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Abstract

This paper investigates the roles of bank and trade credits in a supply chain with a capital constrained retailer facing demand uncertainty. The retailer can borrow credit from a bank (bank credit), and/or from the supplier who allows delayed payment (trade credit). We evaluate the retailer's optimal order quantity and the creditors' optimal credit limits and interest rates in two scenarios where either a single credit or both credits are viable. In the single-credit scenario, we find the retailer prefers trade credit, if the trade credit market is more competitive than the bank credit market; otherwise, the retailer's preference of a specific credit type depends on the risk levels that the retailer would divert trade credit and bank credit to other risky investment. In the dual-credit scenario, if the bank credit market is more competitive than the trade credit market, the retailer first borrows bank credit prior to trade credit, but then switches to borrowing trade credit prior to bank credit as the retailer's internal capital declines. In contrast, if the trade credit market is more competitive, the retailer borrows only trade credit. We further analytically prove that the two credits are complementary if the retailer's internal capital is substantially low but become substitutable as the internal capital grows, and then empirically validate this prediction based on a panel of 674 manufacturing firms in China over the period 2001–2007.

Keywords: trade credit; bank credit; capital constrained; newsvendor; moral hazard

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1 Introduction

Capital is fundamental in a supply chain. Although the extant literature typically assumes abundant capital for supply chains, as evidenced in the Year 2008 world financial crisis, thousands of retailers face shortage of funds to turn around their businesses. For example, the once electronic retailing giant, Circuit City, announced bankruptcy in 2009 partially because of insufficient cash flow.

Capital-constrained retailers traditionally borrow loans from banks, also referred to as *bank credit*. But, caused by the banks' lack of full information on the retailers' investment actions, moral hazard arises, as retailers might divert the bank credit to other projects. This hidden action prompts banks to cap the credit size to avoid substantial financial risks. Consequently, the retailers are unable to reach their optimal ordering levels.

For resolving the above bank credit ceiling impasse, trade credit emerges as an effective alternative (Burkart and Ellingsen, 2004). In a *trade credit* contract, a supplier delivers products to its retailers by allowing them to delay payment with an interest. As documented by Elliehausen and Wolken (1993) in a Federal Reserve Board study, in 1987, trade credit (accounts payable) represented around 15% of the liabilities of non-farm non-financial businesses and around 20% of the liabilities of small firms in the United States. Rajan and Zingales (1995) calculated that, in 1991, trade credit represented 17.8% of total assets for all American firms, and more than a quarter of total corporate assets in European countries, such as Germany, France, and Italy. From the data of 674 manufacturing firms listed in the Shanghai or Shenzhen Stock Exchanges in China over the period from 2001 to 2007, we find that trade credit was approximately 10% of those firms' total assets. Ge and Qiu (2007) also reported that the average ratio of trade credit to total assets is about 13% for a sample of 570 firms in China in 2000.

Undoubtedly, a capital-constrained retailer will be boosted by the availability of both credit types; however, *what is the retailer's optimal borrowing strategy and how is the strategy affected by its internal capital level?* To the best of our knowledge, no theoretical explanation has accounted for the retailer's optimal borrowing strategies in the presence of moral hazard and demand uncertainty when both credit types are viable. In addition, *would the two credit types be substitutable or complementary? Can we support our analytical results with empirical evidence?*

From the perspective of incentive and newsvendor theories, we address the preceding research questions by analyzing a stylized supply chain model that includes a supplier, a capital-constrained retailer with limited liability, and a bank. To maximize its own profit, the retailer, equipped with some but insufficient internal capital, borrows either one or two types of credits to procure products from the supplier and then resells them to end consumers. Due to asymmetric information, the retailer has incentives to divert the credits to other risky projects; as a result, both the supplier and the bank would set credit limits to prevent potential moral hazard. The supply chain game is thus solved by maximizing the retailer's profit in terms of optimal ordering, conditional on the credit limits and interest rates set by the creditors–the bank and the supplier. We then use data from firms in China to verify our theoretical prediction on the substitutability and complementarity of the two credit types.

We first analyze the scenario where only one credit type is viable. We find that, if the internal capital is substantially high, the retailer can make a full investment by borrowing either credit as if there were no capital constraint. But, if the internal capital is substantially low, the retailer's order quantity decreases with the internal capital level. As the retailer's internal capital declines, both the credit limit and the interest rate first increase but then decrease, since the retailer, after exhausting the credit limit, becomes more likely to divert the credit owing to less internal capital. When the trade credit market is more competitive than the bank credit market, the retailer will borrow trade credit, which has a higher credit limit. When the bank credit market becomes more competitive, the retailer's preference of a specific credit type depends on the risk levels that the retailer would divert trade credit and bank credit to other risky investment. Given that the retailer is substantially capital constrained, if the retailer is less likely to divert trade credit, trade credit is preferred because of its higher credit limit; otherwise, bank credit is preferred because of its lower interest rate.

When both credit types are viable, we analyze two cases. In the first case we assume that the bank credit market is more competitive than the trade credit market. As the internal capital declines, our analysis indicates that the retailer first borrows only bank credit because of its lower interest rate. As the internal capital continues to decline, the retailer will then exhaust the bank credit limit and start borrowing some trade credit to boost total investment level. Nevertheless, when the internal capital level is substantially low, the retailer instead exhausts the trade credit limit prior to borrowing bank credit to obtain a higher total credit size. In the second case where the trade credit market is more competitive, we discover an unconventional result that the retailer never borrows bank credit. This is caused by a considerably reduced trade credit limit if the retailer also borrows from the bank. Because, knowing that the retailer is more likely to divert bank credit than trade credit, the supplier will significantly undercut the trade credit limit to avoid increased moral hazard risk. In this circumstance, the retailer adheres to borrowing only trade credit for the benefits of lower interest rate and higher credit limit.

We analytically demonstrate that, if the retailer's internal capital is substantially high, the credit sizes of both credit types are substitutable and complementary otherwise. We then hypothetically test this prediction using empirical evidence from a panel of 674 manufacturing firms listed in the Shanghai and Shenzhen Stock Exchanges in China over the period 2001-2007. We employ a simultaneous equations modeling with panel data approach. Our prediction is supported by statistically significant results.

The remainder of this paper is organized as follows. We review the extant literature in the next section and introduce the model in Section 3. We study the single-credit scenario in Section 4 and then elaborate the dual-credit scenario and discuss the complementarity and substitutability of the two credits in Section 5. We finally use empirical data to support our theoretical prediction regarding the complementarity and substitutability of the two credits in Section 6. We conclude in Section 7. All proofs are relegated to the Appendix–Online Supplements.

2 The Literature Review

Our work is related with the bank and trade credit literature from perspectives of global comparative studies, finance, economics, and especially operations management. Since bank credit has been well studied and understood, our following review focuses on trade credit.

In terms of global comparative studies, Coricelli (1996) argued that trade credit plays a key role in Poland's economic transition. Fafchamps (1997) studied trade credit in Zimbabwean manufacturing sector. McMillan and Woodruff (1999) analyzed the determinants of the prevalent use of trade credit among private firms in Vietnam. Using data from 352 firms in Russia in 1995, Cook (1999) offered evidence to support that trade credit works as a signal and firms that use trade credit have a higher probability of acquiring bank credit. Fisman and Love (2003) found that in countries with relatively weak financial institutions, firms in industries that rely heavily on trade credit have higher growth rates. Other researchers also inspected the development of trade credit in China and concluded that trade credit is an important informal credit channel and is regarded as the substitute credit channel for private sector firms with bank lending constraints (Allen et al., 2005; Cull and Xu, 2003; Ge and Qiu, 2007).

The finance and economics literature on trade credit has been the richest. Researchers have listed numerous merits of trade credit, such as price discrimination benefit (Petersen and Rajan, 1994, 1997), transaction cost reduction (Ferris, 1981; Ng et al., 1999; Nilsen, 2002), increasing financing creditability (Ng et al., 1999; Nilsen, 2002), quality guarantee (Smith, 1987; Long et al., 1993; Deloof and Jegers, 1995; Wilson and Summer, 2002), and others (Atanasova and Wilson, 2003; Emery, 1984; Petersen and Rajan, 1997). Most empirical studies (see Atanasova and Wilson, 2003; Ge and Qiu, 2007; Mateut et al., 2006; Meltzer, 1960; Petersen and Rajan, 1997) supported that trade credit plays a substitutable role to bank credit. For instance, using USA data, Nilsen (2002) showed that companies increase trade credit during monetary contractions. Guariglia and Mateut (2006) suggested that trade credit weakens bank credit in UK. Mateut et al. (2006) confirmed that when monetary policy tightened in UK, bank lending decreased relative to trade credit for manufacturing firms. However, several studies challenged the substitutability hypothesis, and suggested that trade credit plays a complementary role to bank credit. For instance, Biais and Gollier (1997) argued that trade credit acts as a signal to alleviate credit constraints caused by imperfect information, and suggested that trade credit and bank credit are complementary financial resources. Cook (1999) also supported the signaling role of trade credit and provided evidence for the complementary hypothesis. Note that none of the above empirical work has simultaneously documented both substitutability and complementarity between bank and trade credits.

Our work is mostly related to the operations management literature on trade credit. The extant literature has focused on the inventory control model with a newsvendor which our paper inherits. For example, Xu and Birge (2004) analyzed a single-period newsvendor model and showed how a firm's inventory decision is affected by its capital constraint and capital structure (debt/equity ratio). Dada and Hu (2008) considered a capital-constrained newsvendor who can borrow from a bank that acts strategically when choosing the terms (interest rate) of loans. Buzacott and Zhang (2004) studied a deterministic multi-period production/inventory control model and investigated the interplay between inventory decisions and asset-based financing. They concluded that asset-based financing allows retailers to enhance their cash return. In Caldentey and Haugh (2009), the supplier offers a menu of wholesale contracts (with different execution times and wholesale prices) and the retailer chooses the optimal timing to execute the contract. They demonstrated how financial markets can be used as a source of public information, upon which procurements contract can be utilized as a means of financial hedging to mitigate the effects of capital constraint. Other researchers also studied capital constrained supply chain and trade credit from inventory control perspectives (see Chao et al., 2008; Chen and Cai, 2011; Gupta and Wang, 2009; Haley and Higgins, 1973; Huang, 2004). Jing, Chen, and Cai (2012) studied the financing equilibrium in a model with both bank and trade credits. Kouvelis and Zhao (2011) discussed the optimal ordering decision in a newsvendor model where the retailer might go bankruptcy. Different from the above literature, our paper has incorporated both incentive and newsvendor theories to analytically study the retailer's optimal borrowing strategies in scenarios where either a single credit or dual credits are viable. We then use empirical panel data to validate the theoretical prediction regarding the substitutability and complementarity of the two credits.

Probably the mostly related work to ours is Burkart and Ellingsen (2004). Burkart and Ellingsen (2004) used a deterministic model to show that trade credit can be either complementary or substitutable to bank credit. They also explained why trade credit has shorter maturity and is more prevalent in less developed credit markets. Our paper differs from Burkart and Ellingsen (2004) in several aspects. First, Burkart and Ellingsen (2004)'s model is deterministic (i.e., no demand uncertainty). Consequently, they did not consider limited liability for the retailer from a supply chain management perspective. Second, they assumed that trade credit always has a higher interest rate than bank credit. We relax this assumption and discuss both cases where either trade credit or bank credit has a higher interest rate. Third, they did not analyze the single trade credit channel, and thus did not find out that a single trade credit channel could outperform a single bank credit channel, and the trade credit interest rate could be higher even if both credit markets are equally competitive. Neither had they observed that the retailer needs only one credit when their internal capital is substantially high, even if both credits are viable. Fourth, they did not use industry data to support the substitutability and complementarity of the two credits. In contrast, we use empirical panel data to verify the substitutability and complementarity between the two credits. We also analytically and empirically provide other managerial insights.

3 The Model

We consider a supply chain with a supplier, a capital-constrained retailer, and a bank. The retailer, who has limited liability, may borrow a loan/credit from the bank and/or the supplier. In the traditional case of bank credit (BC), the retailer borrows credit from the bank to purchase products, also called inputs, from the supplier.[1] In the trade credit (TC) case, the supplier allows the retailer to delay payment for its order for a certain period of time. In other words, the retailer borrows inputs instead of cash from the supplier. In both cases, there exists moral hazard, since the retailer

[1] In line with the literature, we assume both the bank and the supplier have sufficient capital, which is not necessarily true as evidenced in finance crises but allows tractability for obtaining main managerial insights.

might divert cash and inputs to other risky projects (Burkart and Ellingsen, 2004). Therefore, it is critical for the bank and the supplier to offer a well-designed credit contract, respectively, to reduce the moral hazard risk.

We assume the retailer has an internal capital B and orders inputs in a quantity of Q from the supplier. To obtain tractability, the wholesale price w and retail price p (normalized to 1) are assumed to be exogenously given, which is in line with the extant literature (see Burkart and Ellingsen, 2004; Mateut et al., 2006). The demand of inputs, D, is a random variable following a cumulative distribution function $F(D)$, whose density function is $f(D)$. We denote the investment of total inputs as $I = wQ$. The expected revenue is given by $\Pi(I) = \mathbb{E}p\min\{D, Q\} = \mathbb{E}\min\{D, Q\}$. The interest rate borrowing from the bank is denoted as r_b, while the interest rate borrowing from the supplier is denoted by r_t. The subscripts b and t represent bank credit and trade credit, respectively. Similar to the extant literature (see Burkart and Ellingsen, 2004; Dotan and Ravid, 1985; Mateut et al., 2006), we assume that the bank and the supplier are risk neutral and operate in competitive finance markets. Thus, the bank and the supplier's objectives are to obtain a profit equivalent to that of a risk-free interest rate, $\underline{r}^i, i = b, t$, in their respective market, where $\underline{r}^i$ is defined as

$$\underline{r}^i \equiv \begin{cases} r_f & if\ i = b, \\ r_s & if\ i = t. \end{cases}$$

In the presence of moral hazard, we assume the retailer can realize α (percentage) for every unit of diverted cash, while obtain $\alpha\beta$ for every unit of diverted inputs. Intuitively, diverting cash is easier than diverting inputs, that is, $\beta \in [0, 1)$ (Burkart and Ellingsen, 2004).[2]

We explore two scenarios. In the *single credit channel* scenario, the retailer can borrow only from a single creditor, either the bank or the supplier. The isolation of a particular credit type allows us to more profoundly characterize each individual credit type. In the *dual credit channels* scenario, the retailer may borrow from both the bank and the supplier. Unlike the existing literature assuming $r_f < r_s$ (e.g., Burkart and Ellingsen, 2004; Mateut et al., 2006), we explore both scenarios of $r_f < r_s$ and $r_f \geq r_s$ to more comprehensively investigate the retailer's optimal borrowing strategies in different credit markets.

[2] We assume imperfect legal protection for credits. Otherwise if $\alpha = 0$ and $\alpha\beta = 0$, the legal protection of creditors is perfect.

4 Single Credit Channel

In the single credit channel, the retailer borrows from a single creditor, either the bank or the supplier. We analyze the optimal contract terms, including credit limit and interest rate, for each credit type. Conditional on the retailer's internal capital B, the chosen creditor offers a credit contract $(L_i(B), r_i(B))$ to the capital-constrained retailer, where $i = b$ or t depending on which credit type is used. After accepting the contract, the retailer orders $Q_i(B)$ from the supplier. The credit size is, thus, $L_i(B) = wQ_i(B) - B$. Provided that the financial market is perfectly competitive, a creditor's expected cost equals its expected revenue. Hence, the interest rate $r_i(B)$ is determined by,

$$\mathbb{E}\min\{\min[D, Q_i(B)], L_i(B)(1 + r_i(B)) = L_i(B)(1 + \underline{r}^i)\}.$$

The retailer's objective is to maximize its profit by ordering $Q_i(B)$, which can be described as

$$\pi_i^R = \max_{Q_i(B)} \mathbb{E}\{(\min[D, Q_i(B)] - (wQ_i(B) - B)(1 + r_i(B)))^+\},$$

where superscript R denotes the retailer throughout this paper. Note that the retailer has limited liability, such that it collects a nonnegative profit if $\min[D, Q_i(B)] \geq (wQ_i(B) - B)(1 + r_i(B))$; otherwise, it announces bankruptcy and receives zero profit. For any given r_i, the optimal ordering level $Q_i^*(B)$ can thus be solved from $\bar{F}(Q_i(B)) = w(1 + r_i(B))\bar{F}[(wQ_i(B) - B)(1 + r_i(B))]$.

As the investment information is asymmetric, the retailer however has an incentive to borrow a larger credit and then diverts it to other projects. The retailer's limited liability enhances the incentive. The moral hazard indispensably drives the creditor, either the bank or the supplier, to control the corresponding hidden risk. To the end, the following constraint is enforced:

$$\mathbb{E}\{(\min[D, Q_i(B)] - L_i(B)(1 + r_i(B)))^+\} \geq \psi_i(L_i(B) + B),$$

where

$$\psi_i = \begin{cases} \alpha & if\ i = b, \\ \alpha\beta & if\ i = t. \end{cases}$$

This constraint states that the expected profit obtained from diverting the credit is no more than that without diverting. To make our discussion interesting, we assume $\psi_i > \underline{\psi_i} = \frac{\mathbb{E}\min[D, \frac{I^*(0)}{w}] - I^*(0)}{I^*(0)}$, where $I^*(0) = wQ^N$, and $Q^N = \bar{F}^{-1}(w)$ represents the retailer's optimal order quantity in the classic newsvendor model without capital constraint. That is, the retailer has an incentive to divert the cash/inputs when the cost of borrowing cash/inputs is zero.

Based on the preceding discussion, we summarize the retailer's optimization problem as follows:

$$
\begin{aligned}
&\pi_i^R: && \max_{Q_i(B)} \mathbb{E}\{(\min[D, Q_i(B)] - L_i(B)(1+r_i(B)))^+\} \\
&\text{Subject to (S.t.):} && \mathbb{E}\min\{\min[D, Q_i(B)], L_i(B)(1+r_i(B))\} = L_i(B)(1+\underline{r}^i) \quad (1a) \\
& && \mathbb{E}\{(\min[D, Q_i(B)] - L_i(B)(1+r_i(B)))^+\} \geq \psi_i(L_i(B)+B) \quad (1b) \\
& && \bar{L}_i(B) \geq L_i(B) = wQ_i(B) - B \quad (1c)
\end{aligned} \tag{1}
$$

In equilibrium, we must have $L_i(B) = \bar{L}_i(B)$ in Eqs. (1b); otherwise, the creditor can benefit from a larger $L_i(B)$. Reorganizing the above equations, we can rewrite Eq. (1) as follows:

$$
\begin{aligned}
&\pi_i^R: && \max_{Q_i(B)} \mathbb{E}\min[D, Q_i(B)] - L_i(B)(1+\underline{r}^i) \\
&\text{S.t.}: && \mathbb{E}\min\{\min[D, Q_i(B)], L_i(B)(1+r_i(B))\} = L_i(B)(1+\underline{r}^i) \quad (2a) \\
& && \mathbb{E}\min[D, Q_i(B)] - \bar{L}_i(B)(1+\underline{r}^i) = \psi_i(\bar{L}_i(B)+B) \quad (2b) \\
& && \bar{L}_i(B) \geq L_i(B) = wQ_i(B) - B \quad (2c)
\end{aligned} \tag{2}
$$

Note that the new objective function and Eq. $(2b)$ are obtained from integrating Eq. (1a) into the original objective function and Eq. (1b), respectively. Solving the above problem results in the equilibrium outcome in the single credit channel.

Proposition 1 *In the single credit channel, the optimal credit contract $(L_i^*(B), r_i^*(B))$ for the creditor, either the bank or the supplier, is given by*

$$
L_i^*(B) = \begin{cases} \bar{L}_i(B) & if\ B < \tilde{B}_i(\psi_i), \\ wQ_i^0 - B & if\ B \geq \tilde{B}_i(\psi_i), \end{cases}
$$

and

$$
r_i^*(B) = \begin{cases} r_i^0(B) & if\ B < \tilde{B}_i(\psi_i), \\ r_i^1(B) & if\ B \geq \tilde{B}_i(\psi_i). \end{cases}
$$

The retailer's optimal order quantity is given by

$$
Q_i^*(B) = \begin{cases} \frac{\bar{L}_i(B)+B}{w} & if\ B < \tilde{B}_i(\psi_i), \\ Q_i^0 & if\ B \geq \tilde{B}_i(\psi_i), \end{cases}
$$

where $\tilde{B}_i(\psi_i) = \frac{(1+\psi_i+\underline{r}^i)wQ_i^0 - \mathbb{E}\{\min[D,Q_i^0]\}}{1+\underline{r}^i}$; $Q_i^0 = \bar{F}^{-1}(w(1+\underline{r}^i))$; $\bar{L}_i(B)$ *solves* $\mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) = \psi_i(\bar{L}_i(B)+B)$; $r_i^0(B)$ *solves* $\mathbb{E}\min\{\min[D, \frac{\bar{L}_i(B)+B}{w}], \bar{L}_i(B)(1+r_i(B))\} = \bar{L}_i(B)(1+\underline{r}^i)$; *and* $r_i^1(B)$ *solves* $\mathbb{E}\min\{\min[D, Q_i^0], (wQ_i^0 - B)(1+r_i(B))\} = (wQ_i^0 - B)(1+\underline{r}^i)$.

Proposition 1 describes the optimal order quantity and credit contract terms. It shows that the retailer's internal capital level (B) significantly affects the retailer's investment plan. If the retailer's internal capital is substantially high (i.e., $B \geq \tilde{B}_i(\psi_i)$), a full investment, that is $I_i^* = I_i^0 = wQ_i^0 = L_i^0(B) + B$, is carried out with the aid of the creditor. In this circumstance, the borrowed credit size, $L_i^0(B)$, can be less than the allowable credit limit (i.e., $\bar{L}_i(B)$). If the retailer's internal capital is substantially low (i.e., $B < \tilde{B}_i(\psi_i)$), the imposed credit limit thwarts the retailer's full investment plan.

Theoretically, without moral hazard, the creditor can offer a contract of $(L_i^0(B), \hat{r}_i^0(B))$, where $L_i^0(B) = wQ_i^0 - B$ and $\hat{r}_i^0(B)$ is determined by $\mathbb{E}\min\{\min[D, Q_i^0], (wQ_i^0 - B)(1 + r_i(B))\} = (wQ_i^0 - B)(1 + \underline{r}^i)$. As a result, the retailer can make a full investment with any internal capital (i.e., $I_i^0 = L_i^0(B) + B = wQ_i^0$ which is equivalent to the case with sufficient internal capital). Compared with the preceding situation with moral hazard, the retailer's profit is apparently deteriorated by its intention of diversion, when its internal capital is insufficient.

Corollary 1 *For any ψ_i satisfying our assumption, (i) $r_i^0(B)$ increases with $B \in [0, \tilde{B}_i(\psi_i))$ and $r_i^1(B)$ decreases with $B \in [\tilde{B}_i(\psi_i), wQ_i^0]$; (ii) If $r_f = r_s$ and $B < \hat{B} = \min[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)]$, then $r_b^*(B) \leq r_t^*(B)$ and $\bar{L}_t(B) > \bar{L}_b(B)$.*

As Corollary 1 indicates, the interest rate increases with the internal capital level for both credit types if $B \in [0, \tilde{B}_i(\psi_i))$. This occurs because, after exhausting the credit limit, the retailer becomes more likely to divert the credit owing to the low internal capital and limited liability. When the internal capital is substantially high such that the retailer would not exhaust the credit limit, the interest rate decreases with the internal capital level on account of a lower diversion risk.

Given $r_f = r_s$ and $B < \hat{B} = \min[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)]$, the retailer exhausts the credit limit under either credit type. We find that, in this special case, the supplier charges a higher interest rate than the bank, because the retailer has a lower diversion risk under trade credit than under bank credit. The lower diversion risk leads to a higher credit limit for trade credit, which in turn hikes the risk for the supplier if the retailer defaults. This observation is consistent with the reality that the trade credit interest rate is often higher than that of bank credit (see Burkart and Ellingsen, 2004; Guariglia and Mateut, 2006; Petersen and Rajan, 1997; Smith, 1987).

We further use the following numerical example to highlight some characteristics of the single credit case conditional on $r_f = r_s$. Suppose $w = 0.7$, $r_f = r_s = 0.06$, $\alpha = 0.6$ and $\beta = 0.7$. We assume that demand follows an exponential distribution with a mean value 1000. From Figure 1, we confirm Corollary 1 that the interest rates first increase and then decrease as the internal capital grows. We also observe that the trade credit interest rate is higher than the bank credit interest rate as long as the internal capital is substantially low (i.e., $B < \widetilde{B}_b(\psi_b)$). Figure 2 shows the credit sizes first increase and then decrease as the internal capital grows. Consistent with the above analytical discussion, if $B < \widetilde{B}_b(\psi_b)$, the trade credit limit is significantly higher than the bank credit limit.

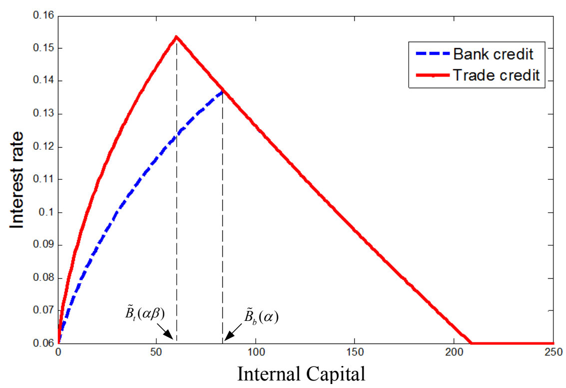

Figure 1: Comparison of r_i w.r.t. B with a single credit channel given $r_f = r_s$.

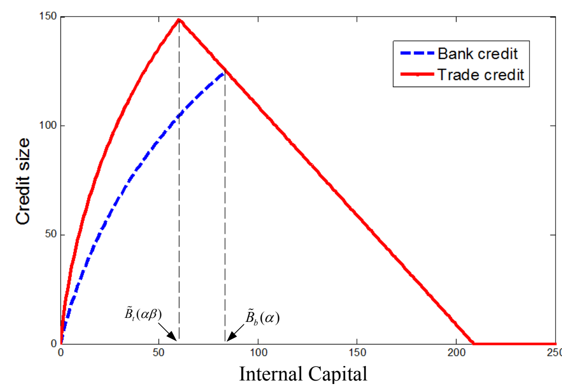

Figure 2: Comparison of L_i w.r.t. B with a single credit channel given $r_f = r_s$.

The preceding special case indicates that trade credit edges out bank credit if $r_f = r_s$; however, would this result sustain if $r_f \neq r_s$? To answer this question, we now analyze a more general case where both credit types are not equally competitive (i.e., $r_f \geq r_s$ or $r_f < r_s$). We find that, if the retailer's internal capital level is substantially high such that the retailer does not need to exhaust either credit limit, the retailer borrows from the credit type with a lower risk-free interest rate. This is intuitive since a lower risk-free interest rate renders the retailer a lower borrowing cost.

Now consider the case where the retailer's internal capital level is in the middle. If $\min[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)] = \tilde{B}_t(\alpha\beta) < B < \tilde{B}_b(\alpha) = \max[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)]$, the retailer borrows trade credit if $\bar{L}_b(B) + B \leq \frac{\mathbb{E}\min[D, Q_t^0] - (wQ_t^0 - B)(1+r_s)}{\alpha}$ and bank credit otherwise. In this situation, whether the retailer borrows only from the trade credit or the bank credit depends on their credit limits and interest rates. If $\min[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)] = \tilde{B}_b(\alpha) < B < \tilde{B}_t(\alpha\beta) = \max[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)]$, the retailer borrows bank credit if $\bar{L}_t(B) + B \leq \frac{\mathbb{E}\min[D, Q_b^0] - (wQ_b^0 - B)(1+r_f)}{\alpha\beta}$ and trade credit otherwise.

What will happen if $B < \hat{B} = \min[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)]$? In this situation, the retailer exhausts the credit limit of either credit type that is chosen. We use the following proposition to characterize the retailer's borrowing strategy.

Proposition 2 *Suppose the retailer will borrow only either bank credit or trade credit. Given* $B < \hat{B} = \min[\tilde{B}_b(\alpha), \tilde{B}_t(\alpha\beta)]$*, we have*

1. *If* $r_f \geq r_s$*, the retailer borrows trade credit;*
2. *If* $r_f < r_s$*, then*
   (a) *If* $\bar{L}_b(B) + B \leq \beta(\bar{L}_t(B) + B)$*, the retailer borrows trade credit;*
   (b) *otherwise, the retailer borrows bank credit.*

Proposition 2 categorizes the retailer's borrowing strategies conditional on a substantially low internal capital level. If the trade credit market is more competitive (i.e., $r_f \geq r_s$), the retailer borrows trade credit. This is consistent with the case where $r_f = r_s$ as discussed in Corollary 1. Given a lower interest rate in addition to a higher credit limit, trade credit eclipses bank credit as the priority for the retailer who is substantially capital constrained. When the bank market is more competitive (i.e., $r_f < r_s$), bank credit gains some momentum against trade credit; thus, the retailer faces a trade-off. On the one hand, the interest rate provided by the bank is lower than that by the supplier. On the other hand, the trade credit limit can be higher. The trade credit limit is negatively affected by the relative diversion risk level β. That is, as β becomes smaller, the retailer has fewer incentives to deviate the inputs; consequently, the trade credit limit enhances. That is, $\bar{L}_t(B)$ increases faster than β decreases. If β is substantially small such that $\bar{L}_b(B) + B \leq \beta(\bar{L}_t(B) + B)$, the retailer borrows trade credit to take advantage of the high credit limit; otherwise, $\bar{L}_b(B) + B > \beta(\bar{L}_t(B) + B)$ and the retailer borrows bank credit to benefit from the lower bank interest rate.

Table 1 illustrates the characteristics indicated by Proposition 2. We adopt a similar setting to the preceding numerical example in which the demand follows an exponential distribution with a mean value of 1000, $B = 20$, $w = 0.7$, and $\alpha = 0.6$. We define $\Delta\pi = \pi_t^R - \pi_b^R$. Table 1 shows that the retailer prefers trade credit financing to bank credit financing if $r_s \leq r_f$. If $r_s > r_f$, its preference also hinges on β, as discussed previously. The case of $r_s = r_f$ is consistent with Corollary 1.

Table 1: The retailer's profit under trade credit minus bank credit in a single credit channel.

r_f & r_s	$\beta = 0.75$	$\beta = 0.80$	$\beta = 0.85$	$\beta = 0.90$	$\beta = 1$
$r_f = 0.01$, $r_s = 0.05$	$\Delta\pi = 4.83$	$\Delta\pi = 2.91$	$\Delta\pi = 1.06$	$\Delta\pi = -0.69$	$\Delta\pi = -3.81$
$r_f = 0.05$, $r_s = 0.05$	$\Delta\pi = 8.64$	$\Delta\pi = 6.72$	$\Delta\pi = 4.87$	$\Delta\pi = 3.12$	$\Delta\pi = 0$
$r_f = 0.05$, $r_s = 0.01$	$\Delta\pi = 15.21$	$\Delta\pi = 12.70$	$\Delta\pi = 10.25$	$\Delta\pi = 7.94$	$\Delta\pi = 3.81$

5 Dual Credit Channels

So far, we have studied cases where only one credit type is viable. In reality, both credit types can coexist. Therefore, it is meaningful to study the scenario where the retailer borrows both bank credit and trade credit. Intuitively, if the retailer's internal capital is not substantially low, the retailer borrows only one type of credit, of which the optimal solutions have been described in Propositions 1. As confined with the credit limit, however, a single credit channel might not provide sufficient funds to the capital-constrained retailer. As a result, the retailer has to borrow from both the bank and the supplier. In a dual-credit case, it is straightforward that the retailer will exhaust the credit limit of the more attractive credit type before borrowing the other one. When the internal capital level is substantially low, depending on which credit market is more competitive, the retailer might exhaust the bank credit limit or the trade credit limit before borrowing the other credit type. Accordingly, we divide our following discussions into two cases based on the comparison of risk-free interest rates.

5.1 Bank Credit Market Is More Competitive

In the dual-credit scenario when bank credit market is more competitive (i.e., $r_f < r_s$), if the internal capital level is substantially high, the retailer does not need to borrow trade credit. In this circumstance, the dual-credit scenario is degenerated into the single-credit scenario as discussed in Section 4, since borrowing only bank credit benefits the retailer with a lower interest rate. When the internal capital level is low, the bank credit limit becomes insufficient for the retailer to make a full investment. Would the retailer always exhausts the bank credit limit at first and then borrows some trade credit before both credit limits are used up? The answer is that it depends on the internal capital level.

To explore the retailer's borrowing strategy, we compare two previously studied single-credit cases with two dual-credit cases that are specified as follows. In the first dual-credit case, called *bank credit priority*, the retailer always exhausts the bank credit limit at first, if necessary, before borrowing trade credit. In contrast, in the second case, called *trade credit priority*, the retailer always exhausts trade credit limit at first, if necessary, before borrowing bank credit. The retailer's preference of any specific case among four cases depends on which case yields the highest profit. It is intuitive that both dual-credit cases occur only if the internal capital is substantially low; otherwise, borrowing only a single credit (i.e., bank credit, provided that $r_f < r_s$) dominates other alternatives. Since we have elaborated the single credit cases in Section 4, we now explore bank credit priority and then trade credit priority as follows.

Bank Credit Priority (BCP)

Given that the internal capital is substantially low, the retailer's *perceived* internal capital before using the trade credit is $\mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_b(B))$, where $Q_t(B)$ is the total order quantity as the retailer starts to borrow trade credit in addition to bank credit. Since the market is competitive, the supplier's objective is to make a profit equal to that with a risk-free interest rate, described as follows:

$$\mathbb{E}\min\{\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_b(B)), L_t(B)(1+r_t)\} = L_t(B)(1+r_s),$$

where $L_t(B) = wQ_t(B) - B - \bar{L}_b(B)$. For the retailer, the credit limit of the bank credit (i.e., $\bar{L}_b(B)$) is used up and some trade credit (i.e., $L_t(B)$) is deployed. Thus, the total investment is $I_t(B) = \bar{L}_b(B) + L_t(B) + B$. The retailer's optimal order quantity, $Q_t(B)$, is determined by maximizing its objective function as follows:

$$\pi^R_{BCP} = \max_{Q_t(B)} \mathbb{E}\{[\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_b(B)) - L_t(B)(1+r_t(B))]^+\}.$$

To avoid moral hazard caused by asymmetric information, the supplier and the bank have to enforce additional constraints to prevent the retailer from diverting all cash or all inputs, respectively, as described below:

$$\mathbb{E}\{[\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)) - L_t(B)(1 + r_t(B))]^+\} \geq \alpha(\bar{L}_b(B) + B),$$

$$\mathbb{E}\{[\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)) - L_t(B)(1 + r_t(B))]^+\} \geq \alpha\beta I_t(B).$$

Summarizing the above objective function and constraints, we can write the optimization problem for the retailer using both bank credit and trade credit as follows:

$$\begin{array}{ll}
\pi^R_{BCP}: & \max\limits_{Q_t(B)} \mathbb{E}\{[\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)) - L_t(B)(1 + r_t(B))]^+\} \\
\text{S.t.:} & \mathbb{E}\min\{\min[D, Q_t(B)], \bar{L}_b(B)(1 + r_b(B))\} = \bar{L}_b(B)(1 + r_f) \quad (3a) \\
 & \mathbb{E}\min\{\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)), L_t(B)(1 + r_t(B))\} = L_t(B)(1 + r_s) \quad (3b) \\
 & \mathbb{E}\{[\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)) - L_t(B)(1 + r_t(B))]^+\} \geq \alpha(\bar{L}_b(B) + B) \quad (3c) \\
 & \mathbb{E}\{[\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)) - L_t(B)(1 + r_t(B))]^+\} \geq \alpha\beta w Q_t(B) \quad (3d) \\
 & \bar{L}_t(B) \geq L_t(B) = wQ_t(B) - B - \bar{L}_b(B) \quad (3e)
\end{array} \tag{3}$$

Based on Eqs. (3a) and (3b), we can rewrite the objective function of Eq. (3) as (see the Appendix for the proof)

$$\mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_f) - L_t(B)(1 + r_s).$$

In equilibrium, the bank would set the credit limit at the level that the retailer would not divert the available cash including the bank credit; thus, Eq. (3c) must hold at equality. So does Eq. (3d) such that the supplier would set the trade credit limit at the level that the retailer would not divert the inputs. As a result, we can rewrite Eq. (3) as

$$\begin{array}{ll}
\pi^R_{BCP}: & \max\limits_{Q_t(B)} \mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_f) - L_t(B)(1 + r_s) \\
\text{S.t.:} & \mathbb{E}\min\{\min[D, Q_t(B)], \bar{L}_b(B)(1 + r_b(B))\} = \bar{L}_b(B)(1 + r_f) \quad (4a) \\
 & \mathbb{E}\min\{\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_b(B)), L_t(B)(1 + r_t(B))\} = L_t(B)(1 + r_s) \quad (4b) \\
 & \mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_f) - L_t(B)(1 + r_s) = \alpha(\bar{L}_b(B) + B) \quad (4c) \\
 & \mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1 + r_f) - L_t(B)(1 + r_s) = \alpha\beta w Q_t(B) \quad (4d) \\
 & \bar{L}_t(B) \geq L_t(B) = wQ_t(B) - B - \bar{L}_b(B) \quad (4e)
\end{array} \tag{4}$$

Trade Credit Priority (TCP)

In this case, the retailer exhausts the trade credit limit before borrowing bank credit. Similar to the bank credit priority case as stated in Eq. (4), the retailer's problem can be described as follows:

$$
\begin{aligned}
\pi^R_{TCP}: \quad & \max_{Q_b(B)} \mathbb{E}\min[D, Q_b(B)] - \bar{L}_t(B)(1+r_s) - L_b(B)(1+r_f) \\
\text{S.t.:} \quad & \mathbb{E}\min\{\min[D, Q_b(B)], \bar{L}_t(B)(1+r_t(B))\} = \bar{L}_t(B)(1+r_s) \quad (5a) \\
& \mathbb{E}\min\{\min[D, Q_b(B)] - \bar{L}_t(B)(1+r_t(B)), L_b(B)(1+r_b(B))\} = L_b(B)(1+r_f) \quad (5b) \\
& \mathbb{E}\min[D, Q_b(B)] - \bar{L}_t(B)(1+r_s) - L_b(B)(1+r_f) = \alpha L_b(B) \quad (5c) \\
& \mathbb{E}\min[D, Q_b(B)] - \bar{L}_t(B)(1+r_s) - L_b(B)(1+r_f) = \alpha\beta w Q_b(B) \quad (5d) \\
& \bar{L}_b(B) \geq L_b(B) = wQ_b(B) - B - \bar{L}_t(B) \quad (5e)
\end{aligned}
\tag{5}
$$

We then solve Eqs. (4) and (5) and compare them with the two single-credit cases. The retailer chooses the case generating the highest profit. The equilibrium solutions for the retailer, the bank, and the supplier are summarized in the following proposition.

Proposition 3 *In the dual-credit scenario when bank credit market is more competitive (i.e., $r_f < r_s$), there exist $\tilde{B}^t_1(\alpha\beta) < \tilde{B}^b_1(\alpha) < \tilde{B}^b_2(\alpha) < \tilde{B}^b_3(\alpha)$ such that the retailer's optimal borrowing strategy is:*

$$
\begin{cases}
\text{if } B \geq \tilde{B}^b_3(\alpha), & \text{Borrows bank credit only;} \\
\text{if } \tilde{B}^b_2(\alpha) \leq B < \tilde{B}^b_3(\alpha), & \text{Borrows bank credit only and exhausts the credit limit;} \\
\text{if } \tilde{B}^b_1(\alpha) \leq B < \tilde{B}^b_2(\alpha), & \text{Exhausts the bank credit limit and borrows trade credit;} \\
\text{if } \tilde{B}^t_1(\alpha\beta) \leq B < \tilde{B}^b_1(\alpha), & \text{Exhausts the trade credit limit and borrows bank credit;} \\
\text{if } B < \tilde{B}^t_1(\alpha\beta), & \text{Exhausts both credits.}
\end{cases}
$$

The optimal order quantity and contract terms are summarized in Table 2, where $\tilde{B}^t_1(\alpha\beta) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I^0_t - \Pi(I^0_t)}{1+r_s}$, $\tilde{B}^b_1(\alpha) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I^0_t - \Pi(I^0_t)}{1+r_f}$, $\tilde{B}^t_2(\alpha\beta) = \frac{[1+r_s+\alpha\beta]I^0_t - \Pi(I^0_t)}{1+r_s}$, $\tilde{B}^b_2(\alpha) = \frac{(1+\alpha+r_f)I^0_t - \Pi(I^0_t)}{1+r_f}$, $\tilde{B}^b_3(\alpha) = \frac{(1+\alpha+r_f)I^0_b - \Pi(I^0_b)}{1+r_f}$, $I^0_b = wQ^0_b$, $I^0_t = wQ^0_t$, $Q^0_b = \bar{F}^{-1}(w(1+r_f))$, and $Q^0_t = \bar{F}^{-1}(w(1+r_s))$. In addition, $\bar{L}^1_b(B)$, $\bar{L}^2_b(B)$, $\bar{L}^3_b(B)$, $\bar{L}^1_t(B)$ and $\bar{L}^2_t(B)$ solve the following equations, respectively,

$$
\begin{aligned}
&\mathbb{E}\min[D, \frac{\bar{L}^1_b(B)+B}{\beta w}] - \bar{L}^1_b(B)(1+r_f) - \frac{(1-\beta)(1+r_s)}{\beta}(\bar{L}^1_b(B)+B) = \alpha(\bar{L}^1_b(B)+B), \\
&\mathbb{E}\min[D, Q^0_t] - \bar{L}^2_b(B)(1+r_f) - (wQ^0_t - \bar{L}^2_b(B) - B)(1+r_s) = \alpha(\bar{L}^2_b(B)+B), \\
&\mathbb{E}\min[D, \frac{\bar{L}^3_b(B)+B}{w}] - \bar{L}^3_b(B)(1+r_f) = \alpha(\bar{L}^3_b(B)+B), \\
&\mathbb{E}\min[D, \frac{\bar{L}^1_t(B)+B}{(1-\beta)w}] - \frac{\beta(1+r_f)}{1-\beta}(\bar{L}^1_t(B)+B) - \bar{L}^1_t(B)(1+r_s) = \frac{\alpha\beta(\bar{L}^1_t(B)+B)}{1-\beta}, \\
&\mathbb{E}\min[D, Q^0_t] - (wQ^0_t - \bar{L}^2_t(B) - B)(1+r_f) - \bar{L}^2_t(B)(1+r_s) = \alpha\beta w Q^0_t.
\end{aligned}
$$

$r_b^0(B)$, $r_b^2(B)$, $r_b^3(B)$, $\tilde{r}_b^1(B)$ *and* $\tilde{r}_b^2(B)$ *solve the following equations, respectively,*

$$\mathbb{E}\min\{\min[D, Q_b^0], (wQ_b^0 - B)(1 + r_b^0(B))\} = (wQ_b^0 - B)(1 + r_f),$$
$$\mathbb{E}\min\{\min[D, Q_t^0], \bar{L}_b^2(B)(1 + r_b^2(B))\} = \bar{L}_b^2(B)(1 + r_f),$$
$$\mathbb{E}\min\{\min[D, \frac{\bar{L}_b^3(B) + B}{w}], \bar{L}_b^3(B)(1 + r_b^3(B))\} = \bar{L}_b^3(B)(1 + r_f),$$
$$\mathbb{E}\min\{\min[D, \frac{\bar{L}_t^1(B) + B}{(1-\beta)w}] - \bar{L}_t^1(B)(1 + r_t^1(B)), \frac{\beta}{1-\beta}(\bar{L}_t^1(B) + B)(1 + \tilde{r}_b^1(B))\}$$
$$= \frac{\beta}{1-\beta}(\bar{L}_t^1(B) + B)(1 + r_f),$$
$$\mathbb{E}\min\{\min[D, Q_t^0] - \bar{L}_t^2(B)(1 + r_t^2(B)), (I_t^0 - \bar{L}_t^2(B) - B)(1 + \tilde{r}_b^2(B))\} = (I_t^0 - \bar{L}_t^2(B) - B)(1 + r_f).$$

$r_t^1(B)$, $r_t^2(B)$ *and* $\tilde{r}_t^2(B)$ *solve the following equations, respectively,*

$$\mathbb{E}\min\{\min[D, \frac{\bar{L}_t^1(B) + B}{(1-\beta)w}], \bar{L}_t^1(B)(1 + r_t^1(B))\} = \bar{L}_t^1(B)(1 + r_s),$$
$$\mathbb{E}\min\{\min[D, Q_t^0], \bar{L}_t^2(B)(1 + r_t^2(B))\} = \bar{L}_t^2(B)(1 + r_s),$$
$$\mathbb{E}\min\{\min[D, Q_t^0] - \bar{L}_b^2(B)(1 + r_b^2(B)), (I_t^0 - \bar{L}_b^2(B) - B)(1 + \tilde{r}_t^2(B))\} = (I_t^0 - \bar{L}_b^2(B) - B)(1 + r_s).$$

Table 2: Optimal solution for the dual-credit scenario when $r_f < r_s$.

B	Credit Type(s)	L_b^*	r_b^*	L_t^*	r_t^*	Q_t^*
$B \geq \tilde{B}_3^b(\alpha)$	BC	$wQ_b^0 - B$	$r_b^0(B)$	N/A	N/A	Q_b^0
$\tilde{B}_2^b(\alpha) \leq B < \tilde{B}_3^b(\alpha)$	BC limit	$\bar{L}_b^3(B)$	$r_b^3(B)$	N/A	N/A	$\frac{\bar{L}_b^3(B)+B}{w}$
$\tilde{B}_1^b(\alpha) \leq B < \tilde{B}_2^b(\alpha)$	BC limit & TC	$\bar{L}_b^2(B)$	$r_b^2(B)$	$I_t^0 - \bar{L}_b^2(B) - B$	$\tilde{r}_t^2(B)$	Q_t^0
$\tilde{B}_1^t(\alpha\beta) \leq B < \tilde{B}_1^b(\alpha)$	TC limit & BC	$I_t^0 - \bar{L}_t^2(B) - B$	$\tilde{r}_b^2(B)$	$\bar{L}_t^2(B)$	$r_t^2(B)$	Q_t^0
$B < \tilde{B}_1^t(\alpha\beta)$	Both limits	$\frac{\beta}{1-\beta}(\bar{L}_t^1(B) + B)$	$\tilde{r}_b^1(B)$	$\bar{L}_t^1(B)$	$r_t^1(B)$	$\frac{\bar{L}_t^1(B)+B}{(1-\beta)w}$

Proposition 3 describes the optimal solution for all players in the dual-credit scenario given $r_f < r_s$. If the internal capital is substantially high (i.e., $B \geq \tilde{B}_3^b(\alpha)$), the retailer borrows only bank credit thanks to its lower interest rate and invests at its highest level (i.e., I_b^0). If the internal capital is less sufficient such that $\tilde{B}_2^b(\alpha) \leq B < \tilde{B}_3^b(\alpha)$, the retailer borrows only bank credit and exhausts its limit. In this situation, the retailer is reluctant to borrow trade credit because of its higher interest rate. As a result, the corresponding investment slides from I_b^0 to I_t^0 because of insufficient capital as B declines. But, as B decreases to the domain of $[\tilde{B}_1^b(\alpha), \tilde{B}_2^b(\alpha))$, the benefit of ordering a larger quantity surpasses the drawback of the higher trade credit interest rate. The retailer thus exhausts the bank credit limit and borrows some trade credit. When $\tilde{B}_1^t(\alpha\beta) \leq B < \tilde{B}_1^b(\alpha)$, the retailer is constrained by the credit limit of bank credit and has to switch to exhausting the trade credit limit–which is significantly higher than that of bank credit–while borrowing some bank credit without exhausting its limit. This furnishes the retailer with a higher total credit size because of the higher trade credit limit. As B shrinks to its lowest level (i.e., $B < \tilde{B}_1^t(\alpha\beta)$), the retailer exhausts both credit limits.

We use the following numerical example to further illustrate Proposition 3. We adopt a market setting of $w = 0.7$, $r_f = 0.04$, $r_s = 0.08$, $\alpha = 0.6$, $\beta = 0.7$, and that the demand follows an exponential distribution with a mean value 1000. Figure 3 demonstrates that, as B increases, the credit sizes in all four cases, including the two single-credit cases and the two dual-credit cases, increase when B is substantially low but decrease when B is substantially high. This is because the credit size is constrained by the credit limits when B is substantially low. When B is substantially high, the retailer's borrowed credit size diminishes with B. This observation supports Proposition 3. The retailer will borrow from the credit type with the lower interest rate (i.e., bank credit in this case where $r_f < r_s$) when B is substantially high. But, the need to borrow more money, even at a higher interest rate, becomes more imperative as B shrinks. As illustrated in Figure 3, this impetus pressures the retailer to first exhaust the bank credit limit and then switch to using up the trade credit limit and finally both credit limits. Accordingly, as shown in Figure 4, the retailer's ordering level decreases piece-wisely as B reduces.

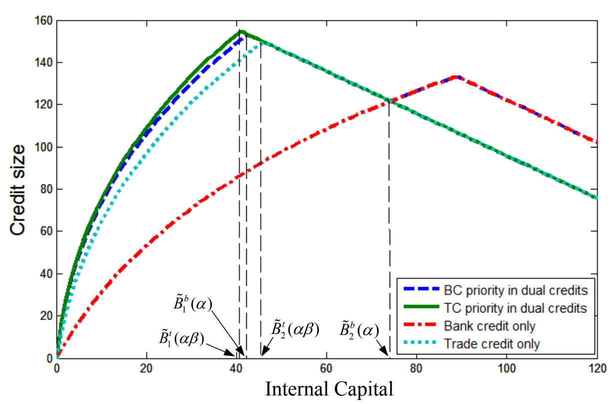

Figure 3: Comparison of L_i w.r.t. B with dual credits given $r_f < r_s$ with bank credit priority.

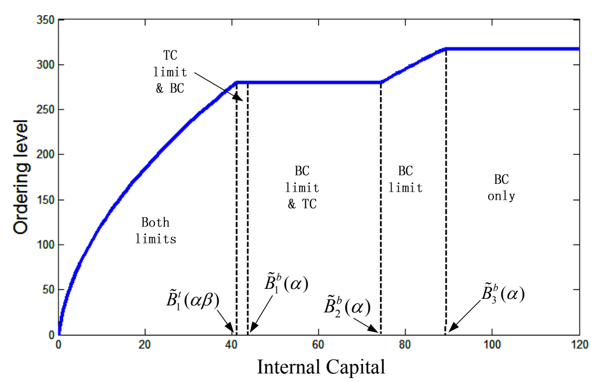

Figure 4: Comparison of Q_i w.r.t. B with dual credits given $r_f < r_s$ with bank credit priority.

From Proposition 3, we also infer the following corollary.

Corollary 2 *Given $r_f < r_s$, the retailer's investment level in single credit channel case is lower* *than that in the dual credit channels case with bank credit priority when $B < \tilde{B}_2^b(\alpha)$ and with trade credit priority when $B < \tilde{B}_2^t(\alpha\beta)$, respectively.*

Corollary 2 suggests that, with bank credit priority, the retailer's borrowing trade credit reduces the risk for the bank, which is empirically supported by Cook (1999). As a result, when the internal capital is substantially low (i.e., $B < \tilde{B}_2^b(\alpha)$), the retailer is able to order more with dual credits than with a single bank credit. We can obtain a similar result for the dual-credit trade credit priority case while comparing it with the single trade credit case when $B < \tilde{B}_2^t(\alpha\beta)$.

5.2 Trade Credit Market Is More Competitive

We now investigate the case where the trade credit market is more competitive than the bank credit market (i.e., $r_f \geq r_s$). This can occur especially when the supplier offers a low-interest trade credit to encourage the retailer to place a bigger order. As demonstrated in Proposition 2, trade credit dominates bank credit for the retailer when $r_f \geq r_s$ in a single-credit channel scenario. Will this phenomenon sustain when both bank credit and trade credit are viable? The answer is yes.

To elaborate, similar to the analysis in Section 5.1, we compare the retailer's profits in two single-credit cases and two dual-credit cases. The dual-credit cases are described in Eqs. (4) and (5), respectively, but conditional on $r_f \geq r_s$. We obtain the following proposition.

Proposition 4 *In the dual-credit scenario when the trade credit market is more competitive (i.e., $r_f \geq r_s$), the capital-constrained retailer borrows only trade credit from the supplier. When $B < \tilde{B}_2^t(\alpha)$, the retailer uses up the trade credit limit. We summarize the optimal order quantity and contract terms in Table 3, where $\tilde{B}_2^t(\alpha) = \frac{(1+\alpha\beta+r_s)I_t^0 - \Pi(I_t^0)}{1+r_s}$, $I_t^0 = wQ_t^0$, $Q_t^0 = \bar{F}^{-1}(w(1+r_s))$, and $\Pi(I_t^0) = \mathbb{E}\min[D, \frac{I_t^0}{w}]$; $\bar{L}_t^1(B)$ solves*

$$\mathbb{E}\min[D, \frac{\bar{L}_t^1(B) + B}{w}] - \bar{L}_t^1(B)(1+r_s) = \alpha\beta(\bar{L}_t^1(B) + B).$$

$r_t^0(B)$ and $r_t^1(B)$ solve the following equations, respectively:

$$\mathbb{E}\min\{\min[D, Q_t^0], (wQ_t^0 - B)(1 + r_t^0(B))\} = (wQ_t^0 - B)(1 + r_s),$$

$$\mathbb{E}\min\{\min[D, \frac{\bar{L}_t^1(B) + B}{w}], \bar{L}_t^1(B)(1 + r_t^1(B))\} = \bar{L}_t^1(B)(1 + r_s).$$

Table 3: Optimal solution for the dual-credit scenario when $r_f \geq r_s$.

B	Credit Type(s)	L_t^*	r_t^*	L_b^*	r_b^*	Q_b^*
$B \geq \tilde{B}_2^t(\alpha)$	TC	$wQ_t^0 - B$	$r_t^0(B)$	N/A	N/A	Q_t^0
$B < \tilde{B}_2^t(\alpha)$	TC limit	$\bar{L}_t^1(B)$	$r_t^1(B)$	N/A	N/A	$\frac{\bar{L}_t^1(B)+B}{w}$

The conventional wisdom suggests that, if the retailer's internal capital is substantially low, the retailer would borrow from more credit channels. This is well supported by Proposition 3 where the bank credit market is more competitive (i.e., $r_f < r_s$). Given $r_f \geq r_s$, however, Proposition 4 demonstrates that the retailer would use only trade credit, even if its internal capital is substantially low. Why wouldn't it be the same when the trade credit market is more competitive (i.e., $r_f \geq r_s$)?

The argument stemming from $r_f < r_s$ bears a certain allure, but does not apply in the context of $r_f \geq r_s$. When the retailer's internal capital is substantially high (i.e., $B \geq \tilde{B}_2^t(\alpha)$), it is straightforward that only trade credit is needed because of its lower interest rate. When the retailer has insufficient internal capital (i.e., $B < \tilde{B}_2^t(\alpha)$), the retailer, after exhausting the trade credit limit, is expected to borrow bank credit. But, if the retailer also borrows from the bank, the supplier would significantly reduce the trade credit limit, since a higher bank credit diversion rate makes the entire investment riskier for the supplier. Therefore, borrowing a higher interest rate bank credit on top of trade credit not only lowers the marginal profit but yields a lower overall credit limit for the retailer.

We use the following numerical example to further illustrate Proposition 4. Suppose $w = 0.7$, $r_f = 0.08$, $r_s = 0.05$, $\alpha = 0.8$, $\beta = 0.6$, and that the demand follows an exponential distribution with a mean value 1000. As depicted in Figure 5, if the retailer borrows bank credit at first and then trade credit (bank credit priority), the retailer obtains a higher overall credit limit than in a single credit channel although at a higher interest rate. But, the combined credit limit is less than the single trade credit limit. If the retailer borrows trade credit at first and then bank credit (trade credit priority), consistent with Proposition 4, the supplier would significantly reduce its credit limit on account of a higher moral hazard risk caused by potential diversion of bank credit. Therefore, given $r_f \geq r_s$, it is more profitable for the retailer to borrow only trade credit. Correspondingly, as shown in Figure 6, the optimal order quantity using only trade credit is no less than that of using both credits.

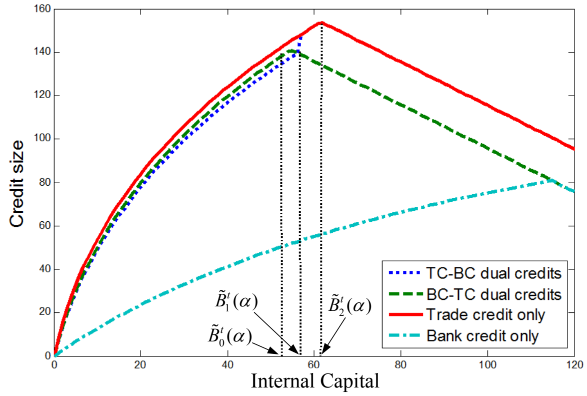

Figure 5: Comparison of credit sizes w.r.t. B with dual credits given $r_f \geq r_s$.

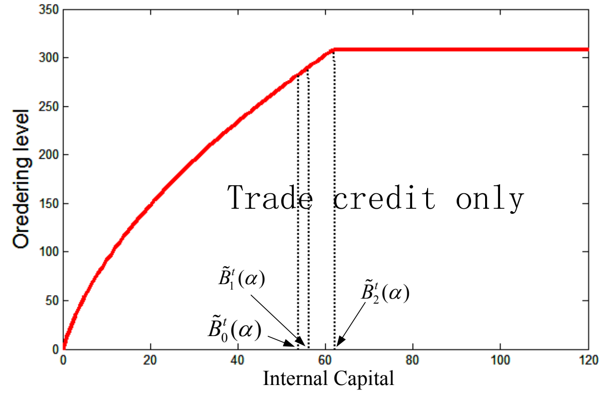

Figure 6: Ordering level as a function of B with dual credits given $r_f \geq r_s$.

5.3 Complementarity and Substitutability between Bank Credit and Trade Credit

Researchers have been arguing on whether bank credit and trade credit are complementary or substitutable (see Biais and Gollier, 1997; Cook, 1999; Guariglia and Mateut, 2006; Mateut et al., 2006; Nilsen, 2002). Notwithstanding, none have simultaneously provided both theoretical analysis and empirical evidence. To fill such a literature gap, we first theoretically show both credit types can be either complementary or substitutable and demonstrate the empirical evidence in the next section.

To the end, we focus on the dual-credit scenario when the bank credit market is more competitive (i.e., $r_s > r_f$). The main reason is that the supplier has incentives to charge a higher interest rate, which is supported by empirical data (see Burkart and Ellingsen, 2004; Guariglia and Mateut, 2006; Petersen and Rajan, 1997). This is also consistent with our preceding discussion that it can be an optimal solution for both credit types to coexist when $r_s > r_f$, while it is not when $r_s \leq r_f$.

Continuing from Proposition 3, we observe the following result.

Proposition 5 *In the dual-credit scenario when $r_f < r_s$, if $B < \tilde{B}_1^t(\alpha\beta)$, the credit sizes of bank credit and trade credit are complementary; otherwise if $\tilde{B}_1^t(\alpha\beta) \leq B \leq \tilde{B}_2^b(\alpha)$, the credit sizes of bank credit and trade credit are substitutable.*

Proposition 5 states that both credit types can be either complementary or substitutable in terms of credit sizes depending on the retailer's internal capital level. According to Proposition 3, if $B < \tilde{B}_1^t(\alpha\beta)$, the retailer uses up both credit limits. As the internal capital grows, the bank and the supplier both extend their credit limits, because a higher credit limit in one credit type reduces the risk for the other one. When $\tilde{B}_1^t(\alpha\beta) \leq B \leq \tilde{B}_2^b(\alpha)$, the retailer no longer needs to exhaust both credit limits. More specifically, if $\tilde{B}_1^t(\alpha\beta) \leq B < \tilde{B}_1^b(\alpha)$, the retailer exhausts only the trade credit limit and borrows some bank credit with trade credit priority. If $\tilde{B}_1^b(\alpha) \leq B \leq \tilde{B}_2^b(\alpha)$, the retailer exhausts only the bank credit limit and borrows some trade credit with bank credit priority. In both cases, the credit limit in either credit type increases as B grows; however, the total credit size diminishes on account of a larger B. Therefore, the credit sizes of both credit types substitute each other.

In the next section, we use empirical data to hypothetically test the above observation. Based on Proposition 5, *we predict that bank credit and trade credit would be complements if a company's internal capital is low, but substitutes if a company's internal capital is high.* To the end, we examine the empirical evidence from a panel of 674 manufacturing firms included in the Shanghai and Shenzhen Stock Exchanges in China over the period of 2001-2007.

6 Empirical Evidence

We now report empirical results to support the preceding theoretical prediction on the complementarity and substitutability between the two credit types and provide additional insights. We adopt a framework of simultaneous equations with panel data modeling. The data of our study come from China's Wind Financial Database, which contains historical observations of publicly traded firms in China.

6.1 Data

We focus on firms in the manufacturing sector with complete data for the period of 2001-2007. To highlight our prediction effect, we also inspect the impact of monetary policy, either tight or loose, on the relationship of trade credit and bank credit. Following Mateut et al. (2006), we use monetary stance (MS) and some specific firm characteristics as explanatory variables. Monetary stance is proxied by the required reserves ratio (RRR), a monetary policy instrument of the People's Bank of

China (PBoC), the central bank of China.[3] During the period from January 1, 2001 to December 31, 2007, RRR in China has been adjusted 15 times. To obtain yearly data for RRR, we average all ratios used in that year using proportion of days as weights. The weighted RRR is given in Table 4. To study the impact of monetary policy on bank and trade credits we divide our panel into two

Table 4: Average Yearly RRR and OLR of China

Year	2001	2002	2003	2005	2006	2007
RRR	6%	6%	6.28%	7%	7.75%	11.07%

periods: (1) 2001-2003 (this is a period when monetary policy was loose); and (2) 2005-2007 (this is a period when monetary policy was tight).[4] Specifically, we introduce a dummy variable d, which equals 1 when monetary condition is loose (period 2001-2003), and 0 when monetary condition is tight (period 2005-2007).

Our dependent variables are bank credit and trade credit. We use short-term debt to measure bank credit and account payable to measure trade credit, which is consistent with the extant literature (see Atanasova and Wilson, 2003; Ge and Qiu, 2007; Mateut et al., 2006; Nilsen, 2002; Petersen and Rajan, 1994, 1997). The firm characteristic explanatory variables are listed as follows:

- SOLVENCY: The ratio of shareholders' equity to total assets. This variable is an indicator of a firm's riskiness.
- ASSETS: The total real assets of a firm. This variable measures the size of a firm.
- INVENTORY: The ratio of inventory to total assets. This variable is an indicator of a firm's short-term assets.

[3] We can also use the official lending rate (OLR) of financial institutions as a proxy for monetary stance. The estimate results are very similar to the usage of RRR as monetary stance.

[4] He and Pauwels (2008) pointed out that PBoC's monetary stance was "implicit" and "hidden behind the policy actions." They proposed a discrete choice model to construct an index for PBoC's monetary stance from its observable policy instruments (such as RRR and OLR) and some other macroeconomic and financial variables. According to their estimation, the implicit monetary stance was "loose" for the period 2001-2003, and "tight" for the period 2004-2007. Our definition of PBoC's monetary stance follows He and Pauwels (2008)'s results, which is corroborated by the fact that the fiscal policy of China was "expansionary and proactive" for the 2001-2003, and was "prudent" for the period 2005-2007 (Yang, 2009).

- SALES: The real total sales. This variable is a proxy for a firm's level of activity.
- CASH: The real cash holding. This variable measures a firm's availability of funds that are substitutes to bank credit and trade credit.

We summarize the descriptive statistics of relevant variables in Table 5.

Table 5: Summary Statistics of Key Variables

Variable	Mean	Std. Dev.	Min	Max
BC	0.474	0.792	2.6×10^{-5}	19.543
TC	0.267	0.686	3.97×10^{-5}	16.389
ASSETS	2.803	6.428	0.04	179.714
BC/ASSETS	0.211	0.169	1.7×10^{-4}	4.220
TC/ASSETS	0.091	0.068	3.27×10^{-5}	0.725
SOLVENCY	0.443	0.472	-13.474	0.946
INVENTORY	31.133	1841.718	1.0×10^{-5}	1.17×10^{5}
SALES	2.289	6.584	1.2×10^{-6}	189.237
CASH	0.363	0.732	2.2×10^{-5}	17.9

Data source: The Wind Financial Database of China.

The total number of firm-year observations is 4044.

The units of BC, TC, ASSETS, CASH and SALES are billion Yuan.

6.2 Estimation Methodology

We use panel data models to characterize the relationship between firm credits variables and their financial characteristics variables. For analyzing individual firm level data, panel data modeling is preferred to cross-sectional data modeling, because of its ability to portrait dynamic relationships and control for unobserved heterogeneity (Wooldridge, 2001; Cameron and Trivedi, 2005). Meanwhile, according to the credit market equilibrium theory, there is an unobserved interrelationship between firms' bank loans and trade loans (Cook, 1999). Therefore, we need to use a simultaneous equations model to control for the endogeneity caused by the simultaneity of bank credit and trade credit. Specifically, we consider the following simultaneous equations model with panel data

$$y_{it1} = a_1 + \alpha_1 y_{it2} + X_{it}\beta_1 + \theta_1 d_t + \mu_{i1} + v_{it1}, \quad (6)$$

$$y_{it2} = a_2 + \alpha_2 y_{it1} + X_{it}\beta_2 + \theta_2 d_t + \mu_{i2} + v_{it2}, \quad (7)$$

where $i = 1, \cdots, N$ is the index for firms, $t = 1, \cdots, T$ is the index for time, y_{it1}, y_{it2} denote bank credit and trade credit, respectively, X_{it} is a collection of independent variables (i.e., log(ASSETS), INVENTORY, MS, SALES, log(SOLVENCY) and YEAR), d_t is the monetary condition dummy variable, μ_{i1}, μ_{i2} denote the unobserved individual firm effects, a_1, a_2 are intercepts, α_1 denotes the effect of trade credit on bank credit, α_2 denotes the effect of bank credit on trade credit, β_1 (β_2) is the regression coefficient which represents the effects of explanatory variables on the dependent variable y_{it1} (y_{it2}), and v_{it1}, v_{it2} represent random errors. We assume that models described in Eqs. (6) and (7) are *random effects* models (i.e., μ_{i1} and μ_{i2} are uncorrelated with all the regressors in Eqs. (6) and (7), respectively).[5]

Note that y_{it2} in Eq. (6) is an endogenous regressor (i.e., it is correlated with the error term v_{it1}). Similarly, y_{it1} in Eq. (7) is an endogenous regressor and is correlated with the error term v_{it2}. Due to endogeneity, Ordinary Least Squares estimation of Eq. (6) and Eq. (7) produces biased estimators of regression coefficients. Our estimation strategy is to estimate Eqs. (6) and (7) separately, each with a G2SLS (Generalized Two-Stage Least Squares) random effects IV regression. The key is to find appropriate instrumental variables for the endogenous variables. By definition, an instrumental variables z for an endogenous variable x must satisfy two criteria: (i) z is closely correlated with x, and (ii) z is uncorrelated with the regression error. For estimating Eq. (6), we use the one-period lagged values of y_{it2} (i.e., $y_{i,t-1,2}$) as the instrument for y_{it2}, and for estimating Eq. (7), we use $y_{i,t-1,1}$ as the instrument for y_{it1}.[6]

[5]We can also assume that models in Eqs. (6) and (7) are *fixed effects* models which allows arbitrary correlation between the individual effect μ_{i1}, μ_{i2} and the regressors in Eqs. (6) and (7), or we can use the Hausman's test to select between the fixed or random effects models based on data. As pointed out by (Baltagi, 2008) pps. 21-22, determining whether a model is fixed or random is a very difficult job. We opt for the random effect models because of the following observations: (1) The Hausman test gave inconsistent results: among all the 4 models we estimated, some are fixed effects and some are random effects by Hausman test; (2) If we make the fixed effect assumption, then in half of our models, none of the regressors are significant, which is quite counter-intuitive.

[6]The usage of two-period lagged values as instruments delivers similar estimation results.

6.3 Estimation Results

The results in our theoretical model depend on the retailer's internal capital level. In this paper we use the logarithm of the real cash holding as the proxy of the retailer's internal capital. We classify firms into two categories according to their real cash holdings: Type L (low cash) if the amount of real cash holding is less than or equal to 40 million yuan, and Type H (high cash) otherwise.[7] For each category, we estimate a simultaneous equations model with panel data as described by Eqs. (6) and (7).

Type L firms

We first consider Type L firms. Specifically, we estimate Eq. (6) and Eq. (7) separately using a Generalized Two-Stage Least Squares (G2SLS) random effects IV regression method. In both regressions, BC (Bank Credit) and TC (Trade Credit) appear in their original scales. The parameter estimates and their precisions are reported in Table 6 and Table 7, respectively.

As we can see from Table 6 and Table 7, for firms with low level of cash holding, the amount of credit that they can obtain from suppliers and the bank have a very significantly positive correlation, after controlling for other important factors that might affect credit rationing. This validates the complementarity prediction in Proposition 5.

A firm's total assets are always a key factor for its credits: the larger a firm, the more credits (both bank credit and trade credit) it can acquire. Solvency is also significant for both kinds of credits: the more leveraged a firm, the more difficult it can get credit from banks or suppliers. Inventory to assets ratio and sales status are only relevant for trade credit acquiring. Other factors such as the levels of Required Reserve Ratio (MS), the status of monetary policy and time are statistically insignificant factors in determining either trade credit or bank credit.

[7] The threshold value of 40 million yuan is approximately the 15% percentile point of all values of real cash holding. The main message from our estimation results does not change if we switch to a different threshold level, as long as it is greater than 30 million yuan and less than 161 million yuan.

Table 6: Dependent Variable = BC , Type L firms

Y	Coef.	Std. Err.
TC	0.997 ***	0.317
log(ASSETS)	0.154***	0.022
INVENTORY	-0.021	0.02
MS	-0.529	0.757
log(SALES)	-0.003	0.007
SOLVENCY	-0.055***	0.012
YEAR	0.007	0.018
d	0.017	0.055
CONSTANT	-17.446	30.115
R^2	0.46	
No. obs.	576	

Data source: The Wind Financial Database of China.

*: significance at 10%.

**: significance at 5%.

***: significance at 1%.

Type H firms

We now estimate the simultaneous equations model in Eq. (6) and Eq. (7) for Type H firms. The estimation method is the same as that for Type L firms. For these two set of regressions, BC and TC appear in their logarithm scales. The parameter estimates and their precisions are reported in Table 8 and Table 9, respectively.

We find from Table 8 and Table 9 that for firms with relatively adequate amount of cash, trade credit and bank credit have a significantly negative correlation, thus validating the substitutability prediction in Proposition 5.

The impact of firm size (total assets) and leverage position (solvency) have the similar effects on trade and bank credits as for Type L firms. This means that regardless of the cash holding situation of a firm, firm size is invariably a positive factor for credits acquiring, and the degree of

Table 7: Dependent Variable = TC , Type L firms

Y	Coef.	Std. Err.
BC	0.085***	0.026
log(ASSETS)	0.025***	0.007
INVENTORY	0.021 ***	0.005
MS	-0.136	0.182
log(SALES)	0.005 ***	0.0016
SOLVENCY	-0.0066**	0.003
YEAR	-0.001	0.004
d	-0.012	0.013
CONSTANT	2.17	8.44
R^2	0.43	
No. obs.	576	

Data source: The Wind Financial Database of China.

*: significance at 10%.

**: significance at 5%.

***: significance at 1%.

riskiness always plays a negative role. For bank credit, only these two factors and the amount of trade credit have explanatory power; while for trade credit, we have two additional explanatory variables that are statistically significant. The higher the Required Reserve Ratio (MS), the more difficult a firm can borrow from its suppliers. And time (year) now plays an active role for trade credit obtaining, suggesting the gradual growth of the shadow banking system in China during that period.

In summary, the above results support our prediction that trade credit plays a complementary role to bank credit for firms with low levels of cash holding, and that it plays a substitutable role to bank credit as firms have more cash.

Table 8: Dependent Variable = log(BC) , Type H firms

Y	Coef.	Std. Err.
log(TC)	-0.200 ***	0.046
log(ASSETS)	1.021***	0.053
INVENTORY	6.15×10^{-5}	2.73×10^{-4}
MS	-2.582	1.768
log(SALES)	0.011	0.012
SOLVENCY	-2.728***	0.128
YEAR	0.058	0.042
d	0.279	0.131
CONSTANT	-113.713	83.943
R^2	0.55	
No. obs.	3468	

Data source: The Wind Financial Database of China.

*: significance at 10%.

**: significance at 5%.

***: significance at 1%.

7 Conclusions

This paper investigates the roles of bank and trade credits in a supply chain with a capital constrained retailer under demand uncertainty. Two scenarios are explored: a single credit channel where only one credit type is viable, and dual credit channels where both credit types are viable. We first utilize the principal-agent and newsvendor theories to theoretically characterize the optimal credit limits, interest rates, and order quantity in both scenarios. We then use panel data to hypothetically test the substitutability and complementarity of the two credits.

In the single-credit scenario, both credit sizes and interest rates first increase and then decrease as the retailer's internal capital grows. If the trade credit market is more competitive than the bank credit market, trade credit outperforms bank credit for the retailer; otherwise, the retailer's preference of credit type hinges on the relative diversion risk level of trade credit over bank credit.

Table 9: Dependent Variable = log(TC) , Type H firms

Y	Coef.	Std. Err.
log(BC)	-0.164 ***	0.032
log(ASSETS)	1.145***	0.035
INVENTORY	-0.003	0.002
MS	-2.287**	1.136
log(SALES)	0.005	0.008
SOLVENCY	-1.628***	0.111
YEAR	0.056**	0.027
d	-0.021	0.085
CONSTANT	-113.409**	54.28
R^2	0.70	
No. obs.	3468	

Data source: The Wind Financial Database of China.

*: significance at 10%.

**: significance at 5%.

***: significance at 1%.

In the dual-credit scenario, when the bank credit market is more competitive than the trade credit market, the retailer borrows bank credit prior to trade credit, but switches to exhausting the trade credit limit prior to borrowing bank credit as the internal capital declines. But if the trade credit market is more competitive, the retailer will borrow only trade credit regardless of the internal capital level.

We also analytically prove that the two credits are complementary if the retailer's internal capital is substantially low, but become substitutable as the internal capital grows. We then use the empirical evidence from a panel of China firms to validate that our above theoretical prediction is statistically significant.

Our work has its limitations. To obtain tractability, we assume exogenous retail and wholesale prices. The same assumption has been very well taken in the existing related literature and allows us to focus on main managerial insights. It is, however, a future research venue to make retail and wholesale prices endogenous and then utilize simulation to explore additional managerial insights. In addition, competition among multiple retailers, multiple banks, and multiple suppliers could be intriguing. Finally, relaxing the unlimited credit capacity assumption is another future research priority.

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Appendix: Online Supplements

Proof of Proposition 1. To prove Proposition 1, we first address two lemmas.

Lemma 1 *In a competitive financial market, a loan* (L_i, r_i) *is feasible if and only if* $\mathbb{E}[\mathcal{L}] \geq L_i(1+\underline{r}^i)$*, where* $\mathcal{L} = \min[D, Q_i]$*. There exists a unique value* $r_i > \underline{r}^i$ *such that* $\mathbb{E}\min[\mathcal{L}, L_i(1+r_i)] = L_i(1+\underline{r}^i)$*.*

Proof: We prove by contradiction. Suppose that we can find a loan (L_i, r_i) satisfying $\mathbb{E}\min[\mathcal{L}, L_i(1+r_i)] = L_i(1+\underline{r}^i)$ when $\mathbb{E}[\mathcal{L}] < L_i(1+\underline{r}^i)$. By Jensen's inequality, we obtain

$$\mathbb{E}[\min\{\mathcal{L}, L_i(1+r_i)\}] \leq \min[\mathbb{E}[\mathcal{L}], L_i(1+r_i)] < L_i(1+\underline{r}^i)$$

for all r_i. Thus, $\mathbb{E}\min[\mathcal{L}, L_i(1+r_i)] = L_i(1+\underline{r}^i)$ cannot be satisfied. Therefore, for a loan (L_i, r_i) to be feasible in a competitive financial market we must have $\mathbb{E}[\mathcal{L}] \geq L_i(1+\underline{r}^i)$.

To prove the uniqueness of r_i satisfying the equality, we define $G(r_i) = \mathbb{E}\min[\mathcal{L}, L_i(1+r_i)] - L_i(1+\underline{r}^i)$ which is a continuous function of r_i. We can further show that $\frac{dG(r_i)}{dr_i} = L_i\bar{F}[L_i(1+r_i)] \geq 0$. Moreover, we have $G(0) \leq 0$ and $G(\infty) \geq 0$ given $\mathbb{E}[\mathcal{L}] \geq L_i(1+\underline{r}^i)$. Therefore, by continuity there exists a unique $r_i \geq \underline{r}^i$ such that $G(r_i) = 0$. Q.E.D.

Lemma 2 *There exists a unique threshold* $\tilde{B}_i(\psi_i) > 0$ *such that* $\bar{L}_i(\tilde{B}_i) + \tilde{B}_i = wQ_i^0$*, where* $\bar{L}_i(\tilde{B}_i)$ *solves* $\mathbb{E}\min[D, \frac{\bar{L}_i(\tilde{B}_i)+\tilde{B}_i}{w}] - \bar{L}_i(\tilde{B}_i)(1+\underline{r}^i) = \psi_i(\bar{L}_i(\tilde{B}_i) + \tilde{B}_i)$ *and* $Q_i^0 = \bar{F}^{-1}(w(1+\underline{r}^i))$*.*

Proof: We first prove that $\bar{L}_i(B)$ increases with B. From Eq. (2), we know that for a given B, $\bar{L}_i(B)$ solves $\mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) = \psi_i(\bar{L}_i(B) + B)$. We now define $G(\bar{L}_i(B), B) \equiv \mathbb{E}\min[D, Q_i] - \bar{L}_i(B)(1+\underline{r}^i) - \psi_i(\bar{L}_i(B) + B)$. Hence, we obtain

$$\frac{dG(\bar{L}_i(B), B)}{d\bar{L}_i(B)} = \Pi'(\bar{L}_i(B)) - (1 + \psi_i + \underline{r}^i),$$

where $\Pi(\bar{L}_i(B)) = \mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}]$ and $\Pi'(\bar{L}_i(B)) = \frac{d\Pi'(\bar{L}_i(B))}{d\bar{L}_i(B)}$. As required by Eq. (2b), we must have $\frac{dG(\bar{L}_i(B),B)}{d\bar{L}_i(B)} < 0$; otherwise, the creditor would continue to increase the value of $\bar{L}_i(B)$. Thus, we have $\Pi'(\bar{L}_i(B)) - (1+\psi_i+\underline{r}^i) < 0$. Also from the definition of $G(\bar{L}_i(B), B)$, we can obtain

$$\frac{d\bar{L}_i(B)}{dB} = -\frac{\frac{\partial G(\bar{L}_i(B),B)}{\partial B}}{\frac{\partial G(\bar{L}_i(B),B)}{\partial \bar{L}_i(B)}} = -\frac{\Pi'(\bar{L}_i(B)) - \psi_i}{\Pi'(\bar{L}_i(B)) - (1 + \psi_i + \underline{r}^i)}.$$

Intuitively, we have $\Pi'(\bar{L}_i(B)) - \psi_i > 0$, since the retailer's marginal profit from selling the product must exceed that of diversion; otherwise, the creditor would refuse lending. Combining the above conditions together, we show that $\frac{d\bar{L}_i(B)}{dB} > 0$; thus $\bar{L}_i(B) + B$ increases with B.

We now prove that there exists a unique $B = \tilde{B}_i$ satisfying $\bar{L}_i(\tilde{B}_i) + \tilde{B}_i = wQ_i^0$. Note that $Q_i^0 = \bar{F}^{-1}(w(1+\underline{r}^i))$ is the optimal order quantity for the retailer without moral hazard. Based on our assumption on ψ_i, we have

$$\psi_i > \underline{\psi_i} = \frac{\mathbb{E}\min[D, Q_i^0] - wQ_i^0}{wQ_i^0} > \frac{\mathbb{E}\min[D, Q_i^0] - wQ_i^0(1+\underline{r}^i)}{wQ_i^0}.$$

Reorganizing the above inequality yields

$$\begin{aligned} wQ_i^0 &> \frac{\mathbb{E}\min[D, Q_i^0] - wQ_i^0(1+\underline{r}^i)}{\psi_i} \\ &> \frac{\mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - (\bar{L}_i(B)+B)(1+\underline{r}^i)}{\psi_i} \\ &= \frac{\psi_i(\bar{L}_i(B)+B)}{\psi_i} \\ &\geq \bar{L}_b(0) + 0 \quad (\text{Since } B \geq 0). \end{aligned}$$

As $\bar{L}_i(B) + B$ increases with B and could bypass wQ_i^0 as B grows, we observe a single-crossing point such that a unique $B = \tilde{B}_i$ satisfies $\bar{L}_i(\tilde{B}_i) + \tilde{B}_i = wQ_i^0$. Obviously, $\bar{L}_i(\tilde{B}_i)$ also solves $\mathbb{E}\min[D, \frac{\bar{L}_i(\tilde{B}_i)+\tilde{B}_i}{w}] - \bar{L}_i(\tilde{B}_i)(1+\underline{r}^i) = \psi_i(\bar{L}_i(\tilde{B}_i) + \tilde{B}_i)$. $\square$

We now prove Proposition 1. From Lemma 2, we have $B = \tilde{B}_i(\psi_i)$ satisfying $\bar{L}_i(\tilde{B}_i) + \tilde{B}_i = wQ_i^0$ and $\mathbb{E}\min[D, Q_i^0] - \bar{L}_i(\tilde{B}_i)(1+\underline{r}^i) = \psi_i(\bar{L}_i(\tilde{B}_i) + \tilde{B}_i)$. Combining the above two equations, we have

$$\tilde{B}_i(\psi_i) = \frac{(1+\psi_i+\underline{r}^i)wQ_i^0 - \mathbb{E}[\min[D, Q_i^0]]}{1+\underline{r}^i}.$$

For any given $B < \tilde{B}_i(\psi_i)$, the retailer orders $Q_i = \frac{\bar{L}_i(B)+B}{w}$, which increases with B and less than Q_i^0 as constrained by the credit limit of $\bar{L}_i(B)$. Based on Eq. (2b), we can solve $\bar{L}_i(B)$ from

$$\mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) = \psi_i(\bar{L}_i(B)+B).$$

Thus, it is straightforward that the retailer would order Q_i^0 if its internal capital is substantially high (i.e., $B \geq \tilde{B}_i(\psi_i)$); otherwise, it would order $Q_i = \frac{\bar{L}_i(B)+B}{w}$ to reach the upper limit as allowed by the creditor. Therefore, the optimal order quantity and the credit limit for the retailer and the creditor, respectively, are given by

$$Q_i^*(B) = \begin{cases} \frac{\bar{L}_i(B)+B}{w} & if\ B < \tilde{B}_i(\psi_i), \\ Q_i^0 & if\ B \geq \tilde{B}_i(\psi_i), \end{cases}$$

$$L_i^*(B) = \begin{cases} \bar{L}_i(B) & if\ B < \tilde{B}_i(\psi_i), \\ wQ_i^0 - B & if\ B \geq \tilde{B}_i(\psi_i). \end{cases}$$

Based on Lemma 1, we can obtain

$$r_i^*(B) = \begin{cases} r_i^0(B) & if\ B < \tilde{B}_i(\psi_i), \\ r_i^1(B) & if\ B \geq \tilde{B}_i(\psi_i), \end{cases}$$

where $r_i(B) = r_i^0(B)$ solves $\mathbb{E}\min\{\min[D, \frac{\bar{L}_i(B)+B}{w}], \bar{L}_i(B)(1+r_i(B))\} = \bar{L}_i(B)(1+\underline{r}^i)$ and $r_i(B) = r_i^1(B)$ solves $\mathbb{E}\min\{\min[D, Q_i^0], (wQ_i^0 - B)(1+r_i(B))\} = (wQ_i^0 - B)(1+\underline{r}^i)$. Q.E.D.

Proof of Corollary 1.

Part (i): Based on Lemma 1, we have $\mathbb{E}\min[\mathcal{L}, L_i(B)(1+r_i(B))] = L_i(B)(1+\underline{r}^i)$ and $(1+r_i(B))\bar{F}(L_i(B)(1+r_i(B))) < 1+\underline{r}^i$. From $\mathbb{E}\min[\mathcal{L}, L_i(B)(1+r_i(B))] = L_i(B)(1+\underline{r}^i)$, we obtain

$$\frac{dL_i(B)}{dr_i(B)} = -\frac{L_i(B)\bar{F}(L_i(B)(1+r_i(B)))}{(1+r_i(B))\bar{F}(L_i(B)(1+r_i(B))) - (1+\underline{r}^i)}.$$

Combining the above equations, we have $\frac{dL_i(B)}{dr_i(B)} > 0$. Subsequently, we have $\frac{dr_i(B)}{dL_i(B)} = 1/\frac{dL_i(B)}{dr_i(B)} > 0$; thus interest rate $r_i(B)$ increases with credit size $L(B)$. When $B \in [0, \tilde{B}_i(\psi_i))$, from the proof in Lemma 2, we obtain $\frac{\bar{L}_i(B)}{dB} \geq 0$ and show that $\bar{L}_i(B)$ increases with B. In this case, the credit size is equal to credit limit and increases with B. As a result, $r_i(B) = r_i^0(B)$ increases with B, since $r_i(B)$ increases with credit size. When $B \in [\tilde{B}_i(\psi_i), wQ_i^0]$, loan size $L_i^0(B) = wQ_i^0 - B$ decreases with B, then $r_i(B) = r_i^1(B)$ decreases with B.

Part (ii): Based on Eq. (2), $\bar{L}_i(B)$ is solved by $\mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) = \psi_i(\bar{L}_i(B)+B)$. Let $G(\bar{L}_i(B), \psi_i) = \mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) - \psi_i(\bar{L}_i(B)+B)$, and we get

$$\begin{aligned} \frac{d\bar{L}_i(B)}{d\psi_i} &= -\frac{\partial G(\bar{L}_i(B), \psi_i)}{\partial \psi_i} / \frac{\partial G(\bar{L}_i(B), \psi_i)}{\partial \bar{L}_i(B)} \\ &= -\frac{-(\bar{L}_i(B)+B)}{\Pi'(\bar{L}_i(B)) - (1+\underline{r}^i+\psi_i)}. \end{aligned}$$

In the proof of Lemma 2, we obtain $\Pi'(\bar{L}_i(B)) - (1+\underline{r}^i+\psi_i) < 0$, and then $\frac{d\bar{L}_i(B)}{d\psi_i} < 0$. For a fixed $\underline{r}^i$ (e.g., $r_f = r_s$), we obtain $\bar{L}_t(B) > \bar{L}_b(B)$ since $\psi_t = \alpha\beta < \alpha = \psi_b$. From Lemma 2, we also get $\bar{L}_b(\tilde{B}_b(\alpha)) + \tilde{B}_b(\alpha) = wQ_b^0$, and $\bar{L}_t(\tilde{B}_t(\alpha\beta)) + \tilde{B}_t(\alpha\beta) = wQ_t^0$. Since $r_f = r_s$ and $Q_b^0 = Q_t^0$, we show $\tilde{B}_t(\alpha\beta) < \tilde{B}_b(\alpha)$. When $B \geq \tilde{B}_t(\alpha\beta)$, $\bar{L}_t(B)$ is independent of $\alpha\beta$, while when $B \geq \tilde{B}_b(\alpha)$, $\bar{L}_b(B)$ is independent of α. Therefore, when $B < \tilde{B}_t(\alpha\beta)$, both bank credit and trade credit reach their limits. When $B \in [\tilde{B}_t(\alpha\beta), \tilde{B}_b(\alpha))$, the bank credit reaches its limit, but the trade credit does not.

Thus, when $B < \tilde{B}_t(\alpha\beta)$, $\bar{L}_t(B,\alpha\beta) = \bar{L}_b(B,\alpha\beta)$ since $r_f = r_s$, and $\bar{L}_b(B,\alpha\beta) > \bar{L}_b(B,\alpha)$ since $\psi_t = \alpha\beta < \alpha = \psi_b$. Then, $\bar{L}_t(B,\alpha\beta) > \bar{L}_b(B,\alpha)$. When $B \in [\tilde{B}_t(\alpha\beta), \tilde{B}_b(\alpha))$, $L_t(B) + B = wQ_t^0$ and $\bar{L}_b(B) + B < wQ_b^0$. Since $Q_t^0 = Q_b^0$, we have $L_t(B) > \bar{L}_b(B)$. As a result, we show that when $B \leq \tilde{B}_b(\alpha)$, the credit size under a single trade credit channel is larger than that under a single bank credit channel. Based on the proof in Part (i) of Corollary 1, we can show that $\frac{dr_i(B)}{dL_i(B)} > 0$. It means a larger credit size induces a higher interest rate. Therefore, if $B < \tilde{B}_b(\alpha)$, we obtain that $r_t^*(B)$, the interest rate under trade credit, is larger than $r_b^*(B)$ of bank credit, since $\bar{L}_t(B)$ or $L_t(B)$ is larger than $\bar{L}_b(B)$. Note that when $B = \tilde{B}_b(\alpha)$, $r_t^*(B) = r_b^*(B)$. Q.E.D.

Proof of Proposition 2: To prove items 1 and 2 of Proposition 2, we first compare the retailer's profits under bank credit and trade credits when $r_f = r_s$. From Eq. (2), we can show that the retailer's profit (i.e., π_i^R) under either bank credit or trade credit increases with the order quantity Q_i in the domain of $[0, \bar{F}^{-1}(w(1+\underline{r}^i)]$, where $\bar{F}^{-1}(w(1+\underline{r}^i)$ is the upper limit of order quantity that the retailer does not need to borrow either credit. From the proof of Proposition 1, the retailer obtains its optimum at $Q_i = \frac{\bar{L}_i(B)+B}{w}$, which is in $[0, \bar{F}^{-1}(w(1+\underline{r}^i))]$. Recall $\frac{d\bar{L}_i(B)}{d\psi_i} < 0$ if $r_f = r_s$ from the proof of Corollary 1. Therefore, we have $Q_t(B) = \frac{\bar{L}_t(B)+B}{w} \geq Q_b(B) = \frac{\bar{L}_b(B)+B}{w}$ since $\psi_t \leq \psi_b$. Following Proposition 1, we define $\pi_t^R(B) = \mathbb{E}\min[D, \frac{\bar{L}_t(B)+B}{w}] - \bar{L}_t(B)(1+r_s)$ and $\pi_b^R(B) = \mathbb{E}\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1+r_f)$ when the retailer orders optimally under either bank credit or trade credit. Consequently, if $r_f = r_s$, $\pi_t^R(B) \geq \pi_b^R(B)$.

To prove item 1, we now compare the retailer's profits under bank credit and trade credits when $r_f > r_s$. Let $\pi_b^R(B, r_f) = \mathbb{E}\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1+r_f)$ and $\pi_b^R(B, r_s) = \mathbb{E}\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1+r_s)$. Based on Eq. (2), $\bar{L}_i(B)$ is solved by $\mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) = \psi_i(\bar{L}_i(B)+B)$. Let $G(\bar{L}_i(B), \psi_i) = \mathbb{E}\min[D, \frac{\bar{L}_i(B)+B}{w}] - \bar{L}_i(B)(1+\underline{r}^i) - \psi_i(\bar{L}_i(B)+B)$, and we get

$$\begin{aligned}\frac{d\bar{L}_i(B)}{d\underline{r}^i} &= -\frac{\partial G(\bar{L}_i(B),\underline{r}^i)}{\partial \underline{r}^i} / \frac{\partial G(\bar{L}_i(B),\underline{r}^i)}{\partial \bar{L}_i(B)} \\ &= -\frac{-\bar{L}_i(B)}{\Pi'(\bar{L}_i(B)) - (1+\underline{r}^i+\psi_i)}.\end{aligned}$$

In the proof of Lemma 2, we have $\Pi'(\bar{L}_i(B)) - (1+\underline{r}^i+\psi_i) < 0$. Thus, we yield $\frac{d\bar{L}_i(B)}{d\underline{r}^i} < 0$. Furthermore, we have $\pi_b^R(B, r_f) = \alpha(\bar{L}_b(B, r_f) + B) < \alpha(\bar{L}_b(B, r_s) + B) = \pi_b^R(B, r_s)$ conditional on $r_f > r_s$. Recall the above proof that $\pi_b^R(B, r_s) \leq \pi_t^R(B, r_s)$. As a result, we have $\pi_b^R(B, r_f) < \pi_b^R(B, r_s) \leq \pi_t^R(B, r_s)$.

We next prove item 2. From the proof of Proposition 1, we have $\pi_t^R(B) = E\min[D, \frac{\bar{L}_t(B)+B}{w}] - \bar{L}_t(B)(1+r_s) = \alpha\beta(\bar{L}_t(B)+B)$ if $B \leq \tilde{B}_t(\alpha\beta)$, and $\pi_b^R(B) = E\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1+r_f) = \alpha(\bar{L}_b(B)+B)$ when $B \leq \tilde{B}_b(\alpha)$. Therefore, we have $\pi_b^R(B) \leq \pi_t^R(B)$ if $\bar{L}_b(B)+B \leq \beta(\bar{L}_t(B)+B)$; otherwise, we get $\pi_b^R(B) > \pi_t^R(B)$. Q.E.D.

Transformation of the objective function in Eq. (3): Note that the objective function in Eq. (3) can be rewritten as

$$\begin{aligned}
&\int_{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)}^{Q_t(B)} DdF(D) + \int_{Q_t(B)}^{\propto} Q_t(B)dF(D) \\
&-\int_{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)}^{\propto} [\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)]dF(D).
\end{aligned}$$

We now rewrite Eq. (3b) as

$$\begin{aligned}
&\int_{\bar{L}_b(B)(1+r_b)}^{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)} [D-\bar{L}_b(B)(1+r_b)]dF(D) + \int_{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)}^{\propto} L_t(B)(1+r_t)dF(D) \\
= \;& L_t(B)(1+r_s).
\end{aligned}$$

We then substitute the third item of the above equation into the same item of the new objective function. Thus, the objective function becomes

$$\begin{aligned}
&\int_{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)}^{Q_t(B)} DdF(D) - \int_{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)}^{\propto} \bar{L}_b(B)(1+r_b)dF(D) \\
&+\int_{Q_t(B)}^{\propto} Q_t(B)dF(D) - L_t(B)(1+r_s) + \int_{\bar{L}_b(B)(1+r_b)}^{\bar{L}_b(B)(1+r_b)+L_t(B)(1+r_t)} [D-\bar{L}_b(B)(1+r_b)]dF(D) \\
= \;& \int_{\bar{L}_b(B)(1+r_b)}^{Q_t(B)} DdF(D) + \int_{Q_t(B)}^{\propto} Q_t(B)dF(D) - L_t(B)(1+r_s) - \int_{\bar{L}_b(B)(1+r_b)}^{\propto} \bar{L}_b(B)(1+r_b)dF(D).
\end{aligned}$$

We also rewrite Eq. (3a) as

$$\int_0^{\bar{L}_b(1+r_b)} DdF(D) + \int_{\bar{L}_b(1+r_b)}^{\propto} \bar{L}_b(1+r_b)dF(D) = \bar{L}_b(1+r_f).$$

We then substitute the second item of the above equation into the latest objective function and rewrite the objective function as follows:

$$\begin{aligned}
&\int_{\bar{L}_b(B)(1+r_b)}^{Q_t(B)} DdF(D) + \int_{Q_t(B)}^{\propto} Q_t(B)dF(D) - L_t(B)(1+r_s) \\
&-[\bar{L}_b(B)(1+r_f) - \int_0^{\bar{L}_b(B)(1+r_b)} DdF(D)] \\
= \;& \int_0^{Q_t(B)} DdF(D) + \int_{Q_t(B)}^{\propto} Q_t(B)dF(D) - L_t(B)(1+r_s) - \bar{L}_b(B)(1+r_f) \\
= \;& \mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_f) - L_t(B)(1+r_s). \text{ Q.E.D.}
\end{aligned}$$

Proof of Proposition 3. To prove Proposition 3, we first address four lemmas regarding borrowing either only a single credit or both credits.

Lemma 3 *Given both dual credit are viable, if the retailer borrows only bank credit, then (i) there exists a unique* $B = \tilde{B}_3^b(\alpha) = \frac{(1+r_f+\alpha)I_b^0 - \Pi(I_b^0)}{1+r_f} > 0$ *such that* $wQ_b^0 = B + \bar{L}_b(B)$ *and* $\mathbb{E}\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1+r_f) = \alpha(\bar{L}_b(B)+B)$*, where* $Q_b^0 = \bar{F}^{-1}(w(1+r_f))$*; (ii) there exists a unique* $B = \tilde{B}_2^b(\alpha) = \frac{(1+r_f+\alpha)I_t^0 - \Pi(I_t^0)}{1+r_f} > 0$ *such that* $wQ_t^0 = B + \bar{L}_b(B)$ *and* $\mathbb{E}\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1+r_f) = \alpha(\bar{L}_b(B)+B)$*, where* $Q_t^0 = \bar{F}^{-1}(w(1+r_s))$*; (iii)* $\tilde{B}_3^b(\alpha) > \tilde{B}_2^b(\alpha)$*.*

Proof of Lemma 3: **Part (i):** The proof is equivalent to Lemma 2 where $i = b$ and $\psi_i = \alpha$. Recall that $Q_b^0 = \bar{F}^{-1}(w(1+r_f))$ is the optimal order quantity in the single bank credit case. Similarly, a unique $B = \tilde{B}_3^b(\alpha)$ solve $\bar{L}_b(\tilde{B}_3^b(\alpha)) + \tilde{B}_3^b(\alpha) = wQ_b^0$ and $\mathbb{E}\min[D, \frac{\bar{L}_b(\tilde{B}_3^b(\alpha))+\tilde{B}_3^b(\alpha)}{w}] - \bar{L}_b(\tilde{B}_3^b(\alpha))(1+r_f) = \alpha(\bar{L}_b(\tilde{B}_3^b(\alpha))+\tilde{B}_3^b(\alpha))$. Combining the above two equations, we obtain $\tilde{B}_3^b(\alpha) = \frac{(1+\alpha+r_f)I_b^0 - \Pi(I_b^0)}{1+r_f}$, where $\Pi(I_b^0) = \mathbb{E}\min[D, Q_b^0]$.

Part (ii): Let $Q_t^0 = \bar{F}^{-1}(w(1+r_s))$ solve the problem of the dual credit channels without moral hazard (i.e., the objective function in Eq. (4)). Since $r_f < r_s$, we have $Q_t^0 < Q_b^0$ for this single bank credit case, and correspondingly $wQ_t^0 < wQ_b^0$. Similarly, we can find a unique $B = \tilde{B}_2^b(\alpha)$ solving $\bar{L}_b(\tilde{B}_2^b(\alpha)) + \tilde{B}_2^b(\alpha) = wQ_t^0$ and $\mathbb{E}\min[D, \frac{\bar{L}_b(\tilde{B}_2^b(\alpha))+\tilde{B}_2^b}{w}] - \bar{L}_b(\tilde{B}_2^b(\alpha))(1+r_f) = \alpha(\bar{L}_b(\tilde{B}_2^b(\alpha)) + \tilde{B}_2^b(\alpha))$. Combining the above two equations, we obtain $\tilde{B}_2^b(\alpha) = \frac{(1+\alpha+r_f)I_t^0 - \Pi(I_t^0)}{1+r_f}$, where $\Pi(I_t^0) = \mathbb{E}\min[D, Q_t^0]$.

Part (iii): Let $G(I_b) = (1+\alpha+r_f)I_b - \Pi(I_b)$, where I_b is the investment with bank credit. From Lemma 2, we have $\Pi'(I_b) - (1+\alpha+r_f)I_b < 0$, where $\Pi'(I_b) = \frac{d\mathbb{E}\min[D, \frac{I_b}{w}]}{dI_b}$. Thus, $G(I_b)$ increases with I_b. Since $I_b(Q_b^0) = wQ_b^0 = I_b^0 > I_b(Q_t^0) = wQ_t^0 = I_t^0$, $G(I_b(Q_b^0)) > G(I_b(Q_t^0))$. Therefore, we have $\tilde{B}_3^b(\alpha) > \tilde{B}_2^b(\alpha)$, since $\tilde{B}_3^b(\alpha) = \frac{(1+\alpha+r_f)I_b^0 - \Pi(I_b^0)}{1+r_f} = \frac{G(I_b(Q_b^0))}{1+r_f}$ and $\tilde{B}_2^b(\alpha) = \frac{(1+\alpha+r_f)I_t^0 - \Pi(I_t^0)}{1+r_f} = \frac{G(I_b(Q_t^0))}{1+r_f}$. $\square$

Lemma 4 *Given* $r_s < \alpha + r_f \leq 1 + r_s + r_f$*, there exists a unique* $B = \tilde{B}_1(\alpha)$ *such that: (i)* $\bar{L}_b(B) + \bar{L}_t(B) + B = wQ_t^0$*, where* $\bar{L}_b(B)$ *and* $\bar{L}_t(B)$ *are determined by* $\mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha(\bar{L}_b(B)+B)$ *and* $\mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\beta(\bar{L}_b(B)+\bar{L}_t(B)+B)$*; (ii)* $B_2^b(\alpha) > B_1^b(\alpha)$*.*

Proof of Lemma 4: **Part (i):** In this case, the retailer uses both bank credit and trade credit. From the incentive compatible conditions of Eqs. (4b) and (4c), we can determine $\bar{L}_b(B)$ and $\bar{L}_t(B)$ from the following equations:

$$\begin{aligned}
&\mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha(\bar{L}_b(B) + B), \\
&\mathbb{E}\min[D, Q_t(B)] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\beta(\bar{L}_b(B) + \bar{L}_t(B) + B). \qquad \text{(A-1)}
\end{aligned}$$

Let $wQ_t(B) = \bar{L}_b(B) + \bar{L}_t(B) + B = I_t(B)$. From the above equations, we get

$$\bar{L}_t(B) = \frac{1-\beta}{\beta}(\bar{L}_b(B) + B) = (1-\beta)I_t(B).$$

Furthermore, we have $\bar{L}_t(B) = (1-\beta)I_t(B)$ and $\bar{L}_b(B) = \beta I_t(B) - B$. Substituting $\bar{L}_t(B) = \frac{1-\beta}{\beta}(\bar{L}_b(B) + B)$ into Eq. (A-1), we can solve $\bar{L}_b(B)$ from

$$\Pi(I_t^0) - \bar{L}_b(B)(1+r_f) - \frac{(1-\beta)(1+r_s)}{\beta}(\bar{L}_b(B) + B) = \alpha(\bar{L}_b(B) + B).$$

Immediately, we can obtain the value of $\bar{L}_t(B)$ as well. Substituting $\bar{L}_t(B) = (1-\beta)I_t(B)$ and $\bar{L}_b(B) = \beta I_t(B) - B$ into Eq. (A-1), we have

$$\Pi(I_t(B)) - [1 + r_s + (\alpha + r_f - r_s)\beta]I_t(B) + B(1+r_f) = 0, \qquad \text{(A-2)}$$

where $\Pi(I_t(B)) = \mathbb{E}\min[D, \frac{I_t(B)}{w}]$. Let $G(\bar{L}_b(B), \bar{L}_t(B), B) = \Pi(I_t(B)) - [1 + r_s + (\alpha + r_f - r_s)\beta]I_t(B) + B(1+r_f)$. Thus, we have

$$K(I_t(B)) = \frac{dG(\bar{L}_b(B), \bar{L}_t(B), B)}{d\bar{L}_b(B)} = \frac{dG(\bar{L}_b(B), \bar{L}_t(B), B)}{d\bar{L}_t(B)} = \Pi'(I_t(B)) - [1 + r_s + (\alpha + r_f - r_s)\beta].$$

As required by Eq. (A-2), we must have $K(I_t(B)) < 0$; otherwise, there is a higher $I_t(B)$ satisfying Eq. (A-2). Moreover, we can solve $\tilde{B}_1^b(\alpha)$ from Eq. (A-2) given $I_t(\tilde{B}_1^b) = wQ_t^0$ as follows:

$$\tilde{B}_1^b(\alpha) = \frac{[1 + r_s + (\alpha + r_f - r_s)\beta]I_t^0 - \Pi(I_t^0)}{1 + r_f}. \qquad \text{(A-3)}$$

To prove that a unique $B = \tilde{B}_1^b(\alpha)$ satisfies $\bar{L}_b(\tilde{B}_1^b(\alpha)) + \bar{L}_t(\tilde{B}_1^b(\alpha)) + \tilde{B}_1^b(\alpha) = wQ_t^0$, we will show that $\bar{L}_b(B) + \bar{L}_t(B) + B$ increases with B and $\bar{L}_b(0) + \bar{L}_t(0) + 0 < I_t^0$ when $B = 0$. Solving implicit differentiation of both equations in Eq. (A-1), we obtain

$$\begin{aligned}
\frac{d\bar{L}_b(B)}{dB} &= -\frac{\Pi'(I_t(B)) - [1 + r_s - \beta(1 + r_s - \alpha)]}{K(I_t(B))}, \\
\frac{d\bar{L}_t(B)}{dB} &= -\frac{(1-\beta)(1+r_f)}{K(I_t(B))}.
\end{aligned}$$

From the first order condition of the objective function in Eq. (4), we have $\Pi'(I_t(B)) \geq (1+r_s)$. Given that $1+r_s \geq \alpha$, we have $\Pi'(I_t(B)) - [1+r_s-\beta(1+r_s-\alpha) > 0$. Therefore, we obtain $\frac{d\bar{L}_b(B)}{dB} > 0$ and $\frac{d\bar{L}_t(B)}{dB} > 0$. This shows $\bar{L}_b(B) + \bar{L}_t(B) + B$ increases with B.

From the above proof, we show that, when the retailer uses up both credit limits, $\bar{L}_t(B) = (1-\beta)I_t(B)$ and $\bar{L}_b(B) = \beta I_t(B) - B$. If $B = 0$, it would borrow $\bar{L}_b(B) = \beta w Q_t^0$ from the bank, and $\bar{L}_t(B) = (1-\beta)wQ_t^0$ from the supplier, where Q_t^0 is the highest order quantity in the case where the retailer uses both credits simultaneously. To make our discussion interesting, we assume

$$\alpha > \underline{\alpha}(\beta) = \frac{\mathbb{E}\min[D, Q_t^0] - [\beta w Q_t^0(1+r_f) + (1-\beta)(1+r_s)wQ_t^0]}{\beta w Q_t^0},$$

which means that the retailer could not benefit from diverting all investment if the retailer has zero internal capital, as required by the constraints in Eq. (4). From this condition, we have

$$\begin{aligned} wQ_t^0 &> \frac{\mathbb{E}\min[D, Q_t^0] - [\beta w Q_t^0(1+r_f) + (1-\beta)(1+r_s)wQ_t^0]}{\alpha(\beta)\beta} \\ &= \frac{\mathbb{E}\min[D, \frac{\bar{L}_b(0)+\bar{L}_t(0)+0}{w}] - [\bar{L}_b(0)(1+r_f) + \bar{L}_t(0)(1+r_s)]}{\alpha(\beta)\beta} (\text{When } B = 0) \\ &= \frac{\alpha(\beta)\beta(\bar{L}_b(0) + \bar{L}_t(0) + 0)}{\alpha(\beta)\beta} \text{ (From } Eq.(A-1) \text{)} \\ &= \bar{L}_b(0) + \bar{L}_t(0) + 0. \end{aligned}$$

As a result, $B = \tilde{B}_1^b(\alpha)$ is the single-crossing point.

Part (ii): As proved in Part (i), when $B = \tilde{B}_1^b(\alpha)$, the retailer exhausts both credit limits and invests $I_t^0 = wQ_t^0 = \bar{L}_b(\tilde{B}_1^b(\alpha)) + \bar{L}_t(\tilde{B}_1^b(\alpha)) + \tilde{B}_1^b(\alpha)$. From Lemma 3, when $B = \tilde{B}_2^b(\alpha)$, the retailer uses up bank credit limit, and invests $I_t^0 = wQ_t^0 = \bar{L}_b(\tilde{B}_2^b(\alpha)) + \tilde{B}_2^b(\alpha)$. Recall that

$$\begin{aligned} \tilde{B}_1^b(\alpha) &= \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_f}, \\ \tilde{B}_2^b(\alpha) &= \frac{(1+r_f+\alpha)I_t^0 - \Pi(I_t^0)}{1+r_f}. \end{aligned}$$

Since

$$\begin{aligned} 1+r_s+(\alpha+r_f-r_s)\beta &= 1+r_f+\alpha-(\alpha+r_f-r_s)(1-\beta) \\ &< (1+r_f+\alpha) \quad (\text{Since} \quad \alpha+r_f > r_s), \end{aligned}$$

we obtain $\tilde{B}_2^b(\alpha) > \tilde{B}_1^b(\alpha)$. $\square$

Lemma 5 *(i) Given the retailer uses only trade credit, there exists a unique internal capital level* $\tilde{B}_2^t(\alpha\beta) = \frac{[1+r_s+\alpha\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$ *such that* $wQ_t^0 = \tilde{B}_2^t(\alpha\beta) + \bar{L}_b(\tilde{B}_2^t(\alpha\beta))$ *and* $\mathbb{E}\min\{D, \frac{\bar{L}_t(\tilde{B}_2^t(\alpha\beta)) + \tilde{B}_2^t(\alpha\beta)}{w}\} - \bar{L}_t(\tilde{B}_2^t(\alpha\beta))(1+r_s) = \alpha\beta(\bar{L}_t(\tilde{B}_2^t(\alpha\beta)) + \tilde{B}_2^t(\alpha\beta))$*, where* $Q_t^0 = \bar{F}^{-1}(w(1+r_s))$*; (ii) Given the retailer uses both bank and trade credits, for any* $\alpha \leq \frac{1-\beta}{\beta}(1+r_f)$*, there exists a unique* $B = \tilde{B}_1^t(\alpha\beta) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$ *solving* $\bar{L}_b(\tilde{B}_1^t(\alpha\beta)) + \bar{L}_t(\tilde{B}_1^t(\alpha\beta)) + \tilde{B}_1^t(\alpha\beta) = wQ_t^0 = I_t^0$ *such that* $\mathbb{E}\min[D, Q_t^0] - \bar{L}_b(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\bar{L}_b(B)$ *and* $\mathbb{E}\min[D, Q_t^0] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\beta(\bar{L}_b(B) + \bar{L}_t(B) + B)$*; (iii)* $\tilde{B}_2^t(\alpha\beta) > \tilde{B}_1^t(\alpha\beta)$*.*

Proof of Lemma 5: **Part (i):** The proof is similar to that of Lemma 3. Let $i = t$ and $\psi_t = \alpha\beta$ in Lemma 2, we obtain that $\bar{L}_t(B) + B$ increases with B. Similarly, $B = \tilde{B}_2^t(\alpha\beta)$ solves $\bar{L}_t(\tilde{B}_2^t(\alpha\beta)) + \tilde{B}_2^t(\alpha\beta) = wQ_t^0$ and $\mathbb{E}\min[D, \frac{\bar{L}_t(\tilde{B}_2^t(\alpha\beta)) + \tilde{B}_2^t(\alpha\beta)}{w}] - \bar{L}_t(\tilde{B}_2^t(\alpha\beta))(1+r_s) = \alpha\beta(\bar{L}_t(\tilde{B}_2^t(\alpha\beta)) + \tilde{B}_2^t(\alpha\beta))$. Combining the above equations, we obtain $\tilde{B}_2^t(\alpha\beta) = \frac{[1+r_s+\alpha\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$.

Part (ii): The proof is similar to that of Lemma 4. To prove a unique $B = \tilde{B}_1^t(\alpha\beta)$ solving $\bar{L}_b(\tilde{B}_1^t(\alpha\beta)) + \bar{L}_t(\tilde{B}_1^t(\alpha\beta)) + \tilde{B}_1^t(\alpha\beta) = wQ_t^0$, we will show $\bar{L}_t(B) + \bar{L}_b(B) + B$ monotonically increases with B. From Eq. (5), we can determine $\bar{L}_t(B)$ and $\bar{L}_b(B)$ through the following equations:

$$\mathbb{E}\min[D, Q_b] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\beta(\bar{L}_b(B) + \bar{L}_t(B) + B), \quad \text{(A-4)}$$

$$\mathbb{E}\min[D, Q_b] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha(\bar{L}_b(B)). \quad \text{(A-5)}$$

Based on Eqs. (A-4) and (A-5), we obtain $\bar{L}_b(B) = \frac{\beta}{1-\beta}(\bar{L}_t(B) + B)$. From $\bar{L}_t(B) + \bar{L}_b(B) + B = I(B)$, we get $\bar{L}_b(B) = \beta I_t(B)$ and $\bar{L}_t(B) = (1-\beta)I_t(B) - B$. Thus, Eq. (A-4) or (A-5) can be rewritten as

$$\Pi(I(B)) - [1 + r_s + \beta(\alpha + r_f - r_s)]I(B) + B(1+r_s) = 0, \quad \text{(A-6)}$$

where $\Pi(I) = \mathbb{E}\min[D, \frac{I(B)}{w}]$. Let $G(\bar{L}_b(B), \bar{L}_t(B), B) = \Pi(I_t(B)) - [1 + r_s + (\alpha + r_f - r_s)\beta]I_t(B) + B(1+r_s)$. Hence, we have $K(I_t(B)) = \frac{dG(\bar{L}_b(B), \bar{L}_t(B), B)}{d\bar{L}_b(B)} = \frac{dG(\bar{L}_b(B), \bar{L}_t(B), B)}{d\bar{L}_t(B)} = \Pi'(I(B)) - [1 + r_s + (\alpha + r_f - r_s)\beta]$. We must have $K(I_t(B)) < 0$ as required by Eq. (A-6); otherwise, there would exist a higher $I_t(B)$ satisfying Eq. (A-6)). Based on Eq. (A-6), we let $I_t(\tilde{B}_1^t(\alpha)) = I_t^0 = wQ_t^0$ and obtain

$$\tilde{B}_1^t(\alpha\beta) = \frac{[1 + r_s + (\alpha + r_f - r_s)\beta]I_t^0 - \Pi(I_t^0)}{1 + r_s}.$$

Taking implicit differentiation on Eqs. (A-4) and (A-5), we further achieve

$$\begin{aligned} \frac{d\bar{L}_t(B)}{dB} &= -\frac{\Pi'(I_t(B)) - \beta(1 + r_f + \alpha)}{K(I_t(B))}, \\ \frac{d\bar{L}_b(B)}{dB} &= -\frac{\beta(1+r_s)}{K(I_t(B))}. \end{aligned}$$

Based on our assumption, we get $\beta(1+r_f+\alpha) \leq \beta(1+r_f+\frac{1-\beta}{\beta}(1+r_f)) = 1+r_f$. Based on the first order condition of the objective function in Eq. (5), we have $\Pi'(I_t(B)) \geq (1+r_f)$. Thus, we have $\Pi'(I_t) - \beta(1+r_f+\alpha) \geq 0$. Together with $K(I_t(B)) < 0$, we yield $\frac{d\bar{L}_t(B)}{dB} \geq 0$ and $\frac{d\bar{L}_b(B)}{dB} \geq 0$. Therefore, $\bar{L}_b(B) + \bar{L}_t(B) + B$ increases with B. Similar to the proof of Part (i) at Lemma 4, we can show that $\bar{L}_b(0) + \bar{L}_t(0) + 0 < wQ_t^0$; thus $B = \tilde{B}_1^t(\alpha\beta)$ is unique.

Part (iii): Recall that

$$\tilde{B}_2^t(\alpha\beta) = \frac{[1+r_s+\alpha\beta]I_t^0 - \Pi(I_t^0)}{1+r_s},$$
$$\tilde{B}_1^t(\alpha\beta) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}.$$

We thus have

$$\tilde{B}_2^t(\alpha\beta) - \tilde{B}_1^t(\alpha\beta) = \frac{(r_s-r_f)\beta I_t^0}{1+r_s} > 0,$$

since $r_f < r_s$. $\square$

Lemma 6 *(i) $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_1^b(\alpha) < \tilde{B}_2^b(\alpha) < \tilde{B}_3^b(\alpha)$; (ii) $\tilde{B}_1^b(\alpha) < \tilde{B}_2^t(\alpha\beta)$; (iii) If $B = \tilde{B}_1^b(\alpha)$, the retailer's payoff with bank credit priority is equal to that with trade credit priority (i.e., $\pi_{BCP}^R(\tilde{B}_1^b(\alpha)) = \pi_{TCP}^R(\tilde{B}_1^b(\alpha))$).*

Proof of Lemma 6: **Part (i):** Since $r_f < r_s$, from Lemmas 3, 4, and 5, we have $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_2^t(\alpha\beta)$ and $\tilde{B}_1^b(\alpha) < \tilde{B}_2^b(\alpha) < \tilde{B}_3^b(\alpha)$. Recall $\tilde{B}_1^t(\alpha\beta) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$ and $\tilde{B}_1^b(\alpha) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_f}$. We obtain $\tilde{B}_1^b(\alpha) > \tilde{B}_1^t(\alpha\beta)$ since $r_f < r_s$. Thus, $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_1^b(\alpha) < \tilde{B}_2^b(\alpha) < \tilde{B}_3^b(\alpha)$.

Part (ii): We have $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_1^b(\alpha)$ and $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_2^t(\alpha\beta)$. Recall $\tilde{B}_1^b(\alpha) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_f}$ and $\tilde{B}_2^t(\alpha\beta) = \frac{[1+r_s+\alpha\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$. We find that $\tilde{B}_1^b(\alpha) = \tilde{B}_2^t(\alpha\beta)$ if $r_f = r_s$. We now show that $\tilde{B}_1^b(\alpha) < \tilde{B}_2^t(\alpha\beta)$ if $r_f < r_s$. To the end, we have

$$\begin{aligned}
\frac{d\tilde{B}_1^b(\alpha, r_f)}{dr_f} &= \frac{\beta I_t^0(1+r_f) - \big[[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)\big]}{(1+r_f)^2} \\
&= \frac{\beta I_t^0 - \tilde{B}_1^b(\alpha)}{1+r_f} \\
&= \frac{\bar{L}_b(\tilde{B}_1^b(\alpha))}{1+r_f} \text{ (From } Eq.\ (A-1)) \\
&> 0.
\end{aligned}$$

That is, $\tilde{B}_1^b(\alpha, r_f)$ increases with r_f. Thus, $\tilde{B}_1^b(\alpha) < \tilde{B}_2^t(\alpha\beta)$ given $r_f < r_s$. Overall, we have $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_1^b(\alpha) < \tilde{B}_2^t(\alpha\beta)$.

Part (iii): Following Lemma 5, with trade credit priority, the retailer exhausts the trade credit limit and borrows bank credit if $\tilde{B}_1^t(\alpha\beta) < B < \tilde{B}_2^t(\alpha\beta)$. The retailer's payoff can be described by, $\pi_{TCP}^R(B) = \mathbb{E}\min[D, Q_t^0] - \bar{L}_t^2(B)(1+r_s) - (wQ_t^0 - \bar{L}_t^2(B) - B)(1+r_f) = \alpha\beta w Q_t^0 = \alpha\beta I_t^0$. Given $\tilde{B}_1^t(\alpha\beta) < B < \tilde{B}_2^t(\alpha\beta)$, the retailer's payoff is a constant value (i.e., $\alpha\beta I_t^0$) and its net profit $\alpha\beta I_t^0 - B$ decreases with B.

From Lemma 4, we can show that, with bank credit priority, the retailer exhausts bank credit limit and borrows trade credit when $\tilde{B}_1^b(\alpha) < B < \tilde{B}_2^b(\alpha)$. The retailer's payoff is $\mathbb{E}\min[D, Q_t^0] - \bar{L}_b^2(B)(1+r_f) - (wQ_t^0 - \bar{L}_b^2(B) - B)(1+r_s) = \alpha(\bar{L}_b^2(B) + B)$. We can also show that the payoff $\alpha(\bar{L}_b^2(B) + B)$ increases with B. Following Lemma 4, we have $\bar{L}_b(B) = \beta I(B) - B$ and $\bar{L}_t(B) = (1-\beta)I(B)$, where $I(B) = \bar{L}_b(B) + \bar{L}_t(B) + B$. When $B = \tilde{B}_1^b(\alpha)$, $\pi_{BCP}^R(\tilde{B}_1^b(\alpha)) = \alpha(\bar{L}_b(\tilde{B}_1^b(\alpha)) + \tilde{B}_1^b(\alpha)) = \alpha\beta I_t^0$.

When $B = \tilde{B}_1^t(\alpha\beta)$, with trade credit priority, the retailer's investment is I_t^0. Its payoff, $\pi_{TCP}^R(B)$, is a constant value (i.e., $\alpha\beta I_t^0$) for $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_2^t(\alpha\beta)]$. As shown previously, with bank credit priority, the retailer has $\pi_t^R(\tilde{B}_1^b(\alpha)) = \alpha\beta I_t^0$. Since $\tilde{B}_1^t(\alpha\beta) < \tilde{B}_1^b(\alpha) < \tilde{B}_2^t(\alpha\beta)$, we can show that $\pi_t^R(\tilde{B}_1^b(\alpha)) = \pi_b^R(\tilde{B}_1^b(\alpha))$. $\square$

We now show the equilibrium solutions for all players with dual credit channels when the bank credit market is more competitive. Intuitively, if the retailer has substantially high internal capital, it would borrow only from the bank. As the internal capital becomes less sufficient, the retailer would borrow both credits. In the following, we categorize our analysis into four cases in terms of the retailer's internal capital.

- Case I: $B \geq \tilde{B}_3^b(\alpha)$. $\tilde{B}_3^b(\alpha) = \frac{(1+r_f+\alpha)I_b^0 - \Pi(I_b^0)}{1+r_f}$. In this case, the retailer has substantially high internal capital level and borrows only bank credit. As discussed in Proposition 1 and Lemma 3, the loan contract (L_b^0, r_b^0) is given by $L_b^0 = wQ_b^0 - B$ and r_b^0 solves

$$\mathbb{E}\min\{\min[D, Q_b^0], (wQ_b^0 - B)(1+r_b^0)\} = (wQ_b^0 - B)(1+r_f),$$

  where Q_b^0 is the retailer's optimal order quantity.

- Case II: $\tilde{B}_2^b(\alpha) \leq B < \tilde{B}_3^b(\alpha)$. As demonstrated in Lemma 3, $B = \tilde{B}_2^b(\alpha) = \frac{(1+r_f+\alpha)I_t^0 - \Pi(I_t^0)}{1+r_f}$, where $I_t^0 = wQ_t^0$ and Q_t^0 solves $\bar{F}(Q) = w(1+r_s)$. In this case, the retailer uses up the bank credit limit; however, it would not borrow trade credit because of relatively higher trade credit interest rate as compared with the bank credit interest rate. If $B \geq \tilde{B}_2^b(\alpha)$, based on Lemma 2, $\bar{L}_b(B) + B$ increases with B. As B reaches $\tilde{B}_3^b(\alpha)$, $\bar{L}_b(\tilde{B}_3^b(\alpha)) + \tilde{B}_3^b(\alpha) = wQ_b^0$. Thus, given $\tilde{B}_2^b(\alpha) \leq B < \tilde{B}_3^b(\alpha)$, the retailer would order between $[Q_t^0, Q_b^0)$. As of the loan contract $(L_b^3(B), r_b^3(B))$, $L_b^3(B)$ solves

$$\mathbb{E}\min[D, \frac{\bar{L}_b^3(B) + B}{w}] - \bar{L}_b^3(B)(1 + r_f) = \alpha(\bar{L}_b^3(B) + B),$$

and $r_b^3(B)$ solves

$$\mathbb{E}\min\{\min[D, \frac{\bar{L}_b^3(B) + B}{w}], \bar{L}_b^3(B)(1 + r_b^3(B))\} = \bar{L}_b^3(B)(1 + r_f).$$

The retailer invests $I_b^3(B) = \bar{L}_b^3(B) + B \in [I_t^0, I_b^0)$.

- Case III: $\tilde{B}_1^b(\alpha) \leq B < \tilde{B}_2^b(\alpha)$. From Lemma 6, with bank credit priority, the retailer's payoff is $\pi_{BCP}^R(B) = \alpha(\bar{L}_b(B) + B)$ for $B \in [\tilde{B}_1^b(\alpha), \tilde{B}_2^b(\alpha)]$ and $\pi_{BCP}^R(\tilde{B}_1^b(\alpha)) = \alpha\beta I_t^0$. We also observe that $\pi_t^R(B)$ increases with $B \in [\tilde{B}_1^b(\alpha), \tilde{B}_2^b(\alpha)]$. However, with trade credit priority, the retailer's payoff $\pi_{TCP}^R(B) = \alpha\beta I_t^0$ for $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_2^t(\alpha\beta)]$. Since $\tilde{B}_1^b(\alpha) < \tilde{B}_2^t(\alpha\beta)$, in this case, bank credit priority yields a higher profit for the retailer than trade credit priority. Thus, the retailer exhausts the bank credit limit and borrows trade credit. The optimal order quantity is Q_t^0. Since $r_f < r_s$, the retailer uses up bank credit limit $\bar{L}_b^2(B)$ and borrows trade credit $L_t^2(B) = I_t^0 - \bar{L}_b^2(B) - B$. As $B \in [\tilde{B}_1^b(\alpha), \tilde{B}_2^b(\alpha))$ increases, the retailer's investment I remains as a constant (i.e., $I_t^0 = wQ_t^0 = \bar{L}_b(B) + \bar{L}_t(B) + B$), but $\bar{L}_b^2(B)$ increases. Furthermore, we have

$$\begin{aligned} & \mathbb{E}\min[D, Q_t^0] - \bar{L}_b^2(B)(1 + r_f) - (wQ_t^0 - \bar{L}_b^2(B) - B)(1 + r_s) \\ \geq \; & \mathbb{E}\min[D, Q_t^0] - \bar{L}_b^2(\tilde{B}_1^b(\alpha))(1 + r_f) - (wQ_t^0 - \bar{L}_b^2(\tilde{B}_1^b(\alpha)) - \tilde{B}_1^b(\alpha))(1 + r_s) \\ = \; & \alpha\beta(wQ_t^0). \end{aligned}$$

Thus, the condition of Eq. (4c) holds as long as $B \in [\tilde{B}_1^b(\alpha), \tilde{B}_2^b(\alpha))$. As a result, we need to consider only the condition of Eq. (4b). Therefore, $\bar{L}_b^2(B)$ is solved by

$$\mathbb{E}\min[D, Q_t^0] - \bar{L}_b^2(B)(1 + r_f) - (wQ_t^0 - \bar{L}_b^2(B) - B)(1 + r_s) = \alpha(\bar{L}_b^2(B) + B).$$

Correspondingly, $r_b^2(B)$ and $\tilde{r}_t^2(B)$ are determined, respectively, by

$$\mathbb{E}\min\{\min[D, Q_t^0], \bar{L}_b^2(B)(1 + r_b^2(B))\} = \bar{L}_b^2(B)(1 + r_f),$$

$$\mathbb{E}\min\{\min[D, Q_t^0] - \bar{L}_b^2(B)(1 + r_b^2(B)), L_t(B)(1 + \tilde{r}_t^2(B))\} = L_t(B)(1 + r_s),$$

where $L_t(B) = wQ_t^0 - \bar{L}_b^2(B) - B$. The retailer orders Q_t^0 and invests $I_t^0 = wQ_t^0$ constantly.

- Case IV: $\tilde{B}_1^t(\alpha\beta) \le B < \tilde{B}_1^b(\alpha)$. Based on Lemma 6, with trade credit priority, the retailer's payoff is $\pi_{TCP}^R(B) = \alpha\beta I_t^0$ for $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_2^t(\alpha\beta)]$; while with bank credit priority, the retailer's payoff is less than $\pi_{BCP}^R(\tilde{B}_1^b(\alpha)) = \alpha\beta I_t^0$ since $\pi_{BCP}^R(B)$ increases with B given $B \le \tilde{B}_1^b(\alpha)$. Consequently, in this case, trade credit priority yields a higher profit for the retailer than bank credit priority. Thus, the retailer exhausts the trade credit limit and borrows bank credit. Particularly, $\bar{L}_t^2(B)$ solves

$$\mathbb{E}\min[D, Q_t^0] - (wQ_t^0 - \bar{L}_t^2(B) - B)(1+r_f) - \bar{L}_t^2(B)(1+r_s) = \alpha\beta w Q_t^0.$$

Correspondingly, $r_t^2(B)$ and $\tilde{r}_b^2(B)$ are determined, respectively, by

$$\begin{aligned}
& \mathbb{E}\min\{\min[D, Q_t^0], \bar{L}_t^2(B)(1+r_t^2(B))\} = \bar{L}_t^2(B)(1+r_s), \\
& \mathbb{E}\min\{\min[D, Q_t^0] - \bar{L}_t^2(B)(1+r_t^2(B)), (I_t^0 - \bar{L}_t^2(B) - B)(1+\tilde{r}_b^2(B))\} \\
= \quad & (I_t^0 - \bar{L}_t^2(B) - B)(1+r_f).
\end{aligned}$$

The retailer orders Q_t^0 and has a constant investment level I_t^0.

- Case V: $B < \tilde{B}_1^t(\alpha\beta)$. From Lemma 5, $\tilde{B}_1^t(\alpha\beta) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$. In this case, the retailer would use up both credit limits with trade credit priority. For a given B, $\bar{L}_t^1(B)$ solves

$$\mathbb{E}\min[D, \frac{\bar{L}_t^1(B)+B}{(1-\beta)w}] - \frac{\beta(1+r_f)}{1-\beta}(\bar{L}_t^1(B)+B) - \bar{L}_t^1(B)(1+r_s) = \frac{\alpha\beta(\bar{L}_t^1(B)+B)}{1-\beta},$$

and $\bar{L}_b^1(B) = \frac{\beta}{1-\beta}(\bar{L}_t^1(B)+B)$. $\tilde{r}_b^1(B)$ and $r_t^1(B)$ solve the following equations, respectively,

$$\begin{aligned}
& \mathbb{E}\min\{\min[D, \frac{\bar{L}_t^1(B)+B}{(1-\beta)w}] - \bar{L}_t^1(B)(1+r_t^1(B)), \frac{\beta}{1-\beta}(\bar{L}_t^1(B)+B)(1+\tilde{r}_b^1(B))\} \\
& = \frac{\beta}{1-\beta}(\bar{L}_t^1(B)+B)(1+r_f), \\
& \mathbb{E}\min\{\min[D, \frac{\bar{L}_t^1(B)+B}{(1-\beta)w}], \bar{L}_t^1(B)(1+r_t^1(B))\} = \bar{L}_t^1(B)(1+r_s).
\end{aligned}$$

The retailer orders $\frac{\bar{L}_t^1(B)+B}{(1-\beta)w}$ and the investment is $I_t^1(B) = \bar{L}_b^1(B) + \bar{L}_t^1(B) + B = \frac{\bar{L}_t^1(B)+B}{1-\beta}$.

Based on the above analysis, we summarize the key results conditional on $r_f < r_s$ in Table 2. Q.E.D.

Proof of Corollary 2. We prove the corollary by considering bank credit priority and trade credit priority separately. We use superscript D to denote dual credit scenario and S to denote single credit scenario.

We first analyze the bank credit priority scenario. There are two cases: 1. $B < \tilde{B}_1^b(\alpha)$ where the retailer exhausts both credit limits; 2. $\tilde{B}_1^b(\alpha) \leq B < \tilde{B}_2^b(\alpha)$ where the retailer exhausts the bank credit limit and borrows some trade credit. Under bank credit priority, let I^S denote the investment under a single trade credit, and I^D for the dual credit channels.

Case 1: $B < \tilde{B}_1^b(\alpha)$**.** With dual credit channels, the retailer uses up both credit limits. We can rewrite the definition of $\tilde{B}_1^b(\alpha)$ in Eq. (A-3) to

$$[1 + r_s + (\alpha + r_f - r_s)\beta]I^D - \Pi(I^D) = B(1 + r_f),$$

and rearrange it to

$$[1 + r_f + \alpha - (\alpha + r_f - r_s)(1 - \beta)]I^D - \Pi(I^D) = B(1 + r_f),$$

where $\Pi(I^D) = \mathbb{E}\min[D, \frac{I^D}{w}]$.

In the single bank credit case, based on Lemma 2, we have $\bar{L}_b(B) + B = wQ = I$ and $\mathbb{E}\min[D, \frac{\bar{L}_b(B)+B}{w}] - \bar{L}_b(B)(1 + r_f) = \alpha(\bar{L}_b(B) + B)$. Combining these two equations, we obtain

$$(1 + r_f + \alpha)I^S - \Pi(I^S) = B(1 + r_f),$$

where $\Pi(I^S) = \mathbb{E}\min[D, \frac{I^S}{w}]$.

We then use the above dual credit channel equation to subtract that of the single credit channel and obtain

$$(1 + r_f + \alpha)I^D - \Pi(I^D) - (1 + r_f + \alpha)I^S - \Pi(I^S) = (\alpha + r_f - r_s)(1 - \beta)I^D.$$

Let $G(I) = (1 + r_f + \alpha)I - \Pi(I)$. We can rewrite the above equation as

$$G(I^D) - G(I^S) = (\alpha + r_f - r_s)(1 - \beta)I^D.$$

From Lemma 2, we know $G(I)$ increases with I. Based on the assumption $\alpha + r_f > r_s$, we have $(\alpha + r_f - r_s)(1 - \beta)I^D \geq 0$. Therefore, we obtain $I^D > I^S$ as required by the above equation.

Case 2: $\tilde{B}_1^b(\alpha) \leq B < \tilde{B}_2^b(\alpha)$**.** From Proposition 3 on the case of dual credit channels, we know the retailer would order Q_t^0, which is the largest order when $B \in [\tilde{B}_1^b(\alpha), \tilde{B}_2^b(\alpha))$. In the single credit channel case, the retailer would order Q_b^0 which is smaller than Q_t^0 without the trade credit support. Thus, we have $I^D \geq I^S$.

We now analyze the trade credit priority scenario. There are also two cases: I. $B < \tilde{B}_1^t(\alpha\beta)$ where the retailer exhausts both credit limits; II. $\tilde{B}_1^t(\alpha\beta) \leq B < \tilde{B}_2^t(\alpha\beta)$ where the retailer exhausts the trade credit limit and borrows some bank credit with trade credit priority. Let I_t^S denote the investment with a single trade credit, and I_t^D denote that with dual credits.

Case I: $B < \tilde{B}_1^t(\alpha\beta)$. Recall the proofs of Propositions 1 and 4, for any internal capital $B < \tilde{B}_1^t(\alpha\beta)$, we have

$$[1 + r_s + \alpha\beta]I_t^S - \mathbb{E}\min[D, \frac{I_t^S}{w}] = B(1 + r_s),$$
$$[1 + r_s + \alpha\beta + \beta(r_f - r_s)]I_t^D - \mathbb{E}\min[D, \frac{I_t^D}{w}] = B(1 + r_s).$$

Subtracting the second equation from the first one, we have

$$[(1 + r_s + \alpha\beta)I_t^S - \Pi(I_t^S)] - [(1 + r_s + \alpha\beta)I_t^D - \Pi(I_t^D)] = \beta(r_f - r_s)I_t^D.$$

Let $G(I_t) = (1 + r_s + \alpha\beta)I_t - \Pi(I_t)$. The above equation can be rewritten as

$$G(I_t^S) - G(I_t^D) = \beta(r_f - r_s)I_t^D.$$

Since $\Pi(I_t) - (1 + r_s + \alpha\beta)I_t$ decreases with I_t as shown in Lemma 2, $G(I_t)$ increases with I_t. Therefore, we can infer that $I_t^S < I_t^D$, because $\beta(r_f - r_s)I_t^D < 0$. This result suggests the retailer should borrow both trade and bank credits instead of only trade credit when its internal capital level is substantially low.

Case II: $\tilde{B}_1^t(\alpha\beta) \leq B < \tilde{B}_2^t(\alpha\beta)$. Similar to Case 2 with bank credit priority, when $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_2^t(\alpha\beta))$, the retailer invests $I_t^0 = wQ_t^0$ with dual credit channels, while invests only $I_t^S(B) \leq I_t^0$ with a single credit channel. Therefore, the retailer borrows both trade and bank credits instead of only trade credit when $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_2^t(\alpha\beta))$. Q.E.D.

Proof of Proposition 4. Before we prove the proposition, we address the following lemmas.

Lemma 7 *Given that the retailer uses only the trade credit, there exists a unique* $\tilde{B}_2^t(\alpha) = \frac{[1+r_s+\alpha\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$ *such that* $wQ_t^0 = \tilde{B}_2^t(\alpha) + \bar{L}_b(\tilde{B}_2^t(\alpha))$ *and* $\mathbb{E}\min[D, \frac{\bar{L}_t(\tilde{B}_2^t(\alpha)) + \tilde{B}_2^t(\alpha)}{w}] - \bar{L}_t(\tilde{B}_2^t(\alpha))(1+r_s) = \alpha\beta(\bar{L}_t(\tilde{B}_2^t(\alpha)) + \tilde{B}_2^t(\alpha))$*, where* $Q_t^0 = \bar{F}^{-1}(w(1+r_s))$*; There also exists a unique* $\tilde{B}_0^t(\alpha) = \frac{[1+r_s+\alpha\beta]I_b^0 - \Pi(I_b^0)}{1+r_s}$ *such that*

$wQ_b^0 = \tilde{B}_0^t(\alpha) + \bar{L}_b(\tilde{B}_0^t(\alpha))$ and $\mathbb{E}\min[D, \frac{\bar{L}_t(\tilde{B}_0^t(\alpha))+\tilde{B}_0^t(\alpha)}{w}] - \bar{L}_t(\tilde{B}_0^t(\alpha))(1+r_s) = \alpha\beta(\bar{L}_t(\tilde{B}_0^t(\alpha)) + \tilde{B}_0^t(\alpha))$, where $Q_b^0 = \bar{F}^{-1}(w(1+r_f))$; $Q_t^0 \geq Q_b^0$.

Proof: The proof is similar to that of Lemma 3. Let $i = t$ and $\psi_t = \alpha\beta$ in Lemma 2, we obtain that $\bar{L}_t(B) + B$ increases with B. Similarly, $B = \tilde{B}_2^t(\alpha)$ solves $\bar{L}_t(\tilde{B}_2^t(\alpha)) + \tilde{B}_2^t(\alpha) = wQ_t^0$ and $\mathbb{E}\min\{D, \frac{\bar{L}_t(\tilde{B}_2^t(\alpha))+\tilde{B}_2^t(\alpha)}{w}\} - \bar{L}_t(\tilde{B}_2^t(\alpha))(1+r_s) = \alpha\beta(\bar{L}_t(\tilde{B}_2^t(\alpha)) + \tilde{B}_2^t(\alpha))$. Combining the above equations, we yield $\tilde{B}_2^t(\alpha) = \frac{[1+r_s+\alpha\beta]I_t^0 - \Pi(I_t^0)}{1+r_s}$. Similarly, we can obtain $\tilde{B}_0^t(\alpha) = \frac{[1+r_s+\alpha\beta]I_b^0 - \Pi(I_b^0)}{1+r_s}$. Since $r_f \geq r_s$, we have $Q_b^0 \leq Q_t^0$, $I_b^0 = wQ_b^0 \leq I_t^0 = wQ_t^0$, and $\tilde{B}_2^t(\alpha) \geq \tilde{B}_1^t(\alpha)$ in this single credit case. $\square$

Lemma 8 *For any $\alpha \leq \frac{1-\beta}{\beta}(1+r_f)$, there exists a unique $B = \tilde{B}_1^t(\alpha) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_b^0 - \Pi(I_b^0)}{1+r_s}$ solving $\bar{L}_b(\tilde{B}_1^t(\alpha)) + \bar{L}_t(\tilde{B}_1^t(\alpha)) + \tilde{B}_1^t(\alpha) = wQ_b^0 = I_b^0$ such that: (i) $\mathbb{E}\min[D, Q_b^0] - \bar{L}_b(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\bar{L}_b(B)$ and $\mathbb{E}\min[D, Q_b^0] - \bar{L}_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\beta(\bar{L}_b(B) + \bar{L}_t(B) + B)$; (ii) $\tilde{B}_1^t(\alpha) > \tilde{B}_0^t(\alpha)$, where $\tilde{B}_0^t(\alpha) = \frac{[1+r_s+\alpha\beta]I_b^0 - \Pi(I_b^0)}{1+r_s}$.*

Proof: **Part (i):** The proof is similar to that of Lemma 4. To prove a unique $B = \tilde{B}_1^t(\alpha)$ solving $\bar{L}_b(\tilde{B}_1^t(\alpha)) + \bar{L}_t(\tilde{B}_1^t(\alpha)) + \tilde{B}_1^t(\alpha) = wQ_b^0$, we need to show $\bar{L}_t(B) + \bar{L}_b(B) + B$ monotonically increases with B, which is true as proved in Lemma 4. Similar to the proof of Part (ii) of Lemma 4, we have $\bar{L}_b(0) + \bar{L}_t(0) + 0 < wQ_b^0$; thus $B = \tilde{B}_1^t(\alpha)$ is unique.

Part ii: Recall that

$$\tilde{B}_0^t(\alpha) = \frac{[1+r_s+\alpha\beta]I_b^0 - \Pi(I_b^0)}{1+r_s},$$
$$\tilde{B}_1^t(\alpha) = \frac{[1+r_s+(\alpha+r_f-r_s)\beta]I_b^0 - \Pi(I_b^0)}{1+r_s}.$$

We thus have

$$\tilde{B}_0^t(\alpha) - \tilde{B}_1^t(\alpha) = \frac{(r_s-r_f)\beta I_b^0}{1+r_s} \leq 0,$$

since $r_f \geq r_s$. $\square$

We now prove the main content. When $B = \tilde{B}_0^t(\alpha)$, the retailer only borrows trade credit and uses up the limit. The corresponding investment is $I_b^0 = wQ_b^0 = \bar{L}_t(\tilde{B}_0^t(\alpha)) + \tilde{B}_0^t(\alpha)$. When $B = \tilde{B}_1^t(\alpha)$, the retailer uses up both bank and trade credit limits, and invests $I_b^0 = wQ_b^0 = \bar{L}_t(\tilde{B}_1^t(\alpha)) + \bar{L}_b(\tilde{B}_1^t(\alpha)) + \tilde{B}_1^t(\alpha)$. From Lemma 8, we show that $\tilde{B}_0^t(\alpha) \leq \tilde{B}_1^t(\alpha)$. Thus, we obtain that although the bank credit is available, the retailer would borrow only trade credit, even if its internal capital is substantially low.

We now further compare the performance of using a single trade credit with that of using both credits. Let $I_t^S = \bar{L}_t(B) + B$ denote the investment with a single trade credit, and $I_t^D = \bar{L}_t(B) + \bar{L}_b(B) + B$ denote that with dual credits. Based on Lemmas 7 and 8, for any internal capital $B \leq \tilde{B}_1^t(\alpha)$, we have

$$[1 + r_s + \alpha\beta]I_t^S - \mathbb{E}\min[D, \frac{I_t^S}{w}] = B(1 + r_s),$$
$$[1 + r_s + \alpha\beta + \beta(r_f - r_s)]I_t^D - \mathbb{E}\min[D, \frac{I_t^D}{w}] = B(1 + r_s).$$

Subtracting the second equation from the first one, we have

$$[(1 + r_s + \alpha\beta)I_t^S - \Pi(I_t^S)] - [(1 + r_s + \alpha\beta)I_t^D - \Pi(I_t^D)] = \beta(r_f - r_s)I_t^D.$$

Let $G(I_t) = (1 + r_s + \alpha\beta)I_t - \Pi(I_t)$. The above equation can be rewritten as

$$G(I_t^S) - G(I_t^D) = \beta(r_f - r_s)I_t^D.$$

Since $\Pi(I_t) - (1 + r_s + \alpha\beta)I_t$ decreases with I_t as shown in Lemma 2. Hence, $G(I_t)$ increases with I_t. Therefore, we obtain $I_t^S \geq I_t^D$, since $\beta(r_f - r_s)I_t^D \geq 0$. This result suggests the investment with only trade credit is no less than of using both bank and trade credits. In other words, if $r_f \geq r_s$, the retailer borrows only trade credit instead of both trade and bank credits.

Based on Eq. (5), we can also show that the retailer's optimal order quantity is less than Q_b^0, if it borrows both trade credit and bank credit. Thus, if $B \geq \tilde{B}_1^t(\alpha)$, the retailer borrows trade credit and the results in Lemma 2 hold. We then can find $B = \tilde{B}_2^t(\alpha)$ and $\bar{L}_t(\tilde{B}_2^t(\alpha)) + \tilde{B}_2^t(\alpha) = wQ_t^0$, where $Q_t^0 = \bar{F}^{-1}(w(1 + r_s)) \geq Q_b^0$ since $r_f \geq r_s$. If $B < \tilde{B}_2^t(\alpha)$, the retailer uses up the trade credit limit $\bar{L}_t^1(B)$. Based on Lemma 2, $\bar{L}_t^1(B)$ is determined by $\mathbb{E}\min[D, \frac{\bar{L}_t^1(B)+B}{w}] - \bar{L}_t^1(B)(1 + r_s) = \alpha\beta(\bar{L}_t^1(B) + B)$, and $r_t^1(B)$ is determined by $\mathbb{E}\min\{\min[D, \frac{\bar{L}_t^1(B)+B}{w}], \bar{L}_t^1(B)(1 + r_t^1(B))\} = \bar{L}_t^1(B)(1 + r_s)$. When $B \geq \tilde{B}_2^t(\alpha)$, the retailer makes a constant investment $I_t = wQ_t^0$ by borrowing $L_t(B) = wQ_t^0 - B$ from the supplier, and r_t^0 is solved by $\mathbb{E}\min\{\min[D, Q_t^0], (wQ_t^0 - B)(1 + r_t^0(B))\} = (wQ_t^0 - B)(1 + r_s)$. The optimal solutions are summarized in Table 3. Q.E.D.

Proof of Proposition 5.

Scenario one: $B < \tilde{B}_1^t(\alpha)$ with trade credit priority.

Recall the proof of Lemma 5, when $B < \tilde{B}_1^t(\alpha\beta)$, we have

$$\begin{aligned}\frac{d\bar{L}_t(B)}{dB} &= -\frac{\Pi'(I_t(B)) - \beta(1+r_f+\alpha)}{K(I_t(B))},\\ \frac{d\bar{L}_b(B)}{dB} &= -\frac{\beta(1+r_s)}{K(I_t(B))}.\end{aligned}$$

Based on the proof of Lemma 5, we obtain $\frac{d\bar{L}_t(B)}{dB} > 0$ and $\frac{d\bar{L}_b(B)}{dB} > 0$. This indicates that the credit sizes of both credits are complementary when $B < \tilde{B}_1^t(\alpha\beta)$.

Scenario two: $\tilde{B}_1^t(\alpha\beta) \le B < \tilde{B}_1^b(\alpha)$ with trade credit priority.

When $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_1^b(\alpha))$, the retailer invests $I(B) = wQ_t^0$. From Proposition 3, we attain

$$\begin{aligned}&\mathbb{E}\min[D, Q_t^0] - L_b(B)(1+r_f) - \bar{L}_t(B)(1+r_s) = \alpha\beta w Q_t^0,\\ &L_b(B) = wQ_t^0 - \bar{L}_t(B) - B. \end{aligned} \tag{A-7}$$

Differentiating the above two equations with respect to $\bar{L}_t(B)$, $L_b(B)$ and B, respectively, and then reorganizing them leads to

$$\begin{aligned}&[\Pi'(I_t(B)) - (1+r_f)]\tfrac{dL_b(B)}{dB} + [\Pi'(I_t(B)) - (1+r_s)]\tfrac{d\bar{L}_t(B)}{dB} + \Pi'(I_t(B)) = 0,\\ &\tfrac{dL_b(B)}{dB} = -(1+\tfrac{d\bar{L}_t(B)}{dB}).\end{aligned} \tag{A-8}$$

Submitting $\frac{dL_b(B)}{dB} = -(1+\frac{d\bar{L}_t(B)}{dB})$ into $\Pi'(I_t(B)) - (1+r_f)]\frac{dL_b(B)}{dB} + [\Pi'(I_t(B)) - (1+r_s)]\frac{d\bar{L}_t(B)}{dB} + \Pi'(I_t(B)) = 0$, we get $\frac{d\bar{L}_t(B)}{dB} = \frac{1+r_f}{r_s-r_f}$. Then submitting $\frac{d\bar{L}_t(B)}{dB} = \frac{1+r_f}{r_s-r_f}$ into $\frac{dL_b(B)}{dB} = -(1+\frac{d\bar{L}_t(B)}{dB})$, we get $\frac{dL_b(B)}{dB} = -\frac{1+r_s}{r_s-r_f}$. Solving Eq. (A-8) results in

$$\begin{aligned}\frac{d\bar{L}_t(B)}{dB} &= \frac{1+r_f}{r_s-r_f} \ge 0,\\ \frac{dL_b(B)}{dB} &= -\frac{1+r_s}{r_s-r_f} < 0.\end{aligned}$$

This indicates that the credit sizes of both credits are substitutable when $B \in [\tilde{B}_1^t(\alpha\beta), \tilde{B}_1^b(\alpha))$.

Scenario three: $\tilde{B}_1^b(\alpha) \le B \le \tilde{B}_2^b(\alpha)$ with bank credit priority

When $B = \tilde{B}_1^b(\alpha)$, recall Eq. (4c) and Proposition 3 and we get

$$\Pi(I_t(B)) - \bar{L}_b(B)(1+r_f) - L_t(B)(1+r_s) - \alpha(\bar{L}_b(B) + B) = 0, \tag{A-9}$$

$$L_t(B) = wQ_t^0 - \bar{L}_b(B) - B. \tag{A-10}$$

In this case, the retailer orders Q_t^0 with dual credit channels. Then $\bar{L}_b(B)$ and $wQ_t^0 - \bar{L}_b(B) - B$ should satisfy the following first order condition of the objective function in Eq. (4).

$$\Pi'(I_t(B)) - (1 + r_s) = 0.$$

Differentiating Eqs. (A-9) and (A-10) with respect to $\bar{L}_b(B)$, $L_t(B)$, and B, respectively, and then reorganizing them yields

$$[\Pi'(I_t(B)) - (1 + r_f + \alpha)]d\bar{L}_b(B) + [\Pi'(I_t(B)) - (1 + r_s)]dL_t(B) + [\Pi'(I_t(B)) - \alpha]dB = 0,$$
$$dL_t(B) + d\bar{L}_b(B) + dB = 0.$$

Then we can rewrite the above equations as

$$\begin{aligned}&[\Pi'(I_t(B)) - (1 + r_f + \alpha)]\tfrac{d\bar{L}_b(B)}{dB} + [\Pi'(I_t(B)) - (1 + r_s)]\tfrac{dL_t(B)}{dB} + [\Pi'(I_t(B)) - \alpha] = 0,\\ &\tfrac{dL_t(B)}{dB} = -(1 + \tfrac{d\bar{L}_b(B)}{dB}).\end{aligned} \tag{A-11}$$

Similarly, solving Eq. (A-11) results in

$$\begin{aligned}\frac{d\bar{L}_b(B)}{dB} &= \frac{1 + r_f}{\alpha + r_f - r_s} - 1 \geq 0,\\ \frac{dL_t(B)}{dB} &= -\frac{1 + r_f}{\alpha + r_f - r_s} < 0.\end{aligned}$$

This indicates that the credit sizes of both credits are substitutable. Q.E.D.