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Sean X. Zhou

Zhijie Tao

Nianbing Zhang

Gangshu (George) Cai
*Santa Clara University*, gcai@scu.edu



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# Procurement with Reverse Auction and Flexible Noncompetitive Contracts

Sean X. Zhou* Zhijie Tao† Nianbing Zhang‡ Gangshu (George) Cai§

**Abstract**

This paper investigates a hybrid procurement mechanism that combines a reverse auction with flexible noncompetitive contracts. A buyer adopts such mechanism to procure multiple units of a product from a group of potential suppliers. Specifically, the buyer first offers contracts to some suppliers who, if accepting the contract, do not participate in the auction while committing to selling a unit to the buyer at the price of the subsequent auction. For the suppliers rejecting the offers, they can join the subsequent auction with the other suppliers to compete on the remaining units. When the buyer offers only one flexible noncompetitive contract, we find the selected supplier may accept the offer regardless of whether he knows his exact cost information. Meanwhile, the buyer can benefit from offering such a contract, as opposed to solely conducting a regular reverse auction or offering a noncompetitive contract that does not allow suppliers declining offers to join the subsequent auction. Moreover, we find that the suppliers' information about their own costs has a significant impact on the buyer's decision. When the buyer makes multiple offers, we analyze the resulting game behavior of the selected suppliers and demonstrate that the buyer can benefit more than just offering one such contract. Therefore, the hybrid procurement mechanism can be mutually beneficial for both the buyer and the selected suppliers.

**Keywords:** Multi-unit reverse auction; procurement; Nash equilibrium; flexible noncompetitive contracts

## 1 Introduction

With the growth of the Internet and e-commerce, e-procurement has delivered tremendous value to enterprises by reducing costs and streamlining their procurement processes. As a widely adopted tool in e-procurement to source products and services, the reverse auction can lower procurement costs considerably. For instance, in 2001, FreeMarkets, the leading auction software and services

*Department of Decision Sciences and Managerial Economics, CUHK Business School, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong. zhoux@baf.cuhk.edu.hk

†School of International Business Administration, Shanghai University of Finance and Economics, Shanghai, China. tao.zhijie@mail.shufe.edu.cn

‡Investment Banking Department, CITIC Securities Co., Beijing, China. zhangnianbing@gmail.com

§Department of Operations Management and Information Systems, Santa Clara University, Santa Clara, California, 95053, USA. gcai@scu.edu

company, reported that it had saved its customers an estimated 20% on a total of $30 billion worth of purchases since 1995 (Sawhney 2003).

Despite its popularity, however, the reverse auction has been criticized for its sole focus on price, and its lack of flexibility to allow buyers to deal with specific suppliers (e.g., the suppliers that the buyer wants to keep a long-term relationship) and, thus, to promote buyer-supplier relationships. Indeed, according to Giampierro and Emiliani (2007), such auctions may even hurt buyer-supplier relationships. Schoenherr and Mabert (2007) identify five of the most common myths associated with online reverse auctions and use the insights and experiences from 30 companies to generate some guidelines. They note that, in many situations, suppliers are reluctant to participate in reverse auctions because of their negative perception of the mechanism.

To preserve the procurement benefit of auctions and at the same time achieve some flexibility in supplier selection, buyers in practice often combine auctions with other tools. In a review of empirical evidence on various practical issues related to online reverse auctions, Jap (2002) shows that firms have adopted a hybrid mechanism combining (reverse) auctions with noncompetitive bids. A similar device is used in the primary Hull-Grimsby fish market in northern England, which allows certain buyers to obtain fish before the auction session opens and to pay for them at the prices established by the subsequent auction (Cassady 1967). Other examples of noncompetitive bidding include emissions trading by the U.K.[1] and the process by which the Bank of England buys back gilts.[2] Theoretically, Engelbrecht-Wiggans (1996) shows a hybrid mechanism, in which some of the units may be sold before being auctioned through noncompetitive contracts, allowing buyers to avoid bidding and information costs by buying noncompetitively, results in efficiency gains. Engelbrecht-Wiggans and Katok (2006) further demonstrate that a similar hybrid mechanism, combining a reverse auction with noncompetitive contracts (hereafter referred to as the EK model), increases competition and can make the buyer better-off.

In the EK model, however, the selected supplier is not allowed to bid in the following auction. One might wonder whether it is beneficial for the buyer to allow the selected supplier to join the auction, if the supplier does not accept the noncompetitive contract. This question is practically relevant, because the addition of contracts before the auction, regardless of whether or not the selected supplier is allowed in the auction, aims to improve the buyer-supplier relationship. Prohibiting a selected supplier who declines the contract from entering the auction could create unwanted pressure between the supplier and the buyer. In fact, the flexible noncompetitive contract is similar to the Most-Favored-Customer (MFC) clauses detailed in the economics literature

[1]http://www.dmo.gov.uk/docs/ETS/etspr230409.pdf
[2]http://www.marketwatch.com/story/bank-england-buys-uk-gilts-begins

(e.g., Crocker and Lyon 1994, Lyon 2002), which are nondiscrimination guarantees that obligate a buyer or seller to treat all trading partners symmetrically in pricing decisions. In addition to the pricing feature, MFC clauses share other features with our mechanism, in that a buyer often offers a contract with MFC clauses ex ante, both parties are committed to the exchange once the contract is accepted, and the buyer also deals with other suppliers, including those who declined the buyer's previous offers. As noted by Lyon (2002), MFC clauses are powerful devices for shaping relationships between buyers and sellers, and can be tailored to specific situations in various ways. For example, the U.S. government sells its treasury securities with some flexibility through both competitive bids limited to 35% of the offering amount for each bidder and noncompetitive bids limited to those big buyers whereby a buyer accepts the rate determined at the auction. [3]

This motivates us to study the following hybrid procurement mechanism: a buyer first selects some suppliers and offers each of them a contract. If the supplier accepts the contract, then he commits to supplying to the buyer at the price set by the auction, but can still join the subsequent auction if he declines the offer. We call this type of contract a *flexible noncompetitive contract*. Then the suppliers declining the offer join the auction together with the other suppliers to compete on the remaining units. This hybrid mechanism preserves the price-setting benefits of auctions while giving the buyer some control over deciding which suppliers to deal with. Moreover, we consider two different information scenarios called "ex ante uninformed supplier" and "ex ante informed supplier," which are differentiated by whether or not the suppliers have exact information about their own costs ex ante.

Our analysis demonstrates that, from the buyer's perspective, offering flexible contracts to suppliers can yield a lower expected cost than offering the contracts without such flexibility as in the EK model. This is particularly so when competition among suppliers is not so intense (that is, when the procurement quantity is close to the number of suppliers) in the ex ante uninformed supplier scenario or competition is very intense in the ex ante informed supplier scenario. Furthermore, the cost-saving benefit to the buyer by adopting our mechanism is larger when the costs of the suppliers are more variable (the buyer has less precise information about suppliers' costs) or when the spot market price is not too high. Our numerical study finds that the buyer can save a significant proportion of its procurement cost in a single-contract case. Note that a small improvement could be very significant for companies such as General Motors (GM), GE, and others that use the reverse auction tool for procurement. For example, GM's net profit margin was 0.3 percent in 2001. A 0.5 percent reduction in annual spending would have increased its profit by $720 million (Simchi-Levi et al. 2008). In this sense, a 0.5% saving in consecutive years from 2001-2008 would have saved

[3] http://www.treasurydirect.gov/instit/auctfund/work/work.htm

GM about $5 billion. Our mechanism can also benefit the selected supplier more than the EK model. In the uninformed supplier scenario, suppliers still prefer to accept rather than decline the contract when competition is not too intense, but in the informed supplier case, suppliers are likely to accept the offer when their costs are low. Thus, the additional flexibility could result in a mutually beneficial outcome for both buyer and selected suppliers. The intuition behind this is that the selected suppliers will have more options to capitalize their supply, and the buyer will benefit from a lower auction price due to the additional flexibility.

We further analyze the case where the buyer offers multiple flexible contracts and characterize the resulting game behavior among suppliers, which is absent in the EK model. It is worth noting that even if multiple contracts are offered in the EK mechanism, there is no game among the selected suppliers because one supplier's decision will not affect the payoffs of the other suppliers due to their assumption that the selected suppliers will not be allowed to join the auction. However, in our setting, a supplier's decision to accept or reject the contract offer changes the auction price and thus affects the payoffs of the other suppliers, giving rise to a Nash game among the selected suppliers. More importantly, our results suggest that offering more contracts can benefit the buyer, especially when the total procurement quantity increases. Our numerical results indicate that offering two contracts can save the buyer's cost significantly over offering a single contract.

By comparing the results in the two information scenarios, we find that whether suppliers know their exact costs has a rather significant impact not only on their own decisions but also on the buyer's cost. The buyer's procurement cost is lower in the ex ante informed (resp., uninformed) supplier scenario when the competition is intense (resp., not intense). This reveals that, on the one hand, a buyer may have an incentive to subsidize the suppliers' cost resulting from the process of learning the exact cost of supplying the product if the competition among suppliers is intense. On the other hand, when the competition is less intense, the buyer may prefer to offer the contract long before the auction so that the suppliers possess less cost information. We also find that the largest possible cost saving that the buyer can achieve by using our mechanism over the EK model is larger in the uninformed supplier case.

The mechanism that we investigate here is related to the reverse auction and hybrid procurement mechanisms. Besides the closely related papers by Engelbrecht-Wiggans (1996) and Engelbrecht-Wiggans and Katok (2006), the applications of reverse auctions in procurement have been extensively discussed from different perspectives in the previous studies. Dasgupta and Suplber (1990) develop a model with a single buyer who seeks to select a supplier, and show that it is optimal for the buyer to specify a schedule of price and quantity and then to conduct a sealed-bid auction in which the suppliers bid on the quantity. Chen (2007) considers a single retailer who procures from multiple suppliers. The retailer first designs a supply contract that specifies the payment for each possible purchase quantity, and then invites the suppliers to bid for the contract. Chen shows that the supply contract auction is optimal. Chen, Seshadri, and Zemel (2008) utilize an "audit-based" approach that combines auctions and profit sharing to demonstrate that a buyer can optimize her profit and coordinate the supply chain. Wan and Beil (2008) consider a manufacturer who uses a "Request for Quotes" reverse auction in combination with supplier qualification screening to determine which qualified supplier to award a contract. Other related studies on online reverse auction include Elmaghraby (2000) and Pinker et al. (2003). We also refer interested readers to Krishna (2002) for comprehensive theoretical details on standard auctions, including reverse auctions.

The research on preferred suppliers in auctions is also related to this paper, in which the buyer deals with a preferred supplier and other potential suppliers in a competitive procurement setting (e.g., Burguet and Perry (2008) and Hua (2007)). The buyer first grants the preferred supplier a strategic ex ante contract (e.g., right-of-first-refusal) and then conducts an auction or negotiates with other suppliers. Many other papers also compare auctions with negotiations or a combination of the two and other procurement strategies. Interested readers are referred to the work of Bajari et al. (2009), Bandyopadhyay et al. (2008), Chen et al (2005), Murthy et al (2004), Salmon and Wilson (2008), Subramanian and Zeckhauser (2005), and Sun et al. (2010) for more details.

The remainder of this paper is organized as follows. The detailed model is introduced in Section 2. In Sections 3 and 4, we provide analyses and results for the case where a buyer offers only one flexible noncompetitive contract under the two information scenarios. We then compare our results with those of the EK model. In Section 5, we discuss the case where a buyer offers multiple flexible noncompetitive contracts, and characterize the resulting game behavior of the selected suppliers. A comprehensive numerical study is presented in Section 6. We conclude our paper with some extensions in Section 7, and provide all technical proofs in Appendix A (Online Supplements).

## 2 The Procurement Model

Consider a model in which a buyer wants to procure $Q$ ($Q \geq 2$) units of a certain product from $N$ ($N \geq Q+1$) potential suppliers, and each supplier can only provide a single unit. Both the buyer and the suppliers are risk neutral. The unit production cost of supplier $i$, $C_i, i = 1, \ldots, N$, is continuous and comes from a common probability distribution $F(\cdot)$ with a corresponding density function $f(\cdot)$ and finite support. Let $\mu = \mathsf{E}[C_i]$. Without loss of generality, we scale the cost such that $C_i \in [0, 1]$ and $F(0) = 0$ and $F(1) = 1$. We denote the generic random cost by $C$.

The buyer uses a hybrid mechanism that comprises a uniform-price reverse auction and a type of strategic contract to determine which suppliers to use. Before the auction, the buyer offers $M$, $1 \leq M < Q$ contracts to $M$ suppliers, offering one to each selected supplier. As the buyer does not know the exact cost of each supplier, the suppliers are thus identical to her theoretically and are selected randomly for the $M$ contracts. In practice, suppliers might be selected based on other attributes that are of interest to the buyer, such as quality, delivery lead time, and supply chain relationship, but this is not the focus of this paper (interested readers are referred to Wan and Beil (2008) for a discussion of supplier qualification screening). Each selected supplier has the option of accepting the offer, thereby avoiding the auction but committing to selling a unit to the buyer at a price that will be determined later at the auction. Selected suppliers who reject the offer can still join the subsequent auction. Committing to some suppliers ex ante but offering them the flexibility of joining the auction demonstrates the buyer's willingness to maintain a long-term relationship with these suppliers. At the same time, selected suppliers who accept the offer could avoid the "bidding" costs associated with an auction (Engelbrecht-Wiggans (1996)).[4]

As mentioned in the introduction, our setting is different from that in the EK model in that we allow suppliers declining the contract offer to participate in the auction. Both the EK model and our mechanism assume that if any one of the $M$ selected suppliers turns down the offer, then the buyer will have to make up the resulting shortfall from the spot market or by producing it in house at a constant cost $c_0$ with $c_0 \geq \mu$.[5]

Given these assumptions, the auction in the EK model is determined regardless of the decisions of the selected suppliers. For example, in the EK model, once the buyer offers $M$ contracts, $N - M$ suppliers compete for $Q - M$ units at auction. In our model, however, the selected suppliers who decline the contract offer can participate in the auction, and consequently the formation of the auction depends on the decisions of the selected suppliers. For the previous example, if $m$, $m \leq M$ suppliers accept the offer, then in the auction $N - m$ suppliers will compete for $Q - M$ units. We will see in the following analysis that the assumption that the buyer must make up the shortfall from the spot market gives the suppliers an incentive to accept the offer, which may also benefit the buyer. We call our contract a *flexible noncompetitive contract*, in contrast to the *noncompetitive contract* proposed in the EK model. For simplicity, in the remainder of the paper, we may occasionally omit

[4] As in the EK model, we do not model this cost explicitly here. But we will provide some discussion on how to incorporate bidding cost in Section 7.

[5] We can easily generalize the model to random $c_0$. In the analysis, we just need to replace $c_0$ with $\mathsf{E}[c_0]$ and all the results continue to hold. Meanwhile, the assumption that $c_0 \geq \mu$ is not a technical assumption; it is to ensure that the buyer will not procure all or part of the units directly from the outside market, which, otherwise, will make the results less interesting. As we will see in the following analysis, if we focus on comparing the flexible contracts with the regular auction, the assumption $c_0 \geq \mu$ can be dropped.

the term "noncompetitive." In both our model and the EK model, the losing suppliers obtain zero payoff.

The reverse auction considered here is equivalent to a multi-unit sealed-bid auction in which, if $N$ suppliers bid for $Q$ units, then the $Q$ suppliers with the lowest bids win the auction and the price at the auction is the $(Q+1)th$ lowest bid. All of the participating suppliers privately know their own costs before the auction, and the optimal strategy for each supplier participating in the auction is to bid truthfully (submitting a bid that equals his own cost). This gives the following lemma.

**Lemma 1** *If $m$ suppliers, $m \leq M$, accept the flexible contract, then $N-m$ suppliers will participate in the auction to compete for $Q-M$ units. The unit auction price will then be the $(Q-M+1)th$ lowest of the $N-m$ competing suppliers' costs.*

This lemma shows that by offering flexible contracts, the buyer essentially intensifies the competition of the subsequent auction. In both our model and the EK model, it is assumed that if the noncompetitive contracts are rejected, the buyer will procure the corresponding units from an outside market instead of putting them back to the auction. It seems reasonable, however, to also put these units back up for auction when the suppliers decline the contracts. In the EK model, if the declined unit is put back to the auction, it will not affect the supplier's behavior because he is not allowed to join the auction while the buyer may be worse off as the auction becomes less competitive. In our mechanism, if the buyer puts the rejected units back up for the auction, it can be shown that the dominant strategy for the selected supplier is to reject the contract and join the auction. This is rather intuitive: if the supplier accepts the contract, the subsequent auction is more competitive and he would be paid a lower price. Hence, he is better-off to simply reject the contract and join the "less competitive" auction. Therefore such non-competitive contract becomes redundant (and so the buyer can instead just hold a regular procurement auction).

Note that, when deciding whether to take the offer, the cost information held by the suppliers plays an important role, and in turn affects the buyer's procurement cost. Depending on what the suppliers know when the decision is made, two scenarios are considered: the "ex ante uninformed supplier" scenario and the "ex ante informed supplier" scenario. In the former, the suppliers do not know the exact value of their costs ex ante (at the time of deciding whether to take the noncompetitive offer), but the distribution $F(\cdot)$ while in the latter case, the suppliers already know their own cost ex ante. Here we provide some justification on the 'ex ante uninformed supplier" scenario. In this case the buyer and the suppliers have symmetric information at the ex ante stage (contract offering). This happens, for example, when the buyer's specification and requirements of the product/service are different from those usually provided by suppliers (and so the supplier needs more time to discover his exact cost until the auction starts), or when the product cost that mainly comes from the cost of the raw materials (for instance, a certain commodity), fluctuates over time. We also want to note that the information structures of these two scenarios are the same as the "no information" and "cost only" scenarios in the EK model, with which we will later compare. The event sequence of the uninformed supplier case is visualized in Figure 1. For the informed supplier case, the event sequence is similar except that the suppliers already know their cost before the buyer offers the contracts.

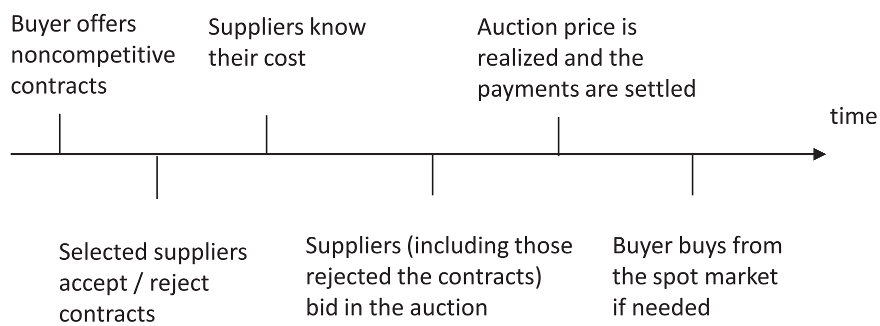



Figure 1: Sequence of events for the ex ante uninformed suppliers case

In the following two sections, we study these two information scenarios separately when the buyer only offers one contract $M = 1$ to gain insights and to make comparisons with the EK model. The analysis also sets the stage for the discussion of the multiple contracts, the $M > 1$ case.

# 3 Single Contract

In this section, we consider the case where $M = 1$. When consider scenarios that suppliers do or do not know their exact cost ex ante when the buyer offers the contract. In either case, if they turn down the contract, they will know their cost when they bid in the auction, so truthful bidding is guaranteed.

### 3.1 Ex Ante Uninformed Supplier

A selected supplier who is offered a contract makes a choice by comparing his expected profits between accepting and rejecting the contract. If he accepts the offer, then he must commit to selling a unit to the buyer without participating in the auction. The price is determined by the auction, in which $N-1$ suppliers bid for $Q-1$ units and the auction price is the $Qth$ lowest cost among the $N-1$ suppliers. We denote $C_{(i,j)}$, $1 \le i \le j$, the $i$th order statistic among $j$ i.i.d. random variables with cdf $F(\cdot)$. For example, $C_{(1,N)} = \min\{C_1, C_2, \ldots, C_N\}$. By David and Nagaraja (2003), the density of $C_{(i,j)}$ is give by $\phi_{(i,j)}(x) = j\binom{j-1}{i-1}F^{i-1}(s)(1-F(s))^{j-i}f(s)$.

The expected payoff of a selected supplier who accepts the offer is given by

$$\pi_u^{ac} = \mathsf{E}[C_{(Q,N-1)} - C]. \tag{1}$$

A selected supplier who declines the offer will join the forthcoming auction (because joining the auction results in a nonnegative profit, whereas quitting leaves him with zero profit). In the resulting auction, there are $N$ suppliers bidding for $Q-1$ units. The selected supplier's payoff is obtained by comparing his cost $C$ with $C_{(Q-1,N-1)}$, which is the $(Q-1)$th lowest cost among the remaining $N-1$ suppliers (Krishna 2002). If $C$ is larger than $C_{(Q-1,N-1)}$, then he loses the auction and receives 0; otherwise, his payoff is $C_{(Q-1,N-1)} - C$. Therefore, his expected payoff when rejecting the contract is

$$\pi_u^{re} \quad = \quad \mathsf{E}[\max\{C_{(Q-1,N-1)} - C, 0\}]. \tag{2}$$

For the supplier who is given the noncompetitive contract, if he rejects the contract and joins the auction, he will always have a non-negative payoff; while if he accepts the contract, it is possible for him to incur a loss. However when he accepts the contract, he effectively makes the competition less intense and the auction outcome will be more favorable to all the suppliers. In this regards, his tradeoff of accepting the contract lies between a more favorable auction price and the possibility of incurring a loss. If the latter effect is weak, particularly, when demand $Q$ is close to the supply $N$ and so the competition is not intense, he will be willing to accept the offer. This leads to the following theorem:

**Proposition 2** *When $Q$ is sufficiently close to $N$ and only one flexible contract is offered, then the selected supplier will accept the contract.*

In the following, assuming that $F(x)$ is uniform, we show that there is a unique cutoff point that characterizes the selected supplier's strategy.

**Proposition 3** *Suppose that $C$ is uniformly distributed. For any given $N$,*

*(a) The supplier will accept the contract if and only if $Q > Q^*(N) = \left\lfloor \frac{2N+3-\sqrt{8N+9}}{2} \right\rfloor$, where $\lfloor x \rfloor$ is the largest integer that is no greater than $x$, and*

*(b) $Q^*(N)$ is nondecreasing in $N$, and in particular, either $Q^*(N+1) = Q^*(N)$ or $Q^*(N+1) = Q^*(N) + 1$.*

It is intuitive that the ratio $Q/N$ indicates the intensity of the competition, whereby the smaller the value of $Q$ (or the larger $N$), the more intense the competition among suppliers. This implies that a supplier is more likely to decline the offer when the competition becomes more intense (i.e., $N$ becomes larger), because there is a higher probability of making a negative profit if he accepts the offer.

We now consider the buyer's problem. As the buyer is rational and has the same information as the suppliers when offering the contract, she can infer $Q^*$[6] and further anticipate the selected supplier's action based on $Q^*$. By analyzing the buyer's expected cost, we aim to determine whether it is in the buyer's interest to offer a flexible contract to the supplier and whether, with the EK model in mind, the buyer should provide the flexibility to allow suppliers declining the offer to join the auction.

We first examine whether the buyer should offer a flexible contract. If $Q > Q^*$, then the selected supplier will accept the offer. The buyer is better-off by offering such a contract, as the resulting auction is more competitive and hence the auction price is lower. However, this may not be the case if $Q < Q^*$. Letting $\Pi_u$ and $\Pi_{au}$ be the buyer's expected costs in our model and an ordinary reverse auction model without contracts, respectively, we derive the following proposition.

**Proposition 4** *For a uniformly distributed $C$ and a given $N$,*

*(a) there exists a threshold $Q_a \leq Q^*$ such that $\Pi_u \leq \Pi_{au}$ if and only if $Q > Q_a$; and*

*(b) $Q_a$ is increasing in $N$ and $c_0$.*

When $Q$ is large, by offering a flexible contract, the buyer can increase the competition of the auction without incurring any additional cost, and thus her expected cost is lower. Sometimes, although aware of that the supplier will reject the contract, if $c_0$ is sufficiently small that $Q^* > \lfloor c_0(N+1)/2 \rfloor$, the buyer will still make an offer, because the benefit gained from increasing the

[6]We suppress $Q^*(N)$ as $Q^*$ in the remainder of the paper for brevity unless confusion may arise. We shall use similar suppressions for other notion.

competition of the auction is greater than the additional cost incurred from acquiring the unit from the spot market. Part (b) of Proposition 4 further shows that when the spot market price $c_0$ becomes larger, the contract is less beneficial to the buyer when the supplier rejects the offer. However, $c_0$ does not affect the supplier's decision to take the offer or not, and once he accepts the offer, $c_0$ no longer has an impact on the buyer's cost.

We next compare the cost of the buyer in our setting with that in the "no information" scenario of the EK model. Let the expected cost of the buyer in the corresponding case of the EK model be denoted by $\Pi_u^{EK}$. In the EK model, the supplier will accept the contract as long as the expected price of the auction is higher than his expected cost. When $M = 1$, the expected unit price established by the auction is $\mathsf{E}[C_{(Q,N-1)}]$. If $F(x)$ is uniformly distributed in $[0, 1]$, it follows that $\mu = 0.5$, and the expected auction price is $Q/N$. The supplier will therefore accept the contract if $Q \geq 1/2N$ in the EK model. Let $Q^*_{EK} = \lfloor N/2 \rfloor$. It is easy to verify that $Q^*_{EK} < Q^*$, or that the supplier is more likely to reject the flexible contract. This result has intuitive appeal. In the EK model, the supplier will accept the contract only if the expected price of the auction is higher than his expected cost, because he makes a positive expected profit if he accepts but zero if he rejects. However in our model, it is possible for the supplier to reject the contract even if the expected price of the auction is higher than his expected cost. This is because he can still enter the auction and may make an even higher profit. This argument also implies that the selected supplier is better-off when he has the flexibility to join the auction.

For the buyer, if $Q > Q^*$, then the supplier accepts the contract in both models, and so it makes no difference for the buyer which type of contract is offered. If $Q \leq Q^*_{EK}$, then the supplier rejects the contract in both cases. However the supplier in our model is allowed to join in the auction thus will make the competition more intense therefore in this case, it is better for the buyer to use flexible contract.

If $Q \in (Q^*_{EK}, Q^*]$, then the supplier in the EK model will accept the contract, but the supplier in our model will reject the contract. This implies that whether or not the buyer should give suppliers the flexibility depends on $c_0$. The cost of the buyer in the EK model is $\Pi_u^{EK} = Q\mathsf{E}[C_{(Q,N-1)}] = Q^2/N$ and the cost of the buyer in our model is $\Pi_u = (Q-1)\mathsf{E}[C_{(Q,N)}] + c_0 = \frac{Q(Q-1)}{N+1} + c_0$. Comparing these two costs, we have

$$\Pi_u^{EK} - \Pi_u = \frac{Q(N+Q)}{N(N+1)} - c_0,$$

which is increasing in $Q$. This implies $\Pi_u^{EK} \geq \Pi_u$ when $Q$ is larger than some threshold. The following proposition summarizes the above discussion.

**Proposition 5** *For a uniformly distributed $C$ and a given $N$,*

*(a) there exists a threshold $Q_b$, such that $\Pi_u \leq \Pi_u^{EK}$ if and only if $Q \notin (Q^*_{EK}, Q_b]$; and*

*(b) $Q_b$ is increasing in $N$ and $c_0$.*

This proposition shows that the buyer may be better-off by offering a flexible contract rather than a contract in the EK model. Furthermore, the buyer may benefit from offering a flexible contract even if she knows that the offer will be rejected, because although she has to procure one unit from the outside market at a high price $c_0$, the resulting more competitive auction leads to a lower auction price. However, when $c_0$ increases, the benefit of giving the supplier flexibility decreases.

Based on Propositions 4 and 5, we can easily obtain the following result.

**Corollary 6** *For a uniformly distributed $C$ and a given $N$, if $Q > \max\{Q_a, Q_b\}$ or $Q \in (Q_a, Q^*_{EK}]$, then $\Pi_u \leq \min\{\Pi_u^{EK}, \Pi_{au}\}$.*

## 3.2 Ex Ante Informed Supplier

In the ex ante informed supplier case, suppose that the selected supplier has a cost $c$ which is realized from $C$. From the supplier's perspective, the expected payoff from accepting the contract is

$$\pi_c^{ac} = \mathsf{E}[C_{(Q,N-1)} - c],$$

and his expected payoff from rejecting the contract is

$$\pi_c^{re} = \mathsf{E}[\max\{C_{(Q-1,N-1)} - c, 0\}].$$

The selected supplier decides whether or not to accept the offer by comparing these two payoffs and his optimal action can then be characterized as follows.

**Proposition 7** *(a) For given $Q$ and $N$, there exists a constant $c^*$ such that the supplier will accept the contract if and only if $c < c^*$; and* [7]

*(b) the threshold $c^*$ is increasing in $Q$ and decreasing in $N$.*

The intuition for Part (a) of Proposition 7 is similar to that of Proposition 2: When the supplier has a small private cost, he is less likely to incur a loss from accepting the contract, therefore he

[7] When $c = c^*$, there is no difference between accepting and rejecting the contract.

can enjoy the more favorable auction price by accepting the contract and making the auction less competitive. Part (b) of Proposition7 implies that, $Q$ and $N$ has an indirect effect on the supplier's decision through $c^*$, in that the selected supplier is more likely to accept the offer when $Q$ increases or $N$ decreases.

Note that in the corresponding "cost only" case of the EK model, there is also a constant $c^*_{EK}$ such that the supplier will accept the contract if and only if his cost $c$ is no more than $c^*_{EK}$. The value $c^*_{EK}$ satisfies $\mathsf{E}[C_{(Q,N-1)} - c^*_{EK}] = 0$. From the previous result, $c^*$ is determined by $\mathsf{E}[C_{(Q,N-1)} - c^*] - \mathsf{E}[\max\{C_{(Q-1,N-1)} - c^*, 0\}] = 0$. It is apparent that $c^* < c^*_{EK}$. In other words, the supplier is more likely to reject the contract in our model than in the EK model. This is again quite intuitive, as the supplier has more flexibility in our model.

We now consider the buyer's problem. In the informed supplier scenario, the buyer's problem will be more complicated than in the uninformed supplier scenario because there is "information asymmetry" in this setting. The buyer does not know the supplier's cost and cannot fully anticipate the supplier's action, but the buyer still knows $c^*$, as solving $c^*$ does not require knowing the exact cost of the selected supplier. Therefore, if the selected supplier rejects the offer, then the buyer will know that his cost is higher than $c^*$ and should update her information about the supplier's cost accordingly. This update is necessary for the buyer, because the supplier will join the auction. Specifically, the buyer knows that if the supplier's cost $C \leq c^*$, then the supplier will accept the contract and her cost will be $QC_{(Q,N-1)}$; otherwise, her cost will be $(Q-1)P_{au} + c_0$, where

$$P_{au} = \begin{cases} C_{(Q-1,N-1)}, & C \leq C_{(Q-1,N-1)}, \\ C, & C_{(Q-1,N-1)} < C \leq C_{(Q,N-1)}, \\ C_{(Q,N-1)}, & C > C_{(Q,N-1)}, \end{cases} \tag{3}$$

is the auction price. Because the selected supplier rejects the offer, his cost $C$ is different from the costs of the other suppliers from the buyer's perspective.

Therefore, the expected cost of the buyer is

$$\begin{aligned} \Pi_c &= F(c^*)Q\int_0^1 x(N-1)\binom{N-2}{Q-1}F^{Q-1}(x)(1-F(x))^{N-Q-1}dF(x) \\ &+ \int_{c^*}^1 \Big[\mathsf{E}[C_{(Q-1,N-1)}\mathbf{1}(x \leq C_{(Q-1,N-1)}) + x\mathbf{1}(C_{(Q-1,N-1)} < x \leq C_{(Q,N-1)}) \\ &\qquad + C_{(Q,N-1)}\mathbf{1}(x > C_{(Q,N-1)})](Q-1) + c_0\Big]dF(x), \end{aligned}$$

where $\mathbf{1}(A) = 1$ if event $A$ is true; otherwise $\mathbf{1}(A) = 0$. The first line represents the expected cost of the buyer if the supplier accepts the offer, and the second and third lines are the expected cost if the supplier rejects the offer.

We next compare the results with those of the "cost only" scenario of the EK model, which shares a same information setting. Let $\Pi_c^{EK}$ be the expected cost of the buyer in the EK model. In the next theorem, we show that from the buyer's perspective, our mechanism can outperform both an ordinary reverse auction and the mechanism proposed in the EK model.

**Proposition 8** *If $Q$ is sufficiently close to $N$ and $C$ is uniformly distributed over $[0,1]$, then the buyer incurs a lower expected cost by offering a flexible noncompetitive contract, i.e., $\Pi_c < \Pi_c^{EK} < \Pi_{au}$ when $c_0 = 1$.*

Proposition8 says that when $Q$ is close to $N$, the buyer is better-off by offering a flexible contract rather than offering a non-flexible contract or using a regular auction. For the selected supplier, there are three possible actions. First, the supplier accepts the noncompetitive offer in both models. Then there is no difference in the subsequent auction, so does the buyer's cost. Second, the supplier rejects the noncompetitive offer in both models. Then in our model, the supplier will join the auction, which makes the auction more competitive and therefore will benefit the buyer (reduce her cost). Third, the supplier rejects the contract under our mechanism while accepts the contract in the EK model. Then in our model the supplier joins the auction, which makes the auction more competitive while the buyer procures the declined unit at a higher price $c_0$. When $Q$ is close to $N$, the buyer is always better-off because a small reduction on the auction price (due to the increased competition) will be applied to all $Q-1$ units and therefore the saving on the total procurement cost outweighs the expected additional cost of procuring from the spot market (when the spot market price is not too high).

We end this section by summarizing our major results into the following table.

Table 1: Summary of main results: single contract

| | Ex ante uninformed suppliers | Ex ante informed suppliers |
|---|---|---|
| Supplier accepts the flexible contract | $Q > Q^*$ | $c < c^*$ |
| The flexible contract is better than the regular auction | $Q > Q_a$ | $Q$ is sufficiently close to $N$ |
| The flexible contract is better than the EK model | $Q \leq Q^*_{EK}$ or $Q > Q_b$ | $Q$ is sufficiently close to $N$ |

# 4 Multiple Contracts

In this section, we first extend our previous analysis of the case where only one flexible contract is offered ($M = 1$) to a more general case where the buyer offers two contracts simultaneously ($M$=2) to two selected suppliers. As in the single-contract case, $Q$, $N$, and the number of contracts are known to the selected suppliers. They need to determine whether to accept the contract at the same time. Meanwhile, it is clear that whether a supplier accepts the contract depends on the other supplier's strategy, since their decisions will affect the number of participants in the subsequent auction and therefore will also affect each other's expected payoff. Hence, we analyze the Nash equilibrium (Fudenberg and Tirole 1991) and discuss how the outcome of the game affects the buyer's expected cost. We then discuss the general case where $M \geq 2$. It should be noted that we only focus on pure-strategy equilibrium. But it is possible to show that there may be a mixed-strategy equilibrium for this game. Again, we discuss two scenarios based on whether or not the suppliers know the exact cost. For the sake of tractability, we assume that $F(x)$ is uniform in the remaining analysis.

## 4.1 Ex Ante Uninformed Supplier

We first consider the case where the suppliers do not know their exact costs at the moment of decision making. We refer to the two selected suppliers as Supplier 1 and Supplier 2, respectively, and denote their action by $a_i$, $i = 1, 2$. Note that the action space for each selected supplier is {reject, accept}. Recall that $Q^*(N)$ is the threshold that determines the supplier's strategy in the case where there are $N$ suppliers and one flexible noncompetitive contract is offered.

In the following theorem, we characterize the pure-strategy Nash equilibrium of the suppliers' game.

**Proposition 9** *Suppose that the buyer offers two contracts ($M = 2$) and $C$ is uniformly distributed. The pure-strategy Nash equilibrium $[a_1^*, a_2^*]$ of the game between two selected suppliers is*

$$[a_1^*, a_2^*] = \begin{cases} \text{[reject, reject]}, & \text{if } Q \leq Q^*(N-1)+1, \\ \text{[accept, accept] or [reject, reject]}, & \text{if } Q^*(N-1)+1 < Q \leq Q^*(N)+1, \\ \text{[accept, accept]}, & \text{if } Q > Q^*(N)+1. \end{cases}$$

When $Q^*(N-1)+1 < Q \leq Q^*(N)+1$, we can further show that [accept, accept] is Pareto optimal for the suppliers between the two Nash equilibria. For either supplier, the expected payoff is $\mathsf{E}[C_{(Q-1,N-2)} - C]$ when both of them accept the contract, whereas it is $\mathsf{E}[(C_{(Q-2,N-1)} - C)^+]$

when both of them reject. As $Q^*(N-1) < Q-1$, and by the definition of $Q^*(\cdot)$,

$$\mathsf{E}[C_{(Q-1,N-2)} - C] > \mathsf{E}[\max\{C_{(Q-2,N-2)} - C, 0\}] \geq \mathsf{E}[\max\{C_{(Q-2,N-1)} - C, 0\}],$$

in which the last inequality holds because $C_{(Q-2,N-2)}$ is larger than $C_{(Q-2,N-1)}$. This shows that the expected payoff for both suppliers under the equilibrium [accept, accept] is better than that under [reject, reject]. Hence, the equilibrium [accept, accept] is Pareto optimal. Despite this, we still cannot predict the exact behavior of the selected suppliers in this case.

Based on the foregoing analysis of the selected suppliers' behavior, we next examine the buyer's total cost under different scenarios specified by the value of $Q$. A related question that we seek to answer is whether it is better for the buyer to offer two contracts or just one. Let $\Pi_u(M)$ ($\Pi_u \equiv \Pi_u(1)$) be the expected cost of the buyer if she offers $M$ flexible contracts.

When $Q \leq Q^*(N-1)+1$, for cases of either $M=1$ or $M=2$, the selected supplier(s) will reject the offer(s), and thus the buyer's expected cost is

$$\Pi_u(2) \quad = \quad (Q-2)\mathsf{E}[C_{(Q-1,N)}] + 2c_0 = \frac{(Q-1)(Q-2)}{N+1} + 2c_0.$$

The relationship between $\Pi_u(2)$ and $\Pi_u$ depends on the values of $N$ and $c_0$. The buyer's cost of offering one flexible contract is lower if

$$\Pi_u = \frac{Q(Q-1)}{N+1} + c_0 \leq \frac{(Q-1)(Q-2)}{N+1} + 2c_0,$$

or equivalently

$$Q \leq \left\lfloor \frac{c_0(N+1)}{2} \right\rfloor + 1;$$

otherwise, if $Q > \lfloor c_0(N+1)/2 \rfloor + 1$, then the buyer should offer two flexible contracts.

If $Q > Q^*(N)+1$, then, in either the $M=1$ or $M=2$ case, the selected supplier(s) will accept the offer(s). Thus,

$$\Pi_u(2) = Q\mathsf{E}[C_{(Q-1,N-2)}] = \frac{(Q-1)Q}{N-1}.$$

It is clear that

$$\frac{(Q-1)Q}{N-1} < \frac{Q^2}{N} = \Pi_u.$$

Therefore, it is better for the buyer to offer two contracts than one.

Let

$$Q_c(N) = \min\left\{\left\lfloor \frac{c_0(N+1)}{2} \right\rfloor, Q^*(N-1)\right\} + 1.$$

If $Q < Q_c(N)$, then the buyer's expected cost is lower than if only one contract is offered. If $Q = Q^*(N-1)+1 = Q^*(N)$, then there are two Nash equilibria and the buyer's expected cost may be higher or lower when $M = 2$ than when $M = 1$ depending on which equilibrium is reached. So if $Q \geq Q_c(N)$ and $Q \notin (Q^*(N-1)+1, Q^*(N)+1]$, then the buyer should offer two contracts. Note that $Q_c(N) \geq Q_a(N)$.

**Proposition 10** *When $C$ is uniformly distributed, there exists $Q_c(N) \leq Q^*(N)$ such that for all $Q \in [Q_c(N), N)$ and $Q \notin (Q^*(N-1)+1, Q^*(N)+1]$, the buyer has a lower expected cost by offering two contracts than one contract.*

Note that if the suppliers follow the Pareto optimal equilibrium, then the buyer has a lower cost with $M = 2$ as long as $Q \geq Q_c(N)$.

## 4.2 Ex Ante Informed Supplier

We now consider the informed supplier scenario, which is expected to be more complex than the uninformed supplier scenario. Again, we first analyze the equilibrium behavior of the two selected suppliers. Let $C_i$ be the cost of Supplier $i$ and $c_i$ be its realization, which is only known to Supplier $i$, $i = 1, 2$. From Proposition7, denote $c_1^* = c^*(N, Q-1)$ and $c_2^* = c^*(N-1, Q-1)$. Note that $c_1^* \leq c_2^*$. The following lemma presents the best response of Supplier $i$ to Supplier $(3-i)$'s strategy, $i = 1, 2$.

**Lemma 11** *Supplier $i$'s best response $a_i(c_i, a_{3-i})$, $i = 1, 2$, can be expressed as follows. If $a_{3-i} =$ accept, then*

$$a_i(c_i, a_{3-i}) = \begin{cases} \textit{accept}, & \textit{if } c_i < c_1^*; \\ \textit{accept} & \textit{if } c_1^* \leq c_i < c_2^*; \\ \textit{reject}, & \textit{if } c_i \geq c_2^*, \end{cases}$$

*and if $a_{3-i} = reject$, then*

$$a_i(c_i, a_{3-i}) = \begin{cases} \textit{accept}, & \textit{if } c_i < c_1^*; \\ \textit{reject}, & \textit{if } c_1^* \leq c_i < c_2^*; \\ \textit{reject}, & \textit{if } c_i \geq c_2^*. \end{cases}$$

If the suppliers know their costs when they decide whether or not to accept the offer, then based on the previous lemma, their equilibrium strategy is described by the following theorem.

**Proposition 12** *Suppose that the buyer offers two contracts. For any given $Q$ and $N$, the pure strategy Nash equilibrium $[a_1^*, a_2^*]$ is*

$$[a_1^*, a_2^*] = \begin{cases} \textit{[reject, reject]}, & \textit{if } c_1 \geq c_1^*, c_2 \geq c_2^* \textit{ or } c_2 \geq c_1^*, c_1 \geq c_2^*; \\ \textit{[accept, accept]}, & \textit{if } c_1 < c_1^*, c_2 < c_2^* \textit{ or } c_2 < c_1^*, c_1 < c_2^*; \\ \textit{[accept, accept] or [reject, reject]}, & \textit{if } c_1^* \leq c_1 < c_2^* \textit{ and } c_1^* \leq c_2 < c_2^*; \\ \textit{[reject, accept]}, & \textit{if } c_1 \geq c_2^*,\ c_2 < c_1^*; \\ \textit{[accept, reject]}, & \textit{if } c_1 < c_1^*,\ c_2 \geq c_2^*. \end{cases}$$

Let us take a closer look at the case when there are two equilibria. We can also show that when $c_1^* \leq c_i < c_2^*, i = 1, 2$, [accept, accept] is Pareto optimal for the suppliers. To this end, we only need to prove that Supplier $i$ obtains a larger expected payoff when both suppliers accept the contracts. Note that if both suppliers accept the offers, then supplier $i$'s expected payoff is

$$\mathsf{E}[C_{(Q-1,N-2)} - c_i] \geq \mathsf{E}[\max\{C_{(Q-2,N-2)} - c_i, 0\}] \geq \mathsf{E}[\max\{C_{(Q-2,N-1)} - c_i, 0\}],$$

for $i = 1, 2$. The first inequality is derived from the definition of $c_2^*$ and $c_i \leq c_2^* = c^*(N-1, Q-1)$, and the second inequality is due to $C_{(Q-1,N-2)}$ being larger than $C_{(Q-1,N-1)}$. Therefore [accept, accept] is the Pareto optimal equilibrium.

As the buyer's problem is very complicated in this case, in the following discussion we only show how to compute the buyer's expected procurement cost. For ease of exposition, we also assume that the suppliers follow the Pareto optimal equilibrium of $[a_1^*, a_2^*] = [accept, accept]$ if $c_1^* \leq c_1 < c_2^*$ and $c_1^* \leq c_2 < c_2^*$ (we can also calculate the buyer's cost by assuming, with a probability $\alpha$, $0 \leq \alpha \leq 1$, that both suppliers will accept, and with a probability $1 - \alpha$ that both will reject). This analysis will enable us to obtain further insights from numerical studies.

For notational convenience, we use $P_{au}(a_1^*, a_2^*)$ to denote the auction price given the equilibrium strategy of the suppliers, and $a_i^* = 1$ means that Supplier $i$ accepts the offer and $a_i^* = 0$ means he rejects it, $i = 1, 2$. Let $\Pi_c(M)$ ($\Pi_c(1) \equiv \Pi_c$) be the buyer's expected cost when offering $M$ contracts. The buyer's total expected cost, from Proposition12, can be written as

$$\begin{aligned} \Pi_c(2) &= \mathsf{E}[QP_{au}(1,1)\mathbf{1}(1,1)] + \mathsf{E}[((Q-1)P_{au}(1,0) + c_0)\mathbf{1}(1,0)] \\ &\quad + \mathsf{E}[((Q-1)P_{au}(0,1) + c_0)\mathbf{1}(0,1)] + \mathsf{E}[((Q-2)P_{au}(0,0) + 2c_0)\mathbf{1}(0,0)], \end{aligned} \tag{4}$$

in which $\mathbf{1}(i,j) = 1$ if $a_1^* = i$ and $a_2^* = j$, $i, j = 0, 1$; otherwise, $\mathbf{1}(i,j) = 0$. We next explain how to compute each term in (4).

Consider the first term. Once both suppliers accept the offers, the buyer gains no extra information for her expectation about the auction price. So,

$$\mathsf{E}[QP_{au}(1,1)\mathbf{1}(1,1)]$$

$$
\begin{aligned}
&= \mathsf{E}[QC_{(Q-1,N-2)}]\Big[Pr(C_1 < c_1^*, C_2 < c_2^* \text{ or } C_1 < c_2^*, C_2 < c_1^*) + Pr(c_1^* \le C_1 < c_2^*, c_1^* \le C_2 < c_2^*)\Big] \\
&= \mathsf{E}[QC_{(Q-1,N-2)}]\Big[Pr(C_1 < c_1^*, C_2 < c_1^*) + Pr(C_1 < c_1^*, c_1^* \le C_2 < c_2^*) + Pr(C_2 < c_1^*, c_1^* \le C_1 < c_2^*) \\
&\quad +Pr(c_1^* \le C_1 < c_2^*, c_1^* \le C_2 < c_2^*)\Big] \\
&= \mathsf{E}[QC_{(Q-1,N-2)}]\Big[F(c_1^*)^2 + 2F(c_1^*)(F(c_2^*) - F(c_1^*)) + (F(c_2^*) - F(c_1^*))^2\Big], \qquad (5)
\end{aligned}
$$

where the second equality from

$$
\begin{aligned}
& (C_1 < c_1^*, C_2 < c_2^* \text{ or } C_1 < c_2^*, C_2 < c_1^*) \\
= \; & (C_1 < c_1^*, C_2 < c_1^* \text{ or } C_1 \le c_1^*, c_1^* \le C_2 < c_2^* \text{ or } C_2 < c_1^*, c_1^* \le C_1 < c_2^*),
\end{aligned}
$$

in which the three events are mutually exclusive.

Now consider the second and third terms in (4). Notice that these two terms are the same because the selected suppliers are symmetric. Therefore, it suffices to show how to compute the second term. When one supplier rejects the contract, the buyer updates her belief about the supplier's cost, and so $P_{au}(1,0)$ and $\mathbf{1}(1,0)$ are no longer independent. Similar to the analysis in Section 4.2,

$$
\begin{aligned}
& \mathsf{E}[((Q-1)P_{au}(1,0) + c_0))\mathbf{1}(1,0)] \\
= \; & \int_0^{c_1^*}\int_{c_2^*}^1 \Big\{ \mathsf{E}\Big[C_{(Q-2,N-2)}\mathbf{1}(\nu \le C_{(Q-2,N-2)}) + \nu\mathbf{1}(C_{(Q-2,N-2)} < \nu \le C_{(Q-1,N-2)}) \\
& +C_{(Q-1,N-2)}\mathbf{1}(\nu > C_{(Q-1,N-2)})\Big](Q-1) + c_0 \Big\} d\nu d\xi \qquad (6)
\end{aligned}
$$

Finally, we turn to the last term in (4). If both suppliers reject the contracts, then the problem becomes much more complicated, as the buyer needs to update her cost information about both suppliers. This will affect her expected cost, because after the suppliers have rejected the offers, they will join the auction to compete for the remaining $Q-2$ units. From Proposition12, both suppliers will reject the contracts if $C_1 > c_1^*, C_2 > c_2^*$, or $C_1 \ge c_2^*, C_2 \ge c_1^*$.

Notice that there are $N$ suppliers competing for $Q-2$ units in the auction, and so the auction price is the $(Q-1)$th lowest cost among the $N$ suppliers. We need to consider Suppliers 1 and 2 separately from the other $N-2$ suppliers because they are different from the buyer's perspective. Figure 2 visualizes different sample paths of $C_1$ and $C_2$ and the corresponding resulting auction price. For instance, for the second row of figure 2, Supplier 1's cost $C_1$ is less than $C_{(Q-3,N-2)}$ and Supplier 2's cost $C_2$ is higher than $C_{(Q-3,N-2)}$ but lower than $C_{(Q-2,N-2)}$, then the resulting auction price will be $C_2$, or the cost of Supplier 2, who actually loses in the auction. Other instances can be similarly explained.

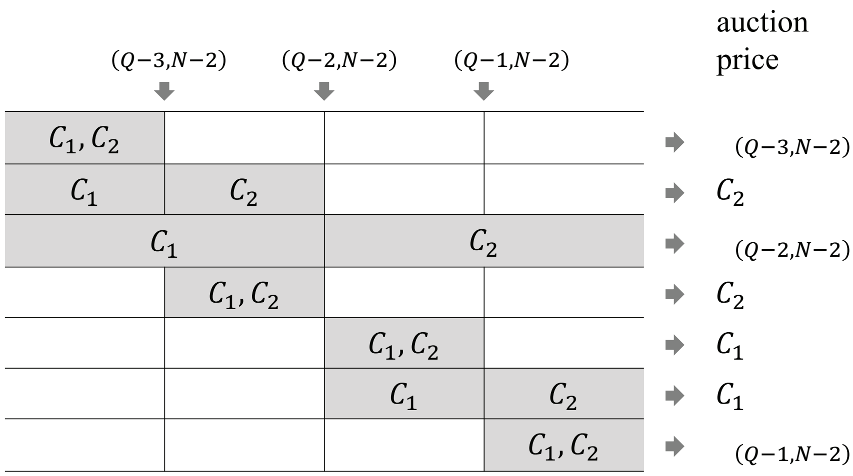



Figure 2: Sample paths of $C_1, C_2$ and auction price given both contracts are rejected

All other scenarios can be similarly analyzed, and we leave the detailed derivation to interested readers. After some algebra, the last term in (4) can be calculated as

$$\begin{aligned} 2\int_{c_2^*}^{1}\int_{c_1^*}^{\nu}\Bigg\{\mathsf{E}\Bigg[&C_{(Q-3,N-2)}\mathbf{1}(\nu < C_{(Q-3,N-2)}) + \nu\mathbf{1}(C_{(Q-3,N-2)} < \nu < C_{(Q-2,N-2)}) \\ &+\xi\mathbf{1}(C_{(Q-2,N-2)} < \xi < C_{(Q-1,N-2)}) + C_{(Q-2,N-2)}\mathbf{1}(\xi < C_{(Q-2,N-2)} < \nu) \\ &+C_{(Q-1,N-2)}\mathbf{1}(\xi > C_{(Q-1,N-2)})\Bigg](Q-2) + 2c_0\Bigg\}d\xi d\nu. \end{aligned} \tag{7}$$

After adding up (5), (7), and twice of (6), we obtain the expected cost of the buyer. As the resulting expression is quite complicated, we omit its detailed expression here. Consequently, we are unable to analytically compare buyer's cost in the flexible noncompetitve contract and in the EK model when $M = 2$. We leave this to our numerical study.

**Remark.** Note that all of the analytical results in this subsection hold for an arbitrary distribution function $F(\cdot)$.

### 4.3 General $M$ Case

Finally, we discuss the case where the buyer offers $2 \le M \le Q-1$ contracts. We refer to the selected suppliers as Supplier 1, Supplier 2,. . ., Supplier $M$. In general, when $M > 2$, the equilibrium behavior of the suppliers is very complicated, and there are multiple equilibria for certain scenarios. However, similar to the results in the previous section, when multiple equilibria exist, we can show that one of them is Pareto optimal.

When the suppliers have to make decisions before knowing their own costs, the following result characterizes their equilibrium behavior, and provides a lower and an upper bound for the optimal number of contracts $M^*$ that the buyer should offer.

**Proposition 13** *Assume that the buyer offers $M$ flexible noncompetitive contracts simultaneously, $2 \leq M \leq Q-1$, and $C$ is uniform. In the uninformed supplier case, for any given $N$ and $Q$,*

*(a) there exist two numbers $\underline{Q}$ and $\overline{Q}$. Each selected supplier rejects the offer if $Q \leq \underline{Q}$ and accepts the offer if $Q > \overline{Q}$, regardless of whether the other suppliers accept or reject their offers;*

*(b) for $\underline{Q} < Q \leq \overline{Q}$, the equilibrium is either that all selected suppliers accept the contracts or all selected suppliers reject the contracts, i.e., [accept,accept,...,accept] and [reject,reject,...,reject] are both equilibria. Furthermore, [accept,accept,...,accept] is Pareto optimal;*

*(c) if the suppliers behave Pareto optimally, then the optimal $M^*$ that minimizes the buyer's expected cost satisfies*

$$Q - Q^*(N - M^* + 1) \leq M^* \leq Q + 1 - \min\left\{Q^*(N - M^* + 1), \left\lfloor \frac{c_0(N+1)}{2} \right\rfloor\right\}.$$

The next proposition characterizes the suppliers' equilibrium behavior when they know their costs at the moment of decision making. Note that we assume that the suppliers will follow the Pareto optimal equilibrium. We denote the cost of Supplier 1 to Supplier $M$ as $c_1, \ldots, c_M$, and let $c_{(1)}, \ldots, c_{(M)}$ be an ordered permutation of $c_1, \ldots, c_M$ such that $c_{(1)} \leq c_{(2)} \leq \ldots \leq c_{(M)}$.

**Proposition 14** *Assume that the buyer offers $M$ flexible noncompetitive contracts simultaneously, $2 \leq M \leq Q-1$. For a general distribution $F$, in the informed supplier case, for any given $N$ and $Q$,*

*(a) there exists a pair of thresholds $\underline{c}$ and $\overline{c}$. Each selected supplier with a cost $c$ should reject the offer if $c > \overline{c}$ and accept the offer if $c \leq \underline{c}$, regardless of whether the other suppliers accept or reject their offers;*

*(b) there exist a sequence of thresholds $c_1^*, \ldots, c_M^*$ such that $c_1^* \leq c_2^* \cdots \leq c_M^*$ and the Pareto optimal equilibrium is suppliers $(1), \ldots, (k)$ accepting the contract and suppliers $(k+1), \ldots, (M)$ rejecting the contract, if $c_{(k)} \leq c_k^*$ and $c_{(k+1)} > c_{k+1}^*, \ldots, c_{(M)} > c_M^*$ for some $k$.*

When the suppliers are informed, the buyer's cost is too complicated to calculate when $M > 2$

even if we only consider the Pareto optimal equilibrium. As a result, we are unable to provide further guidance for the buyer on how to select an optimal $M$ in this case.

# 5 Numerical Study

In this section, we conduct a comprehensive numerical study to examine the effect of the system parameters. We first focus on our mechanism itself, and then investigate and identify the situations where our contract will benefit the buyer by comparing it with the EK model.

Let $\sigma^2$ be the variance of $C$. Throughout this numerical study, the cost distribution of the suppliers is selected from the following:

(1) a uniform distribution, i.e., $f(x) = 1$, $\mu = 0.5$ and $\sigma^2 = 0.08$;

(2) a normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$ on $[0, \infty)$ (a truncated normal) with $\mu = 0.5$, $\sigma = 0.15$, or $\sigma = 0.3$. Note that the variances of these two truncated normal distributions are 0.02 and 0.06, respectively;

(3) a continuous distribution with a quadratic density, i.e., $f(x) = 12(x-0.5)^2$, $\mu = 0.5$, $\sigma^2 = 0.15$; and

(4) a Beta distribution $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{Beta}(\alpha,\beta)}, \alpha, \beta > 0$ , in which $\alpha, \beta$ are specified in the examples.

The rationale for selecting these distributions is that they possess three different shapes, i.e., linear, unimodal, and negative unimodal. Unless otherwise noted, we use $N = 10$ and $c_0 = 1$ in all instances. Note that, as it is clear that $Q$ and $N$ perform a dual role, in that a larger $Q$ has the same impact as a smaller $N$, we only adjust the value of $Q$ to change the intensity of the competition among suppliers.

## 5.1 Insights into the Flexible Noncompetitive Contract

We first provide some insights by investigating our hybrid mechanism. To begin with, we compare our mechanism with a regular reverse auction when $M = 1$. The percentage improvement in the buyer's cost, which is defined as $Improvement_j^{au} = (\Pi_{au} - \Pi_j)/\Pi_j \times 100\%, j = u, c$, with different values of $Q$ and cost distributions of suppliers are reported in Table 2. Note that in Table 2 and the other tables in this section, we do not list the results for all of the values of $Q$ due to space limitations.

Table 2: Improvement over a regular reverse auction

| | $Improvement_u^{au}$ (%) | | | | $Improvement_c^{au}$ (%) | | | |
|---|---|---|---|---|---|---|---|---|
| $F(x)$ | $Q=4$ | 6 | 8 | max ($Q$) | $Q=4$ | 6 | 8 | max ($Q$) |
| Uniform | -13.04 | 2.44 | 2.27 | 3.90 (7) | -7.32 | 1.84 | 1.88 | 2.33 (7) |
| Trunc. Normal(0.5,0.15) | -17.36 | -7.07 | 1.62 | 1.62 (8) | -11.43 | -2.72 | 0.49 | 0.93 (9) |
| Trunc. Normal(0.5,0.3) | -15.07 | -1.78 | 2.23 | 3.32 (7) | -9.34 | -0.08 | 1.63 | 1.63 (8) |
| Quadratic | -7.21 | 12.72 | 1.55 | 12.72 (6) | 0.70 | 5.62 | 1.40 | 7.01 (5) |

We observe that, when the competition among suppliers is not intense (when $Q$ is close to $N$), the buyer benefits from offering a flexible contract in both information scenarios, and the maximum improvement is 12.72%. However, when the competition is intense (e.g. $Q = 4$), the flexible contract will make the supplier more like to reject the contract thus the supplier will suffer from the extra cost of procuring from spot market. For the particular case that $Q = 4$, one unit out of four units is procured from the high-priced spot market if the contract is turned down, therefore the percentage negative improvement is high.

In their experimental study, Engelbrecht-Wiggans and Katok (2006) show that only about 70% of the selected suppliers accept noncompetitive offers, which is less than the theoretical results predict. One possible reason for this that they mention, and we also concur, is that the suppliers may actually be risk averse rather than risk neutral. We also conduct a numerical study on the risk-averse supplier case. For the uninformed supplier case, we observe that, similar to the risk-neutral case, the supplier tends to accept the contract when $Q$ is close to $N$. Meanwhile, for the informed supplier case, we also find that there exists a threshold such that if the supplier's cost is higher than the threshold, he will reject the contract while he will accept the contract otherwise. Due to the complexity of the resulting analysis, these findings are rather challenging to prove analytically.

### 5.1.1 Impact of Information

In this section, we examine the impact of the suppliers' information in the buyer's expected cost. The following figure depicts the difference of the buyer's cost under the informed and the uninformed supplier scenarios $\Pi_c - \Pi_u$.

The figure shows that when the competition is intense (when $Q/N$ is small), the buyer is better-off by dealing with an informed supplier, but she is better-off by dealing with an uninformed supplier when competition is not intense. The intuition behind this observation is as follows. When $Q/N$ is small, an uninformed selected supplier will reject the contract. This hurts the buyer, because

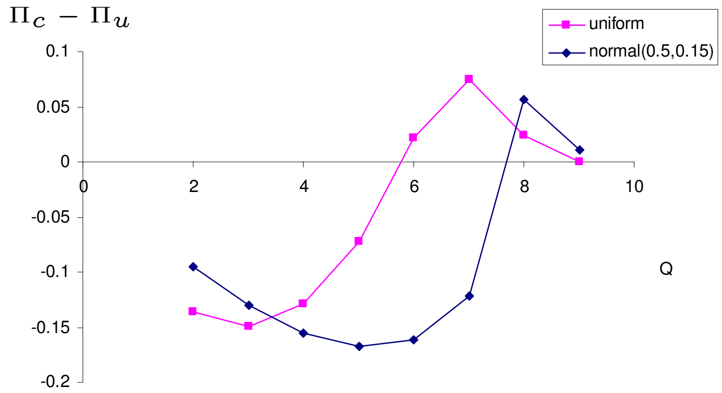



Figure 3: Impact of suppliers' information

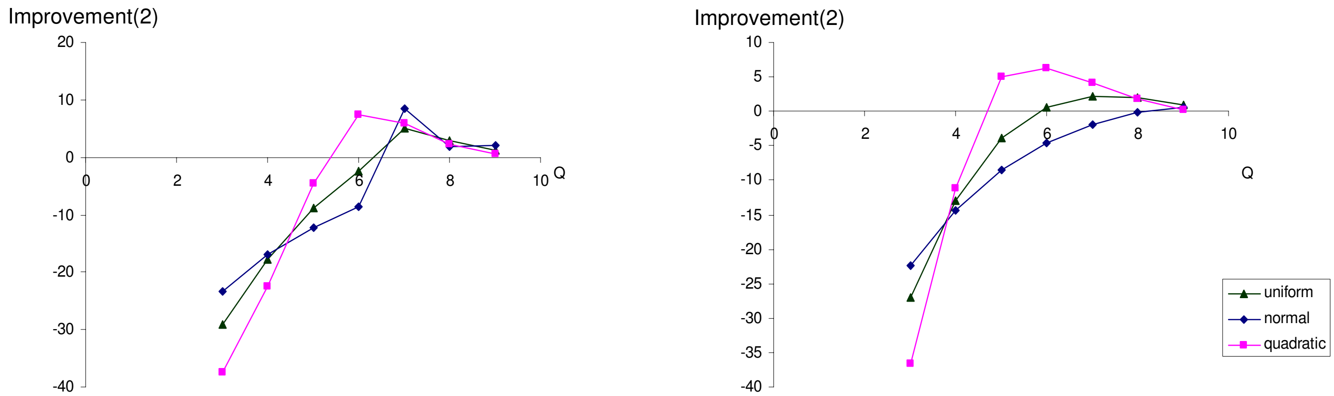
the benefit from a more competitive auction is less than the cost incurred in buying from the spot market. Nevertheless, if the supplier is informed, even though $Q/N$ is small, he might still accept the offer if his cost is low, which means that the buyer's cost decreases. The implication here is that the buyer has an incentive to subsidize the supplier's information cost incurred from the process of learning his exact cost of supplying the product when the competition among suppliers is intense.

### 5.1.2 Single or Multiple Contracts

We compare the buyer's procurement cost under different numbers of contracts. We mainly focus on the comparison between $M = 1$ and $M = 2$. Define $Improvement_j(2)\% = (\Pi_j(1) - \Pi_j(2))/\Pi_j(2) \times 100\%,\ j = u, c$. Figure 4 shows that under both information scenarios, if $Q/N$ is large, then offering one more contract will benefit the buyer. This is intuitive, as the selected suppliers are more likely to accept the contracts, and more contracts will increase the competition at the auction and thus drive down the price.

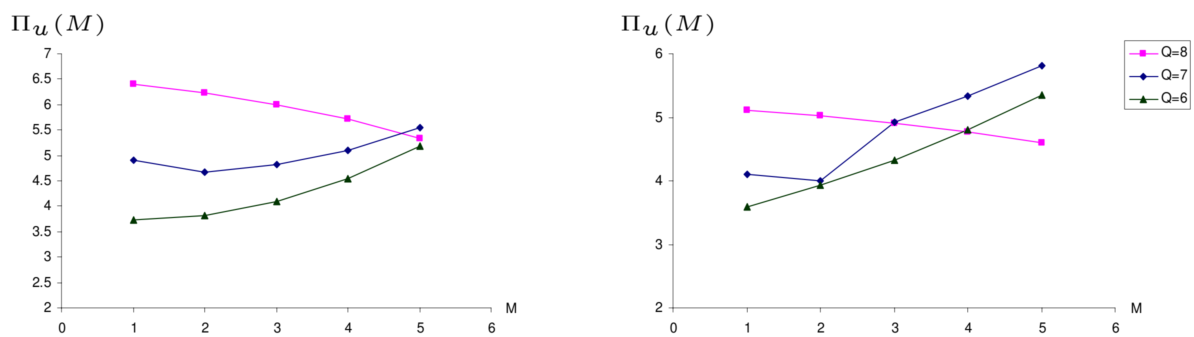
In our numerical results we also find that, compared with an ordinary auction, offering two contracts can save the buyer up to 21.08% of the cost in the uninformed supplier scenario and 12.30% in the informed supplier scenario. Again, the buyer's cost is improved with a larger $Q/N$, or a more intense competition.

For the uninformed supplier scenario, we can calculate the buyer's expected cost with different values of $M$ and the lower and upper bounds for the optimal $M^*$. Figure 5 shows how the buyer's cost changes with $M$. The optimal $M^* = 1, 2, 5$ when $Q = 6, 7, 8$ for both cost distributions, respectively, which lies in $[1, 2]$, $[1, 4]$, $[3, 6]$ from Proposition13. At the same time, the optimal number of




(a) uninformed suppliers (b) informed suppliers

Figure 4: Buyer's cost saving: $M = 2$ vs $M = 1$




(a) Uniform (b) Normal(0.5,0.15)

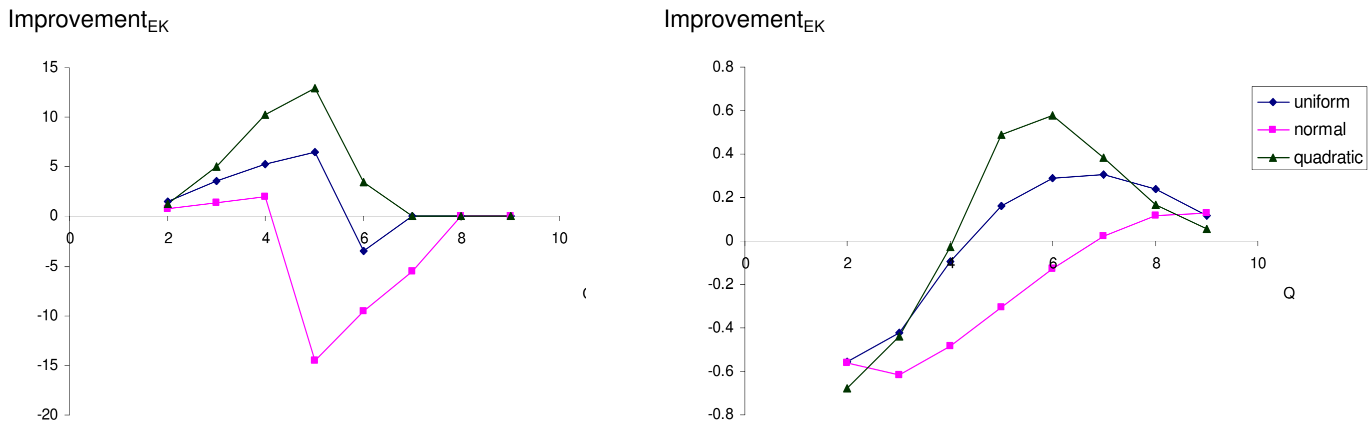
Figure 5: Buyer's cost with multiple contracts: uninformed supplier

contracts $M^*$ increases with $Q$, which implies that when the competition between suppliers is less intense, the buyer should offer more contracts. The saving for the buyer when using an optimal $M^*$ rather than engaging in a regular auction may be significant. For example, when $Q = 8$ and $f(x) = 1$, if the buyer offers $M^* = 5$, then she could save 22.89% of her cost.

## 5.2 The Flexibility Effect: Comparison with the EK Model

This section compares the performance of our mechanism with that of the EK model under various scenarios so that we can identify the conditions under which a buyer should offer flexibility to suppliers

We define the improvement in the buyer's expected cost from our mechanism over that from




(a) uninformed suppliers (b) informed suppliers

Figure 6: Impact of $Q$

the EK model when the buyer offers $M$ contracts as

$$Improvement_j^{EK}(M)\% = \frac{\Pi_j^{EK}(M) - \Pi_j(M)}{\Pi_j(M)} \times 100\%, \ j = u, c.$$

We first provide a general insight into how $Q$ affects the performance improvement. For the uninformed supplier scenario, the numerical results of other cost distributions are in line with Proposition 5, which is based on a uniform distribution, in that the EK model outperforms our model only when $Q/N$ falls within a certain interval. For the informed supplier scenario, the numerical results coincide with Proposition8, i.e., $\Pi_c \le \Pi_c^{EK}$ if $Q$ is relatively close to $N$ when $c_0 = 1$. However, it is noteworthy that the percentage improvement may not increase with $Q$.

We next use the Beta distribution to demonstrate how the improvement changes with the variance and mean of the supplier's cost. With appropriately chosen $\alpha$ and $\beta$, we can obtain a large variety of distribution shapes. We choose $\alpha$ and $\beta$ to match the mean and variance in the following table.

The results in Table 3 show that as the cost variance of the supplier increases, the maximum improvement of our mechanism over the EK model increases. For any given $Q$, the benefit of our mechanism also increases with the variance too. Clearly, our mechanism performs better when the buyer's information about the supplier's cost is more variable.

With respect to the cost mean, we only observe that the maximum improvement decreases with the mean. For a given $Q$, the benefit of our mechanism may increase or decrease with mean as is

Table 3: Comparison of buyer's expected cost from our mechanism and from the EK model: $M = 1$

| | $Improvement_u^{EK}(1)$ (%) | | | | $Improvement_c^{EK}(1)$(%) | | | |
|---|---|---|---|---|---|---|---|---|
| $(\mu, \sigma^2)$ | $Q = 4$ | 6 | 8 | max $(Q)$ | $Q = 4$ | 6 | 8 | max $(Q)$ |
| $(0.5, 0.03)$ | 2.47 | -8.50 | 0 | 2.47 (4) | -0.42 | -0.05 | 0.15 | 0.15 (8) |
| $(0.5, 0.06)$ | 3.98 | -5.56 | 0 | 3.98 (4) | -0.24 | 0.16 | 0.22 | 0.23 (7) |
| $(0.5, 0.09)$ | 5.59 | -2.80 | 0 | 6.89 (5) | -0.05 | 0.32 | 0.24 | 0.32 (6) |
| $(0.5, 0.12)$ | 7.47 | -0.02 | 0 | 9.07 (5) | 0.18 | 0.46 | 0.22 | 0.46 (6) |
| $(0.5, 0.15)$ | 9.69 | 2.79 | 0 | 11.65 (5) | 0.48 | 0.59 | 0.22 | 0.66 (5) |
| $(0.2, 0.05)$ | 4.00 | 11.62 | 0 | 11.62 (6) | -1.64 | -0.08 | 0.56 | 0.56 (8) |
| $(0.35, 0.05)$ | 4.13 | -14.27 | 0 | 5.74 (5) | -0.73 | 0.01 | 0.30 | 0.30 (8) |
| $(0.5, 0.05)$ | 3.48 | -6.49 | 0 | 4.39 (5) | -0.30 | 0.09 | 0.20 | 0.20 (8) |
| $(0.65, 0.05)$ | 3.01 | -1.84 | 0 | 3.01 (4) | -0.04 | 0.14 | 0.14 | 0.16 (7) |
| $(0.8, 0.05)$ | -3.51 | 0.70 | 0 | 2.32 (3) | 0.13 | 0.12 | 0.05 | 0.14 (5) |

the case for the uninformed supplier scenario when $Q = 4$ .

When suppliers are risk-averse, we numerically observe that the performance improvement increases with the suppliers' degree of risk aversion. This can be intuitively explained as follows. As both types of contract are more likely to be rejected by the suppliers when they become more risk averse, our mechanism benefits the buyer more since it allows the suppliers to join the auction, which intensifies the competition and drives the price down.

### 5.2.1 Two Contracts

We compare the performance of our mechanism with that of the EK model when $M = 2$, and draw some comparisons with the previous case of $M = 1$. Notice that when analyzing the buyer's problem, the EK model only focuses on $M = 1$. We assume, in the case of multiple equilibria, that the suppliers behave Pareto optimally. Similar to the $M = 1$ case, our mechanism performs better

Table 4: Comparison of buyer's expected cost from our mechanism and from the EK model: $M = 2$

| | | $Improvement_u^{EK}(2)$ (%) | | | | $Improvement_c^{EK}(2)$ (%) | | | |
|---|---|---|---|---|---|---|---|---|---|
| $F(x)$ | $(\mu, \sigma^2)$ | $Q = 3$ | 5 | 7 | max $(Q)$ | $Q = 3$ | 5 | 7 | max $(Q)$ |
| Uniform | $(0.5, 0.08)$ | 1.85 | 7.84 | 0 | 7.84 (5) | -1.77 | -0.64 | 0.38 | 0.38 (7) |
| Trunc. | $(0.5, 0.02)$ | 0.95 | 3.01 | 0 | 3.01 (5) | -1.51 | -1.52 | -0.63 | -0.26 (9) |
| Normal | $(0.5, 0.06)$ | 1.60 | 5.55 | 0 | 5.55 (5) | -1.64 | -1.08 | 0.01 | 0.16 (8) |
| Quadratic | $(0.5, 0.15)$ | 1.62 | 15.04 | 0 | 15.04 (5) | -3.21 | 0.13 | 0.95 | 1.10 (6) |
| | $(0.5, 0.05)$ | 1.46 | 5.30 | 0 | 5.30 (5) | -1.64 | -1.09 | -0.07 | 0.08 (8) |
| Beta | $(0.35, 0.05)$ | 1.17 | 5.87 | 0 | 5.87 (5) | -2.47 | -2.19 | -0.29 | 0.19 (8) |
| | $(0.65, 0.05)$ | 1.67 | 4.78 | 0 | 4.78 (5) | -0.95 | -0.36 | 0.09 | 0.09 (7) |

than the EK model when $Q/N$ is small in the uninformed supplier scenario, and when $Q/N$ is large under the informed supplier scenario. The maximum improvement that we obtain when $M = 2$ in the uninformed supplier case is 15.04% and in the informed supplier case is 1.10%, both are higher than the corresponding values in the $M = 1$ case. Taken together with the results in the $M = 1$ case, we can conclude that the buyer can obtain a much higher possible cost saving by adopting our mechanism when the suppliers have less information about their costs.

# 6 Conclusion and Discussion

This paper examines a model where a buyer adopts a hybrid procurement mechanism to procure multiple units of a product from a group of suppliers. The mechanism combines a reverse auction with flexible noncompetitive contracts, which preserves the benefits of an auction and meanwhile promotes the buyer-supplier relationship. We investigate the performance of the mechanism and compare it with the EK model under two information scenarios, namely, informed supplier and uninformed supplier. We find that our mechanism benefits the buyer more than a pure reverse auction or the EK model in various cases, for example, when $Q$ is close $N$ and the competition is not intense in the uninformed supplier scenario or competition is very intense in the informed supplier scenario. We also examine the game behavior of suppliers and characterize the Nash equilibrium when the buyer makes multiple offers. We observe that when $Q$ is close to $N$, the buyer may be better off by offering multiple contracts rather than offering only one contract. Therefore our mechanism not only offers flexibility to the specific suppliers that the buyer wants to deal with, but may also reduce the buyer's procurement cost. By comparing the results in the two information scenarios, we find that whether suppliers know their exact costs has a significant impact not only on their own decisions but also on the buyer's cost. The buyer's procurement cost is lower in the informed (resp., uninformed) supplier scenario when the competition is intense (resp., not intense).

Both the EK model and our mechanism assume the suppliers incur no bidding cost in auction. We can show that in the uninformed suppliers case, if all the suppliers pay the bidding fee before knowing their private cost information and such fee is sufficiently small, then all our results in Section 3 are still valid. However, if the suppliers pay the bidding fee after observing their private information, i.e., such fee will explicitly affect their participation of the auction, the situation becomes much more complicated. A preliminary analysis is provided in the appendix and this direction certainly requires more dedicated research.

There are several other related research issues that deserve further study. First, what is the best strategy for a buyer who offers multiple contracts? Shall she offer the contracts one by one or simultaneously, as discussed in this paper? Offering the contracts sequentially might give the buyer more information, but would certainly make the analysis more complicated. Second, we can consider the scenario that one supplier may supply multiple units where multi-unit Vickrey auction mechanism is deployed and each supplier bid a supply curve. In this case, the noncompetitive contract awarded to one particular supplier may also be in the form of a price-supply curve. However how to construct an appropriate noncompetitive contract remains a challenging research question. It is also interesting to investigate whether such noncompetitive contract will be beneficial to the buyer. Finally, although this paper touches on the issue of risk-averse suppliers, a well-designed experimental study along the lines of that of Engelbrecht-Wiggans and Katok (2006) could reveal interesting findings about the behavior of the suppliers and the resulting costs of the buyer.

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# Online Supplements

## Appendix A: Proofs

### Proof of Proposition 2

We show that when $Q = N-1$, $\pi_u^{ac} > \pi_u^{re}$. As $Q = N-1$,

$$\begin{aligned}
\pi_u^{ac} &= \int_0^1 \int_0^1 s(N-1)F^{N-2}(s)f(s)f(x)dsdx - \int_0^1 xf(x)dx \\
&= \int_0^1 x(N-1)F^{N-2}(x)f(x)dx - \int_0^1 xf(x)dx
\end{aligned}$$

and

$$\begin{aligned}
\pi_u^{re} &= \int_{s=0}^1 \int_{x=0}^s s(N-1)(N-2)F^{N-3}(s)(1-F(s))f(s)f(x)dxds \\
&\quad - \int_{s=0}^1 \int_{x=0}^s x(N-1)(N-2)F^{N-3}(s)(1-F(s))f(s)f(x)dxds \\
&= \int_{s=0}^1 s(N-1)(N-2)F^{N-2}(s)(1-F(s))f(s)ds \\
&\quad - \int_{x=0}^1 xf(x) \int_{s=x}^1 (N-1)(N-2)F^{N-3}(s)(1-F(s))f(s)dsdx \\
&= \int_{x=0}^1 x(N-1)(N-2)F^{N-2}(x)(1-F(x))f(x)dx \\
&\quad - \int_{x=0}^1 xf(x)(N-1)\Big(1-F^{N-2}(x)\Big) - xf(x)(N-2)\Big(1-F^{N-1}(x)\Big)dx,
\end{aligned}$$

where the first equality follows from changing the sequence of integrations. Notice that every integrand in the expression of $\pi_u^{ac}$ and $\pi_u^{re}$ has a common factor $xf(x)$. Therefore

$$\begin{aligned}
\pi_u^{ac} - \pi_u^{re} &= \int_0^1 xf(x)\Bigg[(N-1)F^{N-2}(x) - 1 - (N-1)(N-2)F^{N-2}(x)(1-F(x)) \\
&\quad +(N-1)\Big(1-F^{N-2}(x)\Big) - (N-2)\Big(1-F^{N-1}(x)\Big)\Bigg]dx \\
&= \int_0^1 xf(x)(N-2)\Bigg[NF^{N-1}(x) - (N-1)F^{N-2}(x)\Bigg]dx \\
&= (N-2)\int_0^1 xd\Bigg[F^N(x) - F^{N-1}(x)\Bigg] \\
&= (N-2)\int_0^1 \Bigg[F^{N-1}(x) - F^N(x)\Bigg]dx > 0,
\end{aligned}$$

where the last equality follows from integration by part and the inequality follows from $0 \leq F(x) \leq 1$. Hence, accepting the offer yields a higher expected profit for the supplier and so he will accept the contract when $Q = N - 1$.

**Proof of Proposition 3**

For part (a), if $C$ is uniformly distributed, we can write $\pi_u^{ac}$ and $\pi_u^{re}$ in closed form. In this case,

$$\begin{aligned}
\pi_u^{ac} &= \int_0^1 \int_0^1 (s-x)\phi_{ac}(s) f(x) dx ds \\
&= \int_0^1 \int_0^1 (s-x)(N-1)\binom{N-2}{Q-1} F^{Q-1}(s)(1-F(s))^{N-Q-1} f(s) f(x) dx ds \\
&= \int_0^1 \int_0^1 (s-x)(N-1)\binom{N-2}{Q-1} s^{Q-1}(1-s)^{N-Q-1} dx ds \\
&= \int_0^1 (s-\frac{1}{2})(N-1)\binom{N-2}{Q-1} s^{Q-1}(1-s)^{N-Q-1} ds = \frac{Q}{N} - \frac{1}{2}.
\end{aligned}$$

Similarly, $\pi_u^{re} = Q(Q-1)/(2N(N+1))$.

Note that

$$\pi_u^{ac} - \pi_u^{re} = \frac{Q}{N} - \frac{1}{2} - \frac{1}{2}\frac{Q(Q-1)}{N(N+1)} = \frac{-Q^2 + (2N+3)Q - (N^2+N)}{2N(N+1)}.$$

Since $-Q^2 + (2N+3)Q - (N^2+N)$ is a concave parabola, its roots are

$$Q_1(N) = \frac{2N+3-\sqrt{8N+9}}{2} \text{ and } Q_2(N) = \frac{2N+3+\sqrt{8N+9}}{2} > N.$$

So it is clear that $\pi_u^{ac} - \pi_u^{re} < 0$ if $Q < Q_1(N)$ and $\pi_u^{ac} - \pi_u^{re} > 0$, if $Q_1(N) < Q < N$. As $Q_1(N)$ may not be an integer, let $Q^*(N) = \lfloor Q_1(N) \rfloor$. Thus, if $Q \leq Q^*(N)$, $\pi_u^{ac} - \pi_u^{re} \leq 0$, the selected supplier should reject the flexible contract. If $Q > Q^*(N)$, $\pi_u^{ac} - \pi_u^{re} > 0$, the selected supplier should accept the flexible contract.

For part (b),

$$\begin{aligned}
Q^*(N+1) - Q^*(N) &= \left\lfloor \frac{2(N+1)+3-\sqrt{8(N+1)+9}}{2} \right\rfloor - \left\lfloor \frac{2N+3-\sqrt{8N+9}}{2} \right\rfloor \\
&= 1 - \left\lfloor \frac{\sqrt{8N+17}}{2} \right\rfloor + \left\lfloor \frac{\sqrt{8N+9}}{2} \right\rfloor.
\end{aligned}$$

Therefore $0 \leq Q^*(N+1) - Q^*(N) \leq 1$ and the result follows.

**Proof of Proposition 4**

(a) The buyer's expected cost if she conducts an ordinary auction is

$$\Pi_{au} = Q\mathsf{E}[C_{(Q+1,N)}] = Q\int_0^1 xN\binom{N-1}{Q}x^Q(1-x)^{N-Q-1}dx = \frac{Q(Q+1)}{N+1}. \tag{8}$$

If the buyer makes an offer, we consider two cases. If $Q > Q^*(N)$, then the supplier who is given the contract will accept it, and the buyer's cost is

$$\Pi_u = Q\mathsf{E}[C_{(Q,N-1)}] = Q\int_0^1 x(N-1)\binom{N-2}{Q-1}x^{Q-1}(1-x)^{N-Q-1}dx = \frac{Q^2}{N}.$$

It is apparent that $\Pi_u < \Pi_{au}$ as $Q < N$.

Now consider the case where $Q \le Q^*(N)$, i.e., the supplier rejects the contract. The supplier will join the auction, but there are only $Q-1$ items in the auction. Therefore, the unit price will be $C_{(Q,N)}$, and the expected cost of the buyer is

$$\Pi_u = (Q-1)\mathsf{E}[C_{(Q,N)}] + c_0 = (Q-1)\int_0^1 xN\binom{N-1}{-1}x^{Q-1}(1-x)^{N-Q}dx + c_0 = \frac{Q(Q-1)}{N+1} + c_0,$$

as the buyer needs to buy the rejected unit from the spot market. Note that

$$\Pi_{au} - \Pi_u = \frac{Q(Q+1)}{N+1} - \frac{Q(Q-1)}{N+1} - c_0 = \frac{2Q}{N+1} - c_0.$$

If $Q > \lfloor c_0(N+1)/2 \rfloor$, then, $\Pi_{au} > \Pi_u$. Define

$$Q_a(N) = \min\left\{Q^*(N), \left\lfloor \frac{c_0(N+1)}{2} \right\rfloor\right\}.$$

If $Q \le Q_a(N)$, the buyer should buy the products via an ordinary auction without contract; otherwise, it is better for her to offer a flexible noncompetitive contract.

(b) The result is clearly true from the definition of $Q_a(N)$.

**Proof of Proposition 7**

(a) Consider

$$\begin{aligned}\pi_c^{ac} - \pi_c^{re} &= \mathsf{E}[C_{(Q,N-1)} - c] - \mathsf{E}[\max\{C_{(Q-1,N-1)} - c, 0\}] \\ &= -\mathsf{E}[\max\{c - C_{(Q-1,N-1)}, 0\}] + \mathsf{E}[C_{(Q,N-1)}] - \mathsf{E}[C_{(Q-1,N-1)}].\end{aligned} \tag{9}$$

The above expression clearly decreases in $c$, and varies from $\mathsf{E}[C_{(Q,N-1)}] - \mathsf{E}[C_{(Q-1,N-1)}]$, which is positive, to $\mathsf{E}[C_{(Q,N-1)}] - 1$, which is negative, as $c$ increases from 0 to 1. Hence there exists a unique threshold $c^*(Q,N)$, such that, if $c > c^*(Q,N)$, $\pi_c^{ac} < \pi_c^{re}$, and if $c < c^*(Q,N)$, $\pi_c^{ac} > \pi_c^{re}$.

(b) We first prove that for a fixed $N$, $c^*(Q,N)$ is increasing in $Q$. For brevity, we suppress $N$ in $c^*(Q,N)$. Recall from part (a), $c^*(Q)$ is the solution of

$$\mathsf{E}[\max\{c - C_{(Q-1,N-1)}, 0\}] = \mathsf{E}[C_{(Q,N-1)}] - \mathsf{E}[C_{(Q-1,N-1)}]. \tag{10}$$

Following from the definition of $C_{(Q,N-1)}$ and $C_{(Q-1,N-1)}$,

$$\begin{aligned}
\mathsf{E}[\max\{c - C_{(Q-1,N-1)}, 0\}] &= \int_{t=0}^{c} (c-t)d\Pr(C_{(Q-1,N-1)} \le t) \\
&= (c-t)\Pr(C_{(Q-1,N-1)} \le t)|_0^c + \int_{t=0}^{c} \Pr(C_{(Q-1,N-1)} \le t)dt \\
&= \int_{t=0}^{c} \Pr(C_{(Q-1,N-1)} \le t)dt \\
&= \int_{t=0}^{c} \sum_{l=Q-1}^{N-1} \binom{N-1}{l} F^l(t)\bar{F}^{N-1-l}(t)dt,
\end{aligned}$$

where $\bar{F}(\cdot) = 1 - F(\cdot)$. And

$$\begin{aligned}
&\mathsf{E}[C_{(Q,N-1)}] - \mathsf{E}[C_{(Q-1,N-1)}] \\
&= \int_{t=0}^{1} \Pr(C_{(Q,N-1)} \ge t)dt - \int_{t=0}^{1} \Pr(C_{(Q-1,N-1)} \ge t)dt \\
&= \int_{t=0}^{1} \sum_{l=1}^{Q-1} \binom{N-1}{l} F^l(t)\bar{F}^{N-1-l}(t)dt - \int_{t=0}^{1} \sum_{l=1}^{Q-2} \binom{N-1}{l} F^l(t)\bar{F}^{N-1-l}(t)dt \\
&= \int_{t=0}^{1} \binom{N-1}{Q-1} F^{Q-1}(t)\bar{F}^{N-Q}(t)dt.
\end{aligned}$$

Based on the above analysis and after some algebra, we can write (10) as

$$\int_{t=0}^{c^*(Q)} \sum_{l=Q}^{N-1} \binom{N-1}{l} F^l(t)\bar{F}^{N-1-l}(t)dt = \int_{s=c^*(Q)}^{1} \binom{N-1}{Q-1} F^{Q-1}(s)\bar{F}^{N-Q}(s)ds. \tag{11}$$

If we replace $Q$ by $Q+1$ and $c$ by $c^*(Q)$ in (9) and we can show the resulting $\mathsf{E}[C_{(Q+1,N-1)} - c^*(Q)] - \mathsf{E}[\max\{C_{(Q,N-1)} - c^*(Q), 0\}] \ge 0$, or

$$\int_{s=c^*(Q)}^{1} \binom{N-1}{Q} F^Q(s)\bar{F}^{N-Q-1}(s)ds \ge \int_{t=0}^{c^*(Q)} \sum_{l=Q+1}^{N-1} \binom{N-1}{l} F^l(t)\bar{F}^{N-1-l}(t)dt, \tag{12}$$

then we actually prove that $c^*(Q+1) \ge c^*(Q)$ as $\pi_u^{ac} - \pi_u^{re}$ is a decreasing function in $c$. As all the terms in (11) and (12) are positive, multiplying the left (resp., right) hand side on (11) by the left (resp., right) hand side on (12) and rearranging terms yield

$$\int_{t=0}^{c^*(Q)} \int_{s=c^*(Q)}^{1} \sum_{l=Q}^{N-1} \binom{N-1}{l} \binom{N-1}{Q} F^l(t)\bar{F}^{N-1-l}(t)F^Q(s)\bar{F}^{N-Q-1}(s)dtds$$

$$\geq \int_{t=0}^{c^*(Q)} \int_{s=c^*(Q)}^{1} \sum_{l=Q}^{N-2} \binom{N-1}{l+1} \binom{N-1}{Q-1} F^{l+1}(t) \bar{F}^{N-2-l}(t) F^{Q-1}(s) \bar{F}^{N-Q}(s) dtds. \quad (13)$$

To show that (12) is true, it is equivalent to show that the above inequality holds. We compare the integrands of both sides of the inequality. Notice that

$$\frac{\binom{N-1}{l}\binom{N-1}{Q}}{\binom{N-1}{l+1}\binom{N-1}{Q-1}} = \frac{(l+1)(N-Q)}{(N-l-1)Q} > 1, \text{ for } l \geq Q,$$

and

$$\frac{F^l(t)\bar{F}^{N-1-l}(t)F^Q(s)\bar{F}^{N-Q-1}(s)}{F^{l+1}(t)\bar{F}^{N-2-l}(t)F^{Q-1}(s)\bar{F}^{N-Q}(s)} = \frac{\bar{F}(t)F(s)}{F(t)\bar{F}(s)} \geq 1,$$

because $s \geq t$ and $F(\cdot)$ (resp., $\bar{F}(\cdot)$) is an increasing (resp., decreasing) function. Therefore we have shown that each term in the summation on the left hand side of (13) is larger than the corresponding term of its right hand side. As the left hand side of (13) has an extra nonnegative term in the integrand, inequality (13) holds and $c^*(Q+1) \geq c^*(Q)$. Similar arguments can be applied to prove that $c^*(Q,N)$ is decreasing in $N$ for a given $Q$. So we skip its proof here.

**Proof of Proposition 8**

We first show $\Pi_c < \Pi_c^{EK}$ when $Q = N-1$. By the definition of $c^*$ and some simple algebra, $c^*$ is the solution to

$$\frac{1}{N} - (c^*)^{N-1} + \frac{N-2}{N}(c^*)^N = 0, \quad (14)$$

and it is not hard to verify that $c^* < (N-1)/N$. The buyer's expected cost can be simplified to

$$\begin{aligned}
\Pi_c &= c^*\frac{(N-1)^2}{N} + \int_{c^*}^{1}\left[\frac{x^N}{N} + (N-2)\left(\frac{1-x^{N-1}}{N-1} - \frac{1-x^N}{N}\right) + x^{N-1}(1-x)\right](N-1)(N-2)dx \\
&\quad + c_0(1-c^*) \\
&= c^*(N-1)^2\frac{1}{N} + c_0(1-c^*) \\
&\quad + (N-2)(N-1)\left[-\frac{1-c^{*N+1}}{N(N+1)} + \frac{1-c^{*N}}{N(N-1)} + (\frac{1-c^*}{N(N-1)})(N-2)\right] \\
&= (1-c^*)c_0 + \frac{(N-2)(N-1)}{N+1} + \frac{2N-3}{N}c^* - \frac{N-2}{N}c^{*N} + \frac{(N-2)(N-1)}{N(N+1)}c^{*N+1}.
\end{aligned}$$

The buyer's expected cost in the EK model is

$$\Pi_c^{EK} = \frac{c_0}{N} + \left(\frac{N-1}{N}\right)^2 + (N-2)\frac{N-1}{N}. \quad (15)$$

Notice that both $\Pi_c$ and $\Pi_c^{EK}$ are linear in $c_0$. Because $c^* \le (N-1)/N$, the slope of $\Pi_c$(with respect to $c_0$) is larger than $\Pi_c^{EK}$. Therefore we only need to show that when $c_0 = 1$, $\Pi_c < \Pi_c^{EK}$.

When $c_0 = 1$, let

$$\begin{aligned} f(c^*) &= \Pi_c^{EK} - \Pi_c = \frac{1}{N} + \left(\frac{N-1}{N}\right)^2 + (N-2)\frac{N-1}{N} \\ &\quad - \left((1-c^*) + \frac{(N-2)(N-1)}{N+1} + \frac{2N-3}{N}c^* - \frac{N-2}{N}c^{*N} + \frac{(N-2)(N-1)}{N(N+1)}c^{*N+1}\right). \end{aligned}$$

Recall that $c^*$ is the solution to (14). As it is hard to see from the above function directly whether $f(c^*) > 0$, we first calculate $f(c^*)$ for $N = 3, 4, 5, 6$. After solving $c^*$ from (14) and then plugging it into $f(c^*)$, we have

If $N = 3$, $c^* = 0.652704$, $f(c^*) = 0.00688403$;

if $N = 4$, $c^* = 0.733615$, $f(c^*) = 0.0101736$;

if $N = 5$, $c^* = 0.783894$, $f(c^*) = 0.0112284$;

if $N = 6$, $c^* = 0.818193$, $f(c^*) = 0.0113229$.

So $f(c^*) > 0$ for $n < 7$. For $n \ge 7$, we can prove a stronger result that $f(c) > 0$ for all $c \in [0, 1]$. To see this, by some algebra, it is equivalent to prove

$$\begin{aligned} g(c) \equiv (N-2)(N-1)f(c) &= (-N^3 + 3N^2 - 2N)c^{N+1} + (N^3 - N^2 - 2N)c^N \\ &\quad + (-N^3 + 2N^2 + 3N)c + N^3 - 4N^2 + 2N + 1 > 0 \end{aligned}$$

for $N \ge 7$ and $c \in [0, 1]$. First notice that $g(c)$ is a strict convex function, and $g(1) = N + 1$, $g'(1) = N(N+1)$, hence

$$g(c) > g(1) - g'(1)(1-c) = (N+1)\Big(1 - N(1-c)\Big) > 0, \text{ for } c > \frac{N-1}{N}.$$

Next consider

$$\begin{aligned} g'\left(\frac{N-1}{N}\right) &= \frac{N(N+1)}{N-1}\left(-3 + 4N - N^2 + \left(\frac{N-1}{N}\right)^N \Big[2 - 5N + 2N^2\Big]\right) \\ &\le \frac{N(N+1)}{N-1}\left(-3 + 4N - N^2 + e^{-1}\Big[2 - 5N + 2N^2\Big]\right) \\ &= \frac{N(N+1)}{N-1}\Big((2e^{-1} - 1)N^2 + (4 - 5e^{-1})N + 2e^{-1} - 3\Big) < 0, \text{ for } N \ge 7. \end{aligned}$$

The last inequality is due to the fact that the function in the large bracket is a concave parabola with the larger root less than 7. To summarize, we have proved that $g(c) > 0$ for $c > (N-1)/N$ and $g(c)$ is decreasing for $c \le (N-1)/N$, therefore $g(c) > 0$ for all $c \in [0, 1]$.

We next show $\Pi_c^{EK} < \Pi_{au}$ when $c_0 = 1$ or

$$\frac{1}{N} + \left(\frac{N-1}{N}\right)^2 + (N-2)\frac{N-1}{N} < \frac{N(N-1)}{N+1}.$$

Note that

$$\begin{aligned} & \frac{1}{N} + \left(\frac{N-1}{N}\right)^2 + (N-2)\frac{N-1}{N} - \frac{N(N-1)}{N+1} \\ = & \frac{-2N^2 + N + 1 + N^3}{N^2} - \frac{N(N-1)}{N+1} \\ = & \frac{-N^2 + 2N + 1}{N^2(N+1)} < 0, \end{aligned}$$

because $N \geq 3$. Therefore, we have proved that when $Q = N - 1$ and $c_0 = 1$, $\Pi_c < \Pi_c^{EK} < \Pi_{au}$.

**Proof of Proposition 9**

Suppose Supplier 1 accepts the contract. We then consider Supplier 2's best response. He is in the situation where there are $N - 1$ suppliers, $Q - 1$ units would be bought and one flexible contract is offered to him. According to the previous results, if $Q - 1 > Q^*(N - 1)$, he accepts the contract; if $Q - 1 \leq Q^*(N - 1)$, he rejects the contract.

If Supplier 1 rejects the contract, then Supplier 2 is in the scenario where $N$ suppliers are competing for $Q - 1$ units and one flexible contract is offered to him. Similarly, if $Q - 1 > Q^*(N)$, he accepts the contract; otherwise, he turns it down.

Note that $Q^*(N) \geq Q^*(N - 1)$ from Proposition 3. Therefore the best response of Supplier 2 can be written as, if $a_1^*$ =accept,

$$a_2^*(a_1^*) = \begin{cases} \text{reject} & \text{if } Q \leq Q^*(N-1) + 1; \\ \text{accept} & \text{if } Q^*(N-1) + 1 < Q \leq Q^*(N) + 1; \\ \text{accept} & \text{if } Q > Q^*(N) + 1, \end{cases}$$

and if $a_1^*$ =reject,

$$a_2^*(a_1^*) = \begin{cases} \text{reject} & \text{if } Q \leq Q^*(N-1) + 1; \\ \text{reject} & \text{if } Q^*(N-1) + 1 < Q \leq Q^*(N) + 1; \\ \text{accept} & \text{if } Q > Q^*(N) + 1. \end{cases}$$

Because the two suppliers are identical, the best response of Supplier 1, $a_1^*(a_2^*)$ has the same structure as $a_2^*(a_1^*)$. Thus, if $Q \leq Q^*(N-1)+1$, rejecting the contract is the dominant strategy, and if $Q > Q^*(N)+1$, accepting the contract is the dominant strategy. If $Q^*(N-1)+1 < Q \leq Q^*(N)+1$, then the best response is $a_2^*(\text{accept})$ = accept and $a_2^*(\text{reject})$ = reject. Therefore, $[\text{accept}, \text{accept}]$ and $[\text{reject}, \text{reject}]$ are two possible Nash equilibria. However, if $Q^*(N - 1) = Q^*(N)$, this case vanishes and there is always a unique Nash equilibrium.

**Proof of Lemma 11**

It is sufficient to show Supplier 2's best response given Supplier 1's strategy. First suppose Supplier 1 accepts the contract. Supplier 2 is now in the situation where there are $N-1$ suppliers competing for $Q-1$ units and one flexible contract is given to him. According to the previous result, there exists a threshold $c_2^* = c^*(N-1, Q-1)$. If Supplier 2's cost $c_2$ is lower than $c_2^*$, he accepts the contract; otherwise, he rejects the contract.

If Supplier 1 rejects the contract, then Supplier 2 needs to consider the case where $N$ suppliers compete for $Q-1$ units. There exists a threshold $c_1^* = c^*(N, Q-1)$. If Supplier 2's cost $c_2 < c_1^*$, he should accept the contract; otherwise, he should reject the contract.

As $c_1^* < c_2^*$, from Proposition7, Supplier 2 should always reject the flexible contract if $c_2 \geq c_2^*$, and accept the offer if $c_2 < c_1^*$, regardless of Supplier 1's action. And if $c_1^* \leq c_2 < c_2^*$, Supplier 2 should take the same action as Supplier 1.

**Proof of Proposition 12**

If $c_1 \geq c_1^*$ and $c_2 \geq c_2^*$, then by Lemma 11, a dominant strategy for Supplier 2 is to reject the contract. Given Supplier 2 rejects the contract, it is optimal for Supplier 1 to reject the contract because $a_1^*(c_1, \text{reject}) = \text{reject}$. Therefore, [reject, reject] is the unique Nash equilibrium in this case. Because the two suppliers are symmetric, [reject, reject] is also the unique equilibrium if $c_2 \geq c_1^*$ and $c_1 \geq c_2^*$.

If $c_1 < c_1^*$ and $c_2 < c_2^*$, then a dominant strategy for Supplier 1 is to accept the contract. Given that Supplier 1 accepts the contract, it is optimal for Supplier 2 to accept the contract, since $a_2^*(c_2, \text{accept}) = \text{accept}$. Hence [accept, accept] is the unique equilibrium in this case. And by symmetry, it is also the unique equilibrium when $c_2 < c_1^*$ and $c_1 < c_2^*$.

If $c_1^* \leq c_i < c_2^*$, for $i = 1, 2$, suppose $a_1 = \text{accept}$, then $a_2^*(c_2, \text{accept}) = \text{accept}$, because $c_1^* \leq c_1 < c_2^*$. Similarly, if $a_2 = \text{accept}$, then $a_1^*(c_1, a_2) = \text{accept}$. Therefore [accept, accept] is a Nash equilibrium because neither of the two suppliers will deviate from accepting the contract unilaterally. Similar arguments can show that [reject, reject] is another equilibrium.

If $c_1 \geq c_2^*$ and $c_2 < c_1^*$, then a dominant strategy for Supplier 1 is to reject the contract and for Supplier 2 to accept the contract. Then [reject, accept] is the dominant strategy in this case, and therefore it is the Nash equilibrium. By symmetry, [accept, reject] is the dominant strategy and the Nash equilibrium if $c_1 < c_1^*$ and $c_2 \geq c_2^*$.

**Proof of Proposition 13**

We first show part (a). Since all suppliers are identical from the buyer's perspective, let us consider Supplier 1. Besides the contract offered to Supplier 1, there are another $M-1$ flexible contracts. Suppose that $m$ are accepted while the remaining $M-m-1$ offers are rejected. In this case, Supplier 1 faces the situation where the buyer wants to procure $Q-M+1$ units, the total number of bidders is $N-m$, and only one flexible contract is provided to him. Therefore, by our previous analysis, it is optimal for Supplier 1 to reject the contract if and only if $Q-M+1 \leq Q^*(N-m)$, or equivalently, $Q \leq Q^*(N-m)+M-1$. By letting $\underline{Q} = \min_{0\leq m\leq M-1} Q^*(N-m)+M-1 = Q^*(N-M+1)+M-1$ (from Proposition 3), we conclude that no matter what the other suppliers do, Supplier 1 should always reject the contract if $Q \leq \underline{Q}$. Similarly, let $\overline{Q} = \max_{0\leq m\leq M-1} Q^*(N-m) + M - 1 = Q^*(N) + M - 1$, then Supplier 1 should always accept the contract if $Q > \overline{Q}$. So this is the dominant strategy for Supplier 1 as well as others.

For part (b), first we show that [accept,accept,...,accept] and [reject,reject,...,reject] are two equilibria. Suppose among all $M$ suppliers who are given the contracts, $M-1$ suppliers accept the contracts. The remaining supplier then faces the situation where there were $N-M+1$ suppliers competing for $Q-M+1$ units. He will therefore accept the contract if $Q-M+1 > Q^*(N-M+1)$ from Proposition 3. This is indeed the case as we are considering $Q > \underline{Q} = Q^*(N-M+1)+M-1$. Since all suppliers are identical, [accept,accept,...,accept] is one equilibrium. Similar argument can show [reject,reject,...,reject] is another equilibrium.

Next we prove that there is no other equilibrium. We prove this by contradiction. Suppose there is another equilibrium at which $m$ out of $M$ suppliers accept the contracts, $1 \leq m \leq M-1$. A supplier who accepts the contract faces the situation where there are $N-m+1$ suppliers competing for $Q-M+1$ units. Since he accepts the offer, then $Q-M+1 > Q^*(N-m+1)$.

On the other hand, a supplier who rejects the contract faces the situation where there are $N-m$ suppliers competing for $Q-M+1$ units. Because he rejects the contract, then $Q-M+1 \leq Q^*(N-m)$, which clearly contradicts to $Q-M+1 > Q^*(N-m+1)$ as $Q^*(N-m) \leq Q^*(N-m+1)$. Hence, only [accept,accept,...,accept] or [reject,reject,...,reject] are equilibria.

Now we prove that [accept,accept,...,accept] is Pareto optimal. Under the equilibrium, if everyone accepts, each supplier's payoff is $\mathsf{E}(C_{(Q-M+1,N-M)} - C)$ while if everyone rejects, each supplier's payoff is $\mathsf{E}(C_{(Q-M,N-1)} - C)^+$. Since $Q-M+1 > Q^*(N-M+1)$, then by the definition of $Q^*(N-M+1)$, $\mathsf{E}[C_{(Q-M+1,N-M)} - C] \geq \mathsf{E}[(C_{(Q-M,N-M)} - C)^+] \geq \mathsf{E}[(C_{(Q-M,N-1)} - C)^+]$.

To show that part (c) is true, we only need to prove that any $M$ such that $M < Q - \max\{Q^*(N-$

$M+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\}$ or $M > Q+1-\min\{Q^*(N-M+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\}$ cannot be optimal. First suppose at optimum, $M^* < Q - \max\{Q^*(N-M^*+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\}$. Then according to parts (a) and (b), as $Q > Q^*(N-M^*+1)+M^* > Q^*(N-M^*+1)+M^*-1$, all suppliers will accept the contracts. Suppose the buyer offers one more contract, i.e., she offers $M^*+1$ contracts. Because $Q > Q^*(N-M^*+1)+M^*-2$ then $Q > Q^*(N-M^*)+M^*$, then all $M^*+1$ suppliers will still accept the contracts, and the buyer incurs a lower cost. This contradicts with the optimality of $M^*$ and so $M^* \geq Q - Q^*(N-M+1)$.

Next we suppose $M^* > Q+1-\min\{Q^*(N-M^*+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\}$. By part (a), all $M^*$ contracts will be rejected, and the buyer's cost is

$$\Pi_u(M^*) = (Q-M^*)\mathsf{E}[C_{(Q-M^*+1,N)}] + M^*c_0 = \frac{(Q-M^*)(Q-M^*+1)}{N+1} + M^*c_0.$$

Because $Q < \min\{Q^*(N-M^*+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\} + M^* - 1$, $Q \leq \min\{Q^*(N-M^*+2), \lfloor \frac{c_0(N+1)}{2} \rfloor\} + M^* - 2$. Therefore, if the buyer offers one less contract, then all $M-1$ contracts will still be rejected, and the buyer's cost becomes

$$\Pi_u(M^*-1) = (Q-M^*+1)\mathsf{E}C_{(Q-M^*+1,N)} + M^*c_0 = \frac{(Q-M^*+1)(Q-M^*+2)}{N+1} + (M^*-1)c_0.$$

Compare $\Pi_u(M^*)$ and $\Pi_u(M^*-1)$,

$$\begin{aligned} & \Pi_u(M^*-1) - \Pi_u(M^*) \\ = & \frac{(Q-M^*+1)(Q-M^*+2)}{N+1} + (M^*-1)c_0 - \Big(\frac{(Q-M^*)(Q-M^*+1)}{N+1} + M^*c_0\Big) \\ = & \frac{2(Q-M^*+1)}{N+1} - c_0 < 0, \end{aligned}$$

because $Q < \min\{Q^*(N-M^*+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\} + M^* - 1$. This contradicts with the optimality of $M^*$. Therefore, $M^* \leq Q+1-\min\{Q^*(N-M^*+1), \lfloor \frac{c_0(N+1)}{2} \rfloor\}$. Hence, the propositionis proved.

**Proof of Proposition 14**

For part (a), we can use similar arguments as those in the proof of part (a) of Proposition13. Consider Supplier 1. Suppose $m$ of the other $M-1$ contracts are accepted and $M-m-1$ are turned down, then Supplier 1 should accept the contract if and only if his cost $c$ satisfies $c \leq c^*(N-m, Q-(M-1))$. From Proposition7, let $c = \min_{0\leq m\leq M-1} c^*(N-m, Q-(M-1)) = c^*(N, Q-M+1)$ and $\underline{c} = \max_{0\leq m\leq M-1} c^*(N-m, \bar{Q}-(M-1)) = c^*(N-M+1, Q-M+1)$, then the result follows.

Part (b) is more complex and we illustrate the idea using the case $M = 3$. Let $c_1, c_2, c_3$ be the cost of Supplier 1, 2 and 3, respectively. Without loss of generality, we assume $c_1 \le c_2 \le c_3$. Then we need to prove the following. There exist three thresholds $c_1^*, c_2^*, c_3^*$ and $c_1^* \le c_2^* \le c_3^*$. (i) If $c_3 \le c_3^*$, the Pareto optimal equilibrium is [accept,accept,accept]; (ii) if $c_2 \le c_2^*$ and $c_3 > c_3^*$, then the Pareto optimal equilibrium is [accept,accept,reject]; (iii) if $c_1 \le c_1^*, c_2 > c_2^*$ and $c_3 > c_3^*$, the Pareto optimal equilibrium is [accept,reject,reject]; (iv) and if $c_1 > c_1^*, c_2 > c_2^*$ and $c_3 > c_3^*$, the Pareto optimal equilibrium is [reject,reject,reject]. Notice that, if $c_i > c_j$ for $i, j \in \{1, 2, 3\}$ and, at equilibrium, Supplier $i$ accepts the contract, then Supplier $j$ also accepts.

Now we are ready to prove the result case by case. Define $c_1^* = c^*(N, Q-2), c_2^* = c^*(N-1, Q-2)$ and $c_3^* = c^*(N-2, Q-2)$ which satisfy $c_1^* \le c_2^* \le c_3^*$.

(i) If $c_3 \le c_3^*$, then $c_1, c_2 \le c_3^*$. Suppose Supplier 1 and 2 accept the contract, then Supplier 3 is in a situation that there are $N-2$ suppliers competing for $Q-2$ units, and one flexible contract is offered to him. As $c_3 \le c_3^*$, then he would accept the contract. Similarly, Supplier 1 and 2 would have the same best response given the other two suppliers accept and so [accept, accept, accept] is one equilibrium. Now we show that [accept, accept, accept] is Pareto optimal among all other *possible* equilibria, including [accept,accept,reject], [accept, reject, reject] and [reject, reject, reject]. The payoff of Supplier $i$ under different possible equilibrium is listed in the following table. To prove

Table 5: Supplier $i$'s payoff

| *possible* equilibrium | Supplier $i$ accepts | Supplier $i$ rejects |
|---|---|---|
| [accept, accept, accept] | $\mathsf{E}[C_{(Q-2,N-3)} - c_i]$ | N/A |
| [accept, accept, reject] | $\mathsf{E}[C_{(Q-2,N-2)} - c_i]$ | $\mathsf{E}[(C_{(Q-3,N-3)} - c_i)^+]$ |
| [accept, reject, reject] | $\mathsf{E}[C_{(Q-2,N-1)} - c_i]$ | $\mathsf{E}[(C_{(Q-3,N-2)} - c_i)^+]$ |
| [reject, reject, reject] | N/A | $\mathsf{E}[(C_{(Q-3,N-1)} - c_i)^+]$ |

that $\mathsf{E}[C_{(Q-2,N-3)} - c_i]$ is the largest among all other *possible* equilibrium payoffs of Supplier $i$, we only need to show $\mathsf{E}[C_{(Q-2,N-3)} - c_i] \ge \mathsf{E}[(C_{(Q-3,N-3)} - c_i)^+]$. This is true because $c_i \le c_3^*$ and the definition of $c_3^*$. Therefore [accept,accept,accept] is the Pareto optimal equilibrium in this case.

(ii) If $c_2 \le c_2^*$ and $c_3 > c_3^*$, then $c_1 \le c_2^*$. In this case, the dominant strategy of Supplier 3 is to reject the contract as shown in part (a) of the theorem. If Supplier 1 accepts the contract, then Supplier 2 is in the situation that $N-1$ suppliers competing for $Q-2$ units and one contract is offered to him. Then Supplier 2 will accept the contract since $c_2 \le c_2^*$. Therefore [accept,accept,reject] is one equilibrium. Since supplier 3 has a dominant strategy to reject the contract, then the other *possible* equilibria can only be [accept,reject,reject] and [reject,reject,reject]. With a similar approach as in (i), one can see [accept,accept,reject] is Pareto optimal.

(iii) If $c_1 \le c_1^*, c_2 > c_2^*$ and $c_3 > c_3^*$, then Supplier 3 has a dominant strategy to reject the contract, and Supplier 1 has a dominant strategy to accept the contract. Thus, the optimal strategy of Supplier 2 is to reject the contract by the definition of $c_2^*$. Then [accept,reject,reject] is the only equilibrium.

(iv) If $c_1 > c_1^*, c_2 > c_2^*$ and $c_3 > c_3^*$, then Supplier 3 has a dominant strategy to reject the contract. Then if Supplier 1 accepts the contract, Supplier 2 is in the case that there are $N-1$ suppliers and $Q-2$ units, then Supplier 2 will reject the contract because $c_2 > c_2^*$; if Supplier 1 rejects the contract, Supplier 2 is in the case that there are $N$ suppliers and $Q-2$ units, then he will still reject the contract as $c_2 > c_2^*$. Therefore Supplier 2 will reject the contract regardless of Supplier 1's behavior. Given Supplier 2 and 3's action, Supplier 1 will also reject the offer since $c_1 > c_1^*$. Hence [reject,reject,reject] is the only equilibrium.

**Proof of Proposition 15**

(a) In the uninformed supplier case, a supplier's expected payoff of accepting the contract is $\pi_u^{ac} = \mathsf{E}[C_{(Q,N-1)} - C]$ and his expected payoff of rejecting the contract is $\pi_u^{re} = \mathsf{E}[(C_{(Q-1,N-1)} - C, 0)^+] - c_b$. We have shown in Proposition3 that when $Q = N-1$, $\mathsf{E}[C_{(Q,N-1)} - C] > \mathsf{E}[(C_{(Q-1,N-1)} - C, 0)^+]$. Therefore, with the bidding cost, $\pi_u^{ac} > \pi_u^{re}$ when $Q = N-1$ and so the result follows.

(b) When $C$ is uniformly distributed, we can calculate $\pi_u^{ac} = \frac{Q}{N} - \frac{1}{2}$ and $\pi_u^{re} = \frac{Q(Q-1)}{2N(N+1)} - c_b$. Therefore

$$\pi_u^{ac} - \pi_u^{re} = \frac{1}{2N(N+1)}(-Q^2 + (2N+3)Q - (N^2+N)(1-2c_b)).$$

This is a concave parabola, it's roots are

$$Q_1 = \frac{2N+3-\sqrt{8c_bN^2+(8+8c_b)N+9}}{2} \text{ and } Q_2 = \frac{2N+3+\sqrt{8c_bN^2+(8+8c_b)N+9}}{2} > N.$$

Therefore, for $Q \in [2, N-1]$, if $Q > \lfloor Q_1 \rfloor$, then $\pi_u^{ac} > \pi_u^{re}$, and if $Q \le \lfloor Q_1 \rfloor$ then $\pi_u^{ac} \le \pi_u^{re}$. We define $Q^*(N) = \lfloor Q_1 \rfloor$. It is also observed that

$$\begin{aligned} & Q^*(N+1) - Q^*(N) \\ = & \left\lfloor \frac{2(N+1)+3-\sqrt{8c_b(N+1)^2+(8+8c_b)(N+1)+9}}{2} \right\rfloor - \left\lfloor \frac{2N+3-\sqrt{8c_bN^2+(8+8c_b)N+9}}{2} \right\rfloor \\ < & 1. \end{aligned}$$

For (b)-ii., the proof of Proposition 4 can be directly applied here. Therefore we define $Q_a(N) = \min\left\{Q^*(N), \left\lfloor \frac{c_0(N+1)}{2} \right\rfloor\right\}$. Apparently, $Q_a(N)$ increases in $c_0$ and decreases in $c_b$.

For (b)-iii., we consider the EK model with bidding cost $c_b$. Because a supplier is not allowed to join the auction if he turns down the contract, the bidding cost will not affect the supplier's behavior. Therefore when the supplier's cost is uniformly distributed, if $Q > Q^*_{EK} = \lfloor N/2 \rfloor$, the supplier will accept the contract and otherwise reject the contract. We implicitly require that $c_b$ is small such that the expected payoff for the supplier of joining the auction is non-negative, this implies that $Q^*(N) > Q^*_{EK}(N)$ because in our model the supplier compares the payoff of accepting the contract with an alternative of non-negative payoff while in the EK model the supplier compares the payoff of accepting with an alternative of zero payoff. It then follows that all the analysis in the proof of Proposition 5 will go through. And the desired result follows.

## Appendix B: Inclusion of Bidding Cost

Let $c_b > 0$ be the cost when a supplier submits a bid, which, for simplicity, is assumed constant among all suppliers. First note that when the bidding cost is present, "truth-telling" is still a equilibrium for each bidder, because as long as a supplier decides to bid, $c_b$ is a sunk cost (see, for example, Levin and Smith, 1994; Menezes and Monteiro, 2000). Similar to the previous information structure when the suppliers decide whether to take the flexible contracts, we consider that the suppliers either have incurred the bidding cost before knowing their own production cost (e.g., pay the bidding fee long before the auction starts) or have not incurred the bidding cost until they know the production cost. Let us start with the first case. In this case, as long as $\mathsf{E}[C - C_{(Q,N-1)}]^+ \geq c_b$, every supplier will join the auction because all suppliers will pay the fee due to non-negative expected payoff. If $\mathsf{E}[C - C_{(Q,N-1)}]^+ < c_b$, then in equilibrium, some of the suppliers will give up the auction, which reduces the total number of participants in the auction. If we assume $c_b$ is sufficiently small such that $\mathsf{E}[C - C_{(Q,N-1)}]^+ \geq c_b$, such cost will not deter the suppliers from joining the auction.[8] Meanwhile, for the supplier offered the flexible contract, if he accepts the contract, his expected payoff is $\pi_u^{ac} = \mathsf{E}[C_{(Q,N-1)} - C]$ while if he turns down the contract, his expected payoff is $\pi_u^{re} = \mathsf{E}[C_{(Q-1,N-1)} - C]^+ - c_b$. Therefore the supplier will accept the contract if and only if $\pi_u^{ac} \geq \pi_u^{re}$. With these, we can show that all the results obtained in Section 3 continue to hold in this case and we summarize them in the following theorem.

**Theorem 15** *In the uninformed supplier scenario, suppose all suppliers pay the bidding fee before their private cost information reveals and such fee is sufficiently small. If one noncompetitive contract is offered, then*

[8] Engelbrecht-Wiggans (1996) also assume that the bidding cost is sufficiently small that it will not deter the suppliers from joining the auction.

*(a) if $Q$ is sufficiently close to $N$, then the selected supplier will accept the contract;*

*(b) if $C$ is uniformly distributed, then*

*i. there exists a unique threshold $Q^*(N)$ such that the supplier will accept the contract if and only if $Q > Q^*(N)$; and $Q*(N)$ is decreasing in $c_b$;*

*ii. there exists a threshold $Q_a(N) \leq Q^*(N)$ such that $\Pi_u \leq \Pi_{au}$ if and only if $Q > Q_a(N)$ and $Q_a(N)$ is increasing in $N$, $c_0$, and decreasing in $c_b$;*

*iii. there exists a threshold $Q_b(N)$ such that $\Pi_u \leq \Pi_u^{EK}$ if and only if $Q \notin (Q_{EK}(N), Q_b(N))$.*

**Proof.** (a) In the uninformed supplier case, a supplier's expected payoff of accepting the contract is $\pi_u^{ac} = \mathsf{E}[C_{(Q,N-1)} - C]$ and his expected payoff of rejecting the contract is $\pi_u^{re} = \mathsf{E}[(C_{(Q-1,N-1)} - C, 0)^+] - c_b$. We have shown in Proposition3 that when $Q = N-1$, $\mathsf{E}[C_{(Q,N-1)} - C] > \mathsf{E}[(C_{(Q-1,N-1)} - C, 0)^+]$. Therefore, with the bidding cost, $\pi_u^{ac} > \pi_u^{re}$ when $Q = N-1$ and so the result follows.

(b) When $C$ is uniformly distributed, we can calculate $\pi_u^{ac} = \frac{Q}{N} - \frac{1}{2}$ and $\pi_u^{re} = \frac{Q(Q-1)}{2N(N+1)} - c_b$. Therefore

$$\pi_u^{ac} - \pi_u^{re} = \frac{1}{2N(N+1)}(-Q^2 + (2N+3)Q - (N^2+N)(1-2c_b)).$$

This is a concave parabola, it's roots are

$$Q_1 = \frac{2N+3-\sqrt{8c_bN^2+(8+8c_b)N+9}}{2} \text{ and } Q_2 = \frac{2N+3+\sqrt{8c_bN^2+(8+8c_b)N+9}}{2} > N.$$

Therefore, for $Q \in [2, N-1]$, if $Q > \lfloor Q_1 \rfloor$, then $\pi_u^{ac} > \pi_u^{re}$, and if $Q \leq \lfloor Q_1 \rfloor$ then $\pi_u^{ac} \leq \pi_u^{re}$. We define $Q^*(N) = \lfloor Q_1 \rfloor$. It is also observed that

$$\begin{aligned} & Q^*(N+1) - Q^*(N) \\ = & \left\lfloor \frac{2(N+1)+3-\sqrt{8c_b(N+1)^2+(8+8c_b)(N+1)+9}}{2} \right\rfloor - \left\lfloor \frac{2N+3-\sqrt{8c_bN^2+(8+8c_b)N+9}}{2} \right\rfloor \\ < & \ 1. \end{aligned}$$

For (b)-ii., the proof of Proposition 4 can be directly applied here. Therefore we define $Q_a(N) = \min\left\{Q^*(N), \left\lfloor \frac{c_0(N+1)}{2} \right\rfloor\right\}$. Apparently, $Q_a(N)$ increases in $c_0$ and decreases in $c_b$.

For (b)-iii., we consider the EK model with bidding cost $c_b$. Because a supplier is not allowed to join the auction if he turns down the contract, the bidding cost will not affect the supplier's behavior. Therefore when the supplier's cost is uniformly distributed, if $Q > Q^*_{EK} = \lfloor N/2 \rfloor$, the supplier will accept the contract and otherwise reject the contract. We implicitly require that $c_b$

is small such that the expected payoff for the supplier of joining the auction is non-negative, this implies that $Q^*(N) > Q^*_{EK}(N)$ because in our model the supplier compares the payoff of accepting the contract with an alternative of non-negative payoff while in the EK model the supplier compares the payoff of accepting with an alternative of zero payoff. It then follows that all the analysis in the proof of Proposition 5 will go through. And the desired result follows. ∎

Because accepting the flexible contract will save the supplier the bidding cost $c_b$, a positive bidding cost makes the supplier more likely to accept the contract.

Now we consider the scenario where the supplier incurs the bidding cost after observing his production cost. Again, if the bidding cost is small and the suppliers have decided to pay the fee when they are selected by the buyer (i.e., such fee will not affect their participation of the auction), the results in Section 4 will still hold with some minor changes (we omit the details). And again, such cost will incentivize the supplier to accept the flexible contract. However, if the bidding cost is taken into account by the suppliers when deciding whether to join the auction, the analysis will become much more complicated. We provide some initial analysis as follows. Consider a regular auction consists of $N$ suppliers bidding for $Q$ units. When the supplier finds his cost is high, the expected payoff from the auction will be small (as his chance of winning is small) and he will simply not join the auction if such payoff is smaller than the bidding cost. Assume there is a threshold $\tilde{c}$ such that the supplier will join the auction if and only if his cost $c \le \tilde{c}$. Meanwhile, if eventually the total number of auction participants is less than or equal to $Q$, we assume every bidder will be awarded the unit at the price $c_0$, i.e., the outside market price.

At the threshold $\tilde{c}$, the supplier is indifferent between bidding and not bidding, and therefore $\tilde{c}$ satisfies the following equation:

$$\mathsf{E}[\tilde{c} - \tilde{C}_{(Q,N-1)}]^+ - c_b = 0,$$

where $\tilde{C}_{(Q,N-1)}$ is the $Q-$th order statistics of $N-1$ i.i.d random variable $\tilde{C}_i, i = 1, \ldots, N$ and

$$\tilde{C}_i = \begin{cases} C_i & \text{if } C \le \tilde{c}, \\ c_0 & \text{otherwise.} \end{cases}$$

After $\tilde{c}$ is solved, the formation of the auction is known. Based on this, we can then further consider the inclusion of flexible contracts. It can be seen that when the suppliers consider the bidding cost in their participation decision of the auction after knowing the private cost information, even a regular auction becomes much more complicated, let alone the hybrid mechanism considered in this paper. Indeed, some recent researches study the auction with bidding cost (or participation cost) that the bidders learn their private cost before deciding to join the auction. For more details, the readers are referred to Samuelson (1984), Menezes and Monteiro (2000), Tan and Yilankaya

(2006), and Monreno and Wooders (2011). In general they show that for the first-price or second-price auction with biding cost, increasing the number of suppliers may actually harm the buyer. Therefore explicitly considering bidding cost in our model requires more dedicated research and is beyond the scope of this paper. We leave it as our future research.