1999) study a sample of 661 firms in Belgium during 1989-1991, and show that the amount of trade credit is *negatively* affected by the internally generated cash. Similarly, Petersen and Rajan (1997) show that a firm's ability to generate cash internally reduces its demand for trade credit.

7 Conclusion and Discussions

This paper investigates the financing equilibrium between BCF and TCF in a channel where the retailer is capital constrained. When both BCF and TCF are viable, we have shown that TCF (BCF) is the unique financing equilibrium when production cost is below (above) a certain threshold. When production cost is below this threshold, the manufacturer will set a sufficiently high wholesale price for BCF so that the retailer chooses trade credit. Otherwise, the manufacturer will set the wholesale price for BCF at its optimal level to induce the retailer to adopt bank credit. We have also shown that the overall supply chain efficiency is higher under TCF when production cost is low. Therefore, if the bank is a

Pareto player who wants to maximize the overall supply chain efficiency, it will lend loans to the manufacturer (retailer) when production cost is low (high), if the manufacturer is also capital constrained. Furthermore, the equilibrium area of BCF encroaches on that of TCF as demand variability or the retailer's internal capital level grows, which is very well supported by empirical studies (see Cunningham, 2004; Deloof and Jegers, 1999; Petersen and Rajan, 1997).

To the best of our knowledge, our work is the first to analytically characterize the financing equilibrium regions for BCF and TCF in terms of the manufacturer's production cost. Our analysis also provides a guideline for manufacturers to choose between TCF and BCF according to their production costs. If production cost is relatively low, the manufacturer might want to promote TCF to the retailers/buyers; otherwise, it is wiser to let retailers borrow from banks. The latter choice becomes more obvious if demand variability is high and/or the retailer's internal capital level is high.

We conclude by discussing some possible extensions to our current model. First, in our model the retailer has no outside option if the manufacturer prices out BCF. In a separate analysis, we also study the case where the retailer can opt out of the relationship with the manufacturer, so that under TCF the manufacturer has to ensure at least the same profit for the retailer as under BCF. We find that the qualitative result sustains and the equilibrium region of BCF expands. Second, the demand distribution is assumed to be common knowledge. In practice, however, the retailer sometimes has more precise knowledge than creditors about demand conditions. When launching a new product, the manufacturer may possess better information about its quality and hence demand over the retailer and banks. It is worthwhile to analyze the financing equilibrium in channels with such asymmetric information. Finally, our theoretic analysis predicts that a lower margin manufacturer is less likely to offer trade credit; however, it has not been studied empirically. Therefore, collecting relevant industry evidence emerges as a future research priority.

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References

Babich, V. and M. Sobel (2004). Pre-IPO operational and financial decisions. *Management Science 50*, 935–948.

Biais, B. and C. Gollier (1997). Trade credit and credit rationing. *Review of Financial Studies 10(4)*, 903–937.

Booth, L., V. Aivazian, A. Demirguc-Kunt, and V. Maksimovic (2001). Capital structures in developing countries. *Journal of Finance 56(1)*, 87–130.

Boyabatli, O. and L. B. Toktay (2006). Capacity investment in imperfect capital markets: The interaction of operational and financial decision. Working Paper, TM, INSEAD, Fontainebleau, France.

Brennan, M., V. Maksimovic, and J. Zechner (1988). Vendor financing. *Journal of Finance 43(5)*, 1127–1141.

Burkart, M. and T. Ellingsen (2004). In-kind finance: A theory of trade credit. *The American Economic Review 94(3)*, 569–590.

Buzacott, J. A. and R. Q. Zhang (2004). Inventory management with asset-based financing. *Management Science 50(9)*, 1274–1292.

Caldentey, R. and M. Haugh (2009). Supply contracts with financial hedging. *Operations Research 57(1)*, 47–65.

Chao, X., J. Chen, and S. Wang (2008). Dynamic inventory management with financial constraints. *Naval Research Logistics 55(8)*, 758–768.

Chen, X. and G. Cai (2011). Joint logisitics and financial services by a 3pl firm. *European Journal of Operational Research 214(3)*, 579–587.

Cunningham, R. (2004). Trade credit and credit rationing in Canadian firms. *Working paper*.

Dada, M. and Q. J. Hu (2008). Financing the newsvendor inventory. *Operations Research Letters 36(5)*, 369–573.

Dammon, R. M. and L. W. Senbet (1988). The effect of taxes and depreciation on corporate investment and financial leverage. *Journal of Finance 43*, 357–374.

Deloof, M. and M. Jegers (1999). Trade credit, corporate groups, and the financing of Belgian firms. *Journal of Business Finance & Accounting 26(7)*, 945–966.

Ding, Q., L. Dong, and P. Kouvelis (2007). On the integration of productions and financial hedging decisions in global markets. *Operations Research 55(3)*, 470–489.

Dotan, A. and S. A. Ravid (1985). On the interaction of real and financial decisions of the firm under uncertainty. *Journal of Political economy 40*, 501–517.

Guariglia, A. and M. Mateut (2006). Credit channel, trade credit channel, and inventory investment: Evidence from a panel of UK firms. *Journal of Banking & Finance 62*, 2835–2858.

Gupta, D. and L. Wang (2009). A stochastic inventory model with trade credit. *M&SOM 11(1)*, 4–18.

Kouvelis, P. and W. Zhao (2008). Financing the newsvendor: Supplier vs. bank, optimal rates, and alternative schemes. *Working paper*. Olin Business School, Washington University.

Kouvelis, P. and W. Zhao (2009). Supply chain coordination with financial constraints and bankruptcy costs. *Working paper*. Olin Business School, Washington University.

Kouvelis, P. and W. Zhao (2011). The newsvendor problem and price-only contract when bankruptcy costs exist. *Production and Operations Management 20(6)*, 921–936.

Lariviere, M. A. and E. L. Porteus (2001). Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing and Service Operations Management 3(4)*, 293–305.

Petersen, M. and R. Rajan (1997). Trade credit: Theories and evidence. *Review of Financial Studies 10*, 661–691.

Rajan, R. G. and L. Zingales (1995). What do we know about capital structure? Some evidence from international data. *Journal of Finance 50*, 1421–1460.

Xu, X. D. and J. R. Birge (2004). Joint production and financing decisions: modeling and analysis. *Working paper*. Booth School of Business, University of Chicago, available at SSRN: http://ssrn.com/absract=652562.

Zhou, J. and H. Groenevelt (2007). Impacts of financial collaboration in a three-party supply chain. The Simon School, University of Rochester, Working Paper.

Appendix: Online Supplements

Proof of Lemma 1. First, note that $wQ_B(1+r_B^*) < Q_B$ when $w(1+r_B^*) < 1$. We can transform the retailer's objective function under BCF as described in Eq. (2) into

$$\int_{wQ_B(1+r_B^*)}^{Q_B} [D - wQ_B(1+r_B^*)]\, dF(D) + \int_{Q_B}^{\infty} [Q_B - wQ_B(1+r_B^*)]\, dF(D).$$

Expanding the constraint in Eq. (1) and rearranging it leads to

$$\int_{wQ_B(1+r_B^*)}^{\infty} wQ_B(1+r_B^*) dF(D) = wQ_B - \int_0^{wQ_B(1+r_B^*)} D dF(D).$$

Substituting the last expression into the retailer's objective above and collecting terms leads to

$$\int_0^{Q_B} D dF(D) + \int_{Q_B}^{\infty} Q_B dF(D) - wQ_B = E \min[D, Q_B] - wQ_B,$$

which is the standard newsvendor problem without capital constraint. Q.E.D.

Proof of Proposition 1. Parts (1) and (2) follow directly from Lemma 1 and the text preceding the Proposition. The following lemma is useful for proving parts (3) and (4).

Lemma A1. *Consider a scalar $y > 0$ and a random variable W with a positive density function $g(W)$ defined on $[0, \infty)$. There exists a unique $r \geq 0$ such that $y = E[\min\{W, y(1+r)\}]$ if $E[W] > y$.*

Proof of Lemma A1. For $r \in [0, \infty)$, let $\Phi(r) \equiv E[\min\{W, y(1+r)] - y$. We can easily verify that $\frac{d\Phi(r)}{dr} = \int_{y(1+r)}^{\infty} yg(W)dW > 0$, i.e., $\Phi(r)$ is monotonically increasing in r. Suppose $E[W] > y$. We wish to show that there exists a unique $r \geq 0$ so that $\Phi(r) = 0$. We have

$$\Phi(0) = E[\min\{W, y\}] - y \leq \min\{E(W), y\} - y = 0$$

and

$$\Phi(\infty) = \lim_{r \to \infty} E[\min\{W, y(1+r)] - y = E[W] - y > 0.$$

Because $\Phi(r)$ is continuous and strictly increasing in r, by the Intermediate Value Theorem there exists a unique $r \geq 0$ that satisfies $\Phi(r) = 0$. This proves Lemma A1.□

Part (3). Part (1) has shown that the retailer's optimal order quantity is Q_N under bank financing. In Lemma A1, we let $y = wQ_N$ be the loan size and $W = \min\{D, Q_N\}$ be the retail revenue. Since $E[\min\{D, Q_N\}] > wQ_N$ by construction, Lemma A1 thus implies the existence of a unique interest rate that equals the expected return on the loan to its costs. From parts (1) and (2) and the assumption of a competitive bank market, we know that r_B^* uniquely solves $w_B^* Q_N = E[\min\{w_B^* Q_N(1 + r_B^*), \min\{D, Q_N\}\}]$.

Part (4). Since retail revenue is $\min\{D, Q_N\}$, its highest possible value is Q_N. The amount the retailer owes at the end of the period is $w_B^* Q_N(1 + r_B^*)$. Therefore, a necessary condition for the retailer to enter the bank-credit contract is $Q_N > w_B^* Q_N(1 + r_B^*)$, or $w_B^*(1 + r_B^*) < 1$. Q.E.D.

Proof of Proposition 2. *Part (1).* Under TCF, the retailer's profit function $\pi^R(Q_T) \equiv E[\min\{D, Q_T\} - w_T Q_T]^+$ can be rewritten as

$$\pi^R(Q_T) = \int_{w_T Q_T}^{Q_T} [D - w_T Q_T] \, dF(D) + \int_{Q_T}^{\infty} [Q_T - w_T Q_T] \, dF(D).$$

Differentiating π^R with respect to Q_T and collecting terms, we have $\frac{d\pi^R(Q_T)}{dQ_T} = \bar{F}(Q_T) - w_T \bar{F}(w_T Q_T)$. The retailer's optimal order quantity, Q_T^*, satisfies the first-order condition $\bar{F}(Q_T^*) = w_T \bar{F}(w_T Q_T^*)$. We further have

$$\begin{aligned} \frac{d^2\pi^R(Q_T)}{d(Q_T)^2}|_{Q_T = Q_T^*} &= -f(Q_T^*) + (w_T)^2 f(w_T Q_T^*) \\ &= -w_T \bar{F}(w_T Q_T^*)\{h(Q_T^*) - w_T h(w_T Q_T^*)\}. \end{aligned}$$

Since h is an increasing function and $w_T \leq 1$ by assumption, we have $h(Q_T^*) \geq w_T h(w_T Q_T^*)$. Therefore, $\frac{d^2\pi^R(Q_T)}{d(Q_T)^2}|_{Q_T = Q_T^*} < 0$ and Q_T^* is thus unique.

Part (2).

To prove $\frac{dQ_T^*}{dw_T} < 0$, it is equivalent to show $\frac{dw_T}{dQ_T^*} < 0$, where $w_T(Q_T^*)$ is defined by $\bar{F}(Q_T^*) = w_T \bar{F}(w_T Q_T^*)$ which can be inferred from the above first order condition. Multiplying Q_T^* in both sides for the above equation, we have

$$Q_T^* \bar{F}(Q_T^*) = w_T Q_T^* \bar{F}(w_T Q_T^*),$$

where $w_T \leq 1$.

We now continue the proof graphically.[8] Define function $V(Q) = Q\bar{F}(Q)$, as illustrated in Figure 6. The implicit function of $w_T(Q_T^*)$ can then be derived as follows:

$$Q_T^* \bar{F}(Q_T^*) = \tilde{Q}_T^* \bar{F}(\tilde{Q}_T^*), \tag{A-1}$$

where $\tilde{Q}_T^* = w_T(Q_T^*)Q_T^*$ and $\tilde{Q}_T^* \leq Q_T^*$. Note that $\frac{dV(Q)}{dQ} = \bar{F}(Q)(1 - Q\frac{f(Q)}{\bar{F}(Q)}) = \bar{F}(Q)(1 - H(Q))$. Under the assumption of IGFR, we can show that $V(Q)$ is a unimodal function in $Q \in (\underline{D}, \bar{D})$. Since $H(\underline{D}) = 0$ (or assuming it is below 1) and $H(\bar{D}) = \lim_{D \to \bar{D}} H(D) =\propto$, there exists a unique $\hat{Q}$ such that $H(\hat{Q}) = 1$. Thus, there exists a unique $\hat{Q} \in (\underline{D}, \bar{D})$, such that $V(Q)$ is strictly increasing in $Q \in [\underline{D}, \hat{Q})$ and strictly decreasing in $Q \in (\hat{Q}, \bar{D}]$, see Figure 6. As illustrated in Figure 6, Q_T^* must take value in $(\hat{Q}, \bar{D}]$ and $\tilde{Q}_T^*$ in $[\underline{D}, \hat{Q})$. When Q_T^*

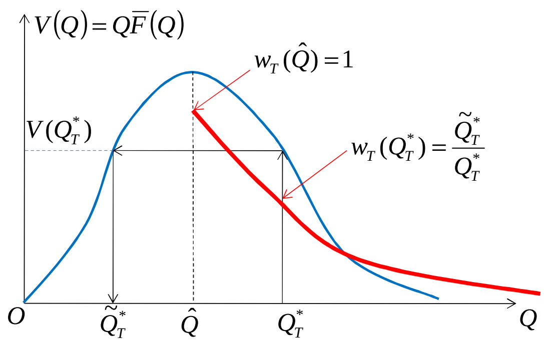

Figure 6: $V(Q)$ as a function of Q

increases, $\tilde{Q}_T^*$ must decrease, and it follows $w_T(Q_T^*) = \frac{\tilde{Q}_T^*}{Q_T^*}$ is strictly decreasing in Q_T^*. As a result, we have $\frac{dQ_T^*}{dw_T} < 0$.

Part (3). Based on Figure 6, we can show, when $w_T = 1$ under TCF, $Q_T^* = \hat{Q}$, which solves $1 - H(Q_T^*) = 0$. Since $1 - H(Q_T^*)$ decreases with Q_T^*, in general we have $1 - H(Q_T^*) \leq 0$ and $Q_T^* \geq \hat{Q}$ for $w_T \in [c, 1]$. Under BCF, Q_N satisfies $\bar{F}(Q_N) - Q_N f(Q_N) = c$. We thus obtain $1 - H(Q_N) = \frac{c}{\bar{F}(Q_N)} \geq 0$. Therefore, when $c = 0$ and $w_T = 1$, we get $1 - H(Q_T^*) = 1 - H(Q_N) = 0$ and $Q_T^* = Q_N$; otherwise, $1 - H(Q_N) > 0 > 1 - H(Q_T^*)$, and we yield $Q_N < Q_T^*$. Q.E.D.

[8]We gratefully thank an anonymous referee for providing this graphic proof and Figure 6.

Proof of Proposition 3. The following discussion is based on the assumption that the retailer is a Pareto or good-willed player. Given that $Q_T^*(w_T)$, is continuous on w_T, we have $Q_T^*(1) = \lim_{w_T \to 1} Q_T^*(w_T)$. Recall Eq. (A-1), $\tilde{Q}_T^* = w_T Q_T^*$, and $Q_T^* \geq \tilde{Q}_T^*$. When w_T increases, $\tilde{Q}_T^*$ is closer to Q_T^*. In Figure 6, we have that when $w_T(\hat{Q}) = 1$, $Q_T^* = \tilde{Q}_T^*$ and $V(Q)$ achieves its maximum $V(\hat{Q})$. Therefore, when $w_T = 1$, $Q_T^* = \hat{Q}$, which solves the function $H(Q_T^*) = 1$.

For $w_T \in [c, 1]$, we have $w_T Q_T^* \leq Q_T^*$. The proof of Proposition 2 has shown that $H(w_T Q_T^*) \leq H(Q_T^*) \leq 1$ for $w_T \in [c, 1]$. Therefore, we have

$$\frac{c[1 - H(w_T Q_T^*)]}{w_T[h(Q_T^*) - w_T h(w_T Q_T^*)]} \geq 0.$$

Consequently, in Eq. (8), the sign of $\frac{d\pi_T^M}{dw_T}$ then depends on that of $[1 - \eta(w_T)]$, where $\eta(w_T) \equiv \frac{\bar{F}(Q_T^*)[1-H(Q_T^*)]}{c[1-H(w_T Q_T^*)]}$. Note that $1 - H(Q_T^*) = 1 - Q_T^* h(Q_T^*)$. Since Q_T^* decreases in w_T (from Proposition 2), $1 - Q_T^* h(Q_T^*)$ increases in w_T. Let $w_T = 1$, we obtain $1 - Q_T^*(1) h(Q_T^*(1)) = 0$. Thus, when $w_T \leq 1$ and $H(Q_T^*) \geq 1$, we have $\eta(w_T) \leq 0$ and, hence, $\frac{d\pi_T^M}{dw_T} \geq 0$. Since the feasible region for w_T is in $[c, 1]$, we yield $w_T^* = 1$. Based on the Proof of Proposition 2, when $w_T^* = 1$, $Q_T^*(w_T^*)$ is equal to $\hat{Q}$, which solves the equation of $H(Q_T^*) = 1$. Q.E.D.

Proof of Proposition 4. We first prove that there exists a unique $c = \hat{c}$ such that the manufacturer is indifferent to TCF versus BCF. Let $\pi_B^M(c)$ and $\pi_T^M(c)$ denote the manufacturer's payoff under BCF and TCF, respectively. We shall show that there exists a unique $c = \hat{c}$ satisfying $\pi_B^M(c) = \pi_T^M(c)$. To the end, we first show $\pi_B^M(c)$ and $\pi_F^M(c)$ monotonically decrease in $c \in [0, 1]$ and then show $\pi_T^M(c) > \pi_B^M(c)$ when $c = 0$ and $\pi_T^M(c) < \pi_B^M(c)$ when $c = 1$.

Part 1: Here we show $\pi_B^M(c)$ and $\pi_F^M(c)$ monotonically decrease in $c \in [0, 1]$. Under TCF, we can rewrite Eq. (7) as

$$\pi_T^M(c) = w_T^* Q_T^* - c Q_T^* - \int_0^{w_T^* Q_T^*} (w_T^* Q_T^* - D) dF(D),$$

given that $w_T^* = 1$ and Q_T^* solved by $H(Q_T^*(w_T^*))|_{w_T^*=1} = 1$ according to Proposition 3. Since w_T^* and Q_T^* are constant in c, we can show that $\frac{d\pi_T^M(c)}{dc} = -Q_T^* < 0$, thus π_T^M is monotonically decreasing in $c \in [0,1]$.

Under BCF, we obtain $\pi_B^M(c) = (w_B^* - c)Q_B^*$, where Q_B^* is solved by $\bar{F}(Q_N) - Q_N f(Q_N) = c$ and $w_B^* = \bar{F}(Q_N)$ based on Proposition 1. Furthermore, we have $\pi_B^M(c) = (\bar{F}(Q_N) - c)Q_N$, and,

$$\begin{aligned}
\frac{d\pi_B^M(c)}{dc} &= \frac{\partial \pi_B^M(c)}{\partial Q_B^*}\frac{dQ_B^*}{dc} + \frac{\partial \pi_B^M(c)}{\partial c} \\
&= [\bar{F}(Q_N) - Q_N f(Q_N) - c]\frac{dQ_N}{dc} - Q_N \\
&= -Q_N \\
&\leq 0,
\end{aligned}$$

and then $\pi_B^*(c)$ is also monotonically decreasing in $c \in [0,1]$.

Part 2: We now show $\pi_T^M(c) > \pi_B^M(c)$ when $c = 0$ and $\pi_T^M(c) < \pi_B^M(c)$ when $c = 1$.

Case I: $c = 0$**.** Under TCF, we can rewrite Eq. (7) as

$$\begin{aligned}
\pi_T^M(c) &= (w_T^* - c)Q_T^* - \int_0^{w_T^* Q_T^*} (w_T^* Q_T^* - D)dF(D) \\
&= Q_T^* - \int_0^{Q_T^*} (Q_T^* - D)dF(D),
\end{aligned}$$

since $c = 0$ and $w_T^* = 1$. Under BCF, similarly, we have

$$\begin{aligned}
\pi_B^M(c) &= (w_B^* - c)Q_N \\
&= w_B^* Q_N \\
&= \int_0^{w_B^*(1+r_B^*)Q_N} DdF(D) + \int_{w_B^*(1+r_B^*)Q_N}^{\infty} w_B^*(1+r_B^*)Q_N dF(D) \\
&= w_B^*(1+r_B^*)Q_N - \int_0^{w_B^*(1+r_B^*)Q_N} (w_B^*(1+r_B^*)Q_N - D)dF(D),
\end{aligned}$$

where the third equation is based on Eq. (1). Combining the following facts: (i) the function $x + \int_0^x (x-y)f(y)dy$ is increasing in x, (ii) $w_B^*(1+r_B^*) < 1$ based on Proposition 1 and (iii) $Q_T^* \geq Q_N$ based on Proposition 2, we yield

$$\pi_T^M(c) = Q_T^* - \int_0^{Q_T^*} (Q_T^* - D)dF(D)$$

$$
\begin{aligned}
&> w_B^*(1+r_B^*)Q_N - \int_0^{w_B^*(1+r_B^*)Q_N} (w_B^*(1+r_B^*)Q_N - D)dF(D) \\
&= \pi_B^M(c).
\end{aligned}
$$

Thus, $\pi_T^M(c) > \pi_B^M$ when $c = 0$.

Case II: $c = 1$. We obtain

$$
\begin{aligned}
\pi_T^M(c) &= (w_T^* - c)Q_T^* - \int_0^{w_T^* Q_T^*} (w_T^* Q_T^* - D)dF(D) \\
&= - \int_0^{w_T^* Q_T^*} (w_T^* Q_T^* - D)dF(D) \\
&< 0 \\
&= (w_B^* - c)Q_N \\
&= \pi_B^M(c),
\end{aligned}
$$

since $Q_N = 0$ when $c = 1$. Thus, $\pi_T^M(c) < \pi_B^M(c)$ when $c = 1$. Therefore, if $c < \hat{c}$, $\pi_T^M < \pi_B^M$, the manufacturer will charge a sufficiently high wholesale price under BCF, such that TCF is the unique subgame perfect equilibrium. Otherwise if $\hat{c} \leq c \leq 1$, $\pi_T^M < \pi_B^M$, and the manufacturer will charge the wholesale price under BCF at its optimal level such that the retailer will choose BCF over TCF, thus, BCF is the subgame perfect equilibrium. Q.E.D.

The manufacturer's problem when it is also capital constrained.

The manufacturer's problem is

$$
\begin{aligned}
\underset{c \leq w_T \leq 1}{Max}\, \pi_T^M(w_T) &= \mathbb{E}(\min[w_T Q_T^*, \min\{D, Q_T^*\}] - c(1+r_s)Q_T^*)^+, \\
\text{subject to} &: \bar{F}(Q_T^*) = w_T \bar{F}(w_T Q_T^*) \\
& \quad \mathbb{E}\min[cQ_T^* r_s, \min\{\min\{D, Q_T^*\}, w_T^* Q_T^*\} - cQ_T^*] = 0.
\end{aligned}
$$

Note that $(x-y)^+ = x - \min\{x, y\}$. The manufacturer's objective can be rewritten as

$\pi_T^M(w_T) = \mathbb{E}[\min[w_T^* Q_T^*, \min\{D, Q_T^*\}] - cQ_T^* - \min[\min\{w_T^* Q_T^*, \min\{D, Q_T^*\}\} - cQ_T^*, cQ_T^* r_s]]$.

Given that $\mathbb{E}\min[cQ_T^* r_s, \min\{\min\{D, Q_T^*\}, w_T^* Q_T^*\} - cQ_T^*] = 0$, the manufacturer's problem is then equivalent to

$$
\underset{c \leq w_T \leq 1}{Max}\, \pi_T^M(w_T) = \mathbb{E}\min[w_T Q_T^*, \min\{D, Q_T^*\}] - cQ_T^*,
$$

$$\text{subject to} : \bar{F}(Q_T^*) = w_T \bar{F}(w_T Q_T^*) \tag{A-2}$$

$$\mathbb{E}\min[cQ_T^* r_s, \min\{\min\{D, Q_T^*\}, w_T^* Q_T^*\} - cQ_T^*] = 0.$$

Proof of Proposition 5: Part (i): We first prove the existence and uniqueness of $\tilde{c}$. Denote $\pi_B^{SC}(c)$ and $\pi_T^{SC}(c)$ as the payoffs of the overall supply chain under BCF and TCF, respectively. In details, we have

$$\begin{aligned}
\pi_B^{SC}(c) &= \mathbb{E}\min\{D, Q_N\} - cQ_N, \\
\pi_T^{SC}(c) &= \mathbb{E}\min\{D, Q_T^*|_{w_T^*=1}\} - cQ_T^*|_{w_T^*=1}.
\end{aligned}$$

Similarly to the proof of Proposition 4, we first show $\pi_B^{SC}(c)$ and $\pi_T^{SC}(c)$ monotonically decrease in $c \in [0, 1]$ and then show $\pi_T^{SC}(c) > \pi_B^{SC}(c)$ at $c = 0$ and $\pi_T^{SC}(c) < \pi_B^{SC}(c)$ at $c = 1$. For $\pi_B^{SC}(c)$, we have

$$\begin{aligned}
\frac{d\pi_B^{SC}(c)}{dc} &= \frac{\partial \pi_B^{SC}(c)}{\partial Q_N}\frac{dQ_N}{dc} + \frac{\partial \pi_B^{SC}(c)}{\partial c} \\
&= [\bar{F}(Q_N) - c]\frac{dQ_N}{dc} - Q_N \\
&= Q_N f(Q_N)\frac{dQ_N}{dc} - Q_N \\
&\leq 0,
\end{aligned}$$

because $\frac{dQ_N}{dc} < 0$ which can be inferred from $\bar{F}(Q_N) - Q_N f(Q_N)) = c$. Similarly,

$$\begin{aligned}
\frac{d\pi_T^{SC}(c)}{dc} &= \frac{\partial \pi_T^{SC}(c)}{\partial Q_T^*}\frac{dQ_T^*}{dc} + \frac{\partial \pi_T^{SC}(c)}{\partial c} \\
&= -Q_T^* \\
&< 0,
\end{aligned}$$

because $\frac{dQ_T^*}{dc} = 0$ based on $H(Q_T^*) = 1$. Thus, $\pi_B^{SC}(c)$ and $\pi_T^{SC}(c)$ decrease monotonically in $c \in [0, 1]$.

When $c = 0$, we can show $\pi_B^{SC}(0) = \mathbb{E}\min[D, Q_N]$ and $\pi_T^{SC}(c) = \mathbb{E}\min[D, Q_T^*]$. Based on Proposition 2, we have $Q_T^* \geq Q_N$, thus, $\pi_T^{SC}(0) \geq \pi_B^{SC}(0)$. When $c = 1$, we now show

$\pi_B^{SC}(1) > \pi_T^{SC}(1)$ as follows.

$$\begin{aligned}
\pi_T^{SC}(1) &= \mathbb{E}\min\{D, Q_T^*\} - Q_T^* \\
&= Q_T^* - \int_0^{Q_T^*} F(D)dD - Q_T^* \\
&= -\int_0^{Q_T^*} F(D)dD \\
&< 0 \\
&= \mathbb{E}\min\{D, Q_N|_{c=1}\} - Q_N|_{c=1} \\
&= \pi_B^{SC}(1),
\end{aligned}$$

because $Q_N = 0$ if $c = 1$. Based on the above single-crossing property, we have proved the existence and uniqueness of $\tilde{c}$.

Part (ii): We now prove $\tilde{c} < \hat{c}$. We prove this by contradiction. Based on the proof in Part (i), we have shown $\pi_B^{SC}(c) < \pi_T^{SC}$ if $c < \tilde{c}$; otherwise, $\pi_B^{SC}(c) > \pi_T^{SC}$. We now suppose $\tilde{c} \geq \hat{c}$. For any $\hat{c} \leq c \leq \tilde{c}$, we can get $\pi_B^{SC}(c) \leq \pi_T^{SC}(c)$. Based on Proposition 4, for the manufacturer $\pi_B^M(c) \geq \pi_T^M(c)$ if $c \geq \hat{c}$. Then, for any $\hat{c} \leq c \leq \tilde{c}$, we have $\pi_B^M(c) \geq \pi_T^M(c)$. Denote $\pi_B^R(c)$ and $\pi_T^R(c)$ as payoffs of the retailer under BCF and TCF, respectively. Based on Propositions 3 and 4, we have $\pi_B^R(c) > \pi_T^R(c) = 0$ for $0 < c < 1$. Consequently we yield $\pi_B^{SC}(c) = \pi_B^M(c) + \pi_B^R(c) > \pi_T^M(c) + \pi_T^R(c) = \pi_T^{SC}(c)$ for $\hat{c} \leq c \leq \tilde{c}$. Therefore, it is a contradiction to $\pi_B^{SC}(c) \leq \pi_T^{SC}(c)$ if $\hat{c} \leq c \leq \tilde{c}$. This proves $\hat{c} > \tilde{c}$. Q.E.D.

Proof of Proposition 6. For the retailer, given $B > 0$, Eq. (9) can be rewritten by

$$\Pi^R(Q_T, B) = \int_{w_T Q_T - B}^{Q_T} [D - (w_T Q_T - B)]dF(D) + \int_{Q_T}^{\infty} [Q_T - (w_T Q_T - B)]dF(D).$$

Differentiating $\Pi^R(Q_T, B)$ with respect to Q_T, we have $\frac{d\Pi^R(Q_T,B)}{dQ_T} = \bar{F}(Q_T) - w_T\bar{F}(w_T Q_T - B)$. Therefore, the optimal order quantity, $Q_T^*(B)$, satisfies the first order condition $\bar{F}(Q_T) = w_T\bar{F}(w_T Q_T - B)$. We further have,

$$\begin{aligned}
\frac{d^2\Pi^R(Q_T, B)}{d(Q_T)^2}\Big|_{Q_T = Q_T^*} &= -f(Q_T^*) + w_T^2 f(w_T Q_T^* - B) \\
&= -\bar{F}(Q_T^*)[h(Q_T^*) - w_T h(w_T Q_T^* - B)].
\end{aligned}$$

Since $h(D)$ increases in D and $w_T \in [c,1]$, we have $h(Q_T^*) \geq w_T h(w_T Q_T^* - B)$. Hence, $\frac{d^2\Pi^R(Q_T,B)}{d(Q_T)^2}\big|_{Q_T=Q_T^*} \leq 0$ and $Q_T^*(B)$ is thus unique.

For the manufacturer, from Eq. (10), we obtain

$$\begin{aligned}\frac{d\Pi_T^M(w_T,B)}{dw_T} &= -\frac{1-w_TQ_T^*h(w_TQ_T^*-B)}{w_Th(Q_T^*)-w_T^2h(w_TQ_T^*-B)}\left[\frac{\bar{F}(Q_T^*)[1-H(Q_T^*)]}{1-w_TQ_T^*h(w_TQ_T^*-B)}-c\right] \\ &= \frac{dQ_T^*}{dw_T}\left[\frac{\bar{F}(Q_T^*)[1-H(Q_T^*)]}{1-w_TQ_T^*h(w_TQ_T^*-B)}-c\right]. \end{aligned} \tag{A-3}$$

Let $\xi(w_T) = \frac{1-H(Q_T^*)}{1-w_TQ_T^*h(w_TQ_T^*-B)}$. Giving $\frac{dQ_T^*}{dw_T}$ is generally nonzero (more specifically, $\frac{dQ_T^*(B)}{dw_T} < 0$ as shown later) and combining the fact that $w_T \in [c,1]$, we thus yield $w_T^*(B) = \min[\tilde{w}, 1]$, where $\tilde{w}$ is implied by $\bar{F}(Q_T^*(\tilde{w}))\xi(\tilde{w}) = c$.

To prove $w_T^*(B)$ is unique, we now show $\Pi_T^M(w_T,B)$ is either a unimodal or monotonically increasing function for $w_T \in [c,1]$. To the end, we first demonstrate $\frac{dQ_T^*(B)}{dw_T} < 0$. Based on $\bar{F}(Q_T) = w_T\bar{F}(w_TQ_T - B)$, we obtain $\frac{dQ_T^*(B)}{dw_T} = -\frac{1-w_TQ_T^*h(w_TQ_T^*-B)}{w_Th(Q_T^*)-w_T^2h(w_TQ_T^*-B)}$. Since the denominator is nonnegative, it is sufficient to show $1 - w_TQ_T^*h(w_TQ_T^* - B) > 0$. Since $\frac{dQ_T^*}{dB} = \frac{h(w_TQ_T^*-B)}{w_T^2h(w_TQ_T^*-B)-w_Th(Q_T^*)} < 0$ for any given w_T, we have $w_TQ_T^*(B)h(w_TQ_T^*(B) - B) \leq w_TQ_T^*(0)h(w_TQ_T^*(0))|_{B=0}$. Therefore, we can show $1 - w_TQ_T^*h(w_TQ_T^* - B) > 0$ if we can show $1 - w_TQ_T^*(0)h(w_TQ_T^*(0)) \geq 0$. Recall Proposition 2, in the case of $B = 0$, $Q_T^*(0)$ solves $\bar{F}(Q_T) = w_T\bar{F}(w_TQ_T)$, and $Q_T^*(0)$ is decreasing in w_T. Thus, $\frac{dw_TQ_T^*(w_T,B)}{dw_T}|_{B=0} = \frac{dw_TQ_T^*(w_T,0)}{dw_T} = \frac{Q_T^*(w_T,0)h(Q_T^*(w_T,0))-1}{h(Q_T^*(w_T,0))-w_Th(Q_T^*(w_T,0)}$. Since $Q_T(w_T,0)$ decreases in $w_T \in [c,1]$ (see Proposition 2), $w_TQ_T^*(w_T,0)$ is a unimodal function of $w_T \in [c,1]$. Define $\dot{w} = \arg\max\{w_TQ_T^*(w_T,0), w_T \in [c,1]\}$ which satisfies the first order condition $\frac{dw_TQ_T^*(w_T,0)}{dw_T}|_{w_T=\dot{w}} = \frac{Q_T^*(\dot{w},0)h(Q_T^*(\dot{w},0))-1}{h(Q_T^*(\dot{w},0))-\dot{w}h(Q_T^*(\dot{w},0)} = 0$. Hence, we get $Q_T^*(\dot{w},0)h(Q_T^*(\dot{w},0)) = 1$. Consequently, following the monotonicity of $h(D)$, we can infer that

$$\begin{aligned} & 1 - w_TQ_T^*(w_T,B)h(w_TQ_T^*(w_T,B) - B) \\ > \;& 1 - w_TQ_T^*(w_T,0)h(w_TQ_T^*(w_T,0)) \\ \geq \;& 1 - \dot{w}Q_T^*(\dot{w},0)h(\dot{w}Q_T^*(\dot{w},0)) \\ \geq \;& 1 - Q_T(\dot{w},0)^*h(Q_T^*(\dot{w},0)) \end{aligned}$$

$$= \quad 0.$$

As a result, we get $\frac{dQ_T^*(w_T,B)}{dw_T} < 0$.

Based on Eq. (A-3), the signal of $\frac{d\Pi_T^M(w_T,B)}{dw_T}$ depends on that of $[\bar{F}(Q_T^*)\xi(w_T) - c]$, where $\xi(w_T) = \frac{1-H(Q_T^*)}{1-w_T Q_T^* h(w_T Q_T^* - B)} \leq 1$. When $w_T = c$, we have $\bar{F}(Q_T^*)\xi(w_T) = c\bar{F}(cQ_T - B)\xi(c) < c$, and $[\bar{F}(Q_T^*)\xi(w_T) - c]$ is thus a negative value. Consequently, $\frac{d\Pi_T^M(w_T,B)}{dw_T}|_{w_T=c} > 0$. Since $\bar{F}(Q_T)$ increases in w_T, we now show $\xi(w_T)$ increases in w_T.

$$
\begin{aligned}
& [\xi(w_T)]' = \left[\frac{1-H(Q_T^*)}{1-w_T Q_T^* h(w_T Q_T^* - B)}\right]' \\
= \quad & \frac{-H'(Q_T)Q_T'[1-H(w_T Q_T - B) - Bh(w_T Q_T - B)]}{[1-H(w_T Q_T - B) - Bh(w_T Q_T - B)]^2} \\
& - \frac{(1-H(Q_T))[-H'(w_T Q_T - B)(w_T Q_T)' - Bh(w_T Q_T - B)(w_T Q_T)']}{[1-H(w_T Q_T - B) - Bh(w_T Q_T - B)]^2} \\
> \quad & \frac{(1-H(Q_T))[-H'(Q_T)Q_T' + H'(w_T Q_T - B)(w_T Q_T)' + Bh(w_T Q_T - B)(w_T Q_T)']}{[1-H(w_T Q_T - B) - Bh(w_T Q_T - B)]^2} \\
> \quad & \frac{(1-H(Q_T))[-H'(Q_T) + H'(w_T Q_T - B) + Bh(w_T Q_T - B)]Q_T'}{[1-H(w_T Q_T - B) - Bh(w_T Q_T - B)]^2} \\
= \quad & \frac{(1-H(Q_T))[(w_T Q_T h'(w_T Q_T - B) - Q_T h'(Q_T)) + (h(w_T Q_T - B) - h(Q_T))]Q_T'}{[1-H(w_T Q_T - B) - Bh(w_T Q_T - B)]^2} \\
> \quad & 0
\end{aligned}
$$

Therefore, $\bar{F}(Q_T)\xi(w_T)$ increases in w_T. If $\bar{F}(Q_T(w_T))\xi(w_T)$ surpasses c before w_T grows up to 1, then w_T^* is determined by $\bar{F}(Q_T(\tilde{w}))\xi(\tilde{w}) = c$; otherwise, w_T^* equals 1. Overall, there exists a unique $w_T^* = \min\{\tilde{w}, 1\}$. Q.E.D.