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Switching interest rate sensitivity regimes of U.S. Corporates

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ABSTRACT

We study interest rate sensitivities of U.S. investment grade BBB-rated and high yield corporate bonds over the period of 2001–2016. Our methodology assesses the capital gains of corporate bond portfolios and risk-free government bond portfolios, using average coupon and blended yield indices for the U.S. market. For both, U.S. BBB and high yield corporate bonds, we evidence the switching, from positive to negative interest rate sensitivity, occurring over the transition from the normal economic conditions to the periods of economic distress and vice-versa. The proposed theoretical explanation of such binary behavior posits an interrelation between interest rate and creditworthiness of issuers, which varies according to the phases of the business cycle. This research advances an economic understanding of interest rate risk management and sheds light on how financial institutions may develop strategies that hedge against downside risk.

1. Introduction

The odds that the Federal Reserve System of the United States will continue to raise interest rates puts a great pressure on the financial system. This is why interest rate (IR) sensitivity of assets has recently received a lot of attention from players in the financial sector and academics alike.

Important issues addressed in academic research include: sensitivity of corporate bond yield spreads to changes in the yield curve (Boulkeroua & Stark, 2013); determinants of bond risk premia (Bauer & Hamilton, 2015; Haddad & Sraer, 2015); flight-to-quality and spreads' changes (Fuerst, McAllister, & Sivitanides, 2015; Gubareva & Borges, 2016a); relationship between interest rates and credit spreads (Dupoyet, Jiang, & Zhang, 2016), and; integrated IR and credit risk management (Gubareva & Borges, 2016b), among others.

The regulatory bodies also try to create widespread awareness of the possible negative impacts of IR changes on bank balance sheets and profitability. For instance, the Basel Committee on Banking Supervision (BCBS) issued “Standards for Interest Rate Risk in the Banking Book” (BCBS, 2016). In this document, the Committee establishes the new rules for the supervision of Interest Rate Risk

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in Banking Book (IRRBB).

The large and mature U.S. corporate bond market offers a great diversity of available data which can be used to study the relationship between the yield spread of corporate bonds and changes in the yield of U.S. government bonds, see, for example: [Davies \(2008\)](#); [Piazzesi and Schneider \(2010\)](#); [Bauer and Hamilton \(2015\)](#) and [Begenau, Piazzesi, and Schneider \(2015\)](#), among others. Our research also utilizes the data-related advantage of the U.S. fixed income capital market.

The possible effects of IR increases – including changes in net interest margins, balance sheet structure, and values of interest-sensitive assets and liabilities, highlight the importance of the correct assessment of IR sensitivity of assets. IR sensitivity can be defined as being the measure of how much the price of a fixed-income asset is affected by changes in IR. The more the price of an asset varies, the more sensitive this asset is to IR. But what is really important for managing IRRBB is how the prices of assets react to medium term downward or upward trends in IR dynamics.

This paper investigates relationship between the capital gains of modeled U.S. corporate bond portfolios and the capital gains of modeled portfolios composed of U.S. Treasury (UST) securities. There are several reasons why it is important to analyze the IR sensitivity of assets in the form of sensitivity of the annual capital gains of a corporate bond portfolio to the capital gains of a government bond portfolio measured over the same time interval.

The first reason is that such capital gain-wise IR sensitivity better enlightens problems and solutions in interest rate risk (IRR) strategies for banking books of financial institutions (i.e., run-on portfolios with “hold-to-collect” and/or “hold to collect and sale” investment strategies), which are different from issues relative to the intra-day or short-term trading. The second reason is that, in real life, an investment fund’s capital gains, either positive or negative, are passed on to the fund’s investors at the end of every year ([Welch, 2016](#)).

However, IR sensitivity of corporate debt is traditionally analyzed in terms of yield sensitivity of corporate bonds to changes in the yield curve of risk-free assets; see [Davies \(2008\)](#), [Manzoni \(2002\)](#), [Landschoot \(2008\)](#), [Boulkeroua and Stark \(2013\)](#), [Neal, Rolph, Dupoyet, and Jiang \(2015\)](#), and associated references. Although spread-to-rate sensitivity issues have already been well studied in the literature, such a traditional approach addresses sensitivity through the prism of daily movements in prices, i.e., daily movements of yields and spreads, which are then averaged or regressed over diverse time windows. However, under certain conditions, such a treatment may appear to be rather senseless from a mid- and long-term investment point of view. For instance, in a volatile market, when daily positive movements of the UST prices can correspond to the daily minimums of corporate bond prices, which, accordingly, behave in the opposite way on a daily basis. Nevertheless, at the same time, both, the UST and corporate bond prices may follow a similar, either positive or negative trend over a longer term, e.g., on a monthly or yearly basis.

The situation is different if the concept of capital gain is applied to study IR sensitivity along longer periods, corresponding to the major moves in the prices of financial assets, as the daily noise does not affect the outcomes from trends anymore. However, the above-mentioned traditional spread-to-rate approach does not address the annual or longer-term capital gains of corporate bond portfolios, because researchers continue to be mainly interested in interest spread-to-rate relationship, which is derived from daily frequency data.

An analysis of financial assets shows that, in order to be able to sell an asset at a certain price, it is necessary to have bought it previously at the price that it had in the past. Therefore, what really matters for assessing IR sensitivity from a medium-to-long term investments perspective is a capital gains measure, which allows for the comparison of the capital gains of risky assets to the capital gains of hedging instruments, which, in our case, are short positions in UST. The net value of these capital gains directly influences the bottom line results of financial institutions. As bottom-line-oriented studies are rather rare, a study of U.S. corporate debt capital gain-wise sensitivity is highly desirable. In this sense, our work proposes a newly developed methodology, which reaches beyond marginal contributions by changing the proper paradigm of how IR sensitivity can be approached in the context of medium-to-long term investments.

Therefore, by focusing on IR sensitivity of capital gains of U.S. investment grade (IG) BBB-rated and high yield (HY) corporate debt portfolios, we attempt to advance empirical research of the joint dynamics of both, risk-free UST securities and risky U.S. corporate bonds. We consider two different cases when the U.S. IG and HY corporate bond portfolios are either hedged by short positions in UST bonds, or not. Our study also contributes to the research on interdependence between credit risk, interest rate risk, and liquidity risk, as it is related to downside risk management and financial stability improvement (see [Gubareva, 2014](#); [Gubareva & Borges, 2014, 2016a, 2016b](#)).

This paper aims to provide an answer to the following question: does it make economic sense to hedge the interest risk of U.S. BBB corporate debt by short positions in UST bonds, or by pay-fixed receive-float interest rate swaps (IRS)? The answer to this question will be of particular importance for IRR management and for dimensioning the amount of economic capital that needs to be allocated to mitigate this type of risk. Interestingly, the capital gain-wise sensitivity coefficient values vary in accordance with different phases of the business cycle. Accordingly, our research is potentially important, not only for the academic community, but also for players in the financial sector and regulatory bodies.

This paper is structured as follows. [Section 2](#) describes the data and details the scope of our study. [Section 3](#) introduces the methodology developed for modeling time series of annual and longer-term capital gains. [Section 4](#) presents the empirical results and [Section 5](#) discusses the implications of the results obtained. [Section 6](#) offers concluding remarks.

2. Data description

The price dynamics of both U.S. IG BBB-rated and U.S. HY corporate bond portfolios is modeled, using a data set of blended yields and average coupons of the two respective sets of corporate bonds. Similarly, the price behavior of a portfolio consisting of U.S.

government bonds is modeled using a data set of blended yields and average coupons of UST. Based on yield and coupon indices, which describe these assets, we investigate the sensitivity of capital gains of modeled U.S. BBB and HY corporate bond portfolios.

To describe the IG BBB-rated corporate debt dynamics we use the Citi Corporate BBB 3 to 7 Sector Yield to Maturity Local (Bloomberg ticker: SAYLYL) and also the Citi Corporate BBB 3 to 7 Average Coupon Local (Bloomberg ticker: SAYLCP) indices, whose constituents are IG BBB-rated bonds with maturity between 3 and 7 years issued by over 200 corporate issuers. The monthly data provided by these BBB-rated yield and coupon indices covers a period of more than 15 years, from March 2001 to August 2016.

Those indices that have a monthly frequency are chosen on purpose, for as we are interested in a medium-to-long term investments perspective, and not intra-day or short-term trading strategies, there is little value added in examining data of a higher frequency, such as daily or weekly data. By using monthly indices, apart of lowering the complexity of non-core calculations, our model attempts to “separate the wheat from the chaff”, especially with regards to daily frequency data, which are subject to a higher level of informational noise, which is naturally partly cancelled out over longer periods, such as monthly ones.

To model the price dynamics of the U.S. HY corporate portfolio, we employ Bloomberg Barclays US Corporate HY Yield to Worst (Bloomberg ticker: LF98YW) bond index, which is composed of USD-denominated securities, which are classified as HY if the middle credit rating of Moody’s, Fitch and S&P rating agencies is, respectively Ba1/BB+/BB+ or below. During the period under analysis from March 2001 to August 2016, the index’s average Macaulay duration of 4.2 years, together with the index’s average yield of 8.8% corresponds to an average constituent bond maturity of approximately 5 years. Based on this information, we also construct the average coupon index, which is calculated as being the mean of the yield, computed over the 5-year long period before the reference date.

On the other hand, to analyze the universe of U.S. government securities, we choose the Citigroup indices for the UST, namely the Citi Treasuries Yield to Maturity Local (Bloomberg ticker: SA14YL) and the Citi Treasuries Average Coupon Local (Bloomberg ticker: SA14CP). The monthly data provided by these UST yield and coupon indices covers a period more than 35 years long, however we use the same time interval as that of the corporate debt data, i.e., from March 2001 to August 2016.

It is worth commenting that the price index would probably have a better choice for analyzing the dynamics of capital gains in debt portfolios, however, to the best of our knowledge, no price indices with similar outstanding and historic series coverage are available in the market. Therefore, instead of researching individual bond price histories and/or developing a range of bond price indices from a selected universe of individual bond data, we opt to use the six above-mentioned indices – three yield indices and three coupon indices. In our research, we do not employ total return indices, as the reinvestment of the net interest income proceeds does not enter the scope of our research.

This paper differs from mainstream research in the field of IR sensitivity and the interdependence of interest rate risk and credit risk, as we do not focus on yield spreads, whether they are analyzed by rating and/or sector, or at individual bond level. The primary focus of our research is to analyze the annual capital gains of bond portfolios, in general, and, in particular, to study the inter-relation between the capital gains of chosen modeled IG BBB-rated and HY U.S. corporate bond portfolios and the capital gains of the respective U.S. government bond portfolio that has a similar average maturity profile.

The next section describes the methodology used to carry out a comprehensive analysis of U.S. corporate and UST bond portfolios, based on the time series of the blended yield and average coupon indices, which is extended to interest rate risk hedging, based on shorting UST.

3. Methodology

The main element of our yield-and-coupon based framework is a conversion of the available index data on blended yields and average coupons into the average price of the modeled portfolios, namely the: U.S. IG BBB-rated corporate; U.S. HY corporate, and UST portfolios, in accordance with the indices employed. The price of the modeled portfolio is generated by discounting future cash flows, consisting of coupon and principal payments. For simplicity’s sake, we consider annual coupon payments and principal redemption on maturity date.

Matured bonds are removed from the indices and newly issued ones are included as time goes on. Those bonds that are downgraded to HY are removed from IG BBB indices, while bonds upgraded to IG are removed from HY indices. We assume that the bond weight composition of our modeled portfolios mimics the weight composition of the index’s eligible debt.

Accordingly, the assumption is that the continuous rebalancing of our modeled portfolios is consistent with the continuous rebalancing of the indices employed. This assumption is frequently used to study risk minimization strategies for portfolio immunization: see for instance [Fong and Vasicek \(2015\)](#). In our case, the “continuous” rebalancing occurs on a monthly basis.

Let us now consider an example of U.S. government debt. As of June 2015, the historical weighted average maturity of marketable debt outstanding from 1980 to June 2015 is equal to 59 months ([U.S. Department of the Treasury, 2015](#)). This figure closely corresponds to the five years, allowing for considering five annual payments of average coupons and the repayment of the amount of principal at maturity.

Therefore, at every point in time, the price of modeled U.S. government debt portfolio with five-year residual maturity (P_{UST}), paying average annual coupon (c_{UST}) and having a face value (p_{UST}), could be written as:

$$P_{UST} = \frac{c_{UST}}{1 + y_{UST}} + \frac{c_{UST}}{(1 + y_{UST})^2} + \frac{c_{UST}}{(1 + y_{UST})^3} + \frac{c_{UST}}{(1 + y_{UST})^4} + \frac{c_{UST} + p_{UST}}{(1 + y_{UST})^5} \quad (1)$$

where y_{UST} is a market defined IR. For simplicity reasons the IR term structure is assumed to be flat. As we are analyzing the UST securities, y_{UST} is the blended yield provided by the SA14YL index. c_{UST} is the average coupon, provided by the SA14CP index. In our

research, we set the principal or face value of our modeled UST portfolio (p_{UST}) as being equal to US\$1000 million.

As we can now price our portfolio of safe assets at any date covered by the employed SA14YL yield and SA14CP coupon historical series, we are thus able to study the dynamics of the annual capital gains. We focus on capital gains, as we are interested in hedging the asset value of U.S. corporate portfolios by shorting UST.

In our research, we consider that the capital gain, or price change, is the difference between the initial price and the final price of our assets, i.e., of the modeled portfolios mimicking compositions of the respective indices, excluding interim coupon payments. As the indices are continuously rebalanced, our assets are continuously changing. In a real world, rebalancing leads to the presence of transaction costs, however, in our study, the transaction gains and expenses are not taken into account, as we are not assessing the efficiency of rebalancing.

The focus on capital gains represents an innovative and distinctive feature of our research, as other researchers almost always work with rates of return, yield, and /or spreads, rather than with capital gains. Rare exceptions are made when the research analyzes taxes, as the tax legislation usually treats capital gains differently from interim payments. Accordingly, given the rather scarce frequency of this term in research on IR sensitivity, it is important to clearly define what is understood by using the term “capital gains” – which, to repeat, is solely the difference between the initial price and the final price of a chosen portfolio.

The possibility of obtaining historical price series for the modeled mirror portfolio of U.S. safe assets enables us to quantify the capital gain that occurred in this portfolio over any chosen period of time as being the difference between portfolio prices subjacent to the two chosen dates:

$$CG_{UST}(t, H) = P_{UST}(t + H) - P_{UST}(t) \quad (2)$$

where CG_{UST} stands for the capital gains of the UST portfolio, t is the initial date of the analyzed time interval, and H stands for the time horizon over which the capital gains are assessed. In our research, we use a time horizon of one year.

The same approach is also applied for the analysis of the capital gains of the BBB corporate portfolio, whereby the average life of the bond basket subjacent to the employed Citi Corporate BBB 3 to 7 Sector Yield to Maturity Local is 4.92 years for the period from March 2001 to August 2016. This figure corresponds to the average value of the Citi Corporate BBB 3–7 Sector Average Life Local index and is very close to the 5Y maturity term. At every point in time, the price of modeled U.S. BBB corporate debt portfolio whose maturity is similar to the residual maturity of UST portfolio can be calculated as:

$$P_{BBB} = \frac{c_{BBB}}{1 + y_{BBB}} + \frac{c_{BBB}}{(1 + y_{BBB})^2} + \frac{c_{BBB}}{(1 + y_{BBB})^3} + \frac{c_{BBB}}{(1 + y_{BBB})^4} + \frac{c_{BBB} + P_{BBB}}{(1 + y_{BBB})^5} \quad (3)$$

where c_{BBB} stands for average annual coupon, P_{BBB} stands for face value of a principal payment, and y_{BBB} is a market IR for the level of riskiness associated with the modeled mirror portfolio of BBB-rated bond universe. For reasons of simplicity, the IR term structure is assumed to be flat. As we are analyzing the U.S. BBB corporate securities, y_{BBB} is the blended yield provided by the SAYLYL index, and c_{BBB} is the average coupon provided by the SAYLCP index. Similar to the case of UST, in our research we set the principal or face value of our modeled BBB-rated corporate portfolio P_{BBB} as being equal to US\$1000 million.

For the U.S. HY portfolio subjacent to the employed U.S. HY FL98YW corporate bond index with average maturity in closed proximity of 5Y point, the same methodology as that used for the U.S. IG BBB-rated portfolio which is provided by Eq. (3) is applied, but this time with the yield and coupon values, which corresponds to the HY debt.

Similar to the UST case, the historical price series for the modeled mirror portfolio of U.S. BBB-rated corporate bonds allow us to quantify the capital gains that occurred in this portfolio over any chosen period of time as being the difference between the two portfolio prices, namely:

$$CG_{BBB}(t, H) = P_{BBB}(t + H) - P_{BBB}(t) \quad (4)$$

where CG_{BBB} stands for the capital gains of the U.S. BBB corporate bond portfolio, t is the initial date of the time interval under analysis, and H stands for a horizon over which the capital gains are assessed.

Likewise, the capital gains of the U.S. HY corporate bond portfolio over the time horizon H , $CG_{HY}(t, H)$ are defined, in line with Eq. (4), as being the difference between the final and initial prices, $P_{HY}(t + H)$ and $P_{HY}(t)$, respectively.

As already mentioned in the discussion following Eq. (1), it is assumed that the yield curves used to calculate prices of UST and BBB corporate securities are flat – see Eqs. (1) and (3), respectively. Although this rather strong model assumption is employed in our intermediary price calculations, an eventual roughness is to a certain degree eliminated from the capital gains as per Eqs. (2) and (4). The proper focus of our approach on capital gains figures leads us to be interested in price changes, rather than in their absolute values – see Eqs. (2) and (4). By calculating a relative value, a flat-curve-caused bias in initial and final prices, in a major part eliminates itself when calculating the capital gain measures.

To evidence that our results are sufficiently robust, we perform robustness checks by relaxing our model flat-curve assumption. Instead of employing this assumption in our calculations, we use the respective risk-free and risky term structures with a linear-slope. As shown in Appendix I, such positively- and negatively-sloped term structures result in fairly similar outcomes to those obtained under the flat curves assumption, which thus corroborates that the proposed methodology enables one to reach robust conclusions related to the capital gains and IR sensitivity of assets under diverse model choices of term structure profiles.

Formulas (2) and (4) are used to calculate the time series of annual capital gains, CG_{UST} and CG_{BBB} , respectively. It is now possible to use the pair of the CG_{UST} and CG_{BBB} time series to assess the sensitivity of the relatively risky U.S. BBB-rated corporate bond portfolio capital gains to the capital gains of the risk-free U.S. government bond portfolio.

This sensitivity can be assessed as being the ratio of a capital gain change ΔCG_{BBB} to the capital gain change ΔCG_{UST} of the

portfolio composed by U.S. government bonds, both provoked by the same shift of a gauging H -year-long time interval from t_1 to t_2 :

$$S_{BBB/UST}(t_2, t_1, H) = \frac{CG(t_2, H)_{BBB} - CG(t_1, H)_{BBB}}{CG(t_2, H)_{UST} - CG(t_1, H)_{UST}} = \frac{\Delta CG(t_2, t_1, H)_{BBB}}{\Delta CG(t_2, t_1, H)_{UST}} \tag{5}$$

where $S_{BBB/UST}(t_2, t_1, H)$ stands for the capital gain-wise sensitivity of BBB-rated corporate bonds when the investment time horizon H is moved forward by the number of days equal to $(t_2 - t_1)$.

As an analogy with the BBB case, we use the same approach to assess HY corporate debt sensitivity:

$$S_{HY/UST}(t_2, t_1, H) = \frac{CG(t_2, H)_{HY} - CG(t_1, H)_{HY}}{CG(t_2, H)_{UST} - CG(t_1, H)_{UST}} = \frac{\Delta CG(t_2, t_1, H)_{HY}}{\Delta CG(t_2, t_1, H)_{UST}} \tag{6}$$

We posit that for a more meaningful assessment of capital gain-wise sensitivity, the capital gain gauging window of the length H should be consecutively dislocated forward in such a manner that its displacement fits the more pronounced moves in capital gains time series of the modeled UST portfolio. We can intuitively say that as a smaller value in the denominator of formulas (5) and (6) can amplify computation uncertainty during the calculation of the ratio, at its limit, the zero capital gain change of the UST portfolio renders the ratio incomputable. To understand more about the techniques employed for identifying the local extrema of time series, see Gubareva and Borges (2016a).

Such an approach to IR sensitivity results from the fact that we are interested in average capital gains over rather extended, 1-year long, time intervals. Our method fits well with the basis IRR hedging strategy, which consists of shorting UST. Accordingly, when assessing the performance of U.S. corporate BBB and HY portfolios, we can assess whether short positions in UST are able to perform the role of an efficient hedge instrument.

The mechanics of our capital gain-wise sensitivity assessment is discussed in more detail in the next sections, which are dedicated to the empirical results, discussions, and their implications.

4. Empirical results

4.1. Analyses of the U.S. Government and U.S. BBB corporate bond indices

Fig. 1 presents the historical time series of the yields and coupons of U.S. government bonds as per the SA14YL and SA14CP indices, respectively.

Analyzing the joint behavior of the blended yield and average coupon of the UST, it can be stated that the average coupon almost always exceeds the corresponding yield rate, with the exception of the two-year long period that precedes the global financial crisis, namely from the second half of 2005 up until the end of the first half of 2007. This phenomenon of coupon values exceeding yield rates is a reflection of the predominantly diminishing risk-free interest rates dynamics that have been observed over the last 30 years, including the period analyzed in our study. This phenomenon occurs because the average coupon of a current UST portfolio is influenced by the history of yields, which were observed in the past, while the constituent UST securities were issued, usually in a closed proximity to par. On the contrary, the rising interest rates of the 2005–2007 period, combined with the falling interest rates during 2007–2008, result in a quasi-flat behavior of the coupon for 2006–2007, when the increasing yield (2006) led to the coupon and yield remaining in close proximity.

Fig. 2 presents the historical time series of the yields and coupons of the U.S. BBB corporate bonds as per the S200YL and S200CP indices, respectively.

During the major part of the time interval under analysis, similar to the case of the UST, the average coupon exceeds the corresponding yield rate, due to the same reasons as those outlined below Fig. 1. The exception to such dynamics is notable for the period at the peak of the global financial crisis, where a spike in the blended yield over the average coupon level is clearly observable. This sharp increase in yield is due to the perceived deterioration of credit quality, brought about by the inherent risks of the deepening of the global economic and financial crisis, whereas the following sharp decrease in yield is due to the diminishing risks experienced along the period of recovery from the crisis. In their turn, coupons, which are influenced by the average level of yields, observed for the issues of the constituents of the U.S. BBB corporate portfolio, does not react to short-term changes in the yield in a

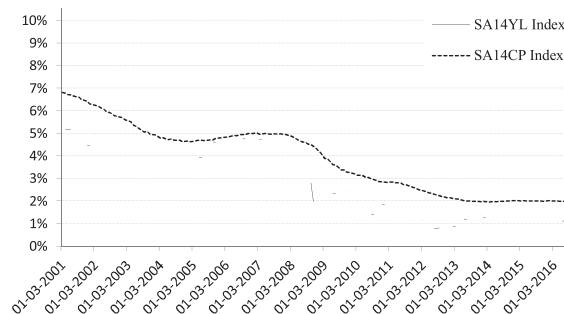


Fig. 1. UST: Blended yield (SA14YL Index) and average coupon (SA14CP Index).

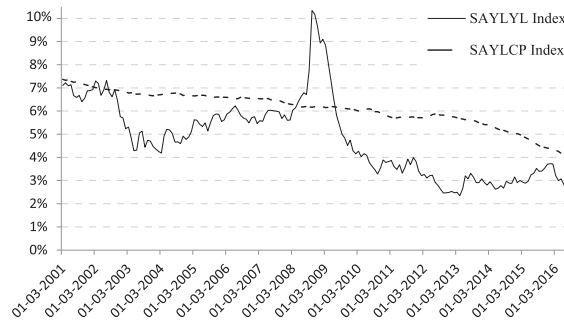


Fig. 2. BBB corporates: blended yield (SAYLYL Index) and average coupon (SAYLCP Index).

significant way, which incorporate perceived changes in the riskiness of the issuers.

Fig. 3 presents the historical time series of the yields and coupons of U.S. HY corporate bonds as per the LF98YW index and LF98CP indices, with the latter resulting from the proprietary calculations, based on averaging the yield values over the 5-year long period preceding the current date.

When compared to the BBB case, the time dynamics of HY bonds' yield and coupon is seemingly less sensitive to the predominantly diminishing risk-free interest rates that have been observed over the last 30 years, including the period under analysis in our research. As the ranges of yield and coupon variations for HY Corporates are situated higher up on the vertical scale, than those of BBB corporates, is it plausible to conclude that the risk-free rate component plays a less significant role in HY corporate yield. Accordingly, HY Corporate debt should be less sensitive to risk-free rates than BBB Corporate debt, which is demonstrated quantitatively later on in this paper.

Similar to the BBB case, the sharp increase in HY yield after the start of the global crisis does not affect the average coupon figures in a significant way. However, such influence is more pronounced in the case of HY, although it remains limited to a rather moderate increase in the coupon values during 2009.

In the next subsection, we use the coupon and yield data to generate price histories for the selected portfolios composed of U.S. corporate and U.S. government bonds.

4.2. Time series of the portfolios' present values

During analyses performed at the level of individual bonds, the relative values of a coupon rate and a yield rate, along with other parameters, define a bond price, which could be useful for classifying a bond as a premium bond, par bond, or discount bond. However, as our price modeling is performed directly at the portfolio level, the price of each modeled portfolio is described as an aggregate asset, which means that it is either a premium asset, a par asset, or a discount asset – in a direct analogy with the classification of an individual bond.

The historic time series for the present values of the risky BBB and HY portfolios and the risk-free UST modeled portfolios are then calculated. We consider the face values of these three portfolios to be constant along the time under analysis, with a value of US\$1000 million. We use the coupon and yield indices data to generate the price series and it is assumed that the weights of bonds in the U.S. BBB corporates, U.S. HY corporates, and UST portfolios mimic the weights of the SAYLYL, LF98YW, and SA14YL indices constituents, respectively.

Fig. 4 represents the price graphs for U.S. BBB-rated corporates, U.S. HY corporates, and UST portfolios.

The comparative analysis of these plots appears to show that the price behavior is quite similar among all the portfolios since the second half of 2003, up until the beginning of the global financial crisis, which occurred during the middle of 2007. Since the second half of 2007, up until the peak of the crisis, a huge flight-to-quality event is observed for both IG BBB-rated and HY corporate

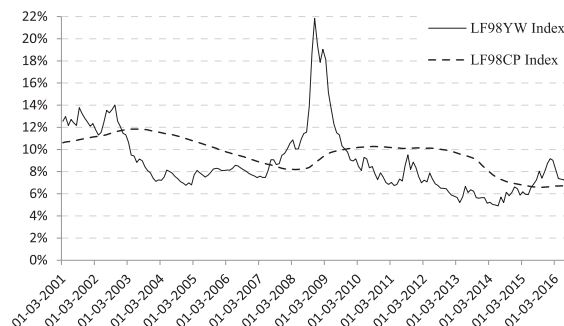


Fig. 3. BBB corporates: blended yield (LF98YW Index) and average coupon (LF98CP Index).

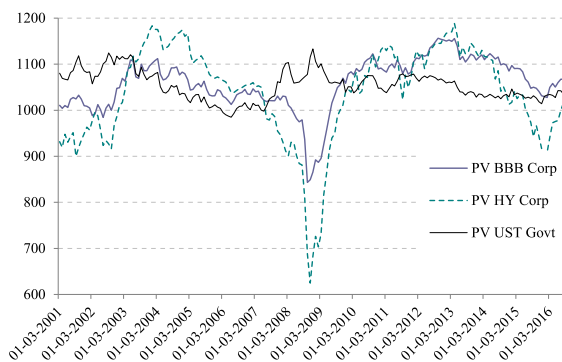


Fig. 4. Present values (PV): U.S. BBB corporates, U.S. HY corporate bonds, and UST portfolios.

portfolios. This corresponds to the rise in prices of safe assets and the decrease in prices of risky bonds. In 2009, all the three curves converged, providing evidence of a recovery from this flight-to-quality.

It is worth noting that the overall ranges of the present values of the modeled portfolios observed along the period of 2003–2009 differ quite considerably. The range of the UST portfolio (circa US\$150 million) is narrower than the respective ranges of the present values of the U.S. BBB-rated corporate bond portfolio (US\$250 million approximately) and also than the U.S. HY corporate bond portfolio (US\$550 million approximately). The periods during which the PV of a modeled portfolio is below/above the nominal of US \$1000 million correspond to the premium/discount classification of the analyzed aggregate asset.

4.3. Modeled historical series of capital gains and losses

In this subsection, we study the dynamics of the historical series of the capital gains and losses in the U.S. BBB and HY corporate bond portfolios, comparing them to the respective price changes of the UST securities. Subtracting the UST-related capital gains from the capital gains of either the U.S. BBB-rated or HY corporate debt portfolio, we obtain the capital gains for both the U.S. BBB-rated and U.S. HY corporate debt portfolios, while hedged by the short positions of the UST.

Fig. 5 represents the time behavior of the 1-year changes in present value for three modeled portfolios, namely: the U.S. BBB corporate bond portfolio; the risk-free UST portfolio, and; the U.S. BBB corporate bond portfolio hedged by shorting the UST portfolio.

What is the meaning of this chart? The points plotted for March 31, 2002, represent the changes in the present values of the respective portfolios that occurred over the 1-year long period, starting on March 31, 2001. It can be observed that the annual capital gains of the BBB corporate bond portfolio and UST bond portfolio are in opposite mode since July 2007 up until December 2010, i.e., during the period affected by the crisis-related financial turmoil. Accordingly, the IR hedging of the U.S. BBB corporate portfolio with the short UST positions does not compensate for the negative impacts, when such set-off are the most needed.

Qualitatively similar conclusions regarding the appropriateness of such short-UST hedging for U.S. HY corporate debt can be also be obtained through the comparative analysis of the capital gains of the respective HY corporate and UST portfolios. Fig. 6 below presents our results for this case.

Similar to the BBB case, the IR hedging of the U.S. HY corporate portfolio with the short UST positions does not compensate for the negative impacts, as such compensation is more desirable, and it even amplifies the volatility of the capital gains of the HY hedged portfolio in comparison with the unhedged investment.

Comparing the amplitudes of the major crisis-induced fluctuations that were suffered by the U.S. BBB-rated and U.S. HY corporate portfolios – see Figs. 5 and 6, respectively, one can conclude that HY debt is impacted by financial and economic turmoil more strongly than in the case of the BBB-rated securities.

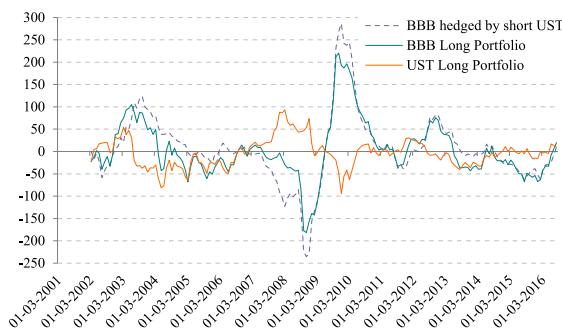


Fig. 5. Annual capital gains (CG): UST and BBB corporate bonds, unhedged, and hedged by UST.

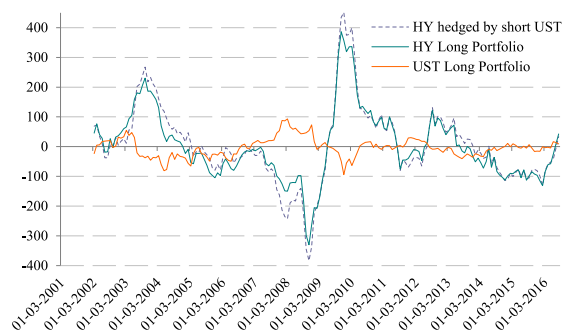


Fig. 6. Annual capital gains (CG): UST and HY corporate bonds, unhedged and hedged by UST.

4.4. Price-wise interest rate sensitivity of U.S. BBB corporates

In this section, we perform a quantitative assessment of the capital gain-wise sensitivity of the modeled U.S. BBB corporate portfolio to changes in prices of the corresponding UST portfolio. Rather than assessing average sensitivity figures for all the available 15-year long data history, we are interested in gauging sensitivity behavior within much shorter time intervals that are determined by the local extrema, i.e. turning points, which separate upward and downward tendencies in UST capital gains dynamics.

Following the big moves of the UST, i.e., rises and falls in capital gains, allows for diminishing denominator-related uncertainty in Eq. (5) and, accordingly, for achieving a more precise assessment of capital gain-wise sensitivity. Applying the methodology developed in Gubareva and Borges (2016a), we identify the local extrema of the modeled historical series of the annual capital gains of the UST portfolio presented in the previous subsection.

We identify 51 considerable rises and falls of the capital gains of the UST portfolio and distribute them among 3 periods. To be able to do this, a statistical analysis is performed to study the homogeneity of the observed sample of the 51 capital-gain wise sensitivities, which correspond to the 51 time-intervals as per Tables 1–3, which are presented further on in the paper.

First, we divide the whole sample into two subsamples, using a clustering approach, based on standard deviation (St.Dev.) minimization. That is to say, we arrange the sensitivities in a chronological order and vary a number of major UST capital gain moves in the subsamples from 1 to 50, i.e., we vary the final date of one trial subsample, which is, in effect, just the initial date of the other subsample. For each of the 50 divisions of the whole spectrum of arrays, the combined St.Dev. of sensitivities is calculated in such a way that, instead of calculating the whole sample average, the respective averages of sub-arrays are used to calculate the deviation of each sensitivity value according to its positioning in one of the sub-arrays. Fig. 7 shows the combined St.Dev. as a function of the date used to separate time interval into two trial sub-intervals.

The behavior of the combined St. Dev. shown in Fig. 7 presents the two major local minima, whose dates, namely May 31, 2007 and September 30, 2012, separate the whole sample into the three most homogeneous subsamples.

As a sanity check, we perform the refined calculations of the St.Dev. combined from the three sample-as-stand-alone St.Dev. values. For instance, as the sub-array border between the second and the third sub-arrays occurs on September 30, 2012, changing the

Table 1

Pre-crisis sensitivity figures per major moves in UST capital gains.

Date	Elapsed time (days)	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Δ US HY Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Sensitivity Δ US HY/ Δ UST
31/03/2002						
30/09/2002	183	43.10	10.97	-19.14	0.25	-0.44
31/10/2002	31	-22.80	-21.47	-29.25	0.94	1.28
31/03/2003	151	57.85	107.93	68.97	1.87	1.19
30/04/2003	30	-17.32	18.13	27.54	-1.05	-1.59
31/05/2003	31	9.51	3.40	11.31	0.36	1.19
31/12/2003	214	-92.94	-47.97	81.69	0.52	-0.88
31/03/2004	91	15.55	0.39	-55.59	0.02	-3.58
31/05/2004	61	-50.99	-91.78	-93.40	1.80	1.83
31/08/2004	92	61.55	65.89	0.86	1.07	0.01
31/03/2005	212	-45.74	-91.02	-92.04	1.99	2.01
30/06/2005	91	62.38	57.86	30.66	0.93	0.49
31/10/2005	123	-45.72	-50.39	-65.01	1.10	1.42
28/02/2006	120	30.25	35.40	-9.82	1.17	-0.32
30/06/2006	122	-30.73	-20.32	23.22	0.66	-0.76
30/11/2006	153	57.77	59.90	56.22	1.04	0.97
31/01/2007	62	-14.15	-24.90	1.70	1.76	-0.12
30/04/2007	89	26.27	25.20	7.72	0.96	0.29
31/05/2007	31	-7.61	-6.13	6.67	0.81	-0.88

Table 2

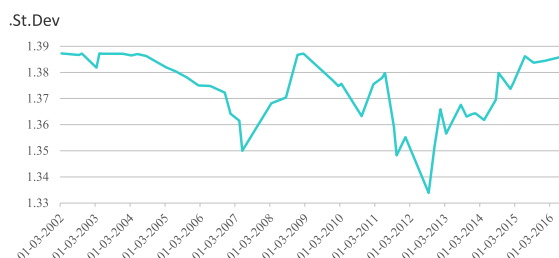
Through-the-crisis sensitivity figures per major moves in UST capital gains.

Date	Elapsed time (days)	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Δ US HY Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Sensitivity Δ US HY/ Δ UST
31/05/2007						
31/03/2008	305	80.28	-38.08	-147.74	-0.47	-1.84
31/08/2008	153	-50.00	-12.51	52.19	0.25	-1.04
31/12/2008	122	29.89	-114.44	-166.39	-3.83	-5.57
28/02/2009	59	-82.25	14.01	56.01	-0.17	-0.68
31/12/2009	306	-84.95	335.05	564.79	-3.94	-6.65
28/02/2010	59	52.42	3.98	-21.18	0.08	-0.40
31/03/2010	31	-21.58	-15.07	0.55	0.70	-0.03
31/10/2010	214	79.97	-114.42	-215.06	-1.43	-2.69
28/02/2011	120	-26.01	-64.10	-22.39	2.46	0.86
31/05/2011	92	12.42	12.89	1.45	1.04	0.12
30/06/2011	30	-12.87	-12.46	-28.36	0.97	2.20
30/09/2011	92	13.51	-37.04	-147.78	-2.74	-10.93
31/10/2011	31	-5.53	3.53	30.75	-0.64	-5.56
31/01/2012	92	32.09	43.27	19.75	1.35	0.62
30/09/2012	243	-39.10	55.68	147.75	-1.42	-3.78

Table 3

Post-crisis sensitivity figures per major moves in UST capital gains.

Date	Elapsed time (days)	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Δ US HY Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Sensitivity Δ US HY/ Δ UST
30/09/2012						
30/11/2012	61	2.97	6.84	-26.21	2.30	-3.13
31/01/2013	62	-13.36	-30.41	-34.70	2.28	2.60
31/03/2013	59	16.40	-7.57	-9.05	-0.46	-0.55
31/08/2013	153	-37.74	-74.15	-67.04	1.96	1.78
31/10/2013	61	15.58	1.67	15.46	0.11	0.99
31/12/2013	61	-11.12	-8.85	-33.07	0.80	2.97
31/01/2014	31	11.94	8.66	-19.11	0.73	-1.60
30/04/2014	89	-9.37	-4.17	-21.22	0.44	2.26
31/08/2014	123	31.44	49.90	35.92	1.59	1.14
30/09/2014	30	-11.33	-24.31	-46.98	2.15	4.15
31/01/2015	123	24.06	-4.56	-15.07	-0.19	-0.63
28/02/2015	28	-11.36	-11.53	8.49	1.02	-0.75
31/03/2015	31	9.89	10.51	-1.01	1.06	-0.10
30/06/2015	91	-14.39	-30.66	-13.62	2.13	0.95
30/09/2015	92	11.48	1.16	5.97	0.10	0.52
31/01/2016	123	-22.32	-19.00	-16.54	0.85	0.74
30/06/2016	151	32.60	59.83	89.24	1.84	2.74
31/08/2016	62	-10.31	-0.96	24.98	0.09	-2.42

**Fig. 7.** St.Dev of sensitivities as a function of the date separating the two trial sub-arrays.

quantity of the constituent time intervals between the first and the second sub-arrays in -1 , 0 , and 1 , results, respectively, in the following combined St.Dev. values: 1.24, 1.23, and 1.25. Similarly, as the sub-array border between the first and the second sub-arrays occurs on May 31, 2007, changing the quantity of the constituent time intervals between the second and the third sub-arrays in -1 , 0 , and 1 , results, respectively, in the following combined St.Dev. values: 1.27, 1.23, and 1.28. Thus, the three statistically selected sub-arrays, with sub-array borders on May 31, 2007 and September 30, 2012 are reconfirmed, which, in fact, represent the most homogeneous sub-samples of sensitivity values, resulting in the minimum possible combined St.Dev. of 1.23.

Accordingly, we allocate the following: 18 UST capital gain moves to the “old normal” pre-crisis period of March 2002 – May

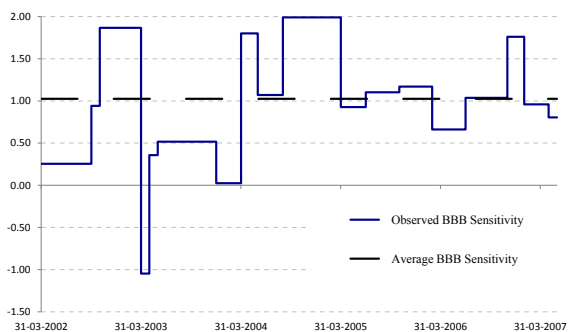


Fig. 8. Price-wise interest rate sensitivity of U.S. BBB corporate bonds prior to the crisis.

2007; 15 moves to the crisis turmoil of May 2007 – September 2012, and; 18 moves to the “new normal” post-crisis period of September 2012 – August 2016. The sensitivity figures are calculated separately for each capital gain move. We analyze IR sensitivity from the portfolio risk management perspective and then attempt to understand how U.S. corporate bond prices respond to the changes in UST prices.

4.4.1. Price-wise interest rate sensitivity during the pre-crisis period

Table 1 shows 18 major moves in capital gains of UST during the “old normal” pre-crisis period. For each time window, we calculate the capital gain deltas for three portfolios: the UST; U.S. BBB, and; U.S. HY modeled portfolios. The sensitivity coefficient is calculated as being the ratio of the capital gain delta observed in the U.S. BBB and HY corporate portfolios to the “inducing” delta observed in the UST portfolio.

Fig. 8 below shows BBB historic sensitivity as per the second from the right column of Table 1.

The dynamics of sensitivity exhibits a certain volatility, especially before 2004. When the value of sensitivity coefficient is below 1, the impact of risk-free IR changes is attenuated, or even inverted if the coefficient turns negative. When the coefficient is above 1, the impact of risk-free IR on U.S. BBB corporate prices is amplified.

Table 1 shows that the length-times-magnitude weighted average of sensitivity is found to be 1.025, which can be seen in the dotted line in Fig. 8.

During this interval of March 2002–May 2007, the use of short positions in UST represents a highly efficient hedge for U.S. BBB corporate bonds, as the sensitivity coefficient is very close to 1. It means that the annual capital gains relative to the U.S. BBB corporate portfolio closely mirrors the annual capital gains of the UST portfolio, i.e., a move in the risk-free IR is entirely passed on to the yield of U.S. BBB corporate bonds.

In an attempt to provide alternative evidence for the above findings, we present the results of a statistical analysis that was performed through the prism of a mainstream linear regression model. Fig. 9 below represents the graph of the relationship between the changes in the capital gains of the UST portfolio (ΔCG_{UST}) and the changes in the capital gains observed over the respective periods in the BBB U.S. Corporate portfolio (ΔCG_{BBB}), as per the respective columns of Table 1.

The slope of the dotted trend-line in Fig. 9 captures the sensitivity that we aim to assess. The regression beta coefficient is statistically significant, as certified by the elevated R-squared value of 0.77. The value of beta coefficient of 1.023, i.e., the sensitivity value obtained through regression, is very similar to the length-times-magnitude weighted average of BBB sensitivity of 1.025, which is calculated directly from Table 1. Nevertheless, we claim that the length-time-magnitude weighting gives a more precise and justified outcome from a hedging perspective, as it places more importance on strong and long-lived trends in UST capital gains,

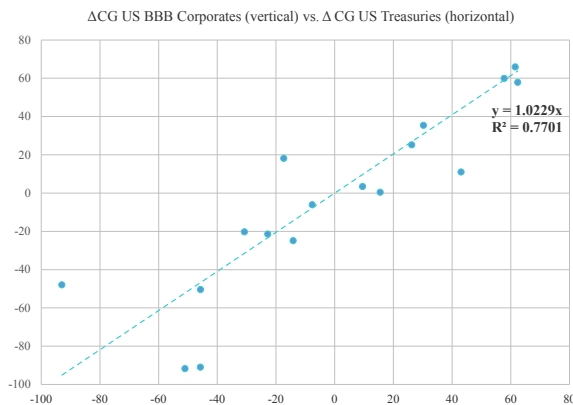


Fig. 9. $\Delta CG_{BBB} - \Delta CG_{UST}$ relationship before the crisis.

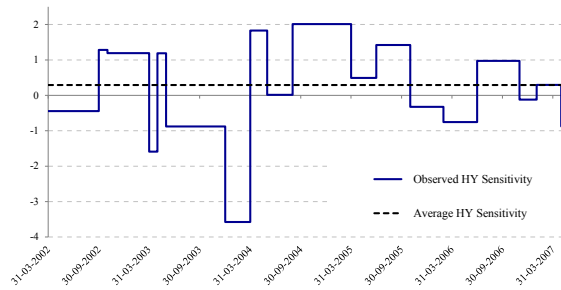


Fig. 10. Price-wise interest rate sensitivity of U.S. HY corporate bonds prior to the crisis.

which seemingly should have a bigger impact on both portfolios than weak and short-lived moves. An additional argument in favor of length-time-magnitude weighting versus the linear regression model approach is that the former, being a polyline-like method by construction, is not reliant on the quite strong and limiting assumption regarding the linearity of the relationship.

Next, we proceed to the consideration of the U.S. HY corporate case. Fig. 10 below shows HY historical sensitivity as per the right-hand column of Table 1.

The dynamics of HY sensitivity, as per Fig. 10, exhibits higher volatility than the BBB sensitivity behavior depicted in Fig. 8.

As per Table 1, the length-times-magnitude weighted average of HY sensitivity is found to be 0.291 – see the dotted line in Fig. 10. This value is considerably smaller than the average BBB sensitivity of 1.025 that was discussed earlier on in this Sub-section. The average HY sensitivity prior to the crisis of 0.291 signifies that a move in the risk-free IR is only partially passed on to the yield of U.S. HY corporate bonds. In other words, this rather low value of HY average sensitivity, which is relatively close to zero, indicates that the U.S. HY corporate debt is only weakly sensitive to the risk-free rates. It means that the capital gains of the HY portfolio somewhat loosely accompany the moves of the UST capital gains, without the tight mirroring, which is verified in the BBB case.

Below we present the results of a statistical analysis of HY debt sensitivity, performed through the prism of a mainstream linear regression model. Fig. 11 shows the graphical representation of the relationship between the changes in the capital gains of the UST portfolio (ΔCG_{UST}) and the changes in the capital gains observed over the respective periods in the HY U.S. Corporate portfolio (ΔCG_{HY}), as per the respective columns of Table 1.

The slope of the dotted trend-line in Fig. 11 is meant to capture the HY sensitivity. However, the plot presents many outlying points, with the regression beta coefficient in this case being rather jeopardized by the low R-squared value of 0.08. Nevertheless, the value of the beta coefficient, which is equal to 0.317, i.e., the sensitivity value obtained through regression, is quite similar to the length-times-magnitude weighted average of HY sensitivity of 0.291, which is calculated directly from Table 1. Considering that the R-squared value is low, we claim that the length-time-magnitude weighting gives a more precise and justified outcome from a hedging perspective, as it places more importance on strong and long-lived trends in UST capital gains, which seemingly ought to have a bigger impact on both UST and HY portfolios, than weak and short-lived moves.

4.4.2. Price-wise interest rate sensitivity through the crisis

Table 2 shows 15 major moves in annual capital gains of UST during the “distressed” through-the-crisis period. We calculate the capital gain deltas for three portfolios: the UST; U.S. BBB, and; U.S. HY portfolios, and also the respective sensitivity values.

Fig. 12 below shows BBB historic sensitivity as per the second from the right column of Table 2.

The length-times-magnitude weighted average of sensitivity is found to be -1.46 . Accordingly, on average during the crisis, the annual capital changes of the U.S. BBB corporate portfolio exhibit inverted behavior when compared to the respective capital changes

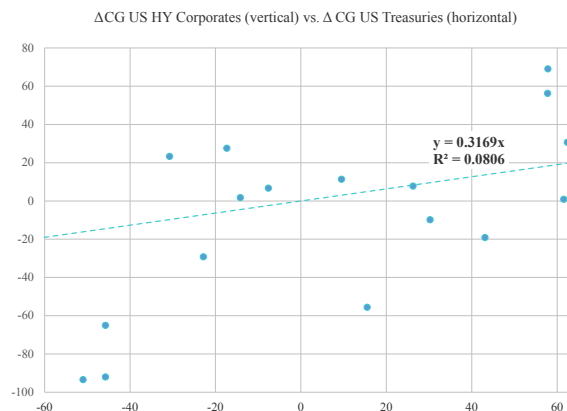


Fig. 11. $\Delta CG_{HY} - \Delta CG_{UST}$ relationship before the crisis.

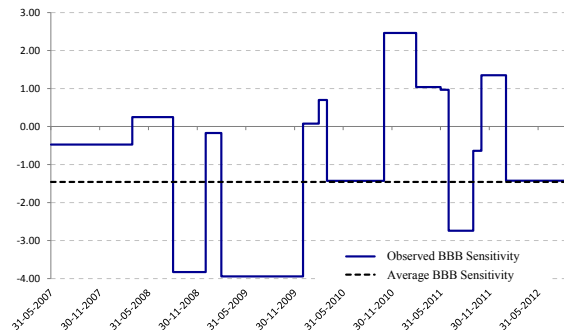


Fig. 12. Price-wise interest rate sensitivity of U.S. BBB corporate bonds through the crisis.

of the UST portfolio.

As the average sensitivity is below -1 , the average impact of risk-free IR on U.S. BBB corporate prices is amplified in an opposite direction, while the average absolute magnitude of the response is equal to 146% of the “inducing” change in UST capital gains. It means that when the yield on a U.S. government debt is rising, the spread of U.S. BBB corporate bonds over the UST yield is narrowing in such a way that it absorbs all the increase in risk-free rates and even causes a decrease in the U.S. BBB yield.

Such behavior corresponds to the outcomes of Merton (1974) structural model, positing the positive influence of IR increases on the creditworthiness of corporate obligors.

To complement the outcomes obtained through the length-times-magnitude weighted averaging, we present below the results of a statistical analysis of BBB debt sensitivity, performed through the prism of a linear regression model. Fig. 13 shows the graphical representation of the relationship between the changes in the capital gains of the UST portfolio ($\Delta CG UST$) and the changes in the capital gains observed over the respective periods in the U.S. BBB Corporate portfolio ($\Delta CG BBB$), as per the respective columns of Table 2.

The slope of the dotted trend-line in Fig. 13 captures the fact that the sensitivity is negative. The regression beta coefficient is somewhat significant, whereby the R-squared value of 0.33 is in the middle of the R-squared low range. The value of the beta coefficient of -1.167 , i.e., the sensitivity value obtained through regression, is, in absolute value, inferior to the absolute value of the length-times-magnitude weighted average of BBB sensitivity of -1.46 , which is calculated directly from Table 2. As in the case of the pre-crisis period, we continue to claim that the length-time-magnitude weighting gives a more precise and justified outcome from a hedging perspective, as it places more importance on strong and long-lived trends in UST capital gains. In addition, contrary to the regression model, the length-time-magnitude weighting approach does not employ the simplifying assumption of the linearity of the relationship between the capital gains of risky and risk-free portfolios.

Moving on, we proceed to the consideration of the U.S. HY corporate case. Fig. 14 below shows HY historic sensitivity as per the right-hand column of Table 2.

The dynamics of HY sensitivity as per Fig. 14, exhibits higher volatility than the BBB sensitivity behavior depicted in Fig. 12, while the average HY sensitivity level of -3.22 is more than twice negative than the respective BBB level.

As in the BBB case, the average sensitivity is below -1 . Therefore, the average impact of risk-free IR on U.S. HY corporate bond prices is amplified in an opposite direction, while the average absolute magnitude of the response is equal to 322% of the “inducing” change in UST capital gains. It means that, while the yield on U.S. government debt is rising, the spread of U.S. HY corporate bonds over the UST yield is narrowing – in such a way as it absorbs all the increase in risk-free rates and even causes a decrease in the U.S. HY debt yield.

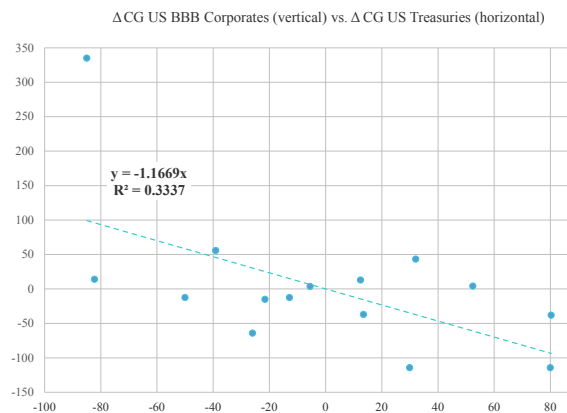


Fig. 13. $\Delta CG BBB - \Delta CG UST$ relationship through the crisis.

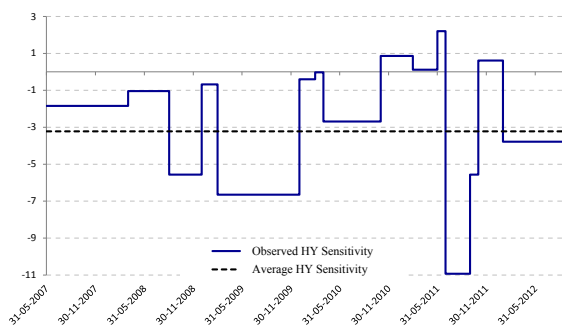


Fig. 14. Price-wise interest rate sensitivity of U.S. HY corporate bonds through the crisis.

Such behavior even stronger than in the BBB case, which corresponds to the outcomes of Merton (1974) structural model, positing the positive influence of IR increases on the creditworthiness of corporate obligors.

To complement the outcomes obtained through the length-times-magnitude weighted averaging, we present below the results of a statistical analysis of the HY debt sensitivity, which is performed through the prism of a linear regression model. Fig. 15 shows the graphical representation of the relationship between the changes in the capital gains of the UST portfolio ($\Delta CG UST$) and the changes in the capital gains observed over the respective periods in the U.S. HY Corporate portfolio ($\Delta CG HY$), as per the respective columns of Table 2.

The slope of the dotted trend-line in Fig. 15 captures the fact that the sensitivity is strongly negative. The regression beta coefficient is somewhat significant, with the R-squared value of 0.54 being in the low range, for the through-the-cycle period, similar to the BBB case, although it is now higher, and thus provides more robustness to the results. The value of the beta coefficient of -2.60 , i.e., the sensitivity value obtained though regression, is, in absolute value, inferior to the absolute value of the length-times-magnitude weighted average of HY sensitivity of -3.223 , which is calculated directly from Table 2. Once again, we evoke our claim that length-time-magnitude weighting gives a more precise and justified outcome from a hedging perspective – as it places more importance on strong and long-lived trends in UST capital gains. However, what is more important, is that both the approaches – the length-time-magnitude weighting and the linear regression, corroborate one another, which thus assures the overall consistency and plausibility of the qualitative conclusions.

4.4.3. Price-wise interest rate sensitivity during the post-crisis period

Table 3 shows 18 major moves in capital gains of UST during the “new normal” post-crisis period. The capital gain deltas for three portfolios: the UST; U.S. BBB, and; U.S. HY portfolios, as well as the respective sensitivity values are presented.

Fig. 16 below shows BBB historic sensitivity as per the second from the right column of Table 3.

The length-times-magnitude weighted average of sensitivity is found to be 1.231. The sign of an average price response of the U.S. BBB corporates is the same as the sign of the “inducing” price change observable in the portfolio of U.S. government bonds, while the amplitude of the price response observed in the U.S. BBB corporates is equal to 123% of the price change relative to UST securities. It means that a move in the risk-free IR is passed on, being amplified, to the yield of U.S. BBB corporate bonds. After the crisis, once again, the BBB sensitivity mostly stays in the positive domain. However, during the “new normal” period, the dynamics of sensitivity is more volatile than under the “old normal” conditions.

We present the results of a statistical analysis of BBB debt sensitivity below, performed through the prism of a linear regression model. Fig. 17 shows the graphical representation of the relationship between the changes in the capital gains of the UST portfolio (Δ

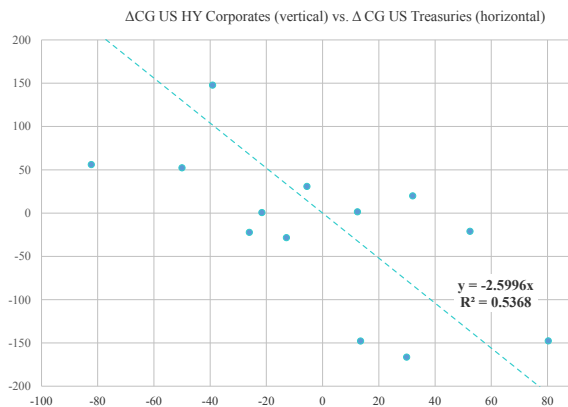


Fig. 15. $\Delta CG HY - \Delta CG UST$ relationship through the crisis.

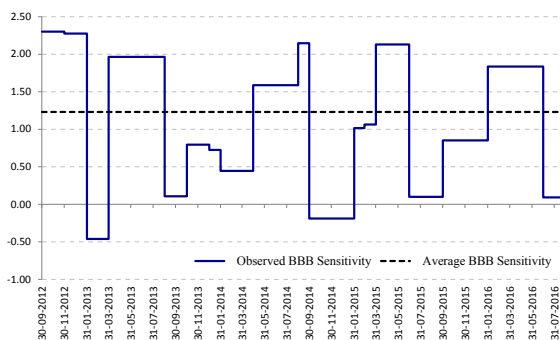


Fig. 16. Price-wise interest rate sensitivity of U.S. BBB corporate bonds after the crisis.

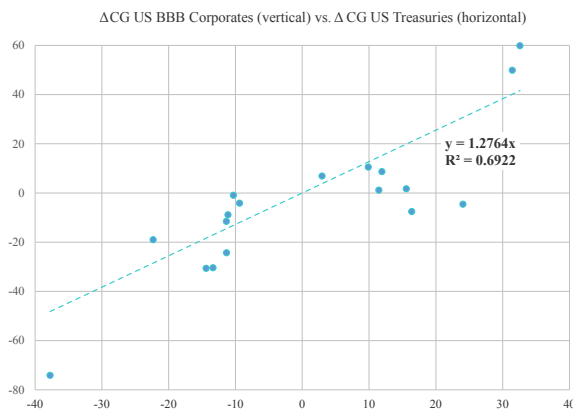


Fig. 17. Δ CG BBB – Δ CG UST relationship after the crisis.

CG UST) and the changes in the capital gains observed over the respective periods in the BBB U.S. Corporate portfolio (Δ CG BBB), as per the respective columns of Table 3.

The slope of the dotted trend-line in Fig. 17 captures BBB sensitivity. The regression beta coefficient is statistically significant, as certified by the R-squared value of 0.69. The value of the beta coefficient of 1.276, i.e., the sensitivity value obtained through regression, is almost the same as the length-times-magnitude weighted average of BBB sensitivity of 1.231, which is calculated directly from Table 3. Nevertheless, the latter value is more precise and it supports the outcome from a hedging perspective, as it places more importance on a strong and long-lived trend in UST capital gains, with larger impacts on both portfolios than the weak and short-lived moves.

Next, we turn to the U.S. HY corporate debt case. Fig. 18 below shows HY historic sensitivity as per the right-hand column of Table 3.

The dynamics of HY sensitivity as per Fig. 18, exhibits higher volatility than the BBB sensitivity behavior depicted in Fig. 16.

As per Table 3, the length-times-magnitude weighted average of HY sensitivity is found to be 1.170 – see the dotted line in Fig. 18. This value is slightly smaller than the average BBB sensitivity of 1.276, which was discussed earlier in this Sub-section. The average HY sensitivity after the crisis of 1.170 signifies that a move in the risk-free IR is passed on to the yield of U.S. HY corporate bonds and is amplified. After the crisis, the HY average sensitivity of 1.170 is much stronger than the average HY sensitivity of 0.291 observed during the pre-crisis period. In other words, before the crisis, risky HY assets were only weakly sensitive to IR, whereas, after the crisis, their average sensitivity results in an amplified response in the same direction as the inducing move in prices of risk-free securities.

We present the results of a statistical analysis of the HY debt sensitivity performed below, through the prism of a mainstream linear regression model. Fig. 19 shows the graphical representation of the relationship between the changes in the capital gains of the UST portfolio (Δ CG UST) and the changes in the capital gains observed over the respective periods in the U.S. HY corporate portfolio (Δ CG HY), as per the respective columns of Table 3.

Due to the volatility of HY sensitivity, as revealed by the existence of many outliers in Fig. 19, a non-elevated value of R-squared metrics, which is equal to 0.4218, makes the information represented by the trend-line less convincing than in the BBB case. Nevertheless, the trend-line slope or the beta coefficient value of 1.2215 seemingly captures the HY sensitivity in the right manner, corroborating with the length-times-magnitude weighted average value of 1.170 – as obtained directly from Table 3. In addition, it is worth noting that the two values of sensitivity obtained from the two different methods are quite similar.

Once again, it is important that the two employed approaches - the length-time-magnitude weighting and the linear regression, corroborate one with another, and thus assure the overall consistency and plausibility of our qualitative conclusions.

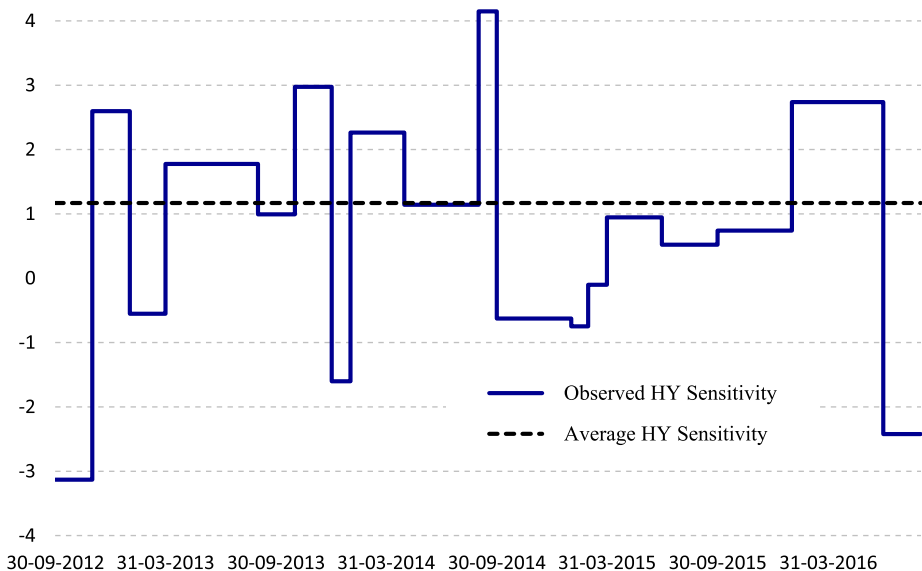


Fig. 18. Price-wise interest rate sensitivity of U.S. HY corporate bonds after the crisis.

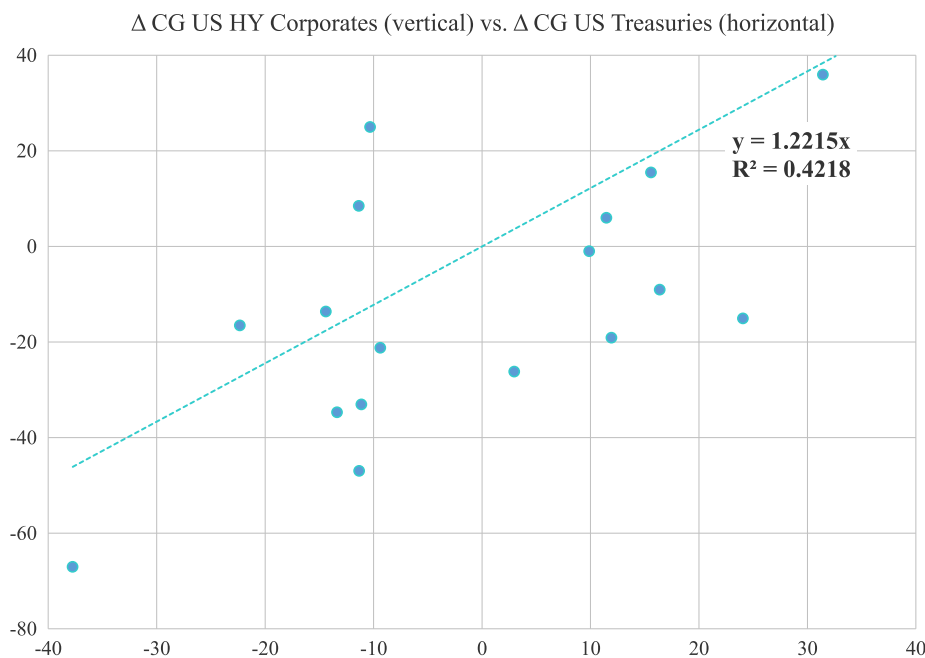


Fig. 19. ΔCG HY – ΔCG UST relationship after the crisis.

4.4.4. Holistic perspective: binary behavior of price-wise interest rate sensitivity

Fig. 20 below shows the historical behavior of BBB on-the-move sensitivity observed during the whole span of our analysis, including “old normal” pre-crisis, crisis, and “new normal” post-crisis intervals, as per the second from the right columns of Tables 1–3. The phase-averaged sensitivity value is constant over the interval, along which it is computed, see the dotted line.

Although on-the-move sensitivity exhibits a certain volatility, the dynamics of the sensitivity coefficient average over the distinctive phases of the business cycle clearly exhibit binary behavior, i.e., during normal economic conditions, the average price-wise IR sensitivity is positive, while during the times of crisis it is negative and amplified.

From the point of view of the economic efficiency of the IR hedge by short positions in UST, all the gains from the hedge registered during the pre-crisis period seem to have been annulled while crossing the phases of crisis downturn and recovery. This implies an inefficiency of the all-weather hedge of the U.S. BBB-rated corporate debt by short positions in UST, or by pay-fixed receive float IRS contracts.

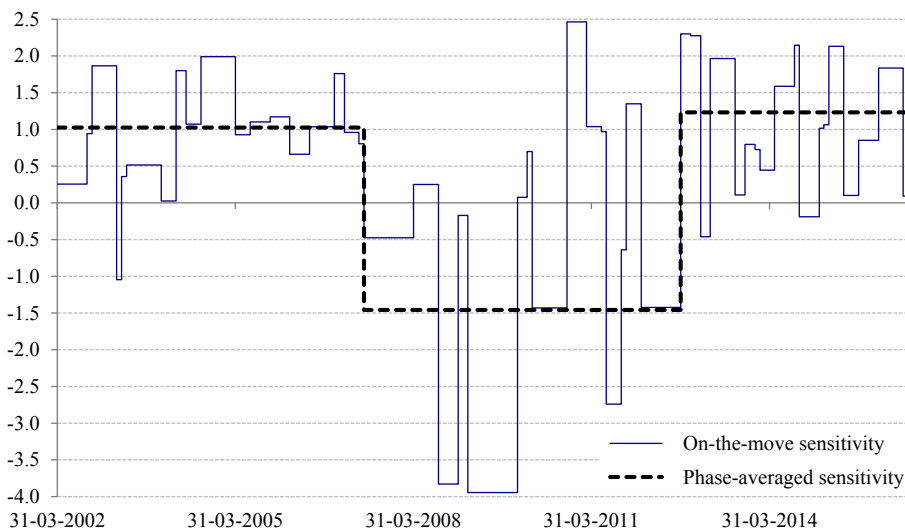


Fig. 20. Binarity of the phase-averaged sensitivity of U.S. BBB corporate bonds, 2002–2016.

Similar to the U.S. BBB case, Fig. 21 below shows the historical behavior of U.S. HY on-the-move sensitivity observed during the whole span of our analysis, as per the right-hand columns of Tables 1–3. The phase-averaged sensitivity value is depicted as being constant over the interval along which it is computed.

Similar to the BBB case, the dynamics of the HY sensitivity coefficient averaged over the distinctive phases of the business cycle exhibits a kind of binary behavior, in the sense that during normal economic conditions the average price-wise IR sensitivity is positive, whereas during the crisis times it is negative and amplified.

As in the previous case, for U.S. HY debt, the economic efficiency of the IR hedge by short positions in UST is even worse than that of BBB-rated corporate debt. This is a result of HY debt is only being weakly sensitive to interest rates during the pre-crisis period, while when crossing the phases of the crisis downturn and recovery, HY sensitivity on average is negative and is amplified more than twice in comparison to BBB sensitivity. Once again, we see inefficiency of the all-weather hedge of U.S. HY corporate debt by short positions in UST.

5. Discussions and implications

5.1. Remarks on an old controversy: Merton’s model vs. Kamin and Kleist approach

Based on the observed binary behavior of IR sensitivities, we revisit an old controversy between Merton (1974) structural model

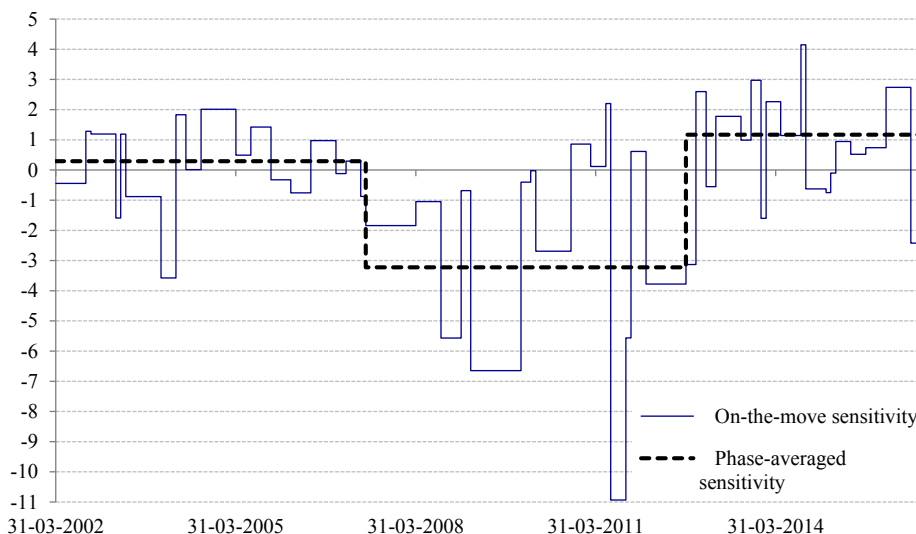


Fig. 21. Binarity of the phase-averaged sensitivity of U.S. HY corporate bonds, 2002–2016.

and [Kamin and Kleist approach \(1999\)](#). The former advocates the negative relation of credit spreads to interest rates while the latter posits that changes in risk-free interest rates are passed on to yields of risky assets with the same magnitude, and thus there is no relation between credit spreads and interest rates.

A negative relation between credit spreads and interest rates is observed in many recent studies - see [Boulkeroua and Stark \(2013\)](#), [Neal et al. \(2015\)](#), and [Dupoyet et al. \(2016\)](#). These empirical studies corroborate [Merton’s model \(1974\)](#). On the other hand, the empirical findings of [Kamin and Kleist \(1999\)](#) for emerging market bonds support their own hypothesis that risk-free rate changes are passed on to yields of risky assets.

Considering the observed binary behavior, we conclude that for a chosen period, there is one particular model, i.e., [Merton \(1974\)](#) model or [Kamin and Kleist \(1999\)](#) approach that better fits our empirical findings.

As an example, for the March 2002–May 2007 “old normal” period, the capital gain sensitivity for U.S. BBB corporate bonds on average is 1 to 1. This is therefore consistent with the theoretical thinking of [Kamin and Kleist \(1999\)](#), as it posits that the probability of default is not affected by changes in risk-free IR, and hence it does not result in a response of credit spreads to risk-free interest rates. The null response corresponds to 1 to 1 sensitivity, as credit spread is immune to the changes in risk-free interest rates. Our findings attest the validity of this model for normal market conditions prior to the crisis and after the crisis in the case of BBB-rated corporate debt, while for HY debt, a slightly positive sensitivity compatible with this model is verified, albeit only for the post-crisis new normal regime.

On the contrary, for the “distressed” through-the-crisis regime, from May 2007 to September 2012, negative sensitivity is observed for both U.S. BBB and U.S. HY corporate bonds. This result is clearly in line with [Merton \(1974\)](#) model, which implies a negative response of credit spreads to interest rates, i.e., the probability of default is reduced by an increase in the risk-free IR.

We also highlight the dangers of long-run averaging, for such algorithms can disguise the otherwise-observable effects and can lead to the risk of “throwing the baby out with the bathwater”.

Let us now analyze this situation in terms of spread – which is merely the difference between corporate yield and the risk-free rate for the BBB case, see [Fig. 22](#) below. In our illustrative scheme, during the normal regime no sensitivity of spreads to the risk-free IR is depicted, i.e., spreads remain constant and the changes in the risk-free IR are passed on to the yield.

On the other hand, during the distressed period, which spans over the contraction and recovery phases, spreads exhibit negative sensitivity to risk-free interest rates. When risk-free rates decrease, spread values increase and vice versa, see [Fig. 22](#). Therefore, under distressed conditions, the changes in risk-free interest rates affect the creditworthiness of obligors.

It now becomes easier to perceive that spanning the window of observations over both, the “normal” market regime with null spread sensitivity and the “distressed” market regime with strong negative sensitivity of spread to risk-free rates, may result, on average, in a slightly negative sensitivity. This phenomenon is observed because strong negative sensitivity during the distressed regime becomes diluted by the null or very weak spread sensitivity within the normal regime.

Accordingly, the aim of our illustrative scheme in [Fig. 22](#) above is to provide conceptual comprehension of what in fact stands behind the findings of [Dupoyet et al. \(2016\)](#), registering a consistently negative relation between credit spread and interest rates over the 1973–2014 period.

In terms of capital gain-wise sensitivities, averaging sensitivities over a long run can disguise the observable effects, as spanning the window of observations over both the “normal” regime with positive sensitivity and the “distressed” regime with negative sensitivity, one observes on average only a predominant sensitivity, which is nevertheless damped by the other. Depending on the

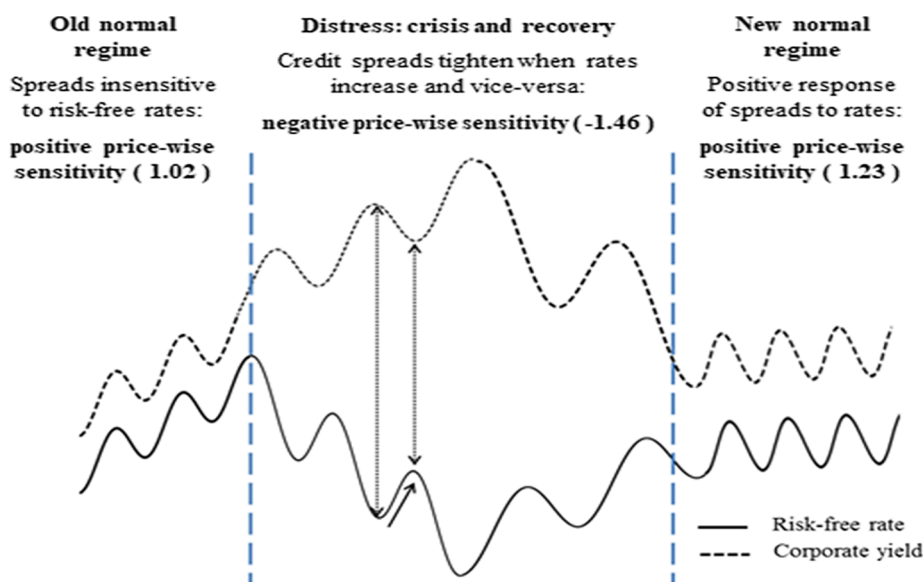


Fig. 22. Illustrative scheme: BBB spread dependence on risk-free rate.

span of the window over the two regimes, it is even possible to observe insensitivity to IR, if, on average, the opposite effects fully cancel each other out.

Although Dupoyet et al. (2016) report a consistently negative relation between credit spread and interest rates for the 1973–2014 period, our research proves that this is not the case for the constituent short run intervals, at least for the case of the U.S. BBB and HY corporate portfolios for 2002–2016. As per Figs. 8, 10, 16 and 18, we observe several time intervals with positive price-wise sensitivities above 1 for the normal regimes. Therefore, these time intervals are all characterized by the positive sensitivities of spreads to risk-free interest rates, i.e., where increases in the risk-free interest rates result in increases in spreads between corporate yields and risk-free rates, and vice versa. Accordingly, it is important to watch out for the danger of long span averaging, which is capable of occulting variation in sensitivities along the constituent short-run intervals, as consequently many important features and trends can remain undetected.

5.2. The business cycle and interest rate sensitivity

Based on our analysis, we propose an explanation for both positive and negative capital gain-wise IR sensitivities of those U.S. corporate bond portfolios observed along the time span of the rising and falling economic scenario.

Every business cycle is different from others; however, certain patterns in growth rates have a tendency to repeat themselves over time. For a typical cycle, we analyze below the four critical phases: an early-cycle phase; mid-cycle phase; late-cycle phase, and; a recession. Alternatively, the business cycle is also frequently divided into the phases of: expansion; peak; contraction, and; trough.

- (i) The typical early-cycle phase represents a sharp recovery from a recession, characterized by a resumption of economic activity, an inflection from negative to positive economic growth, followed by accelerating growth. During this relatively short phase, the accelerating economic activity drives the recovery from the bottom of the recession until full exit is achieved.
- (ii) The typical mid-cycle phase is usually the longest of the phases of the business cycle. This is when the economy passes from recovery to expansion. This phase is characterized by a positive but diminishing rate of economic growth when compared to the early-cycle. Growth reaches a peak, as inventories and sales grow, however, there is a rise in inflationary pressures.
- (iii) During the late-cycle phase, which takes place prior to recession, the growth of the economy gradually becomes saturated and threats of inflation keep increasing. Inventories increase, while sales drop. Economic growth is severely restrained and the economy is poised to slip into a free fall recession.
- (iv) Typically, a recession is the shortest phase of the business cycle. This phase represents a contraction in economic activity, where economic growth stalls. Corporate profits decline and credit is scarce. Inventory levels gradually decrease, despite low sales levels and it is in this way that the recession phase prepares the way for a recovery during the early phase of the next cycle.

It is well acknowledged that performance across asset categories changes in accordance with the phases of the business cycle, see, for example, Mikaelian (2013), Andreasen, Engsted, Møller, and Sander (2016). Below we provide an economic explanation of why IR sensitivity switches in line with phases of the business cycle. As a result, a business cycle approach to IR hedging can add value as part of intermediate-term hedge strategies.

5.2.1. “Normal” interest rate sensitivity: the mid-cycle and late-cycle

Under the “normal” regime of the mid-cycle and late-cycle, the sensitivity of capital gains/losses of U.S. BBB corporate portfolios to the capital gains/losses of the corresponding UST portfolios is positive, i.e., the rises and falls in risk-free IR are passed on to the respective bond yields mostly unchanged, although they can be slightly increased or decreased. Therefore, we posit that moderate and non-abrupt changes in risk-free IR do not affect the level of corporate creditworthiness when considered from the operations point of view.

This is especially true under the “old normal” pre-crisis regime of March 2002–May 2007, and the “new normal” post-crisis regime of September 2012–August 2016, whereby the capital gains/losses of U.S. BBB corporate bonds are related to the capital gains/losses in the corresponding UST portfolio at an approximate ratio of 1 to 1. This implies that market participants interpret the creditworthiness of issuers as being non-dependent on risk-free interest rates.

We ascribe the label of “normal” regime of IR sensitivity to periods of normal to moderate economic growth. This is a plausible statement, as both the mid-cycle phase and the late-cycle phase demonstrate economic expansion, during the initial and saturating phases, respectively. Changes in risk-free IR do not affect the perception of the probability of default and spreads remain virtually unaffected, with capital gains sensitivity being positive.

When discussing the U.S. BBB corporate bond portfolios, we certainly refer to the economic growth dynamics of the US. In addition, we posit that our reasoning is equally applicable for analyses on a global scale, as well as for studies of investment grade corporate debt behavior in isolated geographic zones.

On the other hand, HY debt is characterized by its own specific features, which result in its sensitivity behavior being partially different from the dynamics of BBB-rated corporate bonds. For instance, HY debt is speculative in its nature, as the less certain future of such issuers depends on speculation regarding the perspectives of economic growth, which in turn also affect risk-free interest rates. Prior to the crisis, “the sky was the limit” in the market and no one worried about expansion possibilities that were encouraged by fast-rising risk-free rates. As a result, increases in risk-free rate were partially compensated by the tightening of HY spreads, due to better business perspectives, which, in turn produced a slightly negative average response of HY spreads to rates, albeit resulting in a positive, although attenuated, average capital gain-wise sensitivity level.

After the crisis, the situation changes. The new normal conditions become characterized by doubts by market participants concerning rather limited opportunities for future growth. It is thus during the post-crisis period that increases in risk-free interest rates are passed on to HY spreads, as insufficiently optimistic growth perspectives do not reduce the perception of the risk of default. Consequently, HY spreads on average do not tighten, in fact, they slightly increase, producing, therefore, a weak positive response of spreads to rates. This phenomenon occurs as HY creditworthiness is negatively affected by the adverse changes in the financial costs due to increasing interest rates with no expectable compensation from an operations point of view. Consequently, we observe a positive, above 1, average capital gains-sensitivity level.

5.2.2. “Distressed” interest rate sensitivity: the recession and sharp early-cycle recovery

Next we discuss the negative capital gains/losses sensitivities of the U.S. corporate bond portfolios under the “distressed” regime, i.e., the sharp contraction and recovery of the economy. In recent history, these phases are represented by the global economic crisis and the recovery from the deepest trough of the crisis. In other words, the “distressed” regime is but a transition through the bursting bubble to the bottom of an economic slump, and then further on, through by a sharp recovery of the economy returning to its usual state.

5.2.3. “Distressed” interest rate sensitivity during a recession-driven downturn

During the vicious cycle of a recession phase, markets enter into risk-off mode and risk-free rate behavior exhibits downtrend dynamics, due to the increasing demand for safe assets. To stimulate the investment, which is necessary to turn the economy around, during a recession central banks adopt a policy of reducing interest rates. As a result, yields on risk-free assets drop and capital gains are registered in portfolios comprised of U.S. Treasury securities.

In parallel, the overall worsening of the economy increases the level of credit risk for U.S. corporations through several mechanisms. From an operations point of view, business conditions deteriorate due to a lower demand for products and services because of increasing uncertainty. With regards to the financial side of businesses, companies’ ability to service their debt and obtain external financing worsens and the financing costs of corporations keep growing.

Under such conditions, there is a lot of uncertainty in the market. The demand for safe UST securities causes their yields to drop and their prices to rise. Simultaneously, as a result of flight-to-quality phenomena (Gubareva & Borges, 2016a), investors withdraw their funds from relatively risky corporate bonds. Accordingly, yields of corporate bonds rise and their prices drop. Even though risk-free rates are falling, credit spreads for corporate bonds are widening.

The increase in default risk, which is provoked by the above-mentioned factors, results in credit spreads for corporate bonds rising, to such a degree that yields on risk assets grow even though yields on risk-free assets decrease and, thus, capital losses occur in the case of risky U.S. corporate portfolios. Therefore, capital gain-wise IR sensitivity of U.S. BBB corporate bonds becomes negative under the influence of the sharp deterioration of economic conditions.

As interest rates decrease, credit premia rise, although the rise in credit spreads is greater than the drop in risk-free rates. This is the well-known flight-to-quality phenomenon, which is described here through the prism of a real economy. This phenomenon results in capital losses in risky asset portfolios, such as U.S. BBB and U.S. HY corporate bonds, and simultaneously it also leads to capital gains of UST portfolios.

5.2.4. “Distressed” interest rate sensitivity during a sharp recovery from recession

During the early-cycle phase, which is characterized by recovery from flight-to-quality, i.e., during a “flight-from-quality”, markets enter into the risk-on mode. In periods of accelerated economic growth, the demand for safe assets drops, causing a rise in risk-free IR. As central banks do not usually raise rates when it is likely that corporations will have difficulties to service their debt, these banks tend to raise rates when the economic conditions are on the road to recovery. Under such conditions, central banks are potentially more likely to adopt a tightening of monetary policy by increasing interest rates. As the economy recovers sharply, regulatory increases of risk-free interest rates can even become necessary to avoid the overheating of the economy and as a means of keeping inflation under control. In such times, yields on UST increase, whereas capital losses are registered in UST portfolios.

Simultaneously, this economic recovery results in a decrease in corporate default risk. From an operations point of view, corporations start to benefit from improved consumer confidence, increased demand, and reduced uncertainty. Financing costs also decrease. Credit spreads of corporate bonds decrease, and thus yields on risk assets fall, even though yields on risk-free assets increase, and thus capital gains occur in corporate portfolios.

Similar to during a recession, capital gain-wise IR sensitivity of U.S. BBB and U.S. HY corporate bonds also remains negative during the phase of accentuated recovery from the recession.

Drawing to a close, during the early-cycle phase, when risk-free interest rates rise, credit premia decline. However, credit spreads narrow faster than the risk-free yields increase. This is the recovery from the well-known flight-to-quality phenomenon. This “flight-from-quality” results in capital gains for risky asset portfolios and capital losses for portfolios composed by U.S. Treasury bonds.

Therefore, just for the case of the “distressed” regime, which contains two of the phases of the business cycle – recession and a sharp recovery from the trough – our results are in line with the findings of Dupoyet et al. (2016), which state that the average change in interest rates (credit spreads) is negative (positive) during periods of recession, while the average change in interest rates (credit spreads) is positive (negative) during periods of economic expansion. However, caution is needed – for often the periods of economic expansion referred to above in the cited research end up becoming more like periods of sharp recovery from a recession.

For the two other phases of the business cycle, which are features of normal to moderate economic expansion, namely, the mid-cycle and the late-cycle phases, our results contrast with the research discussed above. We show that under the “normal” regime of

mid-cycle and late-cycle expansion, the negative relationship between interest rates and credit spreads disappears and turns into insensitivity or null responses of spreads to risk-free interest rates.

It should be noted that under the “normal” regime of the mid-cycle and the late-cycle, we observe positive sensitivity of the capital gains of corporate bond portfolios to the capital gains of UST portfolios, while under the distressed regime of recession and a sharp recovery from recession, we observe switching from positive price-wise sensitivity to negative values.

5.3. Additional considerations

We have now come to the time when we need to address the question that we set out to answer. In the end, does it make economic sense to hedge the interest risk of U.S. BBB and HY corporate debt by short positions in UST bonds or by pay-fixed receive-float IRS? As demonstrated in our results, such a hedge for BBB-rated corporate bonds only makes sense during the periods of normal to moderate economic expansion, i.e., during the mid-cycle and late-cycle phases.

In the case of HY debt, the adoption of an IRR hedge by short positions in UST bonds seems to only make sense when subject to the post-crisis new normal conditions, during which the market doubts regarding future perspectives for non-restrained growth. It should be noted that under conditions of good growth expectations during the pre-crisis period and passing through the crisis turmoil and recovery phases, we evidence the lack of economic sense of such traditional hedge strategies based on shorting UST, or by employing pay-fixed receive-float IRS.

On the contrary, to hedge against downside risk in times of economic turmoil, it is advisable to increase exposure to IRR, such as, for example, by contracting pay-float receive-fixed IRS. In summary, we argue that the hedging of IRR and downside risk should not be mechanical, but rather it should be a dynamic process linked to the phases of the business cycle and market perceptions of growth perspectives over the mid to long term.

Should capital gain-wise sensitivity be negative, hedging IRR would not reduce the volatility of profit and loss. Therefore, a business cycle approach to IRR hedging can add value for intermediate-term hedge strategies.

At this point, it is worth commenting on some hot topics that are related to the opportunities and challenges upsurged post recent financial crisis (Lien & Wu, 2014) and to the importance of the Fed’s monetary policy, which are relevant to the field of research described in this paper, namely: zero lower bound (ZLB), quantitative easing (QE), and forward guidance (FG).

For instance, ZLB is a situation that occurs when the short-term nominal interest rate is at, or is near zero. With regards to recent US economic history, ZLB was brought about as a result of the global financial and economic crisis, which resulted in the 2007 recession. In December 2008, the Fed set the federal fund rate in the range from zero to 25 basis points. Short-term interest rates remained ZLB-constrained for a 7-year long period, which began on December 16, 2008, and ended on December 14, 2015, when for the first time since the onset of the crisis, the Fed increased the federal funds rate target. However, there is much evidence in the market data that longer-term interest rates remained sensitive to changes in macroeconomic conditions during this 7-year long period, having been somewhat unconstrained by the above-mentioned ZLB. However, the situation is not that simple and further research of interrelations between ZLB and US corporate debt sensitivity is highly desirable to obtain a deeper understanding of possible low interest rate conditions in the future.

While withstanding ZLB conditions, governments cannot stimulate spending by simply lowering interest rates below zero, as account-holders will start to withdraw their money from their accounts and hold it in cash. Accordingly, as the use of traditional monetary policy becomes severely constrained, the Fed’s few options are to use alternative unconventional approaches, such as rounds of QE, which, in effect, equate to the purchase of government and other securities in order to increase the money supply and stimulate the economy.

The QE carried out by the Fed, which have occurred for a straight 10-year run in November 2018, is the most impressive non-standard policy ever exercised in modern US history. While it is pretty difficult task to correctly evaluate the overall payback of QE, it is easy to agree that a real economy can become overly reliant on financing of asset markets by QE. Therefore, the unforeseen material impacts of the Fed’s monetary policy of quantitative tightening through a series of recent rate hikes and reversal of QE, i.e., quantitative tightening (QT), could potentially affect an interest rate environment, which is deprived of QE support and could arguably provoke certain changes in interest rate sensitivity of U.S. BBB and HY corporate debt.

We suggest that studying how QE can influence US corporate debt sensitivity regimes is an important topic, which should be included in the agendas of future research in this field. The capital gains-based methodology developed here could be included in corporate debt sensitivity studies as it certainly represents a robust tool for addressing these issues, and will be of use for further research.

On the other hand, QE, together with the full implementation of the rules of the Basel III capital accord, has affected the hierarchy and trading activities of various money market interest rates. The pre-crisis liquidity buffers of financial institutions were primarily secured through unsecured interbank lending, whereas the post-crisis liquidity buffers, which are influenced by both Basel III regulation and QE, are represented mostly by liquidity in the form of high quality liquid assets, e.g., sovereign and high-grade corporate bonds, and not by liquidity stored within the proper financial sector. These relatively new circumstances are driving the search for an alternative reference rate and for the Fed’s new policy rate, to account for the recent changes in the financial system. By addressing the changes in interest rate sensitivity by comparing pre-crisis with post-crisis periods, our research provides important insights into the subject. In particular, the empirical implications of our work are that we observe certain quantitative, rather than qualitative changes in interest rate sensitivity patterns when the economy passes from the pre-crisis to the post-crisis period.

In parallel, the search for an effective monetary policy at ZLB (zero lower bound) resulted in ascribing more importance to FG (forward guidance) as a tool, which potentially leads to a lower volatility of the near-term expectations of interest rates. The Fed,

which is concerned with the likelihood that interest rates could repeatedly become stuck at near-zero levels, invested a lot of effort in developing a FG approach based on the so-called “neutral” rate of interest known, also known as “r-star”, i.e., “r*”. This rate is supposed to be a rate that is neither expansionary nor contractionary for the economy in equilibrium.

However, recently in October 2018, two of the leading technical experts of the Federal Open Market Commission expressed the opinion that the r-star had appeared to be the correct guiding star, when the Fed was still far away from it, although it increasingly appears to be barely suitable for FG, as the Fed’s policy shifts closer to it. In other words, at proximity of the r-star, only while economy is going over its development path, and only then does it become possible to infer whether the Fed’s rate targeting is resulting in over- or under-shooting, in which case the nominal rate target must be altered. Therefore, it becomes questionable as to whether such guidance is truly FG.

Nevertheless, according to investors’ perception, as there is a diminishing reliance of the Fed on the r-star, the familiar “trustable” anchor on which the bond market had previously relied, has been removed. When this anchor was undoubtedly in place, changes in expectations regarding short rates had a relatively modest impact on longer-term rate expectations. However, with the apparent removal of the r-star anchor, the long-term rate has increased in response to accelerated economic growth, resulting in the so-called bear steepening of the yield curve, which in turn provoked the sell-off of assets, as observed very recently in October 2018.

As pointed out above, FG raises a number of significant challenges. Furthermore, an open question remains as to what is the right FG. In this context, we believe that research on how diverse FG strategies affect interest rate sensitivity of corporate debt would contribute to the state-of-the-art knowledge in this field, which would also cover recent developments in the Fed’s monetary policy.

6. Conclusions

We develop the necessary framework to assess interest rate sensitivity of corporate bond portfolios, based on average yield and coupon indexes. We apply our approach to the portfolio of U.S. IG BBB and HY corporate bonds. Our research advances well beyond the boundary of widely performed studies regarding the relation between interest rates and credit spreads, as in our research we investigate the bottom line profit and losses of different risky and risk-free model portfolios.

We investigate the impact on annual capital gains of the modeled portfolios and thus address interest risk sensitivity from the point of view of medium-term to long-term investment.

Our quantification of capital gain-wise sensitivities is quite innovative from the point of view of interest rate risk hedging, and also downside risk hedging techniques.

The historical span of our research covers the period of 2001–2016, which enables us to assess interest rate sensitivity of assets during the development, peak, and aftermath of the recent global financial and economic crisis. The results presented in this paper contrast with those of previous empirical research and its theoretical interpretations, as previously, both the expansion and contraction phases of business cycle were used to explain negative responses of credit spreads to interest rates, whereas we show that, in fact this is certainly not the case for U.S. BBB corporates.

In the case of U.S. BBB corporate bonds, we explicitly demonstrate that the mid-cycle and the late-cycle phases of moderate-to-restrained growth correspond to the insensitivities of spreads to interest rates, i.e., the zero response of credit spreads to risk-free interest rates. Negative responses of credit spreads to interest rates were only observed during the recession phase, with a sharp recovery from the bottom. This last statement also applies perfectly to U.S. HY corporates. However, we observed that for the pre-crisis period, HY spread-to-rates sensitivity is negative differently from the spread-to-rate insensitivity observed for the same period in the case of BBB-rated bonds. We posit that the absence of concerns regarding growth possibilities for HY debt companies before the crisis compensates for the negative impacts of increases in IR. During the post-crisis new normal period, for both, BBB and HY corporate sectors, it is common to have overall doubts regarding the future growth, which makes both, BBB and HY corporate spreads fairly insensitive to risk-free rates.

In terms of capital gain-wise sensitivities during the mid-cycle and the late-cycle phases, we observe positive capital gains sensitivity of the risky U.S. corporate bond portfolios, both BBB and HY, to the capital gains of the risk-free U.S. government bond portfolios.

During the recession and sharp early-cycle recovery from recession phases, we observe switching from positive to negative capital gain-wise sensitivities and demonstrate that interest rate sensitivity alternates in accordance with the different phases of the business cycle. We evidence a phenomenon of a binary behavior of interest rate sensitivity along the phases of the business cycle.

During the normal regime of the mid-cycle and the late-cycle phases, the changes in the present value of U.S. BBB and HY corporate bond portfolios are positively related to the changes in present value of the UST bond portfolios and the resulting sensitivity is positive. On the other hand, during the distressed through-the-crisis regime observed during the recession and the sharp recovery from recession, the changes in the present value of U.S. BBB and HY corporate bond portfolios are negatively related to the changes in the present values of UST bond portfolios. Under such conditions, sensitivity is negative. This suggests that the hedging of downside risk should be a dynamic process, which is linked to the phases of the business cycle.

Our approach enabled us to solve an old controversy between [Merton \(1974\)](#) structural model, which advocates the influence of interest rate on creditworthiness of obligors, and the [Kamin and Kleist \(1999\)](#) approach, which argues that changes in risk-free rates are passed on to the yields of risky assets entirely or even increased. We demonstrate that for the mid-cycle and late-cycle phases of moderate to restrained growth, sensitivity is positive, and that the latter approach fits better with our empirical observations, while during the distressed conditions of recession and the sharp recovery from recession, [Merton \(1974\)](#) model provides the theoretical explanation for the observed negative sensitivities. Therefore, the clue to the solution of this controversy resides in the binary behavior of interest rate sensitivities of risky assets, which “privileges” either one or the other model along the cycle.

By examining the behavior of asset sensitivity to interest rate along the phases of business cycles, we corroborate the idea presented in our parallel research, see Gubareva and Borges (2016b, 2018), whereby the integrated treatment of IRR and credit risk enables the optimization of the economic capital of banks and financial institutions, by way of improving risk assessment. This research represents a contribution to the advancement of the discussion regarding the alignment of Pillar 2 methodologies under the Basel III capital accord.

Looking ahead, we can affirm that the applicability of the index-based framework to measure interest rate sensitivity is considerably larger than U.S. BBB and HY corporate debt. Depending on the availability of blended yield and average coupon indices and/or average price indices, this framework could be applied to diverse portfolios that contain fixed-income assets from diverse geographical zones, sectors, and rating categories. Accordingly, further research in this field is highly desirable to positively influence the overall efficiency of the financial system.

Funding

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Appendix I. Robustness checks

This part of our research is dedicated to the robustness checks performed to certify the fairness and similarity of our results, which were obtained using different assumptions regarding the shape of the modeled yield curves. We assess whether our results are fairly robust by reshaping the UST and U.S. corporate yield term structures, i.e., we relax our model flat-curve assumption employed for simplicity reasons in our calculations, by using instead the assumption of linearly-sloped term structures for the analyzed risk-free and risky asset portfolios.

We present below a comparative analysis of the capital gain-wise sensitivities obtained under the flat curves and linear slope assumptions. The upward-sloping, i.e., positive slope curves are obtained by adopting the *pro rata temporis* interpolation of yield values between zero and 5Y point of the respective yield term structure, while the downward-sloping, i.e., negative-slope curves represent their reflections mirrored vertically through the 5Y yield level. They decay downwards with maturity, reaching a flat yield level at 5Y point. Table A1, below, provides the comparison between UST capital gains, U.S. BBB corporate capital gains, and sensitivity values obtained for the pre-crisis period for the three above-mentioned cases: *i*) flat curves (taken from Table 1); *ii*) upward-sloping, and; *iii*) downward-sloping curves. The results for the two latter cases are specifically calculated for this robustness checks exercise.

As could be seen in Table A1, in the case of the pre-crisis period, both the upward- and downward-sloping term structures result in fairly similar outcomes to those obtained under the flat curves assumption. The average length-times-magnitude weighted

Table A1
Pre-crisis sensitivity figures under flat, upward- and downward-sloping curves assumptions.

Date	Elapsed time (days)	Positive slope			Negative slope			Flat curves		
		Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST
31/03/2002										
30/09/2002	183	40.99	10.58	0.26	45.00	11.27	0.25	43.10	10.97	0.25
31/10/2002	31	-21.80	-20.21	0.93	-23.69	-22.49	0.95	-22.80	-21.47	0.94
31/03/2003	151	55.61	103.11	1.85	59.81	111.85	1.87	57.85	107.93	1.87
30/04/2003	30	-16.61	17.44	-1.05	-17.93	18.71	-1.04	-17.32	18.13	-1.05
31/05/2003	31	8.97	3.19	0.36	10.01	3.64	0.36	9.51	3.40	0.36
31/12/2003	214	-89.22	-45.70	0.51	-96.27	-49.85	0.52	-92.94	-47.97	0.52
31/03/2004	91	14.98	0.97	0.06	16.06	-0.11	-0.01	15.55	0.39	0.02
31/05/2004	61	-48.75	-87.45	1.79	-53.02	-95.52	1.80	-50.99	-91.78	1.80
31/08/2004	92	59.83	63.09	1.05	63.09	68.30	1.08	61.55	65.89	1.07
31/03/2005	212	-43.34	-87.24	2.01	-47.91	-94.24	1.97	-45.74	-91.02	1.99
30/06/2005	91	60.57	55.14	0.91	63.98	60.20	0.94	62.38	57.86	0.93
31/10/2005	123	-44.02	-48.33	1.10	-47.22	-52.12	1.10	-45.72	-50.39	1.10
28/02/2006	120	29.76	34.10	1.15	30.68	36.50	1.19	30.25	35.40	1.17
30/06/2006	122	-29.65	-19.78	0.67	-31.67	-20.75	0.66	-30.73	-20.32	0.66
30/11/2006	153	55.86	57.61	1.03	59.42	61.78	1.04	57.77	59.90	1.04
31/01/2007	62	-13.73	-23.71	1.73	-14.51	-25.90	1.79	-14.15	-24.90	1.76
30/04/2007	89	25.07	23.96	0.96	27.30	26.24	0.96	26.27	25.20	0.96
31/05/2007	31	-7.35	-5.82	0.79	-7.83	-6.39	0.82	-7.61	-6.13	0.81

sensitivities figures of 1.025 for flat curves, 1.021 for positive slope, and 1.026 for negative slope – are remarkably similar for the reasons explained in the Methodology section of the main body of this paper.

Table A2, below, provides the comparison between UST capital gains, U.S. BBB corporate capital gains, and sensitivity values obtained for the through-the-crisis period for the three above-mentioned cases: i) flat curves (taken from Table 2); ii) upward-sloping, and; iii) downward-sloping curves. The results relative to the two latter cases are calculated on purpose for this robustness checks exercise.

Table A2

Through-the-crisis sensitivity figures for flat, upward- and downward-sloping curves.

Date	Elapsed time (days)	Positive slope			Negative slope			Flat curves		
		Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST
31/05/2007										
31/03/2008	305	76.88	-37.15	-0.48	83.34	-38.83	-0.47	80.28	-38.08	-0.47
31/08/2008	153	-49.04	-12.23	0.25	-50.84	-12.71	0.25	-50.00	-12.51	0.25
31/12/2008	122	28.47	-109.16	-3.83	31.28	-118.54	-3.79	29.89	-114.44	-3.83
28/02/2009	59	-79.95	13.71	-0.17	-84.35	14.17	-0.17	-82.25	14.01	-0.17
31/12/2009	306	-82.83	321.01	-3.88	-86.93	346.18	-3.98	-84.95	335.05	-3.94
28/02/2010	59	51.31	3.61	0.07	53.47	4.42	0.08	52.42	3.98	0.08
31/03/2010	31	-20.76	-14.44	0.70	-22.36	-15.58	0.70	-21.58	-15.07	0.70
31/10/2010	214	78.64	-109.50	-1.39	81.22	-118.22	-1.46	79.97	-114.42	-1.43
28/02/2011	120	-25.29	-61.88	2.45	-26.70	-66.04	2.47	-26.01	-64.10	2.46
31/05/2011	92	12.28	12.12	0.99	12.55	13.59	1.08	12.42	12.89	1.04
30/06/2011	30	-12.45	-11.95	0.96	-13.27	-12.92	0.97	-12.87	-12.46	0.97
30/09/2011	92	13.29	-35.21	-2.65	13.73	-38.67	-2.82	13.51	-37.04	-2.74
31/10/2011	31	-5.44	3.49	-0.64	-5.61	3.56	-0.63	-5.53	3.53	-0.64
31/01/2012	92	31.35	41.91	1.34	32.79	44.48	1.36	32.09	43.27	1.35
30/09/2012	243	-38.47	54.15	-1.41	-39.69	57.05	-1.44	-39.10	55.68	-1.42

Table A2 shows that for the through-the-crisis period, both, the upward- and downward-sloping shapes of term structures result in quite similar outcomes to those obtained under the flat curves assumption. The average length-times-magnitude weighted sensitivities figures, -1.458 for flat curves, -1.438 for positive slope, and -1.468 for negative slope – are also approximately the same, similar to the situation observed for the pre-crisis period.

Table A3 provides the comparison between UST capital gains, U.S. BBB corporate capital gains, and sensitivity values obtained for

Table A3

Post-crisis sensitivity figures for flat, upward- and downward-sloping curves.

Date	Elapsed time (days)	Positive slope			Negative slope			Flat curves		
		Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST	Δ UST Capital Gains (\$, million)	Δ US BBB Capital Gains (\$, million)	Sensitivity Δ US BBB/ Δ UST
30/09/2012										
30/11/2012	61	3.00	6.35	2.12	2.95	7.29	2.47	2.97	6.84	2.30
31/01/2013	62	-13.06	-29.18	2.23	-13.65	-31.51	2.31	-13.36	-30.41	2.28
31/03/2013	59	16.12	-7.43	-0.46	16.66	-7.69	-0.46	16.40	-7.57	-0.46
31/08/2013	153	-37.01	-71.89	1.94	-38.45	-76.20	1.98	-37.74	-74.15	1.96
31/10/2013	61	15.42	1.54	0.10	15.73	1.79	0.11	15.58	1.67	0.11
31/12/2013	61	-10.92	-8.65	0.79	-11.30	-9.03	0.80	-11.12	-8.85	0.80
31/01/2014	31	11.77	8.22	0.70	12.10	9.06	0.75	11.94	8.66	0.73
30/04/2014	89	-9.18	-3.99	0.43	-9.56	-4.34	0.45	-9.37	-4.17	0.44
31/08/2014	123	31.07	47.68	1.53	31.80	51.94	1.63	31.44	49.90	1.59
30/09/2014	30	-11.14	-23.49	2.11	-11.51	-25.06	2.18	-11.33	-24.31	2.15
31/01/2015	123	23.74	-4.13	-0.17	24.37	-4.95	-0.20	24.06	-4.56	-0.19
28/02/2015	28	-11.17	-11.21	1.00	-11.54	-11.83	1.02	-11.36	-11.53	1.02
31/03/2015	31	9.74	10.09	1.04	10.03	10.90	1.09	9.89	10.51	1.06
30/06/2015	91	-14.19	-29.69	2.09	-14.58	-31.55	2.16	-14.39	-30.66	2.13
30/09/2015	92	11.28	0.66	0.06	11.66	1.62	0.14	11.48	1.16	0.10
31/01/2016	123	-22.00	-18.46	0.84	-22.63	-19.50	0.86	-22.32	-19.00	0.85
30/06/2016	151	32.09	58.00	1.81	33.09	61.49	1.86	32.60	59.83	1.84
31/08/2016	62	-10.15	-0.77	0.08	-10.47	-1.13	0.11	-10.31	-0.96	0.09

the post-crisis period for the three above-mentioned cases: i) flat curves (taken from Table 3); ii) upward-sloping, and; iii) downward-sloping curves. The results of the latter two cases are calculated on purpose for this robustness checks exercise.

As we can see in Table A3, for the post-crisis period, similar to the above-considered cases, both the upward- and downward-sloping shapes of term structures once again result in fairly similar outcomes to those obtained under the flat curves assumption. The average length-times-magnitude weighted sensitivities figures are, 1.231 for flat curves, 1.207 for positive slope, and 1.252 for negative slope – which are also in very close proximity to each other.

Table A4 provides a comparison between the length-times-magnitude weighted sensitivities averaged along the different phases for the three shapes of term structures discussed above.

Table A4 provides evidence of the fair robustness of both our quantitative outcomes and qualitative conclusions, under different parameter choices. The signs of the sensitivities for all the three studied periods do not depend on the term-structure assumptions. The percentage differences in amplitude, i.e., in absolute value of sensitivity coefficient, always remain below the 2% mark. As expected, the upward-sloping curves result in slightly reduced amplitudes, while the downward-sloping shapes result in slightly the increased absolute values of sensitivity coefficients. In conclusion, Table A4 clearly shows that, subject to reasonable diverse assumptions, our conclusions remain robust across all the historical periods.

Table A4

Average sensitivity figures per phases of the cycle under different assumptions.

	Pre-crisis period	Through-the-crisis period	Post-crisis period
Average sensitivity, (A) FLAT CURVES	1.025	-1.458	1.231
Average sensitivity, (B) POSITIVE SLOPE	1.021	-1.438	1.207
Average sensitivity, (C) NEGATIVE SLOPE	1.026	-1.468	1.252
Δ Amplitude Positive, (D) (D = ABS (B) – ABS (A))	-0.004	-0.020	-0.024
Δ Amplitude Negative, (E) (E = ABS (C) – ABS (A))	0.001	0.010	0.020
Δ Amplitude Positive (%), (F) (F = D/ABS (A))	-0.36%	-1.38%	-1.97%
Δ Amplitude Negative (%), (G) (G = E/ABS (A))	0.09%	0.68%	1.65%

However, the advantage of our capital-gains and yield-based approach is that it enables us to reach robust conclusions regarding the quantitative magnitude of IR sensitivities, even without having an exact knowledge of the yield term structures. In other words, the robustness checks presented above corroborate that the proposed methodology enables robust conclusions to be drawn with regards to the capital gains and IR sensitivity of assets under diverse model choices of term structure profiles.

Appendix B. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.najef.2018.11.017>.

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