Winter 2018

Financing Multiple Heterogeneous Suppliers in Assembly Systems: Buyer Finance vs. Finance

Shiming Deng

Chaocheng Gu

Gangshu (George) Cai
Santa Clara University, gcai@scu.edu

Recommended Citation

Deng, S., Gu, C., Cai, G. (George), & Li, Y. (2018). Financing Multiple Heterogeneous Suppliers in Assembly Systems: Buyer Finance vs. Bank Finance. Manufacturing & Service Operations Management, 20(1), 53–69. https://doi.org/10.1287/msom.2017.0677

Financing Multiple Heterogeneous Suppliers in Assembly Systems: Buyer vs. Bank Finance

Shiming Deng * Chaocheng Gu * Gangshu (George) Cai † Yanhai Li *

Abstract

Buyer finance has been practiced by manufacturers/assemblers for years; however, few papers have investigated the efficacy of buyer finance in an assembly system with multiple suppliers. This paper fills the literature gap by comparing buyer finance with bank finance in a supply chain with one assembler and multiple heterogeneous capital-constrained component suppliers. We characterize the equilibrium solutions for different financing schemes (i.e., buyer finance, bank finance, and no finance). We show that in buyer finance the assembler should charge the suppliers the lowest possible interest rate, which may be even below its own unit capital opportunity cost, leading to losses in financing suppliers. However, the assembler can benefit more from enhanced inventory backup and lower component purchasing prices resulted from the low buyer finance interest rate. We further compare the different financing schemes from the perspectives of assembler, (borrowing and non-borrowing) suppliers, and the whole supply chain. Our analysis reveals that the assembler may offer buyer finance even if its own unit capital opportunity cost is higher than banks' risk-free interest rate. We also identify the conditions under which buyer finance is better than bank finance for each party as well as the whole supply chain, and demonstrate how the suppliers' initial capitals, production costs, and their heterogeneities affect the assembler's selection of the optimal financing scheme.

Key words: buyer finance; bank finance; heterogenous suppliers; operations-finance interface; assembly supply chain

1 Introduction

Many manufacturers have been sourcing components globally from suppliers of various sizes. However, small suppliers often have limited working capital and are not accessible to a fair-priced capital

*School of Management, Huazhong University of Science and Technology, Wuhan 430074, China, Email: ccgu.hust@gmail.com, smdeng@mail.hust.edu.cn, 619551185@qq.com; the second author is the corresponding author

†Leavey School of Business, Santa Clara University, CA 95053, Email: gcai@scu.edu

market. Financial distress of suppliers may cause component price increases or even production disruptions, especially for big manufacturers/assemblers with complex Bill of Materials (BOM) structures. According to Bryn and Denton (2010), when an auto parts supplier goes bankrupt, its parts' prices can jump up by 10 to 15 percent.

To address suppliers' financial distress, big manufacturers/assemblers have tried to facilitate their supplier's access to bank loans (hereafter referred to as *bank finance*). For example, Boeing Company has joined a Supply Chain Financing Program guaranteed by the Export-Import (Ex-Im) Bank of the United States since 2012, which allows its hundreds of small suppliers to access affordable loans from affiliated banks (Boeing, 2012). Auto makers PSA and Volkswagen have helped suppliers secure financing during financial crisis after 2008 (Bolduc, 2008).

In contrast to bank finance, some firms have directly provided finance to their suppliers (hereafter referred to as *buyer finance*) in various ways, such as, paying an advance payment for each order, setting up a general finance program for suppliers, or even acquiring the suppliers' stock to alleviate their financial stress. In the automobile industry, as Bolduc (2008) reports, to reduce suppliers' financial hardship during financial downtime following 2008, BMW and PSA pay suppliers in advance for parts, Ford gives loans to suppliers, and Porsche finances suppliers' production tooling. In the aviation industry, Airbus in 2011 purchased 51% share of PFW Aerospace to ensure continued supply of key aircraft components. Boeing in 2009 paid \$590 million to its fuselage supplier, Vought Aircraft Industries, to guarantee its component supply for Boeing 787. The World Bank estimates that in 2008 buyer advance payment represents about 19%-22% (\$3-\$3.5 trillion) of all trade finance arrangements (Chauffour and Malouche, 2011).

Both buyer and bank finance serve to improve suppliers' financial capacity and reinforce supply chain reliability, but they differ in several important ways. In bank finance, assemblers can leverage bank resource and thus reduce their own capital stress and administrative overhead. In buyer finance, the downstream manufacturers/assemblers may earn additional profit from financing their suppliers and benefit from coordinating financial decisions with operational decisions in supply chains.

The distinguished effects of the two financing strategies bring about the forefront research questions: When would manufacturers/assemblers provide buyer finance to suppliers instead of letting them borrow money from banks? How would the suppliers react to the downstream firm's decision? What are the impacts of financial decisions on operational decisions in each financing strategy? How does the complexity of supply chain structure (e.g., the total quantity and heterogeneity of the suppliers' working capitals and production costs) influence the financing decisions?

1.1 Main Findings and Contributions

To answer the aforementioned questions, we consider an assembly supply chain in which a downstream assembler purchases components from multiple capital-constrained suppliers to make an end-product in a single time period. Demand for the end-product is uncertain. The assembly time of the end-product is short, and the assembler makes to order after demand is realized. The lead time of component production, however, is long and the suppliers have to decide their individual stock levels within their capital constraints at the beginning of the time period before demand is realized. Both the assembler and suppliers are risk-neutral expected profit maximizers. We consider two financing strategies: 1) the suppliers get loans from banks at an interest that is competitively priced (bank finance); and 2) the assembler provides direct finance to its suppliers in addition to bank finance and charges them an interest at its discretion (buyer finance).

In both buyer and bank finance, we formulate the interactions in the assembly supply chain as a Stackelberg game, in which the assembler is the leader and the component suppliers are the followers. We identify the equilibrium solutions for the cases of buyer finance, bank finance, and no finance. Interestingly, in buyer finance, it is optimal for the assembler to charge the suppliers an interest rate at its lowest possible value. The interest rate may be even below its unit capital opportunity cost and the assembler actually incurs losses in financing suppliers. However, the assembler reaps operational benefits from raising component inventory levels and reducing component purchasing prices by virtue of the low interest rate offered to the suppliers. The benefits can be greater than the losses in financing the suppliers.

Our analysis reveals that the assembler's financial cost of offering buyer finance is a critical factor in the selection of buyer finance. The assembler's decision follows a threshold policy with regard to the assembler's unit capital opportunity cost. The assembler should offer buyer finance, if and only if its unit capital opportunity cost is below the threshold. We prove that the threshold is larger than the risk-free interest rate. In the range between the risk-free interest rate and the threshold, buyer finance outperforms bank finance, even if the assembler is a less efficient loaner than banks (i.e., the assembler's unit capital opportunity cost is greater than the bank's risk-free interest rate). These benefits of buyer finance manifest the advantage of integrating both finance and operations decisions.

The relative financial efficiency of the assembler and the suppliers (i.e., unit capital opportunity costs) and the total quantity and heterogeneity of the suppliers' initial capitals and production costs also play significant roles in the assembler's choice of buyer finance. When the assembler's unit capital opportunity cost is higher than the maximum of the suppliers' unit capital opportunity costs, the assembler incurs loss in buyer finance. The relative advantage of buyer finance over bank finance increases as the suppliers' total initial capital increases or total production cost decreases, because higher initial capital or lower production cost reduces the assembler's financial burden for carrying the suppliers' default risks in buyer finance.

The impact of heterogeneity in capital/cost on the selection of financing scheme, however, is more complicated and depends on the total amount of initial capital at all suppliers. When the total amount is high, buyer finance may be less attractive to the assembler if the suppliers are more heterogeneous. When the total amount of initial capital is low, the opposite can be true.

When the assembler's unit capital opportunity cost is lower than the maximum of unit capital opportunity costs of all suppliers, financing the suppliers becomes profitable, and the assembler always prefers buyer finance.

We also compare buyer finance and bank finance from the perspective of suppliers. We find that the supplier's preferences are not always aligned with the assemblers. A supplier's preference depends on the assembler's unit capital opportunity cost and whether it borrows money or not in equilibrium. If the assembler's unit capital opportunity cost is sufficiently low, all firms (including all suppliers) are better off in buyer finance, benefiting from the boosted production quantity. However, as the assembler's unit capital opportunity cost grows, there exists a conflicting area that the assembler and the suppliers hold different preferences toward buyer finance. A borrowing supplier may get worse off in buyer finance, because the assembler cutbacks the component purchasing price to alleviate the burden of financing suppliers due to the low interest rate in buyer finance. Consequently, the attraction of buyer finance to the borrowing suppliers downturns, which will eventually hurt the non-borrowing suppliers as the equilibrium production quantity declines. As the assembler's unit capital opportunity cost keeps increasing and crosses the threshold, all firms prefer bank finance to buyer finance.

This paper contributes to the literature in several aspects. First, this work is the first attempt to compare buyer finance and bank finance in an assembly supply chain with one assembler and multiple heterogeneous suppliers. We have characterized the equilibrium solutions of firms in both bank and buyer finance. Second, we identify a threshold condition for the assembler to offer buyer finance. We further demonstrate the impact of the assembler's unit capital opportunity cost, the suppliers' total initial capital/production cost and the suppliers' heterogeneity on the selection of the optimal financing scheme. Third, we also outline both the Pareto zones, in which all parties prefer the same financing scheme, and preference conflicting zones, in which the assembler and the (borrowing or non-borrowing) suppliers may prefer different financing schemes.

1.2 Related Literature

The first stream of relevant research is on the interface of operations and financial decisions. The focus of this line of research is on the impact of financial constraints on operational decisions (see, e.g., Xu and Birge, 2004; Chao et al., 2008; Lai et al., 2009). In these models, traditional stochastic inventory decision-making problems are solved under capital constraints, either in single-period newsvendor settings or in multiple-period dynamic decision-making frameworks. Nevertheless, the issues of how to finance the capital-constrained decision makers are not addressed.

The second stream of relevant research is on the interactions between financial strategies and inventory decision-making for multiple parties in game-theoretic settings. Buzacott and Zhang (2004) discuss the role of asset-based financing in influencing the budget constraints and production activities of a firm. In their model, the capital-constraint newsvendor borrows money from a profit maximizing bank, which is similar to the setting of Dada and Hu (2008) who study the interaction between a budget-constraint retailer and the profit maximizing bank in a Stackelberg game framework. Our paper differs from theirs by focusing on the interaction between the assembler and its component suppliers instead of that between a bank and a firm.

The third closely related research stream is on the role and efficiency of trade credit finance in operations management. This line of research work has grown rapidly. Note that trade credit refers to credit extended by upstream suppliers to downstream firms. On the contrary, we study a scheme in which finance is provided by downstream buyers to capital-constrained upstream suppliers for maintaining availability of component supply. Although the settings are quite different, the results share similar insights. First, the integration of inventory and trade credit finance can increase both the profit of the suppliers and the efficiency of the whole supply chains (Kouvelis and Zhao, 2012; Yang and Birge, 2011; Jing et al., 2012). We also show that offering buyer finance in coordination with purchasing pricing decision can increase the profits of the assembler and the whole supply chain. Second, Kouvelis and Zhao (2012) show that it may be optimal for the supplier to sacrifice financing profit by charging the minimal interest rate in trade credit in order to increase the retailer's order quantity and cultivate the benefit from better demand satisfaction. Similarly, we also demonstrate that the assembler may want to charge the minimal interest rate in buyer finance in order to reduce the purchasing price and increase the inventory levels at suppliers. There are also papers that investigate other issues related to trade credit, such as the impact of credit scores (Kouvelis and Zhao, 2016), the role of risk-sharing in supply chain (Chod, 2016; Yang and Birge, 2016), the effect of trade credit financing on upstream price competition (Peura et al., 2017), and the comparison with other financing schemes (Kouvelis and Zhao, 2012; Cai et al., 2013). For recent comprehensive reviews, we refer readers to Seifert et al. (2013) and Zhao and Huchzermeier (2015).

Different from the above papers on trade credit, our paper investigates both vertical competition and horizontal interaction by employing an assembly supply chain structure. This allows us to examine how the multiple component suppliers, who have heterogeneous production cost and initial capital level, affect each other and how they can benefit from buyer finance.

The fourth focus is on the strategic role of downstream subsidy in alleviating supplier distress. For example, Babich (2010) provides a stochastic dynamic model in which a manufacturer jointly decides its capacity reservation quantity and the financial subsidy offered to suppliers. There is no strategic interactions between firms in this model. Swinney and Netessine (2009) consider a manufacturer contracting with suppliers in a two-period game. They demonstrate that the manufacturer may prefer offering a long-term contract to prevent the supplier from bankruptcy. Their model does not focus on the comparison of bank versus buyer finance.

Other models also investigate the interaction between capital flexibility and resource flexibility (Chod and Zhou, 2014), the impact of bankruptcy on supply chains (Yang et al., 2015), the cash-flow dynamics and its efficiency in 3PL procurement service (Chen et al., 2017), and deferred payment (Rui and Lai, 2015). Our work differs from theirs by considering the interactions between the assembler and component suppliers as well as the interactions among component suppliers.

The work of particular relevance to our paper is Tang et al. (2015), where they compare buyer finance to a particular type of bank finance, that is, purchase order financing under information asymmetry between banks and manufacturers regarding the supplier's effort. They consider a supply chain consisting of one supplier and one manufacturer facing a constant demand. They show that the two finance strategies yield the same profit for the manufacturer if there is no information asymmetry and buyer finance is better than bank finance only if the manufacturer has superior information about the supplier's effort than the bank. In contrast, we consider a more complicated assembly supply chain with one assembler and multiple heterogeneous suppliers facing a stochastic demand. We focus on the interaction between financial and operational decisions under demand uncertainty. Different from the results in Tang et al. (2015), we show that the buyer finance can be better than bank finance without information asymmetry. Furthermore, the structure of the assembly supply chain considered in this paper allows us to explore the impact of supply chain complexity in term of the suppliers' heterogeneity in initial capital and production costs.

Another stream of related literature is on managing assembly supply chains. Our work is partially related to this stream of literature in the way of modeling operational interactions between upstream and downstream firms in assembly supply chains (see, e.g., Song and Zipkin, 2003; Bernstein and DeCroix, 2004, 2006; Fang et al., 2008; Jiang and Wang, 2010). In particular, Wang and Gerchak (2003) discuss a decentralized assembly supply chain that is essentially the same as in our model. However, the aforementioned literature generally do not focus on financial issues.

The remainder of this paper is structured as follows. The next section describes the model. Section 3 characterizes the equilibrium solutions in bank finance and buyer finance. Section 4 presents the assembler's choice of the two financing strategies and Section 5 explores the suppliers' and the whole assembly chain's financing preferences. Section 6 concludes and all proofs are included in the Appendix: Online Supplements.

2 The Model

We consider a stylized assembly supply chain consisting of one assembler and N capital-constrained component suppliers. Demand is uncertain and denoted by D, which follows a cumulative distribution function, $F(\cdot)$, and a probability distribution function, $f(\cdot)$. We assume the demand distribution has an increasing failure rate (IFR), which is a common assumption in the supply chain literature, satisfied by many common distributions (see, e.g., Lariviere and Porteus, 2001). The demand distribution is common knowledge for all firms. The price of the end-product is p and the unit production cost of component supplier i is c^i, where $i = 1, ..., N$. We summarize all notations in Table 1.

Without loss of generality, we assume the assembler needs exactly one unit from each component supplier to assemble one product. The assembler decides the component purchasing price, w^i, for component i, $i = 1, ..., N$. Then, the component suppliers simultaneously select their respective stock levels, q^i, as constrained by their respective initial capital levels, k^i, before demand uncertainty is resolved. Hence, the minimum system stock level is given by $q = \min\{q^i, i = 1, ..., N\}$. After demand is realized at x, the assembler then procures a quantity of $\min(x, q)$ from each component supplier and pays $w^i \cdot \min(x, q)$. For simplicity, the suppliers' unsold components have zero salvage value. Our setting that the assembler sets the component purchasing price first and then the suppliers decide their inventory levels is similar to that in Wang and Gerchak (2003) and Dong and Rudi (2004). This is often the case when the assembler dominates the suppliers in the supply chain.

Due to capital constraints, the suppliers may borrow money from either a bank (bank finance) or the assembler (buyer finance), if both financing schemes are viable. We use subscript bk and br to represent bank finance and buyer finance, respectively. In either financing scheme, each component

Table 1: Notations

N:	the number of component suppliers in the assembly supply chain.
p:	the assembler's retail price of the end-product.
c^i:	the unit production cost of component supplier i.
w^i:	the component purchasing price paid to component supplier i.
q^i:	the production quantity of component supplier i.
q:	the minimum system stock level, $q = min\{q^i, \ \ i = 1, 2, \cdots, N\}$.
k^i:	the initial capital of component supplier i.
B^i:	the loan size borrowed by component supplier i in buyer or bank finance.
r_s^i:	the unit capital opportunity cost of component supplier i.
r_a:	the unit capital opportunity cost of the assembler.
r_{bk}^i:	the real interest rate charged to supplier i in bank finance.
r_f:	the risk-free interest rate
r_{br}^i:	the real interest rate charged to supplier i in buyer finance.
r_b:	the assembler's expected rate of return from buyer finance.
π_n^i:	the expected profit for component supplier i when there is no external finance.
π_{bk}^i:	the expected profit for component supplier i with bank financing.
π_{br}^i:	the expected profit for component supplier i with buyer financing.
Π_n:	the expected profit for the assembler when there is no external finance.
Π_{bk}:	the expected profit for the assembler with bank finance.
Π_{br}:	the expected profit for the assembler with buyer finance.
D:	the random demand of end-product.
$f(\cdot)$:	the probability density function of the demand distribution.
$\bar{F}(\cdot)$:	the complementary cumulative distribution function (CCDF) of the demand distribution

supplier i needs to decide a loan size $B^i \geq 0$ in addition to its decision on production quantity. We assume that supplier i can invest its excess capital (if any) to an alternative project with a return rate (i.e., supplier i's unit capital opportunity cost), r_s^i. After demand is realized, supplier i collects its wholesale revenue, $w^i \cdot min(q^i, D)$, plus the interest revenue from investing its excess capital characterized by $(k^i + B^i - c^i q^i)(1 + r_s^i)$.

Without loss of generality, we assume $r_s^i \leq r_f$ for any $i = 1, ..., N$, where r_f is the bank risk-free interest rate (i.e., $\max_i\{r_s^i\} \leq r_f$); otherwise, supplier i would have no incentive to use its own capital for production but always borrows from banks. When $r_s^i = r_f$, Supplier i is indifferent between borrowing from bank and using its own money. In this case, we assume that the supplier i uses its own money first. These assumptions are also consistent with the pecking order theory: Firms should first exhaust internal funds and then resort to more expensive external debt (see, e.g., Myers and Majluf, 1984).

In line with the extant literature (Xu and Birge, 2004; Kouvelis and Zhao, 2012), we assume that bank loan is competitively priced in bank finance and the risk-neutral bank expects to earn a risk-free interest rate r_f. The real interest rate charged to component supplier i is denoted as r_{bk}^i in bank finance. To achieve the same expected rate of return equal to r_f, the bank generally charges a different real interest rate, r_{bk}^i, to each supplier i, because each supplier has different initial capital,

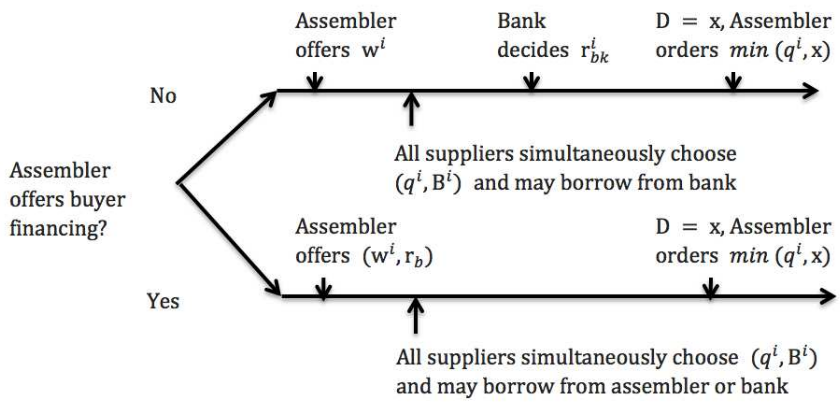


Figure 1: Timing of Events

production cost, and therefore different default risk. Hereafter we also use "bank interest rate" to refer to the bank's expected rate of return (i.e., the risk-free interest rate, r_f) interchangeably.

Different from bank finance, in which the bank interest rate, r_f, is exogenous, the assembler in buyer finance sets the target expected rate of return, r_b, from financing suppliers at its discretion. Moreover, the assembler sets r_b jointly with the component purchase prices, $(w_i, i = 1, 2, ..., N)$ to maximize its expected profit from both buyer finance and product sales. Denote the real interest rate charged to component supplier i in buyer finance as r^i_{br}. It is easy to see that for any supplier i, given other parameters fixed, there is a one-to-one relationship between r^i_{br} and r_b, and if $r_{b1} \geq r_{b2}$, then $r^i_{br1} \geq r^i_{br2}$, where $r^i_{br\xi}$ is the real interest rate corresponding to the expected rate of return, $r_{b\xi}$ for $\xi = 1, 2$. Similar to bank finance, r^i_{br} can be different across suppliers for a given target r_b, because of the heterogeneity of the suppliers. Hereafter we also use "buyer finance interest rate" to refer to the assembler's expected rate of return from buyer finance (i.e., r_b) interchangeably.

If the assembler lends a loan of B^i to supplier i in buyer finance, it incurs a financial cost of $r_a \cdot B^i$, where r_a is the unit capital opportunity cost for the assembler. In reality, companies' unit capital opportunity costs depend on the return of their best alternative investment and can vary widely. The assembler's unit capital opportunity cost can be either higher or lower than r_f depending on the assembler's business status. If it is a dominant player and has many alternative investment projects, its unit opportunity cost could be higher than a typical bank's earning rate. However, if the assembler is small and has few other business opportunities, its opportunity cost can be small.

The sequence of events is illustrated in Figure 1. There are four decisions steps. 1) At the beginning of the time period, facing demand uncertainty, the assembler decides whether to offer buyer finance to its suppliers or not. This step is critical, in that it will lead the game into different paths (finance schemes). 2) If the assembler decides to offer buyer finance, as the Stackelberg leader, it determines r_b and the purchasing price, w^i, for each component i; otherwise, it only sets the purchasing prices. 3) Given the assembler's decisions, the suppliers (Stackelberg followers) then simultaneously decide their productions quantities, q^i, and the loan sizes, B^i, if necessary, in a Nash game. The suppliers can borrow money from either the bank or the assembler, if buyer finance is offered. At the end of the time period, demand is realized. 4) The assembler orders components from its suppliers and assembles them into end products to meet the demand. We solve the game backwards.

3 Equilibrium in Different Financing Schemes

This section starts with no finance as a benchmark. We then solve the game for each individual financial scheme, bank or buyer finance, respectively, and compare the performance with no finance.

3.1 No Finance: The Benchmark

When there is no finance option, the sequence of event is similar to the one described in Figure 1, except that there is not finance decision for the assembler and the suppliers. We solve this game backwards. The expected profit of supplier i is

$$\begin{aligned} \max_{q^i} \quad & \pi_n^i = \mathbf{E}_D[w^i \cdot min(q^i, D) - c^i(1+r_s^i)q^i], \\ s.t. \quad & c^i q^i \leq k^i. \end{aligned}$$

It is easy to prove that $q_c^i = \min\{\bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}), \frac{k^i}{c^i}\}$ is the optimal constrained production quantity for supplier i.

Because of the assembly structure, the assembler will not purchase more than the minimum quantity of all component suppliers' stock levels. Expecting this, no supplier will produce more than the minimum production quantity of other suppliers. We have the following lemma regarding the simultaneous decision by all suppliers given the assembler's decisions.

Lemma 1 *Given the wholesale prices offered by the assembler ($w^i,\ i = 1, \cdots, N$), any $q_1 = q_2 = ... = q_N \in [0, \min_i\{q_c^i, i = 1, \cdots, N\}]$ is a Nash equilibrium and $q_1 = q_2 = ... = q_N = \min_i\{q_c^i, i = 1, \cdots, N\}$ is the unique Pareto-optimal Nash equilibrium.*

The proof of Lemma 1 is straightforward and omitted. Throughout this paper, we consider all component suppliers will choose the unique Pareto-optimal Nash equilibrium.

Given the suppliers' best response functions, the assembler will set the wholesale prices, $\vec{w} = (w_1, w_2, \cdots, w_N)$, to maximize its own expected profit:

$$\Pi_n(\vec{w}) = \mathbf{E}_D\ [(p - \Sigma_{i=1}^N w^i) \cdot min(D, q)],$$

where $q = \min\{q^i(\vec{w}), i = 1, ..., N\}$ is the minimum system stock level and $p - \Sigma_{i=1}^N w^i$ is the unit profit margin for the assembler's end-product. We have the following result for the assembler's equilibrium solution.

Lemma 2 *In equilibrium, the assembler sets $\vec{w}$ such that the system production quantity, q, satisfies the following equation.*

$$q = \bar{F}^{-1}(\frac{c^1(1+r_s^1)}{w^1}) = \bar{F}^{-1}(\frac{c^2(1+r_s^2)}{w^2}) = ... = \bar{F}^{-1}(\frac{c^N(1+r_s^N)}{w^N}) \leq \min_i\{\frac{k^i}{c^i}, i = 1, 2, ..., N\}. \quad (1)$$

Lemma 2 states that the assembler sets the wholesale price for each supplier in such a way that regardless of whether the supplier's capital constraint is tight or not, q^i is always the corresponding newsvendor production quantity, and moreover, q^i is equal to q for all suppliers. Lemma 2 allows us to transfer the assembler's decision from wholesale prices $\vec{w}$ into a single quantity q.

From Lemmas 1 and 2, we can express the component purchasing price for supplier i as a function of q, that is, $w^i = \frac{c^i(1+r_s^i)}{\bar{F}(q)}$. The assembler's optimization problem can thus be transferred to the following:

$$\Pi_n(q) = \mathbf{E}_D\ [(p - \frac{\Sigma_{i=1}^N c^i(1+r_s^i)}{\bar{F}(q)}) \cdot min(q, D)],$$

$$s.t.\ q \leq min_i\{\frac{k^i}{c^i}\}.$$

We can then prove the following lemma.

Lemma 3 $\Pi_n(q)$ *is concave in q and the equilibrium solution, q_n^*, is unique.*

3.2 Bank Finance

Given bank finance is viable, supplier i may borrow a loan B^i from a bank if it wants to produce a quantity of q^i more than what its initial capital allows. In line with the literature (see, e.g., Xu and Birge, 2004; Kouvelis and Zhao, 2012; Cai et al., 2013), supplier i assumes limited liability; that is, at the end of the season, it collects the revenue and is obliged to pay the loan principal and interest, $B^i(1 + r_{bk}^i)$, up to the amount of cash available, where r_{bk}^i is the real interest rate charged in bank finance. We solve the game backwards.

3.2.1 Suppliers' Best Response

The decision problem of supplier i is formulated as

$$\begin{aligned} max_{\{q^i,B^i\}} \quad & \pi^i_{bk} = \mathbf{E}_D \left[w^i min(q^i, D) + (k^i + B^i - c^i q^i)(1 + r^i_s) - \right. \\ & \left. min(w^i \cdot min(q^i, D) + (k^i + B^i - c^i q^i)(1 + r^i_s), B^i(1 + r^i_{bk}))\right], \\ s.t. \quad & c^i q^i \leq k^i + B^i. \end{aligned}$$

The first term in supplier i's objective function represents the wholesale revenue, the second term is the financial return of investing excess capital in alternative projects, and the last term is the expected value of loan payment to the bank.

Recall that bank loans are competitively priced in bank finance. The risk-neutral bank decides the interest rate r^i_{bk}, expecting to earn a risk-free rate r_f, such that,

$$B^i(1 + r_f) = \mathbf{E}_D \left[min(w^i \cdot min(q^i, D) + (k^i + B^i - c^i q^i)(1 + r^i_s), B^i(1 + r^i_{bk}))\right]. \tag{2}$$

With Eq. (2), the objective function of supplier i can be simplified as,

$$max_{\{q^i,B^i\}} \; \pi^i_{bk} = \mathbf{E}_D \left[w^i min(q^i, D) + (k^i + B^i - c^i q^i)(1 + r^i_s) - B^i(1 + r_f)\right].$$

Note that the assumption of $r^i_s \leq r_f$ ensures that supplier i always uses its own capital first and does not use the bank loan for arbitrage. Therefore, the borrowing amount of the supplier will not exceed its capital shortage for producing the quantity q (i.e., $B^i = (c^i q^i - k^i)^+$). Given w^i, we obtain the optimal production quantity for any supplier as follows.

Lemma 4 *In bank finance, supplier i's optimal production quantity for any given w^i is*

$$q^i = G^i_k(w^i) = \begin{cases} \bar{F}^{-1}\left(\frac{c^i(1+r^i_s)}{w^i}\right) & , \quad w^i < w^i_{(1)}, \\ \frac{k^i}{c^i} & , \quad w^i_{(1)} \leq w^i < w^i_{(2)}, \\ \bar{F}^{-1}\left(\frac{c^i \cdot (1+r_f)}{w^i}\right) & , \quad w^i \geq w^i_{(2)}, \end{cases} \tag{3}$$

where $w^i_{(1)} = c^i(1 + r^i_s)/\bar{F}(\frac{k^i}{c^i})$ and $w^i_{(2)} = c^i(1 + r_f)/\bar{F}(\frac{k^i}{c^i})$.

The supplier's optimal production quantity is increasing in w^i when $w^i \in (c^i(1 + r^i_s), w^i_{(1)}) \cup [w^i_{(2)}, \infty)$, but becomes flat when w^i is in the interval $[w^i_{(1)}, w^i_{(2)})$. When $w^i < w^i_{(1)}$, the supplier does not borrow money and her marginal cost is c. When w^i is in the interval $[w^i_{(1)}, w^i_{(2)})$, marginal increase in the component purchasing price does not provide an incentive for the supplier to increase its production quantity, because there is a jump in the supplier's marginal cost if it borrows money. When $w^i \geq w^i_{(2)}$, the component purchasing price is sufficiently high and the supplier starts borrowing. Its marginal cost then becomes $c^i(1 + r_f)$.

Given the best response of each supplier, we can prove that there is a unique Pareto-optimal Nash equilibrium, in which all component suppliers produce the same quantity, $q = \min\{G^i_k(w^i)\}$

for all $i = 1, \cdots, N$ given w^i, $i = 1, \cdots, N$. The proof follows the same argument for Lemma 1.

3.2.2 Assembler's Decision

We now study the assembler's decision in bank finance. The following lemma characterizes the assembler's optimal component purchasing prices.

Lemma 5 *In bank finance, the assembler sets the optimal component purchasing prices, w^{i*}, such that*

$$q^{i*} = G_k^i(w^{i*}) = q^*, \forall i,$$

where q^* is the equilibrium system stock level, q^{i*} is the equilibrium production quantity for supplier i, and G_k^i takes the form of Eq. (3).

The proof follows a similar argument in Lemma 2. Note that the supplier's response curve, $G_k^i(w^i)$, has a flat region, within which the supplier chooses the same q^i. Because of the flat region, the normal inverse function does not exist. However, in this region it is optimal for the assembler to offer the minimum w^i if q^i is the same. Thus, we can define a generalized inverse function $w^i = {G_k^i}^{-1}(q^i) = \inf\{w^i : G_k^i(w^i) \geq q^i\}$. ${G_k^i}^{-1}(q^i)$ returns the minimum value of w^i at which $G_k^i(w^i)$ does not exceed q^i. ${G_k^i}^{-1}(q^i)$ provides a one-to-one mapping between w^i and q^i. Based on Lemma 5, we have $q^i = q$ for all i. We can then simplify the assembler's decisions $(w^i, i = 1, \cdots, N)$ as a single variable q.

Without loss of generality, we assume that component suppliers are ordered by initial capital, $\frac{k^i}{c^i} \leq \frac{k^{i+1}}{c^{i+1}}$, $i = 1, \cdots, N-1$. If the assembler chooses system stock level q such that $\frac{k^i}{c^i} < q \leq \frac{k^{i+1}}{c^{i+1}}$, then supplier j $(j \leq i)$ needs to borrow money (from a bank) in order to produce the quantity of q, while supplier m $(m \geq i+1)$ can produce units of q using its own working capital without external financing. Given the supplier's best response, the assembler's decision problem is equivalent to

$$\max_{q \geq 0} \Pi_{bk} = \mathbf{E}_D\{p - \frac{\sum_{i=1}^N c^i(1 + r_s^i + (r_f - r_s^i) \cdot \delta(c^i q - k^i))}{\bar{F}(q)}\} \cdot min(q, D), \tag{4}$$

where

$$\delta(x) = \begin{cases} 0 & , \quad if\ x \leq 0, \\ 1 & , \quad if\ x > 0. \end{cases}$$

We use Figure 2 to illustrate the assembler's expected profit as a function of the minimum system stock level, q. We can see that the assembler expected profit, Π_{bk}, is piece-wise concave and discontinuous at $q = \frac{k^i}{c^i}$ for $i = 1, \cdots, N$. Note that if $r_s^i = r_f$ for supplier i, the assembler's profit function in bank finance becomes continuous at the point, k_i/c_i.

Let $\Pi_{bk}^{+}(q)$ and $\Pi_{bk}^{-}(q)$ denote the left-hand and right-hand limits of $\Pi_{bk}(q)$, respectively. We have the following Theorem 1 to characterize Π_{bk}.

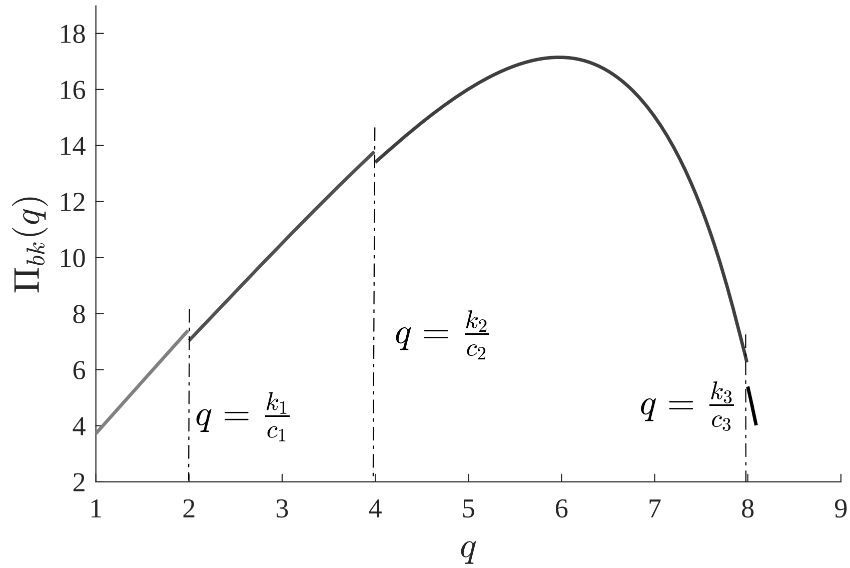


Figure 2: Assembler's Profit with Bank Finance

Theorem 1 *In bank finance, the assembler's expected profit, Π_{bk}, has the following properties:*

1. *Π_{bk} is discontinuous and piece-wise concave in $q \in [0, \infty)$, and continuous and concave in q in the subset of $q \in (\frac{k^i}{c^i}, \frac{k^{i+1}}{c^{i+1}}]$ for any i;*
2. *$\Pi_{bk}^{+}(q) < \Pi_{bk}^{-}(q)$ at the breakpoints $q = \frac{k^i}{c^i}$ for $i = 1, \cdots, N$, where "+" and "−" mean the right and left limit;*
3. *Π_{bk} is left-differentiable and its left derivative satisfies $\frac{\partial_{-}\Pi_{bk}}{\partial q}|_{q=a} > \frac{\partial_{-}\Pi_{bk}}{\partial q}|_{q=b}$ for any $b > a$.*

Theorem 1(3) indicates that the left derivative of $\Pi_{bk}(q)$ is decreasing in q. We can check the left derivative of $\Pi_{bk}(q)$ at each break point and stop at the first break point that has a non-positive left derivative of $\Pi_{bk}(q)$. Suppose that this break point is b_k. The maximum of $\Pi_{bk}(q)$ cannot occur beyond b_k, because $\Pi_{bk}^{+}(q) < \Pi_{bk}^{-}(q)$ at each break point and the derivative $\frac{\partial_{-}\Pi_{bk}}{\partial q} < 0$ for all $q > b_k$. Therefore, we have the following theorem.

Theorem 2 *Suppose b_k is the first break point which has a non-positive left derivative. The maximum of $\Pi_{bk}(q)$ can only occur at one of the break points smaller than b_k or the local maximum in the interval containing b_k.*

Theorem 2 characterizes the assembler's equilibrium solution, q_{bk}^{*}. Note that q_{bk}^{*} may not be unique. Nevertheless, we are still able to provide the following analytical results on how model parameters affect the equilibrium production quantity and the assembler's profit in bank finance. (The proofs are provided in the online supplement.) First, both the equilibrium production quantity q^*_{bk} and the associated assembler profit $\Pi_{bk}(q^*_{bk})$ are decreasing in the production cost, c^i, and the bank interest rate, r_f, but increasing in the retail price p. Second, $\Pi_{bk}(q^*_{bk})$ is increasing in k^i, but q^*_{bk} may not be monotonic in k^i, for $i = 1, 2, ..., N$. The non-monotonicity is due to the discontinuity of the assembler's profit function in q_{bk} at the break point, k_i/c_i.

Comparing bank finance to no finance, we have the following results.

Corollary 1 *The equilibrium in bank finance is Pareto better than that in no finance. Furthermore, if $q^*_{bk} > \min_i\{\frac{k^i}{c^i}\}$, the equilibrium in bank finance is strictly Pareto better than that in no finance.*

It is easy to see that the assembler will not be worse off in bank finance, because the assembler is Stackelberg leader and sets the wholesale prices at its discretion. Pricing the components at the same wholesale prices in no finance is a feasible solution to the assembler in bank finance. However, it is not so intuitive that the suppliers as the followers will always get no worse off. It is easy to show that the components prices (w_i) in bank finance are always no lower than those in no finance. Given a higher w_i, each supplier still has the choice of not borrowing loan from the bank. Therefore, all suppliers are at least not worse off. If any supplier starts borrowing, it must strictly benefit from bank finance.

3.3 Buyer Finance

If the assembler offers buyer finance, suppliers have a new option of borrowing loans from the assembler in addition to bank finance. The expected rate of return from buyer finance, r_b, is a strategic decision to be made jointly with the component purchasing prices to maximize the assembler's total expected profit. Each supplier i's respective interest rate, r^i_{br}, is then determined to achieve the expected rate of return, r_b, following the following equation:

$$B^i(1 + r^i_b) = E_D\ [min\{w^i \cdot min(q^i, D) + (k^i + B^i - c^i q^i)(1 + r^i_s), B^i(1 + r^i_{br})\}]. \tag{5}$$

Because suppliers assume limited liability, the loan payment from supplier i in the RHS of Eq. (5) is the minimum of the supplier's wholesale revenue and the loan principal plus interest. It is easy to see that for any supplier i, there is a one-to-one relationship between r^i_{br} and r_b given other parameters fixed, and if $r_{b1} \geq r_{b2}$, then $r^i_{br1} \geq r^i_{br2}$, where r^i_{brj} is the real interest rate corresponding to the expected rate of return, r_{bj} for $j = 1, 2$. Therefore, given w^i, if r_b in buyer finance is less than the bank risk-free interest rate r_f, then the corresponding real interest is also less than that in bank finance, and the supplier will choose buyer finance at any q^i.

3.3.1 Suppliers' Best Response

Using Eq. (5), we can simplify the supplier's objective function, and it is the same as that in bank finance except that r_f is replaced by r_b, which is determined by the assembler to maximize its expected profit. Similar to Lemma 4 for bank finance, we can obtain supplier i's production quantity, q^i, in buyer finance in terms of r_b as follows.

$$q^i = G_r^i(r_b, w^i) = \begin{cases} \bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}) & , \quad w^i < w_{(1)}^i, \\ \frac{k^i}{c^i} & , \quad w_{(1)}^i \le w^i < w_{(2)}^i, \\ \bar{F}^{-1}(\frac{c^i \cdot (1+r_b)}{w^i}) & , \quad w^i \ge w_{(2)}^i, \end{cases} \tag{6}$$

where $w_{(1)}^i = \frac{c^i(1+r_s^i)}{\bar{F}(\frac{k^i}{c^i})}$ and $w_{(2)}^i = \frac{c^i(1+r_b)}{\bar{F}(\frac{k^i}{c^i})}$. Similar to the argument for suppliers' best response in bank finance, we can show that given w^i and r_b, there is a unique Pareto-optimal Nash equilibrium, in which all component suppliers produces the same quantity, $q^i = q = \min\{G_r^i(r_b, w^i),\ i = 1, \cdots, N\}$, where q is the minimum system stock level.

3.3.2 Assembler's Decision

Based on the suppliers' best response, the assembler's optimal component purchasing prices in the first stage satisfy the following lemma.

Lemma 6 *In buyer finance, the assembler sets the equilibrium component purchasing prices, w_i^*, such that $q^* = G_r^i(r_b, w_i^*)$ for all i, where q^* is the equilibrium minimum system stock level and G_r^i takes the form of Eq. (6).*

Similar to the case of bank finance, we define a generalized reverse function of G_r^i for any given r_b as ${G_r^i}^{-1}(r_b, q^i) = \inf\{w^i : \ G_r^i(r_b, w^i) \ge q^i\}$. ${G_r^i}^{-1}$ provides a one-to-one mapping between (w^i, r_b) and (q^i, r_b). Therefore, we can transform the assembler's decision variables from $(r_b, w^1, \cdots, w^N)$ to $(r_b, q^1, \cdots, q^N)$. Based on Lemma 6, we can simplify the assembler's decisions to only two variables (r_b, q). As a result, the assembler's problem is equivalent to the following:

$$\begin{aligned} \max_{\{q, r_b\}} \Pi_{br} \quad &= \quad [p - \frac{\sum_{i=1}^N c^i(1+r_s^i) + (r_b - r_s^i) \cdot \delta(c^i q - k^i))}{\bar{F}(q)}] \cdot \mathbf{E}_D min(q, D) \\ &+ \sum_{i=1}^N (r_b - r_a) \cdot \delta(c^i q - k^i) \cdot (c^i q - k^i). \end{aligned}$$

$$s.t. \begin{cases} q \ge 0, \\ r_b \ge r_s^i, \ \forall i \in \{1, 2, \cdots, N\}. \end{cases}$$

The first term in the above objective function is the assembler's revenue. The second term represents financial interest earnings in buyer finance. $\delta(x)$ is the indicator function defined in Eq.

(4). The second set of constraints are to prevent the suppliers from arbitraging the loan from the buyer.

If the assembler chooses the system stock level q in the range of $\frac{k^j}{c^j} < q \leq \frac{k_{j+1}}{c_{j+1}}$, after rearranging terms in the assembler's objective function, we have

$$\Pi_{br} = \sum_{i=1}^{j}[(c^i q - k^i) - \frac{c^i}{\bar{F}(q)} \cdot \mathbf{E}_D min(q, D)] \cdot (r_b - r_s^i) \tag{7}$$

$$+(p - \frac{\sum_{i=1}^{N} c^i(1 + r_s^i)}{\bar{F}(q)}) \cdot \mathbf{E}_D min(q, D) - \sum_{i=1}^{j}(r_a - r_s^i)(c^i q - k^i). \tag{8}$$

Solving the assembler's problem leads to the following result.

Theorem 3 *In buyer finance, the assembler should charge an interest rate as low as possible for any given q. To ensure all suppliers using up its own capital first, the assembler should set the equilibrium $r_b^* = \max_i\{r_s^i\}$.*

Note that $r_s^i \leq r_f$ for all $i = 1, ..., N$. We have $r_b^* = \max_i\{r_s^i\} \leq r_f$. The equilibrium interest rate in buyer finance is lower than bank risk-free interest rate. Theorem 3 is consistent with some anecdotal evidences that firms may lend to suppliers at rates equal to or strictly lower than bank interest rate (see, e.g., Tang et al., 2015). Therefore, given the equilibrium component purchasing prices, the suppliers will choose to borrow loans from the assembler instead of banks, if financing is needed.

As we have stated previously, the assembler's unit capital opportunity cost, r_a, can be either greater or lower than r_f. It could also be higher than $\max_i\{r_s^i\}$ (e.g., $r_a > r_f \geq r_s^i$ for all i), in which case the assembler incurs cost in financing suppliers in the equilibrium solution. One might wonder what is the benefit for the assembler to set r_b^* even below its own unit capital opportunity cost, r_a.

A key trade off behind Theorem 3 is between the assembler's financial cost and sales increase due to supply chain operations improvement. On the one hand, charging a lower interest rate causes a bigger financing deficit to the assembler (*interest cost effect*). On the other hand, with the lower interest rate, the assembler has a larger room to command lower component prices in buyer finance, resulting in higher profit margins (*component price effect*). Meanwhile, the low interest rate reduces the suppliers' production cost and provides them incentives for a higher component production level, leading to a higher sales volume (*production level effect*). The double benefits from component price effect and production level effect make the increases in sales revenue outpace the interest costs of financing the suppliers. Therefore, it is better for the assembler to charge a lower interest rate, as long as no supplier arbitrages from buyer finance.

These benefits show that in buyer finance the assembler can better integrate financing and operations decisions to achieve a greater profit. We refer to such benefits in buyer finance as the benefits of finance and operations integration.

Given the optimal $r_b = \max_i\{r_s^i\}$, substituting r_b into Eq. (7). The assembler expected profit is a function of the minimum system stock level q. Note that the first term in Π_{br} is piecewise concave similar to the term in bank finance. The second term is a piecewise linear function. Therefore, Π_{br} is still a piecewise concave function. To find the equilibrium solution, we can first compute the local optimal solutions in all intervals, and choose the one that yields the highest expected profit as the equilibrium q for buyer finance.

If the unit capital opportunity costs of the assembler and the suppliers are the same, the assembler's objective function becomes a smooth concave function (i.e., all the indicator functions vanish). We have the following results.

Corollary 2 *If $r_s^i = r_a$ for all $i = 1, 2, \cdots, N$ and the distribution of demand is IFR, the optimal system stock level in buyer finance, q_{br}^*, is the unique solution to the following first-order condition.*

$$p\bar{F}(q) = \sum_{i=1}^{N} c^i(1 + r_s^i)\left(\frac{f(q)}{[\bar{F}(q)]^2}\int_0^q \bar{F}(x)dx + 1\right).$$

We can show that the impacts of suppliers' costs, assembler's unit capital opportunity cost, and product price on the equilibrium q_{br}^* and $\Pi_{br}(q_{br}^*)$ are similar to those in bank finance. (The proofs are provided in the online supplement). The impact of suppliers' capital on $\Pi_{br}(q_{br}^*)$, however, can be opposite depending on the relative efficiency of the assembler and suppliers in borrowing/lending money (i.e., unit capital opportunity costs).

Corollary 3 *In buyer finance, if $r_a \geq \max_i\{r_s^i\}$, $\Pi_{br}(q_{br}^*)$ are increasing in the supplier i's capital $k^i, i = 1, ..., N$; otherwise, $\Pi_{br}(q_{br}^*)$ can be decreasing in $k^i, i = 1, ..., N$;*

This result is different from that from bank finance, in which the assembler always benefits from the increase of k_i. If the suppliers have the same unit capital opportunity cost (i.e., $r_s^1 = r_s^2 = ... = r_s^N = r_s$), the optimal interest rate in buyer finance is $r_b = \max_i\{r_s^i\} = r_s$ in this case. When $r_a \geq r_s$, the assembler incurs a loss in financing suppliers. Therefore, the assembler's profit and the production quantity increase when suppliers are endowed with more initial capital. When $r_a < r_s$, however, the assembler earns a positive interest return from buyer finance. The less initial capital, the more financing demand from the suppliers, the more interest earnings for the assembler. Therefore, the relative efficiency of the assembler and suppliers in borrowing/lending money (i.e., r_a and r_s^i) play an important role in buyer finance.

We next compare buyer finance with no finance and have the following result, which shares similar intuition to the one for Corollary 1.

Corollary 4 *The equilibrium in buyer finance is Pareto better than that in no finance. Furthermore, if $q_{br}^* > \min_i\{\frac{k^i}{c^i}\}$, the equilibrium in buyer finance is strictly Pareto better than that in no finance.*

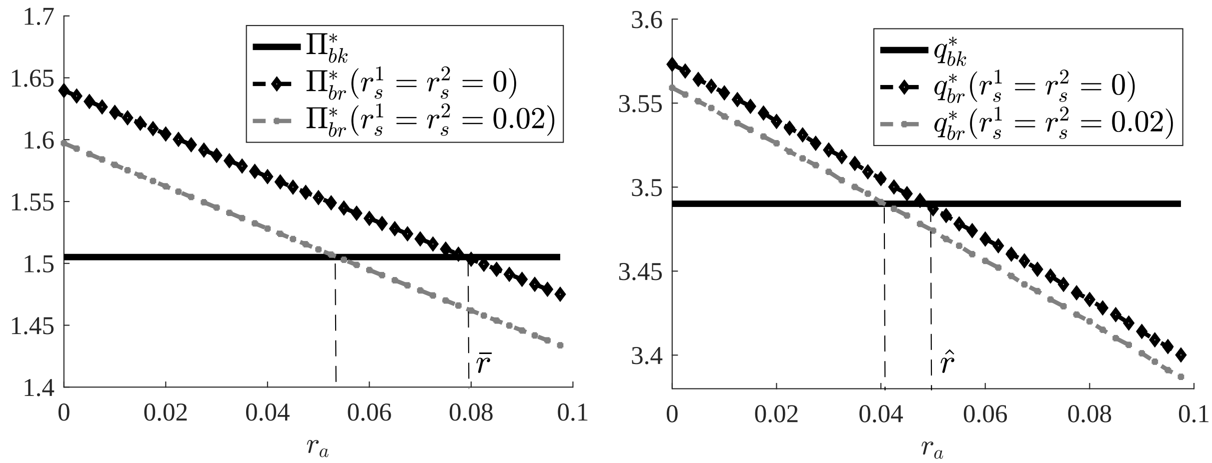
4 Assembler's Choice with Financing Options

We have so far characterized the equilibrium solution for bank finance only and buyer finance (with bank finance), respectively, in Section 3. However, as the initiator of buyer finance, should the assembler offer buyer finance or simply let suppliers borrow money from banks? We have the following sufficient and necessary condition for the assembler's choice of buyer finance vs. bank finance.

Theorem 4 *There exists $\bar{r} > r_f$, such that the assembler prefers buyer finance to bank finance if and only if $r_a \leq \bar{r}$.*

Theorem 4 indicates that the assembler's choice of offering buyer finance follows a threshold policy with regard to its unit capital opportunity cost. It should offer buyer finance if and only if its unit capital opportunity cost is lower than the threshold (i.e., $r_a \leq \bar{r}$). The tradeoff behind this is that, on the one hand, in bank finance, the assembler leverages banks' money and enjoys a free-ride from rising component production level financed by banks. The bank interest rates and the corresponding financing costs are, thus, exogenous to the assembler. On the other hand, in buyer finance, the assembler has a full control on buyer finance interest rate. It can set the interest rate jointly with wholesale prices and enjoy the benefit of finance and operations integration, although it must concurrently bear the capital costs of financing suppliers. As discussed in Subsection 3.3.2, when r_a increases, the assembler's cost of financing its suppliers increases. The benefit of finance and operations integration diminishes in comparison with bank finance. Therefore, the superiority of buyer finance critically depends on r_a and follows a threshold policy.

Interestingly, the threshold value, $\bar{r}$, can be strictly greater than r_f (also illustrated in Figure 3(a)). When r_a is in the range between r_f and $\bar{r}$, the assembler's unit capital opportunity cost is indeed higher than banks' risk-free interest rate. The reason why the assembler still prefers buyer finance is because the benefit of finance and operations integration is still so large that it overshadows the financial cost.



(a) Effect of r_a on Π^*_{br} (b) Effect of r_a on q^*_{br}

Figure 3: Impact of r_a on the Strategy Preference of the Assembler

Compared with bank finance, buyer finance also serves as a better risk-sharing mechanism. Recall that the suppliers have to invest in their inventory before demand uncertainty is resolved. Thus, the suppliers have to take more risk for producing more. The risk is better mitigated by the lower interest rate in buyer finance than in bank finance. However, as r_a increases to a higher level, the assembler incurs a larger financial cost in buyer finance and has to lower component wholesale prices (*component price effect*), which deters the suppliers from producing more components for the assembler. Thus, the risk sharing benefit starts diminishing.

It is worth noting that in Figure 3(b), at $r_a = 0.06$ and $r_s^i = 0$, the suppliers actually produce a quantity even less than that in bank finance (*negative production level effect*). Nevertheless, the assembler still prefers buyer finance to bank finance (as shown in Figure 3(a) at $r_a = 0.06$). This observation reveals a subtlety regarding the relation between the preference of finance scheme and production level. Recall that the lower interest rate in buyer finance leads to two effects: production level effect and component price effect. Although in this situation the assembler can no longer benefit from the (negative) production level effect, the advantage of lower component prices is so conspicuous that the assembler's additional profit margin eclipses the disadvantage of negative production level effect and high financial cost in buyer finance.

As r_a keeps increasing to a very high level (e.g., $r_a > \bar{r}$), the assembler's financial cost, in addition to the negative production level effect, will surpass the relative advantage of higher profit margin. As stated in Theorem 4, the assembler starts preferring bank finance to buyer finance.

Due to the complicated interaction between model parameters in buyer financing, the threshold $\bar{r}$ cannot be expressed explicitly in general. Nevertheless, we can provide a tight lower bound on $\bar{r}$.

Corollary 5 *There exists a lower bound of $\bar{r}$, $\bar{\bar{r}} = \frac{(r_f - \max_i\{r_s^i\}) \cdot E_D min(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)(q_{bk}^* - \frac{\Sigma k^i}{\Sigma c^i})} + \max_i\{r_s^i\}$, such that if $r_a < \bar{\bar{r}}$ the assembler will choose buyer finance. The lower bound is tight because we have $\bar{\bar{r}} = \bar{r}$ if and only if $\max_i r_s^i = r_f$.*

Note that the assembler's selection of buyer or bank finance depends on important parameters in the supply chain such as the supplier's initial capitals and production costs. We next address the impact of these factors on the assembler's preference on financing schemes.

4.1 Impact of Suppliers' Initial Capitals

This subsection discusses how the suppliers' total initial capital and the heterogeneity of initial capital across the suppliers affect the assembler's financing decision.

Impact of Suppliers' Total Initial Capital

Define $K \equiv \Sigma_{i=1}^{N} k^i$ as the total initial capital of all suppliers. Due to the complexity of this equilibrium problem, it is very difficult to obtain analytical results for general cases. However, we can prove the following results for a special case in which the suppliers have the same $\frac{k^i}{c^i}$ (i.e., using only their initial capitals, the suppliers can produce components up to the same inventory level).

Corollary 6 *If all component suppliers are capital constrained (i.e., $\frac{k^i}{c^i} < q_{bk}^*$) and $\frac{k^i}{c^i} = \frac{k^j}{c^j}$ for any $i, j = 1, 2, \cdots, N$, we have*

1. *$\bar{r}$ increases with K;*
2. *If $r_a \geq \max_i\{r_s^i\}$, $\Pi_{br}(q_{br}^*) - \Pi_{bk}(q_{bk}^*)$ increases with K; otherwise, $\Pi_{br}(q_{br}^*) - \Pi_{bk}(q_{bk}^*)$ decreases with K;*
3. *$\bar{r}$ decreases with $\max_i\{r_s^i\}$.*

Corollary 6(1) reveals that when K gets larger, an assembler with a higher unit capital opportunity cost can also benefit from buyer finance. This result occurs because the loan size and the financial cost in buyer finance decreases as the total initial capital gets higher. Corollary 6(2) describes how the benefit of offering buyer finance in comparison to bank finance changes in K. It depends on the relative magnitude of r_a with respect to $max_i\{r_s^i\}$. If $r_a \geq max_i\{r_s^i\}$, the assembler loses money for financing suppliers. Therefore, the higher K, the smaller loan size, the less financial cost in buyer finance, the larger benefit from buyer finance. If $r_a < max_i\{r_s^i\}$, the assembler actually earns a positive interest gain in buyer finance; therefore, the opposite is true. Corollary 6(3) indicates that a higher value of $max_i\{r_s^i\}$ reduces the production level and subsequently undermines the benefit of buyer finance. Financially stronger suppliers actually diminish the value of buyer finance.

Impact of Suppliers' Capital Heterogeneity

In this subsection, we keep the total capital, K, fixed and study how the heterogeneity in suppliers' initial capital affects the assembler's profit. To simplify analysis, we consider the assembler has only two component suppliers. We define the ratio, $\theta \equiv \frac{k^1}{k^2}$, as a measure of the suppliers' capital heterogeneity. We have $\theta \in [0, 1]$. If θ is closer to 1, the suppliers' capitals are more homogeneous; otherwise, more heterogenous. To single out the impact of capital heterogeneity, we assume all other attributes of the two component suppliers are the same (e.g., suppliers have the same unit production cost and unit capital opportunity cost). We can prove the following results.

Theorem 5 *1. In bank finance, $\Pi^*_{bk}(q^*_{bk})$ decreases with θ if $K < q^*_{bk}\Sigma^2_{i=1}c^i$, but increases with θ otherwise.*

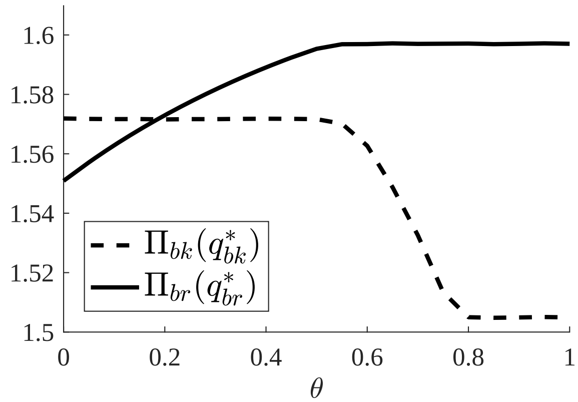
2. In buyer finance, $\Pi^*_{br}(q^*_{br})$ increases with θ if $r_a \geq \max_i\{r^i_s\}$, but decreases with θ otherwise.

Theorem 5(1) says that in bank finance the impact of supplier capital heterogeneity on the assembler's profit actually depends on the total capital level. Figures 4 (a) and (b) show the assembler's expected profits as a function of θ in bank finance (the dash lines) and in buyer finance (the solid lines) for a low level of total initial capital ($K = 2.68$) and a high level of total initial capital ($K = 5.36$), respectively, in the case where $r_a \geq \max_i\{r^i_s\}$.

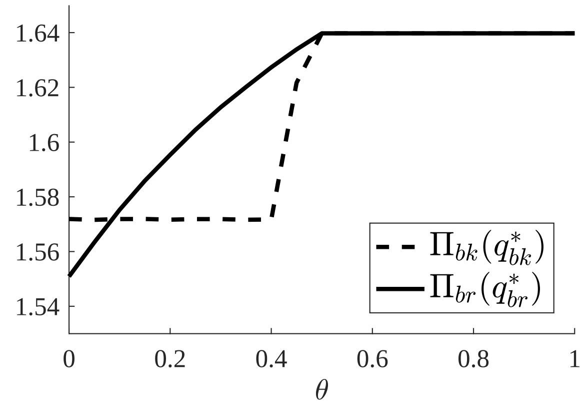
When the aggregated capital level is low (Figure 4(a)), the assembler's profit is decreasing in θ. This is because if the total initial capital is evenly distributed among the two suppliers, neither supplier have enough fund for producing up to the optimal inventory level. Therefore, both suppliers have to borrow money from banks and incur financial costs for two components. However, if the suppliers have heterogeneous initial capital, there may be only one supplier who needs to borrow money from banks and incurs a financial cost for just one component. Therefore, when the aggregated capital level is low, the heterogeneity in initial capitals gives rise to savings on total unit production and financing costs, and thus benefits the assembler.

On the contrary, when the total initial capital level is sufficiently high (Figure 4(b)), the assembler's profit is increasing in θ. The assembler gets more profit because the component suppliers have homogenous initial capital so that no one needs to borrow money from banks.

In buyer finance, Theorem 5(2) says that the assembler's preference over heterogeneity depend on the relative magnitude of r_a and $\max_i\{r^i_s\}$. If $r_a \geq \max_i\{r^i_s\}$, the assembler prefers capital-homogeneous suppliers; otherwise, it prefers capital-heterogeneous suppliers. The rationale behind is that in buyer finance, the assembler bears the financial cost for financing suppliers when $r_a \geq \max_i\{r^i_s\}$. If the initial capital is evenly distributed, the total loan amount is minimized and so



(a) $K = 2.68$



(b) $K = 5.36$

Figure 4: *Buyer Finance vs. Bank Finance with Varying Capital Heterogeneity*

is the total financial interest cost. When $r_a < \max_i\{r_s^i\}$, the assembler earns a positive interest income; thus, it prefers suppliers with heterogeneous initial capital.

We next study the impact of the suppliers' capital heterogeneity on the assembler's financing decision. Figures 4(a) and (b) show the case in which $r_a \geq \max_i\{r_s^i\}$. In both figures, when the two suppliers have heterogeneous initial capital (i.e., θ is small), bank finance dominates buyer finance for the assembler. When the two suppliers have homogeneous initial capital (i.e., θ is close to 1), buyer finance (weakly) dominates bank finance for the assembler.

The reason is as follows. The total loan size required by heterogeneous suppliers in general tends to be larger than that required by homogeneous suppliers, because for heterogeneous suppliers, initial capital is under-utilized at one supplier but running short at the other supplier. If the total initial capital is evenly distributed, it is fully utilized and the total loan size required by the suppliers is minimized. Therefore, when $r_a \geq r_b^* = \max_i\{r_s^i\}$, the total cost of financing heterogenous suppliers is large in buyer finance and may exceed the benefit of finance and operation integration. It is better for the assembler to leverage banks' money instead of its own money. However, when suppliers have homogeneous initial capital and the total loan size is small, the assembler should offer buyer finance.

When $r_a < \max_i\{r_s^i\}$, the assembler gets positive interest earnings and always benefits from financing the suppliers. Note that $\max_i\{r_s^i\} \leq r_f \leq \bar{r}$. By Theorem 4, buyer finance dominates bank finance for all θ.

4.2 Impact of Suppliers' Production Cost

This subsection discusses how the total cost and the cost heterogeneity across suppliers affect the assembler's financing decision.

Impact of Total Production Cost

Because the production costs affect both operations costs and financial expenses, the equilibrium production quantity, the wholesale price and the loan size all depend on costs. Therefore, the interaction of costs with $\bar{r}$ and the assembler's expect profit is much more complicated than initial capitals. However, we can get the following relationship between the total cost and $\bar{\bar{r}}$, the tight lower bound of $\bar{r}$ developed in Corollary 5.

Corollary 7 *If $K = 0$, $\bar{\bar{r}}$ decreases in $\Sigma_{i=1}^N c^i$.*

We have also done thorough numerical studies and all our numerical results (omitted here due to limited space) show that both $\bar{r}$ and $\bar{\bar{r}}$ decrease as $\Sigma_{i=1}^N c^i$ increases, even if $\Sigma_{i=1}^N k^i \neq 0$. This observation implies that the assembler with a high unit capital opportunity cost is less likely to provide buyer finance as the total unit production cost increases. The reason is that a higher unit production cost not only raises the loan size, but also represses the production quantity, which subsequently escalates the assembler's financial risk in buyer finance and curtails its revenue.

Impact of Suppliers' Cost Heterogeneity

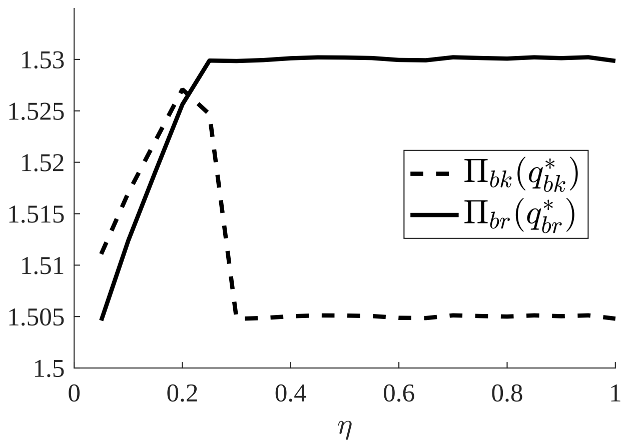
We now characterize the impact of suppliers' cost heterogeneity on the assembler's performance. Again, we consider an assembly system with two component suppliers. To measure the cost heterogeneity, we define $\eta \equiv \frac{c^2}{c^1}$, the ratio of supplier 2's unit cost over that of supplier 1, provided that $C \equiv \Sigma_{i=1}^2 c^i$ is fixed. Thus, $\eta \in [0, 1]$. If η is closer to 1, the suppliers' costs are more homogeneous; otherwise, more heterogenous. To single out the impact of cost heterogeneity, we assume suppliers have the same initial capital and the same unit capital opportunity cost. We have the following results.

Theorem 6 *1. In bank finance, $\Pi_{bk}^*(q_{bk}^*)$ is first increasing and then decreasing in η if $K < Cq_{bk}^*$, but it is always increasing in η if $K \geq Cq_{bk}^*$.*

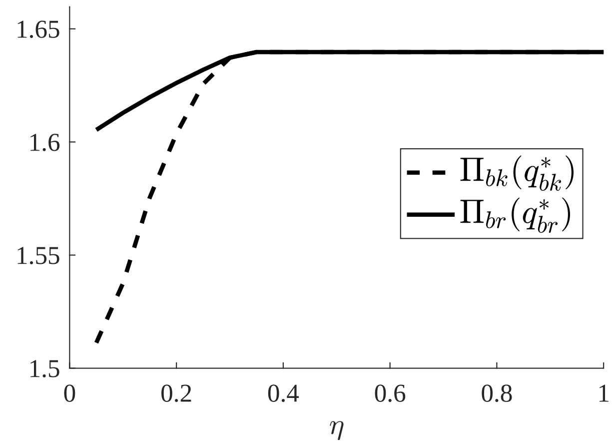
2. In buyer finance, $\Pi_{br}^*(q_{br}^*)$ is increasing in η if $r_a > \max_i\{r_s^i\}$, but decreasing in η if $r_a \leq \max_i\{r_s^i\}$.

The main thrust behind Theorem 6 is that given the same total initial capital for the two suppliers, if the suppliers have very different production costs, then the supplier with a larger cost can produce only a smaller quantity of component and needs a larger amount of loan (due to the larger cost) to build up inventory, while the other supplier may have excess capital left unused even if it can produce components up to the system optimal level. When the two suppliers have the same costs, they can produce up to the same amount of inventory and fully utilized the initial capital. Therefore, in general, the loan size required and therefore the total financial cost in the case of homogeneous costs are smaller than those in the case of heterogeneous costs.

Figures 5 (a) and (b) show the assembler's expected profits as a function of η in bank finance (the dash lines) and in buyer finance (the solid lines) for $K = 2.68$ and $K = 5.36$, respectively, in the case where $r_a \geq \max_i\{r_s^i\}$.



(a) $K = 2.68$



(b) $K = 5.36$

Figure 5: *Buyer Finance vs. Bank Finance with Varying Cost Heterogeneity*

In bank finance, the impact of cost heterogeneity on the assembler's profit depends on the total capital, K. When K is low, the assembler's profit (dash line in Figure 5(a) with $K = 2.68$) first increases and then decreases as η increases. If supplier' costs are more heterogeneous (η is small) , only the more costly supplier (i.e., Supplier 1) borrows money from banks. Its marginal cost is $c_1(1 + r_f)$. As η increases, the marginal cost decreases because c_1 becomes smaller. Therefore, the assembler profit is increasing in η. However, as η keeps increasing (i.e., the supplier's costs becomes more homogeneous), Supplier 2 also starts borrowing from banks and causes a production cost jump for Supplier 2 from c_2 to $c_2(1 + r_f)$. This jump explains why the assembler's expected profit drops sharply in the middle. After that, the assembler's expected profit then becomes relatively flat, because the total cost now is fixed at $(c_1 + c_2)(1 + r_f)$.

When the total capital level is high (Figure 5(b) with $K = 5.36$), the assembler does not like the suppliers' costs to be very different, because the supplier with a larger cost more likely needs to borrow money, even though the total capital is not constraining. In this case, the assembler's profit is always increasing in η (suppliers becomes more homogeneous).

In buyer finance, similarly, the loan size in general is larger when the degree of heterogeneity in suppliers' costs is higher. However, the advantage of buyer finance depends on not only the loan size but also the difference between r_a and $\max_i\{r_s^i\}$. If the difference is positive (i.e., $r_a \leq \max_i\{r_s^i\}$), the assembler is profitable in financing suppliers and prefers a larger loan size; otherwise, it loses money in financing the suppliers and, thus, prefers the loan size to be small. This explains why the assembler's preference is opposite depending on whether $r_b \geq \max_i\{r_s^i\}$.

Combining the above forces behind the impact of suppliers' cost heterogeneity on the assembler's financing decision, Figure 5 shows that in the case $r_a > \max_i\{r_s^i\}$, when the two suppliers are heterogeneous in cost (i.e., η is small), bank finance dominates buyer finance for the assembler. When the two suppliers are homogeneous in costs (i.e., η is close to 1), buyer finance dominates bank finance for the assembler.

If $r_a \leq \max_i\{r_s^i\}$, the assembler simply profits from financing the suppliers. Therefore, buyer finance is always preferred (by Theorem 4).

5 Preferences of Suppliers and Supply Chain

This section investigates the performance of the suppliers and the whole supply chain in bank finance and buyer finance.

5.1 Suppliers' Preference

There are two types of suppliers – those who need to borrow money for production (borrowing suppliers) and others who do not (non-borrowing suppliers). Our following result shows that these two groups of suppliers may have different preferences on bank finance and buyer finance.

Theorem 7 *There exist $\tilde{r}$ and $\hat{r}$, where $\hat{r} \in [r_f, \bar{r})$ and $\tilde{r} < \hat{r} < \bar{r}$, such that non-borrowing suppliers always prefer buyer finance to bank finance if and only if $r_a \leq \hat{r}$, and the borrowing suppliers prefer bank finance to buyer finance if and only if $r_a \geq \tilde{r}$.*

Theorem 7 implies that the suppliers may not benefit from buyer finance even though the assembler does. Moreover, the two types of suppliers can also have conflicting preferences. In particular, when $r_a < \bar{r}$, the assembler prefers buyer finance (Theorem 4). If $r_a < \tilde{r}$, all suppliers also prefer buyer finance. However, if $r_a \in (\tilde{r},\hat{r})$, (*borrowing* and *non-borrowing*) suppliers have different preferences: borrowing suppliers prefer bank finance, whereas non-borrowing suppliers prefer buyer finance. If $r_a \in (\hat{r}, \bar{r})$, all suppliers prefer bank finance.

The rationale behind the above phenomenon is explained as follows. Recall that r_b in buyer finance is lower than the bank interest rate, r_f. Hence, a borrowing supplier has a lower marginal financial cost in buyer finance (*financial saving effect)* than bank finance. But, the assembler has to reduce the component price (*component price effect*) to compensate for the lower interest rate. If the assembler's unit capital opportunity cost is low (i.e., $r_a < \tilde{r}$), the component price effect will be less than the financial saving effect. Consequently, the suppliers produce more in buyer finance than in bank finance. All suppliers benefit from buyer finance. However, if the assembler's unit capital opportunity cost is in a higher category (i.e., $r_a \in (\tilde{r},\hat{r})$), the component price effect surpasses the financial saving effect such that the borrowing suppliers suffer from a lower profit margin while non-borrowing suppliers enjoy a higher production quantity. Therefore, the two types of suppliers hold a conflicting preference. If r_a continues to grow higher (i.e., $r_a \in (\hat{r}, \bar{r})$), both wholesale price and production quantity start to decrease in buyer finance. In this case, all suppliers prefer bank finance.

Based on Theorem 4 and Theorem 7, we can immediately summarize firms' preferences into the following two Pareto zones and two conflicting areas.

Corollary 8 *1. [Buyer finance Pareto Zone] If $r_a \le \tilde{r}$, then all firms prefer buyer finance;*

2. [Borrowing-Supplier Confliction] If $\tilde{r} < r_a \le \hat{r}$, then all firms, except borrowing-suppliers, prefer buyer finance;

3. [Assembler-Supplier Confliction] If $\hat{r} < r_a \le \bar{r}$, then the assembler prefers buyer finance, whereas all suppliers prefer bank finance;

4. [Bank finance Pareto Zone] If $r_a > \bar{r}$, then all firms prefer bank finance.

Corollary 8 reveals that the suppliers may be at odds with the assembler on the preference over buyer finance. However, once the assembler decides to offer buyer finance, the suppliers have to stick with it. This is because, given the same wholesale prices controlled by the assembler as a leader, the interest rate in buyer finance is lower and, thus, more attractive for the suppliers than that in bank finance.

5.2 Supply Chain Efficiency

We now compare the performance of buyer and bank finance from the perspective of supply chain efficiency. Direct comparison of the sum of the assembler and suppliers' profits between the two financing schemes is not fair, because part of the profit in bank finance goes to the banks. Therefore, we extend the supply chain system to include the banks to form a closed system. Given that the retail price is exogenous, the comparison of production quantity is equivalent to that of total supply chain profit, which include the assembler, the suppliers, and the banks.

Because the expected total profit is increasing in q, provided that q is less than the centralized newsvendor optimal solution, we then use the equilibrium quantity q as a measure of supply chain efficiency (see, e.g., Kouvelis and Zhao, 2012). Comparing the optimal production quantity between bank and buyer finance leads to the following result.

Theorem 8 *Comparing the equilibrium quantities in the two financing schemes, we have*

$$\begin{cases} q^{*cen} > q^*_{br} \geq q^*_{bk} & , \quad if\ r_a \leq \hat{r}, \\ q^{*cen} > q^*_{bk} > q^*_{br} & , \quad if\ r_a > \hat{r}, \end{cases}$$

where $q^{*cen} = \bar{F}^{-1}(\frac{\Sigma c^i(1+r_s^i)}{p})$ *is the optimal stock quantity in a centralized supply chain and* $\hat{r} \in [\tilde{r}, \bar{r}]$ *as defined in Theorem 7.*

Theorem 8 indicates that the comparison of supply chain efficiency between the two financing schemes also follows a threshold policy. Because $\hat{r} < \bar{r}$, by Theorem 7, the assembler's incentive to choose buyer or bank finance is not aligned with that of the whole supply chain.

Note that $\hat{r}$ is also the indifference point of non-borrowing suppliers' preference between bank and buyer finance (Theorem 7). Theorem 8 thus delivers an interesting message that the non-borrowing supplier's preference is an indicator of the efficiency of the whole supply chain. This occurs because non-borrowing suppliers are not affected by external interest rates in either financing scheme and their expected profits only rely on the minimum system production quantity. Therefore, non-borrowing suppliers and the whole supply chain are indifferent between buyer finance and bank finance at $\hat{r}$, whereas the assembler prefers buyer finance but borrowing suppliers prefer bank finance.

In general, the expression of $\hat{r}$ is difficult to know, but for symmetric suppliers we can obtain a closed-form expression of $\hat{r}$ and further characterize it as below.

Corollary 9 *For symmetric component suppliers* ($\frac{k^i}{c^i} = \frac{k^j}{c^j}$ *for* $\forall i, j = 1, 2, \cdots, N$) *with constrained initial capital* ($\frac{k^i}{c^i} < q^*_{bk}$), *we have* $\hat{r} = r_f + \frac{f(q^*_{bk})(r_f - \max_i\{r_s^i\}) \cdot E_D min(q^*_{bk}, D)}{[\bar{F}(q^*_{bk})]^2}$ *and*

1. $\hat{r}$ *is independent of* $\Sigma_{i=1}^N k^i$;
2. $\hat{r}$ *decreases with* $\Sigma_{i=1}^N c^i$;
3. $\hat{r}$ *decreases with* $\max_i\{r_s^i\}$.

As shown in Theorem 8, the threshold, $\hat{r}$, characterizes how the assembler's unit capital opportunity cost (r_a) affects the relationship between q^*_{bk} and q^*_{br} ($q^*_{bk} > q^*_{br}$ or $q^*_{bk} \leq q^*_{br}$). Although the assembler's profit in buyer finance ($\Pi_{br}(q^*_{br})$) increases with suppliers' initial capital, the equilibrium quantities q^*_{br} and q^*_{bk} do not depend on $\Sigma_{i=1}^N k^i$ as long as all suppliers need borrow money.

Regarding Corollary 9(2), as the suppliers' production costs increase, the assembler has to pay higher component prices and all firms' profit margins reduce accordingly in both bank and buyer finance. However, in buyer finance, the suppliers' default risk increases (financial cost effect) too. This further burdens the assembler, who in turn forces the suppliers to reduce component prices (component price effect). Thus, as the total production cost increases, the financial cost effect and component price effect together make buyer finance (bank finance) less (more) attractive to the assembler, all the suppliers, and the whole supply chain as well.

Corollary 9(3) shows that $\hat{r}$ is decreasing in $\max_i\{r_s^i\}$. The reason behind is similar to that for the result that threshold value $\bar{r}$ decreases with $\max_i\{r_s^i\}$, as stated in Section 4.1. A higher interest rate in buyer finance leads to a lower production quantity and lower profit margins for suppliers, so it is more likely for all suppliers to prefer bank finance.

6 Conclusions

This paper compares buyer finance with bank finance in a supply chain with one assembler and multiple heterogeneous capital-constrained component suppliers. We characterize the equilibrium solutions for both buyer and bank finance. We show that in buyer finance the assembler should charge the lowest possible interest rate. In this way, the assembler may lose money in financing suppliers, but it can benefit more from enhanced inventory level and lower component prices. We further identify the sufficient and necessary conditions under which the assembler prefers buyer finance to bank finance. Our analysis reveals that the assembler may offer buyer finance even if its own unit capital opportunity cost is higher than bank interest rate. If financing the suppliers is costly, the assembler may prefer buyer finance to bank finance when the suppliers' total initial capital is high, the total production cost is low, or the suppliers are less heterogeneous in capital/cost. If financing the supplier is profitable, buyer finance is always the winner.

We also compare buyer finance with bank finance from the prospective of borrowing and non-borrowing suppliers as well as the whole supply chain. Interestingly, the suppliers' preferences are not always aligned with the assembler's. We completely characterize the Pareto improvement zones and interest conflict zones for the assembler and the suppliers. We also find conditions under which buyer finance benefits the whole supply chain.

Our results deliver several managerial insights. First, we provide a potential theoretical explanation for why firms in practice may lend to suppliers at an interest rate equal to or lower than the bank interest rate. Second, the benefit of buyer finance can be so compelling for assemblers that it may be optimal for the assembler to offer buyer finance even if its own unit capital cost is higher than banks' interest rate. Third, the suppliers and the whole supply chains may not benefit from buyer finance, unless the assembler's unit capital opportunity cost is below a threshold.

There are several limitations in this model, which may be worth further exploration. First, we assume all information is common knowledge among all parties. In reality, this assumption may not be true. Considering information asymmetry in this model can be an interesting direction for future research. Second, buyer finance considered in this paper is non-discriminative, in the sense that the assembler sets the same target expected rate of return for all suppliers, who have equal access to the buyer finance program. However in reality, some suppliers may be more important than others. In this situation, the assembler might give financing priority to the more important suppliers. Studying how the priority for selecting suppliers affects assembly supply chains in buyer finance could be another future research direction.

References

Babich, V. (2010). Independence of capacity ordering and financial subsidies to risky suppliers. *Manufacturing and Service Operations Management 12(4)*, 583–607.

Bernstein, F. and G. DeCroix (2004). Decentralized pricing and capacity decisions in a multi-tier system with modular assembly. *Management Science 50(9)*, 1293–1308.

Bernstein, F. and G. DeCroix (2006). Inventory policies in a decentralized assembly system. *Operations Research 54(2)*, 324–336.

Boeing (2012). Boeing, Citi start export-import bank supplier financing program. http://boeing.mediaroom.com/2012-02-17-Boeing-Citi-Start-Export-Import-Bank-Supplier-Financing-Program. Accessed on Febuary 28, 2016.

Bolduc, A. (2008). Time to say goodbye. *Automative News Europe (December 8), http://www.autonews.com/article/20081208/ANE03/812079905/time-to-say-goodbye*.

Bryn, H.-K. and M. Denton (2010). The new weakest link in your supply chain: Supplier credit. *Report, Oliver Wyman and Association, New York, USA*.

Buzacott, J. A. and R. Q. Zhang (2004). Inventory management with asset-based financing. *Management Science 50 (9)*, 1274–1292.

Cai, G., X. Chen, and Z. Xiao (2013). The roles of bank and trade credits: Theoretical analysis and empirical evidence. *Production and Operations Management 23(4)*, 583–598.

Chao, X., J. Chen, and S. Wang (2008). Dynamic inventory management with cash flow constraints. *Naval Research Logistics 55(8)*, 758–768.

Chauffour, J.-P. and M. Malouche (2011). Trade finance during the 2008-9 trade collapse: Key take aways. *Report, The World Bank, NW Washington, DC, USA*.

Chen, X., G. G. Cai, and J.-S. J. Song (2017). The cash flow advantages of 3pls as supply chain orchestrators. *Manufacturing & Service Operations Management*, Forthcoming. Available at SSRN: https://ssrn.com/abstract=2679357.

Chod, J. (2016). Inventory, risk shifting, and trade credit. *Management Science, epub ahead of print August 22, http://doi.org/10.1287/mnsc.2016.2515*.

Chod, J. and J. Zhou (2014). Resource flexibility and capital structure. *Management Science 60(3)*, 708–729.

Dada, M. and Q. Hu (2008). Financing newsvendor inventory. *Operations Research Letters 36*(5), 569–573.

Dong, L. and N. Rudi (2004). Who benefits from transshipment? exogenous vs. endogenous wholesale prices. *Management Science 50(5)*, 645–657.

Fang, X., K. C. So, and Y. Wang (2008). Component procurement strategies in decentralized assemble-to-order systems with lead time dependent product pricing. *Management Science 54(12)*, 1997–2011.

Jiang, L. and Y. Wang (2010). Supplier competition in decentralized assembly systems with price-sensitive and uncertain demand. *Manufacturing and Service Operations Management 12(1)*, 93–101.

Jing, B., X. Chen, and G. Cai (2012). Equilibrium financing in a distribution channel with capital constraint. *Productions and Operations Management 21(6)*, 1090–1101.

Kouvelis, P. and W. Zhao (2012). Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. *Operations Research 60(3)*, 566–580.

Kouvelis, P. and W. Zhao (2016). Who should finance the supply chain: Impact of credit ratings on supply chain decisions. Working paper, Shanghai Jiaotong University, Shanghai, China.

Lai, G., L. G. Deboa, and K. Sycarab (2009). Sharing inventory risk in supply chain: The implication of financial constraint. *Omega 37(4)*, 811–825.

Lariviere, M. A. and E. L. Porteus (2001). Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing and Service Operations Management 3(4)*, 293–305.

Myers, S. C. and N. S. Majluf (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics 13 (2)*, 187–221.

Peura, H., S. A. Yang, and G. Lai (2017). Trade credit in competition: a horizontal benefit. *Manufacturing & Service Operations Management 19*(2), 263–289.

Rui, H. and G. Lai (2015). Sourcing with deferred payment and inspection under supplier product adulteration risk. *Production and Operations Management 24(6)*, 934–946.

Seifert, D., R. Seifert, and M. Protopappa-Sieke (2013). A review of trade credit literature: Opportunities for research in operations. *European Journal of Operational Research 231(2)*, 245–256.

Song, J. and P. Zipkin (2003). *Supply chain operations: Assemble-to-order systems. Chapter 11 in Handbooks in Operations Research and* Management Science, *Vol. 11: Supply Chain Management. Elsevier, Amsterdam, The Netherlands, 561-596.*, Volume 11. Elsevier.

Swinney, R. and S. Netessine (2009). Long-term contracts under the threat of supplier default. *Manufacturing and Service Operations Management 11(1)*, 109–127.

Tang, C. S., J. Wu, and A. S. Yang (2015). Sourcing from suppliers with financial constraints and performance risk. Working paper. Available at SSRN 2679618.

Wang, Y. and Y. Gerchak (2003). Capacity games in assembly systems with uncertain demand. *Manufacturing and Service Operations Management 5(3)*, 252–267.

Xu, X. and J. Birge (2004). Joint production and financing decisions: Modeling and analysis. Working paper. Available at SSRN 652562.

Yang, A. S. and J. Birge (2011). How inventory is (should be) financed: Trade credit in supply chains with demand uncertainty and costs of financial distress. Working paper. Available at SSRN 1734682.

Yang, A. S. and J. Birge (2016). Trade credit, risk sharing, and inventory financing portfolios.

Yang, A. S., J. Birge, and R. Parker (2015). The supply chain effects of bankruptcy. *Management Science 61*(10), 2320–2338.

Zhao, L. and A. Huchzermeier (2015). Operations–finance interface models: A literature review and framework. *European Journal of Operational Research 244*(3), 905–917.

Appendix

Online Supplements for "Financing Multiple Heterogeneous Suppliers in Assembly Systems: Buyer vs. Bank Finance."

Proof of Lemma 2: To prove that $\vec{w}$ satisfies $q = \bar{F}^{-1}(\frac{c^1(1+r_s^1)}{w^1}) = \bar{F}^{-1}(\frac{c^2(1+r_s^2)}{w^2}) = ... = \bar{F}^{-1}(\frac{c^N(1+r_s^N)}{w^N})$, we use contradiction to show that for any supplier i, $\bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}) > q$ is not optimal. Suppose in Stackelberg equilibrium the assembler would set $\vec{w} = (w^1, \cdots, w^N)$ such that $\bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}) > q$ for some supplier i. Note that $\Pi_n(q) = \mathbf{E_D}\ [(p - \Sigma_{i=1}^N w^i) \cdot \min(D, q)]$. We can improve $\Pi_n(q)$ by keeping q and $(w^1, \cdots, w^{i-1}, w^{i+1}, \cdots, w^N)$ unchanged but lowering w^i so that $\bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}) = q$. This contradicts the assumption that $\vec{w}$ is optimal for the assembler. Therefore, we have $q = \bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}), \forall i$. Similarly, if there exists some supplier i such that $\bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}) > \min_i\{\frac{k^i}{c^i}\}$, then $\Pi_n(q)$ can be improved by lowering w^i but keeping q and $(w^1, \cdots, w^{i-1}, w^{i+1}, \cdots, w^N)$ unchanged. Therefore, in equilibrium we have $\bar{F}^{-1}(\frac{c^i(1+r_s^i)}{w^i}) \leq \min_i\{\frac{k^i}{c^i}\}, \forall i$. Q.E.D.

Proof of Lemma 3: The first order derivative of $\Pi_n(q)$ with respect to q is $\frac{\partial \Pi_n(q)}{\partial q} = -\frac{f(q)\sum_{i=1}^N c^i(1+r_s^i)}{[\bar{F}(q)]^2}\int_0^q \bar{F}(x)dx + p\bar{F}(q) - \sum_{i=1}^N c^i(1+r_s^i)$. Following the IFR assumption, $\frac{f(q)}{\bar{F}(q)}$ increases with q. Note that $\int_0^q \bar{F}(x)dx$ increases with q and $\bar{F}(q)$ decreases with q. Therefore, $\frac{\partial \Pi_n(q)}{\partial q}$ is decreasing in q. Thus $\Pi_n(q)$ is concave in q. Q.E.D.

Proof of Lemma 4: Note that for any supplier i, q^i is increasing in w^i for $w^i \in (c^i(1+r_s^i), w_{(1)}^i) \cup [w_{(2)}^i, \infty)$. This can be seen from the corresponding newsvendor expression of q^i for $w^i \in (c^i(1+r_s^i), w_{(1)}^i)$ and $w^i \in [w_{(2)}^i, \infty)$.

We next use contradiction to prove that $q^i = \frac{k^i}{c^i}$ if $w^i \in [w_{(1)}^i, w_{(2)}^i)$. Assuming there exists $w_0^i \in [w_{(1)}^i, w_{(2)}^i)$ such that it is optimal for the supplier to produce $q_0^i > \frac{k^i}{c^i}$. In this case, the supplier's marginal profit, $w_0^i \cdot \bar{F}(q_0^i) - c^i(1+r_f) < w_0^i \cdot \bar{F}(\frac{k^i}{c^i}) - c^i(1+r_f) = \bar{F}(\frac{k^i}{c^i})[w_0^i - w_{(2)}^i] < 0$. This contradicts the assumption that q_0^i is optimal for supplier i. Q.E.D.

Proof of Lemma 5: The proof follows the same argument for Lemma 2. Q.E.D.

Proof of Theorem 1:

1. For any supplier i, by definition, Π_{bk} is left-continuous at $q = \frac{k^i}{c^i}$. We next show that Π_{bk} is not right-continuous at $q = \frac{k^i}{c^i}$. Note that for each $q = \frac{k^i}{c^i}$, the right-hand limit is $\Pi_+^{bk}(q) = \lim_{q \to (\frac{k^i}{c^i})^+} \Pi_{bk}(q) = \{p - \frac{\sum_{j=i+1}^N c^j + \sum_{j=1}^i (1+r_f)c^j}{\bar{F}(q)}\} \cdot \mathbf{E_D} min(q, D)$, so $\Pi_+^{bk}(q) - \Pi_-^{bk} = -\frac{c^i r_f}{\bar{F}(q)} \cdot \mathbf{E_D} min(q, D) < 0$. Therefore, $q = \frac{k^i}{c^i}$ is a breakpoint $\forall i \in \{1, 2, \cdots, N\}$.

2. In the subset of $q \in (\frac{k^i}{c^i}, \frac{k^{i+1}}{c^{i+1}}]$, $\Pi_{bk} = \{p - \frac{\sum_{j=i+1}^N c^j + \sum_{j=1}^i (1+r_f)c^j}{\bar{F}(q)}\} \cdot \mathbf{E_D} min(q,D)$ resembles Π_n. The proof of concavity of Π_{bk} follows the same argument in the proof of Lemma 3.

3. The proof follows two steps. First, for any $\frac{k^i}{c^i} < a < b \leq \frac{k^{i+1}}{c^{i+1}}$, $\frac{\partial_- \Pi_{bk}}{\partial q}|_{q=a} > \frac{\partial_- \Pi_{bk}}{\partial q}|_{q=b}$ because Π_{bk} is continuous and concave in each subset $(\frac{k^i}{c^i}, \frac{k^{i+1}}{c^{i+1}}]$, which has been proved in part (2). Second, we then show that if $a = \frac{k^i}{c^i}$, for any b such that $a < b \leq \frac{k^{i+1}}{c^{i+1}}$, then $\frac{\partial_- \Pi_{bk}}{\partial q}|_{q=a} > \frac{\partial_- \Pi_{bk}(q)}{\partial q}|_{q=b}$. This result occurs because,

$$
\begin{aligned}
& \frac{\partial_- \Pi_{bk}}{\partial q}|_{q=a} = \frac{\partial_- \Pi^{bk}}{\partial q}|_{q=(\frac{k^i}{c^i})} \\
= \ & \{-\frac{f(q)(\sum_{j=1}^N c^j + \sum_{j=1}^{i-1} r_f c^j)}{[\bar{F}(q)]^2} \int_0^q \bar{F}(x)dx + p\bar{F}(q) - (\sum_{j=1}^N c^j + \sum_{j=1}^{i-1} r_f c^j)\}|_{q=(\frac{k^i}{c^i})} \\
\geq \ & \{-\frac{f(q)(\sum_{j=1}^N c^j + \sum_{j=1}^{i-1} r_f c^j)}{[\bar{F}(q)]^2} \int_0^q \bar{F}(x)dx + p\bar{F}(q) - (\sum_{j=1}^N c^j + \sum_{j=1}^{i-1} r_f c^j)\}|_{q=b} \\
> \ & \{-\frac{f(q)(\sum_{j=1}^N c^j + \sum_{j=1}^{i} r_f c^j)}{[\bar{F}(q)]^2} \int_0^q \bar{F}(x)dx + p\bar{F}(q) - (\sum_{j=1}^N c^j + \sum_{j=1}^{i} r_f c^j)\}|_{q=b} \\
= \ & \frac{\partial_- \Pi_{bk}(q)}{\partial q}|_{q=b}.
\end{aligned}
$$

The first inequality follows from the concavity of Π_{bk} in each subset $q \in (\frac{k^i}{c^i}, \frac{k^{i+1}}{c^{i+1}}]$. The second inequality holds because the summation upper limit is changed from $i-1$ to i and $r_f c^j \geq 0$. To summarize, $\Pi_{bk}(q)$ is globally left-differentiable and $\frac{\partial_- \Pi_{bk}}{\partial q}$ is globally decreasing in q. Q.E.D.

Proof of Theorem 2: By definition of b_k, $\Pi_{bk}(q)$ decreases in q for $q \in (b_k, \infty)$ since $\frac{\partial_- \Pi_{bk}}{\partial q}$ is decreasing in q (Theorem 1). Hence $q^*_{bk} \leq b_k$. Also, $\frac{\partial_- \Pi_{bk}}{\partial q} > 0$ for $q \in [0, \max_i\{\frac{k^i}{c^i}\}|_{\frac{k^i}{c^i} < b_k}]$, implying that q^*_{bk} can not be any internal point of the interval $(\frac{k^{i-1}}{c^{i-1}}, \frac{k^i}{c^i}]$ $\forall i$ such that $\frac{k^i}{c^i} < b_k$. Therefore, q^*_{bk} can be obtained by comparing $\Pi_{bk}(q)$ at $q = \frac{k^i}{c^i}$ for $\forall i$ such that $\frac{k^i}{c^i} < b_k$ and $\Pi_{bk}(q)$ at $q = \bar{q}$ where $\bar{q}$ lies in the interval that includes b_k and $\frac{\partial \Pi_{bk}(q)}{\partial q}|_{q=\bar{q}} = 0$. Q.E.D.

Proof of the sensitivity results in bank finance:

(1) To prove q^*_{bk} decreases in c^i and in r_f but increases in p, we show that $\Pi_{bk}(q, c^i)$ is a submodular function in (q, c^i), a submodular function in (q, r_f), and a supermodular function in (q, p), respectively. To prove that $\Pi_{bk}(q, c^i)$ is submodular in (q, c^i), we can show that $\Pi_{bk}(q, c^i) - \Pi_{bk}(q, c^{i\prime})$ is decreasing in q for $c^i \geq c^{i\prime}$. For a borrowing supplier i, $\Pi_{bk}(q, c^i) - \Pi_{bk}(q, c^{i\prime}) = -(c^i - c^{i\prime})(1 + r_f) \cdot \frac{\mathbf{E_D}[min(q,D)]}{\bar{F}(q)}$ is decreasing in q since $\frac{\mathbf{E_D}[min(q,D)]}{\bar{F}(q)}$ is increasing in q. For a non-borrowing supplier i, $\Pi_{bk}(q, c^i) - \Pi_{bk}(q, c^{i\prime}) = -(c^i - c^{i\prime}) \cdot \frac{\mathbf{E_D}[min(q,D)]}{\bar{F}(q)}$, which decreases in q.

Following a similar argument, we can show that $\Pi_{bk}(q, r_f)$ is submodular in (q, r_f) and $\Pi_{bk}(q, p)$ is supermodular in (q, p).

Note that $\forall q$, $\frac{\partial \Pi_{bk}}{\partial c^i} < 0$ $\frac{\partial \Pi_{bk}}{\partial r_f} < 0$, and $\frac{\partial \Pi_{bk}}{\partial p} > 0$. Having shown q^*_{bk} decreases in c^i and r_f and increases in p, we have $\Pi_{bk}(q^*_{bk})$ decreases in c^i and r_f and increases in p.

(2) We next prove $\Pi_{bk}(q^*_{bk})$ increases in $k^i, \forall i \in \{1, 2, \cdots, N\}$. Assume $k^{i'} < k^{i''}$, for any given q, we have $\Pi_{bk}(q, k^i = k^{i''}) \leq \Pi_{bk}(q, k^i = k^{i'})$ because $\delta(c^i q - k^{i'}) \geq \delta(c^i q - k^{i''})$. Let q'^*_{bk} and q''^*_{bk} denote the equilibrium production quantity when $k^i = k^{i'}$ and $k^i = k^{i''}$, respectively. Therefore, when $k^i = k^{i''}$, the assembler's maximum profit $\Pi_{bk}(q''^*_{bk}, k^i = k^{i''}) \geq \Pi_{bk}(q'^*_{bk}, k^i = k^{i''}) \geq \Pi_{bk}(q'^*_{bk}, k^i = k^{i'})$. Q.E.D.

Lemma 7 *Suppose $f(x)$ has support $[A, B]$, where $A \geq 0$ and B can be ∞. $\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} - q > 0$ for any $q \geq A$ if $\forall \alpha \in (A, B]$, $\{x \in (A, \alpha) : f(x)$ is continuous and $f(x) > 0\} \neq \emptyset$.*

Proof of Lemma 7: Note $q - \frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} = \frac{1}{\bar{F}(q)}[q\bar{F}(q) - \int_0^q \bar{F}(x)dx]$. To prove $\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} - q > 0$, we can instead show $\int_0^q \bar{F}(x)dx > q\bar{F}(q)$. By assumption, $\exists \epsilon \in (A, \alpha) \subset (A, q)$, such that $f(x) > 0$ and is continuous in x for $x \in [\epsilon - \lambda, \epsilon + \lambda]$ for any arbitrary small $\lambda > 0$. We then can show $\bar{F}(\epsilon) = \bar{F}(\epsilon+\lambda) + (\bar{F}(\epsilon) - \bar{F}(\epsilon+\lambda)) = \bar{F}(\epsilon+\lambda) + \int_\epsilon^{\epsilon+\lambda} f(x)dx > \bar{F}(\epsilon+\lambda)$. The last inequality holds because $\int_\epsilon^{\epsilon+\lambda} f(x)dx = \lambda f(\xi) > 0$, where $\xi \in (\epsilon, \epsilon+\lambda)$. We thus have $\bar{F}(\epsilon) > \bar{F}(\epsilon+\lambda) \geq \bar{F}(q)$. We then can show $\int_0^q \bar{F}(x)dx = \int_0^\epsilon \bar{F}(x)dx + \int_\epsilon^q \bar{F}(x)dx \geq \epsilon\bar{F}(\epsilon) + (q-\epsilon)\bar{F}(q) = \epsilon(\bar{F}(\epsilon) - \bar{F}(q)) + q\bar{F}(q) > q\bar{F}(q)$, where the first inequality holds because $\bar{F}(x)$ is nonincreasing. Q.E.D.

Lemma 8 *$\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} - q$ is strictly increasing convex in q.*

Proof of Lemma 8: Note $\partial[\frac{\mathbf{E_D}(q,D)}{\bar{F}(q)} - q]/\partial q = \frac{f(q)}{\bar{F}(q)^2}\mathbf{E_D}(q, D) > 0$ and increases in q. Therefore, $\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} - q$ is strictly increasing convex in q. Q.E.D.

Proof of Corollary 1: Note that $q^*_{bk} \geq \min_i\{\frac{k^i}{c^i}\} \geq q^*_n$, we use this inequality in the following proof. For the assembler, $\Pi_{bk}(q^*_{bk}) \geq \Pi_{bk}(q = \min_i\{\frac{k^i}{c^i}\}) = \Pi_n(q = \min_i\{\frac{k^i}{c^i}\})$, hence the assembler is better off in bank finance than no finance. In bank finance, a borrowing supplier i's profit $\pi^i_{bk}(q^*_{bk}) = c^i \cdot (1 + r_f)(\frac{\mathbf{E_D}(q^*_{bk},D)}{\bar{F}(q^*_{bk})} - q^*_{bk}) + (r_f - r^i_s)k^i$, a non-borrowing supplier i's profit $\pi^i_n(q^*_{bk}) = c^i \cdot (1 + r^i_s)(\frac{\mathbf{E_D}(q^*_{bk},D)}{\bar{F}(q^*_{bk})} - q^*_{bk})$. In no finance, supplier i's profit is $\pi^i_n(q^*_n) = c^i \cdot (1 + r^i_s)(\frac{\mathbf{E_D}(q^*_n,D)}{\bar{F}(q^*_n)} - q^*_n)$. We now compare $\pi^i_n(q^*_n)$, respectively, with $\pi^i_{bk}(q^*_{bk})$ and $\pi^i_n(q^*_{bk})$. Note that $\frac{\mathbf{E_D}(q,D)}{\bar{F}(q)} - q > 0$ (Lemma 7) and increases in q (Lemma 8). Given $q^*_{bk} \geq \min_i\{\frac{k^i}{c^i}\} \geq q^*_n$ and $r_f > r^i_s$, it is then immediately that $\pi_{bk}^i(q_{bk}^*) \geq \pi_n^i(q_n^*)$ and $\pi_n^i(q_{bk}^*) \geq \pi_n^i(q_n^*)$; hence, both borrowing suppliers and non-borrowing suppliers are better off in buyer finance than in no finance.

If $q_{bk}^* > \min_i\{\frac{k^i}{c^i}\}$, $\Pi_{bk}(q_{bk}^*) > \Pi_{bk}(q = \min_i\{\frac{k^i}{c^i}\}) = \Pi_n(q = \min_i\{\frac{k^i}{c^i}\})$, hence the assembler is strictly better off in bank finance. For a borrowing or non-borrowing supplier i, $q_{bk}^* > \min_i\{\frac{k^i}{c^i}\}$ implies that $\pi_{bk}^i(q_{bk}^*) > \pi_n^i(q_n^*)$ and $\pi_n^i(q_{bk}^*) > \pi_n^i(q_n^*)$, hence suppliers are strictly better off. Q.E.D.

Proof of Lemma 6: The proof follows the same argument for Lemma 2. Q.E.D.

Proof of Theorem 3: Rearranging the terms in $\Pi_{br}(r_b)$, we have $\Pi_{br}(r_b) = \sum_{i=1}^{j}[(c^iq - k^i) - \frac{c^i}{\bar{F}(q)} \cdot \mathbf{E}_D min(q,D)] \cdot (r_b - r_s^i) + (p - \frac{\sum_{i=1}^{N} c^i(1+r_s^i)}{\bar{F}(q)}) \cdot \mathbf{E}_D min(q,D) - \sum_{i=1}^{j} (r_a - r_s^i)(c^iq - k^i)$, where the second term and third term are independent of r_b. In the first term, the coefficient of r_b, $\sum_{i=1}^{j}(c^iq - k^i) - \frac{\mathbf{E_D} min(q,D)}{\bar{F}(q)} \cdot \sum_{i=1}^{j} c^i < 0, \forall q$, because $q - \frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} - \frac{k^i}{c^i} < 0$ (by Lemma 7) for every borrowing supplier i. Therefore, $\Pi_{br}(r_b)$ is decreasing in r_b for any q. Because $r_b \geq r_s^i, \forall i \in \{1, 2, \cdots, N\}$, we have $r_b^* = \max_i\{r_s^i\}$. Q.E.D.

Proof of Corollary 2: Given $r^i = r_a, \forall i \in \{1, 2, \cdots, N\}$, substituting $r_b^* = \max_i\{r_s^i\}$ into $\Pi_{br}(q, r_b^*)$, we have $\Pi_{br}(q, r_b^*) = \mathbf{E}_D\ (p - \frac{\Sigma_{i=1}^{N}\ c^i(1+r_s^i)}{\bar{F}(q)}) \cdot min(q,D)$, which is concave in q by Lemma 3. Hence q_{br}^* is uniquely solved from, $p\bar{F}(q) = \sum_{i=1}^{N} c^i(1+r_s^i)(\frac{f(q)}{[\bar{F}(q)]^2}\int_0^q \bar{F}(x)dx + 1)$. Q.E.D.

Proof of the sensitivity results in buyer finance:

To prove q_{br}^* decreases with c^i, we can show $\Pi_{br}(q, c^i) - \Pi_{br}(q, c^{i\prime})$ is decreasing in q for $c^i \geq c^{i\prime}$. For a borrowing supplier i , $\Pi_{br}(q, c^i) - \Pi_{br}(q, c^{i\prime}) = (c^i - c^{i\prime}) \cdot L(q)$, which is decreasing in q, where $L(q) = (r_b - r_a)q - \frac{(1+r_b)\mathbf{E_D}[min(q,D)]}{\bar{F}(q)}$. Hence $\Pi_{br}(q, c^i) - \Pi_{br}(q, c^{i\prime})$ is decreasing in q. For a non-borrowing supplier i, $\Pi_{br}(q, c^i) - \Pi_{br}(q, c^{i\prime}) = -(c^i - c^{i\prime}) \cdot \frac{\mathbf{E_D}[min(q,D)]}{\bar{F}(q)}$, which decreases in q since $\frac{\mathbf{E_D}[min(q,D)]}{\bar{F}(q)}$ increases in q. Following a similar argument, we can show that q_{br}^* decreases in r_a and increases in p. $\Pi_{br}(q_{br}^*)$ increases in p, decreases in c^i, and decreases with r_a. The proof follows the same argument as in the proofs for bank finance.

Proof of Corollary 3:

When $\max_i\{r_s^i\} \leq r_a$, following similar argument in part (2) of the proof for the sensitivity in bank finance, we can show that $\Pi_{br}(q_{br}^*)$ increases in k^i. When $\max_i\{r_s^i\} > r_a$, we introduce a special case, if all $r_s^i, i \in \{1, 2, \cdots, N\}$ are the same, $\Pi_{br} = (p - \frac{\Sigma_{i=1}^{N} c^i(1+r_s^i)}{\bar{F}(q)})\mathbf{E_D}[minD, q] + \Sigma_{i=1}^{N}(\max_i\{r_s^i\} - r_a)(c^iq - k^i)^+$, which decreases in k^i, hence $\Pi_{br}(q_{br}^*)$ decreases in k^i. Q.E.D.

Proof of Corollary 4: The proof follows the same argument for Corollary 1.

Proof of Theorem 4: By Theorem 3, the optimal interest rate in buyer finance is $r_b^* = \max_i\{r_s^i\}$ for any given q. Define $h(q) = \Pi_{br}(q, r_b = r_b^*) - \Pi_{bk}(q)$. We have $h(q) = (r_f -$

$\max_i\{r_s^i\})\ \frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)}\Sigma_{i=1}^j c^i - (r_a - \max_i\{r_s^i\})\Sigma_{i=1}^j(c^i q - k^i)$. If $r_a \le r_f$, we have $h(q) \ge (r_f - \max_i\{r_s^i\})[\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)}\Sigma_{i=1}^j c^i - \Sigma_{i=1}^j(c^i q - k^i)] > 0$, $\forall q \ge 0$, because $\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)}\Sigma_{i=1}^j c^i - \Sigma_{i=1}^j(c^i q - k^i) > 0$ (by Lemma 7). We have $\Pi_{br}(q_{br}^*) \ge \Pi_{br}(q_{bk}^*) > \Pi_{bk}(q_{bk}^*)$. Therefore, $\Pi_{br}(q_{br}^*) > \Pi_{bk}(q_{bk}^*)$ if $r_a \le r_f$. As $\frac{\partial \Pi_{br}(q_{br}, r_a)}{\partial r_a} = -\Sigma_{i=1}^j(c^i q - k^i)^+ < 0$, and $\frac{\partial q_{br}^*(r_a)}{\partial r_a} < 0$, there exists an arbitrary large M such that when $r_a = M$, we have $q_{br}^* = \min_i\{\frac{k^i}{c^i}\}$, and $\Pi_{br}(q_{br}^*) = \Pi_n(q = \min_i\{\frac{k^i}{c^i}\}) \le \Pi_{bk}(q_{bk}^*)$. Because $\Pi_{br}(q_{br}^*(r_a), r_a)$ is continuous in r_a, there exists $\bar{r} > r_f$ such that $\Pi_{br}(q_{br}^*) > \Pi_{bk}(q_{bk}^*)$ when $r_a \le \bar{r}$, and $\Pi_{br}(q_{br}^*) \le \Pi_{bk}(q_{bk}^*)$ when $r_a > \bar{r}$. Q.E.D.

Proof of Corollary 5: We first prove that $\bar{\bar{r}} \ge r_f$. Note that $\bar{\bar{r}} = \frac{(r_f - \max_i\{r_s^i\})\cdot \mathbf{E_D} min(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)(q_{bk}^* - \frac{\Sigma k^i}{\Sigma c^i})} + \max_i\{r_s^i\}$, rearrange the terms, we have $\bar{\bar{r}} = r_f + (\frac{\mathbf{E_D} min(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)(q_{bk}^* - \frac{\Sigma k^i}{\Sigma c^i})} - 1)\cdot(r_f - \max_i\{r_s^i\}) \ge r_f$, where the inequality holds because $\frac{\mathbf{E_D} min(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)(q_{bk}^* - \frac{\Sigma k^i}{\Sigma c^i})} > 1$ (by Lemma 7) and $r_f \ge \max_i\{r_s^i\}$. We then show that $\bar{\bar{r}} \le \bar{r}$. Substituting $r = \bar{\bar{r}}$ and $q = q_{bk}^*$ into $\Pi_{br}(r,q)$, we have $\Pi_{br}(r = \bar{\bar{r}}, q = q_{bk}^*) = \Pi_{bk}(q_{bk}^*)$. Hence, $\Pi_{br}(r = \bar{\bar{r}}, q = q_{br}^*) \ge \Pi_{br}(r = \bar{\bar{r}}, q = q_{bk}^*) = \Pi_{bk}(q_{bk}^*)$. Recall the definition of $\bar{r}$, we have $\bar{\bar{r}} \le \bar{r}$. Q.E.D.

Proof of Corollary 6: When $\frac{k^i}{c^i} = \frac{k^j}{c^j}$ $\forall i, j = 1, 2, \cdots, N$ and $\frac{k^i}{c^i} < q_{bk}^*$, the equilibrium quantity q_{bk}^* and q_{br}^* satisfy the following first-order conditions, respectively,

$$\frac{p\bar{F}(q_{bk}^*)}{\Sigma_{i=1}^N c^i} = (1 + r_f) + \frac{f(q_{bk}^*)(1 + r_f)\mathbf{E_D} min(q_{bk}^*, D)}{[\bar{F}(q_{bk}^*)]^2} \tag{A-1}$$

$$\frac{p\bar{F}(q_{br}^*)}{\Sigma_{i=1}^N c^i} = (1 + r_a) + \frac{f(q_{br}^*)(1 + \max_i\{r_s^i\})\mathbf{E_D} min(q_{br}^*, D)}{[\bar{F}(q_{br}^*)]^2} \tag{A-2}$$

Since $\Pi_{br}(q_{br})$ decreases with r_a, $\Pi_{br}(q_{br}^*)$ decreases with r_a. Let $r_a = \bar{r}$ such that $\Pi_{br}(q_{br}^*) = \Pi_{bk}(q_{bk}^*)$, we have $\bar{r} = \frac{[p - \frac{\Sigma_{i=1}^N c^i(1+\max_i\{r_s^i\})}{\bar{F}(q_{br}^*)}]\mathbf{E_D} min(q_{br}^*, D) - \Pi_{bk}(q_{bk}^*)}{\Sigma_{i=1}^N(c^i q_{br}^* - k^i)} + \max_i\{r_s^i\}$.

1. Suppose $\Sigma_{i=1}^N k^i = K$, by Theorem 4, there exists $r_a = \bar{r} > r_f$ such that $\Pi_{br}(q = q_{br}^*, r_a = \bar{r}, \Sigma_{i=1}^N k^i = K) = \Pi_{bk}(q_{bk}^*)$. By Eq. (A-1) and Eq. (A-2), both q_{bk}^* and q_{br}^* are independent of $k^i, \forall i \in \{1, 2, \cdots, N\}$. Then $\forall K' > K$, there is $\Pi_{br}(q = q_{br}^*, r_a = \bar{r}, \Sigma_{i=1}^N k^i = K') - \Pi_{bk}(q_{bk}^*) = (\bar{r} - \max_i\{r_s^i\})(K' - K) > 0$. Since $\Pi_{br}(q_{br}^*)$ decreases with r_a, there exists $\bar{r}' > \bar{r}$ such that $\Pi_{br}(q = {q_{br}^*}', r_a = \bar{r}', \Sigma_{i=1}^N k^i = K') - \Pi_{bk}(q_{bk}^*) = 0$. Therefore, $\bar{r}$ increases in $\Sigma_{i=1}^N k^i$.

2. In a similar way to item 1, we can prove that $\bar{r}$ decreases in $\max_i\{r_s^i\}$.

3. By Eq. (A-1) and Eq. (A-2), both q_{bk}^* and q_{br}^* are independent of $k^i, \forall i \in \{1, 2, \cdots, N\}$. Therefore, $\Pi_{bk}(q_{bk}^*) = [p - \frac{\Sigma_{i=1}^N c^i(1+r_f)}{\bar{F}(q_{bk}^*)}]\mathbf{E_D} min(q_{bk}^*, D)$ is independent of k^i, while $\Pi_{br}(q_{br}^*) = [p - \frac{\Sigma_{i=1}^N c^i(1+\max_i\{r_s^i\})}{\bar{F}(q_{br}^*)}]\mathbf{E_D} min(q_{br}^*, D) + (\max_i\{r_s^i\} - r_a)\Sigma_{i=1}^N(c^i q_{br}^* - k^i)$, which increases in k^i if $r_a \ge$

$\max_i\{r_s^i\}$. Hence, $\Pi_{br}(q_{br}^*) - \Pi_{bk}(q_{bk}^*)$ increases in $\Sigma_{i=1}^N k^i$ if $r_a \geq \max_i\{r_s^i\}$, decreases in $\Sigma_{i=1}^N k^i$ if $r_a < \max_i\{r_s^i\}$. Q.E.D

Proof of Theorem 5:

1. Given $r_s^1 = r_s^2 = r_s, c^1 = c^2 = \frac{C}{2}$, there is $\Pi_{br}(q,\theta) = (p - \frac{C(1+r_s) + \frac{C(r_f - r_s)}{2}[\delta(\frac{C}{2}q - \frac{\theta K}{1+\theta}) + \delta(\frac{C}{2}q - \frac{K}{1+\theta})]}{\bar{F}(q)}) \cdot \mathbf{E_D}[min(q,D)]$, which depends on θ only because the term $\delta(\frac{C}{2}q - \frac{\theta K}{1+\theta}) + \delta(\frac{C}{2}q - \frac{K}{1+\theta})$ depends on θ. Let $I(q,\theta) = \delta(\frac{C}{2}q - \frac{\theta K}{1+\theta}) + \delta(\frac{C}{2}q - \frac{K}{1+\theta})$, we focus on two scenarios: (1) $K < Cq_{bk}^*$. Since ,$\forall\theta \in (0,1]$, there is $\delta(\frac{C}{2}q_{bk}^* - \frac{\theta K}{1+\theta}) = 1$ and $\delta(\frac{C}{2}q_{bk}^* - \frac{K}{1+\theta})$ increases in θ, thus $I(q_{bk}^*,\theta)$ increases with θ. Therefore, $\Pi_{br}(q_{bk}^*,\theta)$ decreases in θ when $K < Cq_{bk}^*$. (2)$K \geq Cq_{bk}^*$. In this scenario, $\forall\theta \in (0,1]$, there is $\delta(\frac{C}{2}q_{bk}^* - \frac{\theta K}{1+\theta})$ decreases in θ, while $\delta(\frac{C}{2}q_{bk}^* - \frac{K}{1+\theta}) = 0$, hence we have $I(q_{bk}^*,\theta)$ decreases with θ, and $\Pi_{br}(q_{bk}^*,\theta)$ increases in θ when $K \geq Cq_{bk}^*$.

2. Note that we can prove $\Pi_{br}^*(q_{br}^*,\theta)$ increases (decreases) in θ if we have $\Pi_{br}(q,\theta)$ increases (decreases) in θ, $\forall q$. Since $r_s^1 = r_s^2$, the optimal interest rate $r_b^* = \max_i\{r_s^i\} = r_s^i, \forall i \in \{1,2\}$. For any given q, $\Pi_{br}(q,\theta) = (p - \frac{\Sigma_{i=1}^2 c^i(1+r_s^i)}{\bar{F}(q)}) \cdot \mathbf{E_D}[min(q,D)] + (r_b^* - r_a)\cdot((c^1 q - \frac{\theta K}{1+\theta})^+ + (c^2 q - \frac{K}{1+\theta})^+)$, where $\theta \in (0, \frac{c^1}{c^2}]$. We next only need to consider $c^1 < c^2$ and $\theta \in (0, \frac{c^1}{c^2}]$, because if $c^2 < c^1$ we can simply switch the supplier's index. Let $B(q,\theta) = (c^1 q - \frac{\theta K}{1+\theta})^+ + (c^2 q - \frac{K}{1+\theta})^+$, then $\Pi_{br}(q,\theta) = (p - \frac{\Sigma_{i=1}^2 c^i(1+r_s^i)}{\bar{F}(q)}) \cdot \mathbf{E_D}[min(q,D)] + (r_b^* - r_a)\cdot B(q,\theta)$. Note that $\Pi_{br}(q,\theta)$ depends on θ only because $B(q,\theta)$ depends on θ. With $\theta \in (0, \frac{c^1}{c^2}]$, it is then easy to show that $B(q,\theta)$ deceases in θ. Therefore, $\Pi_{br}(q,\theta)$ increases in θ if $r_a \geq \max_i r_s^i$, but decreases in θ if $r_a < \max_i r_s^i$. Q.E.D.

Proof of Corollary 7: When $\Sigma_{i=1}^N k^i = 0$, $\bar{\bar{r}} = \frac{(r_f - \max_i\{r_s^i\})\cdot \mathbf{E_D} min(q_{bk}^*,D)}{\bar{F}(q_{bk}^*)q_{bk}^*} + \max_i\{r_s^i\}$. Since q_{bk}^* decreases in c^i, if we can show $\frac{\int_0^q \bar{F}(x)dx}{q\bar{F}(q)}$ increases in q, then we have $\bar{\bar{r}}$ decreases in $\Sigma_{i=1}^N c^i$. Let $G(q) = \frac{\int_0^q \bar{F}(x)dx}{q\bar{F}(q)}$, then $G^{'}(q) = \frac{\partial G(q)}{\partial q} = \frac{q\bar{F}(q) - (1 - \frac{qf(q)}{\bar{F}(q)})\int_0^q \bar{F}(x)dx}{q^2 \cdot \bar{F}(q)} = \frac{q\bar{F}(q) + (z(q)-1)\int_0^q \bar{F}(x)dx}{q^2\cdot\bar{F}(q)}$, where $z(q) = \frac{qf(q)}{\bar{F}(q)}$. Note that $\forall q \in (-\infty, 0), G(q) = 1$ is constant, and $\forall q \in (\bar{q}, \infty)$, $G^{'}(q) > 0$, where $\bar{q}$ solves from $1 - \frac{qf(q)}{\bar{F}(q)} = 0$. If we show $G(q)$ is increasing in q for $q \in [0, \bar{q}]$, then $G(q)$ is globally increasing in q. We then prove by contradiction. Suppose $G(q)$ is decreasing in $q \in [0,\bar{q}]$ and strictly decreasing in q at least for some subset of $\{q : q \in [0,\bar{q}]\}$, then $G^{'}(q)|_{q=0} = 0$ and $G^{'}(q)|_{q=\bar{q}} = 0$, which implies $\frac{1}{1-z(q)} - G(q)|_{q=0} = 0$ and $\frac{1}{1-z(q)} - G(q)|_{q=\bar{q}} = 0$, this contradicts the facts that $\frac{1}{1-z(q)}$ is increasing in q while $G(q)$ is decreasing in q for $q \in [0,\bar{q}]$. Therefore, $G(q)$ is increasing in q for $q \in [0,\bar{q}]$. Q.E.D

Proof of Theorem 6

1. When $N = 2$, $k^1 = k^2 = K/2$, $r_s^1 = r_s^2 = r_s$ and $\eta = \frac{c^1}{c^2} \in [0,1]$, there is $\Pi_{bk}(q,\eta) =$

$(p - \frac{C[(1+r_s)+\frac{1}{1+\eta}(r_f-r_s)\cdot\delta(\frac{C}{1+\eta}q-K/2)+\frac{\eta}{1+\eta}(r_f-r_s)\cdot\delta(\frac{\eta C}{1+\eta}q-K/2)]}{\bar{F}(q)}) \cdot \mathbf{E_D}[min(q,D)]$. Therefore, $\Pi_{bk}(q,\eta)$ depends on only η, because the term $\frac{\delta(\frac{C}{1+\eta}q-K/2)}{1+\eta} + \frac{\eta\delta(\frac{\eta C}{1+\eta}q-K/2)}{1+\eta}$ depends on η. Let $M(\eta) = \frac{\delta(\frac{C}{1+\eta}q-K/2)}{1+\eta} + \frac{\eta\delta(\frac{\eta C}{1+\eta}q-K/2)}{1+\eta}$. We then focus on two cases: (1) $K \geq Cq^*_{bk}$. In this case, $M(\eta)$ decreases with η, and achieves its minimum at $\eta = 1$ because $\frac{k^1}{c^1} = \frac{k^2}{c^2} \geq q^*_{bk}$ and $\{\delta(\frac{C}{1+\eta}q - K/2)\}|_{\eta=1} = \{\delta(\frac{\eta C}{1+\eta}q - K/2)\}|_{\eta=1} = 0, \forall q \in [0, q^*_{bk}]$. Hence for $\eta \in [0,1]$ and $\forall q \in [0, q^*_{bk}]$, $\Pi_{bk}(q,\eta)$ increases in η. (2) $K < Cq^*_{bk}$. In this case, when $q = q^*_{bk}$, $M(\eta)$ achieves its maximum at $\eta = 1$ because $\frac{k^1}{c^1} = \frac{k^2}{c^2} < q^*_{bk}$ and $\{\delta(\frac{C}{1+\eta}q^*_{bk} - K/2)\}|_{\eta=1} = \{\delta(\frac{\eta C}{1+\eta}q^*_{bk} - K/2)\}|_{\eta=1} = 1$. Since $\delta(\frac{C}{1+\eta}q^*_{bk} - K/2) \geq \delta(\frac{\eta C}{1+\eta}q^*_{bk} - K/2)$, there exists $0 < \bar{\eta} \leq 1$ such that $\delta(\frac{C}{1+\eta}q^*_{bk} - K/2) = 1, \delta(\frac{\eta C}{1+\eta}q^*_{bk} - K/2) = 0$ when $\eta \in (0, \eta_1]$ and $\delta(\frac{C}{1+\eta}q^*_{bk} - K/2) = 1, \delta(\frac{\eta C}{1+\eta}q^*_{bk} - K/2) = 1$ for $\eta \in (\bar{\eta}, 1]$. Therefore, $M(\eta) = \frac{1}{1+\eta}$ for $\eta \in (0, \eta_1]$, and $M(\eta) = 1$ for $\eta \in (\bar{\eta}, 1]$. Hence $M(\eta)$ first decreases and then increases in η, while $\Pi_{bk}(q^*_{bk}, \eta)$ first increases and then decreases in η.

2. The proof of part 2 resembles the proof of part 2 of Theorem 5 and thus omitted. Q.E.D

Proof of Theorem 8: Consider $h(q) = \Pi_{br}(q) - \Pi_{bk}(q)$ as defined in the proof of Theorem 4. Note that $h(q) > 0$, $\forall q \geq 0$ if $r_a \leq r_f$ and $\sum_i k^i \geq 0$. Moreover, if $r_a \leq r_f$, $h(q)$ is also increasing in q because we have $h^{'}(q) = [(r_f - \max_i\{r^i_s\}) \cdot (\frac{(f(q)\cdot\mathbf{E}_D\ min(q,D)}{(\bar{F}(q))^2} + 1) - (r_a - \max_i\{r^i_s\})]\Sigma_i c^i \ > 0$. If we show q^*_{br} cannot be in $[0, q^*_{bk}]$, then there must be $q^*_{br} \geq q^*_{bk}$. By the optimality of q^*_{bk}, $\Pi_{bk}(q) \leq \Pi_{bk}(q^*_{bk})$ for any $q \in [0, q^*_{bk}]$. Furthermore, by the definition of $h(q)$, $\Pi_{br}(q) = \Pi_{bk}(q) + h(q) \leq \Pi_{bk}(q^*_{bk}) + h(q^*_{bk}) = \Pi_{br}(q^*_{bk})$. That is, $\Pi_{br}(q) \leq \Pi_{br}(q^*_{bk})$ for any $q \in [0, q^*_{bk}]$. Therefore, we must have $q^*_{br} \geq q^*_{bk}$. We thus proved the existence of $\hat{r} \geq r_f$ such that $q^*_{br} \geq q^*_{bk}$ if $r_a \leq \hat{r}$. We then prove $r_a \leq \hat{r}$ is not only a sufficient condition but also a necessary condition for $q^*_{br} \geq q^*_{bk}$. To show this, we must have a $\hat{r}$ such that $q^*_{br} < q^*_{bk}$ when $r_a > \hat{r}$. Note that $\frac{\partial q^*_{br}}{\partial r_a} < 0$, given an arbitrary large number M, if $r_a = M$, there is $q^*_{br} = q^*_n < q^*_{bk}$. Since Π_{br} is continuous in both q and r_a, q^*_{br} is also continuous in r_a. Also, q^*_{bk} is independent of r_a, there exists $\hat{r}$ such that: $q^*_{br} > q^*_{bk}$ if $r_a < \hat{r}$, $q^*_{br} = q^*_{bk}$ if $r_a = \hat{r}$, and $q^*_{br} < q^*_{bk}$ if $r_a > \hat{r}$.

For a centralized supply chain, the equilibrium production quantity $q^{*cen} = \bar{F}^{-1}(\frac{\Sigma c^i(1+r^i_s)}{p})$, we thus have $q^*_{br} \leq q^{*cen}$. Similarly, one can show $q^*_{bk} \leq q^{*cen}$. Q.E.D.

Proof of Theorem 7: From the proof of Theorem 8, $q^*_{br} \geq q^*_{bk}$ if and only if $r_a \leq \hat{r}$, where $\hat{r} \geq r_f$. Note that $q^*_{bk} = \bar{F}^{-1}(\frac{c^i\cdot(1+r^i_s+(r_f-r^i_s)\cdot\delta(c^i q^*_{bk}-k^i))}{w^*_i})$, and $q^*_{br} = \bar{F}^{-1}(\frac{c^i\cdot(1+r^i_s+(r^*_b-r^i_s)\cdot\delta(c^i q^*_{br}-k^i))}{w^*_i})$. We next prove non-borrowing suppliers prefer buyer finance if and only if $r_a \leq \hat{r}$, while borrowing suppliers prefer bank finance if and only if $r_a \geq \tilde{r}$.

For a non-borrowing supplier i, $\delta(c^i q_{br}^* - k^i) = 0$. Its profit in bank and buyer financing are, respectively, $\pi_n^i(q_{bk}^*) = \mathbf{E}_D[w_{bk}^{i*} \cdot min(q_{bk}^*, D) - c^i(1+r_s^i)q_{bk}^*] = c^i \cdot (1+r_s^i)(\frac{\mathbf{E_D}(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)} - q_{bk}^*)$, and $\pi_n^i(q_{br}^*) = \mathbf{E}_D[w_{br}^{i*} \cdot min(q_{br}^*, D) - c^i(1+r_s^i)q_{br}^*] = c^i \cdot (1+r_s^i)(\frac{\mathbf{E}_D min(q_{br}^*, D)}{\bar{F}(q_{br}^*)} - q_{br}^*)$. Because $\frac{\mathbf{E}_D min(q,D)}{\bar{F}(q)} - q$ is strictly increasing in q (Lemma 8), by Theorem 8, we have $\pi_n^i(q_{bk}^*) < \pi_n^i(q_{br}^*)$ if $r_a < \hat{r}$; $\pi_n^i(q_{bk}^*) = \pi_n^i(q_{br}^*)$ if $r_a = \hat{r}$; and $\pi_n^i(q_{bk}^*) > \pi_n^i(q_{br}^*)$ if $r_a > \hat{r}$.

For a borrowing supplier i, $\pi_{bk}^i(q_{bk}^*) = c^i \cdot (1+r_f)(\frac{\mathbf{E_D}(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)} - q_{bk}^*) + (r_f - r_s^i)k^i$ is independent of r_a because q_{bk}^* does not dependent r_a. $\pi_{br}^i(q_{br}^*) = c^i \cdot (1+\max_i\{r_s^i\})(\frac{\mathbf{E_D}(q_{br}^*, D)}{\bar{F}(q_{br}^*)} - q_{br}^*) + (\max_i\{r_s^i\} - r_s^i)k^i$ decreases in r_a since $\frac{\partial q_{br}^*}{\partial r_a} < 0$ (by Part (1) of the proof for sensitivity results in buyer finance) and $\frac{\mathbf{E_D}(q,D)}{\bar{F}(q)} - q$ increases in q (By Lemma 8). By Theorem 8, when $r_a \geq \hat{r}$, there is $q_{bk}^* > q_{br}^*$ and hence $\pi_{bk}^i(q_{bk}^*) > \pi_{br}^i(q_{br}^*)$. As $\pi_{br}^i(q_{br}^*)$ is continuous and decreases in r_a, there exists $\tilde{r} < \hat{r}$ such that when $r_a \geq \tilde{r}$, $\pi_{bk}^i(q_{bk}^*) \geq \pi_{br}^i(q_{br}^*)$. Because $\frac{\mathbf{E_D}(q,D)}{\bar{F}(q)} - q$ is increasing convex in q (By Lemma 8) and $\frac{\partial q_{br}^*}{\partial r_a} < 0$, there must be a sufficient small r_a such that $[\frac{\mathbf{E_D}(q_{br}^*, D)}{\bar{F}(q_{br}^*)} - q_{br}^*]/[\frac{\mathbf{E_D}(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)} - q_{bk}^*] > (1+r_f)/(1+\max_i\{r_s^i\}) + (r_f - \max_i\{r_s^i\})/(\frac{\mathbf{E_D}(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)} - q_{bk}^*)$ and thus $\pi_{br}^i(q_{br}^*) > \pi_{bk}^i(q_{bk}^*)$. To conclude, there exists $\tilde{r}$ such that $\pi_{br}^i(q_{br}^*) > \pi_{bk}^i(q_{bk}^*)$ when $r_a < \tilde{r}$, while $\pi_{br}^i(q_{br}^*) \leq \pi_{bk}^i(q_{bk}^*)$ when $r_a \geq \tilde{r}$. Q.E.D.

Proof of Corollary 9: When $\frac{k^i}{c^i} = \frac{k^j}{c^j}$ $\forall i, j = 1, 2, \cdots, N$ and $\frac{k^i}{c^i} < q_{bk}^*$, the equilibrium quantity q_{bk}^* and q_{br}^* , respectively, are solved from Eq. (A-1) and Eq. (A-2). By Eq. (A-2), q_{br}^* decreases in r_a. Let $\hat{r} = r_f + \frac{f(q_{bk}^*)(r_f - \max_i\{r_s^i\}) \cdot \mathbf{E_D} min(q_{bk}^*, D)}{[\bar{F}(q_{bk}^*)]^2}$ and substitute $r_a = \hat{r}$ into Eq. (A-2). We then have $q_{br}^* = q_{bk}^*$. Since q_{br}^* is monotonically decreasing in r_a, we have $q_{br}^* \geq q_{bk}^*$ when $r_a \leq \hat{r} = r_f + \frac{f(q_{bk}^*)(r_f - \max_i\{r_s^i\}) \cdot \mathbf{E_D} min(q_{bk}^*, D)}{[\bar{F}(q_{bk}^*)]^2}$.

1. From the above argument, when $\frac{k^i}{c^i} = \frac{k^j}{c^j}$ $\forall i, j = 1, 2, \cdots, N$ and $\frac{k^i}{c^i} < q_{bk}^*$, the equilibrium quantity in bank finance q_{bk}^* does not depend on $k^i, \forall i \in \{1, 2, \cdots, N\}$, hence $\hat{r} = r_f + \frac{f(q_{bk}^*)(r_f - \max_i\{r_s^i\}) \cdot \mathbf{E_D} min(q_{bk}^*, D)}{[\bar{F}(q_{bk}^*)]^2}$ is also independent of k^i and $\Sigma_{i=1}^N k^i$.

2. We first show $\hat{r} = r_f + \frac{f(q_{bk}^*)(r_f - \max_i\{r_s^i\}) \cdot \mathbf{E_D} min(q_{bk}^*, D)}{[\bar{F}(q_{bk}^*)]^2}$ is increasing in q_{bk}^*, which is true since both $\frac{f(q_{bk}^*)}{\bar{F}(q_{bk}^*)}$ and $\frac{\mathbf{E_D} min(q_{bk}^*, D)}{\bar{F}(q_{bk}^*)}$ are increasing in q_{bk}^*. Note that $\hat{r}$ depends on $\Sigma_{i=1}^N c^i$ only because q_{bk}^* depends on $\Sigma_{i=1}^N c^i$, and by Eq. (A-1), q_{bk}^* decreases in $\Sigma_{i=1}^N c^i$. Therefore, $\hat{r}$ decreases in $\Sigma_{i=1}^N c^i$.

3. $\hat{r} = r_f + \frac{f(q_{bk}^*)(r_f - \max_i\{r_s^i\}) \cdot \mathbf{E_D} min(q_{bk}^*, D)}{[\bar{F}(q_{bk}^*)]^2}$ decreases in $\max_i\{r_s^i\}$ because q_{bk}^* is independent of $\max_i\{r_s^i\}$. Q.E.D.