Approximate Coupling Terms In Seismoelectric Theory: From Frequency Domain To Time Domain

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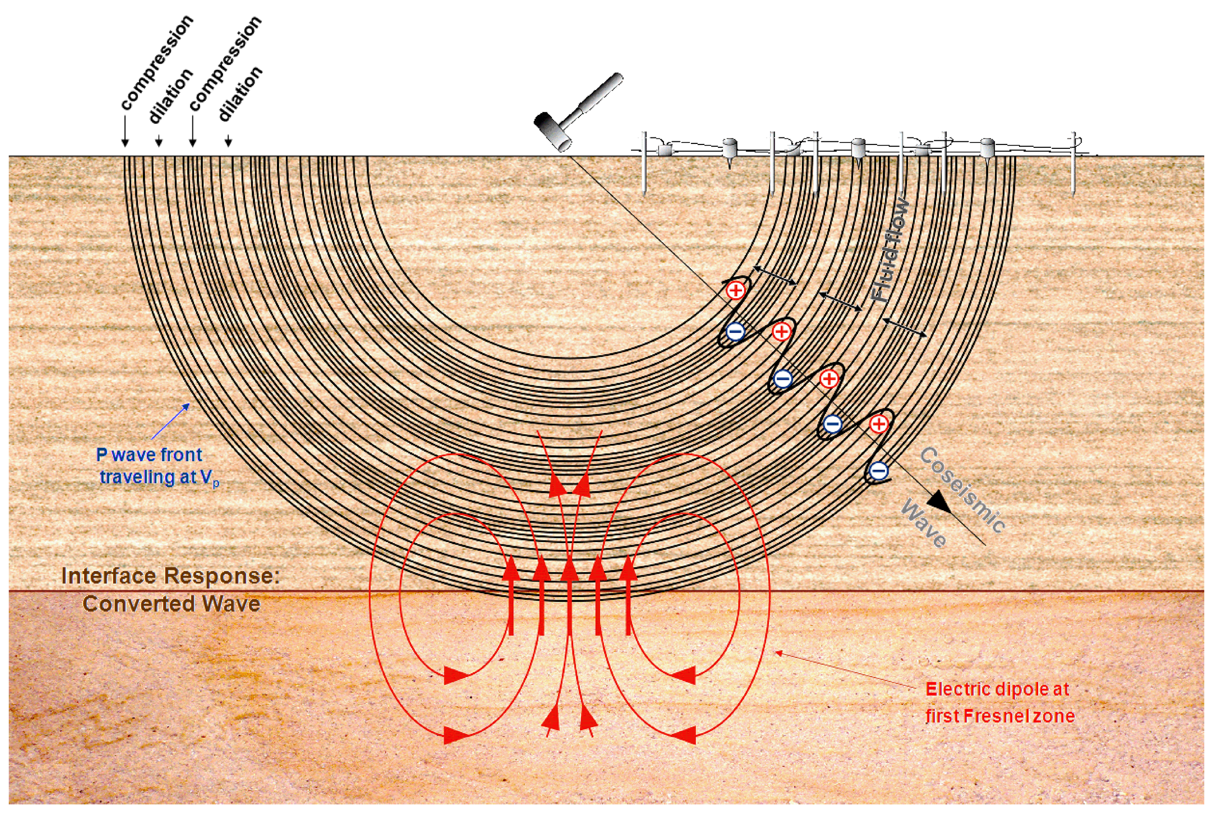
Seismoelectric effects

Electro-kinetic coupling

Porous media can be seen as a solid matrix containing gaps filled with fluid or gas. When these phases contain charge differences, the propagation of a seismic wave generates an electromagnetic wave. Indeed, the propagation of a mechanical wave agitates charged particles, which creates electrical current.



Modeling seismoelectric effects

A model of the coupling phenomenon was provided by Pride [1] that consists in coupling Maxwell's (electromagnetics) and Biot's (elastodynamics) equations followed by Pride and Haarsten [2] with a focus on electroseismic effects.

These phenomena are inherently multi-scale. On the one hand, they arise from a coupling at a small scale. But, on the other, these seismic-wave-induced electromagnetic perturbations may occur at depths such as 300m underground. Features of the solution depend both on macro scale data such as scattering at geological layer interfaces and on micro scale coupling.

This effect occurs in the Earth. If we take a look at the constitution of the Earth (sand/sandstone) at large scale we have seeds/pores that are charged in excess (usually negative) and when the wave propagates there is a fluid-solid motion transporting counterions that induces an electrical current.

[1] Pride, S. R. (1994). Governing equations for the coupled electromagnetics and acoustics of porous media. Physical Review B, 50(21), 15678-15696.
[2] Haartsen, M. W., & Pride, S. R. (1997). Electroseismic waves from point source in layered media. Journal of Geophysical Research: Solid Earth, 102(B11), 24745-24769.

Pride's equations

The model introduced by Pride in [1] reads :

$$
\begin{aligned}
i\omega\rho_a\mathbf{u} + i\omega\rho_f\mathbf{w} &= \nabla\cdot\tau + f_u\\
i\omega\rho_f\mathbf{u} + i\omega\rho_{dyn}(\omega)\mathbf{w} &= -\nabla\mathbf{p} + i\omega\rho_{dyn}(\omega)L(\omega)\mathbf{E} + f_\omega\\
i\omega\tau + i\omega\alpha p &= \mathbf{C} : \epsilon(u)\\
i\omega p &= -M\nabla\cdot\mathbf{w} - M\alpha : \epsilon(u)\\
i\omega\delta_0\mathbf{E} &= \nabla\times\mathbf{H} - \mathbf{J} + \mathbf{f}_C\\
i\omega\mu_0\mathbf{H} &= -\nabla\times\mathbf{E}\\
\mathbf{J} &= \sigma\mathbf{E} - L(\omega)(\nabla p + i\omega\rho_f\mathbf{u})
\end{aligned}
\tag{1}
$$

$\mathbf{u}$	frame velocity	ρ_a	averaged density
$\mathbf{w}$	relative fluid velocity	ρ_f	fluid density
p	fluid pressure	ρ_{dyn}	dynamic density
τ	stress tensor	α	effective stress tensor
$\mathbf{E}$	electric density field	f_u, f_w, f_C	exterior forces
$\mathbf{H}$	magnetic intensity field	$L(\omega), K(\omega)$	coupling coefficient, permeability
J	electric current density	$\mathbf{C}$	stiffness tensor
		$\epsilon(u)$	strain tensor
		M	fluid-solid coupling modulus p 25 thÃ¨se Rose
		δ_0	electric factor
		μ_0	electric permeability
		σ	conductivity

Uknowns (left) and physical parameters (right) in the Pride equations

$$L\tilde{E}(\vec{x},t) = L_0\int_{\mathbf{R}}(1-i\frac{\omega}{\omega_t})^{-1/2}E(\vec{x},\omega)e^{i\omega t}d\omega \tag{2}$$

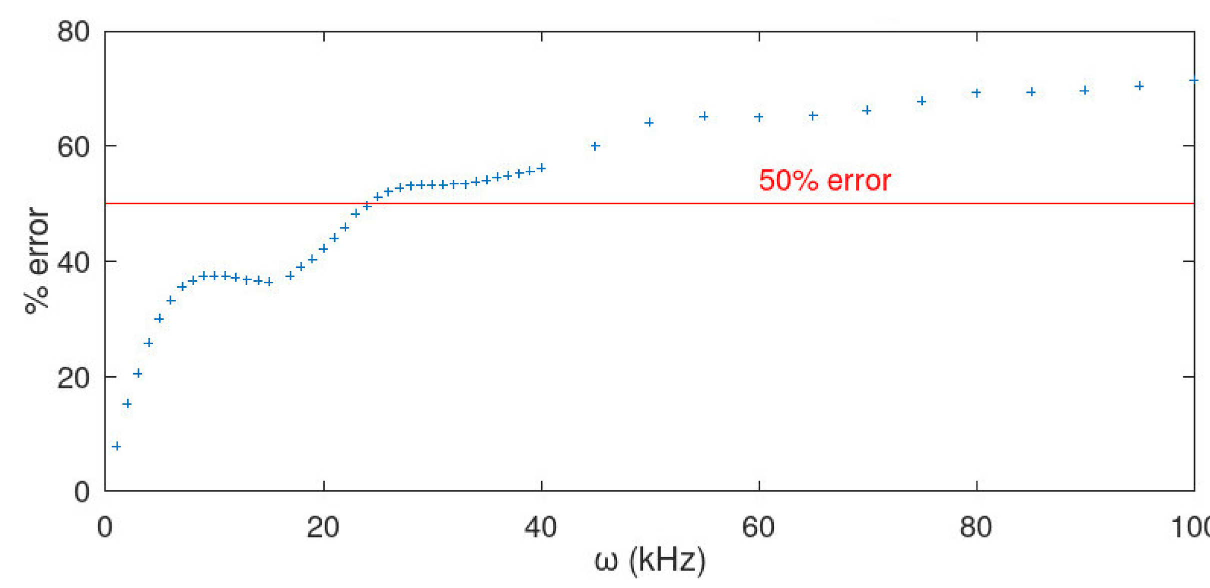
If L is polynomial or rational, $Lu(t)$ is a differential operator that is simple to write. The bottom-left panel introduces polynomial (Taylor) and rational (Padé) approximations of L leading to simplified coupling operators in time domain.

Approximations of the coupling term

The electrokinetic term L couples Maxwell's and Biot's equations. It depends on the frequency :

$$L(\omega) = L_0\left(1-i\frac{m}{4}\frac{\omega}{\omega_t}\right)^{-1/2} \quad \text{with} \quad L_0 = \frac{\phi\epsilon_0\epsilon_r^f\zeta^p}{\alpha_\infty\eta}\underbrace{\left(1-2\frac{d^l}{\lambda}\right)}_{\simeq 1} \tag{3}$$

In time domain, Pride's equations are solved with $L(\omega)\simeq L_0$. This is a low-frequency approximation that only applies to field-type simulations. We show that, at frequencies encountered in laboratory experiments (>20kHz), the effect of frequency on coupling is too significant to be neglected.



Relative error between E computed with $L(\omega)$ and L_0

In the following, we provide approximations of the coupling term L that are polynomial and rational. The rational approximation is based on Padé approximants which, even at low orders, provide very accurate approximations for a wide range of frequencies.

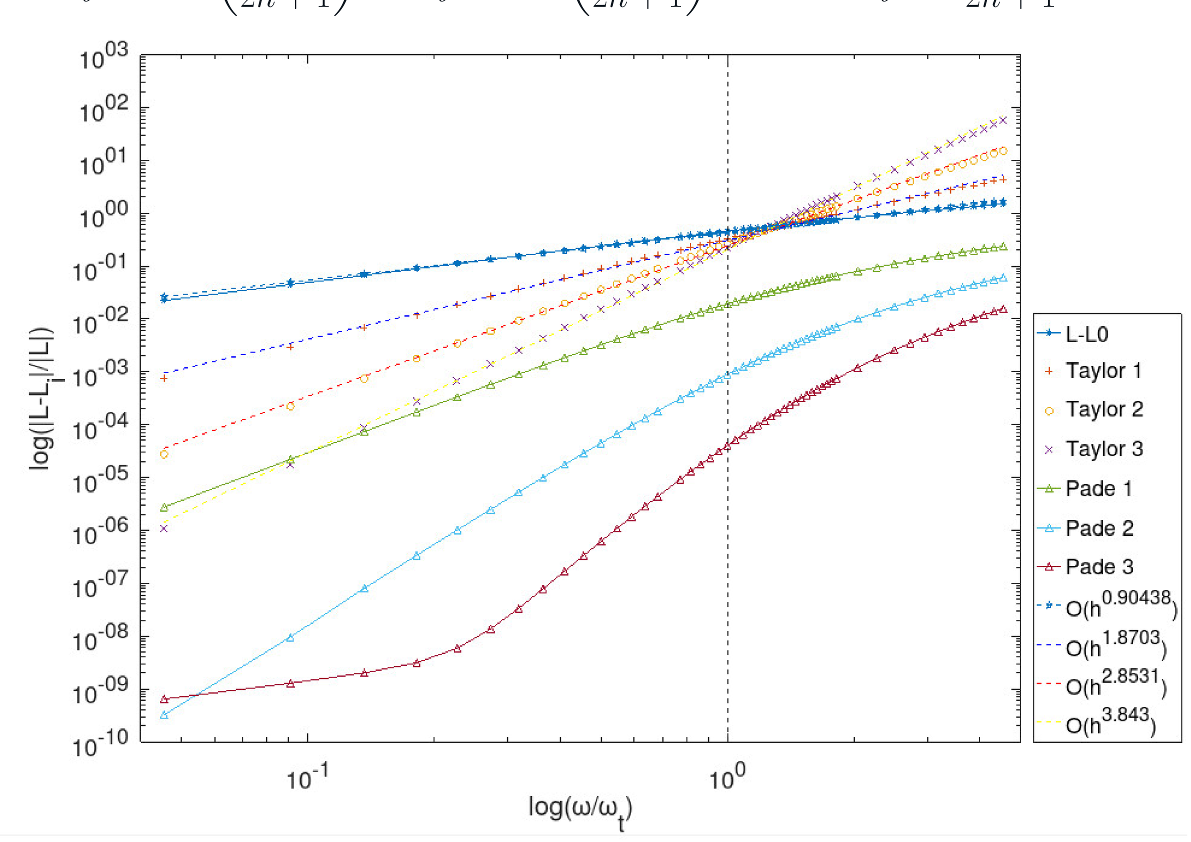
Property: Assuming that ω/ω_t is small enough,

$$L(\omega) = L_0\left(1+\frac{1}{2}i\frac{m}{4}\frac{\omega}{\omega_t} - \frac{3}{8}\frac{m^2}{16}\left(\frac{\omega}{\omega_t}\right)^2 - \frac{5}{16}i\frac{m^3}{64}\left(\frac{\omega}{\omega_t}\right)^3 + \mathcal{O}\left(\left(\frac{\omega}{\omega_t}\right)^4\right)\right)$$

$$\mathcal{L}_n(x) = 1 + \sum_{j=1}^{n}\frac{a_j^{(n)}x}{1+b_j^{(n)}x} = \prod_{j=1}^{n}\frac{1+c_j^{(n)}x}{1+b_j^{(n)}x} \tag{4}$$

where

$$b_j^{(n)} = \cos\left(\frac{j\pi}{2n+1}\right)^2,\quad c_j^{(n)} = \sin\left(\frac{j\pi}{2n+1}\right)^2,\quad \text{and} \quad a_j^{(n)} = \frac{2c_j^{(n)}}{2n+1}$$



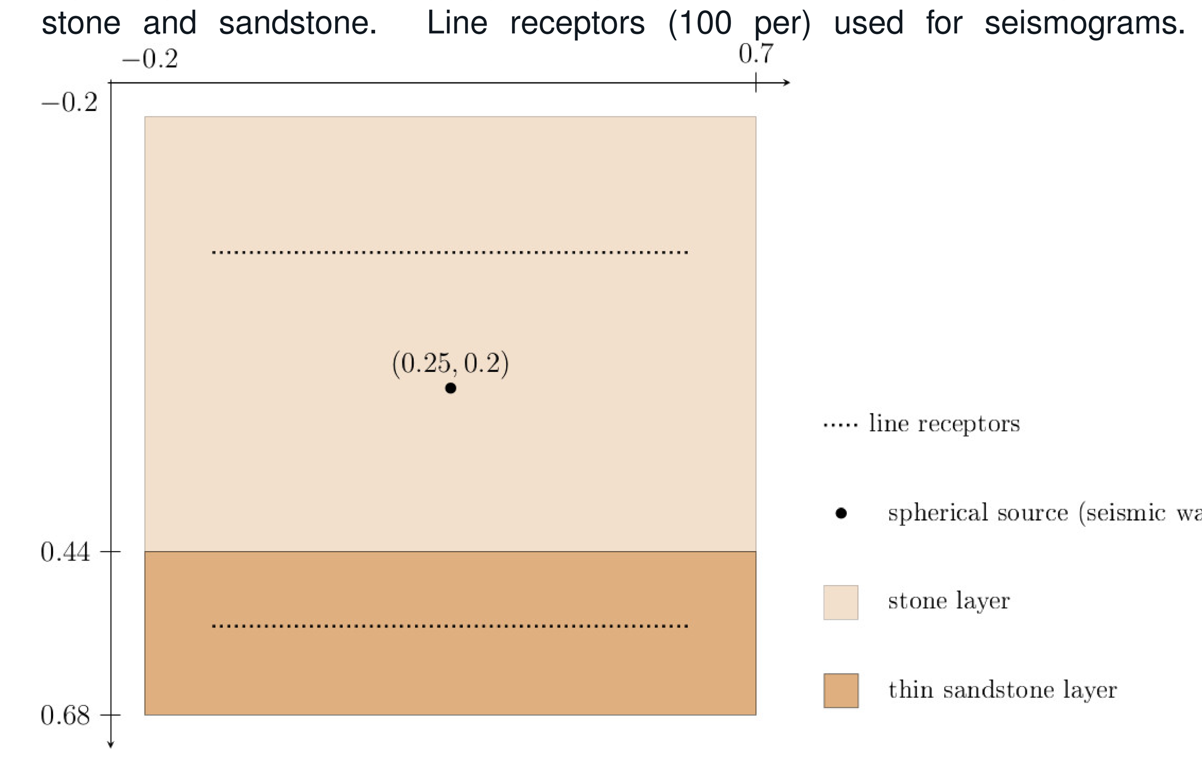
Log errors between $L(\omega)$ and L_0, L_1, L_2, L_3 and Padé approximants

At low frequency, error using L_0 is very low ($\simeq 5\%$). It can be used at low frequencies instead of $L(\omega)$. At high frequencies, Padé approximants can be considered with $\simeq 1\%$ error. In the top-right panel we show the influence of these errors on the solution fields.

Field sensitivity to the coupling term

Two-layered test case

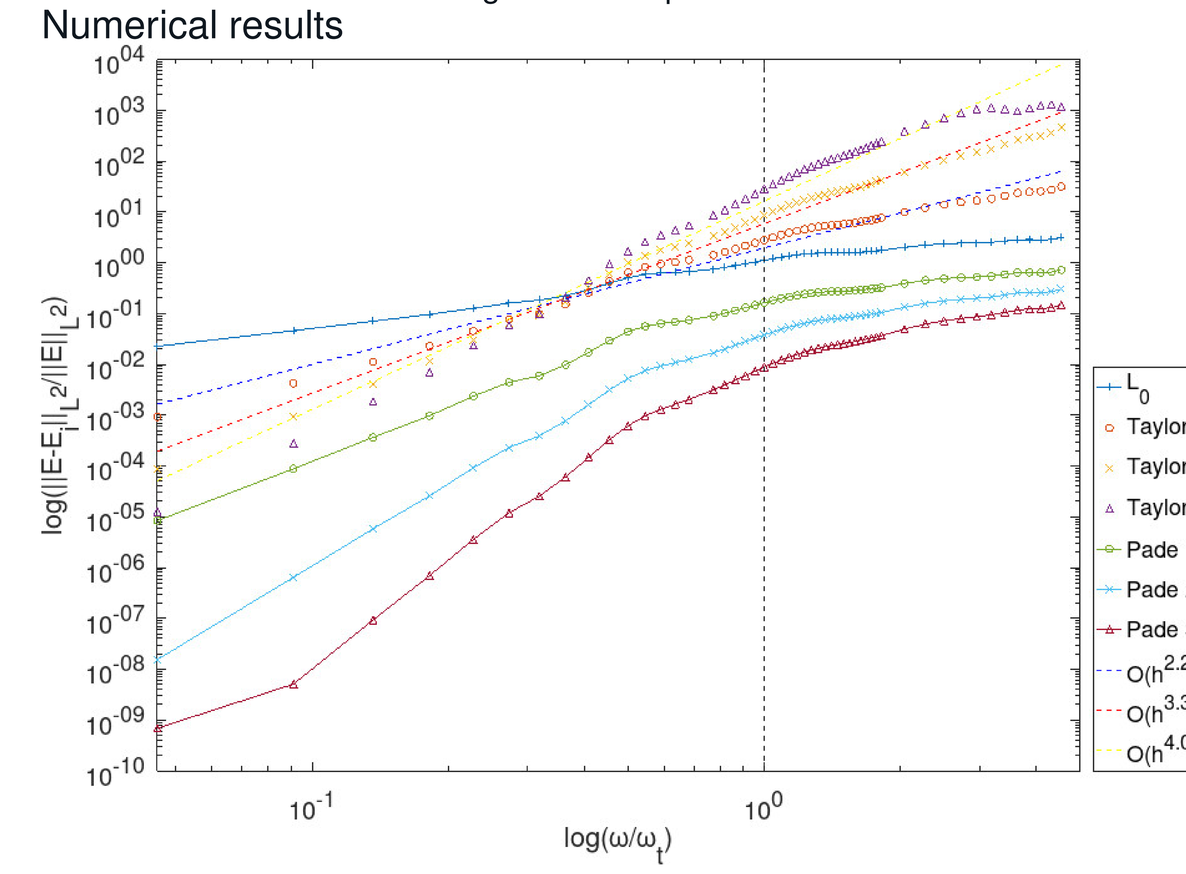
Two-layered test case with spherical seismic source. Lab-scale modeling of porous media. Physical parameters considered correspond to stone and sandstone. Line receptors (100 per) used for seismograms.



Hou10ni2d

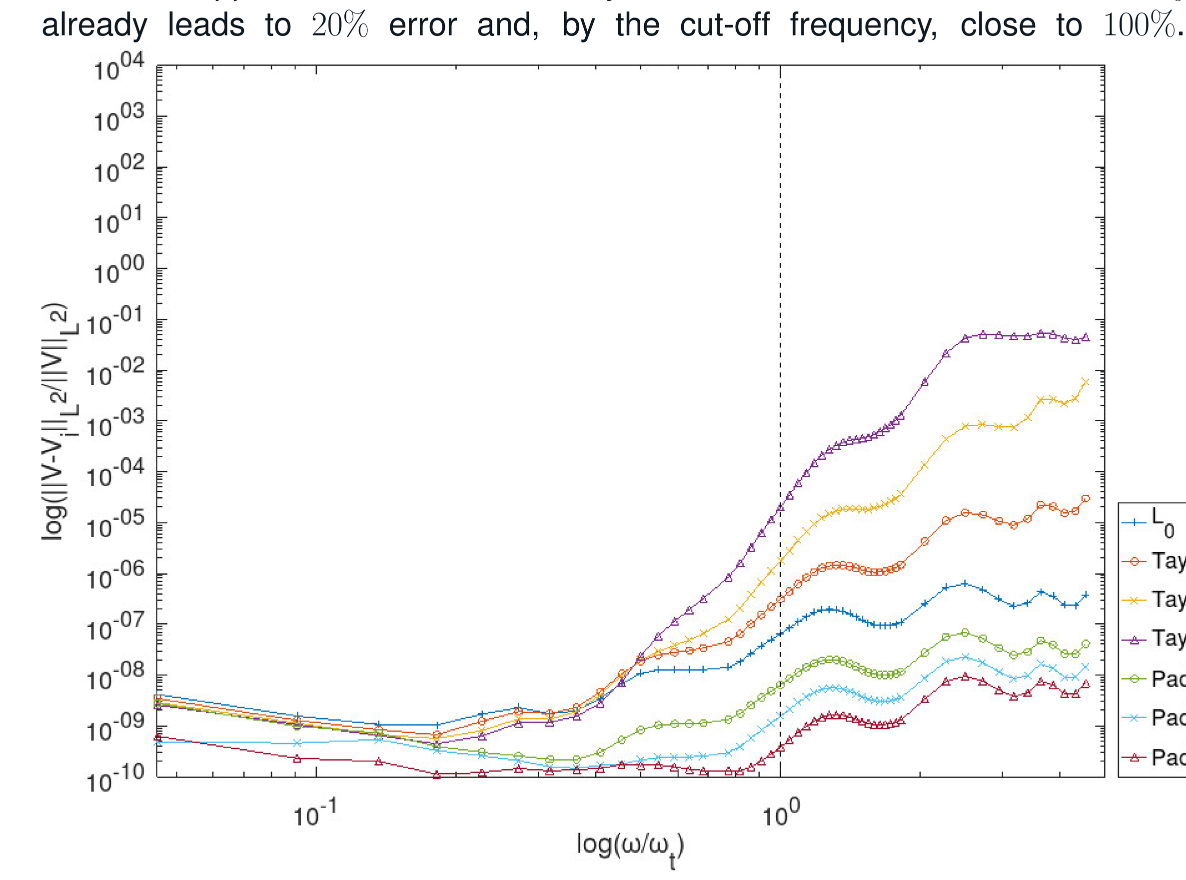
HDG-based unstructered high-order solver for Pride's equations (Biot +Maxwell) in 2D and 3D. Parallelized using MPI and OpenMP.

Numerical results



Error on $\mathbf{E}$ as a function of ω/ω_t for each L approximant

Error on electric field exhibits similar behaviour as on $L(\omega)$. A the cut-off frequency (22kHz), Padé at order 1 only leads to 10% error, and Padé at order 3 to only 0.5% error. Even at very high frequencies (100kHz), the Padé approximant at order 3 only leads to 10% error. At $6.6kHz$, L_0 already leads to 20% error and, by the cut-off frequency, close to 100%.



Error on $\mathbf{V}$ as a function of ω/ω_t for each L approximant

Kinetic fields such as $\mathbf{V}$ do not exhibit the same response. Errors remain very low even up to the cut-off frequency at $1e-5$ in the worst case. Padé approximants produce error close to $1e-8$ even at the highest frequencies(100kHz). The seismic source possibly leads to mostly kinetic $\rightarrow$ eletric coupling. Amplitude of resulting electro-magnetic terms is too weak to influence kinetic fields.

Conclusion and Perspectives

L_0 is a good approximation at low frequency(on the field) while $L_n(\omega)$ should be used at higher frequency (labaratory) with very good accuracy when using Padé. We have done the same analysis for the permeability.

Time-domain coupling operators

Padé approximants lead to small errors on the fields even at high frequencies. In time domain, the last coupled equation becomes, with the 1-st order diagonal Padé approximant:

$$(\omega_t - b_j^{(1)}\partial_t)\mathbf{J} = (\omega_t - b_j^{(1)}\partial_t)\sigma\mathbf{E} - L_0(\omega_t - c_j^{(1)}\partial_t)(\nabla p + \partial_t\rho_f\mathbf{u}) \tag{5}$$

Additional time derivates are introduced by the Padé approximation.